

NOT FOR QUOTATION  
WITHOUT PERMISSION  
OF THE AUTHOR

**COST ALLOCATION IN WATER RESOURCES—  
SIX GAMING EXPERIMENTS  
IN POLAND AND BULGARIA**

Ingolf Stahl  
Ryszard Wasniowski  
Isak Assa

July 1981  
WP-81-83

*Working Papers* are interim reports on work of the International Institute for Applied Systems Analysis and have received only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute or of its National Member Organizations.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS  
2361 Laxenburg, Austria

## PREFACE

This paper is one in a series of reports on experiments with a game concerning cost allocation in water resources. The ultimate purpose of the game is to be an aid in finding better methods for allocating joint costs in projects when several parties, e.g., municipalities, join together to save costs by building a larger facility instead of several smaller ones.

Gaming, that is, the actual playing of games, is seen here as a complement to other, more deductive, methods, mainly based on game theory. Since the idea is that the planners involved shall really want to use the allocation scheme, it is important that the scheme is congruent with the planner's own thinking. Gaming can first of all be used as an "acid test" of the proposed game theoretic suggestion. If some theory is not appealing in an experimental setting, it is more likely not so in real application either. Furthermore, gaming can be seen as a direct way of finding out what ideas of distribution are really held by planners.

The experiments reported on in this working paper were carried out in Poland and Bulgaria. Together with earlier experiments in Sweden and Italy, they were also intended to give some indication of whether planners in different countries are similar enough that it is reasonable to move models or methodology from one country to another and from one economic system to another. The experiments could in this regard have specific relevance for the IIASA Regional Development Task, where similar planning approaches are applied to regions in the four countries mentioned above.

## ABSTRACT

This paper reviews six gaming experiments with a game on cost allocation in water resources, carried out with water planners, scientists and advanced students in Poland and Bulgaria.

The results are similar to those obtained from the games played in Sweden and Italy. The similarity is particularly noticeable when the comparison is limited to games involving planners.

## CONTENTS

INTRODUCTION	1
THE TWO GAMES IN POLAND	5
Background	5
Game with Scientists	5
Game with Water Planners	6
THE FOUR GAMES IN BULGARIA	8
Background	8
Game with Water Planners	9
Game with Scientists	9
Game with Young Management Students	10
Game with Old Management Students	10
COMPARISONS BETWEEN RESULTS AND METHODS	11
REFERENCES	17

**COST ALLOCATION IN WATER RESOURCES—  
SIX GAMING EXPERIMENTS  
IN POLAND AND BULGARIA**

Ingolf Stahl, Ryszard Wasniowski and Isak Assa

**INTRODUCTION**

This paper reports on two gaming experiments in Poland and four in Bulgaria. These six experiments are part of a series of experiments with a game concerning cost allocation in water resources.

The following gives the background information of this game.

In Sweden, six blocks of municipalities in Southwestern Skane have a choice of strategy as regards their long term water planning. Such a municipality block,\* henceforth simply called municipality, can either solve its water supply problem on its own, or it can join together with several municipalities, up to the grand coalition involving all six municipalities. Due to "economies of scale", there are in most cases cost savings when municipalities join together and the grand coalition leads to the lowest total costs.

The question is now, how costs should be allocated. The cost allocation problem arises from the fact that the fixed costs of construction of the plant cannot be assigned to the municipalities in any obviously unique way. One can only propose various principles on which such allocation should depend and on the basis of these principles formulate methods for allocating these costs.

---

\*Although in reality there are 18 municipalities in this region, it was found practical and realistic to group these into six units which for this purpose can be regarded as acting as independent municipalities.

Young, Okada, Hashimoto (1980), analyzed seven different methods:

1. Allocation in proportion to population
2. Allocation in proportion to demand
3. The SCRB-method (separable costs--remaining benefits), a method used in practice
4. The Shapley Value, a well-known game theoretic method, based on the principle that the parties join a coalition one after the other
5. The Ordinary Nucleolus
6. The Weak Nucleolus
7. The Proportional Nucleolus

The three Nucleoli-methods all lead to a solution in what is called the core, implying that they fulfill the two principles of Individual and Group rationality.

Individual rationality implies that the allocation of the costs of the grand coalition is such that no municipality should be better off going alone.

Group rationality implies that the sum of payments made by the members of every coalition which is smaller than the grand coalition should not be larger than the cost that this coalition incurs if it is working on its own, i.e., no group of municipalities shall have an incentive to leave the grand coalition and form a smaller one.

Regarding these two principles as important and noting that only these core methods fulfill these principles in this game, Young makes the final choice between these three core methods. For this he uses two other principles. "If there is a cost overrun, no party shall pay less", and "a player who never contributes to any costs savings when joining with other parties or coalitions should not realize any cost savings above his go alone costs". The Proportional Nucleolus is suggested because it is the only one of the three core methods which always fulfills the principles.

The question now is if water planners really want to use this method. Is the allocation scheme congruent with the planners' mode of thinking? In this connection gaming can be seen as an "acid test" of the proposed game theoretic model. If a theory is not appealing in an experimental setting, it is most likely not so in real application either. Furthermore, gaming can be used as a direct way of finding out what ideas of distribution are really held by planners: How do intelligent decision makers, arrive at a compromise between different concepts, such as efficiency and equity, in negotiations of this type?

With these questions in mind we ran our first gaming experiment with water planners in Skane in November 1979. This experiment is reported on in IIASA WP-80-38. The game was then repeated in April 1980 with regional planners in Tuscany, Italy. This is reported on in WP-80-82. Although the negotiation process was quite different in the two countries as regards the amount of verbal communication, the actual results were very close. In fact, if one used the outcome in the Swedish game as a method for predicting the outcome in Italy it would have been a far better predictor than anyone of the seven methods above.

This then raised the question of whether this similarity in results obtained in two countries was coincidental. To answer this question, more international gaming comparisons were needed.

It was especially interesting to see if similar results could be obtained in countries with different economic systems to the market economies of Sweden and Italy such as in the socialist countries.

The IIASA Regional Development Task carries out case studies in Poland, Bulgaria, Italy and Sweden. For this reason Poland and Bulgaria were the natural choice amongst the Socialist countries to continue our study.#

Since similar model elements were planned to be used in these case studies, the question then naturally arose to what extent was there enough similarity in behavior to warrant the use of similar methodologies in these countries. Also of interest was the water cost allocation problem. The Swedish case is closely connected to the original water problem. The water supply problem is of importance in the regions of the Polish and Bulgarian case studies.

The games in Sweden and Italy were played with planners in the regions studied by IIASA; Skane and Tuscany. For practical reasons we could not play the games in Notec and Silistra which are the regions covered by the case studies in Poland and a Bulgaria. It was namely of paramount importance to have a Polish and Bulgarian gamer with local knowledge who could organize the games. Thus R. Wasniowski (RW) from the Forecasting Research Center of the Technical University of Wroclaw organized the game playing in Wroclaw, Poland, while I. Assa (IA) from the Institute of Social Management but on secondment to IIASA organized the game in Sofia, Bulgaria.

The games played in Poland and Bulgaria were virtually identical to that played in Italy and thus very similar to that played in Sweden.\* The Italian game is presented in detail in WP-80-82, therefore we shall only outline the main points of the game.

There are six players. Each player is the representative of one of the six municipalities: A, H, K, L, M or T. The players are seated around a small table. Each player has a sheet of paper showing the costs of every conceivable coalition, such as table 1. On the basis of this table the outcome according to the different methods has been computed in table 2.

Each municipality can now enter into a coalition with some other municipality or municipalities. When forming a coalition, the players reach an agreement on how much each of the participants in the formed coalition shall pay of the total costs of the whole coalition. As soon as a coalition has been formed and an agreement has been reached as to the allocation of the total costs of this coalition among its members, they register the coalition with the game leader. He will record the coalition participants, as well as the payment each of them would make toward the total costs of the coalition.

---

#Further information on this task can be found in Snickars 1981.

\*For details on how the games differed see pp.6 and 8.

Table 1. Total cost of each possible coalition.

A	21.95	AHK	40.74	AHKL	48.95
H	17.08	AHL	43.22	AHKM	60.25
K	10.91	AHM	55.50	AHKT	62.72
L	15.88	AHT	56.67	AHLM	64.03
M	20.81	AKL	48.74	AHLT	65.20
T	21.98	AKM	53.40	AHMT	74.10
		AKT	54.85	AKLM	63.96
AH	34.69	ALM	53.05	AKLT	70.72
AK	32.86	ALT	59.81	ALMT	73.41
AL	37.83	AMT	61.36	HKLM	48.07
AM	42.76	HKL	27.26	HKLT	49.24
AT	43.93	HKM	42.55	HKMT	59.35
HK	22.96	HKT	44.94	HLMT	64.41
HL	25.00	HLM	45.81	KLMT	56.61
HM	37.89	HLT	46.98	AKMT	72.27
HT	39.06	HMT	56.49	AHKLM	69.76
KL	26.79	KLM	42.01	AHKMT	77.42
KM	31.45	KLT	48.77	AHLMT	83.00
KT	32.89	KMT	50.32	AHKLMT	70.93
LM	31.10	LMT	51.46	AKLMT	73.97
LT	37.86			HKLMT	66.46
MT	39.41				
				AHKLMT	83.82

Table 2: Allocations in Millions of Swedish Crowns

	A	H	K	L	M	T
Population Proportional	10.13	21.00	3.19	8.22	34.22	7.07
Demand Proportional	13.07	16.01	7.30	6.87	28.48	12.08
SCRB	19.54	13.28	5.62	10.90	16.66	17.82
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Nucleolus	20.35	12.06	5.00	8.61	18.32	19.49
Weak Nucleolus	20.03	12.52	3.94	9.07	18.54	19.71
Proportional Nucleolus	20.36	12.46	3.52	8.67	18.82	19.99



A coalition does not come into force, however, until 15 minutes has elapsed since its registration, and then, only provided that none of its members has been registered in another coalition during this period. Hence a player can leave one coalition and join another in order to decrease the amount of his payment. Furthermore, a coalition dissolves by registering a new coalition with additional members.

The game continues until all participants are members of a coalition which has come into force (with the possible exception of a single "left-over" participant). After 90 minutes from the time of its start, it will nevertheless be brought to an end and the coalitions registered but not broken, will come into force.

We shall next proceed to report on the individual games: those in Poland in the next section, and those in Bulgaria in section after and in the final section we shall compare the outcome of these games with the outcome of the games in Sweden and Italy as well as with the theoretical methods.

## **THE TWO GAMES IN POLAND**

### **Background**

RW made contact with managers at the Wroclaw Water Works. Their initial attitude to a game of this type was somewhat sceptical, since the game was not regarded as adequate to Polish conditions involving hierarchical structures of decision making, i.e., with subsystems subject to a controlling organ. Hence our original plans were that one would play two games in Wroclaw: the original game with scientists and doctoral students and a revised version of the game, more adapted to the centralized decision making in Poland, with the water planners. During our period of planning for the game, the Polish government announced changes in the organization of the economic system, aiming at increasing the independence of the enterprises. After this the attitude of the water managers changed somewhat and it was also considered possible to play the original version with the water planners. Hence we planned two games for 20 September 1981, one with water managers, and one with scientists and doctoral students. Only three of the invited water planners could come on that day; therefore, only the game with the researchers took place. The game with the water planners was played on 20 October 1980 instead and then handled only by RW.

### **Game with Scientists**

The first game in Wroclaw was hence a game with six scientists and doctoral students at the Technical University of Wroclaw. The scientists were connected with the Forecasting Research Institute. One had been engaged in research on another game in Poland dealing with local planning. Both the scientists and the doctoral students had fairly solid backgrounds in quantitative methods.

The same seating arrangement was used as in the earlier games. The players were randomly assigned to a municipality role. The same instructions were used as in Italy, with the exception that we, for administrative reasons only, gave a prize (a bottle of good brandy) to the best player and not as in Italy a small amount of real money to each player in proportion to the amount of fictitious money saved in the game. The instructions had furthermore been translated into Polish.

After a brief introduction to the game, not requiring any special comments, the game started.

After only three minutes the two party coalition HL was formed with 11.90 to L and 13.10 to H. The total savings of 7.96 were divided equally.

After 15 minutes from the start the coalition HKL was formed with H getting 10.54, K 6.38 and L 10.34. This division implies that compared to the one party coalition costs, H saves 6.54, K 4.54 and L 5.54. K thus got 2 less than H because he is the party joining the coalition.

After another 7 minutes the remaining three players A, M and T formed a coalition with 20.50 to A, 20.46 to M and 20.40 to T. The idea behind this division was that each one should get roughly the same cost and that the figures should be rounded off.

After another 15 minutes, i.e., after 37 minutes from the start, the grand coalition AHKLMT was formed with 19.02 to A, 10.54 to H, 5.98 to K, 9.94 to L, 19.78 to M and 18.56 to T. The ideas behind this division were as follows: First, an agreement was reached on the concept that the coalition AMT should receive the major part since their costs would still be higher than those of HKL. Hence out of the total savings of 4.8, AMT got 4 and HKL 0.8. HKL divided their 0.8 into two parts of 0.4, going to K and L, on the grounds that H got such a big part of the savings in the preceding coalition formation step. The savings of 4 going to AMT were divided in a more complicated fashion with A getting 1.44, M 0.62 and T 1.94. The main idea behind this was that the party with the higher water demand--M--should get the smallest savings and thus relatively highest costs, while the party with the lowest water demand of the three--T--should get the greatest savings and thus comparatively lower costs.

It should be noted that the result was in the core. Furthermore it should be stressed that the agreement within 37 minutes was the earliest agreement of all water cost allocation games played.

#### Game with Water Planners

The institutional set up, i.e., the instructions, seating arrangement, prize structure etc., were the same in this game as in the game with scientists-doctoral students.

After initial attempts at dividing costs in accordance with water demand, two coalitions were formed after around fifteen minutes:

1. AH giving A 19.69 and H 15
2. LM giving L 13 and M 18.10

The distribution partly follows the original one-party coalition costs, but with one party getting a round number and the other party the rest.

After another quarter of an hour a three-party coalition AHL was formed with 17.69 to A, 13 to H and 12.53 to L. A and H each save 2 compared to the earlier coalition, while L gets the remaining savings.

Again after roughly a quarter of an hour another coalition was formed, the four party coalition HKLM with H obtaining 12, K 7.5, L 12 and M 16.57. We see that H and L agree on equal costs and that the principle of using round numbers is still used.

This coalition was dissolved after only a few minutes by the formation of the coalition AHL, with 20.22 to A and 11.5 to both H and L. Party A being left out in the previous coalition was now willing to take a considerable part of the costs in order to come into a coalition again.

After only a few minutes this coalition was also broken with the coalition HKL being formed with 10.5 to each of H and L and 6.26 to K. H and L each saved 1 compared to the preceding coalition.

Very soon afterwards M and T joined this coalition to form HKLMT with 10 to H, 6 to K, 10 to L, 20.46 to M and 20 to T. H, K and L here go down to the nearest integer.

Finally after 70 minutes from the start of the game the grand coalition AHKLMT was formed with A obtaining 18.8, H 10.19, K 5.80, L 9.42, M 19.81 and T 19.80. This division came into force as the final agreement.

As regards this final division of the costs it should first of all be noticed that this division does not lie in the core. Group rationality is violated with regard to the subcoalition MT. This subcoalition could on its own get away with 39.41, but in the final division M and T together paid 39.61. It appears that M and T were not aware of this fact. The difference of 0.2 is, however, so small that one cannot rule out that, even if the players had been aware of this violation of group rationality, they might not have cared to break up the grand coalition.

The reasoning behind the final distribution is difficult to establish. It appears that the exact form was the result of pure haggling. However every party except H got away with lower costs than he had in the previous coalition that he had been part of. The reason why just H had to raise his costs and now pay more than L, might be dependent on the fact that H has both a higher water demand and higher initial costs than L.

After the game there was a discussion with the managers. They stated that the game could be a good exercise for training, e.g. coalition formation for the realization of joint enterprises. The managers also suggested as an extension of the game to introduce an additional player who would evaluate the behavior of the six ordinary players. They also discussed the possible reconstruction of the game into a hierarchical two level game. In this, one would on the top level conduct the present game. On a lower level one would have a game with several players within each of the six "municipality teams."

## THE FOUR GAMES IN BULGARIA

### Background

In October 1980, I. Assa arranged for the playing of the games in Bulgaria with four teams: one with water planners, one with scientists and two with management students.

All six participants in the water planners' team were engineers and came from the institute called "Vodproekt". The major task of this institute is to work on projects for the construction of water nets in Bulgaria (irrigation, dams, pumping stations etc.) They are also taking part in the Silistra case study and are involved in the implementation of IIASA water models in Bulgaria. Their ages range from 40-45.

Three of the participants in the scientists team were from the Institute for Social Management; two were mathematicians and one a statistician. All are lecturers and work in the department of Quantitative Methods and Modeling in Social Management.

Two of the other participants are economists from the Economic Management Center of the Academy for Social Sciences in Sofia, and lecture in the department of Management of the Industry Sectors. The final participant works in the laboratory of Economic Modeling at the Institute of Economics of the Bulgarian Academy of Science. The age of these scientists is between 30-40.

All twelve participants on the management students' team, aged between 25-35, have a university degree in engineering and/or economics. At present they are following a two year course in management leading to a degree similar to an M.Sc. In this program they are trained to become managers in different enterprises or municipalities.

The management students divided themselves into two teams. It appeared that the younger ones went to one team and the older ones to the other. Although the age difference was not so very great, we shall for the sake of simplicity refer to the two teams as young management students and old management students. It is probable that the participants in the latter team had more practical experience than the members of the former team.

The game played was identical to that played in Poland with two minor differences: A prize was given only to the "best" player of all four teams, while in Poland, where the game was played with the two teams on two separate occasions, a prize was given to the best in each team. However, it was thought that by not giving a prize to one in each team, intra-team competition might be lower. Furthermore, while in Poland, just as in Italy, information was given only on the water demand for each municipality, in Bulgaria we also gave information on the population of each municipality. By doing so we could, in our total series of experiments have roughly half of the games with only demand information and half with both population and demand information.\* It appears, however, that the addition of information regarding population does not play any part in

---

\*It should also be mentioned that prior to playing the games in Poland and Bulgaria we had also played the game five times with young scientists and doctoral students visiting IIASA. This is reported on in WP-80-134 and WP-81-21.

influencing the actual outcome of itself.

#### Game with Water Planners

In their negotiations the water planners started by trying to form as large a coalition as possible directly and they discussed basing the division on either population or water demand. These efforts failed, since these methods would mean that individual municipalities, like M, would have to pay more than their go alone costs.

Next the discussions were directed towards the formation of one coalition HKLM and one coalition AT. The water planners realized, however, that M does not contribute any cost savings. Hence H, K and L instead formed a temporary coalition, which was not formally registered with the game leader. The costs of this coalition were then distributed according to the go alone costs of H, K and L implying 10.63 for H, 6.82 for K and 9.81 for L. This also became the payments of these parties in the grand coalition, which was next formed. The planners agreed that this was favorable enough, in comparison to the go-alone costs, for these parties. The remaining costs  $83.82 - 27.26 = 56.56$  were then divided among AMT in proportion to the go alone costs of these three parties, implying a cost of 19.23 at A, 18.10 to M and 19.23 for T. An agreement on this division was registered after 63 minutes from the start.

It should be noted that this solution does not violate the core concept. It is exactly on the boundary line of the core, since H,K and L, as noted above, can get the same cost sum 27.26 by forming the three party coalition HKL. Since, as noted above, the core concept was violated in the other three games, the prize in Bulgaria was given to the water planners.

#### Game with Scientists

After 21 minutes H and L in the scientists group formed a coalition with 12.95 to H and 12.05 to L. This division implies that the parties divided the costs in direct proportion to their go alone costs.

After 51 minutes from the start a four party coalition HKLM was formed with 12.69 to H, 8.11 to K, 11.80 to L and 15.47 to M. Here the costs of the coalition HKLM were allocated to the members in proportion to their go-alone costs. Two things here are noteworthy. Contrary to the planners, the scientists did not seem to notice that player M did not contribute to any cost savings and that H, K and L would have been better off by only forming the three party coalition HKL. Secondly, the allocation of costs were based on the parties go alone costs. H and L would have been better off, if their shares of the costs instead had been based on the costs they obtained in the two party coalition HL which had just been formed.

Finally, after 67 minutes from the start, the grand coalition AHKLMT was formed with A: 17.87, H: 12.56, K: 8.10, L: 11.80, M: 15.40, and T: 17.99.\* This solution does not lie in the core. H, K and L, for example together pay 32.46, i.e, more than the cost of 27.26 of the coalition AHKL.

---

\*It should be noted that this totals 83.72 instead of 83.82.

### Game with Young Management Students

After 38 minutes from the start H and K registered a coalition HK with H: 15.05 and K: 7.91. Of the total savings of 5.03 compared to the parties go alone costs, H got 2.03 and K 3. Here we could not establish any other principle than one party getting a round number of savings. The fact that K got a considerably larger part of the savings than H could possibly be due to the fact of K having a lower water demand.

Next, after 55 minutes from the start an attempt was made to form the coalition AHKL, with 16.79 to A, 14.08 to H, 10.91 to K and 7.17 to L. Here the costs were said to be divided partly in accordance with water demand. K, realizing that this division would give him far less than the just registered coalition HK, refused to join. Instead, after 61 minutes from the start an other coalition AHKL was registered with A: 16.79, H: 14.08, K: 7.90 and L: 10.18. Thus A and H kept their costs unchanged, K reduced his share, so it would be profitable for him to leave the coalition HK, while L absorbed the whole decrease in K's costs.

Finally, after 66 minutes from the start, M and T formed a two party coalition with 18.91 to M and 19.50 for T.\* This is roughly in proportion to their going alone costs.

After this agreement, the parties after a while broke up the negotiations, when they realized that the planners and scientists had already finished. Thus, the young management students did not form the grand coalition. The two coalitions AHL and MT together pay 88.36, that is 4.54 more than the grand coalition. The prize structure of the game (see page 8) might partly have caused this failure to agree on total minimum costs. Since the grand coalition was not formed, the solution is obviously not in the core.

### Game with Old Management Students

The "old" management students started to discuss a division on the basis of water demand. First they tried to form the five party coalition AHKLT leaving M out probably due to the problem that a six party allocation based on demand would give M much higher costs than he would get if he played alone.

After 45 minutes from the start they were, however, trying to form a four party coalition HKLM for the division. They used the following procedure:

Looking at water demand, they saw that L has the lowest demand and regarded his demand of 3.53 as one "standardized demand unit". Next they divided the costs of H, K, L and M by 3.53, obtaining coefficients 2.33, 1.06, 1 and 4.15 respectively, i.e., a sum of 8.54 standardized demand units. Next the total costs of HKLM, i.e., 48.07, were divided by 8.54 giving a cost of 5.62 per standardized demand unit. Then they assigned a cost of 5.62 to each standardized demand unit.

---

\*This involves an error by adding up to 38.41 instead of 39.41.

This seemingly complicated procedure simply implied that the costs of HKLM were distributed in direct proportion to the water demand of each party. This led to a division H 13.09, K 5.96, L 5.62 and M 23.32. At the last minute M quit this coalition attempt realizing that he would be better off alone.

Next H, K and L proceeded to form a coalition. Finding that L had a large population, in relation to its water demand, H and K started to insist on L taking a larger part of the cost. After some bargaining not based on specific principles, HKL arrived at a distribution H 13.50, K 5.50 and L 8.26. The basis for H's and K's payments seemed to be the earlier attempted distribution giving H 13.09 and K 5.96; H with a relatively large population going up to the nearest round 0.5 and K with a relatively low population going down to the nearest round 0.5. L finally paid the remaining 8.26. This coalition was registered after 59 minutes from the start.

After this, attempts continued to form a larger coalition involving A, M and T. These three parties did not, however, on their own try to form a separate coalition. After another 15 minutes H, K and L did not regard it as worthwhile to continue the negotiations so the coalition H, K and L came into force.

A, M and T then also wanted to quit, remaining as one party coalitions. These decisions to quit were made after the groups with water planners and scientists had discontinued the game, having formed grand coalitions.

In this context it should be noted that the coalitions AM and AT are meaningless since they do not lead to any cost savings. The same applies to AMT, if MT has been formed. Hence the only possible meaningful coalition remaining for these three was MT. A division of the costs of the coalition MT in proportion to either population or demand would, however, have violated individual rationality for M. A strong focus on allocating the costs according to either demand or population, combined with a lack of interest, due to the prize structure of the game (see p.8), appears to explain why, for example, the coalition MT was not formed at this point.

Since the grand coalition was not formed the solution is obviously not in the core.

## COMPARISONS BETWEEN RESULTS AND METHODS

We start by summing up all the experimental results of the six games in table 3, which can then be compared with the suggested outcome according to the various methods in table 2.

In order to see how well the theoretical allocations fit these experimental values, we use three measures of difference:

1. The sum of absolute differences. With T as the theoretical value and E as the experimental value the measure is:

$$\sum_{i=1}^6 | T_i - E_i |$$

Table 3. Summary of outcomes of experiments

GAME	A	H	K
Polish Scientists	19.02	10.54	5.98
Polish Water Planners	18.80	10.19	5.80
Bulgarian Water Planners	19.23	10.63	6.82
Bulgarian Scientists	17.87	12.56	8.10
Bulgarian "Young" Scientists	16.79	14.08	7.90
Bulgarian "Old" Scientists	21.95	13.50	5.50
	L	M	T
Polish Scientists	9.94	19.78	18.56
Polish Water Planners	9.42	19.81	19.80
Bulgarian Water Planners	9.81	18.10	19.23
Bulgarian Scientists	11.80	15.40	17.99
Bulgarian "Young" Scientists	10.18	18.91	19.50
Bulgarian "Old" Scientists	8.26	20.81	21.98

- 2 The sum of the squared differences, i.e.,

$$\sum_{i=1}^6 (T_i - E_i)^2$$

Compared to measure 1, this gives a higher relative weight to large discrepancies.

- 3 The sum of the relative squared differences, i.e., of the squared differences after dividing each difference by the theoretical value, i.e.,

$$\sum_{i=1}^6 (T_i - E_i)^2 / T_i$$



The idea behind this measure is that a difference is more important if it is relatively large in comparison with the "expected" value.

As an additional method of forecasting the outcome of each game we also include the result of the original game played in Skane with Swedish water planners. We then obtain the results presented in tables 4-9 below.

Comparing tables 4-9, we find the following common traits in all six games:

1. Cost allocation in proportion to population is the worse prediction method.
2. Allocation in proportion to water demand is the second worse prediction method.
3. The Nucleolus is a better predictor than the Proportional Nucleolus.

It should be stressed that these three conclusions have also held in all earlier games.

Besides these three conclusions holding for all six games, the results seem to differ. It here appears suitable to divide the games into two groups: one group consisting of the two Polish games and the game with the Bulgarian water planners; the other group consisting of the other three Bulgarian games.

In the first group the Swedish game, the Shapley Value and the Nucleolus were the three best predictors, with the Weak Nucleolus on the fourth place. The results were thus very similar to those obtained, not only in Sweden, but also in Italy and in the games with international groups of scientists at IIASA, (see WP-81-21).

For the second group the Swedish game was not such a good predictor while the SCRB did quite well, at least in two of the games. It is furthermore noteworthy that the difference measure values were much higher in this group than in the first. The results of the second group were more like those of two games played with Swedish doctoral students at IIASA (see WP-80-134).

One can thus, incorporating the earlier games, distinguish for example between one group of games involving planners with similar results and one with students. One might in this connection ask why there was such a difference in results between the planners and the students. Some possible explanations come to mind:

Firstly, planners might be less motivated by prizes than students and out of professional pride attempt to make as good a job as possible without thinking of the prize structure of the game. As mentioned above, part of the discrepancy of the result as regards the Bulgarian management students might have been influenced by the prize structure, making the students at the end of the game less interested in continuing to attempt to form the grand coalition.

Secondly, the planners are more likely to have thought about these types of cost allocation problems earlier. They are also more likely to have a more clear conception of what kind of goals and principles apply in a real situation.

In general it appeared in the games both with the Swedish doctoral students and the Bulgarian Management Students, that students would be somewhat less prone to defend the interests of their own municipality than the planners.

Finally, planners appeared to be more efficient in their use of time and spend more time on trying to form coalitions and on discussing general principles, while students would spend more time calculating specific cost divisions. (See WP-80-134.)

The most interesting result from the playing of the game in Poland and Bulgaria is the similarity in outcome between the games played with planners in Sweden, Italy, Poland and Bulgaria. Although it must be stressed that we have results from only four games and that behavior in a game might very well be different from behavior in real life, the result is encouraging. It is at least not contradicting the hypothesis that planners in different countries and different systems are similar enough that it is reasonable to attempt to use similar types of planning models in different countries.

Table 4. Polish Scientists

	Difference measures		
	1	2	3
Swedish Game	6.18	8.82	0.52
Shapley Value	5.68	10.04	0.63
Nucleolus	7.55	9.81	0.84
Weak Nucleolus	8.29	12.71	1.65
SCRB	8.44	19.11	1.30
Proportional Nucleolus	9.38	16.11	2.44
Water Demand	30.99	194.17	12.32
Population	49.79	539.72	40.58

Table 5. Polish Water Planners

	Difference measures		
	1	2	3
Swedish Game	5.10	8.07	0.42
Nucleolus	6.83	9.51	0.74
Shapley Value	6.98	11.91	0.79
Weak Nucleolus	7.13	12.15	1.49
Proportional Nucleolus	8.04	14.36	2.13
SCRB	10.62	26.16	1.77
Water Demand	31.99	210.23	13.46
Population	50.43	569.98	44.28

Table 6. Bulgarian Water Planners

	Difference measures		
	1	2	3
Shapley Value	2.84	2.32	0.15
Swedish Game	4.92	5.87	0.43
Nucleolus	6.05	8.17	1.07
Weak Nucleolus	7.23	13.48	2.50
SCRB	8.1	13.80	1.14
Proportional Nucleolus	8.88	17.91	3.63
Water Demand	32.49	234.63	14.01
Population	52.97	613.77	46.24

Table 7. Bulgarian Scientists

	Difference measures		
	1	2	3
SCRB	7.20	11.88	1.45
Shapley Value	9.64	16.05	1.30
Nucleolus	13.69	36.96	4.01
Weak Nucleolus	13.95	42.24	6.13
Swedish Game	15.42	41.74	3.49
Proportional Nucleolus	15.72	52.69	8.22
Water Demand	32.97	265.90	15.03
Population	54.41	641.50	45.64

Table 8. Bulgarian "Young" Management Students

	Difference measures		
	1	2	3
SCRB	10.48	21.80	1.87
Shapley Value	10.36	27.41	2.07
Nucleolus	10.65	27.98	2.95
Weak Nucleolus	10.45	30.03	4.84
Proportional Nucleolus	11.66	37.08	6.56
Swedish Game	12.26	43.26	3.62
Water Demand	26.55	175.52	10.71
Population	47.99	507.17	42.78

Table 9. Bulgarian "Old" Management Students

	Difference measures		
	1	2	3
Nucleolus	8.87	17.41	1.02
Proportional Nucleolus	9.00	15.62	1.75
Weak Nucleolus	9.81	18.04	1.49
Swedish Game	10.16	28.14	2.28
Shapley Value	14.62	40.05	2.83
SCRB	13.70	47.37	2.95
Water Demand	32.15	247.17	17.33
Population	49.99	603.44	59.89

## REFERENCES

- Snickars, F. 1981. Comparative Studies of Regional Planning Models- with Special Emphasis on a Case Study of Southwestern Skane. WP-81-49 International Institute for Applied Systems Analysis, Laxenburg, Austria
- Stahl, I. 1980a. A Gaming Experiment on Cost Allocation in Water Resources Development. WP-80-38. International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Stahl, I. 1980b. The Application of Game Theory and Gaming to Conflict Resolution in Regional Planning. WP-80-82. International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Stahl, I. 1980c. Cost Allocation in Water Resources--Two Gaming Experiments with Doctoral Students. WP-80-134. International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Stahl, I. 1981. Cost Allocation in Water Resources--Three Gaming Experiments with Doctoral Students. WP-81-21. International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Young, H. P., Okada, N., and T. Hashimoto, 1980. Cost Allocation in Water Resources Development--A Case Study of Sweden. RR-80-32. International Institute for Applied Systems Analysis, Laxenburg, Austria.