NOT FOR QUOTATION
WITHOUT PERMISSION
OF THE AUTHOR

DRAMOS: A MULTI-CATEGORY SPATIAL RESOURCE ALLOCATION MODEL FOR HEALTH SERVICE MANAGEMENT AND PLANNING

L.D. Mayhew

March 1981
WP-81-39

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

THE AUTHOR

Leslie Mayhew is an IIASA research scholar working within the Health Care Task of the Human Settlements and Services Area. He is on secondment from the Operational Research Unit of the Department of Health and Social Security, UK.

FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

The work presented in this paper brings together two resource allocation models (RAMOS and DRAM), which deal respectively with allocation problems in space and between patient categories and resource types, to produce a new model DRAMOS (Disaggregated Resource Allocation Model Over Space). This new model has not only a similar mathematical basis but also a substantially enhanced range of potential applications. The limitations of the parent models are first reviewed before the new model is developed and tested in detail.

Related publications in the Health Care Systems Task are listed at the end of the paper.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ACKNOWLEDGMENTS

The author would like to thank the four Thames Regional Health Authorities in the United Kingdom on whose data the results in this paper are based. Thanks go also to P. Kitsul (Health Care Systems Task) and G. Leonardi (Public Facility Location Task) for their valuable comments on an earlier draft. Any errors or omissions, however, are solely the author's. In addition I am indebted to Lucy Tomsits for typing and collating the material presented and to Maria Rogers for her editorial comments.

ABSTRACT

This paper combines the main mathematical features and underlying theory of two IIASA health care resource allocation models, DRAM (Disaggregated resource allocation model) and RAMOS (Resource allocation model over space), to produce a more general model with an enhanced range of possible applications. Although these models were developed independently, and for entirely separate reasons, the fact that the amalgamation can be achieved so simply implies an encouraging consistency in their respective formulations. The paper starts with a short critique of the current limitations of both models, in particular in those applications in which serious errors may arise. For each limitation described a remedy is proposed before the new model, DRAMOS, is developed in detail. It is shown in what way the new model departs from the parent models, how it can be calibrated to give it operational potential, and finally, how it may be applied in practice. For the latter, three elementary planning scenarios are developed to illustrate the potential of the model when it is applied in a region, London, that is known to have highly complex planning problems. The paper concludes with some proposals for further development.

CONTENTS

DRAMOS: A MULTI-CATEGORY SPATIAL RESOURCE ALLOCATION MODEL FOR HEALTH SERVICE MANAGEMENT AND PLANNING

1. INTRODUCTION

The extent of the contribution of health care services to the health and well-being of the human population is not known with certainty, but it is certain that we would be much worse off without it. Our lack of knowledge concerning the impact of such services is nevertheless a cause for concern. For a variety of reasons, the amounts of resources allocated to health care provision is increasing yearly in most countries (WHO, 1978 p.18). Almost nothing is known, however, regarding the changes, if any, these increases are having on the health status of the population. Indeed, it is not even clear how the health status of people should be measured in relation to the resources spent.

In spite of our partial ignorance and the imperfections in our judgements, decisions still have to be taken by the principal providers of health care - at the treatment level by doctors, nurses, and others; at the administrative level by managers and planners; and at the government level by politicians. In carrying out their duties, the variety of questions posed by each of these groups collectively affect the type and operational characteristics of the resultant health care system (HCS). Many of these questions turn out to be allocative in kind: that is they address the problem of who gets what, where and how much of it they receive. For doctors and nurses the answers to such questions are often subjective or implicit, having been guided mainly by a mixture of medical judgement, scientific knowledge, and ethical considerations. At the administrative level, however, there is a better defined set of decision-making criteria. In a rationally organized system this set normally takes into account the size and characteristics of the population served (age, sex, and socio-economic characteristics), the mix and availability of resources (types and sizes of hospitals, clinics, emergency facilities, beds, and other medical equipment), the quality and availability of manpower (doctors, nurses, and auxiliaries) and, of course, the sources of income (government finance, insurance, charitable donations, and personal payments). At the government level the problem of resource allocation may be addressed directly or indirectly. An example of a direct allocation would be a proposal to transfer resources from education to health care. An indirect example would be the institutional framework, which fixes through legislation the operational parameters of the health care system, but does not control either the detailed allocations of individual agents or a large proportion of the revenue.

1.1. Models of Health Care Resource Allocation

At all these levels in the HCS it seems desirable to formulate criteria of resource allocation that in some sense are judged rational (e.g. RAWP, 1974), to evolve methods that can anticipate the effects of distributional change (as, say, in the demand characteristics of the population or the resource mix over time and space), and generally to develop data-based models that provide decision makers with high quality information that will enable a more efficient use of scarce resources

(Feldstein, 1967). It is in the allocation aspects of health service management and planning at the administrative level described above that most advances have been made in recent years, and to which the model presented in this paper is addressed.

1.2. Allocation Modeling at IIASA

The two main groups of resource allocation models being developed at IIASA are DRAM (Disaggregated resource allocation model) and RAMOS (Resource allocation model over space). The first of these, DRAM, is a set of behavioral models that simulates what happens in the HCS when resource levels change to the number of patients admitted to a hospital or related institution in each treatment category (diagnosis, specialty group), to the quality of care the patients receive (amounts of each resource consumed), and to the modes of care in which they are treated (out-patient, in-patient, home care, etc.). The theoretical basis and the model have been formulated and discussed in Gibbs (1978) and developed further in Hughes and Wierzbicki (1980). DRAM assumes two things: firstly, that there are not enough resources to satisfy all the demands made on the HCS; and secondly, that the HCS behaves as if it were maximising an aggregate utility function whose parameters are consistent with past allocative behavior or some other criteria.

RAMOS, by contrast, is based on a type of gravity model (Wilson, 1974) that explores what happens in a region to service levels (numbers of patients treated) as a result of one or more of the following: hospital building programs, treatment trends in in-patient care, population changes, or transport developments affecting the accessibility of the population to health care supply. RAMOS takes as inputs an index of the projected potential demand in each area of the region, a 'test' configuration of health care facilities aggregated into treatment districts, and data on patient accessibility. It then gives as outputs the anticipated hospitalization rates by area of residence as well as further information that is of potential value to the decision maker. The basis for the model shows a similarity with DRAM in that it is behavioral and it usually assumes that there are insufficient resources in the HCS to meet all the demands placed upon it. Additional details of RAMOS and its range of possible applications in a planning context are contained in Mayhew and Taket (1980) and are developed further in Mayhew (1980).

1.3. Scope of the Paper

As with all models neither DRAM nor RAMOS can answer all the questions decision makers may want to pose. There is the danger, too, that they may be applied in situations for which, strictly speaking, they were not designed. The purpose of this paper is to present a short critique outlining the limitations of the two models. It will be argued that these weaknesses may be overcome by combining the basic features and assumptions of both to produce a hybrid model that, it is hoped, will enhance their range of application and appeal to a larger number of potential users. This hybrid model is called DRAMOS (Disaggregated resource allocation model over space). It can be regarded either as DRAM with space introduced or as RAMOS with different patient categories and standards of treatment incorporated. The behavioral bases of the former models are, therefore, unchanged by this marriage of ideas. In addition, the previous scopes of the models are retained, while the outputs are potentially more detailed.

Following a short critique of DRAM and RAMOS, the new model is developed in detail and preliminary tests are carried out on data from London in south east England. It is shown how to calibrate the model for this example, by giving values to the main parameters, and how it performs when applied to three hypothetical planning scenarios for this particular region. The results are preliminary in that experiments must yet be carried out to see how the model performs when it is stretched for particular parameter sets. Also, the marriage of the models is not fully accomplished in this paper. The full integration must be postponed until more data are available on the different resources used in each mode of care. Nevertheless, these extensions would seem possible on the basis of previous theoretical research (Hughes and Wierzbicki, 1980).

2. DRAM, RAMOS: A SHORT CRITIQUE

Critiques of models tend to address one of two things: either the basic assumptions and theoretical underpinnings or the relevance and accuracy of the results. For current purposes, attention focuses here only on the latter as each model is taken in turn.

2.1. DRAM

For DRAM there are essentially four types of limitation that stem directly from the exclusion of space. These constrain its applicability and, in circumstances to be described, may influence the validity of the predictions. These are now discussed below, and specific proposals that correct for them are made.

2.1.1. Aggregate versus disaggregate utility

DRAM assumes an aggregate utility function to describe the allocative behavior of agents in the HCS in a region or country. Sub-regions in this system make resource allocations that are hypothesized as being consistent with this function. Indeed, data on these allocations are used to estimate the parameters of the model so that it can be used in a predictive way (Hughes,1978; Aspden, 1980a,b).

The difficulty with this reasoning is that many regions are really collections of heterogeneous sub-regions with different sets of allocative priorities and hence utilities. Furthermore, the extent of the heterogeneity *increases* with a *decreasing* size of sub-region. It seems wrong to compare, for example, the allocative behavior of a highly prestigious teaching hospital in health district A with that of a small community hospital in health district B. The roles in the HCS fulfilled by each are substantially different.

At a more macro level (as in past applications of DRAM), these differences may even out, so that the assumption of an aggregate utility function seems more valid. The difficulty with this formulation, however, is that decision makers are often more interested in what happens at the local scale - at, say, the health district level described above. In this case the use of an aggregate utility function is flawed. The solution offered here to this problem is to argue that, while all places of treatment have the same utility function, the parameter set and hence their allocative priorities are different. The first modification introduced, therefore, is to say that *if there are J treatment districts then there are also J different sets of utilities*, each district striving to maximize its own utility.

2.1.2. Ideal levels

Essential in the model for each patient category and resource type are a set of *ideal admission levels* (measured in numbers treated per head of population) *and standards* (resources supplied per patient). Necessarily, because of the model's assumptions, the satisfied demands - the actual levels and standards achieved - fall short of these ideals since there are insufficient resources in the HCS to go round. At issue here are not the concepts of ideal levels, standards or demand insatiability, but the way in which one set of these parameters, the ideal levels, are introduced in the model.

Although several methods have been proposed (Gibbs, 1978), it has been more common practice to date to infer the ideal levels *endogenously* from past resource allocations using a computer algorithm (Aspden,1980a,b). The problem with this approach is that past allocations in a region are partially a response to the demand potential, which in turn is a function of the age, sex, and morbidity structure of the population at a particular time. If potential demand is changing rapidly in time and space - as is the case, say, in large urban regions - then the ideal levels, and hence predictive capabilities of the model, will be seriously impaired unless some allowance is made (Pauly 1981, p.4). The second modification we make to DRAM is designed to avoid this problem: it is to *include specific information on the population structure, in particular the potential demand for medical care in the region at risk.*

2.1.3. The measurement of reference populations

Inputs and outputs to the model are estimates and predictions of the ideal admission levels and the numbers treated in each category of care *per head of population.* The problem here is what is meant by the measure "population". Is it the administered population or the catchment population (i.e. the population in places of residence from which patients are actually attracted)? In a densely populated urban region, for example, patient cross-flows between administered health districts are frequently observed (Mayhew and Taket, 1980). Aspden (1980a, p.6)recognized this problem noting that patient categories should be chosen that are locally self-sufficient. This excluded for consideration many important regional specialties and, in this application, limited the scope of the model. When catchment populations are chosen as a basis, the results may still be in error because these populations themselves are products of a particular resource configuration (Mayhew 1980, p.38). The third modification introduced here, therefore, is to base the reference populations on catchment populations *corrected for changes in the geographical pattern of* *resource allocation and population.*

2.1.4. *The measurement of accessibility*

Currently, the accessibility costs for patients to health care supply are not a variable in DRAM. These costs, however, can sharply influence the uptake in services in a region. This is one reason why there can be large variations in the numbers of patients generated in different areas, despite similarities in potential demand (Mayhew and Taket, 1980). The exclusion of accessibility costs also influences the determination of the ideal admission levels. The perception of what the admissions should be in a district's HCS are higher in the immediate locality of that HCS than in neighboring districts where demands are satisfied by adjoining facilities. The fourth modification made to the model, therefore, is to say that the number of ideal admissions from a place of residence i to a place of treatment j are *positively related to the size of the patient demand potential in i but are negatively related to the accessibility costs of getting from i to j.*

2.2. RAMOS

In the second model, RAMOS, there are essentially two features that potentially benefit from the merger with DRAM.

2.2.1. *Interactions between patient categories*

In RAMOS specialty groups are modeled separately. The choice of parameters is determined by the patterns of patient flow existing in a region between places of residence and places of treatment. This separation of specialty groups or patient categories into independently modeled streams, however, may be too rigid for the levels of detail required in some planning applications. For example the HCS can transfer resources from one specialty group to another depending on the type of resource and the possibilities for substitution. The first modification made to RAMOS, therefore, is to *allow for interaction between patient categories and their demands for resources.*

2.2.2. Inclusion of treatment standards

Treatment standards are the average amounts of resources (e.g., length of stay in hospital) supplied in each patient category. Currently, these are modeled exogenously by RAMOS (Mayhew, 1980) using a mixture of information that is based mostly on time-series analyses but also on expert medical opinion. There is, however, a case for making treatment standards *endogenous* in the model to observe more realistically the balance that is struck in the HCS between treating fewer patients with more resources per patient or more patients with less. The second proposed modification in RAMOS is,hence, *to make treatment standards endogenous in the model.*

3. DRAMOS: THE MODEL

The model description begins with a definition of the model subscripts: these characterize the potential demand for health care by where patients live (places of residence), where they are treated (health districts), and the patient categories in which they fall. The first part of the subsequent analysis follows very closely the previous theoretical developments in DRAM.

3.1. Basic Definitions

i = place of residence (origin zone), $i = \overline{1,I}$
j = place of treatment (destination zone), $j = \overline{1,J}$
k = patient category (e.g. specialty, diagnosis), $k = \overline{1,K}$

We want DRAMOS to predict the following two variables

T_{ijk} = the number of hospital admissions from origin zone i treated in zone j in patient category k

l_{jk} = the average length of stay in days in zone j for each patient category k (i.e. treatment standards)

There are three sets of contraints observed in the model governing the values of T and l*. The first is the resources available in each place of treatment. Suppose there are a total of R_j bed-days available in the hospitals located in j. The model finds the values of T and l that satisfy the following identity:

$$\sum_k D_{jk}\, l_{jk} = R_j \qquad \forall_j \tag{1}$$

D_{jk} in equation (1) is the number of hospital admissions in the place of treatment in patient cateogry k. D_{jk} and T_{ijk} are related thus

$$D_{jk} = \sum_i T_{ijk} \qquad \forall_j \tag{2}$$

There are J identities of the kind shown in equation(1) and JK in (2); the first set simply states therefore that all the resources in every place of treatment j are used.

---

*When variables are used without subscripts, it is implicit that they refer to a matrix or vector with dimensions of their subscripted counterparts, e.g. $\{T = T_{ijk}, \; i = \overline{1,I}, \; j = \overline{1,J}, \; k = \overline{1,K}\}$

The second and third sets of constraints are

$$0 < T_{ijk} < \phi_{ijk} \tag{3}$$

$$0 < l_{jk} < L_{jk} \tag{4}$$

Equations (3) and (4) define the bounds of the model variables T and l in relation to ϕ and L, the ideal admission levels and treatment standards. More precisely

ϕ_{ijk} = the ideal number of patients in patient category k in place i perceived to require treatment in j

L_{jk} = the ideal treatment standard measured in length of hospital stay in patient category k for each individual treated in j

Furthermore, it is hypothesized that

$$\phi_{ijk} = f(W_{ik}, c_{ij}, \omega_{jk}) \tag{5}$$

where the terms on the right-hand side of (5) are defined as follows:

W_{ik} — an index of the relative demand potential for medical care in i for patient category k. It is normally given by $\sum_{l} P_{il} U_{lk}$, where P_{il} is the population in i in age-sex category l and U_{lk} is the national rate in terms of hospital admissions. W_{ik} is analogous in sense (and use) to the patient generating factor (p.g.f.) defined in Mayhew (1980)

c_{ij} — expresses the difficulty of someone in i obtaining treatment in j. It is an accessibility measure based usually on travel time, distance or some other surrogate

ω_{jk} — a scaling factor, to be estimated, positively related to the importance of the treatment facilities in j in category k

A suitable form of equation (5) is given below.

3.2. Model Solution

The utility function that the various agents in the HCS seek to maximize in each place of treatment is taken to be

$$U_j(T,l) = \sum_k \sum_i g(T_{ijk}) + \sum_k \sum_i T_{ijk}\, h(l_{jk}) \tag{6}$$

where

$$g(T_{ijk}) = - \frac{\phi_{ijk} C_j L_{jk}}{\alpha_{jk}} \left(\frac{T_{ijk}}{\phi_{ijk}} \right)^{-\alpha_{jk}} \tag{7}$$

$$h(l_{jk}) = \frac{L_{jk}}{\gamma_{jk}} \left[1 - \left(\frac{l_{jk}}{L_{jk}} \right)^{-\gamma_{jk}} \right] \tag{8}$$

and where

α_{jk}, γ_{jk} are strictly positive constants dependent on both the place of treatment and the patient category

C_j is the marginal cost of a bed-day in each place of treatment

In these equations, α is a parameter measuring the relative importance of treating the ideal number of individuals ϕ, while γ is a parameter measuring the relative importance of achieving the ideal length of stay L. The utility function in (6) depicts the behavior of the HCS in which the agents are striving to attain the ideal levels of service (ϕ) and standards of care (L), but where the desire to increase the actual levels (T,l) decreases according to the parameters α and γ. Exactly as in DRAM, the utilities in each place of treatment for treating more patients and for treating each patient with more resources are considered to be monotonically increasing, and additive across patient categories.

The costs of treatment, C_j, can also be introduced, so that the marginal increases in U when ideal levels are achieved ($T = \phi$, $l = L$) equal the marginal resource costs. For the empirical example given later, however, we assume that C_j is everywhere the same, and so it is dropped from the remaining analysis. Although it is not discussed here, the utility function may also be given a strong economic interpretation (Gibbs, 1978); this helps reduce any potential unease concerning this particular functional form for the model.

The maximization problem solved below is now almost the same as that considered in earlier versions of DRAM. It is to maximize the utility function on (6) subject to the resource constraint in equation (1). Attaching the Lagrange multiplier, λ_j, in the usual way, we obtain an expression for the Lagrangian in j, H_j, which is

$$H_j(T,l,\lambda) = U_j(T,l) + \lambda_j(R_j - \sum_k D_{jk} l_{jk}) \qquad (9)$$

where λ is a vector of Lagrange multipliers $\{\lambda_j, j = \overline{1,J}\}$. In order to find the values of T and l that maximize H_j, it is necessary to solve the $J\{K(I + 1)+1\}$ equations

$$\frac{\partial H_j}{\partial T_{ijk}} = 0 \qquad \forall_{ijk} \qquad (10)$$

$$\frac{\partial H_j}{\partial l_{jk}} = 0 \qquad \forall_{kj} \tag{11}$$

$$\frac{\partial H_j}{\partial \lambda_j} = 0 \qquad \forall_j \tag{12}$$

From equations (6), (9) and (11) we have

$$\lambda_j = h'(l_{jk}) \tag{13}$$

where h' is the derivative of h. By rearranging terms, we can obtain from equation (8)

$$l_{jk} = L_{jk}\, \lambda_j^{-\frac{1}{(\gamma_{jk}+1)}} \tag{14}$$

From equations (6), (9), and (10), we arrive at

$$g'(T_{ijk}) = \lambda_j l_{jk} - h(l_{jk}) \tag{15}$$

and from equations (7) and (8), it is seen on substitution in (15) that

$$T_{ijk} = \phi_{ijk}(\mu_{jk})^{-\frac{1}{(\alpha_{jk}+1)}} \tag{16}$$

where

$$\mu_{jk} = \frac{1}{\gamma_{jk}} \left[(\beta_{jk}+1)\lambda_j^{\frac{\gamma_{jk}}{(\gamma_{jk}+1)}} - 1 \right] \tag{17}$$

Substitution of (14) and (15) in the constraint equation gives an expression for λ_j. This must be solved for λ_j in the case

$$f(\lambda_j) = 0 \qquad \forall_j \tag{18}$$

This expression is

$$f(\lambda_j) = -R_j + \sum_k \sum_i \phi_{ijk} L_{jk} (\mu_{jk})^{-\frac{1}{(\alpha_{jk}+1)}} (\lambda_j)^{-\frac{1}{(\gamma_{jk}+1)}} \tag{19}$$

Because it is impossible to make λ_j the subject of this equation, λ_j must be determined by a numerical technique such as the Newton-Raphson method. In this method, an improved value of λ_j is found by the rule.

$$\lambda_{j(n+1)} = \lambda_{j(n)} - \frac{f\left(\lambda_{j(n)}\right)}{f'\left(\lambda_{j(n)}\right)} \tag{20}$$

where n is the iteration number and $f'(\lambda_j)$ is the derivative of (19). In equation (20), solutions are being sought in the range

$$\lambda_j > 1 \qquad \forall_j$$

since only these will satisfy

$$0 < T_{ijk} < \phi_{ijk}\,, \qquad 0 < l_{jk} < L_{jk}$$

Within this specified range, it can be shown (for example, Hughes, 1978), that function (19), and its first derivative, are analytic and that it has only one real solution. For a large λ_j it means that, *ceteris paribus*, the available resources in j are further from satisfying the ideal levels than if λ_j were small.

3.3. DRAM, RAMOS: The Linkage

Consider the case when there is only one patient category (i.e. K = 1). Dropping the subscript k from equation (1), we find

$$R_j = \sum_i T_{ij} \, l_j \tag{21}$$

Substituting from (16) and rearranging terms, we get

$$(\mu_j)^{-\frac{1}{(\alpha_j + 1)}} = \frac{R_j}{l_j \sum_i \phi_{ij}} \tag{22}$$

from which it is seen

$$T_{ij} = \frac{R_j \, \phi_{ij}}{l_j \sum_i \phi_{ij}} \tag{23}$$

Noting that, by definition

$$\frac{R_j}{l_j} = D_j \tag{24}$$

where D_j is the number of cases treated annually in j, and letting

$$\phi_{ij} = \omega_j W_i e^{-\beta c_{ij}} \tag{25}$$

where the right-hand side combines ω_j, the scaling factor, potential demand, and β, a gravity parameter measuring the discounting effects of accessibility, we find, on substitution in (23),

$$T_{ij} = B_j D_j W_i e^{-\beta c_{ij}} \tag{26}$$

an attraction constrained gravity model (Wilson, 1974) that is analogous to RAMOS, where

$$B_j = \left[\sum_i W_i e^{-\beta c_{ij}} \right]^{-1} , \tag{27}$$

which is a balancing factor that ensures $\sum_i T_{ij} = D_j$.

Equations (21) to (26) link RAMOS and DRAM for one patient category. For K categories, there are K interlinked gravity models. From (1)

$$R_j = \sum_k \sum_i T_{ijk} l_{jk}$$

$$= \sum_i \phi_{ijm} l_{jm} \mu_{jm} + \sum_{\forall k \neq m} \sum_i \phi_{ijk} l_{jk} \mu_{jk} \tag{28}$$

where m is a particular category. Repeating steps (22) and (23), we obtain the same type of gravity model, but for a particular patient category m.

$$T_{ijm} = \frac{(R_j - \sum_{k \neq m} \sum_i T_{ijk} l_{jk})}{l_{jm}} \cdot \frac{\phi_{ijm}}{\sum_i \phi_{ijm}} \tag{29}$$

$$= \frac{R_{jm}\phi_{ijm}}{l_{jm}\sum_i \phi_{ijm}} \tag{30}$$

$$= B_{jm} D_{jm} W_{jm} e^{-\beta_m c_{ij}} \tag{31}$$

In (30), R_{jm} are the resources in bed-days allocated to a particular category, m(=k). They in turn are dependent on the resources available to the other $k \neq m$ patient categories in the gravity model through the mechanism

$$R_{jm} = R_j - \sum_{k \neq m} \sum_i T_{ijk} l_{jk} \tag{32}$$

Except, however, in the special case when K=1, it is necessary to base calculations of T_{ijk} on equation (16) above, as this variable occurs both on the left and right side of equation (29).

4. A PARAMETER ESTIMATION PROCEDURE

A set of equations has been derived linking the numbers of patients admitted from each place of residence i to each place of treatment j and in different patient categories k to the actual resources received (measured by length of hospital stay in j) and to the total resources available in j (total bed-days). These equations provide a powerful tool for simulating the behavior of the HCS when potential demand and resource availability are changing in time and space.

To give the model practical meaning and operational potential, however, values must be given to the variables and model parameters. Fortunately, the problems of estimating these values have already been mostly solved, but in separate contexts and using different methods during the development phases of DRAM and RAMOS. The main difficulty is to link the procedures in a logical sequence. The model parameters for which values are required are ω (not ϕ), L, α, β, and γ. The exogenous variables for which data are required comprise $\overline{R}_j$ (resources), $\overline{c}_{ij}$ (accessibility costs), and $\overline{W}_{ik}$ (demand potentials). In addition, the parameter estimation process requires that observations be available on $\overline{T}_{ijk}$ (the observed matrix counterpart of T_{ijk}), and $\overline{I}_{jk}$ (lengths of stay)[†].

4.1. Estimation of the discount parameter, β_k

It is logical to start with the estimation of β_k, the spatial discount parameter, which allocates patients to different destinations. In fact, since β_k is outside the control of the HCS, mainly reflecting the characteristics of the regional transport system that connects patients with their places of treatment, it can be estimated independently of the other parameters using one of the methods detailed in Mayhew and Taket (1980) or elsewhere (e.g., Hyman, 1969). These methods —— usually based

[†]Where a bar appears over a variable, it indicates that actual observations are being discussed.

on maximum likelihood or a type of regression analysis - try to find a value of β_k such that the model most accurately recreates an origin-destination matrix of observed patient flows $\{\overline{T}_{ijk}\}$. Since these methods are not relevant to what follows, their application has been assumed, and for the empirical example presented later a trial value of β_k is used that was first derived in the above reference.

4.2. Other Estimations

For estimating the remaining parameters, there are several choices depending on the amount of information available to the user. In some instances, the estimates may be decided on the basis of expert medical opinion and other factors; in others, they may reflect previous patterns of allocative behavior within the same region. In the present application, the second possibility is considered; that is, no *a priori* knowledge of the parameters is assumed. The method that is now described as an example follows very closely the one developed by Gibbs (1978, case 2). The first step in this method is to carry out a series of longitudinal regressions in each place of treatment j of the following form, based on past observations on patient admissions and lengths of stay in the region of interest

$$\log \overline{I}_{jkt} = \hat{a}^{1} + \hat{b}^{1}_{jk} \log \overline{R}_{jt} + u_{jt} \tag{33}$$

$$\log \overline{D}_{jkt} = \hat{a}^{D} + \hat{b}^{D}_{jk} \log \overline{R}_{jt} + z_{jt} \tag{34}$$

where u_{jt} and z_{jt} are stochastic error terms in time t. $\hat{a}^1$ and $\hat{a}^D$ are constants, and $\overline{I}_{jkt}$, $\overline{D}_{jkt}$ and $\overline{R}_{jt}$ are actual observations. We are interested in $\hat{b}^1_{jk}$ and $\hat{b}^D_{jk}$, the slope coefficients or *elasticities* of the lengths of stay and the

hospital admissions in each patient category. As Gibbs (1978) shows, these coefficients can be related to the elasticity parameters α_{jk} and γ_{jk}, as follows:
For γ_{jk}

$$\gamma_{jk} = \frac{V_j}{\hat{b}^1_{jk}} - 1 \tag{35}$$

$$\text{where } V_j = \frac{-\bar{R}_j}{\lambda_j f'(\lambda_j)} \tag{36}$$

and $f'(\lambda)$ is the derivative of (19);

for α_{jk}

$$\alpha_{jk} = \frac{V_j \ \gamma_{jk}}{\hat{b}^D_j \left(\gamma_{jk}+1-\lambda_j^{-\frac{\gamma_{jk}}{\gamma_{jk+1}}} \right)} - 1 \tag{37}$$

Next, it is necessary to find expressions for L_{jk}, the ideal lengths of stay, and ω_{jk}, the scaling factor in the ideal levels, ϕ_{ijk}.

From (14), rearranging terms

$$L_{jk} = \bar{l}_{jk} \ \lambda_j^{\frac{1}{(\gamma_{jk}+1)}} \tag{38}$$

and from (16), and (25)

$$\phi_{ijk} = T_{ijk} \ (\mu_{jk})^{\frac{1}{(\alpha_{jk}+1)}} \tag{39}$$

whence

$$\omega_{jk} = \frac{T_{ijk}}{W_{ik}e^{-\beta_k c_{ij}}} (\mu_{jk})^{\frac{1}{(\alpha_{jk}+1)}} \tag{40}$$

Examining the right-hand side of (40), we can substitute T_{ijk} with (31) and obtain

$$\omega_{jk} = B_{jk} \overline{D}_{jk} (\mu_{jk})^{\frac{1}{(\alpha_{jk}+1)}} \tag{41}$$

an equation that is no longer dependent on i and that uses the gravity model variables B_{jk} and $\overline{D}_{jk}$ directly. Here B_{jk}, the balancing factor, has been derived as an output from the procedure in section 4.1. above.

4.3. Restriction on Empirical Elasticities

A restriction on the input coefficients in these steps concerns the values of $\hat{b}^{l}_{jk}$ and $\hat{b}^{D}_{jk}$, the empirically determined elasticities. Suppose that estimates of these coefficients have been obtained in the manner described in section 4.2. The derivatives of the expected values of $\hat{l}_{jk}$ and $\hat{D}_{jk}$ with respect to R_j are given by

$$\frac{d\hat{l}_{jk}}{dR_j} = \frac{\hat{b}^{l}_{jk}\hat{l}_{jk}}{R_j} \tag{42}$$

and

$$\frac{d\hat{D}_{jk}}{dR_j} = \frac{\hat{b}^{D}_{jk}D_{jk}}{R_j} \tag{43}$$

From the constraint equation in (1), we know

$$\sum_k D_{jk}\, l_{jk} = R_j \tag{1}$$

Differentiating this with respect to R_j, we obtain

$$\sum_k \left(D_{jk} \frac{dl_{jk}}{dR_j} + l_{jk} \frac{dD_{jk}}{dR_j} \right) = 1 \tag{44}$$

Substituting (42) and (43), the empirical estimates of the derivatives, the condition arises that

$$\sum_k \left(\hat{b}^l_{jk} + \hat{b}^D_{jk} \right) = 1 \tag{45}$$

This states that the empirical elasticities in every j summed over k equal 1.0. In the event that this fails to occur (because of statistical reasons in the regression analysis, or through errors in the data), certain consistency problems may arise in subsequent procedures used by the model. A simple correction factor to remedy for this is therefore given by

$$p_j = \frac{\bar{R}_j}{\sum_k \bar{D}_{jk} l_{jk} (\hat{b}^l_{jk} + \hat{b}^D_{jk})} \tag{46}$$

This factor, p_j, is used to scale the empirical elasticities on input into the calibration procedure and ensures that the condition in (45) holds true.

4.4. Additional Considerations

From equations (35) to (41), expressions have been derived for each of the parameters in terms of the empirical elasticities, $\hat{b}^l_{jk}$ and $\hat{b}^D_{jk}$. Unfortunately, this is an equation system for which in every j there are three more unknowns. V_j, λ_j $f'(\lambda_j)$, than there are available equations (= 4K, the number of patient categories); thus, there are an infinite number of solutions and, hence, possible parameter sets. Nevertheless, in applying this method of parameter estimation the experience of Gibbs (1978) has been borne out. That is, for a reasonably wide range of arbitrary initial values of V_j and λ_j ($\tilde{V}$ and $\tilde{\lambda}$), the model outputs, T and l, are not affected to any important degree. Some additional comments on this step, including suitable starting values, for these constants are given in section 5.7 below.

4.5. Summary of Main Steps

The whole parameter estimation process for the model can now be summarized in the following five steps:

1. Estimate β_k, B_{jk}, and T_{ijk} from an attraction constrained gravity model with input data on $\overline{T}_{ijk}$, $\overline{W}_{ik}$, and $\overline{c}_{ij}$ as in Mayhew and Taket (1980)
2. Estimate using log-linear regression, with observations on $\overline{I}_{jkt}$, $\overline{D}_{jkt}$ and $\overline{R}_{jt}$, the empirical elasticities $\hat{b}^l_{jk}$ and $\hat{b}^D_{jk}$
3. Assume initial values for $V_j (=\tilde{V})$ and λ_j $(=\tilde{\lambda})$.
4. Determine the parameter values γ_{jk}, α_{jk}, L_{jk}, and ω_{jk} using $\tilde{V}$, $\tilde{\lambda}$, β_k, $\overline{W}_{ik}$, B_{jk}, $\overline{D}_{jk}$, and $\overline{c}_{ij}$ in the sequence shown in section 4.2

5. With these parameter values predict l_{jk} and T_{ijk} solving $f(\lambda_j) = 0$ for different combinations of potential demand, W_{ik} and resources, R_j

After step 5 a variety of additional outputs can be calculated that are of value in condensing the results and in interpreting the outputs. They include

a. Hospitalization rates by place of residence (and patient category if desired)

$$HR_i = \left(\sum_k \sum_j T_{ijk}/P_i \right) \times 10^3 \tag{47}$$

where P_i is the population of i and units are cases per thousand resident population

b) Catchment populations for each j (see also Mayhew and Taket 1980, p.22)

$$CP_j = \sum_i E_{ij} \, P_i \tag{48}$$

where

$$E_{ij} = \sum_k T_{ijk} / \sum_k \sum_j T_{ijk} \tag{49}$$

c) Admission rates for each place of treatment

$$A_j = \left(\sum_k \sum_i T_{ijk}/CP_j \right) \times 10^3 \tag{50}$$

where $\sum_k \sum_i T_{ijk} (= \sum_k D_{jk})$ are the number of cases admitted to j annually and units are in cases per thousand catchment population

5. MODEL APPLICATION

Preliminary tests using the above model have been carried out on in-patient hospital data from the London region in the United Kingdom using a purpose-written set of computer programs. These tests are presented in the form of three planning scenarios that have been designed to test the response of the HCS to regional variations in resource availability and demand potential.

5.1. The London Problem

London forms a particularly interesting case-study for the application of this model for three reasons: Firstly, there are a large number of complicated cross flows between districts in the city that would create problems in a conventional application of DRAM. These are caused partly by an overconcentration of hospitals in the city center, and partly by the ready availability of transport services that facilitate travel between different areas (Mayhew, 1979). Secondly, because of a relatively rapid changing demographic structure, London (and many other cities like it) undergo frequent shifts in their potential demand for health care services, thereby making it difficult to keep track of the relative level of resources needed for each part of the city. Thirdly, the London hospital system is a national and international center for medical training and research with many long-term obligations in these activities (LHPC, 1979) that also have to be taken into account.

The three chosen scenarios, which have been kept purposely simple for illustrative reasons, are as follows:

1. A 10% reduction in available bed-days (R_j) in each place of treatment
2. An increase of 10% in available bed-days in each place of treatment
3. An increase of 10% in bed-days in each place of treatment, 10% loss in demand potential in central areas of residence, and a 10% increase in peripheral areas

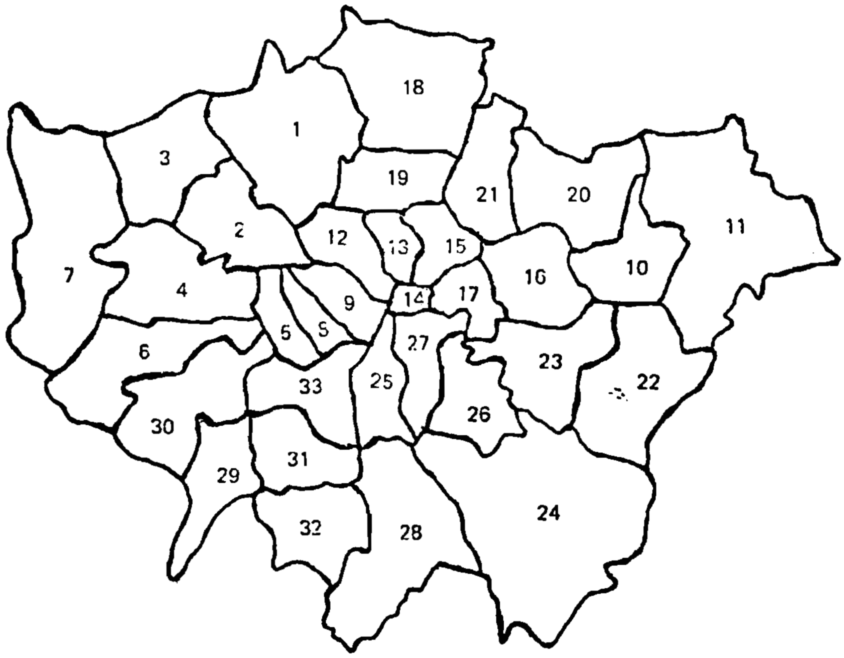
5.2. The Regional Dimensions

We require the model to tell us what the consequences will be of these changes on the admissions to hospitals and the treatment standards in all parts of the city.

For these scenarios, the same set of origins and destinations is used as that in Mayhew (1980). This set is shown in the map in Figure 1; a key to the numbering system that appears is provided in Table 1. The zones depicted corresponds to the region covered by the Greater London Council (GLC), an area containing approximately seven million residents where nearly one million acute in-patient hospital cases were treated in 1977 (the year for which the data-set applies). In all, there are 33 origin zones and 36 destination zones, plus one external zone to "close" the system.

There are two limitations in this data-set that have prevented a full investigation of the model's potential, and these must be borne in mind in the interpretation of the results. They are the absence of detailed information on different patient categories and the unavailability of estimates for the empirical elasticities at each location. It is hoped to remedy these deficiencies at an early date, as soon as the data become available. Instead, a one-category (K=1) model (based on an aggregation of 23 acute patient specialties) is developed using an assumed set of empirical elasticities. For this reason the k subscript is no longer required and is thus

A) Origin zones (places of residence)



B) Destination zones (places of treatment)

Figure 1. The Greater London Council: definition of zones.

Table 1. Key to Figure 1.

Origin				Destination			
1	Barnet	24	Bromley	1	Barnet	24	West Roding
2	Brent	25	Lambeth	2	Edgware	25	Bexley
3	Harrow	26	Lewisham	3	Brent	26	Greenwich
4	Ealing	27	Southwark	4	Harrow	27	Bromley
5	Hammersmith	28	Croydon	5	Hounslow	28	St.Thomas†
6	Hounslow	29	Kingston	6	South Hammersmith	29	Kings
7	Hillingdon	30	Richmond	7	North Hammersmith	30	Guys
8	Kens & Chelsea	31	Merton	8	Ealing	31	Lewisham
9	Westminster	32	Sutton	9	Hillingdon	32	Croydon
10	Barking	33	Wandsworth	10	KCW Northwest*	33	Kingston
11	Havering	34	Other	11	KCW Northeast	34	Roehampton
12	Camden			12	KCW South	35	Wandsworth/East Merton
13	Islington			13	Barking	36	Sutton
14	City			14	Havering	37	Other
15	Hackney			15	North Camden		
16	Newham			16	South Camden		
17	Tower Hamlets			17	Islington		
18	Enfield			18	City		
19	Haringey			19	Newham		
20	Redgridge			20	Tower Hamlets		
21	Waltham Forest			21	Enfield		
22	Bexley			22	Haringey		
23	Greenwich			23	East Roding		

*K/C/W = Kensington, Chelsea, and Westminster

†Destinations 28, 29, 30 are named after teaching hospitals within the districts.

dropped form the remaining analysis. Clearly, a one-category model is the simplest of all cases, and a potential user may prefer to use other, simpler methods of dealing with it. Its presentation here is simply for illustrative purposes - to indicate the potential power of this approach.

A partial justification for the relative magnitudes of the values used for the elasticity set ($\hat{b}_j^1$, and $\hat{b}_j^D$) is provided in two scattergrams, based on cross-sectional data, for the same places of treatment in 1977 (Figures 2 and 3). They show plots of length of stay and patient admissions on bed-days in each location. It is plain from these scatters that the average resources received (lengths of stay) are substantially less elastic than the patient admissions. This actual behavior has thus influenced (but not determined) the choice of these parameters. In addition, however, some further variability has been allowed for, since each location is known to exhibit a slightly different behavior with respect to these variables.

5.3. Model Calibration

The model functions in two modes: a calibration mode in which values are given the parameters, and a predictive mode in which the consequences of resource reallocations are tested. Table 2 shows the results obtained during a calibration run for the particular set of observations on resources ($\bar{R}_j$) and lengths of stay ($\bar{I}_j$). The large variation in parameter values shown is itself a justification for making the hypothesized utility function [equation (6)] dependent on location, providing the assumed elasticities are a fair representation of (actual) behavior.

One of the most interesting features in these results is the wide variation in the value of ω_j. This parameter acts as a scale on the patient generating factor and accessibility (W_i and c_{ij}) in the calculation of ϕ_{ij}, the ideal number of admissions from i. When ω_j is large it can be

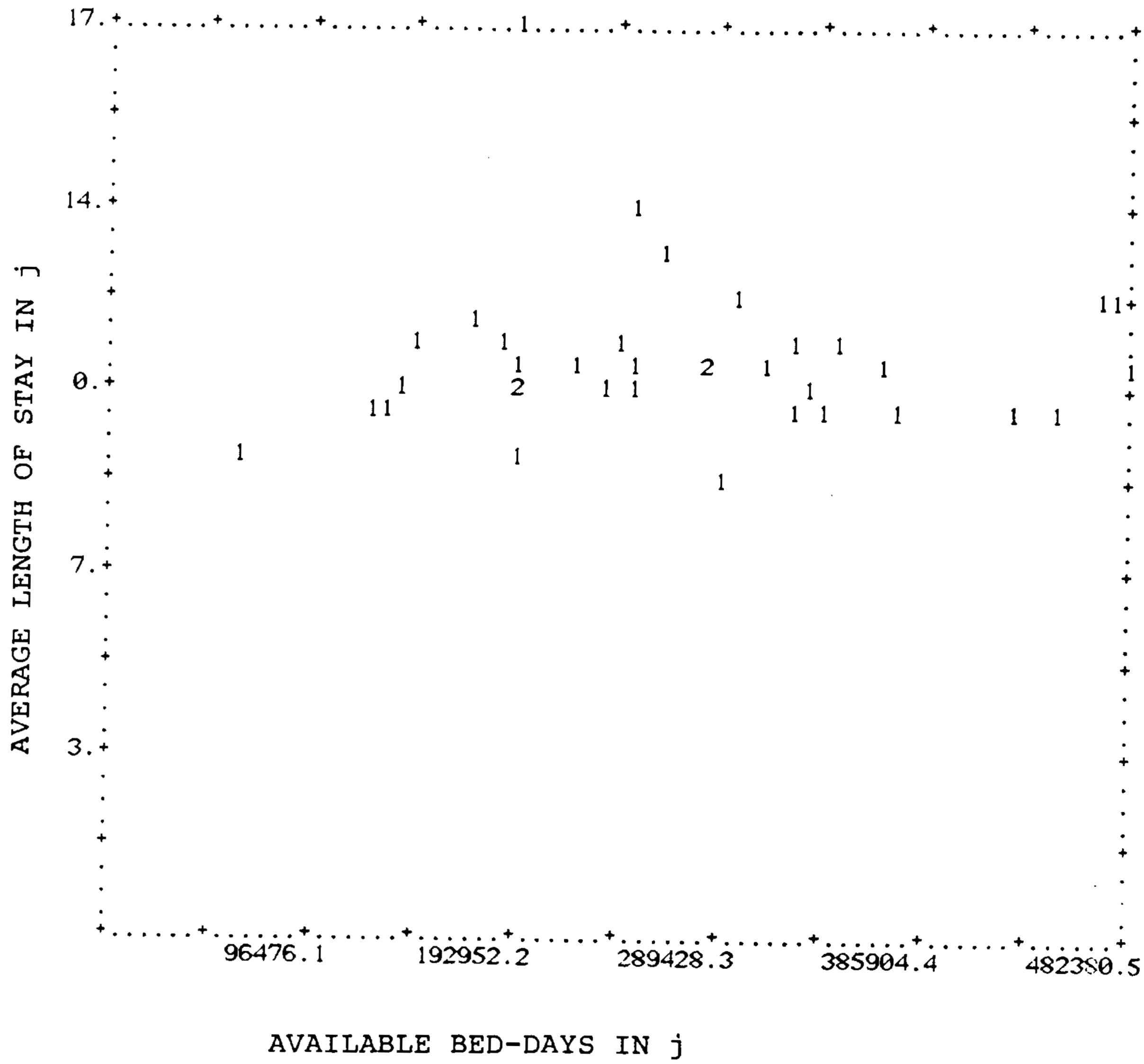


Figure 2. A plot of average length of hospital stay on available bed-days by place of treatment in Greater London area, 1977. Shows length of stay is generally inelastic to bed supply.

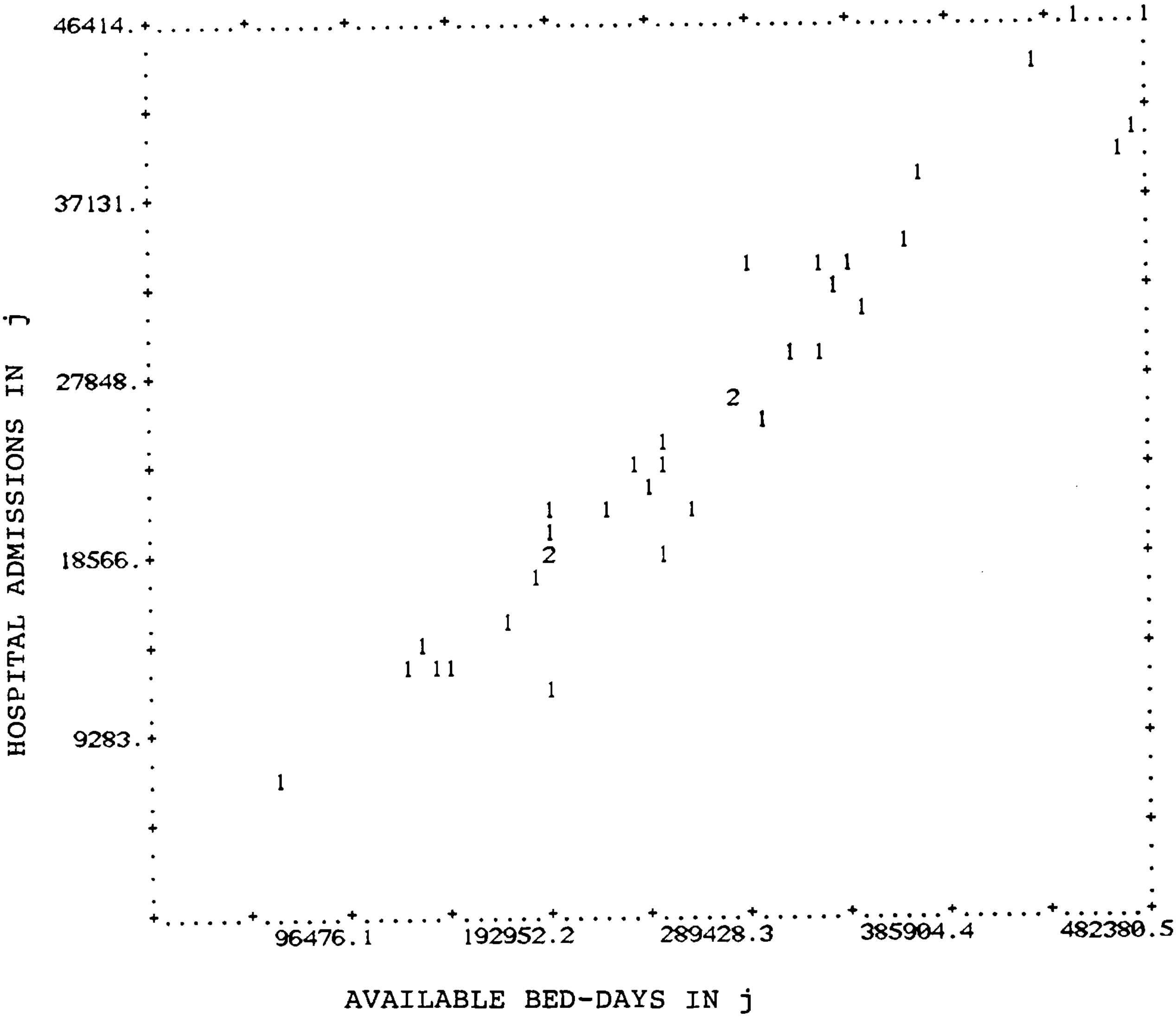


Figure 3. A plot of hospital admissions on available bed-days by place of treatment in Greater London area, 1977. As is seen hospital admissions are very elastic to bed supply.

Table 2. Resource availability, elasticity, and parameter estimates by place of treatment.

Zone	Name	R_j	l_j	L_j	ω_j	α_j	γ_j	$\hat{b}^D_j$	$\hat{b}^l_j$
1	barnet+	136547.	10.30	12.10	2.78	2.19	12.64	0.88	0.22
2	edgware	264024.	12.90	16.92	2.92	3.25	7.11	0.63	0.37
3	brent	194494.	10.20	12.99	1.56	3.05	8.09	0.67	0.33
4	harrow	189703.	8.90	8.97	2.62	2.02	299.00	0.99	0.01
5	hounslow	237147.	10.30	13.81	1.91	3.42	6.50	0.60	0.40
6	s hamm	248602.	10.60	14.42	3.31	3.54	6.14	0.58	0.42
7	n hamm	174461.	11.30	14.39	3.18	3.05	8.09	0.67	0.33
8	ealing	65056.	9.10	10.08	0.51	2.34	20.43	0.86	0.14
9	hilligdn	363752.	10.50	10.65	15.31	2.04	149.00	0.98	0.02
10	kcw nw	330120.	10.00	14.01	6.95	3.82	5.52	0.54	0.46
11	kcw ne	290902.	8.70	12.10	24.28	3.75	5.67	0.55	0.45
12	kcw s	482381.	10.50	12.61	28.13	2.71	11.00	0.75	0.25
13	barking	248117.	10.10	10.63	5.13	2.16	41.86	0.93	0.07
14	havering	192831.	17.00	19.25	11.64	2.43	16.65	0.83	0.17
15	n camden	243479.	11.10	14.03	4.35	3.00	8.37	0.68	0.32
16	s camden	450216.	9.70	14.30	21.69	4.41	4.66	0.47	0.53
17	islingtn	346269.	11.00	15.98	4.06	4.23	4.88	0.49	0.51
18	city +	477333.	11.90	15.26	8.81	3.10	7.82	0.66	0.34
19	newham	250925.	13.50	14.63	1.56	2.26	26.27	0.89	0.11
20	t hamlet	467492.	11.90	19.44	5.79	6.36	3.48	0.33	0.67
21	enfield	185636.	11.00	11.84	2.29	2.24	29.00	0.90	0.10
22	haringey	298535.	11.90	14.19	3.10	2.67	11.50	0.76	0.24
23	e roding	127315.	9.60	12.14	2.89	3.00	8.38	0.68	0.32
24	w roding	312638.	10.80	12.97	2.80	2.71	11.00	0.75	0.25
25	bexley	144782.	11.00	18.91	3.09	8.12	3.05	0.26	0.74
26	greenwch	373929.	9.80	14.86	5.81	6.09	4.28	0.38	0.61
27	bromley	282073.	10.70	14.45	6.25	3.48	6.32	0.59	0.41
28	st thoms	282452.	10.80	18.43	16.88	7.81	3.11	0.27	0.73
29	kings	324341.	9.80	12.48	9.89	3.05	8.09	0.67	0.33
30	guys	323534.	11.20	14.90	22.06	3.36	6.69	0.61	0.39
31	lewisham	222761.	10.80	13.75	2.10	3.05	8.09	0.67	0.33
32	croydon	195580.	10.00	11.66	1.64	2.56	13.29	0.79	0.21
33	kingston	192666.	10.60	14.31	3.29	3.48	6.32	0.59	0.41
34	roehamtn	132077.	9.60	10.18	4.66	2.18	36.50	0.92	0.08
35	wands+em	427015.	9.80	9.94	10.11	2.04	149.00	0.98	0.02
36	sutton +	336719.	9.90	14.28	8.19	4.14	5.00	0.50	0.50
37	others	29422450.	9.00	13.46	8.90	4.62	4.45	0.45	0.55

$\tilde{V} = 3.0$ $\tilde{\lambda} = 9.0$

$\beta = 0.367$ (derived from "RAMOS: A Resource Allocation Model over Space", Mayhew and Taket, 1980) taken as a sign of the relative importance of the treatment facilities in j in that they expect to attract ω_j times more than their demand potential would suggest. Looking at the results we see the largest values are concentrated mostly in treatment districts in central parts of the city where the large teaching hospitals are located. In effect this means that these hospitals have a much greater patient attracting power than would be normally expected from a consideration of their demand potential alone. In contrast, many peripheral districts have small values of ω_j, and hence the hospitals here have only local significance. It must also be stressed that the magnitudes of the parameters are strongly dependent on the starting values for $\tilde{V}$ and $\tilde{\lambda}$, but the model outputs are not. Suitable starting values for $\tilde{V}$ and $\tilde{\lambda}$ are discussed in a sensitivity analysis below (section 5). First, however, predictions made under each scenario are discussed in turn.

5.4. Scenario 1

In scenario 1, the availability of resources has been reduced by 10% in every place of treatment. What happens to the number of patients admitted to hospitals and the average standards of care received depends on the relative strength of the elasticity parameters, α and γ. Low α_j relative to γ_j implies an inelasticity in patient admissions with respect to treatment standards and vice versa. As is seen (Table 3), all locations experience a fall in treatment standards, but because of the choice of empirical elasticities the negative impact on admissions has generally been more significant, bearing out the evidence in Figures 2 and 3.

In more detail, it is noticed, for example, that in district 4 (a very low assumed elasticity with respect to treatment standards) the percentage fall in length of stay in

Table 3. Scenario 1: A 10% decrease in resource availability in each place of treatment.

Zone	Name	$l_{j(a)}$†	$l_{j(b)}$	% change	$D_{j(a)}$	$D_{j(b)}$	% change
1	barnet+	10.3	10.1	-2.09	13257.0	12185.6	-8.08
2	edgware	12.9	12.4	-3.83	20467.0	19153.3	-6.42
3	brent	10.2	9.9	-3.42	19068.0	17768.9	-6.81
4	harrow	8.9	8.9	-0.11	21315.0	19204.6	-9.90
5	hounslow	10.3	9.9	-4.14	23024.0	21616.0	-6.12
6	s hamm	10.6	10.1	-4.33	23453.0	22062.5	-5.93
7	n hamm	11.3	10.9	-3.42	15439.0	14387.5	-6.81
8	ealing	9.1	9.0	-1.46	7149.0	6529.4	-8.67
9	hilligdn	10.5	10.5	-0.21	34643.0	31244.4	-9.81
10	kcw nw	10.0	9.5	-4.74	33012.0	31189.1	-5.52
11	kcw ne	8.7	8.3	-4.64	33437.0	31556.3	-5.62
12	kcw s	10.5	10.2	-2.60	45941.0	42451.6	-7.60
13	barking	10.1	10.0	-0.74	24566.0	22273.6	-9.33
14	havering	17.0	16.7	-1.78	11343.0	10393.3	-8.37
15	n camden	11.1	10.7	-3.32	21935.0	20419.5	-6.91
16	s camden	9.7	9.2	-5.44	46414.0	44176.0	-4.82
17	islingtn	11.0	10.4	-5.24	31479.0	29898.4	-5.02
18	city +	11.9	11.5	-3.52	40112.0	37418.3	-6.72
19	newham	13.5	13.3	-1.15	18587.0	16923.6	-8.95
20	t hamlet	11.9	11.1	-6.82	39285.0	37945.9	-3.41
21	enfield	11.0	10.9	-1.05	16876.0	15348.8	-9.05
22	haringey	11.9	11.6	-2.50	25087.0	23156.1	-7.70
23	e roding	9.6	9.3	-3.32	13262.0	12345.8	-6.91
24	w roding	10.8	10.5	-2.60	28948.0	26749.6	-7.59
25	bexley	11.0	10.2	-7.51	13162.0	12808.1	-2.69
26	greenwch	9.8	9.2	-6.30	38156.0	36648.2	-3.95
27	bromley	10.7	10.2	-4.23	26362.0	24774.9	-6.02
28	st thoms	10.8	10.0	-7.41	26153.0	25421.9	-2.80
29	kings	9.8	9.5	-3.42	33096.0	30842.3	-6.81
30	guys	11.2	10.7	-4.03	28887.0	27091.4	-6.22
31	lewisham	10.8	10.4	-3.42	20626.0	19221.6	-6.81
32	croydon	10.0	9.8	-2.18	19558.0	17995.3	-7.99
33	kingston	10.6	10.2	-4.24	18176.0	17082.6	-6.02
34	roehamtn	9.6	9.5	-0.84	13758.0	12487.3	-9.24
35	wands+em	9.8	9.8	-0.21	43573.0	39296.9	-9.81
36	sutton +	9.9	9.4	-5.14	34012.0	32270.4	-5.12
37	others	9.0	8.5	-5.64	3269161.0	3118044.0	-4.62

† (a) current
(b) predicted

relatively small (-0.11%) as compared with district 20 (allocated a high elasticity), where the change is more substantial (-6.82%). If we compare the numbers of cases admitted to these two districts, however, the picture is reversed. The admissions to district 4 are down by 9.90%, while those in district 20 are only 3.41% lower. These examples show how the model is working. With more data on different patient categories, it is straight-forward to visualize the potential degree of detail available using this approach. We now compare these results with those obtained in scenario 2.

5.5. Scenario 2

In scenario 2, the resources in each place of treatment have been increased by 10% (results in Table 4). As would be expected the results show completely the opposite pattern - an all-round improvement in treatment standards and the number of patients treated. Again because of the choice in empirical elasticities, the proportionate increases in the in the latter are generally higher but they still vary between treatment districts.

For comparative purposes, it is noted that in districts 4 and 20, the actual increases are 0.09% and 6.58% in terms of length of stay and 9.90% and 3.20% for patient admissions. This is hence an almost exact reversal of the results described in scenario 1 above.

Table 4. Scenario 2: A 10% increase in resource availability in each place of treatment.

Zone	Name	$l_{j(a)}^{\dagger}$	$l_{j(b)}$	% change	$D_{j(a)}$	$D_{j(b)}$	% change
1	barnet+	10.3	10.5	1.92	13257.0	14307.6	7.93
2	edgware	12.9	13.4	3.59	20467.0	21736.0	6.20
3	brent	10.2	10.5	3.19	19068.0	20325.9	6.60
4	harrow	8.9	8.9	0.09	21315.0	23425.2	9.90
5	hounslow	10.3	10.7	3.87	23024.0	24381.8	5.90
6	s hamm	10.6	11.0	4.08	23453.0	24785.8	5.68
7	n hamm	11.3	11.7	3.19	15439.0	16458.0	6.60
8	ealing	9.1	9.2	1.35	7149.0	7759.4	8.54
9	hilligdn	10.5	10.5	0.19	34643.0	38034.6	9.79
10	kcw nw	10.0	10.4	4.47	33012.0	34758.6	5.29
11	kcw ne	8.7	9.1	4.38	33437.0	35238.8	5.39
12	kcw s	10.5	10.8	2.41	45941.0	49346.5	7.41
13	barking	10.1	10.2	0.67	24566.0	26843.6	9.27
14	havering	17.0	17.3	1.63	11343.0	12276.9	8.23
15	n camden	11.1	11.4	3.09	21935.0	23405.0	6.70
16	s camden	9.7	10.2	5.17	46414.0	48545.0	4.59
17	islingtn	11.0	11.5	4.97	31479.0	32987.5	4.79
18	city +	11.9	12.3	3.29	40112.0	42717.5	6.50
19	newham	13.5	13.6	1.05	18587.0	20232.8	8.85
20	t hamlet	11.9	12.7	6.58	39285.0	40543.8	3.20
21	enfield	11.0	11.1	0.96	16876.0	18387.1	8.95
22	haringey	11.9	12.2	2.32	25087.0	26971.3	7.51
23	e roding	9.6	9.9	3.09	13262.0	14150.8	6.70
24	w roding	10.8	11.1	2.41	28948.0	31094.2	7.41
25	bexley	11.0	11.8	7.29	13162.0	13494.0	2.52
26	greenwch	9.8	10.4	6.03	38156.0	39582.8	3.74
27	bromley	10.7	11.1	3.98	26362.0	27888.6	5.79
28	st thoms	10.8	11.6	7.20	26153.0	26837.3	2.62
29	kings	9.8	10.1	3.19	33096.0	35280.7	6.60
30	guys	11.2	11.6	3.78	28887.0	30618.8	6.00
31	lewisham	10.8	11.1	3.19	20626.0	21987.7	6.60
32	croydon	10.0	10.2	2.03	19558.0	21086.7	7.82
33	kingston	10.6	11.0	3.97	18176.0	19229.6	5.80
34	roehamtn	9.6	9.7	0.76	13758.0	15019.3	9.17
35	wands+em	9.8	9.8	0.19	43573.0	47837.1	9.79
36	sutton +	9.9	10.4	4.87	34012.0	35676.2	4.89
37	others	9.0	9.5	5.37	3269161.0	3412706.5	4.39

† (a) current
(b) predicted

5.6. Scenario 3

In scenario 3, the added abilities of DRAMOS are most clearly demonstrated by giving the predicted outcomes of what happens to service levels when there are simultaneous changes in resource levels and potential demand. On the supply side resources have been increased by 10% in every location; on the demand side, the potential uptake in services has been reduced by 10% in the inner-city and increased by 10% on the city periphery.

The resultant effects on the treatment standards and patient admissions are shown in Table 5. Why are these results substantially different from scenario 2 in which resources were increased by the same amount? The reason is the changed pattern of potential demand. In inner-city areas, where potential demand has been reduced 10%, the increases in patient admissions are small, and in two cases, slightly negative. On the other hand, the increase in resource levels (+10%), which is contrary in trend to the demand potiential, creates relative surplus of bed-days in these parts of the city, thus enabling higher treatment standards to result (over 10% on two lengths of stay in zones 20 and 28). In peripheral areas, however, the opposite predictions are made. The added demand potential induces proportionately more patient admissions (since more resources are available), but causes treatment standards to change only little (since there is a higher demand potential). These results are precisely the types of predictions that neither DRAM nor RAMOS can make if they are applied separately to this problem.

Some further contrasts with the prediction given in scenario 2 are shown in the following two diagrams (Figures 4 and 5), which help to indicate the magnitude of probable error in admission/hospitalization rates were information in the model on demand potential excluded. As is seen these differences could be as high as ±8% in each i and j for this particular scenario, with the over-estimates concentrated in the inner-city and under-estimates on the periphery.

Table 5. Scenario 3: Patient admissions and treatment standards resulting from an increase in resource availability and a redistribution of demand potential.

Zone	Name	$l_{j(a)}$ †	$l_{j(b)}$	% change	$D_{j(a)}$	$D_{j(b)}$	% change
1	barnet+	10.3	10.3	0.04	13257.0	14577.1	9.96
2	edgware	12.9	12.9	0.04	20467.0	22506.8	9.97
3	brent	10.2	10.2	0.16	19068.0	20941.9	9.83
4	harrow	8.9	8.9	-0.00	21315.0	23447.4	10.00
5	hounslow	10.3	10.3	0.01	23024.0	25322.9	9.98
6	s hamm	10.6	11.2	5.71	23453.0	24405.5	4.06
7	n hamm	11.3	11.6	2.32	15439.0	16597.6	7.50
8	ealing	9.1	9.1	0.03	7149.0	7862.0	9.97
9	hilligdn	10.5	10.5	0.00	34643.0	38107.1	10.00
10	kcw nw	10.0	10.7	6.66	33012.0	34045.9	3.13
11	kcw ne	8.7	9.2	5.85	33437.0	34746.6	3.92
12	kcw s	10.5	10.9	4.09	45941.0	48550.4	5.68
13	barking	10.1	10.1	0.03	24566.0	27014.3	9.97
14	havering	17.0	17.0	0.01	11343.0	12476.0	9.99
15	n camden	11.1	11.5	3.92	21935.0	23217.5	5.85
16	s camden	9.7	10.4	7.14	46414.0	47651.6	2.67
17	islingtn	11.0	12.0	8.71	31479.0	31852.6	1.19
18	city +	11.9	12.6	5.88	40112.0	41673.4	3.89
19	newham	13.5	13.8	1.89	18587.0	20066.4	7.96
20	t hamlet	11.9	13.3	11.50	39285.0	38755.7	-1.35
21	enfield	11.0	11.0	0.15	16876.0	18535.8	9.84
22	haringey	11.9	12.2	2.58	25087.0	26900.7	7.23
23	e roding	9.6	9.6	0.36	13262.0	14535.7	9.60
24	w roding	10.8	10.8	0.43	28948.0	31706.7	9.53
25	bexley	11.0	11.0	0.15	13162.0	14456.7	9.84
26	greenwch	9.8	10.0	1.55	38156.0	41330.8	8.32
27	bromley	10.7	10.7	0.42	26362.0	28877.7	9.54
28	st thoms	10.8	12.2	12.85	26153.0	25492.3	-2.53
29	kings	9.8	10.4	5.68	33096.0	34448.4	4.09
30	guys	11.2	11.8	5.59	28887.0	30094.7	4.18
31	lewisham	10.8	11.4	5.16	20626.0	21575.7	4.60
32	croydon	10.0	10.0	0.02	19558.0	21509.0	9.98
33	kingston	10.6	10.6	0.14	18176.0	19966.3	9.85
34	roehamtn	9.6	9.7	0.86	13758.0	15004.9	9.06
35	wands+em	9.8	9.8	0.26	43573.0	47806.1	9.72
36	sutton +	9.9	9.9	0.12	34012.0	37369.9	9.87
37	others	9.0	9.5	5.37	3269161.0	3412706.5	4.39

† (a) current
(b) predicted

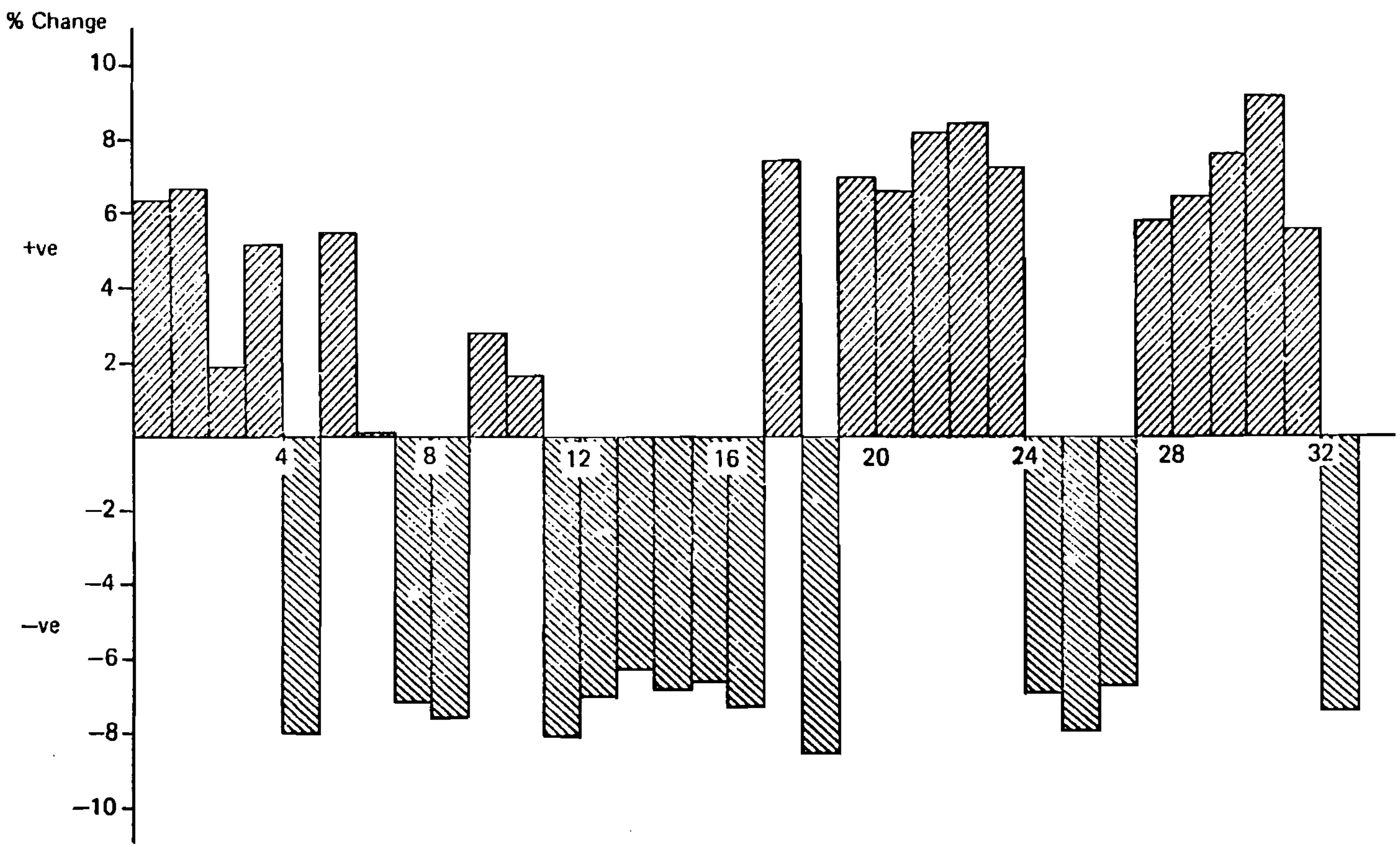


Figure 4. Changes in hospitalization rates: Scenario 2 and Scenario 3.

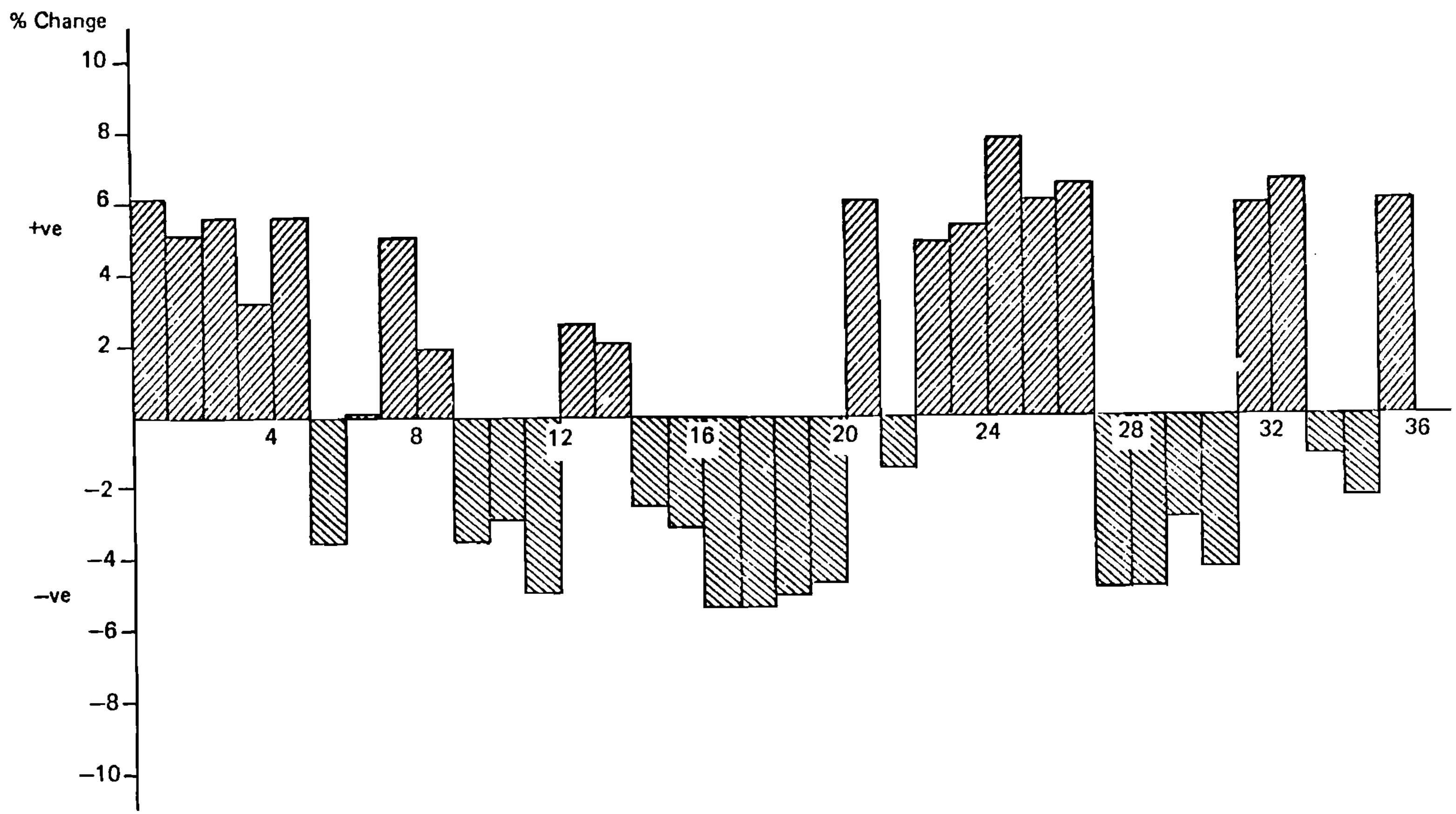


Figure 5. Changes in admission rates: Scenario 2 and Scenario 3.

We may ask the model finally whether the strategy of increasing the resources by 10% in all locations is a good one, given the changed configuration in potential demand. In Figures 6 and 7 two scatter diagrams are shown based also on the outputs of the model. On the horizontal axes are the indices of potential demand (W_i) scaled by θ (where $\theta = \sum_j D_j / \sum_i W_i$); on the vertical axes is satisfied demand (total patients generated in i, $\sum_j T_{ij}$). For a more equitable resource configuration, these points would be on a straight line (Mayhew, 1980). As is seen, the simulated effects of the proposed reallocation (Figure 7) create no improvement at all (in fact, the correlation coefficient shows a fractional fall). On this basis, therefore, the unsatisfactory predictions would probably lead to the rejection of this planned set of allocations, and the creation of another option probably involving proportionately larger allocations to peripheral areas.

5.7. Sensitivity to $\tilde{V}$ and $\tilde{\lambda}$

In the model there is some arbitrariness due to the values of $\tilde{V}$ and $\tilde{\lambda}$ that are assumed at the outset of the calibration steps. A sequence of combined calibration-prediction runs, however, showed only very small (< 0.01%) variation in the outputs when $\tilde{V}$ and $\tilde{\lambda}$ were allowed to vary over a range from 3.0 to 15.0. This confirms the earlier finding of Gibbs (1978) using a similar method of calibration. Since the parameter values are all strictly positive a suitable starting value for $\tilde{V}$ is found from an inspection of equations (35), and (37); namely

$$\gamma_{jk} = \frac{V_j}{\hat{b}^1_{jk}} - 1 \qquad (35)$$

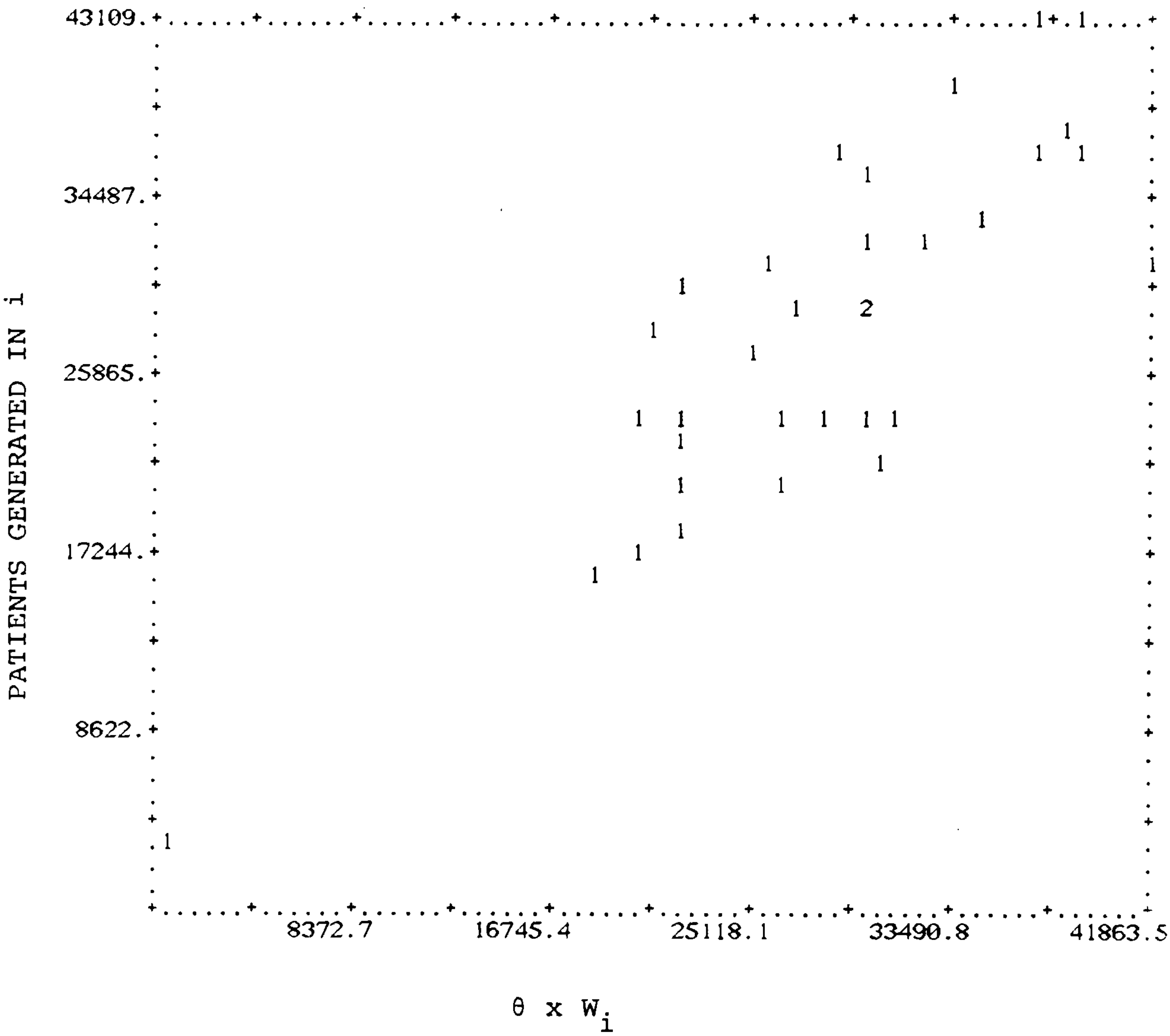


Figure 6. Plot of patients generated in i ($= \sum_j T_{ij}$) on the relative demand in i scaled by θ, where

$\theta = \sum_j D_j / \sum_i W_i$, for existing resource configuration and

demand potentials. For an efficient resource configuration the result should be a straight line 45° to the origin (Mayhew, 1980). Here the correlation coefficient is 0.832 (instead of 1.0) and the slope, 0.892 (1.0). Conclusion: Current distribution of resources not very equitable.

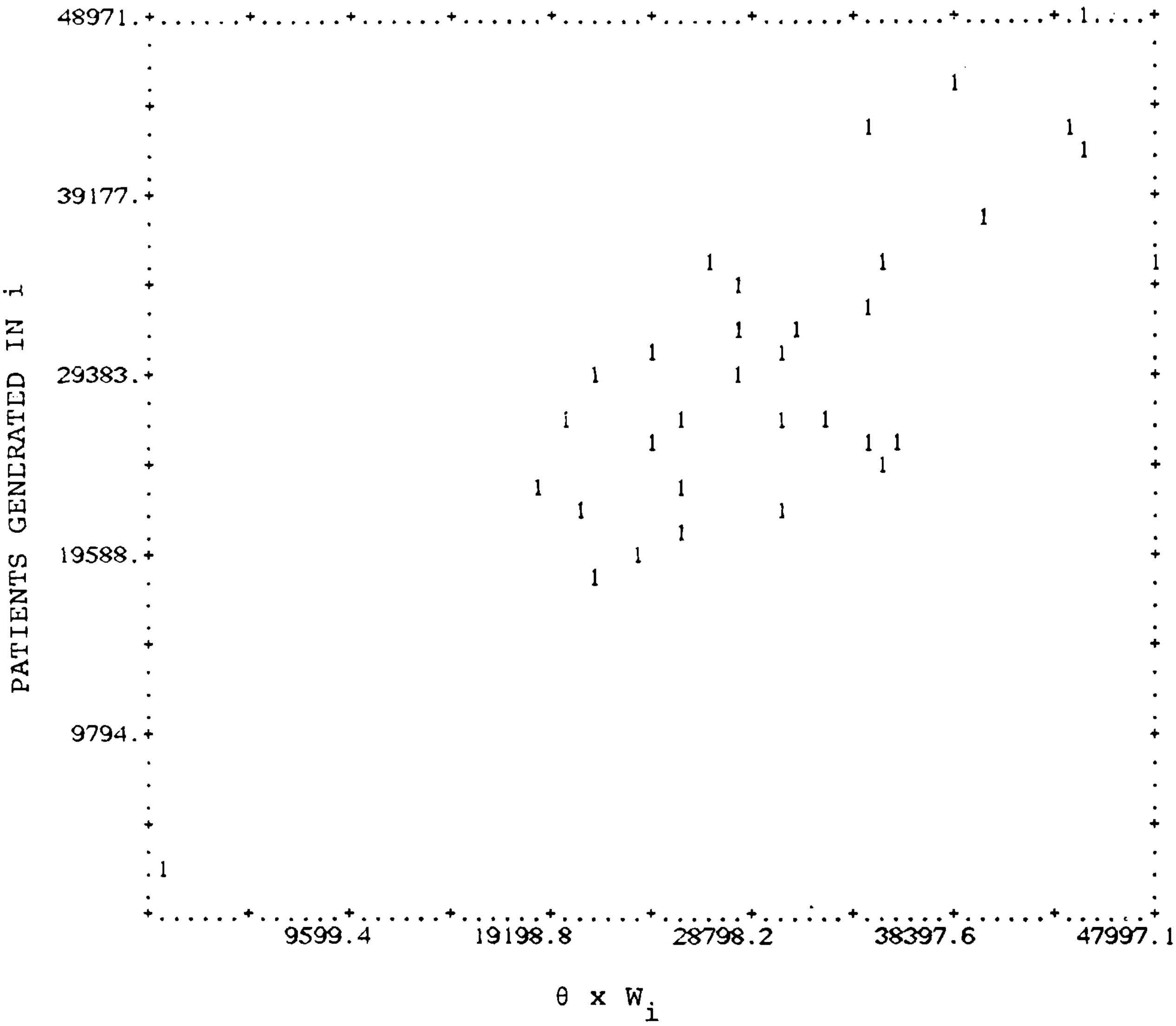


Figure 7. Plot of patients generated in i on the relative demand in i based on the distribution of demand potentials and resource configuration presumed in scenario 3. The correlation coefficient is 0.808, and the slope, 0.786. Conclusion: The test resource configuration is slightly poorer than that existing at present.

$$\alpha_j = \frac{V_j \; \gamma_j}{\hat{b}^D_j \left(\gamma_j + 1 - \lambda_j^{\;-\frac{\gamma_j}{\gamma_j + 1}} \right)} - 1 \qquad (37)$$

Ignoring the subscript j on V (the same value can be used everywhere) the above suggest

$$\tilde{V}_{min} > \max_j \left\{ \hat{b}^D_j , \;\; \hat{b}^1_j \right\} \qquad (51)$$

For reasons given in Gibbs (1978), a suitable value for $\tilde{\lambda}$ is approximately given by

$$\tilde{\lambda} \approx 2^c \qquad (52)$$

Computational experience here has shown this to be a rough though useful guideline. In solving $f(\lambda)=0$ for a new set of resources/ demands the convergence procedure usually reaches a solution in an average of about five iterations for each place of treatment j depending on the accuracy required. A sample set of iterations (from scenario 1) is shown in Table 6 as an illustration of the method. The tolerance value $\in$ in $f(\lambda) = 0 \pm \in$, has been set in this example to 0.005.

Table 6. Scenario 3: A typical iteration sequence in destination zones 9 to 11 to solve $f(\lambda_j)=0$ for one patient category.

zone	destination		iter no	f(lamda)	lamda	omega
10	kcw	nw	1	-.4761e+05	0.9000e+01	0.1333e+03
10	kcw	nw	2	0.1696e+05	0.5866e+01	0.6882e+02
10	kcw	nw	3	0.1165e+04	0.6465e+01	0.8001e+02
10	kcw	nw	4	0.6187e+01	0.6513e+01	0.8092e+02
10	kcw	nw	5	-.2558e-02	0.6513e+01	0.8093e+02

10	kcw	nw	5	-.2558e-02	0.6513e+01	0.8093e+02

theta = 0.19229e+02
fdash =-0.24301e+05
alpha = 0.25510e+01
gamma = 0.40000e+01

zone	destination		iter no	f(lamda)	lamda	omega
11	kcw	ne	1	-.3804e+05	0.9000e+01	0.5574e+02
11	kcw	ne	2	0.7991e+04	0.7179e+01	0.4094e+02
11	kcw	ne	3	0.2391e+03	0.7439e+01	0.4299e+02
11	kcw	ne	4	0.2337e+00	0.7448e+01	0.4305e+02
11	kcw	ne	5	-.2468e-02	0.7448e+01	0.4305e+02

11	kcw	ne	5	-.2468e-02	0.7448e+01	0.4305e+02

theta = 0.79079e+01
fdash =-0.28797e+05
alpha = 0.10410e+01
gamma = 0.23330e+01

zone	destination		iter no	f(lamda)	lamda	omega
12	kcw	s	1	-.7875e+05	0.9000e+01	0.1390e+03
12	kcw	s	2	0.3787e+05	0.5079e+01	0.6914e+02
12	kcw	s	3	0.3921e+04	0.5919e+01	0.8338e+02
12	kcw	s	4	0.4993e+02	0.6027e+01	0.8525e+02
12	kcw	s	5	0.1111e-01	0.6028e+01	0.8527e+02

12	kcw	s	5	0.1111e-01	0.6028e+01	0.8527e+02

theta = 0.17291e+02
fdash =-0.35382e+05
alpha = 0.20420e+01
gamma = 0.90000e+01

KEY

gamma = γ_j

alpha = α_j

theta = $\Theta_j = \lambda_j^{\alpha_i/\gamma_j+1}\left[\alpha_j + \gamma_j + 1.0 - \left(\frac{\alpha_j+1}{\gamma_j+1}\right)\lambda_j^{-\gamma_j/\gamma_j+1}\right]$

omega = $\Omega_j = (\gamma_j+1)\lambda_j^{(\alpha_j+\gamma_j+1)/(\gamma_j+1)} - \lambda_j^{(\alpha_j+1)/(\gamma_j+1)}$

$$f'_j(\lambda) = -\frac{L_j \sum_i \phi_{ij}}{(\alpha_j+1)} \gamma_j^{1/(\alpha_j+1)} \Omega_j^{-(\alpha_j+2)/(\alpha_j+1)} \Theta_j$$

j = destination zone (place of treatment)

6. CONCLUSIONS

In this paper two IIASA health care models, DRAM and RAMOS, have been merged to produce an enhanced resource allocation model called DRAMOS. This merger was achieved with very little modification to the theoretical structures of either model, implying a consistency in their assumptions and formulation. This is highly encouraging since both models were developed entirely independently and for completely different reasons. In the future it is hoped to test the new model on a larger set of data containing more than one patient category in order to develop more fully the potential of this approach.

Naturally, this merger has caused a very substantial increase in data requirements: for example, observations are generally required on the matrix $\{\overline{T}_{ijk}\}$, the observed patient flows between i and j in category k and this information may not be readily available in some health care systems. On the other hand, this work has shown that it is possible to take general information on resource availability, treatment standards, and population characteristics in a region and to form meaningful predictions concerning service standards at the local level of health care delivery. This is most important since it is at this level that the providers and consumers of health care alike generally guage and discuss the efficacy of the system, and at which models of this type can be usefully employed to show in a simple way the consequences of different actions in terms of population and treatment trends.

In the future, there are several interesting avenues of development for this model. They mainly involve improvements in the allocative submodels to take account of theoretical developments by Hughes and Wierzbicki (1980) in the DRAM component, and similarly, a greater recognition of recent advances that are relevant on the RAMOS side (Mayhew, 1980; Walsh and Gibberd, 1980; Leonardi, 1980).

REFERENCES

Aspden, P. (1980a) *The IIASA Health Care Resource Allocation Submodel: Model Calibration for Data from Czechoslovakia*, WP-80-53. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Aspden, P. (1980b) *The IIASA Health Care Resource Allocation Submodel: DRAM Calibration for Data from the South West Health Region, UK* , WP-80-115. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Feldstein, M.S. (1967) *Economic Analysis for Health Service Efficiency.* Amsterdam: North Holland.

Gibbs, R.J. (1978) *The IIASA Health Care Resource Allocation Model: Mark 1*, RR-78-8. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J. (1978) *The IIASA Health Care Resource Allocation Submodel: Estimation of Parameters*, RM-78-67. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J., and A. Wierzbicki (1980) *DRAM: A Model of Health Care Resource Allocation*, RR-80-23. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hyman, G.M. (1969) The Calibration of Trip Distribution Models. *Environment and Planning* 1:105-112.

Leonardi, G. (1980) *A Multiactivity Location Model with Accessibility-and Congestion-Sensitive Demand,* WP-80-124. Laxenburg, Austria: International Institute for Applied Systems Analysis.

LHPC (1979) *Acute Hospital Services in London.* A profile by the London Health Planning Consortium, Her Majesty's Stationery Office (HMSO), London, UK.

Mayhew , L.D. (1979) *The Theory and Practice of Urban Hospital Location.* Unpublished Ph.D. thesis. Department of Geography, Birkbeck College, University of London.

Mayhew, L.D. (1980) *The Regional Planning of Health Care Services: RAMOS and RAMOS^{-1},* WP-80-166. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Mayhew, L.D., and A. Taket (1980) *RAMOS: A Model of Health Care Resource Allocation in Space,* WP-80-125. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Pauly, M.V. (1981) *Adding Demand, Incentives, Disequilibrium, and Disaggregation to Health Care Models,* WP-81-4. Laxenburg, Austria: International Institute for Applied Systems Analysis.

RAWP (1976) *Sharing Resources for Health in England.* Report of the Resource Allocation Working Party. HMSO, London, UK.

Walsh, P.K., and R.W. Gibberd (1980) Developments of an entropy model for residential location with maximum zonal population contraints. *Environment and Planning* 12(11):1253-1266.

WHO (1978) *Financing of Health Services.* Technical Report Series. World Health Organization, Geneva.

Wilson, A.G. (1974) *Urban and Regional Models in Geography and Planning.* London: Wiley.

APPENDIX: Glossary of Main Terms used in DRAMOS

Variable Name	Notation*	Remarks
Accessibility costs	c_{ij}	Expresses the difficulty of someone in i obtaining treatment in j
Admission rate	A_j	Patients admitted in j per head of catchment population: $(\sum_k \sum_j T_{ijk}/CP_j)\times 10^3$
Alpha	α_{jk}	A parameter measuring the relative importance of treating the ideal number of individuals in j, category k
Balancing factor	B_{jk}	$= \left[\sum_i W_{ij} e^{-\beta_k c_{ij}} \right]^{-1}$
Catchment population	CP_j	Resident population dependent on treatment facilities in j: $\sum_i E_{ij} P_i$ where $E_{ij} = \sum_k T_{ijk} / \sum_k \sum_j T_{ijk}$
Cases treated	D_{jk}	Patient admissions in j, category k $(= \sum_i T_{ijk})$
Compound variable	μ_{jk}	$= \frac{1}{\gamma_{jk}} \left[(\beta_{jk}+1)\lambda_j^{\frac{\gamma_{jk}}{(\gamma_{jk}+1)}} - 1 \right]$

*Bars over variables in this paper indicate that actual observations are being discussed; hats indicate regression estimates.

Variable Name	Notation	Remarks
Constrained utility function	$H_j(T,l,\lambda)$	The utility function in that agents in the HCS seek to maximize given constraints on resource availability.
Empirical constants	$\hat{a}^D, \hat{a}^l$	Constants in the linear regressions: $\log \overline{l}_{jkt} = f(\log \overline{R}_{jt}) + u_{jt}$ and $\log \overline{D}_{jkt} = f(\log \overline{R}_{jk}) + z_{jt}$, where $\overline{l}$, $\overline{D}$ and $\overline{R}$ are observations in time t and u and z are stochastic error terms.
Empirical elasticity w.r.t length of stay	$\hat{b}^l_{jk}$	Input into the model to determine γ_{jk}. Coefficients in above regression equation (see empirical constants).
Empirical elasticity w.r.t patient admissions	$\hat{b}^D_{jk}$	Input into the model to determine α_{jk}. Coefficients in above regression equation (see empirical constants).
Gamma	γ_{jk}	A parameter measuring the relative importance of achieving the ideal length of stay, L_{jk}.
Gravity parameter	β_k	Behavioral parameter estimated from actual patient flows ($\overline{T}_{ijk}$) and accessibility costs ($\overline{c}_{ij}$) and potential demand ($\overline{W}_{ik}$).
Hospitalization rate	HR_i	The number of patients per head of population in i admitted to hospital: $= (\sum_k \sum_i T_{ijk}/P_i) \times 10^3$
Ideal patient flow	ϕ_{ijk}	The ideal number of patients generated in i, treated in j in category k.
Ideal treatment Standard	L_{jk}	The ideal length of hospital stay in j, category k.
Multiplier	λ_j	The Lagrange multiplier in $H_j\ (T,l,\lambda)$

Variable Name	Notation	Remarks
Patient category	k	A clinical specialty, disease category or combinations thereof, $k=\overline{1,K}$.
Patient generating factor	W_{ik}	An index of relative potential patient demand in i category k.
Place of residence	i	Zone i, $i = \overline{1, I}$; an administrative subset of the same region as i.
Place of treatment	j	Zone j, $j = \overline{1, J}$; a health district and subset of a region.
Predicted patient flow	T_{ijk}	Patients generated in i, treated in j, in patient category k.
Resource availability	R_j	Bed-days available in $j \quad (= \sum_k \sum_i T_{ijk} \, l_{jk})$.
Service/demand ratio	Θ	The ratio of total cases treated to total potential demand, $\sum_j D_j / \sum_i W_i$
Scaling factor	ω_{jk}	Scales the demand potential on j discounted by accessibility costs . A measure of the relative importance of j used for calculating ϕ_{ijk}.
Starting constants in calibration procedure	$\tilde{V}, \tilde{\lambda}$	Abitrary constants used in the parameter estimation process.
Time	t	See "empirical constants".
Treatment standards	l_{jk}	Length of hospital stay in j, patient category k, where $l = \{l_{jk}\}$.
Treatment costs	C_j	The marginal cost of a bed-day in each place of treatment.
Utility function	$U_j(T,l)$	The utility function in j that agents in the HCS seek to maximize.

RECENT PUBLICATIONS IN THE HEALTH CARE SYSTEMS TASK

Hughes, D.J. (1979) *A Model of the Equilibrium Between Different Levels of Treatment in the Health Care Systems: Pilot Version* (WP-79-15).

Fleissner, P. (1979) *Chronic Illnesses and Socio-Economic Conditions: The Finland Case 1964 and 1968* (WP-79-29).

Shigan, E.N., D.J. Hughes, and P. Kitsul (1979) *Health Care Systems Modeling at IIASA: A Status Report* (SR-79-4).

Rutten, F.F.H. (1979) *Physician Behaviour: The Key to Modeling Health Care Systems for Government Planning* (WP-79-60).

A Committee Report (1979) to IIASA by the participants in an Informal Meeting on *Health Delivery Systems in Developing Countries* (CP-79-10).

Shigan, E.N., P. Aspden, and P. Kitsul (1979) *Modeling Health Care Systems: June 1979 Workshop Proceedings* (CP-79-15).

Hughes, D.J., E. Nurminski, and G. Royston (1979) *Nondifferentiable Optimization Promotes Health Care* (WP-79-90).

Rousseau, J.M., R. J. Gibbs (1980) *A Model to Assist Planning the Provision of Hospital Services* (CP-80-3).

Fleissner, P., K. Fuchs-Kittowski, and D.J. Hughes (1980) *A Simple Sick-Leave Model used for International Comparison* (WP-80-42).

Aspden, P., R. Gibbs, and T. Bowen (1980) *DRAM Balances Care* (WP-80-43).

Aspden, P., and M. Rusnak (1980) *The IIASA Health Care Resource Allocation Submodel: Model Calibration for Data from Czechoslovakia* (WP-80-53).

Kitsul, P. (1980) *A Dynamic Approach to the Estimation of Morbidity* (WP-80-71).

Shigan, E.N., and P. Kitsul (1980) *Alternative Approaches to Modeling Health Care Demand and Supply* (WP-80-80).

Hughes, D.J., and A. Wierzbicki (1980) *DRAM: A Model of Health Care Resource Allocation* (RR-80-23).

Aspden, P. (1980) *The IIASA Health Care Resource Allocation Submodel: DRAM Calibration for Data from the South West Health Region, UK* (WP-80-115).

Mayhew, L., and A. Taket (1980) *RAMOS: A Model of Health Care Resource Allocation in Space* (WP-80-125).

Mayhew, L.D. (1980) *The Regional Planning of Health Care Services: RAMOS and* $RAMOS^{-1}$ (WP-80-166).

Pauly, M.V. (1981) *Adding Demand, Incentives, Disequilibrium, and Disaggregation to Health Care Models* (WP-81-4).