Working Paper

MODEL SCHEDULES IN MULTISTATE DEMOGRAPHIC ANALYSIS: THE CASE OF MIGRATION

Andrei Rogers
Luis J. Castro

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Prepared for presentation at the Conference on Multidimensional Demography, Washington, D.C., March 23-25, 1981

International Institute for Applied Systems Analysis
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PREFACE

Interest in human settlement systems and policies has been a central part of urban-related work at IIASA since its inception. From 1975 through 1978 this interest was manifested in the work of the *Migration and Settlement Task*, which was formally concluded in November 1978. Since then, attention has turned to dissemination of the Task's results and to the conclusion of its comparative study, which is carrying out a comparative quantitative assessment of recent migration patterns and spatial population dynamics in all of IIASA's 17 NMO countries.

This paper is part of the Task's dissemination effort and is the third of several to focus on the age patterns of migration exhibited in the data bank assembled for the comparative study. It begins with a comparative analysis of over 500 observed migration schedules and then develops, on the basis of this analysis, a family of hypothetical "synthetic" schedules for use in instances where migration data are unavailable or inaccurate.

Reports, summarizing previous work on migration and settlement at IIASA, are listed at the back of this paper. They should be consulted for further details regarding the data base that underlies this study. A technical appendix listing the parameters and variables of over 600 model migration schedules is available on request.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ACKNOWLEDGMENTS

The authors are grateful to the many national collaborating scholars who have participated in IIASA's Comparative Migration and Settlement Study. This paper could not have been written without the data bank produced by their collective efforts. Thanks also go to Richard Raquillet for his contributions to the early phases of this study and to Walter Kogler for his untiring efforts on our behalf in front of a console in IIASA's computer center.

ABSTRACT

This paper draws on the fundamental regularity exhibited by age profiles of migration all over the world to develop a system of hypothetical "synthetic" model migration schedules that can be used to carry out multiregional population analyses in countries that lack adequate migration data.

CONTENTS

MODEL SCHEDULES IN MULTISTATE DEMOGRAPHIC ANALYSIS: THE CASE OF MIGRATION

1. INTRODUCTION

The age-specific fertility and mortality schedules of most human populations exhibit remarkably persistent regularities; consequently demographers have found it possible to summarize and codify such regularities by means of hypothetical schedules called *model* schedules. Although the development of model fertility and mortality schedules has received considerable attention, the construction of model migration schedules has not, even though the techniques that have been successfully applied to treat the former can readily be extended to deal with the latter. The same may be said of model schedules of labor force entry and exit, and of marriage, divorce, and remarriage.*

In this paper we consider the notion of model multistate schedules, focusing in particular on the development of a family of model migration schedules for use in situations where the available migration data are inadequate or inaccurate. We begin by examining regularities in age profile that are exhibited by empirical schedules of migration rates. Expressing this regularity in a mathematical form called a model migration schedule,

*There are a few notable exceptions, however, such as the paper on model divorce schedules by Krishnan and Kayani (1973).

we go on to examine the patterns of variation that occur in a large data bank of such schedules. Drawing on this comparative analysis, we then outline two alternative approaches for generating families of hypothetical "synthetic" model migration schedules and conclude that further work is needed if such approaches are to be of practical use in migration studies carried out in Third World population settings.

2. MIGRATION AGE PATTERNS

Migration measurement can usefully apply concepts borrowed from both mortality and fertility analysis, modifying them where necessary to take into account aspects that are peculiar to spatial mobility. From mortality analysis, migration studies can borrow the notion of the life table, extending it to include increments as well as decrements, in order to reflect the mutual interaction of several regional cohorts (Rogers, 1973a, b, and 1975; Rogers and Ledent, 1976). From fertility analysis, migration studies can borrow well-developed techniques for graduating age-specific schedules (Rogers, Raquillet, and Castro, 1978). Fundamental to both "borrowings" is a workable definition of the migration rate.

2.1 Migration Rates and Migration Rate Schedules

During the course of a year, or some such fixed interval of time, a number of individuals living in a particular community change their regular place of residence. Let us call such persons *mobiles* to distinguish them from those individuals who have not changed their place of residence, i.e., the *non-mobiles*. Some of the mobiles will have moved to a new community of residence; others will simply have transferred their household to another residence within the same community. The former may be called *movers*, the latter, *relocators*. A few in each category will have died before the end of the unit time interval.

Assessing the situation with respect to the start and the end of the unit time interval, we may divide movers who *survived* to the end of the interval into two groups: those living in the same community of residence as at the start of the interval and those living elsewhere. The first group of movers will be referred to as *surviving returnees*, the second will be called *surviving migrants*. An analogous division may be made of movers who died before the end of the interval to define *nonsurviving returnees* and *nonsurviving migrants*.

A *move*, then is an *event*: a separation from a community. A *mover* is an individual who has made a move at least once during a given interval of time. A *migrant* (i.e., a surviving or nonsurviving migrant), on the other hand, is an individual who at the end of a given time interval no longer inhabits the same community of residence as at the start of the interval. (The act of separation from one state is linked with an addition to another state.) Thus paradoxically, a multiple mover may be a nonmigrant by our definition; that is, if a particular mover returns to the initial place of residence before the end of the unit time interval, no "migration" is said to have taken place.*

The simplest and most common measure of migration is the crude migration rate, defined as the ratio of the *number of migrants*, leaving a particular population located in space and time, to the average *number of persons* (more exactly, the number of *person-years*) exposed to the risk of becoming migrants.**

Because migration is highly age selective, with a large fraction of migrants being the young, our understanding of migration patterns and dynamics is aided by computing migration rates for each single year of age. Summing these rates over all ages of life gives the *gross migraproduction rate* (GMR), the migration analog of fertility's gross reproduction rate.

Figure 2.1 indicates that age-specific annual rates of residential mobility among whites and blacks in the U.S. during 1966-1971 exhibited a common *profile*. Mobility rates among infants and young children mirrored the relatively high rates of their parents, young adults in their late twenties. The mobility of adolescents was lower but exceeded that of young

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*We define migration to be the transition between states experienced by a migrant.

**Because data on nonsurviving migrants are generally unavailable, the numerator in this ratio generally excludes them.

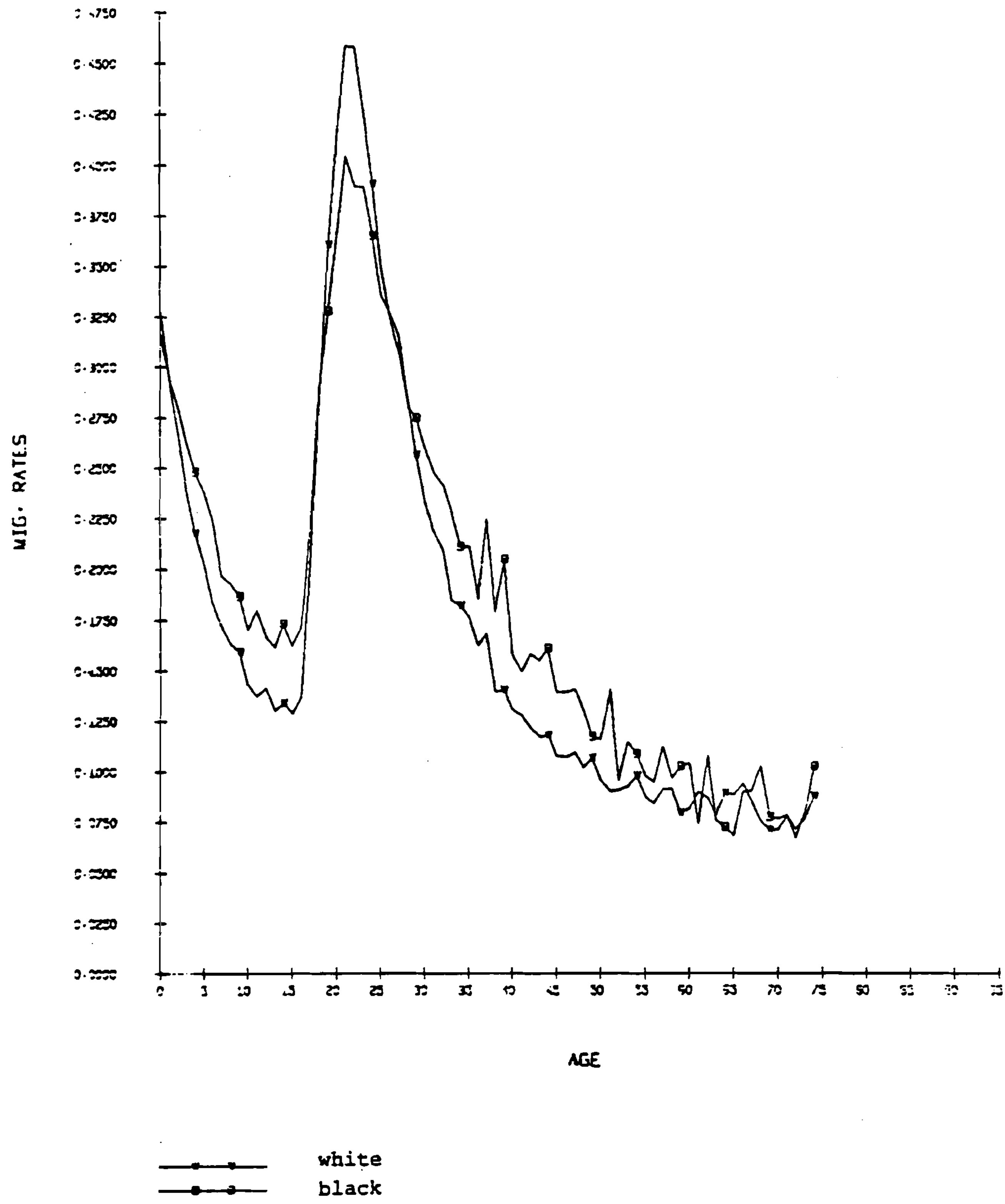
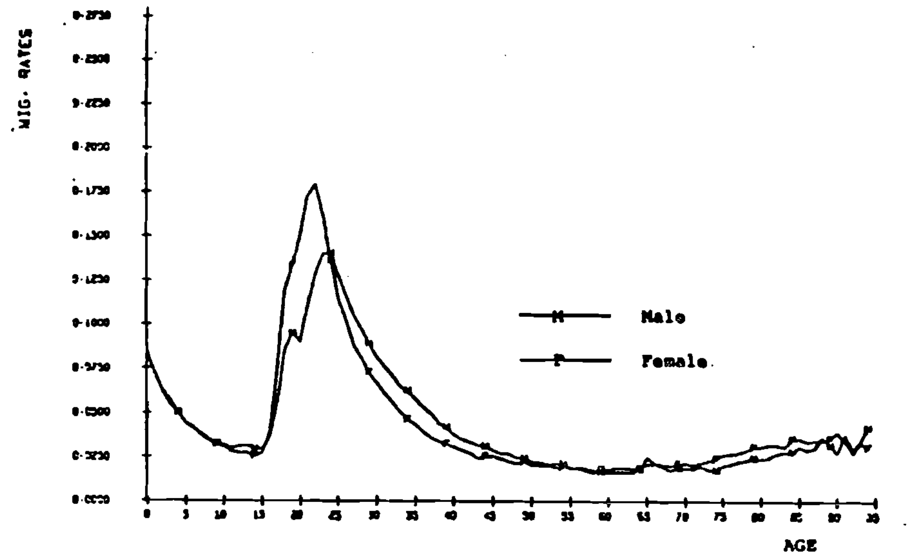


Figure 2.1 Observed annual migration rates by color and single years of age: the United States, 1966-1971.

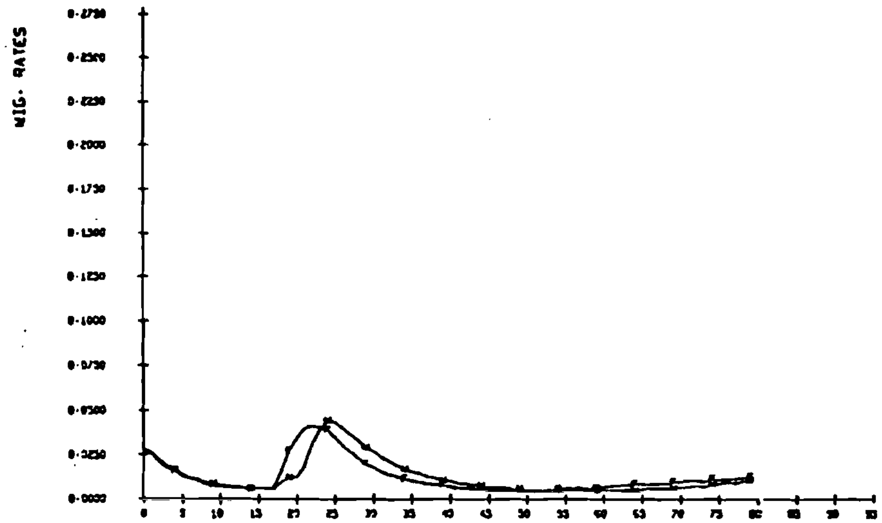
teens, with the latter showing a local *low point* around age fifteen. Thereafter mobility rates increased, attaining a *high peak* at about age twenty two and then declining monotonically with age to the ages of retirement. The mobility *levels* of both whites and blacks were roughly similar, with whites showing a gross migraproduction rate of about 14 moves and blacks one of approximately 15 over a lifetime undisturbed by mortality before the end of the mobile ages.

Although it has been frequently asserted that migration is strongly sex selective, with males being more mobile than females, recent research indicates that sex selectivity is much less pronounced than age selectivity and that it is less uniform across time and space. Nevertheless, because most models and studies of population dynamics distinguish between the sexes, most migration measures do also.

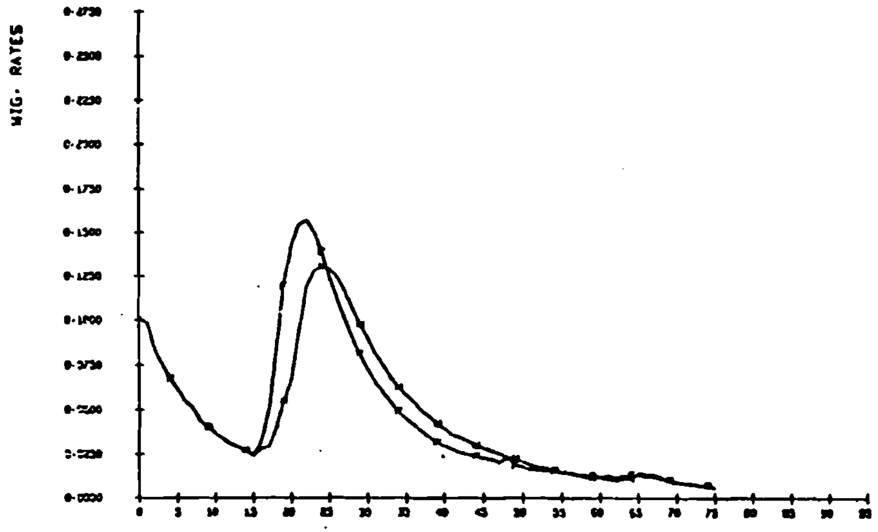
Figure 2.2 illustrates the age profiles of male and female migration schedules in four different countries at about the same point in time between roughly comparable areal units: communes in the Netherlands and Sweden, voivodships in Poland, and counties in the U.S. The migration levels for all but Poland are similar, varying between 3.5 and 5.3 moves per lifetime; and the levels for males and females are roughly the same. The age profiles, however, show a distinct, and consistent, difference. The high peak of the female schedule always precedes that of the male schedule by an amount that appears to approximate the difference between the average ages at marriage of the two sexes.



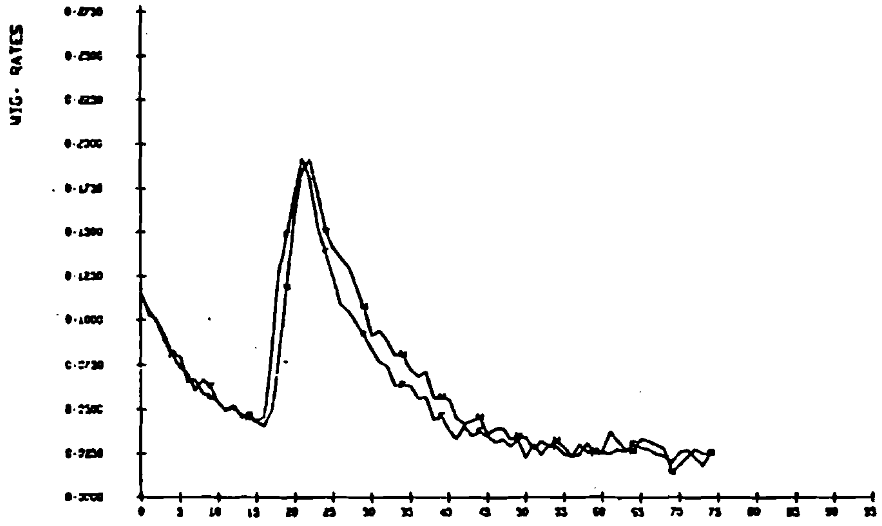
A. The Netherlands, 1972



B. Poland, 1973



C. Sweden, 1968-1973



D. The United States, 1966-1971

Figure 2.2 Observed annual migration rates by sex and single years of age: the Netherlands, Poland, Sweden, and the United States, around 1970.*

*Intercommunal migration in the Netherlands and Sweden; intervoivodship migration in Poland; intercounty migration in the United States.

Under normal statistical conditions, point-to-point movements are aggregated into streams between one civil division and another; consequently, the level of interregional migration depends on the size of the areal unit selected. Thus, if the areal unit chosen is a minor civil division such as a county or a commune, a greater proportion of residential location will be included as migration than if the areal unit chosen is a major civil division such as a state or a province.

Figure 2.3 presents the age profiles of female mobility and migration schedules as measured by different sizes of areal units: 1) all moves from one residence to another, 2) changes of residence within county boundaries, 3) migration between counties, and 4) migration between states. The respective four gross migraproduction rates (GMRs) are 14.3, 9.3, 5.0, and 2.5, respectively. The four age profiles appear to be remarkably similar, indicating that the regularity in age pattern persists across areal delineations of different size.

Finally, migration occurs over time as well as across space; therefore, studies of its patterns must trace its occurrence with respect to a time interval, as well as over a system of geographical areas. In general, the longer the time interval, the larger will be the number of return movers and nonsurviving migrants and, hence, the more the count on *migrants* will understate the number of inter-area *movers* (and, of course, also of *moves*). Philip Rees, for example, after examining the ratios of one-year to five-year migrants between the Standard Regions of Great Britain, found that

> the number of migrants recorded over five years in an interregional flow varies from four times to two times the number of migrants recorded over one year. (Rees, 1977, p.247).

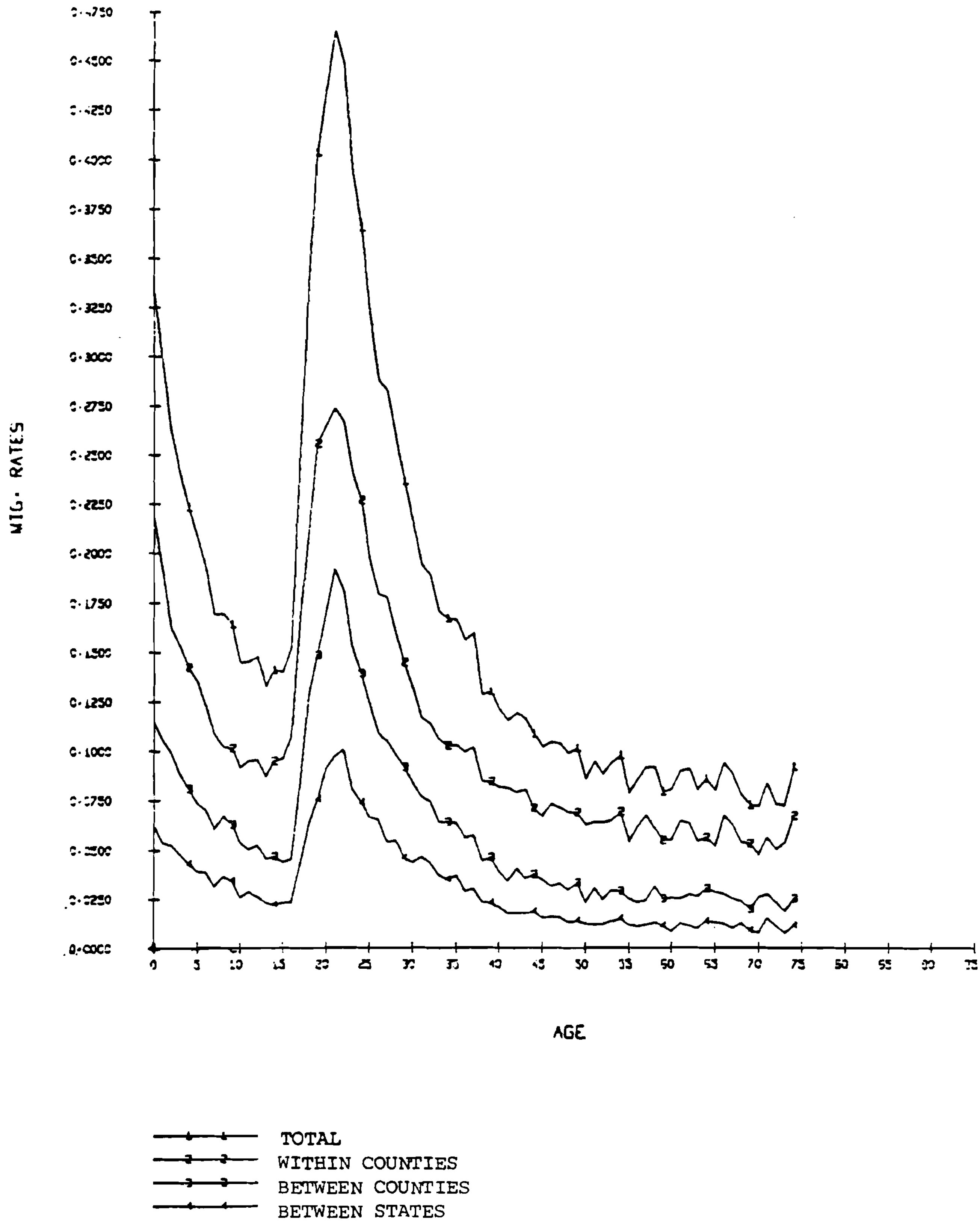


Figure 2.3 Observed female annual migration rates by levels of areal aggregation and single years of age: the United States, 1966-1971.

2.2 Model Migration Rate Schedules

It appears that the most prominent regularity found in empirical schedules of age-specific migration rates is the selectivity of migration with respect to age. Young adults in their early twenties generally show the highest migration rates and young teenagers the lowest. The migration rates of children mirror those of their parents; hence the migration rates of infants exceed those of adolescents. Finally, migration streams directed toward regions with warmer climates and into or out of large cities with relatively high levels of social services and cultural amenities often exhibit a "retirement peak" at ages in the mid-sixties or beyond.

Figure 2.4 illustrates a typical *observed* age-specific migration schedule (the jagged outline) and its graduation by a *model* schedule (the superimposed smooth outline) defined as the sum of four components:

1) a single negative exponential curve of the *pre-labor force* ages, with its rate of descent, α_1
2) a left-skewed unimodal curve of the *labor force* ages positioned at μ_2 on the age axis and exhibiting rates of ascent, λ_2, and descent, α_2.
3) an almost bell-shaped curve of the *post-labor force* ages positioned at μ_3 on the age axis and exhibiting rates of ascent, λ_3, and descent, α_3
4) a constant curve, c, the inclusion of which improves the quality of fit provided by the mathematical expression of the schedule

The decomposition described above suggests the following simple sum of four curves (Rogers, Raquillet, and Castro, 1978):*

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*Both the labor force and the post-labor force components in equation (1) are described by the "double exponential" curve formulated by Coale and McNeil (1972) for their studies of nuptiality and fertility.

$$M(x) = a_1 e^{-\alpha_1 x} + a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}} + a_3 e^{-\alpha_3 (x-\mu_3) - e^{-\lambda_3 (x-\mu_3)}} + c \quad , \; x = 0,1,2,\ldots,z \qquad (1)$$

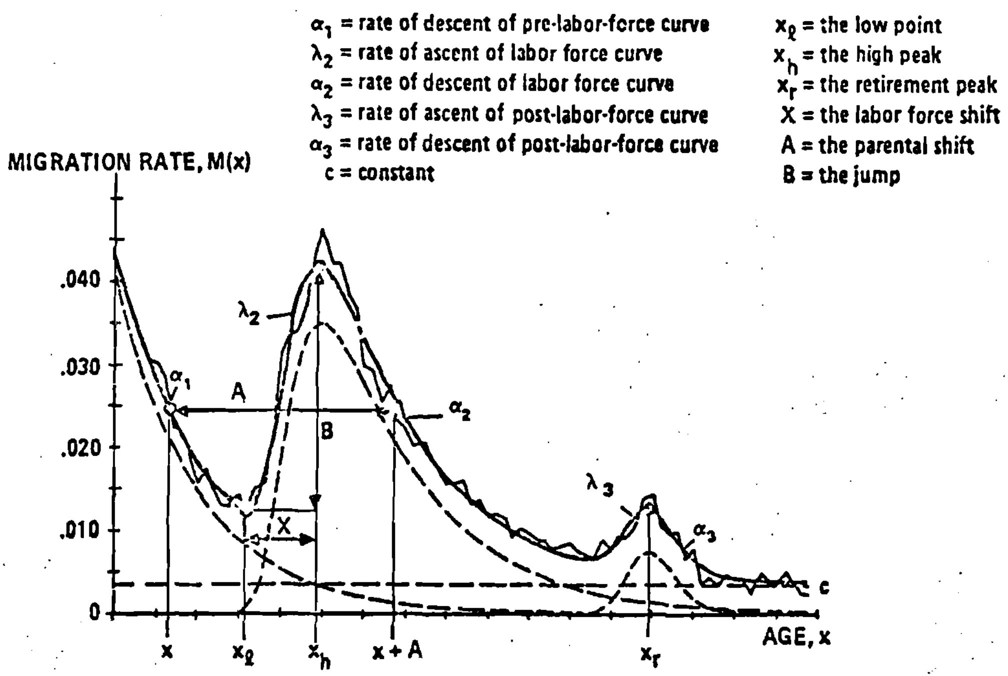


Figure 2.4 The model migration schedule.

The "full" model schedule in equation (1) has eleven parameters: a_1, α_1, a_2, μ_2, α_2, λ_2, a_3, μ_3, α_3, λ_3, and c. The *profile* of the full model schedule is defined by seven of the eleven parameters: α_1, μ_2, α_2, λ_2, μ_3, α_3, and λ_3. Its *level* is determined by the remaining four parameters: a_1, a_2, a_3, and c. A change in the value of the gross migraproduction rate of a particular model schedule alters proportionally the values of the latter but does not affect the former. However, as we shall see in the next section, certain aspects of the profile also depend on the allocation of the schedule's level among the labor, pre-labor, and post-labor force age components, and on the share of the total level accounted for by the constant term, c. Finally, migration schedules without a retirement peak may be represented by a "reduced" model with seven parameters, because in such instances the third component of equation (1) is omitted.

Table 2.1 sets out illustrative values of the basic and derived measures presented in Figure 2.4. The data refer to 1974 migration schedules for an eight-region disaggregation of Sweden (Andersson and Holmberg, 1980). The method chosen for fitting the model schedule to the data is a functional-minimization procedure known as the modified Levenberg-Marquardt algorithm.* Minimum chi-square estimators are used instead of least squares estimators. The differences between the two parametric estimates tend to be small, and because the former give more weight to age groups with smaller rates of migration, we use minimum chi-square estimators in the remainder of the paper.

To assess the quality of fit that the model schedule provides when it is applied to observed data, we calculate the "mean absolute error as a percentage of the observed mean":

$$E = \frac{\frac{1}{n}\sum_x \left|\hat{M}(x) - M(x)\right|}{\frac{1}{n}\sum_x M(x)} \cdot 100 \quad . \tag{2}$$

*See Appendix A and Brown and Dennis (1972), Levenberg (1944), and Marquardt (1963).

Table 2.1 Parameters and variables defining observed model migration schedules: Swedish regions, 1974.

Parameters and Variables	1. Stockholm M	F	2. East Middle-Sweden M	F	3. South Middle-Sweden M	F	4. South M	F
GMR*	1.45	1.43	1.44	1.48	1.33	1.41	0.87	0.84
a_1	.033	.041	.035	.039	.032	.033	.025	.021
α_1	.097	.091	.088	.108	.096	.106	.117	.104
a_2	.059	.067	.079	.096	.091	.112	.066	.067
μ_2	20.80	19.32	20.27	18.52	19.92	18.49	21.17	19.88
α_2	.077	.094	.090	.109	.104	.127	.115	.129
λ_2	.374	.369	.406	.491	.404	.560	.269	.442
a_3	.000	.000						
μ_3	76.55	85.01						
α_3	.776	.369						
λ_3	.145	.072						
c	.003	.003	.003	.004	.003	.004	.002	.002
$\bar{n}$	31.02	29.54	29.17	28.38	28.29	27.96	28.26	28.14
%(0-14)	25.61	25.95	22.81	22.59	21.40	20.67	22.76	21.93
%(15-64)	64.49	65.10	70.38	69.48	72.47	71.73	70.73	70.76
%(65+)	9.90	8.94	6.81	7.94	6.13	7.60	6.51	7.31
δ_{1c}	13.56	13.06	12.14	9.79	12.26	8.90	13.27	9.93
δ_{12}	.716	.604	.446	.403	.350	.293	.377	.312
δ_{32}	.003	.003						
β_{12}	1.26	.977	.981	.993	.921	.883	1.02	.809
σ_2	4.86	3.94	4.52	4.49	3.88	4.40	2.34	3.43
σ_3	.187	.196						
x_ℓ	16.39	14.81	15.92	14.80	15.41	15.07	14.52	15.61
x_h	24.68	22.70	23.78	21.46	23.12	21.06	24.16	22.58
x_r	64.80	61.47						
X	8.29	7.89	7.86	6.66	7.71	5.99	9.64	6.97
A	27.87	25.49	29.99	27.32	29.93	27.27	29.90	27.87
B	.029	.030	.040	.022	.044	.059	.026	.032

*The GMR, its percentage distribution across the three major age categories (i.e., 0-14, 15-64, 65+), and the mean age, $\bar{n}$, all are calculated with a model schedule spanning an age range of 95 years.

Table 2.1 Parameters and variables defining observed model migration schedules: Swedish regions, 1974 (cont.)

Parameters and Variables	5. West M	F	6. North Middle-Sweden M	F	7. Lower North-Sweden M	F	8. Upper North-Sweden M	F
GMR	0.80	0.82	1.22	1.33	1.33	1.46	1.03	1.24
a_1	.021	.022	.031	.027	.034	.031	.024	.023
α_1	.090	.106	.104	.102	.123	.119	.135	.128
a_2	.046	.055	.084	.116	.109	.141	.079	.116
μ_2	20.36	19.36	19.75	18.18	19.62	17.93	19.47	17.62
α_2	.091	.114	.103	.139	.118	.148	.114	.143
λ_2	.416	.442	.437	.561	.427	.701	.449	.711
c	.001	.002	.002	.004	.003	.004	.003	.004
$\bar{n}$	28.49	28.39	28.09	28.17	28.24	27.93	29.91	28.99
%(0-14)	23.54	23.18	21.52	19.40	19.84	18.26	18.29	16.40
%(15-64)	70.34	69.03	72.51	72.45	73.61	73.65	73.46	74.56
%(65+)	6.12	7.79	5.97	8.15	6.55	8.09	8.25	9.04
δ_{1c}	14.42	10.11	13.34	7.27	11.38	7.41	8.29	5.84
δ_{12}	.457	.395	.369	.237	.310	.219	.305	.198
β_{12}	.979	.926	1.00	.730	1.04	.801	1.19	.890
σ_2	4.55	3.87	4.23	4.03	3.63	4.74	3.95	4.95
x_ℓ	16.11	15.23	15.56	14.71	15.19	15.07	15.21	14.77
x_h	23.80	22.30	22.93	20.60	22.56	20.12	22.47	19.85
X	7.69	7.07	7.37	5.89	7.37	5.05	7.26	5.08
A	29.57	27.42	29.92	27.01	30.15	26.94	31.61	28.30
B	.023	.027	.042	.059	.053	.077	.040	.063

This measure indicates that the fit of the model to the Swedish data is reasonably good, the eight indices of goodness-of-fit being 6.87, 6.41, 12.15, 11.01, 9.31, 10.77, 11.74, and 14.82, for males and 7.30, 7.23, 10.71, 8.78, 9.31, 11.61, 11.38, and 13.28 for females. Figure 2.5 illustrates graphically this goodness-of-fit of the model schedule to the observed regional migration data for Swedish females.

Model migration schedules of the form specified in equation (1) may be classified into *families* according to the ranges of values taken on by their principal parameters. For example, we may order schedules according to their migration *levels* as defined by the values of the four level parameters in equation (1), i.e., a_1, a_2, a_3, and c (or by their associated gross migraproduction rates). Alternatively, we may distinguish schedules with a retirement peak from those without one, or we may refer to schedules with relatively low or high values for the rate of ascent λ_2 or the mean age $\bar{n}$. In many applications, it is also meaningful to characterize migration schedules in terms of several of the fundamental measures illustrated in Figure 2.4, such as the *low point,* x_ℓ, the *high peak,* x_h, and the *retirement peak,* x_r. Associated with the first pair of points is the labor force shift, X, which is defined to be the difference in years between the ages of the high peak and the low point, i.e., $X = x_h - x_\ell$. The increase in the migration rate of individuals aged x_h over those aged x_ℓ will be called the *jump,* B.

The close correspondence between the migration rates of children and those of their parents suggests another important shift in observed migration schedules. If, for each point x on the post-high-peak part of the migration curve, we obtain (where it exists) by interpolation the age, $x - A_x$ say, with the identical rate of migration on the pre-low-point part of the migration curve, then the average of the values of A_x, calculated incrementally for the number of years between zero and the low-point x_ℓ, will be defined to be the observed *parental shift,* A.

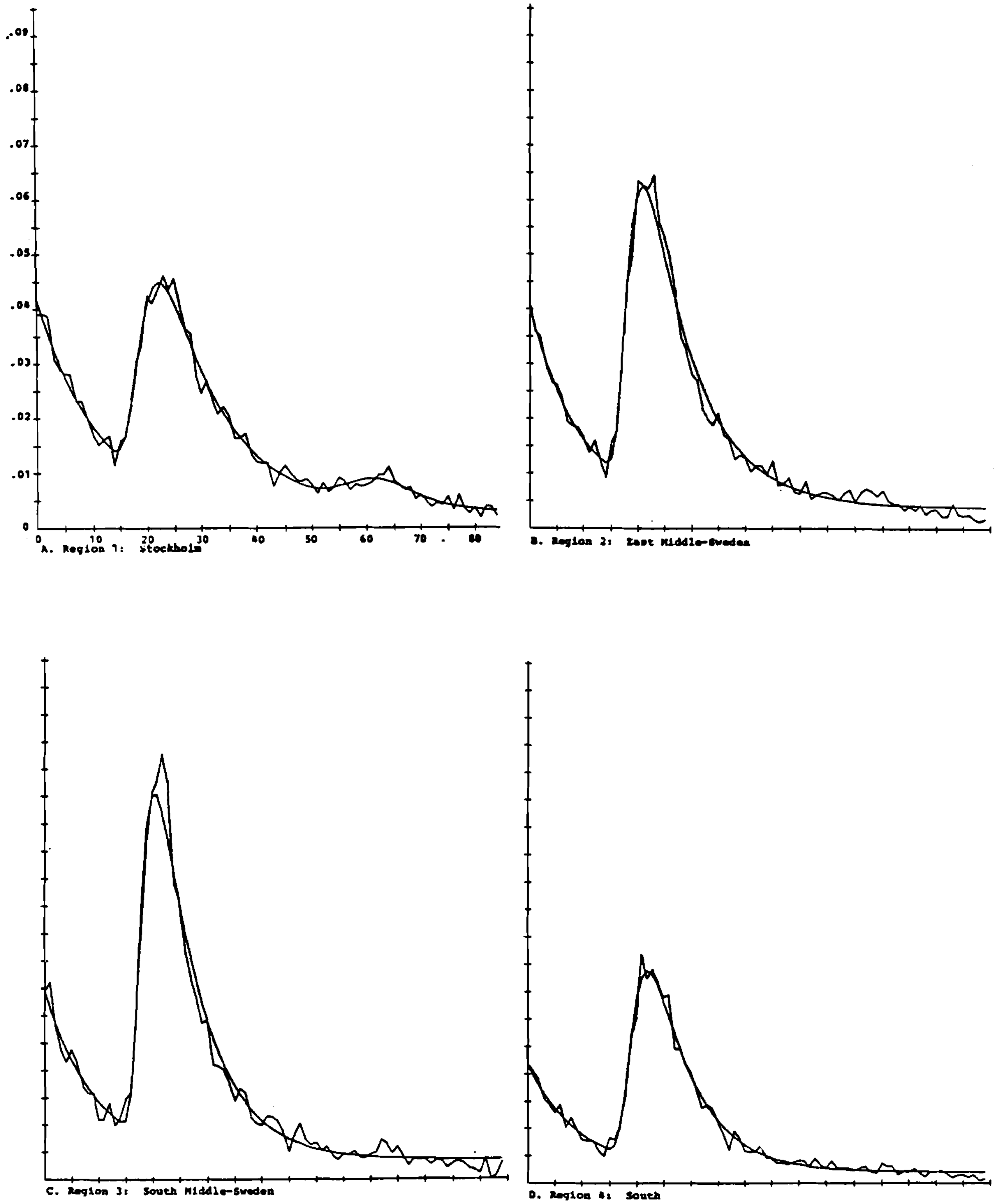


Figure 2.5 Observed and model migration schedules: females, Swedish regions, 1974.

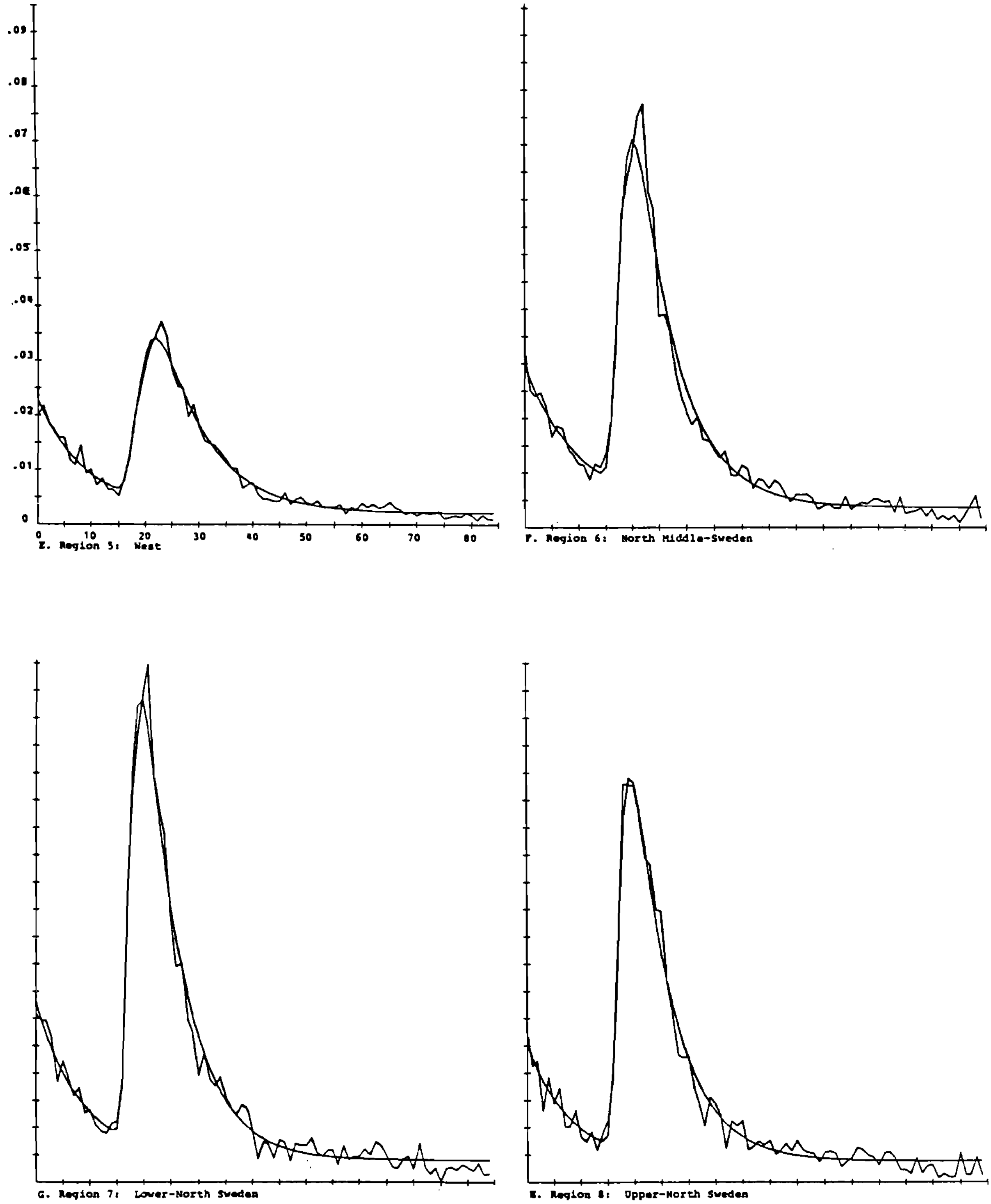


Figure 2.5 Observed and model migration schedules: females, Swedish regions, 1974 (continued).

An observed (graduated) age-specific migration schedule may be described in a number of useful ways. For example, references may be made to the heights at particular ages, to locations of important peaks or troughs, to slopes along the schedule's age profile, to ratios between particular heights or slopes, to areas under parts of the curve, and to both horizontal and vertical distances between important heights and locations. The various descriptive measures characterizing an age-specific model migration schedule may be conveniently grouped into the following categories and sub-categories:

Basic measures (the 11 fundamental parameters and their ratios)

heights : a_1, a_2, a_3, c

locations: μ_2, μ_3

slopes : $\alpha_1, \alpha_2, \lambda_2, \alpha_3, \lambda_3$

ratios : $\delta_{1c} = a_1/c,\ \delta_{12} = a_1/a_2,\ \delta_{32} = a_3/a_2,$

$\beta_{12} = \alpha_1/\alpha_2,\ \sigma_2 = \lambda_2/\alpha_2,\ \sigma_3 = \lambda_3/\alpha_3$

Derived measures (properties of the model schedule)

areas : GMR, %(0-14), %(15-64), %(65+)

locations: $\bar{n}, x_\ell, x_h, x_r$

distances: X, A, B

A convenient approach for characterizing an observed *model* migration schedule [i.e., an empirical schedule graduated by equation (1)] is to begin with the central labor force curve and then to "add-on" the pre-labor and post-labor force components and the constant component. This approach is represented graphically in Figure 2.6.

One can imagine describing a decomposition of the model migration schedule along the vertical and horizontal dimensions, e.g., allocating a fraction of its level to the constant component and then dividing the remainder among the other three (or two) components. The ratio $\delta_{1c} = a_1/c$ measures the former allocation, and $\delta_{12} = a_1/a_2$ and $\delta_{32} = a_3/a_2$ reflect the latter division.

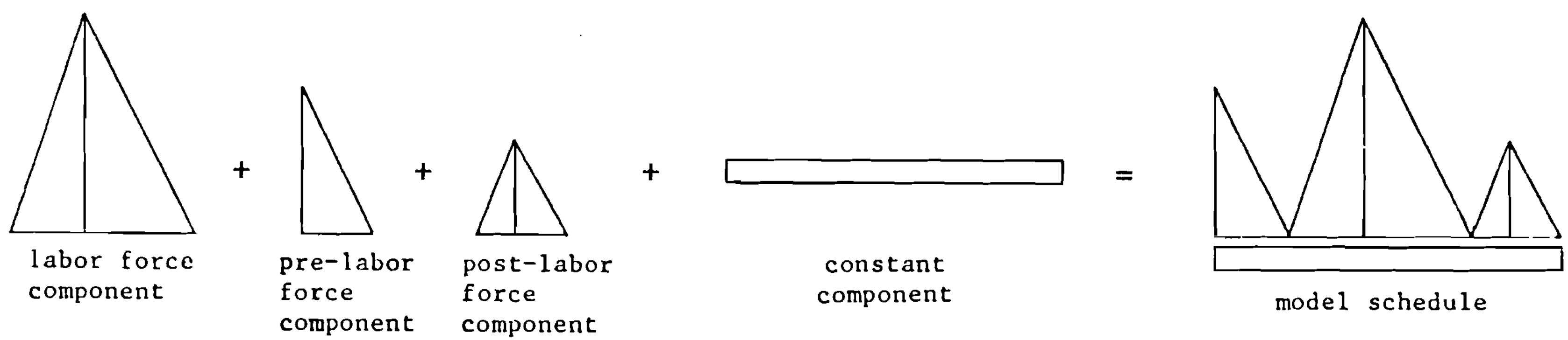
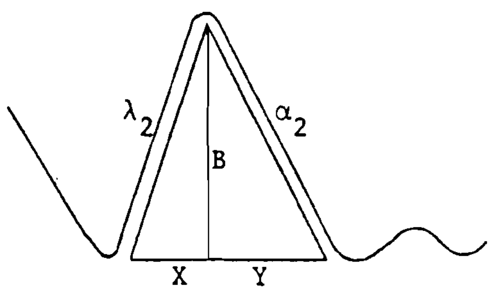


Figure 2.6 Schematic diagram of the fundamental components of the model migration schedule.

The heights of the labor force and pre-labor force components are reflected in the parameters a_2 and a_1, respectively, therefore the ratio a_2/a_1 indicates the degree of "labor dominance", and its reciprocal, $\delta_{12} = a_1/a_2$, the *index of child dependency*, measures the *level* at which children migrate with their parents. Thus the lower the value of δ_{12}, the lower is the degree of child dependency exhibited by a migration schedule and, correspondingly, the greater is its labor dominance. This suggests a dichotomous classification of migration schedules into *child dependent* and *labor dominant* categories.

An analogous argument applies to the post-labor force curve, and $\delta_{32} = a_3/a_2$ suggests itself as the appropriate index. However it will be sufficient for our purposes to rely simply on the value taken on by the parameter λ_3, with positive values pointing out the presence of a retirement peak and a zero value indicating its absence. High values of λ_3 will be interpreted as identifying *retirement dominance*.

Labor dominance reflects the relative migration levels of those in the working ages relative to those of children and pensioners. *Labor asymmetry* refers to the shape of the skewed bell-shaped curve describing the profile of labor-force-age migration. Imagine that a perpendicular line, connecting the high peak with the base of the bell-shaped curve (i.e., the jump, B), divides the base into two segments X and Y as, for example, in the schematic diagram:



Clearly, the ratio Y/X is an indicator of the degree of asymmetry of the curve. A more convenient index, using only two parameters of the model schedule is the ratio $\sigma_2 = \lambda_2/\alpha_2$, the *index of labor asymmetry*, Its movement is highly correlated with that of Y/X, because of the approximate relation:

$$\sigma_2 = \frac{\lambda_2}{\alpha_2} \sim \frac{B}{X} \div \frac{B}{Y} = \frac{Y}{X} \quad ,$$

where $\sim$ denotes proportionality. Thus σ_2 may be used to classify migration schedules according to their degree of labor asymmetry.

Again, an analogous argument applies to the post-labor force curve, and $\sigma_3 = \lambda_3/\alpha_3$ may be defined to be the *index of retirement asymmetry*.

When "adding-on" a pre-labor force curve of a given *level*. to the labor force component, it is also important to indicate something of its *shape*. For example, if the migration rates of children mirror those of their parents, then α_1 should be approximately equal to α_2, and $\beta_{12} = \alpha_1/\alpha_2$, the *index of parental-shift regularity*, should be close to unity.

The Swedish regional migration patterns described in Figure 2.5 and in Table 2.1 may be characterized in terms of the various basic and derived measures defined above. We begin with the observation that the outmigration levels in all of the regions are similar, ranging from a low of 0.80 for males in Region 5 to a high of 1.48 for females in Region 2. This similarity permits a reasonably accurate visual assessment and characterization of the profiles in Figures 2.5.

Large differences in gross migraproduction rates give rise to slopes and vertical relationships among schedules that are non-comparable when examined visually. Recourse then must be made to a standardization of the areas under the migration curves, for example, a general re-scaling to a GMR of unity. Note that this difficulty does not arise in the numerical data in Table 2.1 because, as we pointed out earlier, the principal slope and location parameters and ratios used to characterize the schedules are not affected by changes in levels. Only heights, areas, and vertical distances, such as the jump, are level-dependent measures.

Among the eight regions examined, only the first two exhibit a definite retirement peak, the male peak being the more dominant one in each case. The index of child dependency is highest in Region 1 and lowest in Region 8, distinguishing the latter region's labor dominant profile from Stockholm's child dependent outmigration pattern. The index of labor asymmetry varies from a low of 2.34, in the case of males in Region 4 to a high of 4.95 for the female outmigration profile of Region 8. Finally, with the possible exception of males in Region 1 and females in Region 6, the migration rates of children in Sweden do indeed seem to mirror those of their parents. The index of parental-shift regularity is 1.26 in the former case and .730 in the latter; for most of the other schedules it is close to unity.

3. A COMPARATIVE ANALYSIS

The preceding section demonstrated that age-specific rates of migration exhibit a fundamental age profile that can be expressed in mathematical form called a model migration schedule, which is defined by a total of 11 parameters. In this section we seek to establish the ranges of values typically assumed by each of these parameters and their associated derived variables. This exercise is made possible by the availability of a relatively large data base collected by the Comparative Migration and Settlement Study, recently concluded at the International Institute for Applied Systems Analysis (IIASA) in Laxenburg, Austria (Rogers, 1976a, 1976b, 1978; Rogers and Willekens, 1978, and Willekens and Rogers, 1978). The migration data for each country included in this study are set out in the individual national reports.

3.1 Data Preparation, Parameter Estimation, and Summary Statistics

The age-specific migration rates that were used to demonstrate the fits of the model migration schedule in the last section were single-year rates. Such data are very scarce at the regional level and, in our comparative analysis, are available only for Sweden. All other region-specific migration data are reported for five-year age groups only and, therefore, must be interpolated to provide the necessary input data by single years of age. In all such instances the region-specific migration schedules were first scaled to a gross migraproduction rate of unity (GMR = 1) before being subjected to a cubic spline interpolation (McNeil, Trussell, and Turner, 1977).

Starting with a migration schedule with a GMR of unity and rates by single years of age, the nonlinear parameter estimation algorithm ultimately yields a set of estimates for the model schedule's parameters.* Table 2.1 in Section 2 presented the results that were obtained using the data for Sweden. Since these data were available for single years of age, the influence of the interpolation procedure could be assessed. Table 3.1

*See Appendix A for details.

contrasts the estimates for female schedules in Table 2.1 with those obtained when the same data are first aggregated to five year age groups and then disaggregated to single years of age by a cubic spline interpolation. A comparison of the parameter estimates indicates that the interpolation procedure gives generally satisfactory results.

Table 3.1 refers to results for rates of migration from each of eight regions to the rest of Sweden. If these rates are disaggregated by region of destination, then $8^2 = 64$ interregional schedules need to be examined for each sex, complicating comparisons across several nations. To resolve this difficulty we shall associate a "typical" schedule with each collection of national rates by calculating the mean of each parameter and derived variable. Table 3.2 illustrates the results for the Swedish data.

To avoid the influence of unrepresentative "outlier" observations in the computation of averages defining the typical national schedule, it was decided to delete approximately 10 percent of the "extreme" schedules. Specifically, the parameters and derived variables were ordered from low value to high value; the lowest 5 percent and the highest 5 percent were defined to be extreme values. Schedules with the largest number of low and high extreme values were discarded, in sequence, until only about 90 percent of the original number of schedules remained. This reduced set then served as the population of schedules for the calculation of various summary statistics. Table 3.3 illustrates the average parameter values obtained with the Swedish data. Since the median, mode, standard deviation-to-mean ratio, and lower and upper bounds are also of interest, they are included as part of the more detailed computer outputs reproduced in Appendix B.

Table 3.1 Parameters defining observed model migration schedules and those obtained with a cubic spline interpolation: Swedish regions, females, 1974*.

Parameters	1. Stockholm		2. East Middle-Sweden		3. South Middle-Sweden		4. South		5. West		6. North Middle-Sweden		7. Lower North-Sweden		8. Upper North-Sweden	
	1 yr	5 yr	1 yr	5 yr	1 yr	5 yr	1 yr	5 yr	1 yr	5 yr	1 yr	5 yr	1 yr	5 yr	1 yr	5 yr
a_1	.029	.028	.026	.026	.023	.023	.025	.025	.027	.025	.021	.022	.021	.021	.019	.021
α_1	.091	.089	.108	.106	.106	.105	.104	.106	.106	.095	.102	.115	.119	.130	.128	.160
a_2	.047	.049	.065	.070	.080	.087	.080	.085	.067	.069	.087	.097	.096	.118	.094	.106
μ_2	19.32	19.69	18.52	18.99	18.49	18.93	19.88	20.23	19.36	19.72	18.18	18.57	17.93	19.11	17.62	18.00
α_2	.094	.098	.109	.117	.127	.136	.129	.135	.114	.121	.139	.145	.148	.172	.143	.150
λ_2	.369	.313	.491	.351	.560	.375	.442	.367	.442	.395	.561	.345	.701	.305	.711	.330
c	.002	.002	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003
a_3	.000	.000														
μ_3	85.01	81.20														
α_3	.369	.364														
λ_3	.072	.080														

*Observed data are for single years of age (1yr); the cubic-spline-interpolated inputs are obtained from observed data by five-year age groups (5yr).

Table 3.2 Mean values of parameters defining the full set of observed model migration schedules: Sweden, 8 regions*, 1974 observed data by single years of age until 84 years and over.

	Sweden			
	Males		Females	
Parameters	Without retirement peak (52 schedules)	With retirement peak (11 schedules)	Without retirement peak (58 schedules)	With retirement peak (5 schedules)
a_1	0.029	0.025	0.027	0.023
α_1	0.126	0.080	0.114	0.087
a_2	0.066	0.050	0.078	0.051
μ_2	21.09	21.52	19.13	19.20
α_2	0.113	0.096	0.133	0.101
λ_2	0.459	0.439	0.525	0.377
c	0.003	0.002	0.003	0.003
a_3		0.0012		0.0017
μ_3		75.45		72.07
α_3		0.797		0.688
λ_3		0.294		0.192

*Region 1 (Stockholm) is a single-commune region and hence there exists no "intraregional" schedule for it, leaving $(8)^2 - 1 = 63$ schedules.

The comparison, in Table 3.1, of estimates obtained using one-year and five-year age intervals for the same Swedish data indicated that the interpolation procedure gave satisfactory results. However, it also suggested that the parameter λ_2 was consistently underestimated with five-year data. To confirm this, the results of Table 3.3 were replicated with the Swedish data base, using an aggregation with five-year age intervals. The results, set out in Table 3.4, show once again that λ_2 is always underestimated by the interpolation procedure. Although the degree of underestimation is not large, this tendency should be noted and kept in mind.

It is also important to note the erratic behavior of the retirement peak, apparently due to its extreme sensitivity to the loss of information arising out of the aggregation. Thus, although we shall continue to present results relating to the post-labor force ages, they will not be a part of our search for families of schedules.

3.2 National Contrasts

Tables 3.3 and 3.4 of the preceding subsection summarized average parameter values for 57 male and 57 female Swedish model migration schedules. In this subsection we shall expand our analysis to include a much larger data base, adding to the 114 Swedish model schedules, another 164 schedules from the United Kingdom (Table 3.5); 114 from Japan and 20 from the Netherlands (Table 3.6); 58 from the USSR, 8 from the USA, and 32 from Hungary (Table 3.7).* Summary statistics for these 510 schedules are set out in Appendix B; of those, 206 are male schedules, 206 are female schedules, and 98 are for the combination of both sexes (males plus females).*

*This total does not include the 56 schedules excluded as "extreme" schedules. During the process of fitting the model schedule to these more than 500 interregional migration schedules a frequently encountered problem was the occurrence of a negative value for the constant c. In all such instances the initial value of c was set equal to the lowest observed migration rate and the nonlinear estimation procedure was started once again.

Table 3.3 Mean values of parameters defining the reduced set of observed model migration schedules: Sweden, 8 regions, 1974, observed data by single years of age until 84 years and over. *

	Sweden			
	Males		Females	
Parameters	Without retirement peak (48 schedules)	With retirement peak (9 schedules)	Without retirement peak (54 schedules)	With retirement peak (3 schedules)
a_1	0.029	0.026	0.026	0.024
α_1	0.124	0.085	0.108	0.093
a_2	0.067	0.051	0.076	0.055
μ_2	20.50	21.25	19.09	18.87
α_2	0.104	0.093	0.127	0.106
λ_2	0.448	0.416	0.537	0.424
c	0.003	0.002	0.003	0.003
a_3		0.0006		0.0001
μ_3		76.71		74.78
α_3		0.847		0.938
λ_3		0.158		0.170

*Region 1 (Stockholm) is a single-commune region and hence there exists no intraregional schedule for it, leaving $(8)^2 - 1 = 63$ schedules, of which 6 were deleted.

Table 3.4 Mean values of parameters defining the reduced set of observed model migration schedules: Sweden, 8 regions, 1974, observed data by five years of age until 80 years and over.*

	Sweden			
	Males		Females	
Parameters	Without retirement peak (49 schedules)	With retirement peak (8 schedules)	Without retirement peak (54 schedules)	With retirement peak (3 schedules)
a_1	0.028	0.026	0.026	0.026
α_1	0.115	0.088	0.108	0.077
a_2	0.068	0.052	0.080	0.044
μ_2	20.61	20.26	19.52	19.18
α_2	0.105	0.084	0.133	0.089
λ_2	0.396	0.390	0.374	0.341
c	0.002	0.001	0.002	0.002
a_3		0.0017		0.0036
μ_3		77.47		77.72
α_3		0.603		0.375
λ_3		0.148		0.134

*Region 1 (Stockholm) is a single-commune region and hence there exists no intraregional schedule for it, leaving $(8)^2 - 1 = 63$ schedules, of which 6 were deleted.

A significant number of schedules exhibited a pattern of migration in the post-labor force ages that differed from that of the 11-parameter model migration schedule defined in equation (1). Instead of a retirement peak, the age profile took on the form of an "upward slope". In such instances the following 9-parameter modification of the basic model migration was introduced

$$
\begin{aligned}
M(x) = \; & a_1 e^{-\alpha_1 x} \\
& + a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}} \\
& + a_3 e^{\alpha_3 x} \\
& + c \qquad\qquad , x = 0, 1, 2, \ldots, z \qquad (3)
\end{aligned}
$$

The right half of Table 3.6, for example, sets out the mean parameter estimates of this modified form of the model migration schedule for the Netherlands.

Tables 3.3 through 3.7 present a wealth of information about national patterns of migration by age. The parameters, set out in columns, define a wide range of model migration schedules. Four refer only to migration *level*: a_1, a_2, a_3, and c. Their values are for a GMR of unity; to obtain corresponding values for other levels of migration, these four numbers need to be multiplied by the desired level of GMR. For example, the observed GMR for female migration out of the Stockholm region in 1974 was 1.43. Multiplying $a_1 = 0.029$ by 1.43 gives 0.041, the appropriate value of a_1 with which to generate the migration schedule having a GMR of 1.43.

The remaining model schedule parameters refer to migration age *profile*: α_1, μ_2, α_2, λ_2, μ_3, α_3, and λ_3. Their values remain constant for all levels of the GMR. Taken together, they define the age profile of migration from one region to another. Schedules without a retirement peak yield only the four profile parameters: α_1, μ_2, α_2, and λ_2, and schedules with a retirement slope have an additional profile parameter, α_3.

A detailed analysis of the parameters defining the various classes of schedules is beyond the scope of this report, nevertheless a few basic contrasts among national average age profiles may be usefully highlighted.

Let us begin with an examination of the labor force component defined by the four parameters a_2, μ_2, α_2, and λ_2. The national average values for these parameters generally lie within the following ranges:

$$0.05 < a_2 < 0.10$$

$$17 < \mu_2 < 22$$

$$0.10 < \alpha_2 < 0.20$$

$$0.25 < \lambda_2 < 0.60$$

In all but two instances, the female values for a_2, α_2, and λ_2 are larger than those for males. The reverse is the case for μ_2, with two exceptions, the most important of which is exhibited by Japan's females who consistently show a high peak that is older than that of males.

The two parameters defining the pre-labor force component, a_1 and α_1, generally lie within the ranges of 0.01 to 0.03 and 0.08 to 0.12, respectively. The exceptions are the Soviet Union and Hungary, which exhibit unusually high values for α_1. Unlike the case of the labor force component, consistent sex differentials are difficult to identify.

Average national migration age profiles, like most aggregations, hide more than they reveal. Some insight into the ranges of variations that are averaged out may be found by consulting the lower and upper bounds and standard-deviation-to-mean ratios listed in Appendix B for each set of national schedules. Additional details are available in the technical appendix to this report*. Finally, Table 3.8, illustrates how parameters vary in

*The technical appendix entitled "638 Model Migration Schedules: A Technical Appendix" is available on request.

several *unaveraged* national schedules, by way of example. The model schedules presented there describe migration flows out of and into the capital regions of each of six countries: Finland, Hungary, Japan, the Netherlands, Sweden, and the United Kingdom. The former schedules describe capital outflow profiles, the latter define capital inflow profiles. All are illustrated in Figure 3.1.

The most apparent difference between the age profiles of the capital outflow and inflow migration schedules is the dominance of young labor force migrants in the latter, that is, proportionately more migrants in the young labor force ages appear in capital inflow schedules. Indicating this labor dominance are the larger values of the product $a_2\lambda_2$ in the inflow schedules and of the ratio $\delta_{12} = a_1/a_2$ in the outflow schedules.

A second profile attribute is the degree of asymmetry in the labor force component of the migration schedule, i.e., the ratio of the rate of ascent λ_2, to the rate of descent α_2, defined as σ_2 in Section 2. In all but the Japanese case, the labor force curve of the capital outflow profile is more asymmetric than that of the corresponding inflow profile. We refer to this characteristic as labor asymmetry.

Examining the observed rates of descent of the labor and pre-labor force curves, α_2 and α_1, respectively, we find, for example, that they are close to being equal in the outflow schedules of Helsinki and Stockholm and are highly unequal in the cases of Budapest, Tokyo, and Amsterdam. In four of the six capital inflow profiles $\alpha_2 > \alpha_1$. Profiles with significantly different values for α_2 and α_1, are said to be irregular.

In conclusion, the empirical migration data of six industrialized nations suggest the following hypothesis. *The migration profile of a typical capital inflow schedule is, in general, more labor dominant and more labor symmetric than the migration profile of the corresponding capital outflow schedule.* No comparable hypothesis can be made regarding its anticipated degree of irregularity.

Table 3.5 Mean values of parameters defining the reduced set of observed model migration schedules: United Kingdom, 10 regions, 1970.*

Parameters	United Kingdom			
	Males		Females	
	Without retirement peak (59 schedules)	With retirement peak (23 schedules)	Without retirement peak (61 schedules)	With retirement peak (21 schedules)
a_1	0.021	0.016	0.021	0.018
α_1	0.099	0.080	0.097	0.089
a_2	0.059	0.053	0.063	0.048
μ_2	22.00	20.42	21.35	21.56
α_2	0.127	0.120	0.151	0.153
λ_2	0.259	0.301	0.327	0.333
c	0.003	0.004	0.003	0.004
a_3		0.007		0.002
μ_3		71.11		71.84
α_3		0.692		0.583
λ_3		0.309		0.403

*No intraregional migration data were available; hence $(10)^2 - 10 = 90$ schedules were analyzed and 8 were deleted.

Table 3.6 Mean values of parameters defining the reduced set of observed model migration schedules: Japan, 8 regions, 1970 and the Netherlands, 12 regions, 1974.*

Parameters	Japan		Netherlands	
	Males	Females	Males	Females
	Without retirement peak (57 schedules)	Without retirement peak (57 schedules)	With retirement slope (10 schedules)	With retirement slope (10 schedules)
a_1	0.014	0.021	0.013	0.012
α_1	0.095	0.117	0.080	0.098
a_2	0.075	0.085	0.063	0.084
μ_2	17.63	21.32	20.86	20.10
α_2	0.102	0.152	0.130	0.174
λ_2	0.480	0.350	0.287	0.307
c	0.002	0.004	0.003	0.004
a_3			0.00001	0.00004
α_3			0.077	0.071

*Region 1 in Japan (Hokkaido) is a single-prefecture region and hence there exists no intraregional schedule for it, leaving $(8)^2 - 1 = 63$ schedules, of which 6 were deleted. The only migration schedules available for the Netherlands were the migration rates out of each region without regard to destination; hence only 12 schedules were used, of which 2 were deleted.

Table 3.7 Mean values of parameters defining the reduced set of observed model migration schedules: USSR, 8 regions, 1974, USA, 4 regions, 1970-71, and Hungary, 6 regions, 1974.*

	USSR	USA	Hungary	
	Total (Males plus Females)	Total (Males plus Females)	Total (Males plus Females)	Total (Males plus Females)
	Without retirement peak (58 schedules)	With retirement peak (8 schedules)	Without retirement slope (7 schedules)	With retirement slope (25 schedules)
a_1	0.005	0.021	0.010	0.015
α_1	0.302	0.075	0.245	0.193
a_2	0.126	0.060	0.090	0.099
μ_2	19.14	20.14	17.22	18.74
α_2	0.176	0.118	0.130	0.159
λ_2	0.310	0.569	0.415	0.274
c	0.004	0.002	0.004	0.003
a_3		0.002		0.00032
μ_3		81.80		
α_3		0.430		0.033
λ_3		0.119		

*Intraregional migration was included in the USSR and Hungarian data but not in the USA data; hence there were $(8)^2 = 64$ schedules for the USSR, of which 6 were deleted, $(6)^2 = 36$ schedules for Hungary, of which 4 were deleted, and $(4)^2 - 4 = 12$ schedules for the USA, of which 2 were deleted because they lacked a retirement peak and another 2 were deleted because of their extreme values.

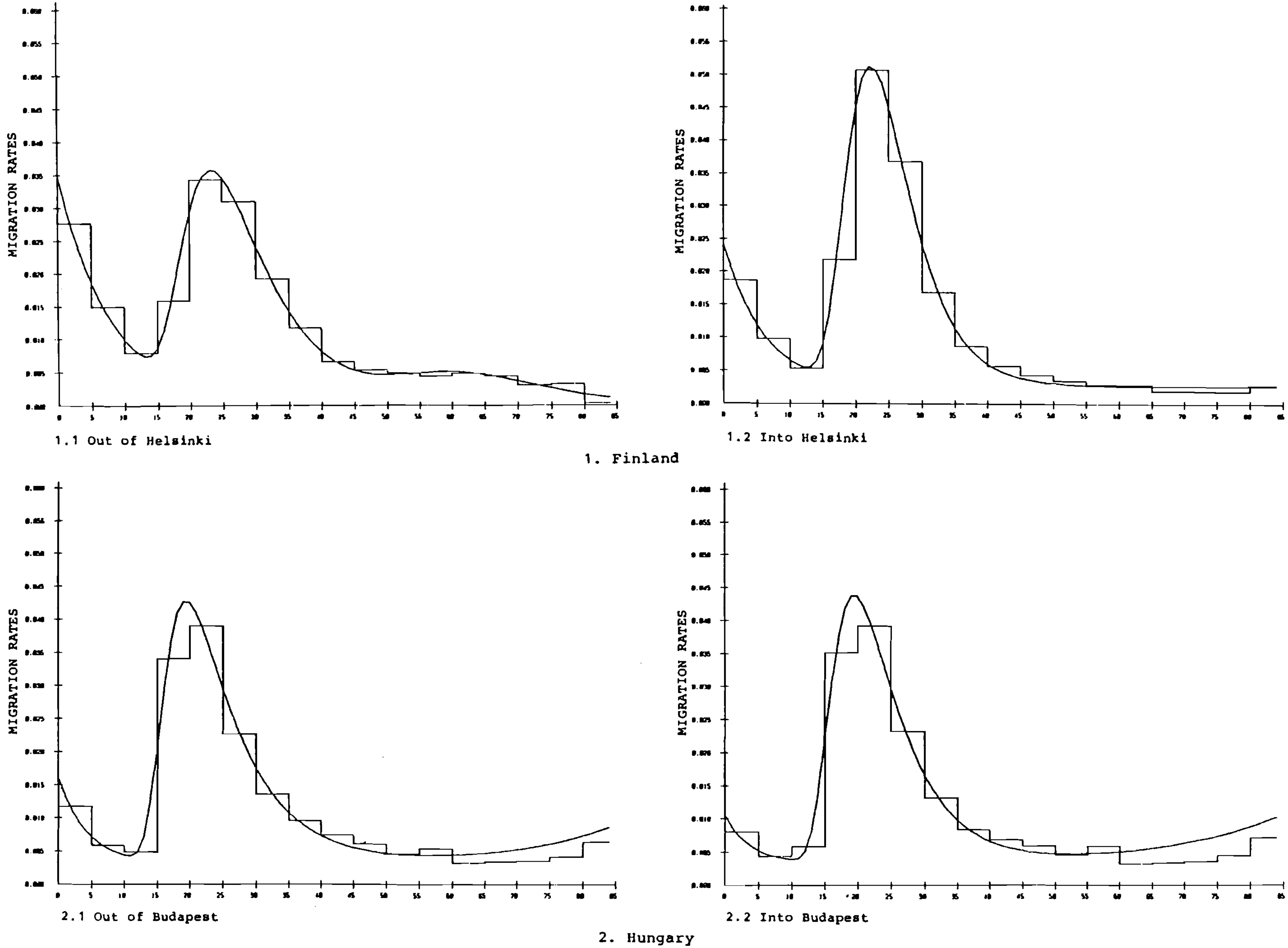


Figure 3.1 Migration age profiles of capital region outflows and inflows

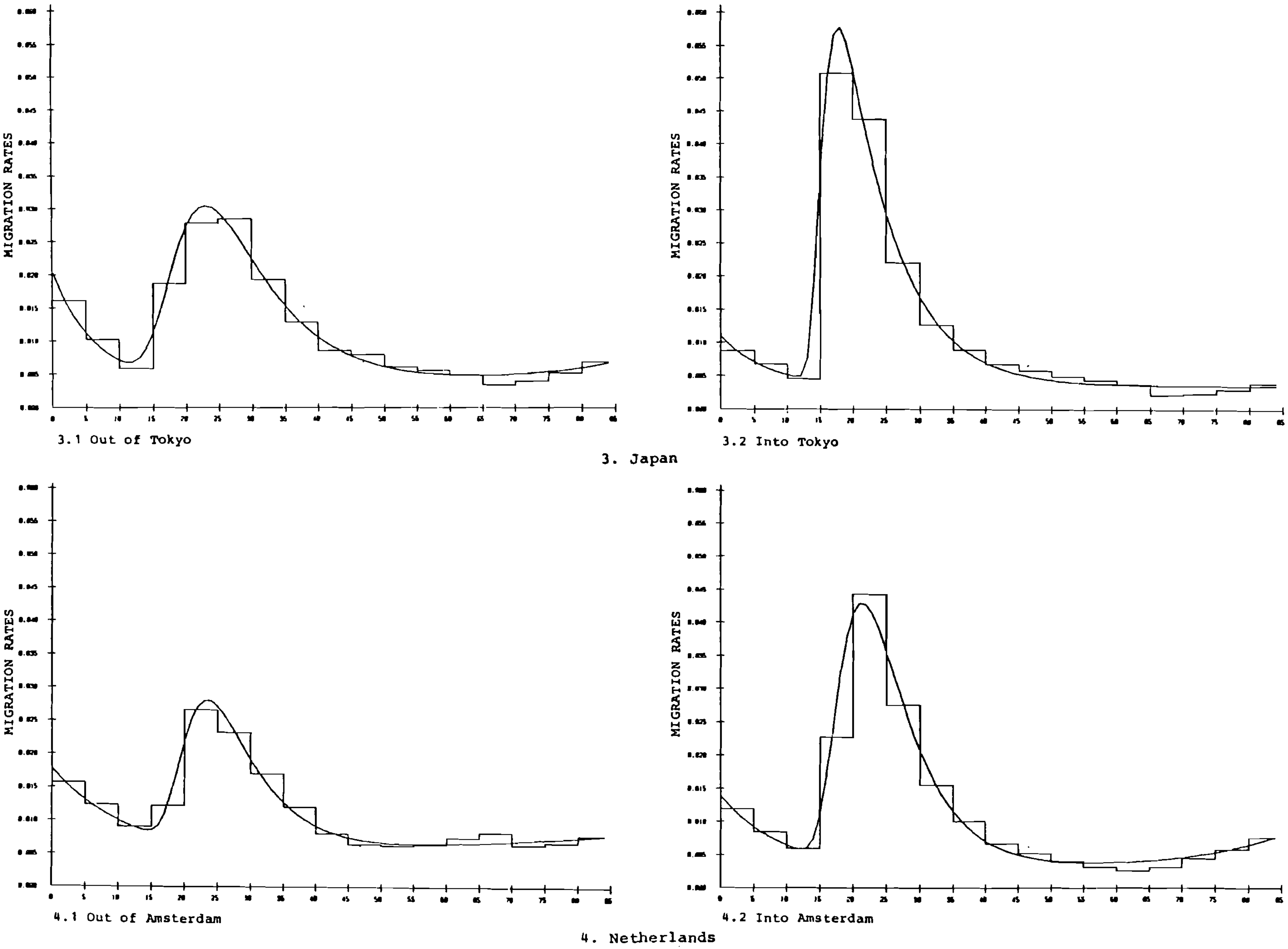
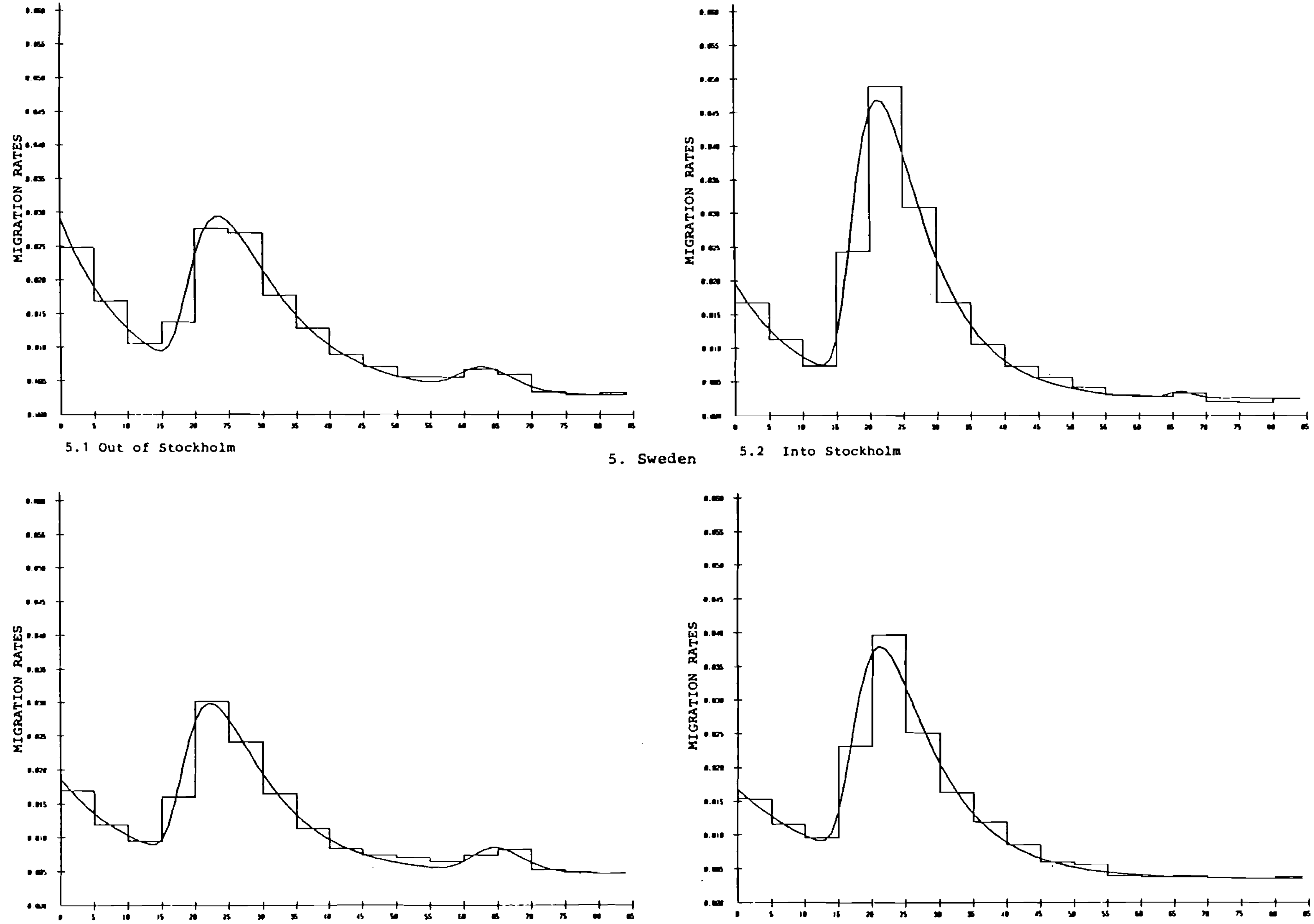


Figure 3.1 Migration age profiles of capital region outflows and inflows (continued)



5.1 Out of Stockholm

5.2 Into Stockholm

5. Sweden

6.1 Out of London

6.2 Into London

6. United Kingdom

Figure 3.1 Migration age profiles of capital region outflows and inflows (continued)

Table 3.8 Parameters defining observed total (males plus females) model migration schedules for capital-region flows: Finland, 1974, Hungary, 1974, Japan, 1970, Netherlands, 1974, Sweden, 1974, United Kingdom, 1970.

	Finland		Hungary		Japan	
Parameters	From Helsinki	To Helsinki	From Budapest	To Budapest	From Tokyo	To Tokyo
a_1	0.037	0.024	0.015	0.008	0.019	0.008
α_1	0.127	0.170	0.239	0.262	0.157	0.149
a_2	0.081	0.130	0.082	0.094	0.064	0.096
μ_2	21.42	22.13	17.10	17.69	20.70	15.74
α_2	0.124	0.198	0.130	0.152	0.111	0.134
λ_2	0.231	0.231	0.355	0.305	0.204	0.577
c	0.000	0.003	0.003	0.003	0.003	0.002
a_3	0.00027		0.00001	0.00005	0.00002	0.00131
μ_3	99.32					
α_3	0.204		0.072	0.059	0.061	0.000
λ_3	0.042					

Table 3.8 Parameters defining observed total (males plus females) model migration schedules for capital-region flows: Finland, 1974; Hungary, 1974; Japan, 1970; Netherlands, 1974; Sweden, 1974; United Kingdom, 1970 (continued).

Parameters	Netherlands		Sweden		United Kingdom	
	From Amsterdam	To Amsterdam	From Stockholm	To Stockholm	From London	To London
a_1	0.015	0.012	0.028	0.018	0.015	0.014
α_1	0.085	0.108	0.098	0.102	0.090	0.072
a_2	0.050	0.093	0.046	0.093	0.048	0.067
μ_2	21.62	19.66	20.48	19.20	19.65	18.81
α_2	0.141	0.150	0.095	0.134	0.111	0.123
λ_2	0.284	0.288	0.322	0.323	0.327	0.320
c	0.002	0.003	0.003	0.002	0.005	0.004
a_3	0.00229	0.00002	0.00004	0.00003	0.00003	
μ_3			80.32	73.19	81.13	
α_3	0.012	0.066	0.616	1.359	0.676	
λ_3			0.105	0.255	0.112	

3.3 Families of Schedules

Three sets of model migration schedules have been defined in this paper: the 11-parameter schedule with a retirement peak, the alternative 9-parameter schedule with a retirement slope, and the simple 7-parameter schedule with neither peak nor slope. Thus we have at least three broad families of schedules.

Additional dimensions for classifying schedules into families are suggested by the above comparative analysis of national migration age profiles and the basic measures and derived variables defined in Section 2. These dimensions reflect different locations on the horizontal and vertical axes of the schedule, as well as different ratios of slopes and heights.

Of the 528 model migration schedules studied in this Section, 416 are sex-specific and, of these, only 336 exhibit neither a retirement peak nor a retirement slope. Because the parameter estimates describing the age profile of post-labor force migration are unreliable, we shall restrict our search for families of schedules to these 164 male and 172 female model schedules, summary statistics for which are set out in Tables 3.9 and 3.10.

An examination of the parametric values exhibited by the 336 migration schedules summarized in Tables 3.9 and 3.10 suggests that a large fraction of the variation exhibited by these schedules is a consequence of changes in the values of the following four parameters and derived variables: μ_2, δ_{12}, σ_2, and β_{12}.

Migration schedules may be early or late peaking, depending on the location of μ_2 on the horizontal (age) axis. Although this parameter generally takes on a value close to 20, roughly 3 out of 4 observations fall within the range of 17 to 25. We shall call those below age 19 as *early peaking* schedules and those above 22 as *late peaking* schedules.

The ratio of the two basic vertical parameters, a_1 and a_2, is a measure of the relative importance of the migration of children in a model migration schedule. The index of child dependency, $\delta_{12} = a_1/a_2$, tends to exhibit mean value of about a third with 80 percent of the values falling between one-fifth and four-fifths. Schedules with an index of one-fifth or less will be said to be *labor dominant*; those above two-fifths will be called *child dependent*.

Migration schedules with labor force components that take the form of a relatively symmetrical bell-shape will be said to be *labor symmetrical*. These schedules will tend to exhibit an index of labor asymmetry, $\sigma_2 = \lambda_2/\alpha_2$, that is less than 2. *Labor asymmetric* schedules, on the other hand, will usually assume values for σ_2 of 5 or more. The average migration schedule will tend to show a σ_2-value of about 4, with approximately 5 out of 6 schedules exhibiting a σ_2 within the range of 1 to 8.

Finally, the index of parental-shift regularity in many schedules is close to unity, with approximately 70 percent of the values lying between one-third and four-thirds. Values of $\beta_{12} = \alpha_1/\alpha_2$ that are lower than four-fifths or higher than six-fifths will be called *irregular*.

Thus we may image a 3 by 4 cross-classification of migration schedules that defines a dozen "average families."

Measures / Schedule	Peaking	Dominance	Symmetry	Regularity
	$\mu_2=20$	$\delta_{12}=1/3$	$\sigma_2=4$	$\beta_{12}=1$
Retirement Peak	+	+	+	+
Retirement Slope	+	+	+	+
Reduced Form	+	+	+	+

Introducing a low and a high value for each parameter gives rise to 16 additional families for each of the three classes of schedules. Thus we may conceive of a minimum set of 60 families, equally divided among schedules with a retirement peak, schedules with a retirement slope, and schedules with neither a retirement peak nor a retirement slope.

JAPAN+SWEDEN+UK MALES N =164

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.00539	1.81309	0.22642	0.13176	0.09578	0.27380	1.20928
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.75751	62.98674	16.22228	13.10527	13.49189	9.95789	0.61384
A1	0.00173	0.04891	0.02084	0.01992	0.01824	0.00879	0.42204
ALPHA1	0.00009	0.40526	0.10491	0.10390	0.10138	0.05358	0.51077
A2	0.01559	0.22707	0.06716	0.06471	0.06846	0.02578	0.38391
MU2	14.68744	43.96579	20.04227	19.67385	19.07919	3.95015	0.19709
ALPHA2	0.03471	0.29735	0.11164	0.10618	0.10037	0.04389	0.39316
LAMBDA2	0.06051	1.76712	0.39110	0.37244	0.31650	0.21146	0.54068
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00003	0.00704	0.00266	0.00263	0.00248	0.00130	0.48947
MEAN AGE	24.71596	40.53283	30.71751	30.41339	30.25187	2.72144	0.08860
%(0-14)	4.92484	29.69068	18.93871	19.02262	18.54605	4.91304	0.25942
%(15-64)	60.27293	86.29065	72.08085	71.29800	66.77736	5.10213	0.07078
%(65+)	1.35294	17.31658	8.98045	8.71650	8.53658	3.49047	0.38867
DELTA1C	0.37762	712.88135	14.36314	6.79034	36.00280	56.75620	3.95152
DELTA12	0.02274	1.53679	0.35774	0.33571	0.24985	0.20221	0.56523
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.00092	7.47530	1.11318	1.02442	1.12208	0.81866	0.73542
SIGMA2	0.30349	24.23831	4.27564	3.42123	3.89371	3.26113	0.76272
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	6.91004	18.26030	13.72508	13.34019	12.01766	2.14485	0.15627
X HIGH	17.11028	28.14053	22.50278	22.95041	23.17692	2.14731	0.09542
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	2.90007	16.93039	8.77770	8.38019	7.81068	2.28557	0.26038
A	22.33532	102.41312	32.97422	31.54365	34.34699	7.58660	0.23008
B	0.01107	0.07343	0.02994	0.02775	0.02666	0.01036	0.34609

Table 3.9 Estimated summary statistics of parameters and variables associated with reduced sets of observed model migration schedules for Sweden, the United Kingdom, and Japan: Males, 164 schedules*

*A list of definitions for the parameters and variables appears in Appendix B.

JAPAN+SWEDEN+UK FEMALES N =172

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.00388	1.59564	0.19909	0.11590	0.08347	0.24085	1.20973
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.17964	60.83579	15.42092	12.26192	7.01245	9.85544	0.63910
A1	0.00526	0.04496	0.02259	0.02209	0.01916	0.00851	0.37664
ALPHA1	0.01585	0.41038	0.10698	0.10883	0.11448	0.05091	0.47587
A2	0.02207	0.18944	0.07426	0.06935	0.06391	0.02693	0.36263
MU2	15.06610	37.76019	20.63237	19.88280	18.47021	3.50346	0.16980
ALPHA2	0.05467	0.33556	0.14355	0.13434	0.12489	0.04993	0.34784
LAMBDA2	0.08367	1.49869	0.40032	0.37870	0.29592	0.19248	0.48081
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00012	0.00685	0.00347	0.00350	0.00315	0.00139	0.39940
MEAN AGE	24.51402	37.86541	30.65265	30.53835	29.18701	2.69720	0.08799
%(0-14)	9.37675	31.87480	20.93872	20.68939	19.50087	4.26504	0.20369
%(15-64)	60.55278	81.17286	68.65491	68.07751	67.76981	4.34828	0.06334
%(65+)	1.46164	19.56255	10.40638	10.32867	9.60705	3.40400	0.32711
DELTA1C	0.89359	192.60318	9.39987	5.95681	10.47907	16.22441	1.72602
DELTA12	0.02828	0.90435	0.34847	0.32367	0.33490	0.17420	0.49989
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.09121	2.48385	0.81472	0.84944	0.92863	0.37720	0.46298
SIGMA2	0.38917	12.23371	3.26434	2.89784	2.16585	2.12718	0.65164
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	10.32012	21.79038	14.51330	14.75022	14.33471	1.95309	0.13457
X HIGH	17.03028	30.92059	22.49959	22.46040	21.89189	2.14262	0.09523
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	2.89007	15.09035	7.98629	7.61017	7.16017	2.11207	0.26446
A	23.73040	37.24700	28.50972	28.17807	27.10955	2.47098	0.08667
B	0.00831	0.09111	0.03118	0.02970	0.02901	0.01149	0.36845

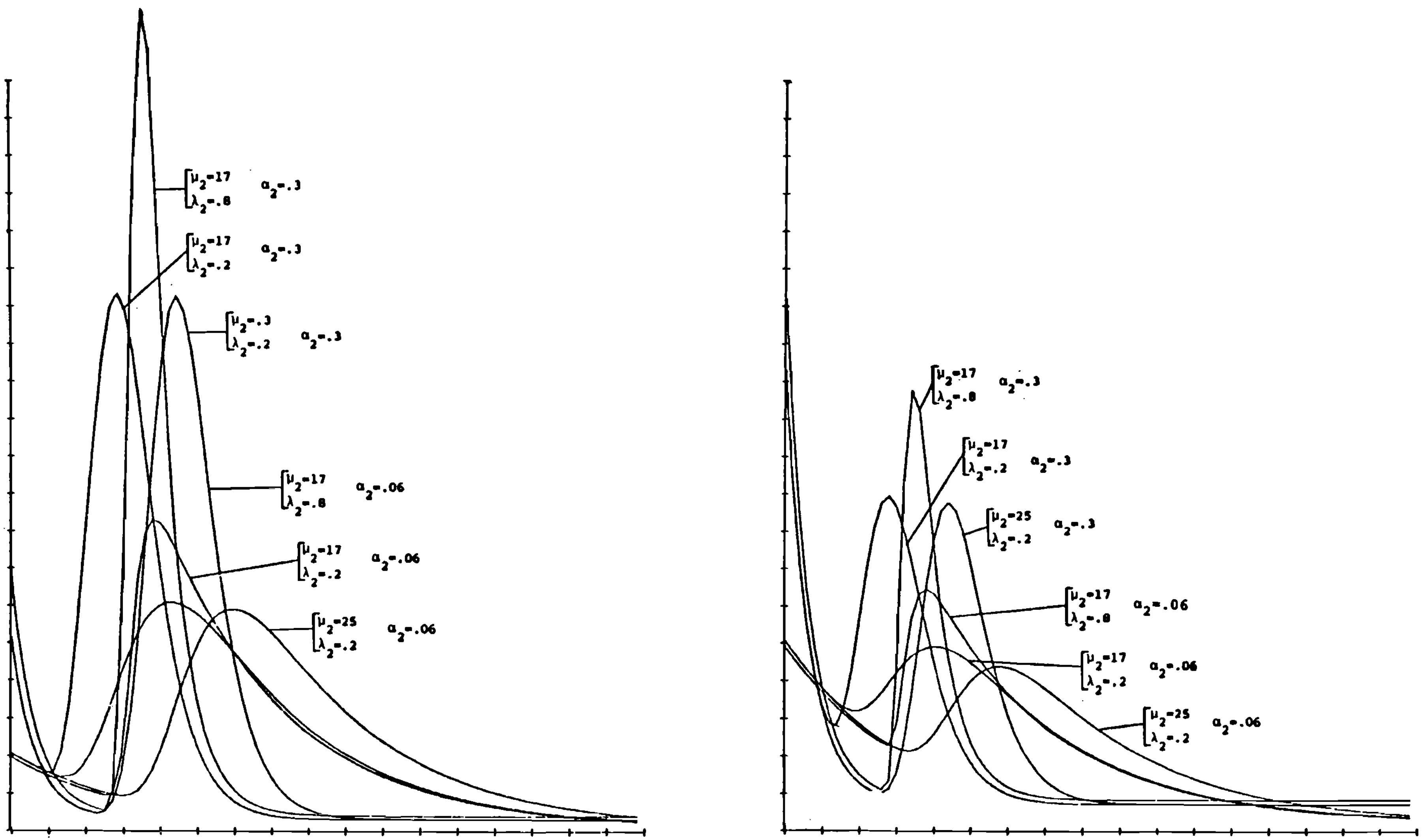
Table 3.10 Estimated summary statistics of parameters and variables associated with reduced sets of observed model migration schedules for Sweden, the United Kingdom, and Japan: Females, 172 schedules*

*A list of definitions for the parameters and variables appears in Appendix B.

To complement the above discussion with a few visual illustrations, we present in Figure 3.2A six labor dominant profiles, with δ_{1c} fixed at 22. The tallest three exhibit a steep rate of descent α_2 = .3; the shortest three show a much more moderate slope of α_2 = .06. Within each family of three curves, one finds variations in μ_2 and in the rate of ascent, λ_2. Increasing the former shifts the curve to the right along the horizontal axis; increasing the latter parameter raises the relative height of the high peak.

The six schedules in Figure 3.2B depict the corresponding two families of child dependent profiles. The results are generally similar to those in Figure 3.2A, with the exception that the relative importance of migration in the pre-labor force age groups is increased considerably. The principal effects of the change in δ_{12} are: (1) a raising of the intercept $a_1 + c$ along the vertical axis, and (2) a simultaneous reduction in the height of the labor force component in order to maintain a constant area of unity under each curve.

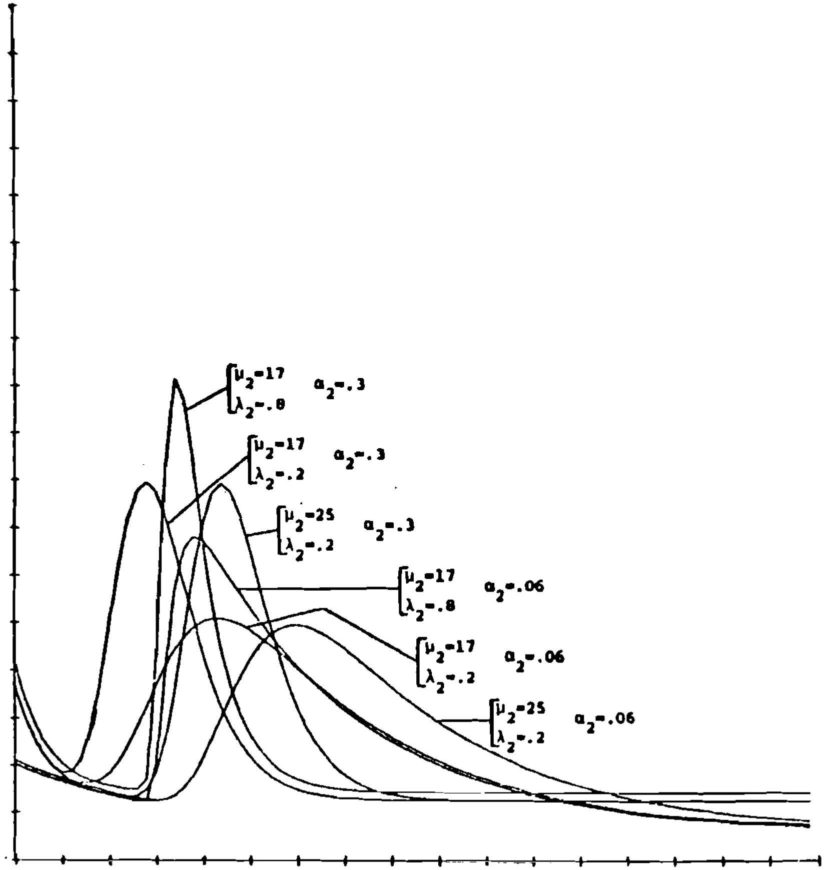
Finally, the dozen schedules in Figures 3.2C and 3.2D describe similar families of migration curves, but in these profiles the relative contribution of the constant component to the unit GMR has been increased significantly (i.e., δ_{1c} = 2.6). It is important to note that such "pure" measures of profile as x_ℓ, x_h, X, and A remain unaffected by this change, whereas "impure" profile measures, such as the mean age of migration, $\bar{n}$, now take on a different set of values.



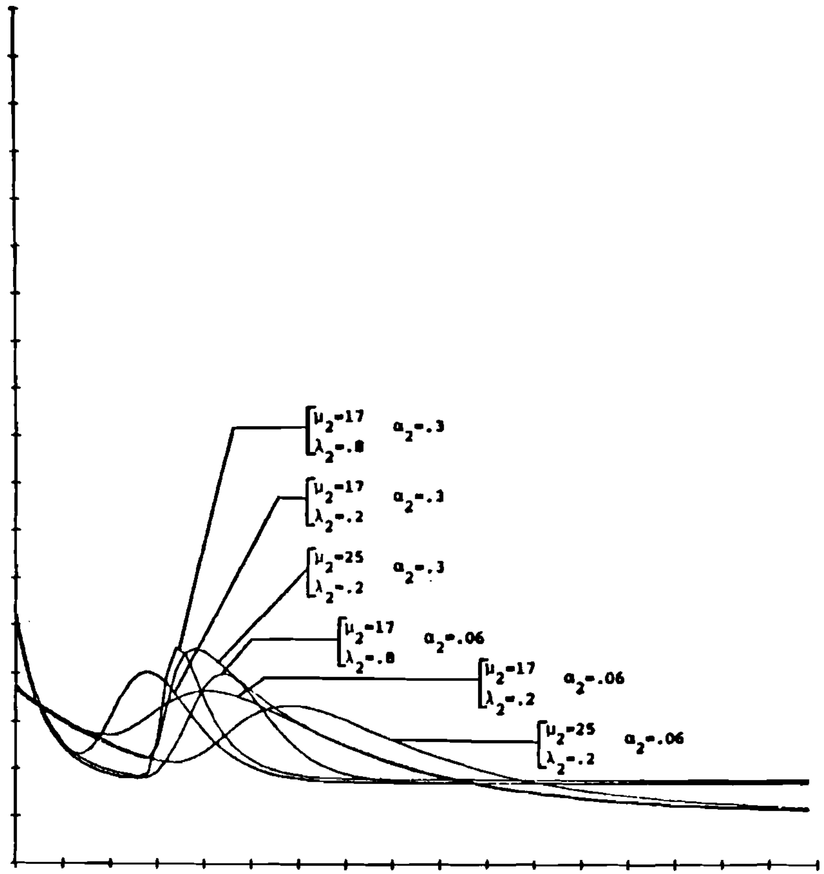
A. Labor dominant schedule with $\delta_{1c}=22.0$ and $\delta_{12}=.2$

B. Child dependent schedule with $\delta_{1c}=22.0$ and $\delta_{12}=.8$

Figure 3.2 Hypothetical model migration schedules with unit gross migraproduction rates and different parameter combinations.



C. Labor dominant schedule with δ_{1c}=2.6 and δ_{12}=.2



D. Child dependent schedule with δ_{1c}=2.6 and δ_{12}=.8

Figure 3.2 Hypothetical model migration schedules with unit gross migraproduction rates and different parameter combinations (continued).

3.4 Sensitivity Analysis

The preceding subsections have focused on a comparison of the fundamental parameters defining the model migration age profiles of a number of nations. The comparison yielded ranges of values within which each parameter may be expected to fall and suggested a classification of schedules into families. We now turn to an analytic examination of how changes in several of the more important parameters become manifested in the age profile of the model schedule. For analytical convenience we begin by focusing on the properties of the double exponential curve that describes the labor force component:

$$f_2(x) = a_2 \, e^{-\alpha_2(x-\mu_2)-e^{-\lambda_2(x-\mu_2)}} \tag{4}$$

We begin by observing that if α_2 is set equal to λ_2 in the above expression, then the labor force component assumes the shape of a well-known extreme value distribution used in the study of flood flows (Gumbel, 1941; Kimball, 1946). In such a case the function $f_2(x)$ is symmetrical around its mean $x_h = \mu_2$ and reaches its maximum, y_h, at that point. To analyze the more general case where $\alpha_2 \neq \lambda_2$, we may derive analytical expressions for both of these variables by differentiating equation (4) with respect to x, setting the result equal to zero, and then solving to find

$$x_h = \mu_2 - \frac{1}{\lambda_2} \, ln \left(\frac{\alpha_2}{\lambda_2}\right) \tag{5}$$

an expression that does not involve a_2, and

$$y_h = a_2 \left(\frac{\alpha_2}{\lambda_2}\right)^{\frac{\alpha_2}{\lambda_2}} e^{-\frac{\alpha_2}{\lambda_2}} \quad , \tag{6}$$

an expression that does not involve μ_2.

Note that if $\lambda_2 > \alpha_2$, which is almost always the case, then $x_h > \mu_2$. And observe that if $\alpha_2 = \lambda_2$, then the above two equations simplify to

$$x_h = \mu_2$$

and

$$y_h = \frac{a_2}{e}$$

Since μ_2 affects x_h only as a displacement, we may focus on the variation of x_h as a function of α_2 and λ_2. A plot of x_h against α_2, for a fixed λ_2, shows that increases in α_2 lead to decreases in x_h. Analogously, increases in λ_2, for a fixed α_2, produce increases in x_h but at a rate that decreases rapidly as the latter variable approaches its asymptote.

The behavior of y_h is independent of μ_2 and varies proportionately with a_2. Hence its variation also depends fundamentally only on the two variables α_2 and λ_2. A plot of y_h against α_2, for a fixed λ_2, gives rise to a U-shaped curve that reaches its minimum at $\alpha_2 = \lambda_2$. Increasing λ_2 widens the shape of the U.

The introduction of the pre-labor force component into the profile generally moves x_h to a slightly younger age and raises y_h by about $a_1 e^{-\alpha_1 x_h}$, usually a negligible quantity. The addition of the constant term c, of course, affects only y_h, raising it by the amount of the constant. Thus the migration rate at age x_h may be expressed as

$$M(x_h) \doteq a_1 e^{-\alpha_1 x_h} + y_h + c \quad .$$

A variable that interrelates the pre-labor and labor force components is the parental shift, A. To simplify our analysis of its dependence on the fundamental parameters, it is convenient to assume that α_1 and α_2 are approximately equal. In such instances, for ages immediately following the high peak x_h, the labor force component of the model migration schedule is closely approximated by the function

$$a_2 \, e^{-\alpha_2 (x_2 - \mu_2)} \quad .$$

Recalling that the pre-labor force curve is given by

$$a_1 \, e^{-\alpha_2 x_1}$$

when $\alpha_1 = \alpha_2$, we may equate the two functions to solve for the difference in ages that we have called the parental shift:

$$A = x_2 - x_1 = \mu_2 + \frac{1}{\alpha_2} \, ln \, \frac{1}{\delta_{12}} \quad . \tag{7}$$

This equation shows that the parental shift will increase with increasing values of μ_2 and will decrease with increasing values of α_2 and δ_{12}. Table 3.11 compares the values of this analytically defined "theoretical" parental shift with the corresponding observed parental shifts presented earlier in Table 2.1 for Swedish males and females. The two definitions appear to produce similar numerical values, but the analytical definition has the advantage of being simpler to calculate and analyze.

Consider the rural-to-urban migration age profile defined by the parameters in Table 3.12. In this profile the values of α_2 and λ_2 are almost equal making it a suitable illustration of several points raised in the above discussion.

First, calculating x_h with equation (5) gives

$$x_h = 21.10 - \frac{1}{.270} \, ln \left(\frac{.237}{.270}\right) = 21.58$$

as against the $x_h = 21.59$ set out in Table 3.11. Deriving y_h with equation (6) gives

$$y_h = 0.187 \left(0.878\right)^{0.878} e^{-0.878} = 0.069$$

where $\alpha_2/\lambda_2 = 0.237/0.270 = 0.878$. Thus M(21.59) is approximately equal to $y_h + c = 0.069 + 0.004 = 0.073$. The value given by the model migration schedule equation is also 0.073.

REGIONS OF SWEDEN								
The Parental Shift	1. Stockholm	2. East Middle-Sweden	3. South Middle-Sweden	4. South	5. West	6. North Middle-Sweden	7. Upper North-Sweden	8. Upper North-Sweden
Observed,[a] males	27.87	29.86	29.91	29.89	29.57	29.92	30.15	31.61
Theoretical,[b] males	26.67	28.97	29.63	29.74	28.84	29.43	29.74	30.59
Observed,[a] females	25.47	27.21	27.26	27.87	27.42	27.09	26.94	28.36
Theoretical,[b] females	24.49	26.33	27.51	28.21	27.19	27.69	27.53	28.59

Table 3.11 Observed and theoretical values of the parental shift: Swedish regions, 1974. ([a] Source: Table 2.1; [b]Source: Rogers, Raquillet, and Castro [1978], p. 497.)

Since $\alpha_1 \neq \alpha_2$, we cannot adequately test the accuracy of equation (7) as an estimator of A. Nevertheless, it can be used to help account for the unusually large value of the parental shift. Substituting in the values for μ_2, α_2, and δ_{12} into equation (7), we find

$$A = 21.10 + \frac{1}{.237} \ln\left(\frac{1}{.011}\right)$$

$$= 21.10 + \frac{4.51}{.237} = 40.13 \quad .$$

And although this is an underestimate of 45.13, it does suggest that the principal cause for the unusually high value of A is the unusually low value of δ_{12}. Had this latter parameter the value found for Stockholm's males, for example, the parental shift would exhibit the much lower value of 22.52.

Table 3.12 Parameters and variables defining observed urban/rural model migration schedules for urban/rural flows: USSR, 1974.

Variables and Parameters	USSR Total (Males plus Females) Urban to Rural	Rural to Urban
GMR	0.74	3.41
a_1	0.005	0.002
α_1	0.313	0.431
a_2	0.127	0.187
μ_2	19.26	21.10
α_2	0.177	0.237
λ_2	0.286	0.270
c	0.005	0.004
$\bar{n}$	33.66	31.24
%(0-14)	8.63	5.59
%15-64)	78.30	84.60
%(65+)	13.07	9.81
δ_{1c}	0.977	0.548
δ_{12}	0.038	0.011
β_{12}	1.77	1.82
σ_2	1.61	1.14
x_ℓ	11.09	11.38
x_h	20.94	21.59
X	9.85	10.21
A	42.30	45.13
B	0.045	0.063

4. SYNTHETIC MODEL MIGRATION SCHEDULES: I. THE CORRELATIONAL PERSPECTIVE

A *synthetic* model schedule is a collection of age-specific rates that is based on patterns observed in various populations other than the one being studied and some incomplete data on the latter. The justification for such an approach is that age profiles of fertility, mortality, *and migration* vary within predetermined limits for most human populations. Birth, death, and migration rates for one age group are highly correlated with the corresponding rates for other age groups, and expressions of such interrelationships form the basis of model schedule construction. The use of these regularities to develop synthetic (hypothetical) schedules that are deemed to be close approximations of the unobserved schedules of populations lacking accurate vital and mobility registration statistics has been a rapidly growing area of contemporary demographic research.

4.1 Introduction: Alternative Perspectives

The earliest efforts in the development of model schedules were based on only one parameter and hence had very little flexibility (United Nations, 1955). Demographers soon discovered that variations in the mortality and fertility regimes of different populations required more complex formulations. In mortality studies greater flexibility was introduced by providing families of schedules (Coale and Demeny, 1966) or by enlarging the number of parameters used to describe the age pattern (Brass, 1975). The latter strategy was also adopted in the creation of improved model fertility schedules and was augmented by the use of analytical descriptions of age profiles (Coale and Trussell, 1974).

Since the age patterns of migration normally exhibit a greater degree of variability across regions than do mortality and fertility schedules, it is to be expected that the development of an adequate set of model migration schedules will require a greater number both of families and of parameters. Although many alternative methods could be devised to summarize regularities in the form of families of model schedules defined by several parameters, three have received the widest popularity and dissemination:

1. the regression approach of the Coale-Demeny model life tables (Coale and Demeny 1966)
2. the logit system of Brass (Brass, 1971), and
3. the double-exponential graduation of Coale, McNeil and Trussell (Coale, 1977, Coale and McNeil, 1972, and Coale and Trussell, 1974)

The regression approach embodies a *correlational* perspective that associates rates at different ages to an index of level, where the particular associations may differ from one "family" of schedules to another. For example, in the Coale-Demeny model life tables the index of level is the expectation of remaining life at age 10, and a different set of regression equations is established for each of four "regions" of the world.*

Brass's logit system reflects a *relational* perspective in which rates at different ages are given by a standard schedule where shape and level may be suitably modified to be appropriate for a particular population.

The Coale-Trussell model fertility schedules are relational in perspective (they use a Swedish standard first-marriage schedule), but they also introduce an analytic description of the age profile by adopting a double exponential curve that defines the shape of the age-specific first-marriage function.

In this section and the next we mix the above three approaches to define two alternative perspectives for creating synthetic model migration schedules to be used in situations where only inadequate or defective data on internal (origin-destination) migration flows are available. Both perspectives rely on the analytic (double plus single exponential) graduation defined by the basic model migration schedule set out in Section 1 of this paper; they differ in the method by which a synthetic schedule is identified as being appropriate for a particular population.

*Each of the four regions (North, South, East, and West) defines a collection of similar mortality schedules that are more uniform in pattern than the totality of observed life tables.

The first, the regression approach, associates variations in the parameters and derived variables of the model schedule to each other and then to age-specific migration rates. The second, the logit approach, embodies different relationships between the model schedule parameters in several standard schedules and then associates the logits of the migration rates in the standard to those of the population in question.

4.2 The Correlational Perspective: The Regression Migration System

A straightforward way of obtaining a synthetic model migration schedule from limited observed data is to associate such data to the basic model schedule's parameters by means of regression equations. For example, given estimates of the migration rates of infants and young adults, M(0 - 4) and M(20 - 24) say, we may use equations of the form

$$Q_i = b_0 [M(0-4)]^{b_1} \; [M(20-24)]^{b_2}$$

to estimate the set of parameters Q_i that define the model schedule. However, the comparative analysis in Section 3 showed that the parameters of the fitted model schedules were not independent of each other. For example, higher than average values of λ_2 were associated with lower than average values of a_1. The incorporation of such dependencies into the regression approach would surely improve the accuracy and consistency of the estimation procedure. An examination of empirical associations among model schedule parameters and variables, therefore, is a necessary first step.

Regularities in the covariations of the model schedule's parameters suggest a strategy of model schedule construction that builds on regression equations embodying these covariations. For example, if a_2 increases linearly with increasing values of α_2, then the linear regression equation

$$a_2 = b_0 + b_1 \alpha_2$$

may adequately capture this pattern of covariation. For Swedish females this equation is estimated to be

$$a_2 = -.006 + 0.645\,\alpha_2$$

The correlation coefficient is 0.92, and the t-statistic of the regression coefficient associated with α_2 is 17.51.

Table 4.1 presents regression equations, such as the one above, fitted to Swedish data on males and on females. The particular choice of variables and parameters included there is, of course, only one of many possible alternatives, and it reflects a particular sequence of steps by which a complete model schedule with unit GMR can be inferred on the basis of estimates for: δ_{12}, x_ℓ, and x_h. Given values for these three variables, one can proceed to estimate μ_2, λ_2, σ_2, and β_{12}. Since $\sigma_2 = \lambda_2/\alpha_2$ we obtain, at the same time, an estimate for α_2, which we then can use to find a_2. With a_2 established, a_1 may be estimated by drawing on the definitional equation $\delta_{12} = a_1/a_2$, and α_1 may be found with the similar equation $\beta_{12} = \alpha_1/\alpha_2$. An initial estimate of c is obtained by setting $c = a_1/\delta_{1c}$, where δ_{1c} is estimated by regressing it on δ_{12}, and a_1, a_2, and c are scaled to give a GMR equal to unity.

Conceptually, this approach to model schedule construction begins with the labor force component and then appends to it the pre-labor force part of the curve. The value given for δ_{12} reflects the relative weights of these two components, with low values defining a labor dominant curve and high values pointing to a family dominated curve. (The behavior of the post-labor force curve is here assumed to be treated exogenously.)

We begin the calculations with μ_2 to establish the location of the curve on the age axis; is it an early or late peaking curve? Next, we turn to the determination of its two slope parameters λ_2 and α_2 by determining whether or not it is a labor symmetric curve. Values of σ_2 between 1 and 2 generally characterize a labor symmetric curve; higher values describe an asymmetric age profile.

Table 4.1 The Swedish regression equations: males and females.

Dependent Variables	Regression Coefficients of Independent Variables* Intercept	δ_{12}	α_2	x_ℓ	x_h	Multiple Correlation Coefficient r
MALES						
μ_2	-5.037	-2.886 (-4.85)		0.134 (2.25)	1.052 (13.06)	0.90
σ_2	32.884	9.351 (4.36)		1.193 (5.55)	-2.164 (-7.45)	0.82
β_{12}	5.211	2.000 (8.00)		-0.186 (-7.44)	-0.085 (-2.52)	0.83
λ_2	2.239	0.172 (1.40)		0.104 (8.43)	-0.148 (-8.90)	0.87
a_2	0.007		0.576 (11.19)			0.86
δ_{1c}	9.725	-0.631 (-0.13)				0.02
FEMALES						
μ_2	-1.080	-2.527 (-6.71)		0.086 (1.57)	0.914 (15.71)	0.92
σ_2	8.054	8.019 (7.20)		1.592 (9.88)	-1.423 (-8.28)	0.88
β_{12}	2.407	1.594 (6.81)		-0.147 (-4.33)	0.005 (0.14)	0.77
λ_2	1.759	0.192 (2.38)		0.155 (13.27)	-0.169 (-13.52)	0.93
a_2	-0.006		0.645 (17.51)			0.92
δ_{1c}	5.959	11.553 (1.93)				0.26

*Values in parentheses are t-statistics.

The regression of a_2 on α_2 produces the fourth parameter needed to define the labor force component. With values for μ_2, λ_2, α_2, and a_2 the construction procedure turns to the estimation of the pre-labor force curve, which is defined by the two parameters α_1 and a_1. Its relative share of the total unit area under the model migration schedule is set by the value give to δ_{12}.

Exhibit 4.1 demonstrates the sequence of calculations with the Stockholm model migration schedule for females. Figure 4.1 illustrates the resulting fit.*

4.3 The Basic Regression Equations

The collection of regression equations set out in Exhibit 4.1 may be defined to represent the "child dependency" set, inasmuch as their central independent variable δ_{12} is the index of child dependency. It is, of course, also possible to replace this independent variable with others, such as σ_2 or β_{12}, for example, to create a "labor asymmetry" or a "parental regularity" set, respectively. Table 4.2 presents regression coefficients for all three variants, obtained using the age-specific interregional migration schedules (scaled to unit GMR) of Sweden, the United Kingdom, and Japan. Deleting schedules with a retirement peak, leaves a total of 163 for males and 172 for females.

Tests of the 3 variants of the basic regressions using the data on Sweden, the United Kingdom, and Japan produced relatively satisfactory results, with the goodness-of-fit index E generally lying in the range between 5 and 35. Of the three variants, the child dependency set gave the best fits in about a half of the female schedules tested, whereas the parental regularity set was overwhelmingly the best fitting variant for the male schedules.

4.4 Using the Basic Regression Equations

To use the basic regression equations presented in Table 4.2, one first needs to obtain estimates of δ_{12}, x_ℓ, and x_h. Values for these three variables may be selected to reflect informed guesses, historical data, or empirical regularities between such model schedule variables and observed migration data.

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*The retirement peak is introduced exogenously by setting its parameters equal to those of the "observed" model migration schedule.

A. INPUTS

$\delta_{12} = 0.604$ $\quad x_\ell = 14.81$ $\quad x_h = 22.70$

B. OUTPUTS

B.1 Labor force component

$\mu_2 = -1.080 - 2.527\ \delta_{12} + 0.086\ x_\ell + 0.914\ x_h$
$= 19.42$

$\sigma_2 = 8.054 + 8.019\ \delta_{12} + 1.592\ x_\ell - 1.423\ x_h$
$= 4.17$

$\lambda_2 = 1.759 + 0.192\ \delta_{12} + 0.155\ x_\ell - 0.169\ x_h$
$= 0.334$

$\alpha_2 = \lambda_2/\sigma_2 = 0.080$

$a_2 = -\ 0.006 + 0.645\ \alpha_2$

$= 0.046$

B.2 Pre-labor force component

$a_1 = a_2\ \delta_{12} = 0.028$

$\beta_{12} = 2.407 + 1.594\ \delta_{12} - 0.147\ x_\ell + 0.005\ x_h$
$= 1.31$

$\alpha_1 = \alpha_2\ \beta_{12} = 0.104$

B.3 Constant component

$\delta_{1c} = 5.959 + 11.553\ \delta_{12}$
$= 12.94$

$c = a_1/\delta_{1c} = 0.028/12.937 = 0.002$

C. GOODNESS OF FIT*

$E = 8.50$

Exhibit 4.1 The calculation sequence with the Swedish regressions: Stockholm females, GMR = 1

* The goodness-of-fit index E is the mean absolute error expressed as a percentage of the observed mean. It is defined in equation (2) of Section 2.

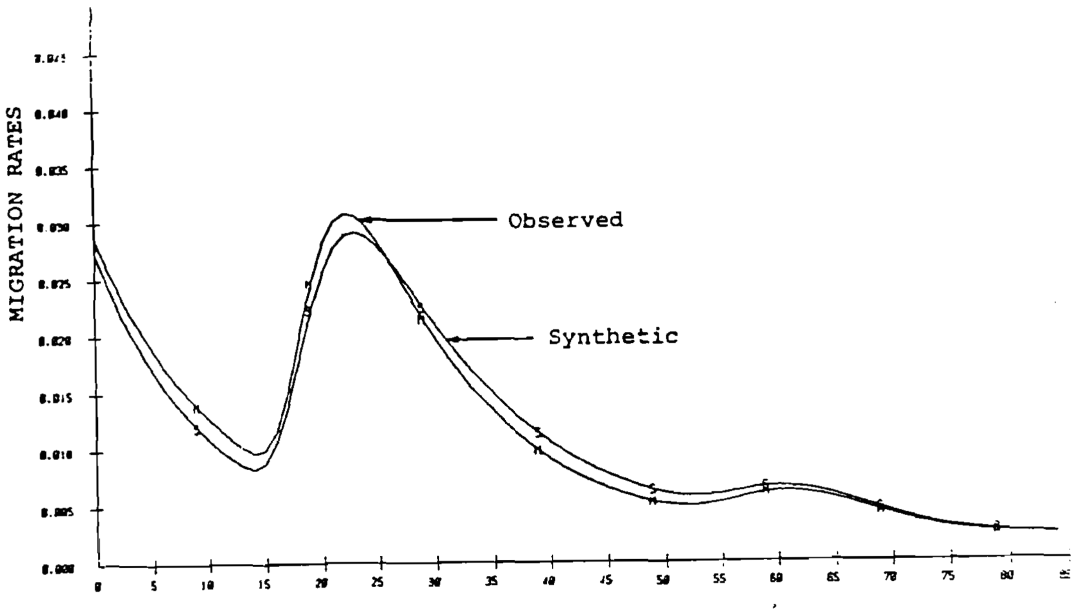


Figure 4.1 The fit of the synthetic model migration schedule based on Swedish national regression equations; Stockholm females, 1974.

Table 4.2 A basic set of regression equations.

A. MALES

Dependent Variables	Inter-cept	δ_{12} / σ_2 / β_{12}	α_2	x_ℓ	x_h	Multiple Correlation Coefficient r
	Regression Coefficients of Independent Variables*					
Child Dependency Set (δ_{12})						
σ_2	16.42682	5.59390 (5.23)		0.89435 (9.54)	-1.17441 (-11.14)	0.72
β_{12}	1.90489	1.33191 (3.60)		-0.02651 (-0.82)	-0.04019 (-1.10)	0.28
λ_2	1.30848	0.15118 (3.16)		0.07617 (18.15)	-0.08963 (-19.00)	0.88
Labor Asymmetry Set (σ_2)						
δ_{12}	-1.14777	0.02610 (5.23)		-0.01384 (-1.74)	0.07039 (9.01)	0.64
β_{12}	-1.42236	0.18826 (8.70)		-0.19178 (-5.57)	0.19388 (5.72)	0.57
λ_2	----------same equation as in the child dependency set----------					
Parental Regularity Set (β_{12})						
δ_{12}	-0.88605	0.05634 (3.60)		0.01179 (1.78)	0.04530 (6.85)	0.60
σ_2	10.38013	1.70652 (8.70)		0.97656 (11.77)	-0.95133 (-11.47)	0.78
λ_2	1.16111	0.02563 (2.58)		0.07816 (18.58)	-0.08316 (-19.77)	0.87
Equations Common to All Sets (δ_{12})						
μ_2	-3.26006	3.27947 (2.77)		-0.67070 (-6.46)	1.39248 (11.93)	0.77
a_2	0.03398		0.29713 (7.46)			0.51
δ_{1c}	9.41424	13.83372 (0.63)				0.05

*Values in parentheses are t-statistics.

Table 4.2 A basic set of regression equations (continued).

B. FEMALES

Dependent Variables	Inter-cept	$\frac{\delta_{12}}{\sigma_2}$ / β_{12}	α_2	x_ℓ	x_h	Multiple Correlation Coefficient r
		Regression Coefficients of Independent Variables*				
Child Dependency Set (δ_{12})						
σ_2	10.96834	6.05257 (9.85)		0.63402 (11.47)	-0.84512 (-16.16)	0.82
β_{12}	1.82060	1.42203 (9.04)		-0.04282 (-3.02)	-0.03911 (-2.92)	0.58
λ_2	1.19343	0.12937 (2.98)		0.07635 (19.57)	-0.08650 (-23.45)	0.90
Labor Asymmetry Set (σ_2)						
δ_{12}	-1.03192	0.06046 (9.85)		-0.02597 (-3.66)	0.06933 (10.81)	0.72
β_{12}	0.28708	0.09485 (5.35)		-0.08643 (-4.22)	0.06544 (3.53)	0.39
λ_2	-----------same equation as in the child dependency set----------					
Parental Regularity Set (β_{12})						
δ_{12}	-0.81011	0.22998 (9.04)		0.02297 (4.12)	0.02835 (5.60)	0.70
σ_2	5.92233	1.53566 (5.35)		0.77520 (12.34)	-0.67378 (-11.80)	0.75
λ_2	1.09905	0.01926 (1.08)		0.07916 (20.28)	-0.08282 (-23.33)	0.89
Equations Common to All Sets (δ_{12})						
μ_2	-7.69222	-2.14239 (-2.37)		-0.52726 (-6.49)	1.63218 (21.25)	0.86
a_2	0.03850		0.24908 (6.79)			0.46
δ_{1c}	0.18996	26.42951 (3.85)				0.28

*Values in parentheses are t-statistics.

For example, suppose that a fertility survey has produced a crude estimate of the ratio of infant to parent migration rates: M = M(0-4)/M(20-24), say. A linear regression of δ_{12} on this M-ratio gives, for Swedish females,

$$_F\hat{\delta}_{12} = -0.05562 + 0.79321\ M$$

and a correlation coefficient of 0.92. Enlarging the data set to also include the United Kingdom and Japan reduces the correlation coefficient to 0.66, and gives

$$_F\hat{\delta}_{12} = 0.10311 + 0.40811\ M \quad .$$

Estimating the corresponding equation for males yields

$$_M\hat{\delta}_{12} = -0.02066 + 0.68602\ M$$

and a correlation coefficient of 0.80. And repeating the above two regression calculations using data for single years of age (that is, M = M(0-1)/M(20-21)) gives

$$_F\hat{\delta}_{12} = 0.18224 + 0.20346\ M \qquad (r = 0.60)$$

and

$$_M\hat{\delta}_{12} = 0.09318 + 0.35022\ M \qquad (r = 0.74)$$

The correlation coefficients indicate that the fits for the five-year age groups are somewhat better for both males and females, and such data are generally more readily available. Moreover, tests of both pairs of regressions with data for Sweden, the United Kingdom, and Japan consistently show that the two pairs produce virtually identical age profiles for fixed values of x_ℓ

and x_h. Consequently we shall restrict our attention to the five-year age interval regression equations.

Figure 4.2 illustrates examples of the quality of fit provided by the synthetic schedules to the observed model migration data. Two sets of synthetic schedules are shown: those with the observed index of child dependency (δ_{12}) and those with the estimated index ($\hat{\delta}_{12}$), calculated using the five-year age group regressions.

4.5 Applications

A closer examination of the basic set of regression equations reveals several weaknesses. The equation for estimating β_{12} in the child dependency set has a low coefficient of multiple correlation, r=0.28. It would seem prudent to simply set β_{12} equal to a fixed value, say unity. A similar justification may be made for setting c equal to 0.003 say.

The male and female regression equations to calculate a_2 are similar enough to lead one to combine them to define the unisexual equation

$$a_2 = 0.04 + 0.27\ \alpha_2$$

The regression equations for calculating μ_2, σ_2, and λ_2 remain as set out in Table 4.2.

Simplification of the M-ratio regression also is possible. Forcing the regression through the origin gives

$${}_F\hat{\delta}_{12} = 0.549\ M$$

and

$${}_M\hat{\delta}_{12} = 0.654\ M$$

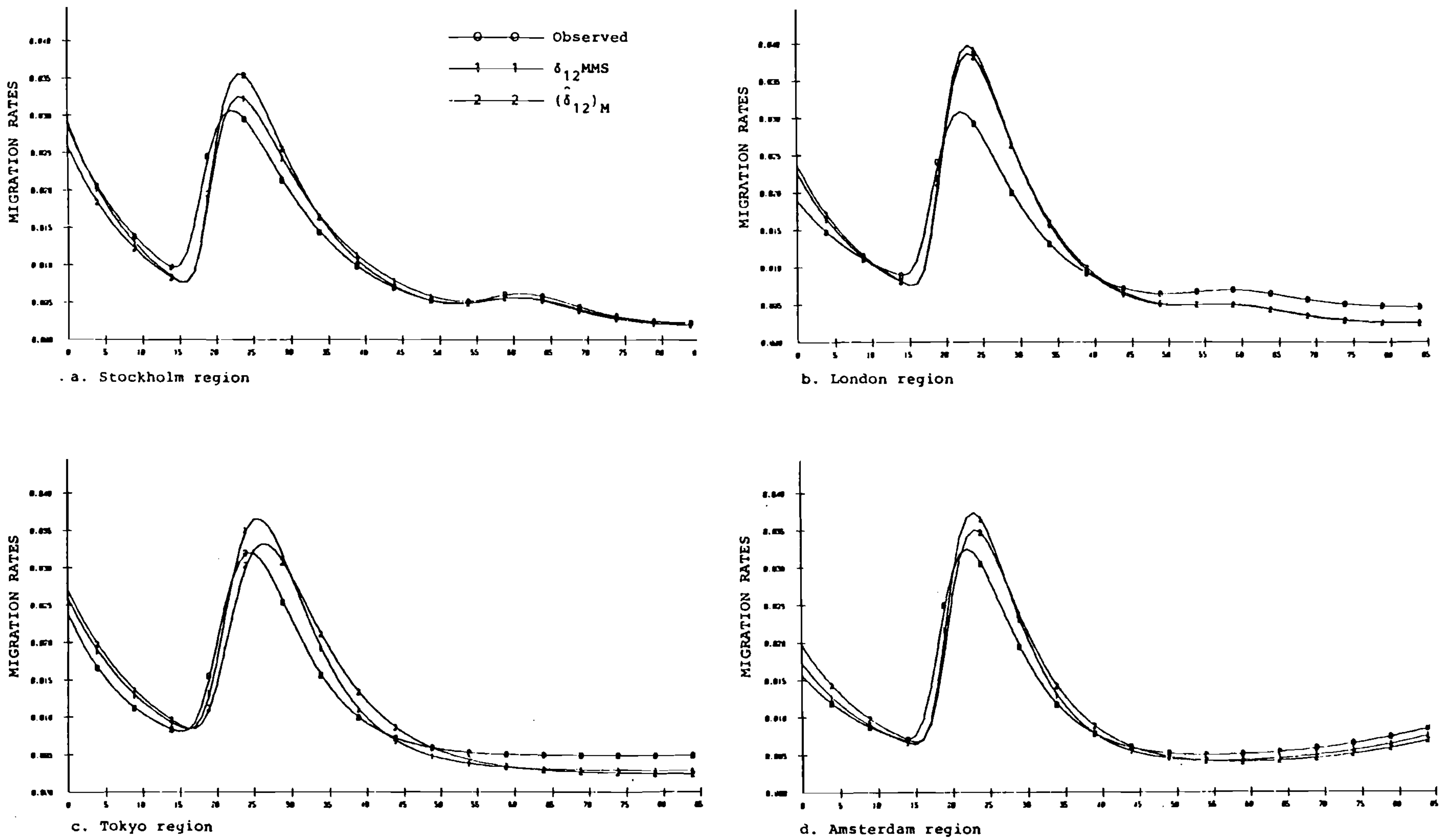


Figure 4.2 The fits of correlational synthetic model migration schedules to data for the female populations of Stockholm, London, Tokyo, and Amsterdam.

Exhibit 4.2 presents the calculation sequence that uses the above equations to produce the synthetic model migration schedule for Philippine males illustrated in Figure 4.3. The result is not very satisfactory and suggests that further research on the development of a basic set of regressions appropriate to Third World countries is needed.

A. INPUTS

$\frac{M(0-4)}{M(20-24)} = \frac{0.051}{0.132} = 0.386$ (from del Mar Pernia 1977, p. 114)

$\hat{\delta}_{12} = a_1 \frac{M(0-4)}{M(20-24)} = 0.654(0.386) = 0.252$

$x_\ell = 13.50$

$x_h = 23.00$

B. OUTPUTS

B.1 Labor force component

$\mu_2 = -3.260 + 3.279\ \delta_{12} - 0.671\ x_\ell + 1.392\ x_h = 20.525$

$\sigma_2 = 16.427 + 5.594\ \delta_{12} + 0.894\ x_\ell - 1.174\ x_h = 2.906$

$\lambda_2 = 1.308 + 0.151\ \delta_{12} + 0.076\ x_\ell - 0.090\ x_h = 0.302$

$\alpha_2 = \frac{\lambda_2}{\sigma_2} = 0.104$

$a_2 = 0.04 + 0.27\ \alpha_2 = 0.068$

B.2 Pre-labor force component

$a_1 = a_2\delta_{12} = 0.017$

$\beta_{12} = 1$

$\alpha_1 = \alpha_2\beta_{12} = \alpha_2 = 0.104$

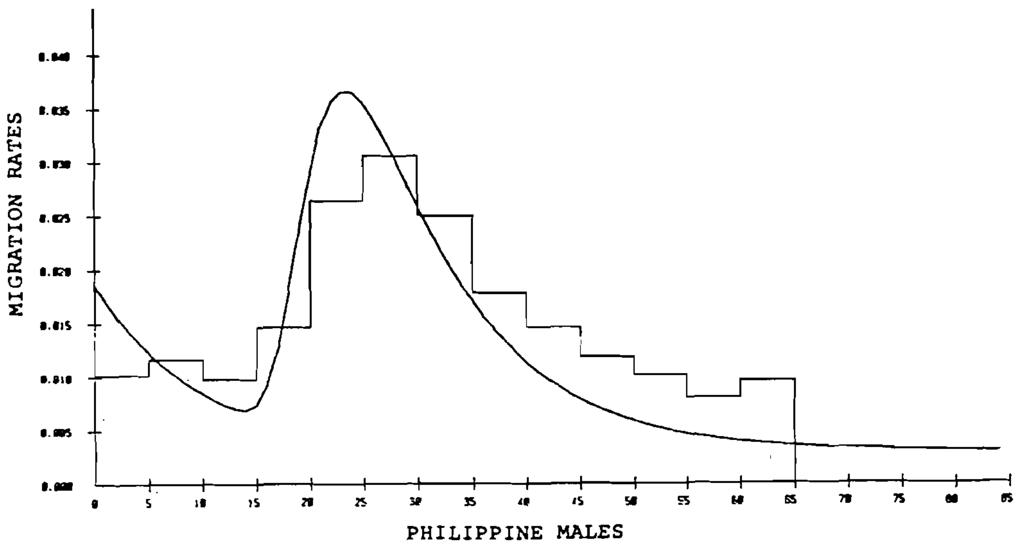
B.3 Constant component

$c = 0.003$

C. GOODNESS OF FIT

$E = 35.10$

Exhibit 4.2 The calculation sequence with the simplified version of the basic regressions: Philippine males.



Source: del Mar Pernia (1977)

Figure 4.3 A synthetic model migration schedule for Philippine males: the correlational approach

5. SYNTHETIC MODEL MIGRATION SCHEDULES: II. THE RELATIONAL PERSPECTIVE

Two alternative perspectives for identifying an appropriate synthetic model migration schedule for a regional population with inadequate data were outlined at the beginning of the preceding section. Both ultimately depend on the availability of some limited data to obtain the appropriate model schedule, for example, at least two age-specific rates such as M(0-4) and M(20-24) and informed guesses regarding the values of a few key variables, such as the low and high points of the schedule.

Although the appropriate alternative will always depend on the particular situation at hand, it seems reasonable to expect that the relational logit system may turn out to be the more suitable approach in some particular instances. Therefore, we shall continue our discussion of synthetic schedules, in this section, by focusing on the development of a logit migration system.

5.1 Introduction: The Logit Approach

Among the most popular methods for estimating mortality from inadequate or defective data, is the so-called "logit system" developed by William Brass about twenty years ago and now widely applied by demographers all over the world.* The logit approach to model schedules is founded on the assumption that different mortality schedules can be related to each other by a linear transformation of the logits of their respective survivorship probabilities. That is, given an observed series of survivorship probabilities $\ell(x)$ for ages $x = 1,2,\ldots,\omega$, it is possible to associate these with a "standard" series $\ell_s(x)$ by means of the linear relationship

$$\text{logit}\ [1 - \ell(x)] = \gamma + \rho\ \text{logit}\ [1 - \ell_s(x)]$$

*Brass (1971), Brass and Coale (1968), Carrier and Hobcraft (1971), Hill and Trussell (1977), and Zaba (1979).

where

$$\text{logit}\,[y(x)] = \frac{1}{2}\ln\left[\frac{y(x)}{1-y(x)}\right] = Y(x)\text{ , say , } \quad 0 < y(x) < 1\text{ ,}$$

or

$$Y(x) = \gamma + \rho\, Y_s(x)$$

The inverse of this function is

$$\ell(x) = \frac{1}{1 + e^{2Y(x)}}$$

The principal results of this mathematical transformation of the nonlinear $\ell(x)$ function is a more nearly linear function in x, with a range of minus and plus infinity rather than unity and zero.

Given a standard schedule, such as the set of standard logits, $Y_s(x)$, proposed by Brass, a life table can be created by selecting appropriate values for γ and ρ. In the Brass system γ reflects the level of mortality and ρ defines the relationship between child and adult mortality. The closer γ is to zero and ρ to unity, the more like the standard is the synthetically created life table.

5.2 The Relational Perspective: The Logit Migration System

As before, let ${}_uM(x)$ denote the age-specific migration rates of a schedule scaled to a unit gross migraproduction rate (GMR), and let ${}_uM_s(x)$ denote the corresponding standard schedule. Taking logits of both sets of rates gives the logit migration system

$${}_uY(x) = \gamma + \rho\ {}_uY_s(x)$$

and

$$_{u}M(x) = \frac{1}{1 + e^{-2[\gamma + \rho \, _{u}Y_{s}(x)]}}$$

where, for example,

$$\text{logit}\,[_{u}M_{s}(x)] = {}_{u}Y_{s}(x) = \frac{1}{2} \ln \frac{_{u}M_{s}(x)}{1 - {}_{u}M_{s}(x)} \quad , \quad 0 < {}_{u}M_{s}(x) < 1$$

The selection of a particular migration schedule as a standard reflects the belief that it is broadly representative of the age pattern of migration in the multiregional population system under consideration. To illustrate a number of calculations carried out with several sets of multiregional data, we shall adopt the national age profile as the standard in each case and strive to estimate regional outmigration age profiles by relating them to the national one. Specifically, given an m by m table of interregional migration flows for any age x, we divide each origin-destination-specific flow $O_{ij}(x)$ by the population in the origin region $K_i(x)$ to define the age-specific migration rate $M_{ij}(x)$. For the corresponding national rate, we define

$$M_{..}(x) = \frac{\sum_{i}\sum_{j} O_{ij}(x)}{\sum_{i} K_i(x)} \qquad \text{for all } i \neq j \ .$$

Scaling all schedules to unit GMR gives

$$_{u}M_{ij}(x) = \frac{M_{ij}(x)}{\sum_{x} M_{ij}(x)} = \frac{M_{ij}(x)}{GMR_{ij}} \quad , \quad i \neq j \ ,$$

and

$$_{u}M_{..}(x) = \frac{M_{..}(x)}{\sum_{x} M_{..}(x)} = \frac{M_{..}(x)}{GMR_{..}} \ .$$

Figure 5.1a illustrates the national migration rate schedule of Swedish males and females in 1974, scaled to unit GMR. The rates are for single years of age and describe transfers across the regional boundaries of the eight-region system adopted in the comparative study.

Figure 5.1b graphs the age pattern of the logit values, $Y_s(x)$, of the national migration rates.* Regressing the set of 85 age-specific outmigration rates from Stockholm to the rest of the nation, on these two standard schedules of logits, gives

$$ {}_uY(x) = -0.4871 + 0.7664\ Y_s(x) $$

for males, and

$$ {}_uY(x) = -0.3317 + 0.8362\ Y_s(x) $$

for females.

Alternatively, fitting the model migration schedule to the national standard with GMR set equal to unity, taking logits of these standard rates, and regressing Stockholm's model schedule outmigration rates (with GMR also equal to unity) on the standard logits, gives

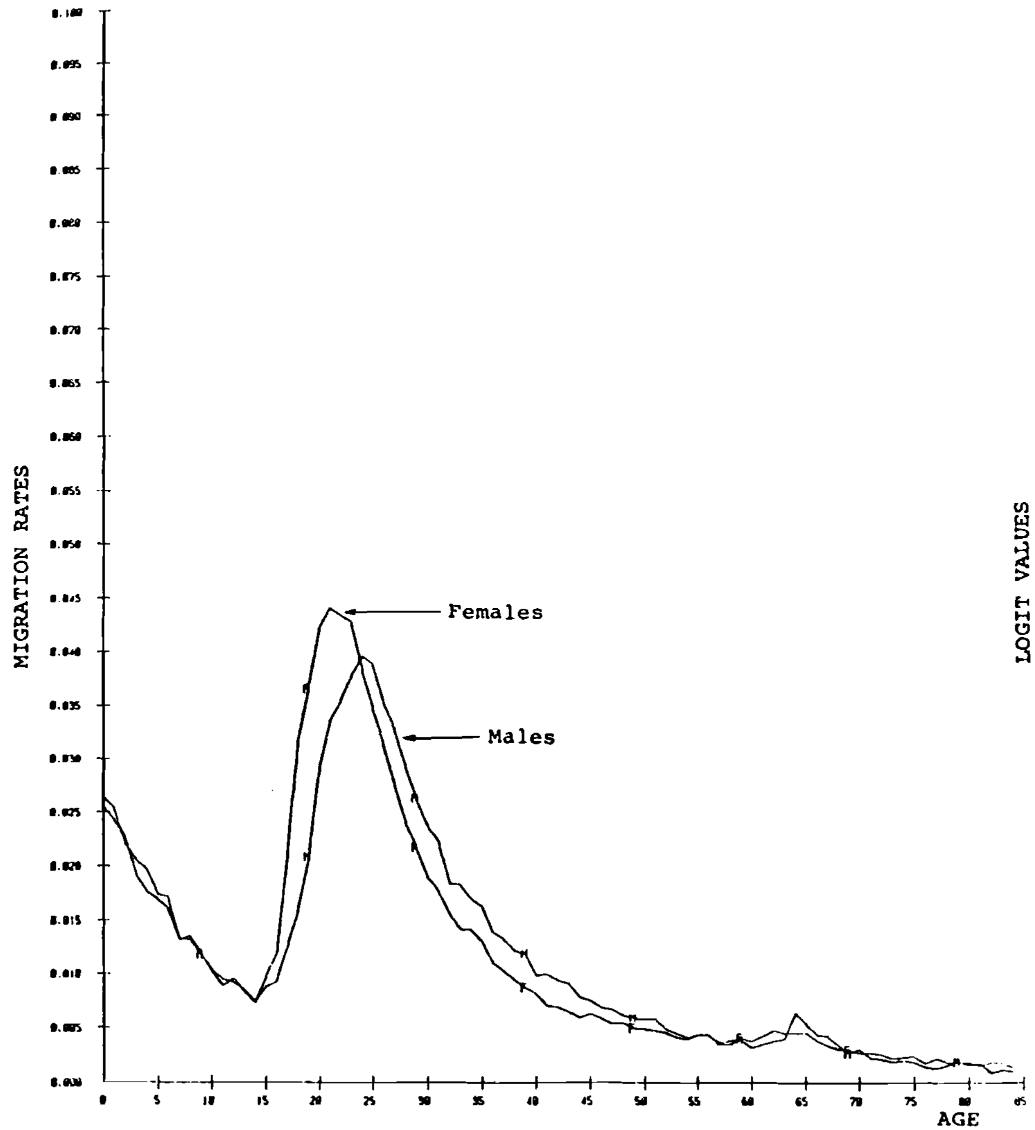
$$ {}_uY(x) = -0.4978 + 0.7612\ Y_s(x) $$

for males, and

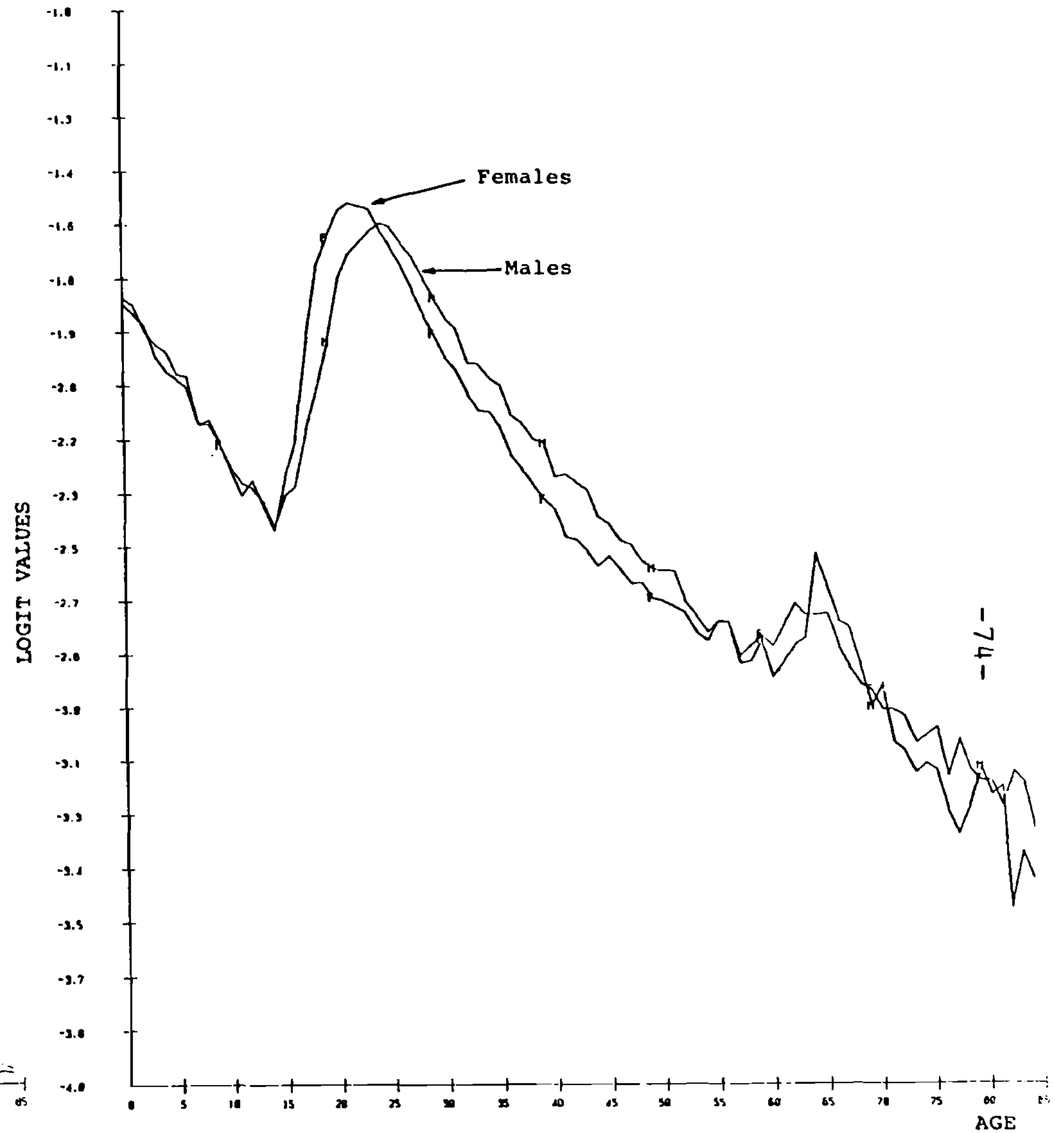
$$ {}_uY(x) = -0.3358 + 0.8345\ Y_s(x) $$

for females. The differences are minor for most of the Swedish data and so are their consequences for the fits of the synthetic Stockholm model schedules to the observed data and its graduated expression. Figure 5.2 illustrates both pairs of fits for Stockholm.

*Our standard schedules shall always have a unit GMR; hence the left subscript on ${}_uY_s(x)$ will be dropped henceforth.



a. National age specific migration rates.



b. Standard logit values

Figure 5.1 National age-specific migration rates and standard logit values: Sweden 1974.

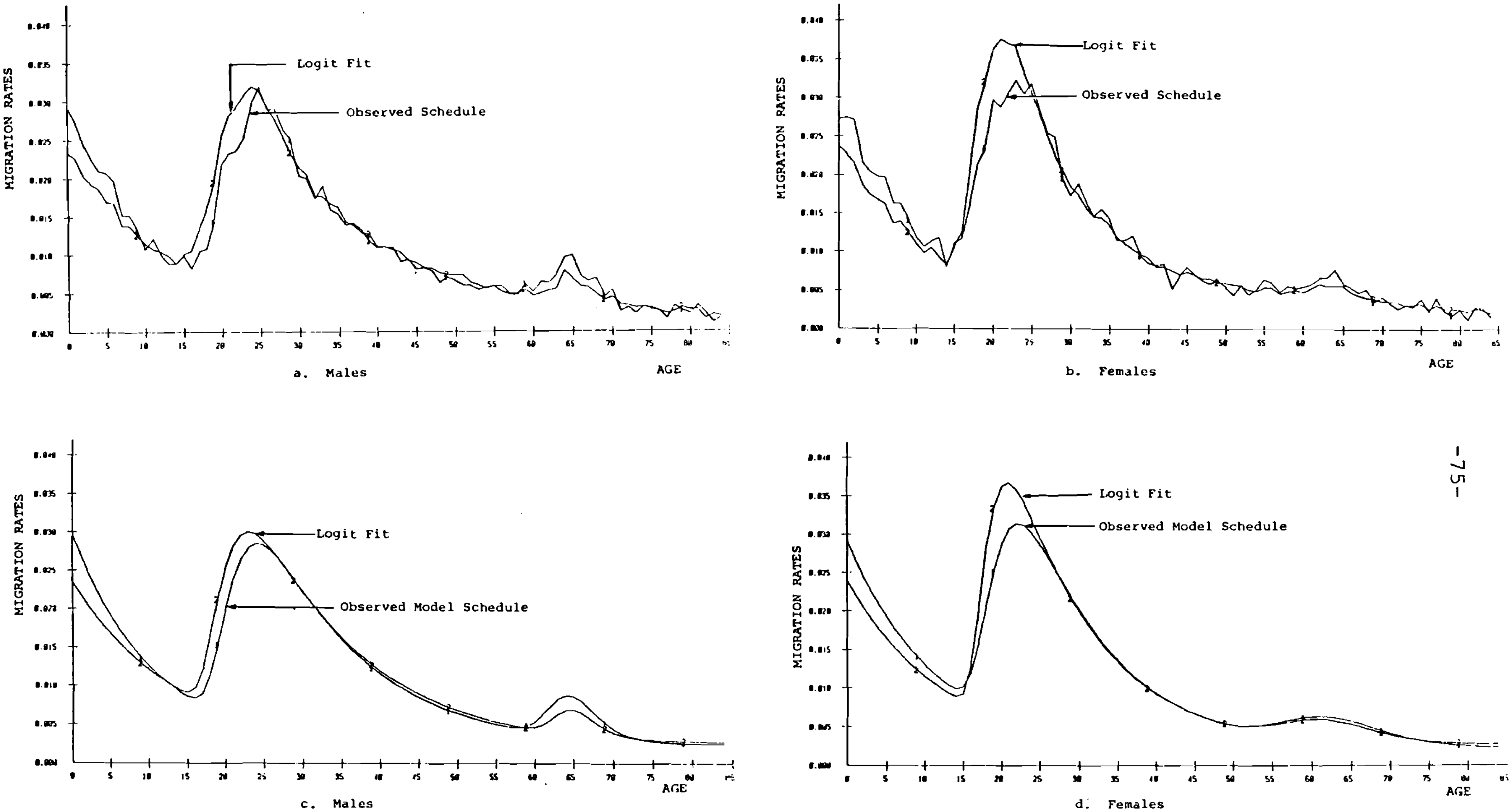


Figure 5.2 Logit fits to observed and model migration schedules for males and females leaving Stockholm, 1974.

Henceforth we shall deal only with graduated fits inasmuch as all of our non-Swedish data are for five-year age intervals and therefore need to be graduated first in order to provide single-year profiles by means of interpolation.

Figure 5.3 presents male national standards for Sweden, the United Kingdom, Japan, and the Netherlands. The differences in age profile are marked. Only the Swedish and the U.K. standards exhibit a retirement peak. Japan's profile is described without one because the age distribution of migrants given by the census data ends with the open interval of 65 years and over. The data for the Netherlands, on the other hand, show a definite upward slope at the post-labor force ages and therefore have been graduated with the 9-parameter model schedule with an "upward slope".

Regressing the logits of the age-specific outmigration rates of each region on those of its national standard (the GMRs of both first being scaled to unity) gives the estimated values for γ and ρ that are set out in Table 5.1. Reversing the procedure and combining selected values of γ and ρ with a national standard of logit values, produces the GMRs set out in Table 5.2. The latter table identifies the following important regularity: *whenever* $\gamma = 2(\rho - 1)$ *then the GMR of the synthetic model schedule is approximately unity.* Regressions of the form

$$\gamma = d_0 + d_1 \rho$$

fitted to our data for Sweden, the U.K., Japan, and the Netherlands, consistently produce estimates for d_0 and d_1 that are approximately equal to 2 in magnitude and that differ only in sign, i.e.,

$$\hat{d}_0 = -2 \qquad \text{and} \qquad \hat{d}_1 = +2$$

Thus

$$\gamma = -2 + 2\rho = 2(\rho - 1)$$

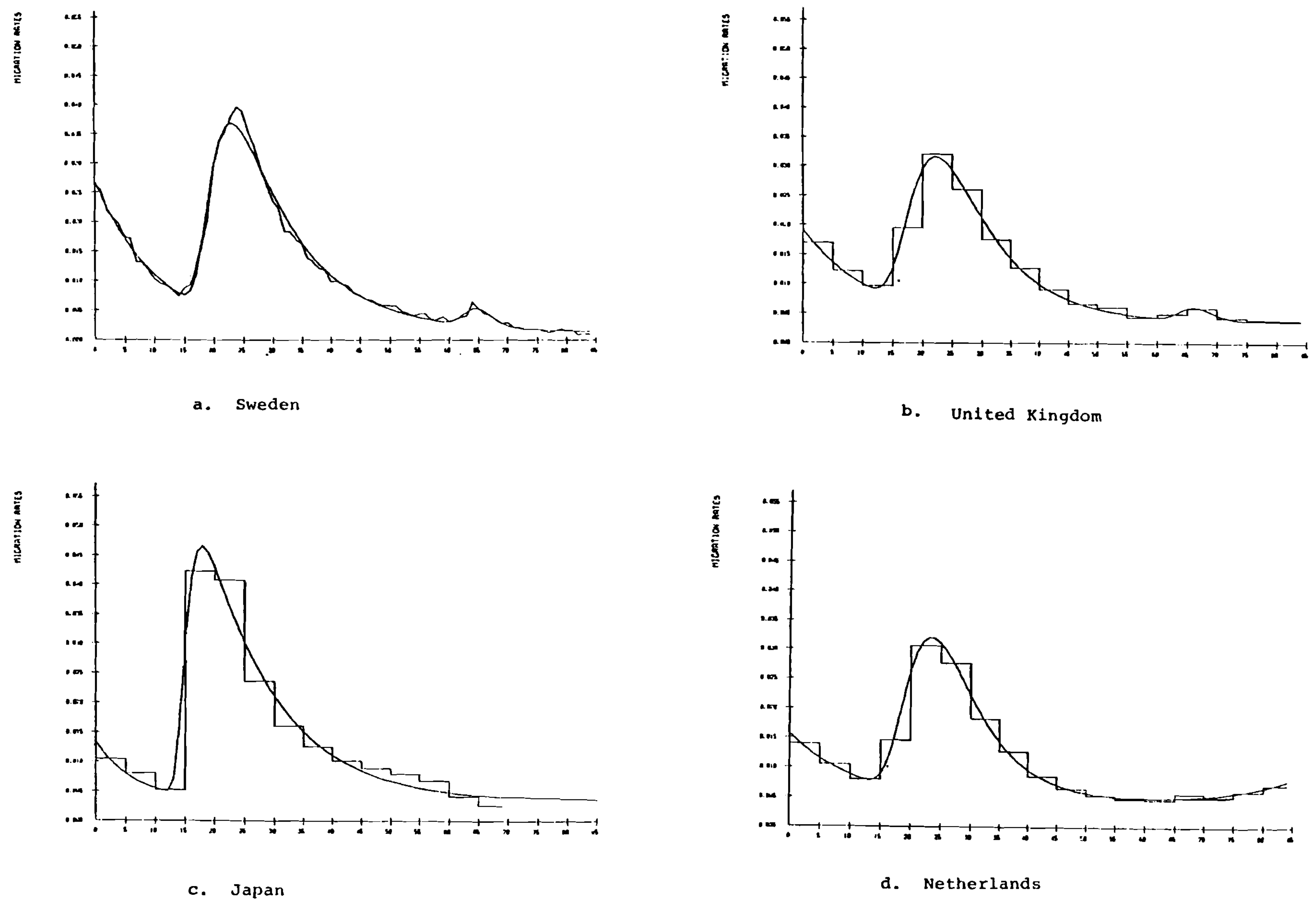


Figure 5.3 National male standard schedules: Sweden, the United Kingdom, Japan, the Netherlands.

Table 5.1. Estimated logit model parameters.

a.	Sweden	γ		ρ	
		Males	Females	Males	Females
1.	Stockholm	-0.4978	-0.3358	0.7612	0.8345
2.	East Middle	-0.1719	-0.0943	0.9214	0.9588
3.	South Middle	-0.0341	-0.0129	0.9939	1.0053
4.	South	-0.0669	-0.0005	0.9773	1.0090
5.	West	-0.0724	-0.0787	0.9697	0.9665
6.	North Middle	-0.0130	-0.0738	1.0051	0.9801
7.	Lower North	-0.0706	-0.0693	0.9852	0.9901
8.	Upper North	-0.2946	-0.2004	0.8768	0.9278

b.	United Kingdom	γ		ρ	
		Males	Females	Males	Females
1.	North	0.0604	0.0259	1.0326	1.0154
2.	Yorkshire	0.1464	0.2303	1.0699	1.1097
3.	North West	-0.2577	-0.0480	0.8826	0.9789
4.	East Middle	0.2730	0.1774	1.1276	1.0828
5.	West Middle	0.1768	0.1300	1.0816	1.0614
6.	East Anglia	0.0838	0.1966	1.0389	1.0918
7.	South East	-0.3324	-0.2959	0.8449	0.8626
8.	South West	0.3395	0.1247	1.1625	1.0588
9.	Wales	0.1416	-0.0144	1.0717	0.9976
10.	Scotland	0.5269	0.8599	1.2512	1.4074

Table 5.1. Estimated logit model parameters (continued).

c. Japan	γ		ρ	
	Males	Females	Males	Females
1. Hokkaido	-0.1075	-0.4254	0.9519	0.7931
2. Tohoku	-0.6740	0.0975	0.7008	1.0747
3. Kanto	-0.5251	-0.7071	0.7581	0.6753
4. Chubu	0.2507	0.0984	1.1351	1.0509
5. Kinki	0.1971	-0.3372	1.0916	0.8418
6. Chugoku	0.3687	0.2055	1.1973	1.1066
7. Shikoku	-0.0356	0.1680	1.0098	1.1009
8. Kyushu	-0.2333	0.3389	0.9009	1.1738

d. Netherlands	γ		ρ	
	Males	Females	Males	Females
1. Groningen	0.1434	0.1136	1.0705	1.0550
2. Friesland	0.0222	-0.1122	1.0160	0.9507
3. Drenthe	0.1835	-0.0103	1.0920	0.9982
4. Overijssel	0.2430	0.2902	1.1445	1.1403
5. Gelderland	0.1714	0.1103	1.0945	1.0541
6. Utrecht	-0.0493	0.1539	1.0000	1.0762
7. Noord-Holland	-0.1172	-0.2586	0.9549	0.8778
8. Zuid-Holland	-0.1746	-0.2075	0.9292	0.9014
9. Zeeland	0.3046	-0.0224	1.1537	0.9907
10. Noord-Brabant	0.2353	0.0135	1.1427	1.0092
11. Limburg	0.2923	0.1657	1.1679	1.0830

Table 5.2. Estimated GMRs for different logit parameter values and male standard schedules.

Sweden		ρ = 0.75	ρ = 1.00	ρ = 1.25
γ	-0.5	1.04	0.37	0.14
γ	0	2.74	1.00	0.37
γ	0.5	6.91	2.63	1.00

United Kingdom		ρ = 0.75	ρ = 1.00	ρ = 1.25
γ	-0.5	1.07	0.37	0.13
γ	0	2.82	1.00	0.36
γ	0.5	7.15	2.64	0.96

Japan		ρ = 0.75	ρ = 1.00	ρ = 1.25
γ	-0.5	1.04	0.37	0.14
γ	0	2.75	1.00	0.37
γ	0.5	6.94	2.62	1.00

Netherlands		ρ = 0.75	ρ = 1.00	ρ = 1.25
γ	-0.5	1.08	0.37	0.13
γ	0	2.87	1.00	0.35
γ	0.5	7.32	2.65	0.94

We have noted before that when $\gamma = 0$ and $\rho = 1$, the synthetic model schedule is identical to the standard. Moreover since the GMR of the standard is always unity, values of γ and ρ that satisfy the equality $\gamma = 2(\rho - 1)$ guarantee a GMR of unity for the synthetic schedule. What are the effects of other combinations of values for these two parameters?

Figure 5.4 illustrates how the Swedish male standard schedule is transformed when γ and ρ are assigned particular pairs of values. Figure 5.4a shows that fixing $\gamma = 0$ and increasing ρ from 0.75 to 1.25 lowers the schedule, giving migration rates that are smaller in value than those of the standard. On the other hand, fixing $\rho = 0.75$, and increasing γ from -1 to 0 raises the schedule, according to Figure 5.4b. Finally, fixing the GMR = 1 by selecting values of γ and ρ that satisfy the equality $\gamma = 2(\rho - 1)$ shows that as γ and ρ both increase, so does the degree of labor dominance exhibited by the synthetic schedule. For example, moving from a synthetic schedule with $\gamma = -0.5$ and $\rho = 0.75$ to one with $\gamma = 0.5$ and $\rho = 1.25$ does not alter the area under the curve (GMR = 1) but it does increase its labor dominance (Figure 5.4c).

Figure 5.5 compares the behavior of the Swedish male standard with those of the U.K., Japan, and the Netherlands, as γ and ρ are assigned values of -0.5, 0, +0.5 and 0.75, 1.0, 1.25, respectively. In all cases, increases in γ and ρ values lead to more labor dominant profiles. Note that, whereas the Swedish curve shows three points of intersection, the Japanese profile exhibits only two. This suggests that it might be useful to distinguish families of standard schedules according to the number and locations along the age axis of such intersection points.

5.3 The Basic Standard Schedule

The comparative analysis of national and interregional migration patterns carried out in Section 3 identified at least three distinct families of age profiles. First, there was the 11-parameter *basic model migration schedule* with a retirement peak that described adequately a number of interregional flows, for

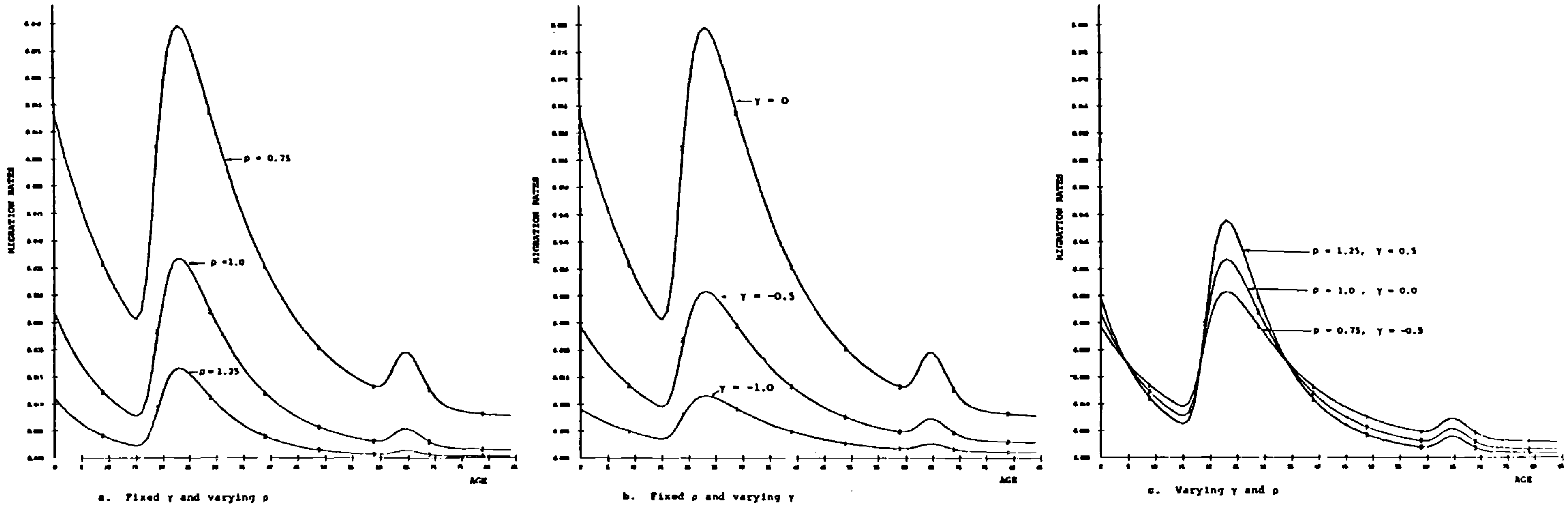


Figure 5.4 Sensitivity of logit model schedule to variations in γ and ρ: Swedish male standard schedules.

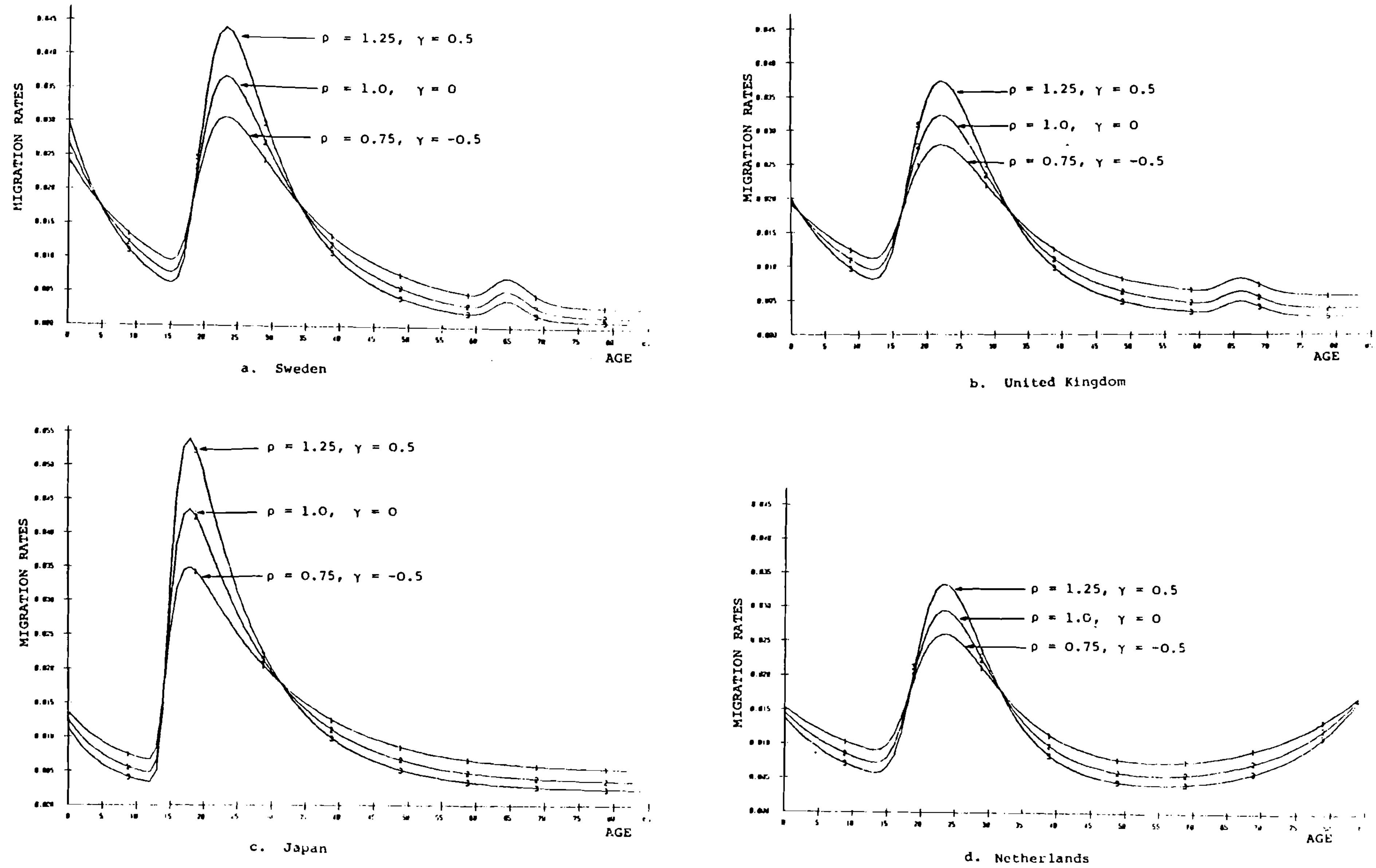


Figure 5.5 Sensitivity to logit model schedule to variations in γ and ρ: four national male standard schedules.

example, the age profiles of outmigrants leaving capital regions such as Stockholm and London. The elimination of the retirement peak gave rise to the 7-parameter *reduced form* of this basic schedule, a form that was used to describe a large number of labor dominant profiles and the age patterns of migration schedules with a single open-ended age interval for the post-labor force population, for example, Japan's migration schedules. Finally, the existence of a monotonically rising tail in migration schedules such as those exhibited by the Dutch data led to the definition of a third profile: the 9-parameter *upward-sloping model migration schedule*. These three fundamental age profiles define our three families of model migration schedules.

Within each family of schedules, a number of key parameters or variables may be put forward in order to further classify different categories of migration profiles. For example, in Section 3 we identified the special importance of the following aspects of shape and location along the age axis:

1. PEAKING: early peaking vs. late peaking (μ_2)
2. DOMINANCE: child dominance vs. labor dominance (δ_{12})
3. SYMMETRY: labor symmetry vs. labor asymmetry (σ_2)
4. REGULARITY: parental regularity vs. parental irregularity (β_{12})

These fundamental families and 4 key parameters give rise to a large variety of standard schedules. For example, even if the 4 key parameters are restricted to only dichotomous values, one already needs $2^4 = 16$ standard schedules. If, in addition, the sexes are to be differentiated, then 32 standard schedules are a minimum. A large number of standard schedules would make the logit approach a less desirable alternative. Hence we shall examine the feasibility of adopting only a single standard for each sex and assume that the shape of the post-labor force part of the schedule may be determined exogenously.*

*In tests of our logit migration system, therefore, we shall always set the post-labor force retirement peak or upward slope equal to observed model schedule values.

Table 5.3 presents the median parameter values of the 164 male and 172 female model schedules that have no retirement peak, among the interregional schedules calculated for Sweden, the United Kingdom, and Japan. This data base is precisely the same one that was used in Section 4 to develop the basic regression equations set out in Table 4.2.

5.4 Using the Basic Standard Schedule

Given a standard schedule and a few observed rates, such as M(0 - 4) and M(20 - 24), for example, how can one find estimates for γ and ρ, and with those estimates go on to obtain the entire synthetic schedule?

First, taking logits of the two observed migration rates gives Y(0 -4) and Y(20 - 24) and associating these two logits with the pair of corresponding logits for the standard gives

$$Y(0-4) = \gamma + \rho\, Y_s(0-4)$$

$$Y(20-24) = \gamma + \rho\, Y_s(20-24)$$

Solving these two equations in two unknowns gives crude estimates for γ and ρ, and applying them to the standard schedule's full set of logits results in a set of logits for the synthetic schedule. From these one can obtain the migration rates, as shown in subsection 5.2. However, tests of such a procedure with the migration data for Sweden, the United Kingdom, Japan, and the Netherlands indicate that the method is very erratic in the quality of the fits that it produces and, therefore, more refined procedures are necessary. Such procedures (for the case of mortality) are described in the literature on the Brass logit system (for example, in Brass 1975 and Carrier and Goh, 1972).

Table 5.3 The basic standard median migration schedule.

A. MALES

δ_{12} = 0.33571		μ_2 = 19.67385
σ_2 = 3.42123	a_1 = 0.01992	a_2 = 0.06471
β_{12} = 1.02442	α_1 = 0.10390	α_2 = 0.10618
δ_{1c} = 6.79034	c = 0.00263	λ_2 = 0.37244

B. FEMALES

δ_{12} = 0.32367		μ_2 = 19.88280
σ_2 = 2.89784	a_1 = 0.02209	a_2 = 0.06935
β_{12} = 0.84944	α_1 = 0.10883	α_2 = 0.13434
δ_{1c} = 5.95881	c = 0.00350	λ_2 = 0.37870

A reasonable first approximation to an improved estimation method is suggested by the regression approach in Section 4. Imagine a regression of ρ on the M-ratio, M(0 - 4)/M(20 - 24). Starting with the basic standard median migration schedule and varying ρ within the range of observed values, one may obtain a corresponding set of M-ratios. Associating ρ and the M-ratio in this way, one may proceed further and use the relational equation to estimate $\hat{\gamma}$ from $\hat{\rho}$:

$$\hat{\gamma} = 2(\hat{\rho} - 1)$$

Using the basic median standard, for example, gives the following regression equation:

$$\hat{\rho}_F = 2.690 - 3.062\ M$$

for females, and

$$\hat{\rho}_M = 2.510 - 2.983\ M$$

for males.

A further simplification can be made by forcing the regression line to pass through the origin, as in Section 4. Since the resulting regression coefficient has a negative sign and the intercept exhibits roughly the same absolute value, but with a positive sign, the regression equations take on the form

$$\hat{\rho}_F = 2.226\ (1 - M)$$

for females, and

$$\hat{\rho}_M = 2.101\ (1 - M)$$

for males.

Given a standard schedule and estimates for γ and ρ, one can proceed to compute the associated synthetic model migration schedule. Figure 5.6 illustrates representative examples of the quality of fit obtained using this procedure. Two synthetic schedules are illustrated with each observed model migration schedule: those calculated with the interpolated 85 single-year of age observations and the resulting least-squares estimates of γ and ρ, and those computed using the above regression equations of ρ on the M-ratio.

5.5 Applications

The male and female median standard schedules set out in Table 5.3 are similar, and a simplified logit system could use their average parameter values to define a unisexual standard. A rough rounding of these averages would simplify matters even more. In applying the logit migration system to data on the Philippines, we shall adopt both of the above simplifications and use the simplified basic standard migration schedule presented in Table 5.4.

The simplified basic schedule in Table 5.4 together with estimates of $\hat{\rho}$ obtained with the pair of M-ratio regressions set out earlier produced the synthetic schedules for the Philippines illustrated in Figure 5.7. As in Section 4, the results are unsatisfactory and underscore the need to define a more appropriate standard for Third World countries.

Table 5.4 The simplified basic standard migration schedule.

$\delta_{12} = 1/3$		$\mu_2 = 20$
$\sigma_2 = 4$	$a_1 = 0.02$	$a_2 = 0.06$
$\beta_{12} = 1$	$\alpha_1 = 0.10$	$\alpha_2 = 0.10$
$\delta_{1c} = 6$	$c = 0.003$	$\lambda_2 = 0.40$

The values of a_1, a_2, and c are initial values only and need to be scaled proportionately to ensure a unit GMR.

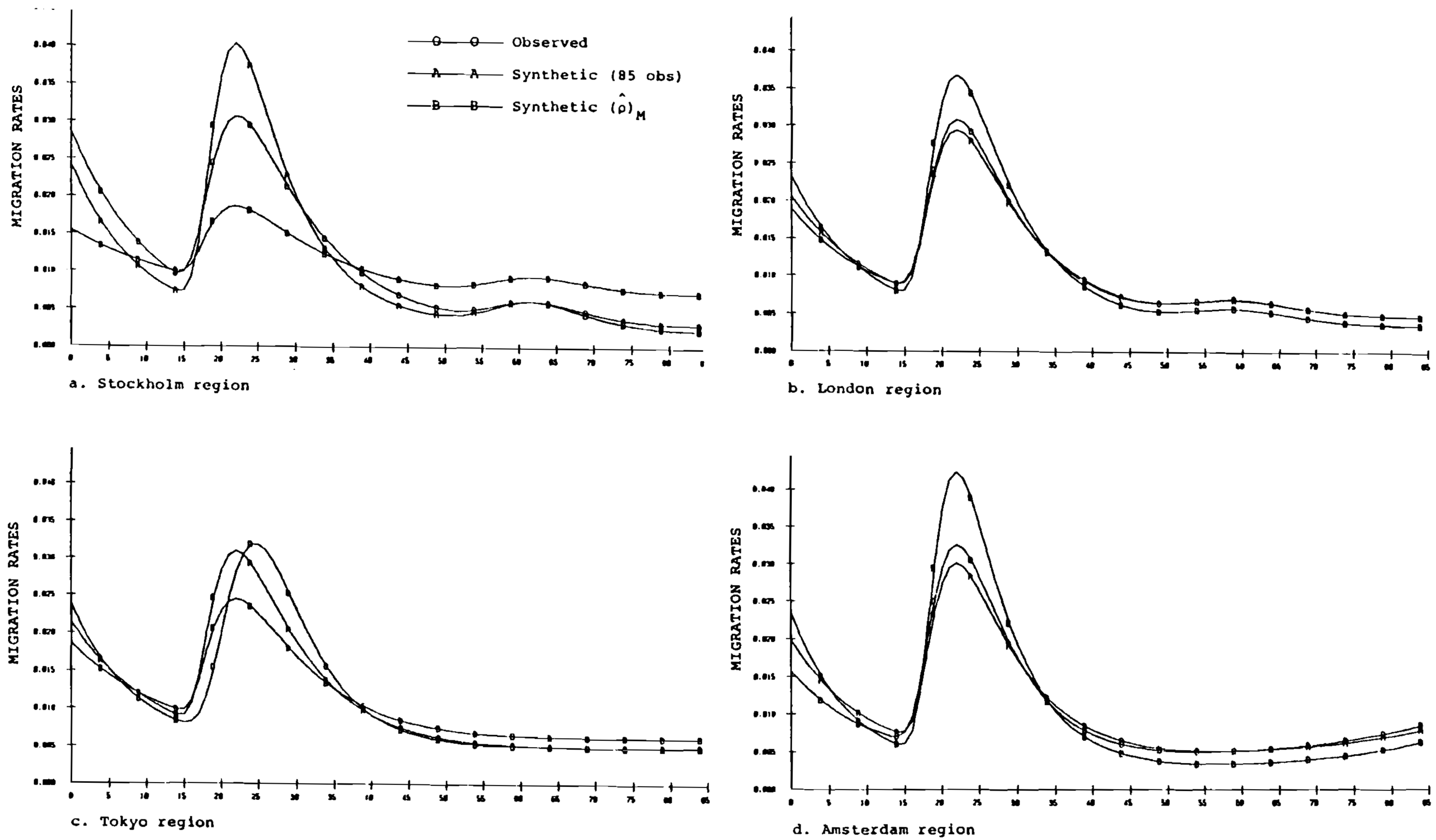
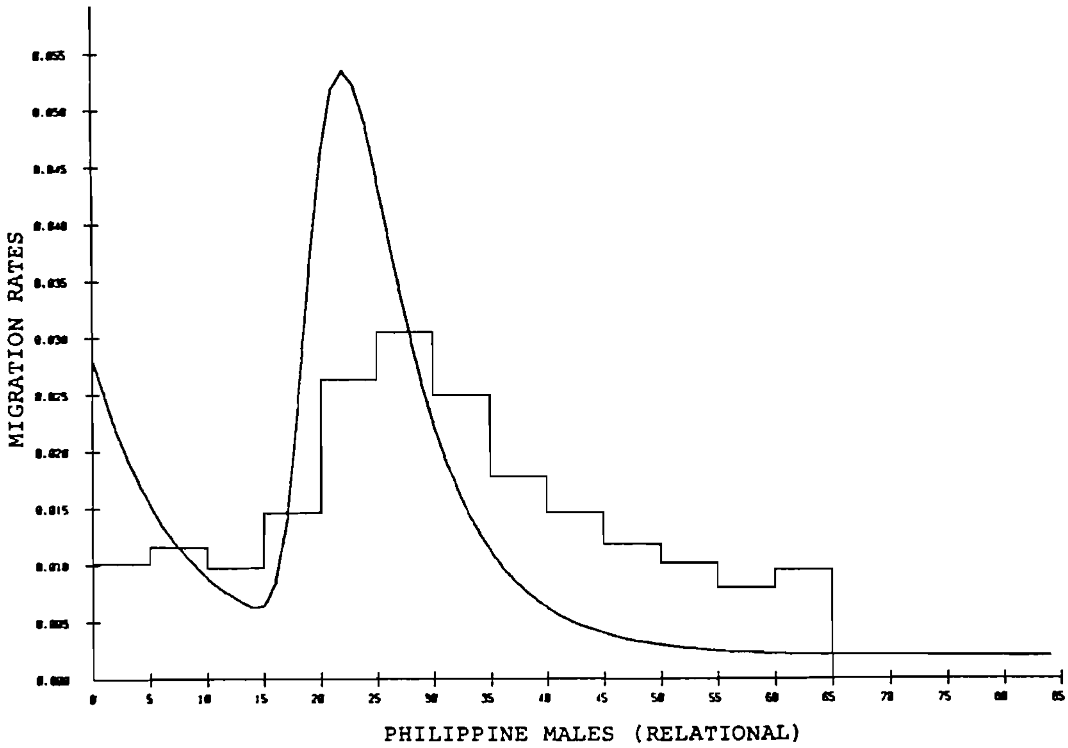


Figure 5.6 The fits of relational synthetic model migration schedules to data for the female populations of Stockholm, London, Tokyo and Amsterdam.



Source: del Mar Pernia (1977)

Figure 5.7 A synthetic model migration schedule for Philippine males: the relational approach.

6. CONCLUSION

This paper began with the observation that empirical regularities characterize observed multistate schedules in ways that are no less important than the corresponding well-established regularities in observed fertility or mortality schedules. Section 2 was devoted to defining mathematically such regularities in observed migration schedules in order to exploit the notational, computational, and analytical advantages that such a formulation provides. Section 3 reported on the results of an examination of over 500 migration schedules that underscored the broad generality of the model migration schedule proposed and helped to identify a number of families of such schedules.

Regularities in age profiles lead naturally to the development of hypothetical or synthetic model migration schedules that might be suitable for studies of populations with inadequate or defective data. Drawing on techniques used in the corresponding literature in fertility and mortality, Sections 4 and 5 outlined two alternative perspectives for inferring migration patterns in the absence of accurate migration data.

Of what use, then, is the model migration schedule defined in this study? What are some of its concrete practical applications?

The model migration schedule may be used to *graduate* observed data, thereby smoothing out irregularities and ascribing to the data summary measures that can be used for comparative analysis. It may be used to *interpolate* to single years of age, observed migration schedules that are reported for wider age intervals. Assessments of the *reliability* of empirical migration data and indications of appropriate strategies for their *correction* are aided by the availability of standard families of migration schedules. Finally, such schedules also may be used to help resolve problems caused by *missing data.*

The analysis of national migration age patterns reported in this study seeks to demonstrate the utility of examining the regularities in age profile exhibited by empirical schedules of interregional migration. Although, data limitations have restricted some of the findings to conjectures, a modest start has been made. It is hoped that the results reported here will induce others to devote more attention to this topic.

APPENDIX A

APPENDIX A

NONLINEAR PARAMETER ESTIMATION IN MODEL MIGRATION SCHEDULES

This appendix will attempt to briefly illustrate the mathematical programming procedure used to estimate the parameters of the model migration schedule. The nonlinear estimation problem may be defined as the search for the "best" parameter values for the function:

$$\begin{aligned} M(x) &= a_1 e^{-\alpha_1 x} \\ &+ a_2 e^{-\alpha_2(x-\mu_2)-e^{-\lambda_2(x-\mu_2)}} \\ &+ a_3 e^{-\alpha_3(x-\mu_3)-e^{-\lambda_3(x-\mu_3)}} \\ &+ c \quad ; \end{aligned} \tag{A1}$$

best in the sense that a pre-defined objective function is minimized when the parameters take on these values.

This problem is the classical one of nonlinear parameter estimation in unconstrained optimization. All of the available methods start with a set of given initial conditions, or initial guesses of the parameter values, from which they begin a search for better estimates following specific convergence criteria. The iterative sequence ends after a finite number of iterations, and the solution is accepted as giving the "best" estimates for the parameters.

The problem of selecting a "good" method has been usefully summarized by Bard (1974, p.84) as follows:

> ...no single method has emerged which is best for the solution of all nonlinear programming problems. One cannot even hope that a "best" method will ever be found, since problems vary so much in size and

nature. For parameter estimation problems we must seek methods which are particularly suitable to the special nature of these problems which may be characterized as follows:

1. A relatively small number of unknowns, rarely exceeding a dozen or so.
2. A highly nonlinear (though continuous and differentiable) objective function, whose computation is often very time consuming.
3. A relatively small number (sometimes zero) of inequality constraints. Those are usually of a very simple nature, e.g., upper and lower bounds.
4. No equality constraints, except in the case of exact structural models (where, incidentally, the number of unknowns is large). ...

For computational convenience, we have chosen the Marquardt method (Levenberg, 1944; Marquardt, 1963). This method seeks out a parameter vector P* that minimizes the following objective function:

$$\phi(P) = ||f_P||_2^2 \qquad \text{(A2)}$$

where f_P is the residual vector and $||\cdot||_2$ represents the known Euclidean vector norm. For the case of a model schedule with a retirement peak, vector P has the following elements:

$$P^T = [\, a_1, \alpha_1, a_2, \alpha_2, \mu_2, \lambda_2, a_3, \alpha_3, \mu_3, \lambda_3, c\,] \quad . \qquad \text{(A3)}$$

The elements of the vector f_P can be computed by either of the following two expressions:

$$f_P(x) = (M(x) - \hat{M}_P(x)) \qquad \text{(A4)}$$

or

$$f_P(x) = (M(x) - \hat{M}_P(x)) \Big/ \hat{M}_P^2(x) \qquad \text{(A5)}$$

where M(x) is the observed value at age x and $\hat{M}_P(x)$ is the estimated value using equation (A1) and a given vector P of parameter estimates.

By introducing equation (A4), in the objective function set out in equation (A2), the sum of squares is minimized; if, on the other hand, equation (A5) is introduced instead, the chi-square statistic is minimized.

In matrix notation, the Levenberg-Marquardt method follows the next iterative sequence:

$$P_{q+1} = P_q - \{J_q^T J_q + \lambda_q D_q\}^{-1} J_q^T f P_q$$

where λ is a non-negative parameter adjusted to ensure that at each iteration the function (A2) is reduced, J_q denotes de Jacobian matrix of $\phi(P)$ evaluated at the q iteration, and D is a diagonal matrix equal to the diagonal of $J^T J$.

The principal difficulty in nonlinear parameter estimation is that of convergence, and this method is no exception. The algorithm starts out by assuming some initial parameters, and then a new vector P is estimated according to the value of λ, which in turn is also modified following some gradient criteria. Once some given stopping values are achieved, vector P* is assumed to be the optimum. However, in most cases, this P* reflects local minima that may be improved with better initial conditions and a different set of gradient criteria.

Using the data described in this paper, several experiments were carried out to examine the variation in parameter estimates that can result from different initial conditions (assuming Newton's gradient criteria).* Among the cases studied, the most significant differences were found for the vector P with eleven parameters, principally among the parameters of the retirement component. For schedules without the retirement peak, the vector P* shows no variation in most cases.

The impact of the gradient criteria on the optimal vector P* was also analyzed, using the Newton and the Steepest Descent methods. The effects of these two alternatives were reflected in

*For a complete description of gradient methods, see Fiacco (1968) and Bard (1974).

the computing times but not in the values of the vector P*. Nevertheless, Bard (1974) has suggested that both methods can create problems in the estimation, and therefore they should be used with caution, in order to avoid unrealistic parameter estimates. It appears that the initial parameter values may be improved by meansof an interactive approach suggested by Benson (1979).

APPENDIX B

APPENDIX B

ESTIMATED SUMMARY STATISTICS OF NATIONAL PARAMETERS AND VARIABLES OF THE REDUCED SETS OF OBSERVED MODEL MIGRATION SCHEDULES

Symbols

GMR (OBS)	Observed gross migraproduction rate
GMR (MMS)	Unit gross migraproduction rate
MAE%M	Goodness-of-fit index*
A1	a_1
ALPHA1	α_1
A2	a_2
MU2	μ_2
ALPHA2	α_2
LAMBDA2	λ_2
A3	a_3
MU3	μ_3
ALPHA3	α_3
LAMBDA3	λ_3
C	c
MEAN AGE	Mean age of migration schedule
%(0-14)	Percentage of GMR in 0-14 age interval
%(15-64)	Percentage of GMR in 15-64 age interval
%(65+)	Percentage of GMR in 65 and over age interval
DELTA1C	$\delta_{1c} = a_1/c$
DELTA12	$\delta_{12} = a_1/a_2$
DELTA32	$\delta_{32} = a_3/a_2$
BETA12	$\beta_{12} = \alpha_1/\alpha_2$
SIGMA2	$\sigma_2 = \lambda_2/\alpha_2$
SIGMA3	$\sigma_3 = \lambda_3/\alpha_3$
X LOW	x_ℓ = the low point
X HIGH	x_h = the high point
X RET.	x_r = the retirement peak
X SHIFT	X = the labor force shift
A	A = the parental shift
B	B = the jump

*Mean absolute error as a percentage of the observed mean.

SWEDEN MALES 1 WITHOUT RETIREMENT PEAK N = 48

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GHR (OBS)	0.02478	0.83908	0.20509	0.15766	0.14693	0.16162	0.78806
GHR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	6.32477	62.98674	26.09850	22.98794	20.49026	11.93332	0.45724
A1	0.01829	0.04891	0.02894	0.02750	0.02595	0.00708	0.24465
ALPHA1	0.06495	0.40526	0.12372	0.11137	0.11600	0.05466	0.44179
A2	0.03624	0.12465	0.06739	0.06832	0.06718	0.01913	0.28392
MU2	16.05684	23.99384	20.50230	20.36539	20.42221	1.43641	0.07006
ALPHA2	0.05701	0.18775	0.10439	0.10426	0.10277	0.02843	0.27233
LAMBDA2	0.19407	1.76712	0.44762	0.38743	0.43003	0.26230	0.58598
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00000	0.00704	0.00264	0.00279	0.00246	0.00134	0.50760
MEAN AGE	24.71596	36.54450	29.73375	29.58655	30.03880	2.05835	0.06923
%(0-14)	13.88474	27.75659	22.20945	22.27053	21.51425	3.36488	0.15151
%(15-64)	61.50196	77.42499	69.71529	69.65226	71.85192	3.44397	0.04940
%(65+)	1.35294	17.31658	8.07528	8.23866	8.53658	2.82110	0.34935
DELTA1C	0.00000	33.70855	9.43123	8.72132	8.42714	5.85991	0.62133
DELTA12	0.17064	0.89970	0.46595	0.45039	0.57162	0.17371	0.37280
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.66868	3.51656	1.22123	1.14700	0.81107	0.47585	0.38965
SIGMA2	1.16055	24.23831	4.86348	3.94838	2.31444	3.98036	0.81842
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	8.72009	18.26030	15.62129	15.72025	15.87525	1.67033	0.10693
X HIGH	20.86036	26.19049	23.57146	23.67043	23.79193	1.25751	0.05335
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	2.90007	12.34028	7.95018	7.99018	8.09218	1.87450	0.23578
A	26.54375	37.28526	30.27044	29.85372	29.22913	2.00217	0.06614
B	0.01625	0.05504	0.03036	0.02954	0.02983	0.00762	0.25106

SWEDEN MALES 1 WITH RETIREMENT PEAK N = 9

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.05726	0.24937	0.16343	0.16041	0.23976	0.06846	0.41891
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	15.31033	39.60669	22.46128	18.73808	16.52515	8.75361	0.38972
A1	0.02010	0.03547	0.02644	0.02749	0.02745	0.00441	0.16694
ALPHA1	0.04069	0.12939	0.08476	0.08059	0.08061	0.02637	0.31112
A2	0.03431	0.08440	0.05139	0.04441	0.04182	0.01534	0.29844
MU2	19.79847	25.50892	21.24856	20.88873	20.08399	1.73551	0.08168
ALPHA2	0.07750	0.11222	0.09306	0.09343	0.07924	0.01123	0.12071
LAMBDA2	0.16894	0.61686	0.41581	0.43068	0.41530	0.13718	0.32991
A3	0.00001	0.00390	0.00056	0.00013	0.00020	0.00126	2.26618
MU3	71.79685	85.71539	76.71105	75.07949	73.88464	4.57307	0.05961
ALPHA3	0.27276	1.26871	0.84724	0.94211	1.11932	0.35752	0.42198
LAMBDA3	0.09179	0.20566	0.15819	0.18034	0.19997	0.04584	0.28979
C	0.00039	0.00453	0.00218	0.00181	0.00143	0.00126	0.57877
MEAN AGE	27.38409	34.12481	30.76871	30.73515	30.41742	2.07682	0.06750
%(0-14)	19.83781	26.52260	23.83921	24.40201	24.18293	2.06681	0.08670
%(15-64)	59.15461	74.10361	66.60196	67.11652	67.37656	4.57156	0.06864
%(65+)	6.05858	14.32279	9.55884	8.64010	6.47179	2.96092	0.30976
DELTA1C	6.06509	60.22449	17.91566	13.51922	14.18900	16.23569	0.90623
DELTA12	0.27933	0.80125	0.55066	0.53239	0.46200	0.16816	0.30538
DELTA32	0.00036	0.08854	0.01207	0.00240	0.00477	0.02871	2.37846
BETA12	0.42608	1.46937	0.92460	0.81842	0.79123	0.31735	0.34323
SIGMA2	1.60498	7.95960	4.60178	4.48710	3.82910	1.83530	0.39882
SIGMA3	0.14795	0.41012	0.20853	0.18449	0.18728	0.07805	0.37427
X LOW	15.47024	17.78029	16.49360	16.42026	16.50976	0.75926	0.04603
X HIGH	22.80041	27.76052	24.46156	23.97043	23.54443	1.50376	0.06147
X RET.	63.16779	68.95871	65.63027	64.87784	64.61552	2.00638	0.03057
X SHIFT	6.01014	12.19028	7.96796	7.47017	7.55517	1.88117	0.23609
A	25.07877	30.40369	28.66785	29.00578	28.53997	1.68724	0.05885
B	0.01345	0.03986	0.02360	0.02375	0.01741	0.00789	0.33434

SWEDEN FEMALES 1 WITHOUT RETIREMENT PEAK N = 54

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.02256	0.87818	0.20644	0.16573	0.15090	0.15964	0.77331
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	8.11798	60.83579	25.05564	20.65920	21.29676	11.07337	0.44195
A1	0.00952	0.04464	0.02648	0.02774	0.02884	0.00728	0.27500
ALPHA1	0.02108	0.19659	0.10800	0.11278	0.11761	0.03713	0.34382
A2	0.04018	0.18944	0.07616	0.06995	0.06257	0.02600	0.34134
MU2	17.33270	21.31304	19.09371	18.99365	18.72582	0.86976	0.04555
ALPHA2	0.07664	0.24522	0.12696	0.12185	0.11879	0.03726	0.29351
LAMBDA2	0.25622	1.49869	0.53687	0.48282	0.44259	0.19779	0.36842
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00000	0.00579	0.00288	0.00296	0.00318	0.00123	0.42521
MEAN AGE	24.51402	33.18372	28.98599	28.88618	28.41539	1.80056	0.06212
%(0-14)	9.37675	28.91071	22.04352	22.26965	20.12043	3.63470	0.16489
%(15-64)	61.93792	81.17286	69.30895	69.01508	68.67014	3.42040	0.04935
%(65+)	1.46164	14.17442	8.64754	8.77672	8.45367	2.40189	0.27775
DELTA1C	0.00000	34.70223	10.45738	8.68991	8.67556	7.10051	0.67899
DELTA12	0.05026	0.72119	0.38938	0.39909	0.41927	0.15910	0.40859
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.13332	1.53044	0.90442	0.92119	1.04145	0.33065	0.36559
SIGMA2	1.13861	12.23371	4.57128	3.97896	2.80288	2.14015	0.46817
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	13.19019	17.64029	15.25968	15.11023	14.74773	0.93022	0.06096
X HIGH	18.83032	23.70043	21.72038	21.71038	21.50888	1.03422	0.04762
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	2.89007	8.59020	6.46070	6.65015	6.02514	1.17260	0.18150
A	23.73040	30.35461	27.22177	27.26609	26.71129	1.47430	0.05416
B	0.01932	0.09111	0.03586	0.03357	0.03009	0.01126	0.31401

SWEDEN FEMALES 1 WITH RETIREMENT PEAK N = 3

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.13278	0.47590	0.28125	0.23508	0.14994	0.17616	0.62633
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	10.57396	20.49792	15.52629	15.50700	11.07016	4.96201	0.31959
A1	0.01944	0.03060	0.02384	0.02149	0.02000	0.00594	0.24915
ALPHA1	0.08182	0.10413	0.09284	0.09256	0.08294	0.01116	0.12018
A2	0.04146	0.07787	0.05491	0.04541	0.04328	0.01998	0.36383
MU2	18.17883	19.33387	18.86767	19.09032	18.23658	0.60886	0.03227
ALPHA2	0.09427	0.12621	0.10640	0.09871	0.09587	0.01730	0.16262
LAMBDA2	0.27430	0.58193	0.42440	0.41696	0.28968	0.15395	0.36275
A3	0.00001	0.00014	0.00009	0.00013	0.00013	0.00007	0.77509
MU3	73.38062	76.25882	74.78143	74.70483	73.52454	1.44063	0.01926
ALPHA3	0.90737	0.96737	0.93753	0.93784	0.91037	0.03000	0.03200
LAMBDA3	0.15760	0.18530	0.17028	0.16794	0.15899	0.01400	0.08220
C	0.00269	0.00444	0.00337	0.00297	0.00278	0.00094	0.27921
MEAN AGE	26.79165	33.03862	30.71901	30.32676	29.00400	2.15048	0.07000
%(0-14)	19.06055	26.38641	23.02162	23.61790	19.42684	3.69915	0.16068
%(15-64)	62.63004	72.57767	66.03382	62.89375	63.12742	5.66867	0.08584
%(65+)	8.36178	13.75206	10.94456	10.71983	8.63129	2.70216	0.24690
DELTA1C	4.83614	10.29016	7.45207	7.22991	5.10884	2.73379	0.36685
DELTA12	0.24967	0.67379	0.48056	0.51823	0.27088	0.21455	0.44646
DELTA32	0.00019	0.00320	0.00214	0.00302	0.00305	0.00169	0.79014
BETA12	0.73337	1.10458	0.88095	0.82889	0.75193	0.19275	0.21683
SIGMA2	2.77878	4.61088	3.93750	4.42285	2.87038	1.00788	0.25597
SIGMA3	0.16804	0.19155	0.18156	0.18508	0.16922	0.01214	0.06689
X LOW	13.17019	15.30024	14.44355	14.86023	13.27669	1.12450	0.07785
X HIGH	20.74036	22.63040	21.90372	22.34040	20.83486	1.01788	0.04647
X RET.	64.39774	64.81783	64.60445	64.59778	64.41875	0.21012	0.00325
X SHIFT	5.88013	9.17021	7.46017	7.33017	6.04463	1.64889	0.22103
A	25.02372	27.84035	26.11944	25.49425	25.16455	1.50881	0.05777
B	0.01454	0.04145	0.02575	0.02126	0.01589	0.01401	0.54391

U.K. MALES WITHOUT RETIREMENT PEAK N = 59

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.02521	1.05541	0.15658	0.09630	0.07672	0.18257	1.16594
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	5.59109	25.51109	11.66710	10.93198	10.57109	4.25471	0.36468
A1	0.00852	0.04154	0.02073	0.01979	0.01678	0.00665	0.32070
ALPHA1	0.02167	0.26591	0.09937	0.09878	0.10715	0.04812	0.48427
A2	0.01559	0.11192	0.05946	0.06078	0.06857	0.01676	0.28177
MU2	14.68744	43.96579	22.00013	20.11916	19.07919	5.36015	0.24364
ALPHA2	0.06427	0.27413	0.12654	0.11611	0.09575	0.04760	0.37617
LAMBDA2	0.06051	0.90653	0.25947	0.24042	0.27202	0.15062	0.58048
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00000	0.00587	0.00286	0.00280	0.00205	0.00155	0.54198
MEAN AGE	25.15435	36.36529	30.65815	30.45968	30.19927	2.60321	0.08491
%(0-14)	15.19911	29.69068	20.88979	20.46826	18.82200	3.45535	0.16541
%(15-64)	60.27293	78.68406	69.70760	69.30323	66.71683	3.85501	0.05530
%(65+)	1.35734	16.64217	9.40261	9.56441	6.70703	3.74348	0.39813
DELTA1C	0.00000	108.15191	10.09796	6.40383	5.40760	16.02651	1.58710
DELTA12	0.13305	1.53679	0.39065	0.34557	0.20324	0.22076	0.56511
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.08403	2.64845	0.89863	0.69816	0.46869	0.56755	0.63157
SIGMA2	0.39349	11.98600	2.50122	2.07064	0.88762	2.01686	0.80635
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	6.71004	17.19028	12.70424	12.61017	12.56417	1.82025	0.14328
X HIGH	17.11028	28.14053	23.16957	22.82041	22.07389	1.81849	0.07849
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	4.50010	16.93039	10.46532	10.35024	10.09373	2.21174	0.21134
A	22.33532	34.75360	30.56486	30.77489	31.64904	2.64842	0.08665
B	0.01107	0.04390	0.02347	0.02331	0.02256	0.00595	0.25341

U.K. MALES WITH RETIREMENT PEAK N = 23

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.04391	0.43105	0.14234	0.11835	0.06327	0.09731	0.68369
GRR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.50555	11.74034	7.37499	7.19781	7.03773	1.81126	0.24559
A1	0.01006	0.02859	0.01629	0.01595	0.01099	0.00455	0.27924
ALPHA1	0.03347	0.13892	0.07963	0.07539	0.07038	0.02759	0.34650
A2	0.03143	0.07553	0.05305	0.05147	0.05127	0.01226	0.23102
MU2	16.66712	28.29313	20.42433	19.52551	19.57362	2.85049	0.13956
ALPHA2	0.07522	0.22997	0.11999	0.10031	0.09843	0.04072	0.33934
LAMBDA2	0.13783	0.59710	0.30095	0.27547	0.25265	0.12285	0.40820
A3	0.00003	0.02391	0.00657	0.00036	0.00122	0.00821	1.25102
MU3	60.14665	78.18250	71.11082	72.81990	73.67355	5.26965	0.07410
ALPHA3	0.09157	1.46849	0.69225	0.73938	0.29811	0.39070	0.56440
LAMBDA3	0.14822	0.79255	0.30877	0.20966	0.18044	0.19968	0.64670
C	0.00135	0.00581	0.00350	0.00319	0.00336	0.00120	0.34311
MEAN AGE	29.52324	39.42478	33.68306	33.08322	31.99862	3.02730	0.08988
%(0-14)	14.81974	24.25047	19.54684	19.24777	18.12050	2.41459	0.12353
%(15-64)	60.63728	73.54926	66.53702	66.50820	65.15647	3.79339	0.05701
%(65+)	5.96680	21.22636	13.91614	14.18742	14.35956	4.14668	0.29798
DELTA1C	1.97896	21.21980	5.66514	5.07938	2.94100	4.05781	0.71628
DELTA12	0.17540	0.54374	0.31838	0.31866	0.30432	0.09968	0.31309
DELTA32	0.00041	0.76076	0.14960	0.00712	0.03843	0.21305	1.42410
BETA12	0.19783	1.38213	0.72696	0.71787	0.61234	0.31032	0.42740
SIGMA2	0.66296	6.31024	2.82047	2.64495	2.07478	1.49891	0.53144
SIGMA3	0.16168	8.65535	1.14756	0.20633	0.58636	2.04738	1.78411
X LOW	10.51013	16.36026	13.37237	13.18019	11.97266	1.77361	0.13241
X HIGH	19.28033	26.80050	22.82693	22.69040	22.66441	1.60085	0.07013
X RET.	61.57806	68.02851	65.83775	66.87827	67.06094	2.13126	0.03237
X SHIFT	5.77013	12.31020	9.45457	9.30021	8.71320	1.87231	0.19803
A	22.40042	35.62120	29.79299	29.76039	29.67185	2.67844	0.08990
B	0.01281	0.02976	0.02141	0.02178	0.02552	0.00515	0.24049

U.K. FEMALES WITHOUT RETIREMENT PEAK N = 61

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.02365	1.01236	0.14575	0.09184	0.07309	0.17830	1.22333
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.17964	35.50578	10.91377	9.55528	8.87856	4.72799	0.43321
A1	0.00813	0.04496	0.02104	0.01983	0.01365	0.00826	0.39241
ALPHA1	0.01585	0.41038	0.09690	0.08956	0.07503	0.06900	0.71205
A2	0.02207	0.11110	0.06266	0.06204	0.06213	0.01709	0.27274
MU2	17.63140	30.57491	21.34874	20.45384	19.57293	2.83357	0.13273
ALPHA2	0.05467	0.33556	0.15079	0.14175	0.12489	0.06028	0.39976
LAMBDA2	0.09786	0.71288	0.32671	0.30048	0.25162	0.14006	0.42869
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00000	0.00685	0.00348	0.00345	0.00308	0.00157	0.45136
MEAN AGE	25.52103	37.86541	31.58546	32.08269	32.31044	2.95593	0.09359
%(0-14)	14.64687	31.87480	21.59961	20.53595	18.95385	3.76920	0.17450
%(15-64)	62.06953	76.41191	66.97395	66.34695	65.65512	3.41943	0.05106
%(65+)	3.64517	19.56255	11.42645	11.65862	13.99147	3.93660	0.34452
DELTA1C	0.00000	72.47650	8.64625	5.24755	3.62383	10.60588	1.22665
DELTA12	0.08424	0.90435	0.36713	0.32109	0.28927	0.18290	0.49818
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.09121	2.48385	0.72317	0.67343	0.68937	0.46099	0.63746
SIGMA2	0.49564	10.36208	2.73345	2.09932	0.98896	2.07345	0.75855
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	10.32012	17.72029	14.24906	14.20021	15.13023	1.70798	0.11987
X HIGH	20.83036	25.98048	22.94304	22.74041	22.63291	1.19496	0.05208
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	5.56013	13.55031	8.69397	8.44019	7.55767	1.93305	0.22234
A	23.79711	34.79032	28.09603	27.65704	27.64474	2.59165	0.09224
B	0.00831	0.04026	0.02497	0.02519	0.02269	0.00573	0.22964

U.K. FEMALES WITH RETIREMENT PEAK N = 21

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.04829	0.34301	0.14933	0.13736	0.09250	0.08348	0.55901
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.74971	22.13955	9.20055	8.04962	5.61920	4.27702	0.46487
A1	0.00805	0.04165	0.01794	0.01517	0.00973	0.00821	0.45765
ALPHA1	0.02459	0.24502	0.08924	0.09505	0.03561	0.05465	0.61239
A2	0.01233	0.07618	0.04833	0.04547	0.04106	0.01745	0.36113
MU2	18.00047	36.08138	21.55869	19.77335	18.90451	4.96641	0.23037
ALPHA2	0.08835	0.49309	0.15341	0.13615	0.10859	0.09143	0.59595
LAMBDA2	0.09244	0.51326	0.33265	0.33593	0.28181	0.13022	0.39148
A3	0.00000	0.00854	0.00203	0.00017	0.00043	0.00289	1.42077
MU3	60.61970	90.38014	71.84245	70.90856	71.03586	8.32396	0.11586
ALPHA3	0.01154	1.62553	0.58313	0.40945	0.09224	0.46984	0.80572
LAMBDA3	0.05481	1.56080	0.40293	0.20234	0.13011	0.42518	1.05522
C	0.00171	0.00692	0.00381	0.00389	0.00405	0.00133	0.35041
MEAN AGE	26.72770	40.77051	34.04731	34.46955	34.45125	3.48995	0.10250
%(0-14)	15.85610	31.41287	19.86567	18.90520	18.18962	3.63649	0.18305
%(15-64)	60.30930	71.40600	65.92708	65.93875	60.86414	3.32773	0.05048
%(65+)	6.56363	22.01840	14.20725	14.98109	15.06375	3.89729	0.27432
DELTA1C	1.17883	17.45453	5.77446	4.68926	1.99262	4.24057	0.73437
DELTA12	0.16936	0.87399	0.40947	0.34529	0.27505	0.20123	0.49145
DELTA32	0.00006	0.33792	0.04819	0.00499	0.01695	0.08349	1.73242
BETA12	0.05347	2.77330	0.71114	0.65679	0.73343	0.57360	0.80659
SIGMA2	0.29251	5.73387	2.78827	2.95765	2.74112	1.53409	0.55019
SIGMA3	0.13237	93.39887	8.39149	0.18624	4.79570	21.50307	2.56249
X LOW	10.77013	15.86025	13.91878	13.92020	14.07871	1.26210	0.09068
X HIGH	21.15037	24.31044	22.50659	22.30040	21.62438	0.91016	0.04044
X RET.	52.01966	70.26899	63.13780	62.21795	62.05679	4.21923	0.06683
X SHIFT	6.01014	13.54031	8.58782	8.15019	7.89268	1.81185	0.21098
A	23.49932	37.58021	26.55560	28.10036	27.01954	3.23951	0.11345
B	0.01172	0.03499	0.02252	0.02300	0.02452	0.00594	0.26384

JAPAN MALES WITHOUT RETIREMENT PEAK N = 57

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GHR (OBS)	0.00539	1.81309	0.31666	0.17186	0.09578	0.38464	1.21466
GHR (HMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.75751	37.80335	12.62047	11.18192	9.71439	5.62913	0.44603
A1	0.00173	0.02405	0.01412	0.01527	0.01624	0.00592	0.41931
ALPHA1	0.00009	0.25947	0.09480	0.09977	0.14275	0.05486	0.57890
A2	0.03492	0.22707	0.07492	0.06809	0.04453	0.03483	0.46486
MU2	15.10364	22.61861	17.62831	17.11779	15.47939	1.94362	0.11026
ALPHA2	0.03471	0.29735	0.10232	0.09257	0.07411	0.04706	0.45997
LAMBDA2	0.16413	0.90290	0.47975	0.47975	0.49658	0.14115	0.29422
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00000	0.00634	0.00247	0.00225	0.00159	0.00146	0.59006
MEAN AGE	26.02538	40.53283	31.60737	31.44515	31.10299	3.05673	0.09671
%(0-14)	4.92484	20.91126	14.16486	13.95493	12.11873	3.46604	0.24469
%(15-64)	62.99514	86.29065	76.52940	77.34460	75.80768	4.42163	0.05778
%(65+)	3.43647	16.09360	9.30574	8.72926	11.66360	3.64293	0.39147
DELTA1C	0.00000	712.88135	22.93119	5.58207	35.64407	94.88651	4.13788
DELTA12	0.02274	0.52642	0.23256	0.24896	0.24940	0.12682	0.54531
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.00092	7.47530	1.24426	1.16222	1.12208	1.16237	0.93419
SIGMA2	1.02500	12.92530	5.61729	5.00862	2.81011	2.82837	0.50351
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	10.91014	17.71029	13.18492	12.28017	11.93016	1.77209	0.13440
X HIGH	17.64029	24.98046	20.91265	20.47035	18.74131	2.15590	0.10309
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	4.63011	12.96030	7.72772	7.58017	6.71266	1.57475	0.20378
A	0.00000	102.41312	37.74501	35.85357	35.84459	11.86458	0.31434
B	0.02035	0.07343	0.03630	0.03335	0.02831	0.01191	0.32812

JAPAN FEMALES WITHOUT RETIREMENT PEAK   N = 57

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GRR (OBS)	0.00388	1.59564	0.24922	0.11912	0.08347	0.33651	1.35027
GRR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	5.01904	28.38801	11.11674	10.35964	6.18749	5.10822	0.45951
A1	0.00526	0.04003	0.02056	0.02091	0.02091	0.00874	0.42507
ALPHA1	0.01953	0.21084	0.11681	0.11836	0.12475	0.03604	0.30852
A2	0.03340	0.18839	0.08486	0.07980	0.07215	0.03158	0.37210
MU2	15.06610	37.76019	21.32339	21.16880	16.20080	4.98334	0.23370
ALPHA2	0.06431	0.28581	0.15151	0.14412	0.14184	0.04493	0.29654
LAMBDA2	0.08367	0.80120	0.34973	0.32355	0.26305	0.16910	0.48352
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00012	0.00656	0.00401	0.00399	0.00366	0.00135	0.33511
MEAN AGE	25.92860	37.10249	31.23327	30.88583	28.72207	2.41142	0.07721
%(0-14)	10.63559	29.12714	19.18479	20.40160	20.80594	4.79971	0.25018
%(15-64)	60.55278	79.84567	69.63420	69.05502	65.37601	5.40643	0.07742
%(65+)	2.99108	16.75492	10.98102	10.64606	10.56119	2.97760	0.27116
DELTA1C	0.89359	192.60318	9.20455	5.02601	10.47907	25.26971	2.74535
DELTA12	0.02828	0.72176	0.28974	0.27999	0.06295	0.16540	0.57085
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.10464	1.52050	0.82773	0.85703	0.88336	0.29367	0.35478
SIGMA2	0.38917	7.64776	2.59435	2.25908	2.20382	1.57000	0.60516
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	11.36015	21.79638	14.08898	12.58017	11.88166	2.62811	0.18654
X HIGH	17.03028	30.92059	22.76322	23.37042	23.28092	3.25665	0.14307
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	4.60011	15.09035	8.67423	8.58020	7.22267	2.24611	0.25894
A	25.13712	37.24700	30.17262	29.88948	29.37558	2.18864	0.07254
B	0.01296	0.06495	0.03339	0.02891	0.02596	0.01340	0.40134

NETHERLANDS MALES MX WITH RETIREMENT SLOPE N = 10

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	3.17845	4.81395	3.91493	3.81677	3.58732	0.53446	0.13652
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	3.02542	6.41094	5.25190	5.30331	5.22601	1.04352	0.19869
A1	0.01065	0.01574	0.01265	0.01234	0.01090	0.00187	0.14779
ALPHA1	0.04667	0.10277	0.07955	0.08613	0.08874	0.01595	0.20047
A2	0.05424	0.07066	0.06319	0.06621	0.05506	0.00582	0.09204
MU2	19.46053	22.93296	20.86084	20.69522	20.32864	0.95922	0.04598
ALPHA2	0.11257	0.14982	0.12984	0.12854	0.11443	0.01338	0.10304
LAMBDA2	0.22094	0.35961	0.28665	0.30015	0.29721	0.03995	0.13936
A3	0.00000	0.00005	0.00001	0.00001	0.00000	0.00002	1.38535
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.05744	0.10053	0.07651	0.07588	0.07683	0.01292	0.16892
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00104	0.00422	0.00343	0.00389	0.00343	0.00093	0.27251
MEAN AGE	37.73109	41.49833	38.94663	39.31461	37.91945	1.27571	0.03276
%(0-14)	13.69166	17.27305	15.41468	15.15449	17.09398	1.28401	0.08330
%(15-64)	59.97063	66.26878	63.02232	63.92394	60.28554	2.28423	0.03624
%(65+)	18.80301	25.63899	21.56301	22.35854	19.14481	2.27409	0.10546
DELTA1C	2.52201	14.47297	4.51612	3.75886	3.11956	3.55875	0.78801
DELTA12	0.15677	0.27627	0.20271	0.18714	0.17470	0.04189	0.20665
DELTA32	0.00001	0.00095	0.00020	0.00012	0.00006	0.00028	1.40678
BETA12	0.41455	0.80146	0.61474	0.63704	0.62735	0.12439	0.20234
SIGMA2	1.49832	3.19446	2.23921	2.23897	2.26158	0.45391	0.20271
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	12.72018	14.77022	14.08921	14.21021	14.25771	0.53618	0.03806
X HIGH	22.50040	24.86045	23.44342	23.38042	22.85441	0.75102	0.03204
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	8.19019	10.47024	9.35422	9.24021	9.21622	0.73500	0.07857
A	29.53608	33.37366	31.44317	32.11462	30.11172	1.41603	0.04503
B	0.02060	0.02722	0.02408	0.02394	0.02292	0.00213	0.08845

NETHERLANDS FEMALES MX WITH RETIREMENT SLOPE N = 10

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	3.52109	4.92170	4.13650	4.26010	4.29143	0.47133	0.11394
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	5.40977	11.05379	8.04365	8.90725	5.69197	2.05565	0.25556
A1	0.00994	0.01413	0.01228	0.01273	0.01266	0.00128	0.10426
ALPHA1	0.06176	0.11502	0.09830	0.10605	0.11236	0.01628	0.16562
A2	0.06480	0.10439	0.08382	0.09071	0.06678	0.01317	0.15718
MU2	19.75573	20.57280	20.10061	20.04311	19.79658	0.27033	0.01345
ALPHA2	0.14553	0.20475	0.17375	0.18125	0.14849	0.01982	0.11408
LAMBDA2	0.26334	0.35494	0.30683	0.30909	0.26792	0.02847	0.09280
A3	0.00000	0.00019	0.00004	0.00003	0.00001	0.00006	1.40559
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.03847	0.11854	0.07134	0.07127	0.05048	0.02375	0.33289
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00315	0.00457	0.00374	0.00374	0.00322	0.00063	0.16751
MEAN AGE	37.57629	39.77856	38.81507	39.19236	39.22799	0.78790	0.02030
%(0-14)	13.21536	16.78795	14.56102	14.46851	13.39399	1.27618	0.08764
%(15-64)	59.85442	65.44514	62.67490	63.07958	62.92931	1.63127	0.02603
%(65+)	20.13247	25.10497	22.76408	23.30609	23.36459	1.50698	0.06620
DELTA1C	2.17413	4.04725	3.36279	3.61493	3.95359	0.60866	0.18100
DELTA12	0.10707	0.20471	0.15107	0.13879	0.12172	0.03540	0.23431
DELTA32	0.00000	0.00202	0.00046	0.00030	0.00010	0.00064	1.38423
BETA12	0.33449	0.65607	0.57057	0.60265	0.60783	0.09931	0.17406
SIGMA2	1.31773	2.34448	1.79960	1.77764	1.57442	0.35266	0.19596
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	12.75018	14.47022	13.49520	13.45019	12.83618	0.66204	0.04906
X HIGH	21.24037	22.63040	21.86338	21.80038	21.30987	0.51774	0.02368
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	7.93018	9.06021	8.36819	8.35019	8.21269	0.32111	0.03837
A	27.02269	29.90750	28.73727	28.99037	29.18630	0.77992	0.02714
B	0.02568	0.03485	0.03036	0.03316	0.03347	0.00369	0.12143

USSR TOTAL WITHOUT RETIREMENT PEAK N = 58

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.00815	3.90378	0.66532	0.19186	0.20293	1.00916	1.51681
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	10.18453	24.94810	17.93700	17.82011	15.35178	3.17316	0.17691
A1	0.00105	0.01283	0.00486	0.00437	0.00282	0.00262	0.53817
ALPHA1	0.17472	0.60651	0.30245	0.27777	0.28267	0.10223	0.33799
A2	0.06952	0.19473	0.12579	0.12539	0.12586	0.03256	0.25885
MU2	16.81462	23.78566	19.13940	18.96427	19.25448	1.68024	0.08779
ALPHA2	0.08706	0.29517	0.17642	0.17852	0.09747	0.05590	0.31684
LAMBDA2	0.19184	0.44446	0.31015	0.30346	0.30552	0.06112	0.19708
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00146	0.00664	0.00427	0.00431	0.00483	0.00106	0.24795
MEAN AGE	28.33398	36.51470	32.81122	32.80619	32.83338	1.71835	0.05237
%(0-14)	3.47014	12.07090	8.23203	8.62854	9.06063	2.09048	0.25394
%(15-64)	72.46465	92.28165	80.16578	79.63146	77.41890	4.21266	0.05255
%(65+)	4.24821	17.13380	11.60220	11.85933	10.04673	2.50298	0.21573
DELTA1C	0.28231	3.81763	1.16415	0.99382	0.81261	0.64231	0.55174
DELTA12	0.00561	0.08434	0.04012	0.04178	0.04891	0.01967	0.49028
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	0.76316	6.05851	1.92186	1.69760	1.02793	1.06543	0.55438
SIGMA2	0.67698	4.52200	2.08855	1.57544	1.25373	1.14313	0.54733
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	9.82011	12.22017	11.23893	11.27014	11.14015	0.50767	0.04517
X HIGH	19.57033	22.06039	20.81760	20.84036	20.93987	0.54317	0.02609
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	8.08018	11.72027	9.57867	9.48022	9.35421	0.73888	0.07714
A	30.17941	85.90950	45.68583	43.46015	38.53893	12.28768	0.26896
B	0.03600	0.06988	0.04773	0.04670	0.04786	0.00791	0.16568

U.S. TOTAL WITH RETIREMENT PEAK N = 8

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.17654	0.67502	0.39920	0.46159	0.20146	0.17155	0.42974
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	6.35763	12.44090	8.76274	9.02917	6.66179	2.18925	0.24984
A1	0.01496	0.02682	0.02128	0.02078	0.02623	0.00461	0.21667
ALPHA1	0.03284	0.11438	0.07537	0.07852	0.11030	0.02920	0.38745
A2	0.04074	0.08871	0.05965	0.06023	0.06233	0.01414	0.23705
MU2	19.37771	21.05273	20.13819	20.12657	19.96397	0.54137	0.02688
ALPHA2	0.08742	0.17384	0.11764	0.10559	0.09174	0.03137	0.26662
LAMBDA2	0.44557	0.75143	0.56910	0.62537	0.46086	0.11553	0.20299
A3	0.00003	0.00658	0.00192	0.00057	0.00036	0.00269	1.39900
MU3	71.87231	90.13589	81.80841	88.02872	72.78548	8.55974	0.10464
ALPHA3	0.21260	0.66147	0.43023	0.46137	0.23504	0.16264	0.37804
LAMBDA3	0.08569	0.22924	0.11914	0.10588	0.10722	0.04554	0.38223
C	0.00103	0.00387	0.00233	0.00229	0.00202	0.00087	0.37436
MEAN AGE	28.73096	32.64307	30.83244	31.18867	32.44746	1.53949	0.04993
%(0-14)	20.08696	23.59063	21.67020	21.29156	20.26214	1.33968	0.06182
%(15-64)	63.85034	72.09166	67.76926	68.11514	64.26241	2.63248	0.03884
%(65+)	7.11183	13.39558	10.56054	11.47903	8.68277	2.24953	0.21301
DELTA1C	4.45463	18.98871	10.49483	10.30285	8.08815	4.77822	0.45529
DELTA12	0.24833	0.52458	0.36772	0.39712	0.28977	0.09357	0.25446
DELTA32	0.00045	0.11427	0.03477	0.01391	0.00614	0.04826	1.38804
BETA12	0.21316	1.08323	0.67544	0.62885	0.60469	0.29015	0.42957
SIGMA2	2.56309	7.48964	5.19982	5.27043	7.24331	1.82038	0.35009
SIGMA3	0.17089	0.96724	0.34780	0.23084	0.21071	0.27618	0.79409
X LOW	16.27026	17.44028	16.70652	16.80027	16.32876	0.38767	0.02320
X HIGH	22.18039	23.40042	22.80541	22.70041	22.48540	0.46221	0.02027
X RET.	62.74786	72.68951	68.93377	71.36922	71.19826	3.91673	0.05682
X SHIFT	5.13012	7.07016	6.09889	6.43015	5.61513	0.66965	0.10980
A	25.06041	29.67035	28.01789	28.28370	29.43986	1.52249	0.05434
B	0.02010	0.04069	0.03144	0.03289	0.02731	0.00650	0.20663

HUNGARY TOTAL WITHOUT RETIREMENT PEAK N = 7

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GMR (OBS)	0.13064	2.13464	0.71087	0.47229	0.23084	0.75975	1.06875
GMR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAF%M	10.07720	18.81879	12.89946	10.86141	10.51428	3.68319	0.28553
A1	0.00330	0.01593	0.01045	0.01240	0.00393	0.00522	0.49965
ALPHA1	0.17236	0.37358	0.24483	0.24450	0.18242	0.07000	0.28591
A2	0.07082	0.10192	0.08996	0.09241	0.07237	0.01028	0.11428
MU2	15.62418	18.95611	17.22307	17.53528	15.79078	1.42781	0.08290
ALPHA2	0.09495	0.15195	0.13046	0.13107	0.14910	0.02138	0.16388
LAMBDA2	0.24078	0.59629	0.41459	0.37163	0.32966	0.12926	0.31177
A3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00326	0.00428	0.00381	0.00373	0.00423	0.00042	0.11055
MEAN AGE	31.10266	33.15700	31.96349	32.00492	31.20538	0.71427	0.02235
%(0-14)	8.28110	13.84877	11.23482	12.43240	8.55948	2.46649	0.21954
%(15-64)	73.97253	81.60341	77.70294	77.24712	75.11716	2.94927	0.03796
%(65+)	9.90099	12.17871	11.06224	10.81083	12.06482	1.03134	0.09323
DELTA1C	0.79678	4.03978	2.73490	3.32747	0.95893	1.28987	0.47163
DELTA12	0.03906	0.16216	0.11447	0.12569	0.15600	0.05245	0.45818
DELTA32	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BETA12	1.14650	3.09410	1.95821	1.82489	1.24388	0.73893	0.37735
SIGMA2	1.58466	6.28032	3.40439	2.63601	2.28901	1.66463	0.48897
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	10.62013	13.09019	11.68301	11.67015	11.73166	0.77332	0.06619
X HIGH	18.47031	20.99037	19.84177	20.33035	18.59631	1.08176	0.05452
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	6.64015	10.22023	8.15876	7.90018	6.81915	1.22065	0.14961
A	31.98261	55.53356	41.49559	35.42572	33.16016	9.56251	0.23045
B	0.03795	0.04959	0.04272	0.04177	0.04086	0.00373	0.08740

HUNGARY TOTAL WITH RETIREMENT SLOPE   N = 25

	LOWEST VALUE	HIGHEST VALUE	MEAN VALUE	MEDIAN	MODE	STD. DEV.	STD. DEV. / MEAN
GHR (OBS)	0.08771	3.80248	0.92281	0.35561	0.27345	1.15148	1.24781
GHR (MMS)	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000	0.00000
MAE%M	4.89345	12.97295	8.51940	8.36460	8.52923	2.26151	0.26545
A1	0.00505	0.02273	0.01497	0.01474	0.01477	0.00448	0.29931
ALPHA1	0.12606	0.33951	0.19268	0.17129	0.15808	0.05620	0.29168
A2	0.07316	0.12793	0.09908	0.09790	0.09781	0.01350	0.13624
MU2	17.23109	20.77004	18.73634	19.02641	19.17752	1.04162	0.05559
ALPHA2	0.09383	0.20285	0.15866	0.15747	0.14289	0.02715	0.17111
LAMBDA2	0.20185	0.37486	0.27448	0.26804	0.26240	0.03984	0.14516
A3	0.00001	0.00178	0.00032	0.00019	0.00010	0.00039	1.22796
MU3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
ALPHA3	0.00436	0.06211	0.03339	0.03045	0.03035	0.01448	0.43345
LAMBDA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
C	0.00091	0.00486	0.00265	0.00240	0.00229	0.00098	0.37183
MEAN AGE	29.63155	39.95061	34.14457	33.49084	33.24322	2.51858	0.07376
%(0-14)	8.28035	18.86661	13.41424	13.58471	14.10279	2.80661	0.20925
%(15-64)	65.67160	75.63367	70.82892	71.06123	71.15075	2.70972	0.03826
%(65+)	10.51482	23.40787	15.75685	15.42784	15.02739	3.12025	0.19802
DELTA1C	1.32364	21.93596	6.97334	6.33304	6.47672	4.57756	0.65644
DELTA12	0.04552	0.24074	0.15406	0.15763	0.11385	0.05060	0.32847
DELTA32	0.00014	0.01554	0.00318	0.00167	0.00091	0.00367	1.15420
BETA12	0.79696	2.00363	1.22783	1.17916	0.85729	0.32345	0.26343
SIGMA2	0.99508	3.58419	1.82030	1.70936	1.38345	0.59299	0.32577
SIGMA3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X LOW	10.22012	12.12016	11.19414	11.24014	11.64515	0.54132	0.04836
X HIGH	19.65034	21.61038	20.62596	20.66036	20.14035	0.51739	0.02508
X RET.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
X SHIFT	8.30019	10.64024	9.43182	9.30021	9.11921	0.52667	0.05584
A	27.61854	39.83525	32.99152	32.40030	36.78108	3.58805	0.10876
B	0.02970	0.04574	0.03738	0.03683	0.03692	0.00349	0.09324

REFERENCES

Andersson, Å. and I. Holmberg (1980) *Migration and Settlement: 3. Sweden*. RR-80-5. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Bard, Y. (1974) *Nonlinear Parameter Estimation*. New York: Academic Press.

Benson, M. (1979) Parameter Fitting in Dynamic Models. *Ecological Modelling* 6:97-115.

Brass, W. and Ansley Coale (1968) Methods of Analysis and Estimation. *Demography of Tropical Africa*, ed. by William Brass. Princeton, New Jersey: Princeton University Press: 88-150.

Brass, W., ed. (1971) On the Scale of Mortality. *Biological Aspects of Demography*. London: Taylor and Francis, Ltd.: 69-110.

Brass, W. (1975) *Methods for Estimating Fertility and Mortality from Limited and Defective Data*. Chapel Hill, North Carolina: Laboratories for Population Statistics, The University of North Carolina at Chapel Hill.

Brown, K.M. and J.E. Dennies (1972) Derivative Free Analogues of the Levenberg-Marquardt and Gauss Algorithms for Nonlinear Least Squares Approximations. *Numerische Mathematik* 18:289-297.

Carrier, N. and J. Hobcraft (1971) *Demographic Estimation for Developing Societies*. London: Population Investigation Committee, London School of Economics.

Carrier, N. and T. Goh (1972) The Validation of Brass's Model Life Table System. *Population Studies* 26(1):29-51.

Coale, A. and P. Demeny (1966) *Regional Model Life Tables and Stable Populations*. Princeton, New Jersey: Princeton University Press.

Coale, A.J. and D.R. McNeil (1972) The Distribution by Age of the Frequency of First Marriage in a Female Cohort. *Journal of the American Statistical Association* 67:743-749.

Coale, A. and J. Trussell (1974) Model Fertility Schedules: Variations in the Age Structure of Childbearing in Human Populations. *Population Index* 40(2):185-206.

Coale, A. (1977) The Development of New Models of Nuptiality and Fertility. *Population, numéro spécial*:131-154.

Fiacco, A. and G. McCormick (1968) *Nonlinear Programming: Sequential Unconstrained Minimization Techniques*. New York: Wiley.

Gumbel, E.J. (1941) The Return Period of Flood Flows. *Annals of Mathematical Statistics* 12:163-190.

Hill, K. and J. Trussell (1977) Further Developments in Indirect Mortality Estimation. *Population Studies* 31(2):313-334.

Kimball, B.F. (1946) Sufficient Statistical Estimation Functions for the Parameters of the Distribution of Maximum Values. *Annals of Mathematical Statistics* 17:299-309.

Krishnan, P. and A.K. Kayani (1973) Model Divorce Tables. *Genus* 32:109-127.

Levenberg, K. (1944) A Method for the Solution of Certain Nonlinear Problems in Least Squares. *Q. Appl. Math* 2:164-168.

Marquardt, D.W. (1963) An Algorithm for Least-Squares Estimation of Nonlinear Parameters. *SIAM, J. Numer. Anal.* 11:431-441.

McNeil, D.R., T.J. Trussell, and J.C. Turner (1977) Spline Interpolation of Demographic Data. *Demography* 14:245-252.

Rees, P.H. (1977) The Measurement of Migration, from Census Data and Other Sources. *Environment and Planning A* 1:247-260.

Rogers, A. (1973a) The Multiregional Life Table. *The Journal of Mathematical Sociology* 3:127-137.

Rogers, A. (1973b) The Mathematics of Multiregional Demographic Growth. *Environment and Planning* 5:3-29.

Rogers, A. (1975) *Introduction to Multiregional Mathematical Demography*. New York: Wiley.

Rogers, A. (1976a) *The Comparative Migration and Settlement Study: A Summary of Workshop Proceedings and Conclusions*. RM-76-01. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rogers, A. (1976b) *Two Methodological Notes on Spatial Population Dynamics in the Soviet Union*. RM-76-48. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rogers, A., ed. (1978) Migration and Settlement: Selected Essays. *Environment and Planning A* 10(5):469-617.

Rogers, A. and J. Ledent (1976) Increment-Decrement Life Tables: A Comment. *Demography* 13:287-290.

Rogers A., R. Raquillet and L.J. Castro (1978) Model Migration Schedules and Their Applications. *Environment and Planning A* 10:475-502.

Rogers, A. and F. Willekens (1978) *Migration and Settlement: Measurement and Analysis*. RR-78-13. Laxenburg, Austria: International Institute for Applied Systems Analysis.

United Nations (1955) *Age and Sex Patterns of Mortality: Model Life Tables for Underdeveloped Countries*. New York.

Willekens, F. and A. Rogers (1978) *Spatial Population Analysis: Methods and Computer Programs*. RR-78-18. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Zaba, B. (1979) The Four-Parameter Logit Life Table System. *Population Studies* 33(1):79-100.

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