MISINFORMATION AND EQUILIBRIUM IN INSURANCE MARKETS

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October 1981
CP-81-30

Paper presented at a Conference on the
Economics of Regulation
International Institute of Management
Berlin (West)
July 1981

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ABSTRACT

This paper focuses on the role of misinformation by firms and consumers with respect to the selling and buying of insurance. For example, the reader may wish to think of automobile insurance when the firms do not know the accident probabilities for each of their customers and insured individuals in turn, may misperceive the probabilities of being involved in an accident.

Our point of departure is the model developed by Rothschild-Stiglitz which demonstrated that firms could distinguish between different types of risks by offering a set of policies consisting of a premium per dollar and a stated amount of coverage. We will investigate two types of equilibrium concepts in the spirit of this model: a traditional *Nash equilibrium* where each firm determines the set of policies it will offer under the assumption that all other firms make no changes in their offerings and a *Wilson equilibrium* where firms look far enough ahead in the future to evaluate the consequences of a new policy offering on the profitability of current policies.

The paper contrasts the Nash and Wilson equilibria for cases where consumers correctly perceive the probability of a loss as well as when they misperceive this probability. We focus attention on the case where there are two risk groups in order to highlight significant differences between Nash and Wilson equilibria through graphical procedures. The final portion of the paper generalizes the results to n risk groups and discusses the welfare implications of consumer misperceptions.

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MISINFORMATION AND EQUILIBRIUM IN INSURANCE MARKETS

Paul Kleindorfer and Howard Kunreuther[1]

I. INTRODUCTION

This paper is concerned with the role of misinformation by firms and consumers with respect to the selling and buying of insurance. Our interest is in the relationship between the accuracy of consumer beliefs and the relative performance of the market system and social programs. Such an investigation requires us to determine under what conditions a stable market equilibrium exists and, if it does, what type of insurance policies are offered to consumers. We can then contrast these market outcomes with premium regulation or some form of required insurance

[1]The research in this paper is supported in part by the Bundesministerium fuer Forschung und Technologie, F.R.G., contract no. 321/7591/RGB 8001. While support for this work is gratefully acknowledged, the views expressed are the authors' and not necessarily shared by the sponsor. The authors would also like to express their appreciation to Uday Apte for his computational assistance and to Michael Rothschild and Joseph Stiglitz, and the participants in the Conference on the Economics of Regulation in Berlin, for their helpful comments on an earlier draft.

coverage.

There are two reasons we are focusing on consumer misinformation in this paper. First, there is considerable evidence from recent laboratory experiments and field survey data that individuals systematically misestimate probabilities particularly when they are relatively low, the type of situation where insurance is most relevant (see Kunreuther et al. 1978; Fischhoff, Slovic and Lichtenstein 1978). Secondly, economists have focused almost entirely on firm misinformation and implicitly assumed that consumer misperception only affects individuals adversely but has little impact on market behavior.

The results of our analysis suggest that this may not be the case. We show that the existence and efficiency of competitive insurance markets can be affected by consumer (mis)perceptions of the risks that are being insured against. For example, we show that underestimation of the probability of insurable events by high-risk individuals may adversely affect the existence of competitive equilibrium.

The paper is in the spirit of recent work in economics which deals with accuracy and asymmetries in information between the consumer and the firm where insurance is used as a prototype example (see Arrow 1963; Williamson 1975). If the consumer knows more about his risk than the supplier, then problems of adverse selection may result where only the highest risk group is offered coverage unless special steps are taken by the insurer. Such adverse selection problems brought on by insurers' lack of information on customer characteristics will be one feature of the model developed below. Our specific interest is to determine what additional problems consumer misperceptions may cause in such a world.

As an example, the reader may wish to think of automobile insurance under conditions where firms do not know the accident probabilities for each of their customers. Alternatively, firms may be prevented by law from using information (e.g., geographic location) which would properly classify customers according to their respective risk class. Customers, in turn, may misperceive their probabilities of being involved in an accident. Under these conditions, and assuming free entry and exit (no fixed costs) of insurance firms into this market, we are interested in determining what sort of policies, if any, would be marketed in the absence of regulation.

Rothschild and Stiglitz (1976) suggest an ingenious way to overcome the adverse selection problem. Rather than specifying a premium rate per dollar, firms would offer a set of policies $\{<p_j, Q_j>| j=1,2,\ldots,J\}$, consisting of a premium per dollar (p_j) and a stated amount of coverage (Q_j).[2] In this way the rate could differ between high and low coverage. Given this system of insurance they investigate under what situations a stable (Nash) competitive equilibrium exists. One of the most important results of their analysis is that there cannot be a pooled equilibrium (i.e., a single market-wide policy) which is stable. In the case of two risk groups, an equilibrium, if it exists, consists of two separate policies with different premiums and different stated coverage. Wilson (1977) independently investigated the same problem as Rothschild and Stiglitz (hereafter referred to as R-S), but utilized a different definition of equilibrium which involved some foresight on the part of firms.

[2]The results derived below also hold if the Q_j are interpreted as maximum or minimum coverage limits and are allowed to vary according to the premium.

The contrast between the two types of equilibrium is instructive. In a traditional Nash equilibrium, each firm (including potential entrants) is assumed to determine the set of policies it will offer under the assumption that all other firms make no changes in their current offerings. Each policy is on the actuarially fair odds line so that no firm can enter the market and make a profit by offering a policy to either high- or low-risk group. In the Wilson equilibrium each firm determines its optimal set of policy offerings under the assumption that any currently marketed policies which become unprofitable as a result of new offerings will no longer be offered in the market place. Thus, an equilibrium in Wilson's sense requires that firms look ahead far enough to evaluate the consequences of new policy offerings on the profitability of all currently marketed policies. Clearly, it would be empirically and theoretically of interest to determine when the Wilson assumptions on firm behavior are justified. In this paper, however, we shall simply analyze the implications of the Nash and Wilson assumptions when consumer misperceptions are present. This is intended as a prelude to an empirical study of other aspects of firm and consumer decision processes in insurance markets (see, e.g., Kleindorfer and Kunreuther 1981, and Finsinger 1981 in this regard).

Miyasaki (1977) has studied the Wilson equilibrium in detail. When there are two risk classes, a high- and a low-risk group, the Miyasaki results imply that the Wilson equilibrium will not be a pooled equilibrium, as R-S and Wilson had both originally, but erroneously, thought. Rather it will consist of a pair of contracts, one directed toward the high-risk and the other directed toward the low-risk individuals. This is just the same as for the Nash equilibrium, which (for this case with two risk groups) also consists of a pair of policies whenever it exists. Indeed, whenever the separating Nash equilibrium exists, it coincides with the Wilson equilibrium and all consumers pay actuarially fair premia. When the Nash separating equilibrium fails to exist, however, the Wilson equilibrium will still exist, but it will now entail a pair of insurance policies being marketed such that low-risk individuals will subsidize high-risk persons.

Miyazaki's work is related to labor market theory where firms could not distinguish between high productivity (i.e., low risk) and low productivity (i.e., high risk) workers. Spence (1978) translated Miyasaki's model to the insurance market context and generalized the model to accommodate n different groups. His analysis provides a parsimonious mathematical framework for analyzing the existence of Nash-Wilson equilibria and the associated cross-subsidization issues involved. More recently Dahlby (1980) provided a graphical procedure for determining Nash-Wilson equilibria and for analyzing the degree of subsidy which the low risk group provides to the high-risk group in equilibrium. Our discussion of the characteristics of an equilibrium will build on these three papers, (henceforth referred to as M-S-D). None of these studies investigated the implications of consumer misperceptions on market stability and welfare.

There is an underlying rationale in back of equilibrium analysis which enables us to generalize the above studies to the case where there are n different risk groups and within each group there may be multiple subgroups having different misperceptions of the probability of a loss. This rationale consists of two very simple principles: first, existing firms must offer policies yielding zero expected profits and which maximize the perceived expected utility of the lowest risk group over all feasible (i.e., zero-profit) sets of policies. In addition, they must choose policies which prevent new firms from entering and making positive profits.

In the case of a Nash equilibrium, where there is no foresight by existing firms, all equilibrium policies *must* be actuarially fair, whether or not consumers misperceive the probabilities of risks. As we shall see, the region of stability will be determined by the perceived expected utility of individuals, in each of the different risk classes. For the case of a Wilson equilibrium, where firms are assumed to have a special type of foresight, optimal policies are determined by the *perceived* expected utility of individuals. The degree of cross-subsidization between individuals will thus be a function of the misperceptions of the probabilities of a loss. We will illustrate how these two basic principles of a stable equilibrium apply to each of the cases discussed in the paper.

For ease of exposition and graphical convenience we develop our analysis in Sections II-IV by assuming that there are only two risk classes—high and low—each of whom face the same loss X.[3] Section II spells out the appropriate definitions and assumptions. We then briefly review in Section III the case where consumers have correct perceptions of the probability of a loss, in order to contrast the Nash and Wilson equilibria. These results also serve as a useful benchmark for investigating the problem of consumer misperceptions in Section IV. Generalizations of these results to n risk groups and their welfare implications are discussed in Section V.

[3]The analyses by R-S, Wilson, Miyasaki and Dahlby all make these same assumptions.

II. DEFINITIONS AND ASSUMPTIONS

Our simplified world consists of N consumers divided into high (H) and low (L) risk groups of sizes N_H and N_L respectively. Each consumer faces a risk involving a potential loss (X) which he correctly estimates.[4] Each group $i = H, L$ has their own perception of the probability of a loss (φ_i) which may differ from the true probability (Φ_i). Neither X nor the φ_i can be influenced by consumer actions, so moral hazard problems do not exist. The initial wealth of consumers in the high and low-risk groups is given by W_H and W_L respectively. Unless otherwise specified we assume $W_H = W_L = W$. If a loss does not occur, then the wealth level of group i is given by W_1; a loss results in wealth level of W_2. An uninsured individual in group i with perfect information on the probability of X thus faces a lottery yielding outcomes $W_1 = W_i$ and $W_2 = W_i - X$ with probabilities $1 - \varphi_i$ and φ_i respectively.

The insurance industry consists of n identical firms which offer different insurance policies to consumers. Each firm is unable to distinguish between low and high-risk consumers who express an interest in purchasing insurance. Since we are interested in the stability conditions of equilibrium it is irrelevant whether firms have a correct perception of Φ_i. Equilibrium insurance policies reflect the condition of zero expected profits for each firm, so that the true probabilities of a loss will reveal themselves through a long-run adjustment policy.[5] Each policy j consists

[4] The case where consumers misestimate X is also discussed briefly below and turns out to be analogous to the case where consumers misestimate Φ. We thank Robert Willig for raising this point.

[5] Firms' possible misperceptions of probabilities (or losses) do become important in analyzing the dynamics of the industry adjustment process in attaining equilibrium.

of a premium per dollar (p_j) and a specified amount of coverage (Q_j) which we denote by $< p_j, Q_j >$. If a policy is only offered to group i because the firm can differentiate between consumers, then it is denoted by $< p_j^i, Q_j^i >$. We are assuming that consumers are not allowed to purchase more than X dollars of insurance[6] and that claims are monitored to enforce this restriction. A consumer in group i selects from among the insurance policies offered him the one which maximizes his expected utility $[E\,(U_i)]$ where U_i is a von Neuman-Morgenstern utility function. We assume $U_i' > 0$, $U_i'' < 0$ so consumers are risk averse. If a person chooses policy $< p_j, Q_j >$ based on the perceived probability φ_i, then his *ex ante* perceived utility is

$$E[U_i\,(\varphi_i)] = (1 - \varphi_i)\,U_i\,[W_i - p_j Q_j] + \varphi_i U_i\,[W_i - X + (1 - p_j)\,Q_j]. \quad (1)$$

In measuring consumer welfare, we will be careful to distinguish between perceived and actual welfare, depending on whether φ_i or Φ_i is used in computing $E\,[U_i]$ in (1).

The primary interest of this study is on the impact of imperfect information on the stability of equilibrium and the welfare implications of alternative regulatory measures. In the next sections we will address the following questions with respect to the case where consumers correctly estimate Φ_i and the case where they misperceive these probabilities (i.e., $\varphi_i \neq \Phi_i$):

[6]This assumption is not critical for our analysis. For example, if consumers estimate $\varphi_H > \Phi_H$ and firms offer actuarially fair premiums then consumers will purchase $Q_H > X$ if such a policy were offered.

1. What are the relevant conditions with respect to (a) true and perceived probabilities of a loss and (b) number of consumers in the high and low risk groups which lead to a stable Nash Equilibrium?

2. What are the characteristics of a Wilson equilibrium and how does it compare to a Nash equilibrium if it exists?

3. What are the welfare implications of consumer misperceptions as these impact on own-group welfare and on other-group welfare at the market (Nash-Wilson) equilibrium?

III. CORRECT PERCEPTIONS BY CONSUMERS

Our analysis of resulting equilibrium with correct and incorrect perceptions of Φ by consumers will parallel the graphical methods introduced by R-S. They note that the implications of any insurance policies offered in the market can be reflected by their impact on consumer wealth in the two relevant states: no loss and loss. Denote by (W_1, W_2) consumer wealth in these two states respectively. If $<p, Q>$ were an insurance policy offered to either consumer group, then the representation of this policy in (W_1, W_2) space is seen from (1) to be:

$$W_1 = W - pQ \quad \text{and} \quad W_2 = W - X + (1-p)Q \quad . \tag{2}$$

Similarly, any point in (W_1, W_2) space corresponds to an insurance policy which might be marketed. Consumer decisions regarding the choice between insurance policies will be determined by maximizing their expected utility, so that the traditional iso-utility curve analysis applies.

PROPERTIES OF A NASH EQUILIBRIUM

Since firms cannot differentiate between high and low-risk consumers, then adverse selection problems may arise and a market equilibrium may or may not exist. R-S first show that no single policy (i.e., pooled) equilibrium can exist. They then discuss conditions under which a separating Nash equilibrium, consisting of two policies $< p_H, Q_H >$ and $< p_L, Q_L >$, can exist. Two conditions are necessary. First, high-risk consumers must be offered full insurance at actuarial rates $< \Phi_H, X >$; second, low-risk consumers must be offered an actuarially fair policy $< \Phi_L, Q_L >$ whose utility to the high-risk consumers is identical to the policy $< \Phi_H, X >$. In this case there is no incentive for the high-risk group to purchase a low-risk policy (which would create negative profits for firms). The resulting (potential) equilibrium is shown as α_H, α_L in Figure 1. These conditions are equivalent to having the firm maximize expected utility of the low-risk consumer while ensuring that there is no incentive for a high risk individual to buy a policy offered to a low-risk consumer. Hence, they conform to the basic principles for an equilibrium outlined in Section I. To see whether α_H, α_L is actually a stable Nash equilibrium, we must consider whether new entrants can make a positive profit if all firms continue to offer the above two policies. We first construct the market fair odds for pooled policies (i.e., EF in Figure 1). Since there are N_H and

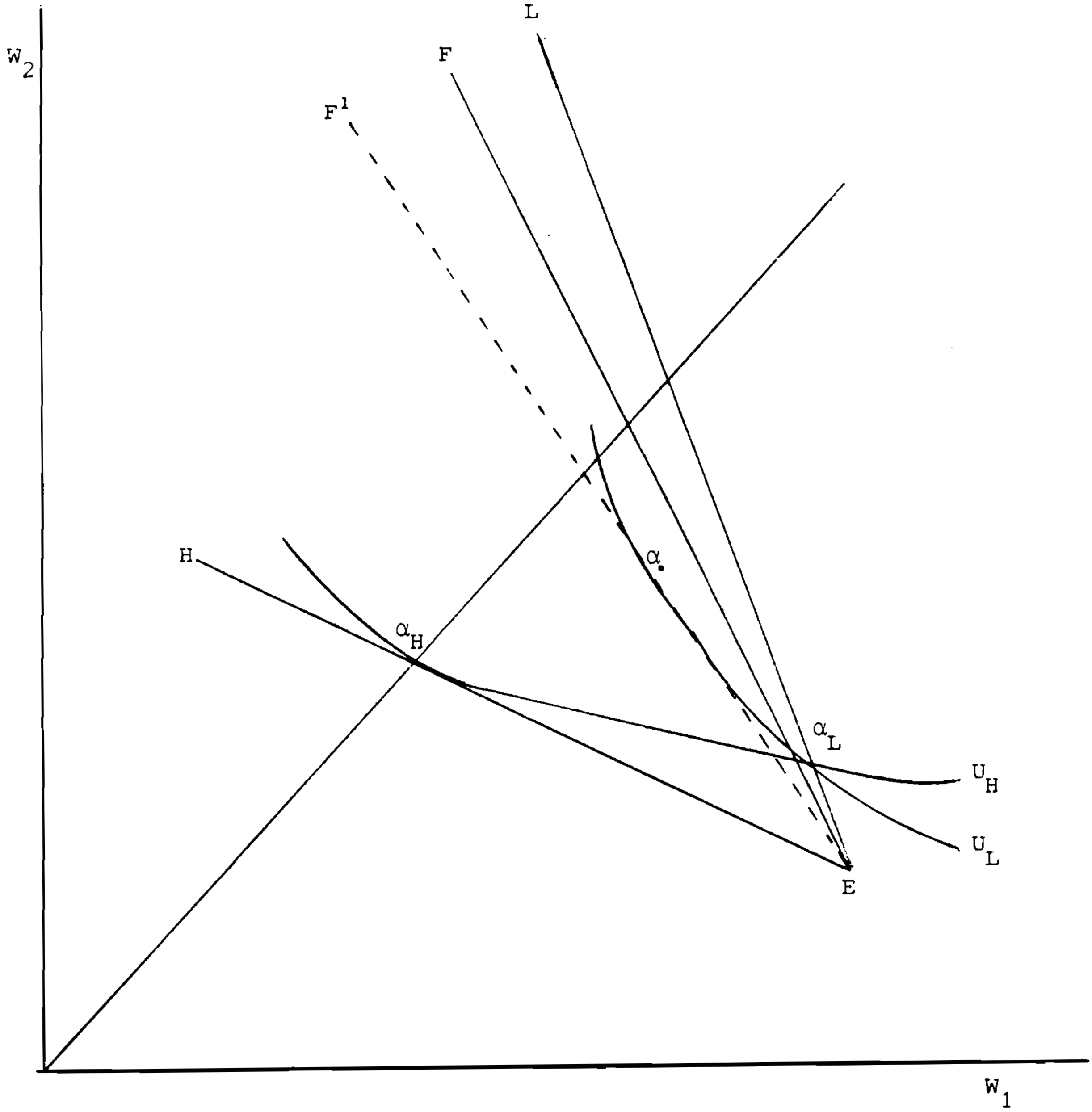

Figure 1: Nash equilibrium.

N_L consumers in each risk class, the slope of this line is given by $-(1-\Phi)/\Phi$, where

$$\Phi = \frac{N_H\Phi_H + N_L\Phi_L}{N_H + N_L} = \frac{\Phi_H + R\Phi_L}{1+R}, \tag{3}$$

and $R = N_L/N_H$. We then determine whether the iso-utility line, U_L, which passes through α_L allows a point such as γ in Figure 1 above it and below the fair market odds line. If such a point exists, some enterprising new firm will make positive profits by offering this policy to consumers. This point is preferred by all consumers to both α_H and α_L. Whether or not such a point exists depends on the ratio of low to high-risk consumers in the market (i.e., on R in (3)). As R decreases, the market odds line EF moves in the direction of EH and the area of instability decreases. Of interest is the maximal R, denoted by R^*, for which the separating equilibrium α_H, α_L is stable. In Figure 1 this would be the R corresponding to the market fair odds line EF' which is tangent to U_L.

To set the stage for our analysis of equilibrium when consumers have imperfect information and as a matter of interest in its own right, we consider a few examples illustrating how R^* varies as objective data changes. To be concrete we use the exponential utility function, although the qualitative results given are more general.

In Figure 2 we depict iso-R^* contours as a function of Φ_H and Φ_L when $X = 500$, $W = 1000$, and $U_i(w) = -\exp(-C_i w)$ with $C_H = C_L = .01$. Several points regarding the figure are worth noting. As the value of Φ_L increases, the maximum value R^* at which the Nash equilibrium is stable decreases. For example, if $\Phi_H = .05$ and $\Phi_L = .01$ then $R^* = 2.59$; when

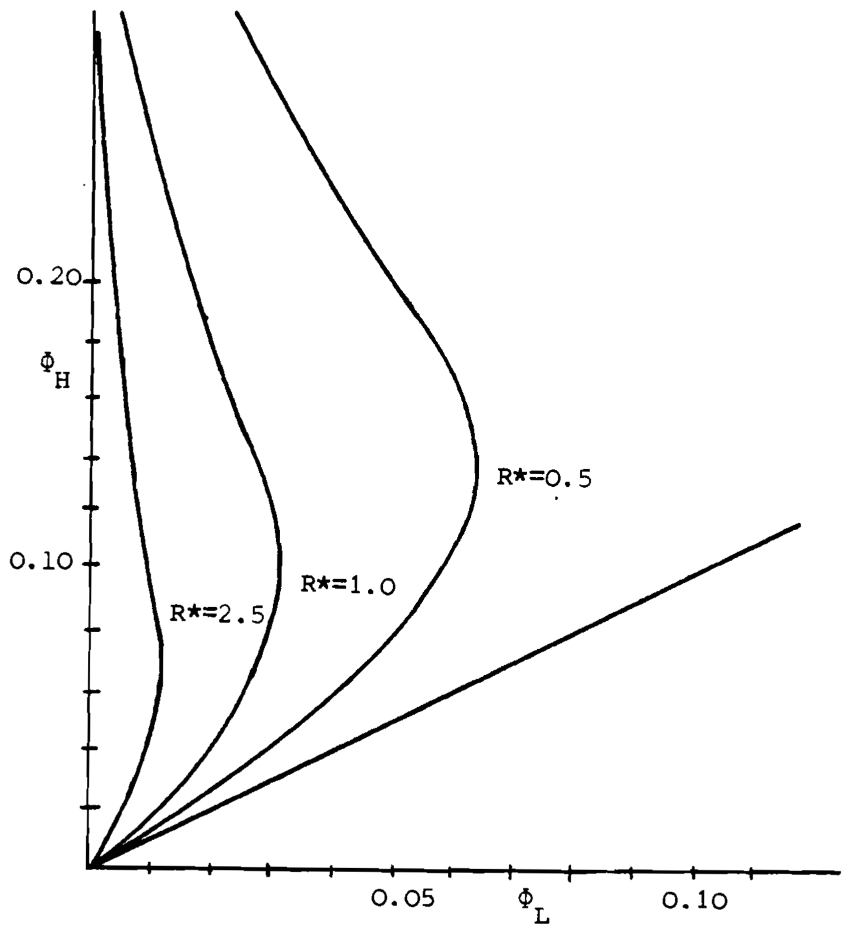

Figure 2: Effects of Φ_i on stability.

$\Phi_H = .05$ and $\Phi_L = .03$ then R^* decreases to .63. The same pattern occurs along any ray $\{(\Phi_H, \Phi_L) / \Phi_H = t\Phi_L, t > 1\}$.

For Φ_H the situation is a bit more complicated. For any given Φ_L the value of R^* increases to a critical value as Φ_H increases and thereafter R^* decreases. The analysis of Figure 2 thus operationalizes the conjectures of R-S concerning the effects of Φ_i on the stability of equilibrium, while at the same time demonstrating that no simple conclusions regarding differences between Φ_H and Φ_L and resulting stability emerge.

Changes in risk aversion also affect stability. As the consumer becomes more risk averse he is willing to give up more W_1 for the same increase in W_2. Thus as the high-risk consumer becomes more risk averse the curve U_H becomes less steep so that α_L moves up on the fair odds line EL in Figure 1. This increases the region of stability. On the other hand, an increase in C_L causes U_L to become less steep which decreases the region of stability. A similar analysis can be undertaken with respect to the affect of changes in the loss X on stability. Higher losses reduce stability because the uninsured point E in Figure 1 is shifted downward with consequent downward shifts in α_H and α_L.

PROPERTIES OF A WILSON EQUILIBRIUM

When a separating, Nash equilibrium does not exist, one may argue (as do Rothschild-Stiglitz) that the market is likely to fail in the absence of regulation. Alternatively, one may proceed as in Wilson (1977) and Spence (1978) to analyze competitive equilibrium by assuming a stronger equilibrium concept, one which attributes foresight and restraint to firms. Such assumptions raise a number of empirical questions which we will not pursue here. We will simply point out the implications of these alternative Nash-Wilson assumptions for the resulting market adjustment processes and equilibria.

In a traditional Nash equilibrium, each firm (including potential entrants) is assumed to determine the set of policies by maximizing its expected profits under the assumption that all other firms make no changes in their current offerings. In the Wilson equilibrium, each firm determines its optimal set of policy offerings under the assumption that any currently marketed policies which become unprofitable as a result of their new offerings will no longer be offered in the marketplace. Clearly, if a Nash equilibrium exists it is also a Wilson equilibrium.

Figure 3 depicts a case where a stable separating (Nash) equilibrium does not exist. A Wilson equilibrium will, nonetheless, always exist for this case (as Miyasaki, 1977, proved). Following Dahlby (1980), the construction of the Wilson equilibrium proceeds as follows. First, the dotted line CD in the Figure is constructed as follows. To each point on BD, like X, the unique point Y on the high-risk iso-utility contour U_H^1 passing through X is determined for which the policies X and Y together achieve zero profits when the high-risk consumers buy X and the low-risk consumers buy Y. Thus, the dotted line CD is the locus of low-risk policies necessary to achieve zero profits if high-risk consumers are offered full insurance. It can then be verified (see Spence 1978) that the Wilson equilibrium is the pair of policies $\alpha^\omega = (\alpha_H^\omega, \alpha_L^\omega)$, illustrated in Figure 3, where the low-risk consumer maximizes his expected utility along CD.

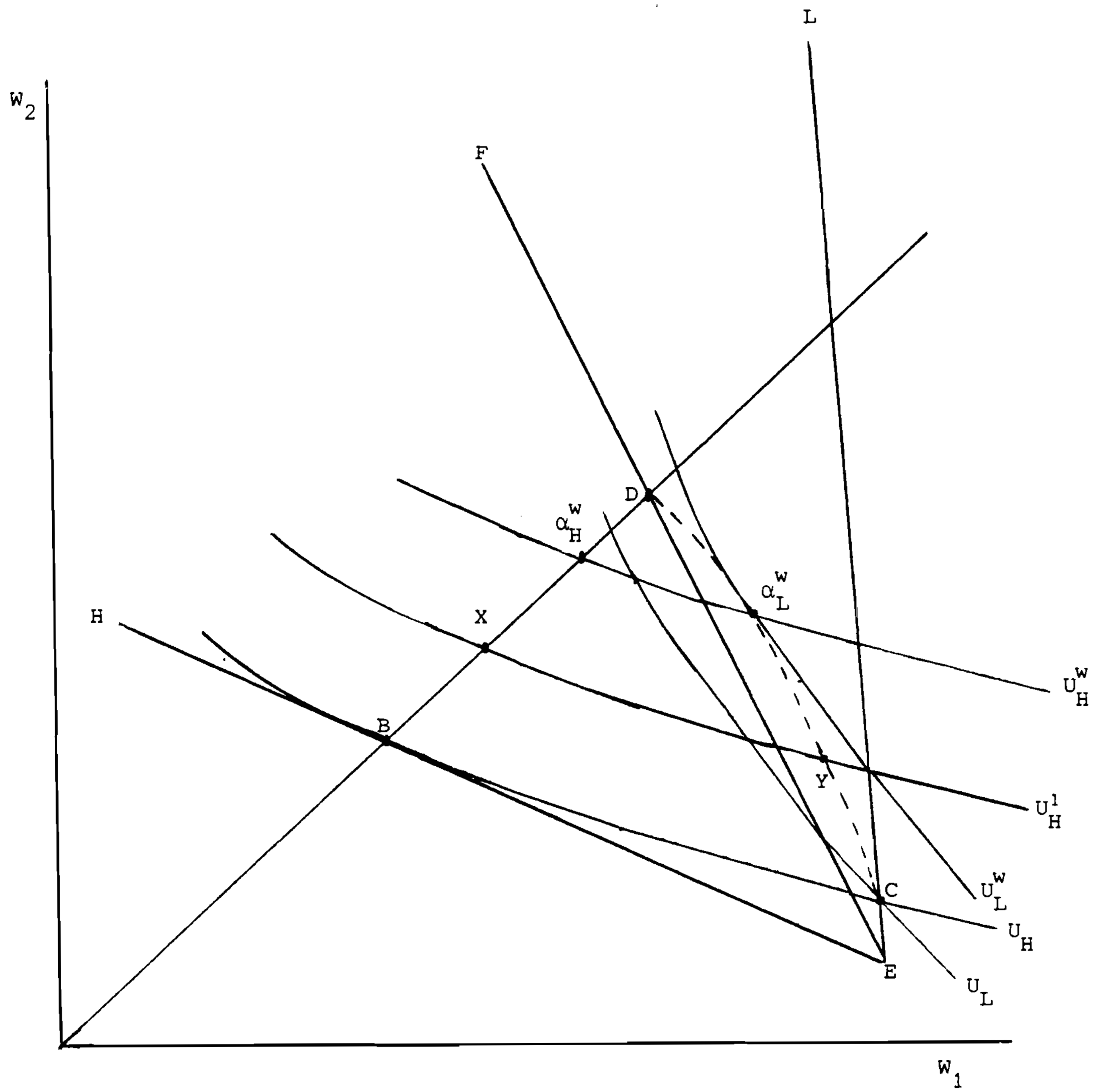

Figure 3: Wilson equilibrium.

The logic establishing that α^{ω} is indeed a Wilson equilibrium is that the only way for a firm to possibly make profits on a policy deviating from α^{ω} is if all remaining firms continue to offer α^{ω}, suffering losses in the process. Given the Wilson assumptions, the deviant policy would never be offered in the first place. The resulting equilibrium α^{ω} turns out to be unique (see M-S-D). As is apparent from Figure 3, a Wilson equilibrium always involves subsidies from low-risk to high-risk consumers whenever it does not coincide with the Nash equilibrium.

IV. MISPERCEPTIONS BY CONSUMERS

In this section we will develop equilibrium results for the case where consumers misperceive the probability of a loss, assuming it to be $\varphi_i \neq \Phi_i$. To motivate the analysis and to provide a contrast with the previous section we will first look at the case where firms have perfect knowledge of the risk facing each of their customers so that they do not have adverse selection problems and a Nash equilibrium exists. We will then turn to the case where firms cannot distinguish between high and low-risk customers, still maintaining the assumption that either or both groups of insured misperceives the probability of a loss.

PERFECT INFORMATION BY FIRMS

Suppose consumers misperceive Φ_i, believing it to be φ_i; firms continue to have perfect information on Φ_i. Hence the consumer's iso-utility curves are based on φ_i instead of Φ_i. Two cases are possible: either $\varphi_i > \Phi_i$ or $\varphi_i < \Phi_i$. We depict both of these situations in Figure 4. Let us concentrate first on the high risk group. Suppose that a consumer estimates $\varphi_H = \varphi_H^1 > \Phi_H$. He is then willing to purchase full insurance (e.g., the policy λ) at more than the actuarially fair price as shown by the perceived iso-utility curve U_H^1. This curve is tangent to the consumer's perceived odds line, EH_1, which is below EH because $\varphi_H > \Phi_H$.[7] Firms offering the policy λ would make positive profits, thus inducing entry by others at a lower premium. Price will continue to fall until it reaches an equilibrium at α_H.

If consumers underestimate the risk so that $\varphi_H = \varphi_H^2 < \Phi_H$ their perceived odds line, EH_2, is above EH. They will only want to purchase full coverage if $p_H = \varphi_H < \Phi_H$. Firms offering such a policy will thus lose money, so that equilibrium will be established at the point where the consumer's perceived iso-utility curve, U_H^2, is tangent to the objective fair odds line EH. This point α_H^2 indicates that the resulting market equilibrium will provide a policy with less than full coverage when firms have perfect information and consumers underestimate Φ_H. The analogous situation holds for low-risk consumers who underestimate their risk as

[7] Iso-utility contours of the perceived utility function $E[(U)] = (1-\varphi)U(W_1) + \varphi U(W_2)$ are all tangent to the perceived fair odds line $W_2 = -[(1-\varphi)/\varphi]W_1 + (W/\varphi) - X$ where full coverage occurs (i.e., where $W_1 = W_2$). The proof follows by implicit differentiation of the iso-utility contours along the full-insurance line $W_1 = W_2$.

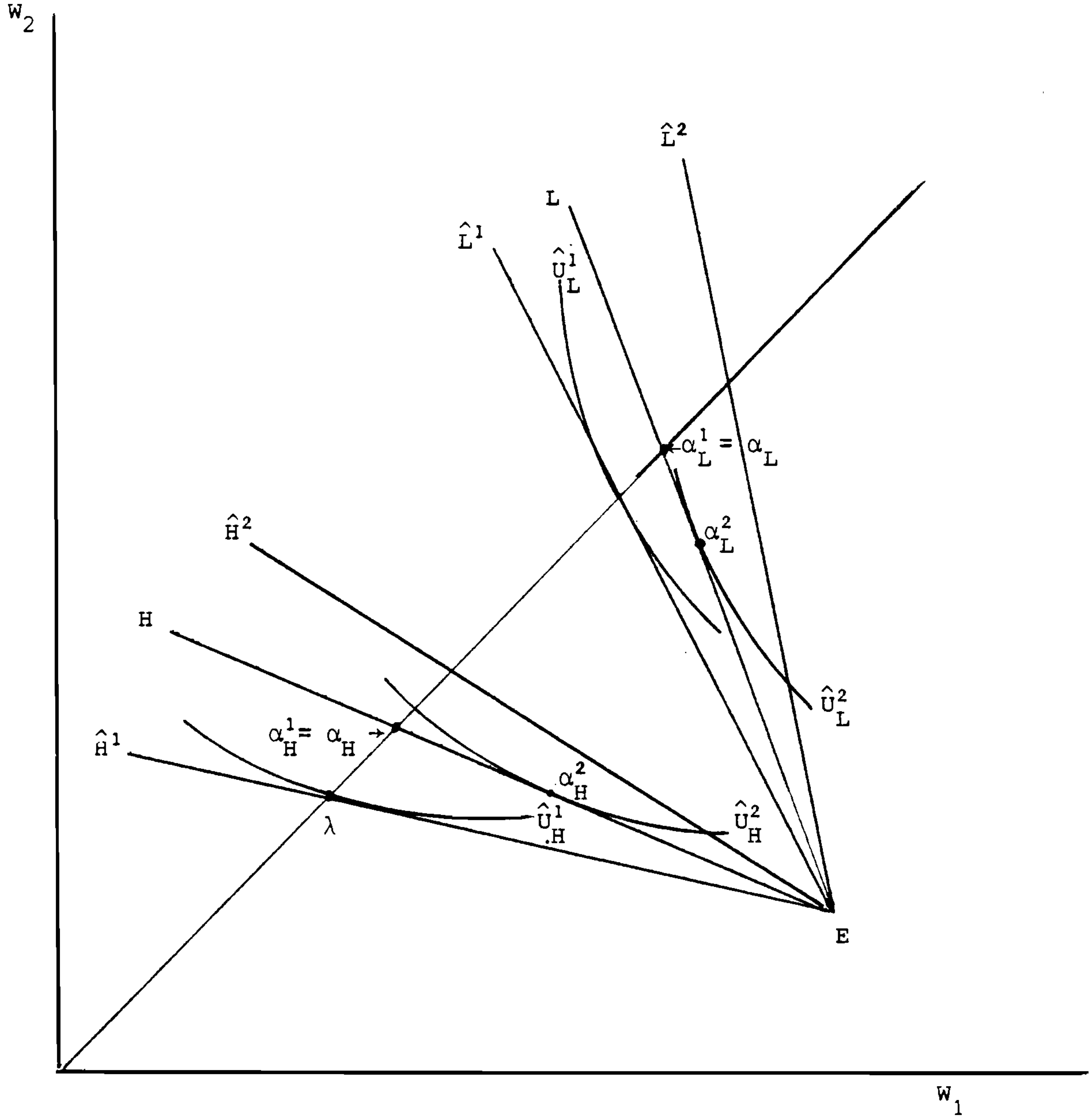

Figure 4: Equilibrium under consumer misperceptions

shown by the equilibrium point α_2^L in Figure 4.

To illustrate the impact of consumer misperception on equilibrium consider an example using an exponential function $U_1 = -be^{C_i w}$ where C_i is the risk aversion coefficient. The relevant objective data are:

$$\Phi_H = .04 \quad \Phi_H = .02 \quad X = 500 \text{ and } W = 1000$$

The equilibrium insurance policies are

$$<p_H, Q_H> = <.04, 500> \quad <p_L, Q_L> = <.02, 500>$$

if consumers have perfect information. In this case α_H and α_L are respectively (980, 980) and (990, 990) no matter how risk averse any individual may be. To examine the impact of misperception on equilibrium values it is instructive to vary not only φ_H and φ_L but also C_H and C_L. The following Table presents illustrative results. The first row represents the case where probabilities are either known perfectly or overestimated. The equilibrium policy is always full insurance. When probabilities are underestimated the equilibrium policy will deviate increasingly from full insurance as the consumer becomes less risk averse. For example if $\varphi_L = .01$ and $C_L = .02$ then $\alpha_H = (990.7, 955.7)$ compared with $\alpha_H = (991.4, 921.4)$ when $C_L = .01$.

We close this section by noting that the above analysis goes through unchanged if consumers also misestimate the magnitude of the loss X. In this case, their estimate of X, say X_i for group i, replaces X in equation (1) in computing perceived expected utility. Under- (over)estimates of X then have the same effect on perceived iso-expected utility contours and resulting market equilibrium as under-(over)estimates of Φ.

Table: The effects of consumer misperceptions.

Consumer Perceptions	Objective Data			
	$\Phi_H = 0.04$	$\Phi_H = 0.02$	$X = 500$	$W = 1000$
	$C_H = 0.01$	$C_H = 0.02$	$C_L = 0.01$	$C_L = 0.02$
	α_H	α_H	α_L	α_L
$\phi_H \geq \Phi_H$	(980.0, 980.0)	(980.0, 980.0)	--	--
$\phi_L \geq \Phi_L$	--	--	(990.0, 990.0)	(990.0, 990.0)
$\phi_H = 0.03$	(981.2, 951.2)	(980.6, 965.6)	--	--
$\phi_L = 0.01$	--	--	(991.4, 921.4)	(990.7, 955.7)

IMPERFECT INFORMATION BY FIRMS

Suppose that firms cannot distinguish between high- and low-risk consumers, and that consumers may misperceive the probability of a loss. Just as we proceeded with informed firms (see Figure 4), so here too the only change required to incorporate consumer misperceptions into the analysis of the previous section is to substitute *perceived* for *actual* expected utility contours in the analysis. We restrict our attention here to misperceptions of Φ, although a similar analysis applies for misperceptions of X, as discussed just above.

Nash Equilibrium

We illustrate in Figure 5 the process for determining the Nash separating equilibrium if it exists. Of fundamental concern to us are the effects of misinformation on market stability and the resulting equilibrium policies. It is relatively straightforward to determine what impact deviations of φ_H and φ_L from the true parameters will have on these characteristics. In the case of the high risk group we note that if $\varphi_H > \Phi_H$ then full coverage will be offered. Furthermore as Φ_H increases, the point α_L moves up the fair odds line EL and increases the region of stability. We illustrate these points in Figure 5 by constructing two iso-utility curves U_H^1, U_H^2 which correspond to $\varphi_H^1 > \Phi_H$ and $\varphi_H^2 < \Phi_H$ respectively. If all other data remain constant, then we see that the policies $\{\alpha_H^1 , \alpha_L^1\}$ associated with φ_H^1 will be stable. On the other hand, the policy

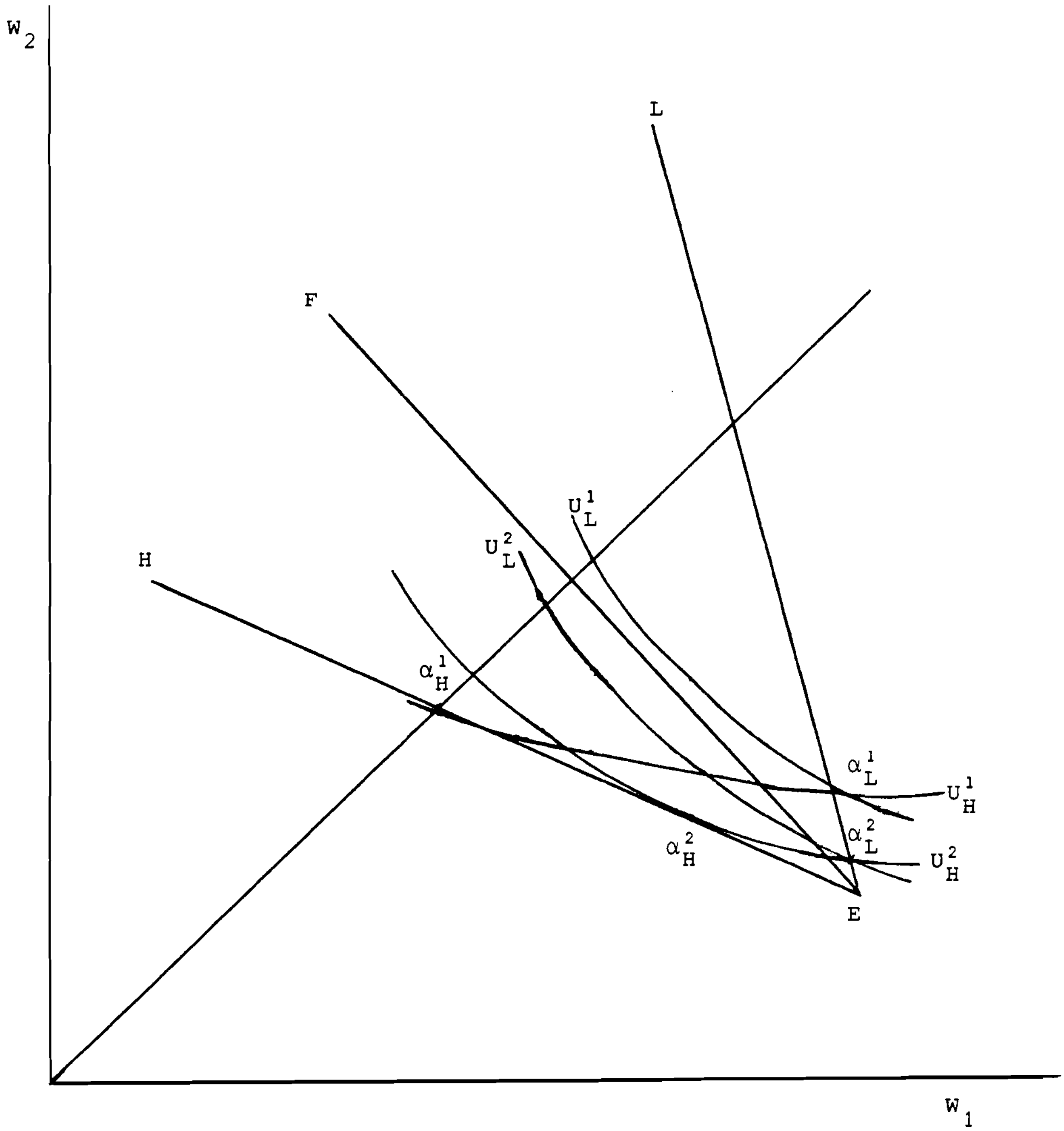

Figure 5: Illustrating the effects of misperceptions of Φ_H.

$\{\alpha_H^2, \alpha_L^2\}$ is unstable because U_L^2 is below the market fair odds line EF, thus enabling a new firm to enter and make positive profits in the short-run. Note that we can generate similar affects on stability by changing the high-risk consumer's risk aversion: increasing the degree of risk aversion produces the same affect as increasing φ_H.

Misperception by low-risk consumers is illustrated in Figure 6 for the case where $\varphi_H = \Phi_H$. It should be noted that α_L will not be affected by misperception on the part of the low-risk consumer because it is determined solely by the high-risk iso-utility curve associated with α_H, in this case U_H. If low-risk consumers overestimate Φ_L (i.e., $\varphi_L^1 > \Phi_L$) then the iso-utility curve is given by U_L^1. The curve U_L^2 represents the case where $\varphi_L^2 < \Phi_L$. In general, the region of stability is increased as φ_L decreases. In Figure 6 a stable separating equilibrium exists when $\varphi_L = \varphi_L^2$ but not when $\varphi_L = \varphi_L^1 > \varphi_L^2$. The impact of misperception of Φ_L produces similar effects in stability as changes in risk aversion. As the consumer becomes more risk averse his iso-utility curve become less steep reflecting a willingness to sacrifice more W_1 for the same amount of W_2. This is similar to the effects just discussed for increases in φ_L.

The above illustrative examples assumed that all consumers in the high and low-risk groups had the same misestimates of the probability. If there were a whole spectrum of misestimates then the procedure for determining whether a stable equilibrium exists is based on similar principles. There would be a range of policies offered to high-risk individuals ranging from full coverage for all those who perceive $\varphi_H \geq \Phi_H$ to the lowest tangency of the U_H curve to the fair odds line. Figure 7 depicts the three

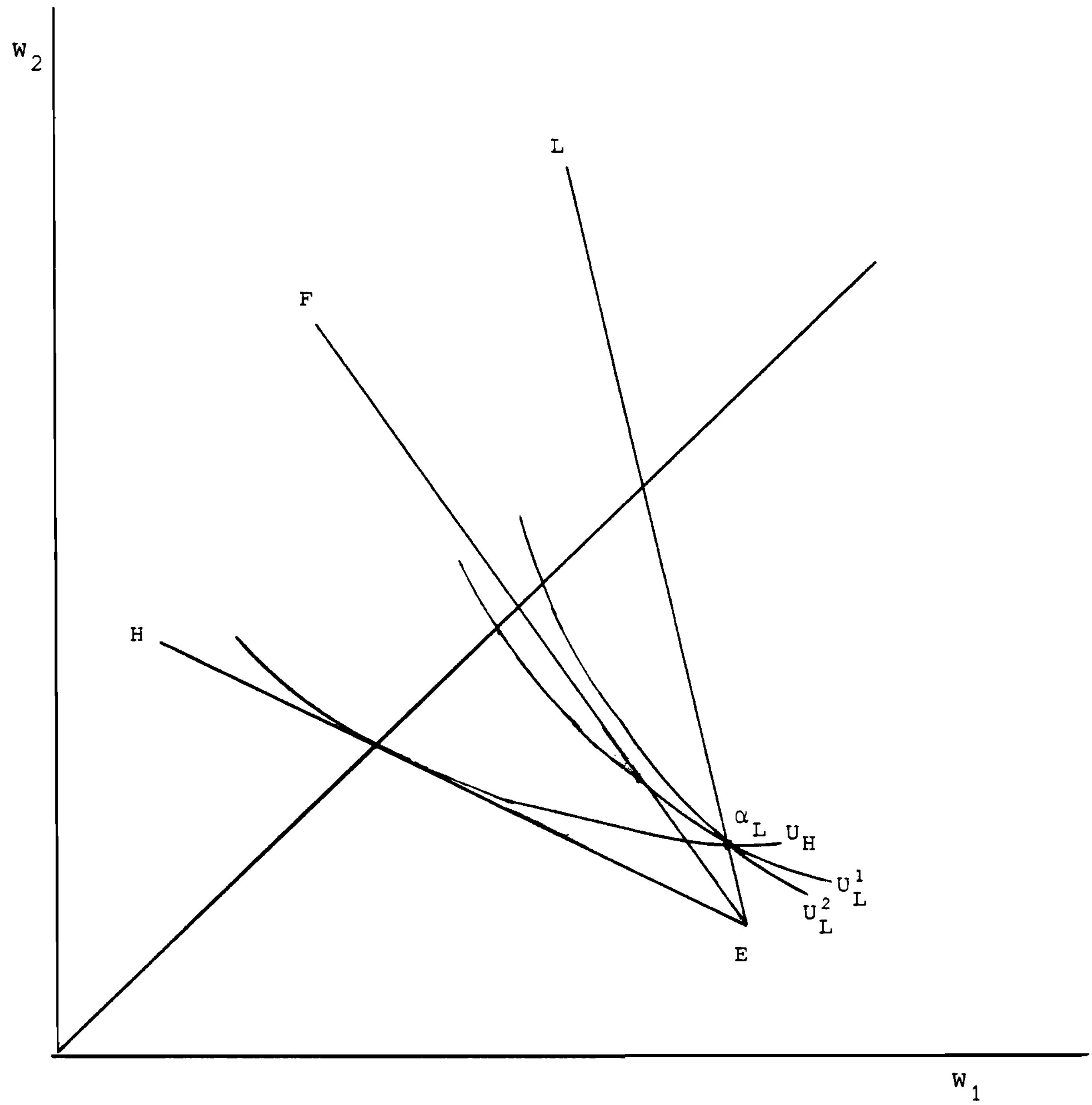

Figure 6: Illustrating the effects of misperceptions of Φ_L.

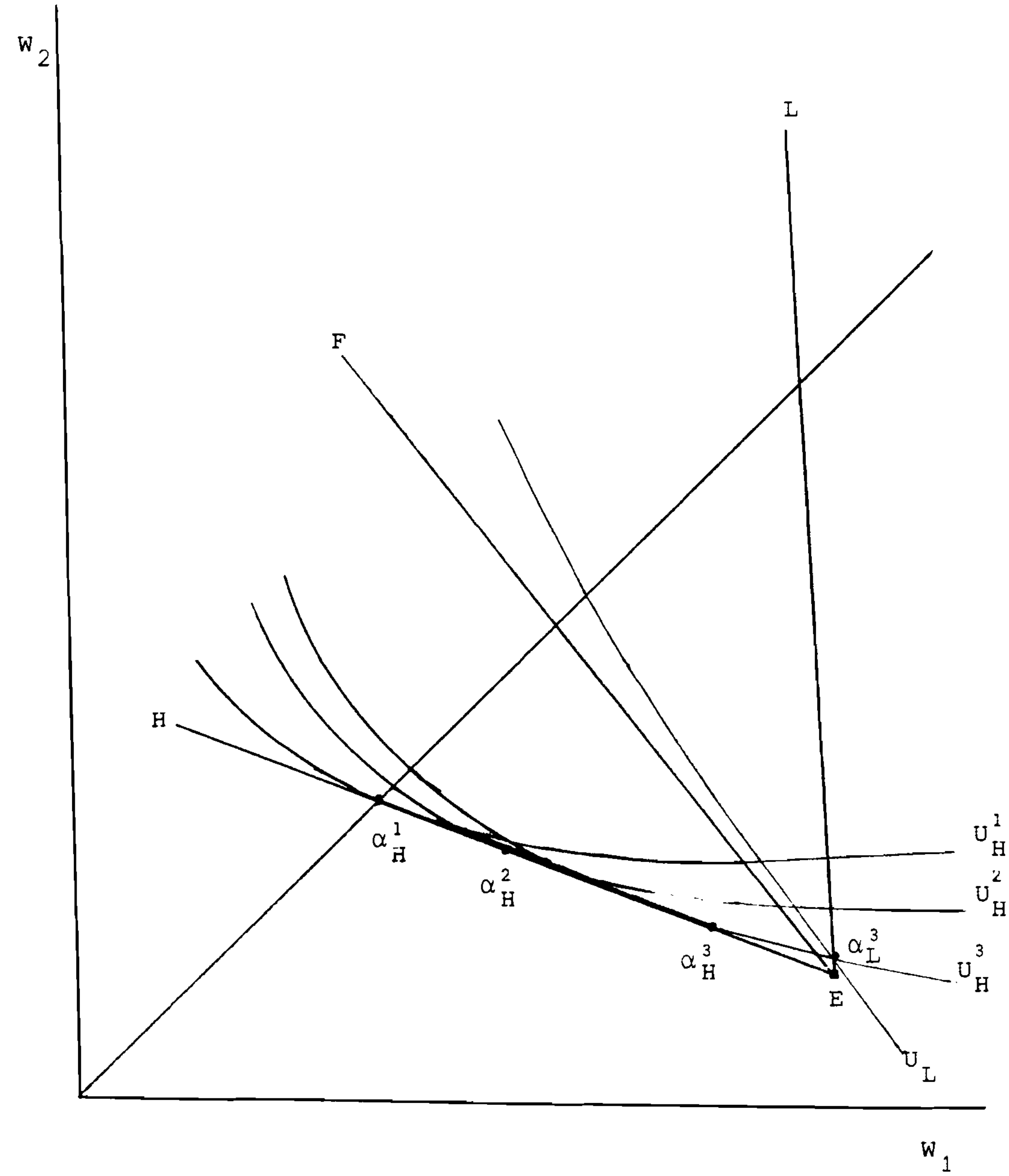

Figure 7: Nash equilibrium with consumer misperceptions.

policies α_1^H, α_2^H, α_3^H, for the case where two high-risk groups (2 and 3) underestimate Φ_H and one (group 1) correctly estimates it. There is only one policy for the low-risk group--α_3^L which is determined by the intersection of U_3^H with EL. This construction guarantees that every high-risk consumer is provided a policy which maximizes his perceived expected utility at actuarial rates (i.e., along EH), and all low-risk consumers are offered the highest coverage which, at actuarial rates (along EL), is consistent with not attracting any high-risk consumers to the policy intended for low-risk consumers.

We thus see that low-risk individuals are penalized by those high risk consumers underestimating φ_H--they are offered less coverage than if they had estimated $\varphi_H = \Phi_H$. The stability of equilibrium is determined by looking at the position of the *lowest* U_L curve (i.e., the U_L curve corresponding to the minimum φ_L) to the fair odds line. The situation least likely to lead to stability is if some high-risk consumers grossly underestimate Φ_H and some low-risk consumers overestimate Φ_L. It should now be clear why stability of Nash equilibrium is so sensitive to perceptions of Φ_i by consumers. Suppose low-risk consumers estimate $\varphi_L \gg \Phi_L$ and one high risk consumer estimates $\varphi_H \ll \Phi_H$. What otherwise may have been a stable Nash equilibrium is now unstable. This observation suggests that the existence of a Nash equilibrium can be sensitive to consumer (mis)perceptions of the risk involved[8].

[8]Of course, we assume away fixed costs of entry in this perfectly competitive model. If these were present, they would naturally dampen the entry threat to existing firms resulting from changes of the above sort, where only one (or a few) high-risk consumers' perceptions changed.

Wilson Equilibrium

As we will see, the Wilson equilibrium behaves somewhat more smoothly in response to consumer misperceptions. The procedure for determining the Wilson equilibrium set of policies parallels the one described above for the Nash equilibrium. In particular, if $\varphi_H^{\min}$ is the lowest estimate of Φ_H by high-risk individuals and $\varphi_L^{\max}$ is the highest estimate of Φ_L by low-risk individuals, then the set of optimal policies for all individuals is based on these two extreme groups just as in the Nash equilibrium case. The one critical difference between the two cases is that in a Nash equilibrium there are no cross-subsidies, whereas the Wilson equilibrium entails such subsidies just as in the case where consumer perceptions are accurate.

We restrict our attention here to deriving the Wilson equilibrium when there are just four groups,[9] denoted H_1, H_2, L_1, L_2, where

$$\varphi_H^1 \geq \Phi_H > \varphi_H^2 ; \qquad \varphi_L^1 \geq \Phi_L > \varphi_L^2 .$$

Thus, groups $\{H_1 , L_1\}$ overestimate and groups $\{H_2 , L_2\}$ underestimate their respective accident probabilities. Our first concern will be to determine, for any fixed level of total subsidy from low- to high-risk consumers, what high-risk policies can be offered in the market. Thereafter we analyze what level of subsidy is compatible with market equilibrium under the Wilson assumptions on firm adjustment.

[9]A more formal derivation for multiple sub-groups is contained in Kleindorfer and Kunreuther (1981).

We begin by noting that if a subsidy is provided any high-risk group, then it must be provided in such a manner that the *perceived* expected utility of the group in question is maximized over all policies offering the same level of subsidy. Otherwise a new entrant could offer the group a policy it would prefer and which would entail a lower subsidy. Now the set of all constant-subsidy policies for the high-risk group is easily represented in (W_1, W_2) space by the transformation

$$(W_1, W_2) \to (W_1 + S, W_2 + S),$$

where S is the subsidy involved. In Figure 8, the parallel lines S_0, S_1, S_2 indicate sets of policies with increasing levels of subsidy to the high-risk group, where S_0, the zero-subsidy line, is just the fair odds line for the high-risk group.

The condition that perceived expected utility be maximized for each high-risk sub-group along the iso-subsidy lines just derived is reflected in Figure 8. For group 2 this yields the locus L_H^2 which is the set of policies obtained through the tangency of the iso-perceived utility contours to the iso-subsidy lines. For group 1, the maximizing policy along any iso-subsidy line is just the full-insurance policy since all our consumers are risk averse and φ_H^1 is no smaller than Φ_H.

Now what we have noted above is that only policies on $\mathcal{L}_H^1$ or $\mathcal{L}_H^2$ can be offered to groups 1 and 2 respectively. A further feasibility restriction is that whatever is offered to group 2 must not be preferred by group 1 to the policy intended for them. For example, suppose the policies (A_1, B_2) in Figure 8 were offered on the market. Clearly *all* of the high-risk consumers would prefer B_2 to the policy A_1. A new entrant

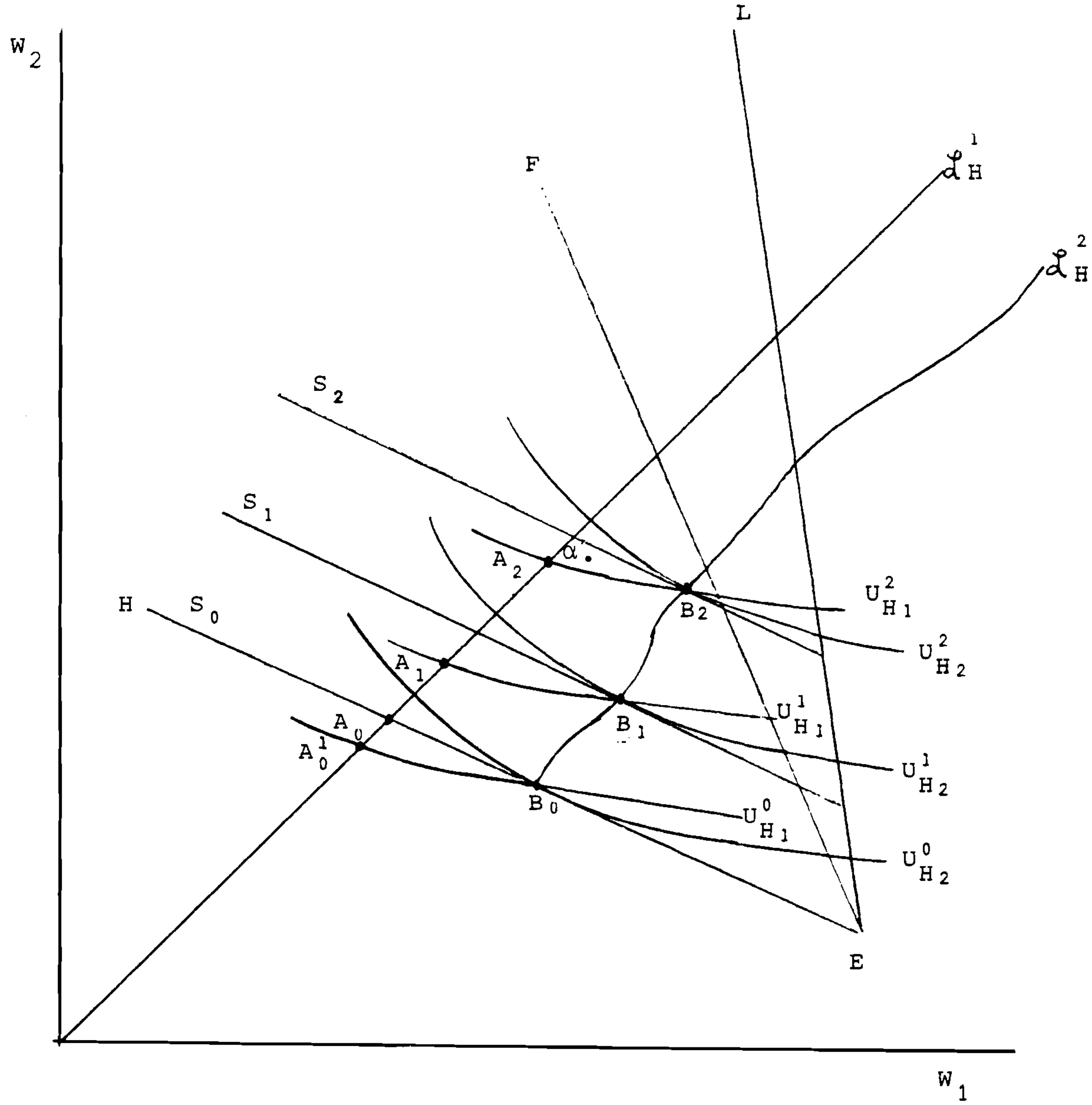

Figure 8: Constructing a Wilson equilibrium.

could then offer the policy γ and attract only individuals from group 1. Such a new entrant would thus pay the high-risk group as a whole a smaller subsidy. Thus, if B_2 is to be offered at all, competition (to minimize total subsidies to the high-risk group) will push the solution to the pair (A_2, B_2). A similar argument holds for the pair (A_1, B_1). Note for (A_0, B_0), however, that the point A_0', if offered, would offer positive profits, thus inducing entry and pushing the solution to (A_0, B_0).

We see from the above discussion that only policies (α_H^1, α_H^2) satisfying the following conditions can qualify as candidates for the policies offered to the high-risk groups: first, each of the α_H^i must belong to the respective perceived utility maximizing curve $\mathcal{L}_H^i$; and secondly, the condition

$$E\left[U_H^1(\alpha_H^1)\right] = E\left[U_H^1(\alpha_H^2)\right]$$

must obtain unless this implies a positive subsidy to group 1, in which case group 1 is offered the actuarially fair, full-coverage policy.

We note that the above procedure provides us, for any pre-specified subsidy S to the high-risk group, with a unique pair of policies $< \alpha_H^1 (S), \alpha_H^2 (S) >$ which can be marketed to these two high-risk groups.

Having determined feasible policy offerings for the high-risk groups for any specified subsidy level S, we can now proceed to determine the amount of subsidy to the high-risk groups which is compatible with a Wilson equilibrium. We proceed as in Figure 3 to construct a locus of low-risk policies which, if purchased together with the pair

$< \alpha_H^1\ (S)\ ,\ \alpha_H^2\ (S) >$, will provide a subsidy of S to the high-risk groups and are such that no one in the high-risk group finds it attractive to switch to the policy intended for the low-risk groups. Figure 9 summarizes this process, paralleling that described in Figure 3. As before, the desired locus of low-risk policies is labeled *CD*. Thus, the policies $(A_i\ ,\ B_i\ ,\ C_i)$, $i = 0, 1, 2$, are constructed so that (a) zero profits are achieved and (b) at the level of subsidy implied for each high-risk group the utility of that group is maximized; and (c) no one prefers the policy intended for any other group to his own.

The final piece of the Wilson equilibrium puzzle can now be put into place, namely the determination of the particular policy (there will only be one) offered to the low-risk group. Just as in Figure 3, so here also, it is easy to see that the policy offered to the low-risk group must be such as to maximize their welfare along the zero-profits contour *CD* in Figure 9. Moreover, since the low-risk group with the highest misperception will be the easiest to "skim" off, it must in fact be this group whose perceived utility is maximized along *CD*. Putting all of this together we obtain the Wilson equilibrium depicted in Figure 10 as the policy $< \alpha_H^1\ ,\ \alpha_H^2\ ,\ \alpha_L >$.

We may note immediately that as the maximal overestimate of any low-risk group increases (i.e., as φ_L^i increases), the Wilson equilibrium moves up the zero profit contour *CD*. Thus, the policy α_L' might correspond to the Wilson equilibrium policy offered to all low-risk consumers if φ_L^1 were to increase (or if there were another sub-group within the low-risk group with higher overestimates of Φ_L than those of sub-group L_1). Thus, overestimates by any sub-group in the low-risk group costs

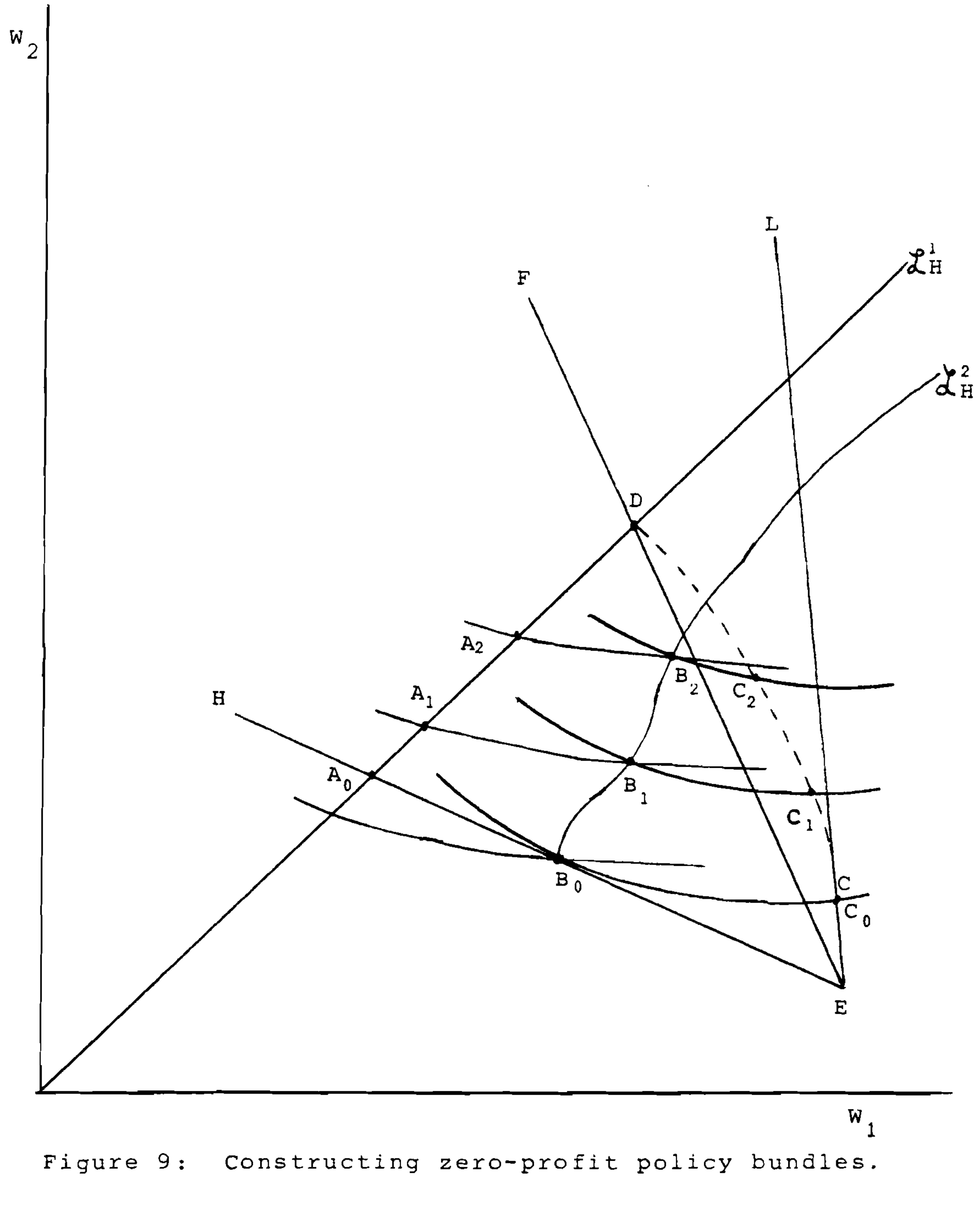

Figure 9: Constructing zero-profit policy bundles.

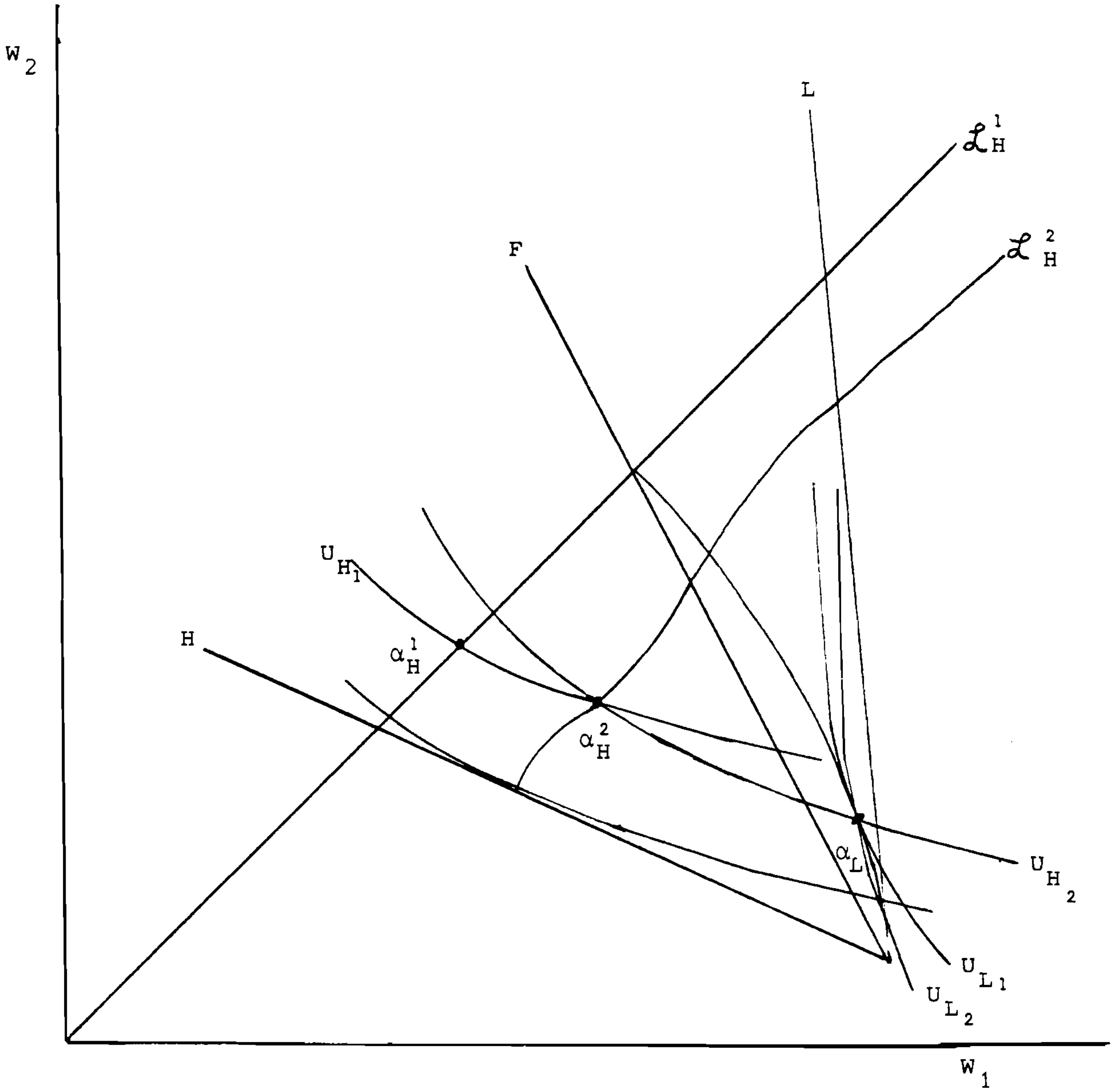

Figure 10: Wilson equilibrium.

everyone in the low-risk group additional taxes which flow to the high-risk group as subsidies.

Similarly, a more pronounced underestimation by the high-risk group reduces the amount of coverage low-risk groups are offered in equilibrium. Thus, we see that information imperfections on the part of consumers can affect both welfare outcomes associated with market equilibrium as well as the nature and existence of such equilibria.

V. GENERALIZATIONS AND WELFARE IMPLICATIONS

The graphical procedure described above for investigating stability of Nash equilibrium and characterizing Wilson equilibria is quite general. If there are n different risk groups and a range of misperceptions within each one of these groups the same general principles above apply:

(1) There can be multiple policies offered to the highest risk group (denoted H) depending on the extent of their misperceptions,

(2) There is only one policy offered to the lowest risk group (denoted L) based on $\varphi_L^{\max}$,

(3) All risk groups between H and L have their policies determined so that an individual in a higher risk group has *no* incentive to purchase a policy designed for a lower risk group. Naturally this incentive is based on perceived expected utility.

(4) With respect to the stability of a Nash equilibrium one must determine whether there is any pooled policy which is more attractive to adjacent paired groups than the proposed separat-

ing policies and at the same time yields a profit to any firm offering such a policy.

The above principles are studied analytically in Kleindorfer and Kunreuther (1981) following the M-S-D framework. Assuming their validity for the moment, the following welfare implications, which we have analyzed here for the case of two groups, may be conjectured in general:

(W1) An increasing underestimation of risk by the higher risk groups reduces the amount of coverage low-risk individuals are offered (under either a stable Nash equilibrium or a Wilson equilibrium) and also reduces the (*ex ante* objective) expected utility of the resulting policy offered to low-risk individuals.

(W2) Increasing overestimation of risk by the lower risk groups increases the tax paid by these low-risk people because they demand more insurance.

Besides verifying W1 and W2 generally, several additional welfare and regulatory matters are of interest. For example, when is compulsory insurance a welfare-improving regulation?[10] How is learning incorporated into both firms' knowledge of consumers' as well as consumers knowledge of the risks against which they are insuring themselves. Finally, and perhaps most importantly, there is the question of the applicability of the price-quantity framework we have been using here.

In contrast with the price-quantity framework, one might suppose that insurance policies are specified through a premium (price per dollar of coverage) as in Pauly (1974) and Kunreuther and Pauly (1981), where

[10] See Dahlby (1980) for an analysis of this question when consumers are perfectly informed.

each customer then determines the total coverage he or she will purchase at the stated premium. The primary reason why such pricing policies may be a better model of actual insurance markets than price-quantity policies was already recognized by R-S, viz. price-quantity policies require a central monitoring system for the entire insurance industry if they are to function. If, for example, a policy $< p , Q >$ is offered with $p < \Phi_H$ and $Q < X$, then high-risk consumers would buy several such policies (with total coverage approximating X) from different firms, thus undermining the intended self-selection mechanism inherent in offering less than full coverage. The only way to prevent this is to monitor all (high-risk) consumer purchases to ensure that only one policy is purchased. Such a central monitoring system is problematical in a competitive market. Moreover, if such a monitoring system could be set up at low cost, it is also likely that, at little additional cost, sophisticated statistical techniques could be used to classify customers over time according to their risk class. Each customer could then be offered the socially optimal policy of full coverage at actuarial rates.

The broader issue here is the empirical question of which forms of policy are actually offered to the consumer in various insurance contexts as well as how firms and consumers gather and process information relating to these policies. This issue has both institutional as well as decision theoretic characteristics (e.g., involving insurance agents' behavior in representing available policies). As we have seen in this paper, informational and behavioral differences resulting from the mutual interaction of firms and consumers have interesting implications for market equilibrium, regulation, and welfare.

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