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Rising Longevity, Increasing the Retirement Age, and the Consequences for Knowledge-based Long-run Growth

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We assess the long-run growth effects of rising longevity and increasing the retirement age when growth is driven by purposeful research and development. In contrast to economies in which growth depends on learning-by-doing spillovers, raising the retirement age fosters economic growth. How economic growth changes in response to rising life expectancy depends on the retirement response. Employing numerical analysis, we find that the requirement for experiencing a growth stimulus from rising longevity is fulfilled by the USA, nearly met by the average OECD economy, but missed by the European Union and by Japan.

INTRODUCTION

Rich countries have been facing unprecedented increases in life expectancy over the past decades. For example, life expectancy in the USA increased from about 69 years in the 1950s to about 79 years in 2019, while it increased by even more in countries such as France and Germany, from about 67 years in the 1950s to more than 80 years in 2019 (United Nations 2022). This development undoubtedly raises individual wellbeing (Kuhn and Prettner 2016; Baldanzi *et al*. 2019; Frankovic *et al*. 2020; Bloom *et al*. 2021). However, it also comes with certain economic concerns. If people live longer for a constant retirement age, then the resulting increase in economic dependency could lead to a reduction in economic growth and pose a threat to the sustainability of social security systems and pension funds (Gruber and Wise 1998; World Economic Forum 2004; *Economist* 2009, 2011; Bloom *et al*. 2010). Whether such adverse effects materialize, however, depends on the extent to which individuals change their savings behaviour in response to increasing longevity and on the extent to which retirement policies are adjusted to cope with demographic change (Bloom *et al*. 2007, 2010).

The economic effects of changing life expectancy and changing retirement policies have been analysed, for example, by Futagami and Nakajima (2001), Heijdra and Romp (2009), Bloom *et al*. (2007, 2014), Heijdra and Mierau (2011), d'Albis *et al*. (2012), Prettner and Canning (2014), Kuhn *et al*. (2015) and Sánchez-Romero *et al*. (2016). Either these papers assume a partial equilibrium perspective in which the interest rate and economic growth do not react to individual decisions, or they are based on models in which growth is driven by physical capital accumulation—either with decreasing returns to capital accumulation à la Solow (1956), Cass (1965), Koopmans (1965) and Diamond (1965), or with constant returns to capital accumulation due to learning-by-doing spillovers à la Romer (1986) and Rebelo (1991). However, long-run economic growth in rich countries is driven mainly by purposeful research and development (R&D) investments (Romer 1990; Aghion and Howitt 1992; Jones 1995; Kortum 1997). To analyse the effects of rising life expectancy and changing the retirement age for these economies, we therefore integrate a demographic structure of overlapping generations in the vein of Blanchard (1985) into the endogenous R&D-based growth model of Romer (1990) with a given retirement age.

In a first step, we show that increasing the retirement age raises economic growth and the equilibrium interest rate because the positive growth effect of the larger workforce implied by a longer working life overcompensates for the negative growth effect of reduced savings. Thus raising the retirement age may be an appropriate policy response to the phenomenon of secular stagnation in which economic growth is sluggish and the equilibrium interest rate is stuck below zero (Eggertsson *et al*. 2019, 2020).

The savings channel—and the savings channel only—also features in Futagami and Nakajima (2001) and Heijdra and Mierau (2011), who find that an increase in the retirement age leads to a *reduction* of capital accumulation and therefore to a *reduction* of economic growth in a Romer (1986) model in which growth is driven by capital accumulation with learning-by-doing spillovers. While their results are highly relevant for economies with exogenous technological progress (e.g. small economies adopting technologies from abroad), our results imply the opposite effect in countries that drive the worldwide technological frontier such as Germany, Japan and the USA. The opposing results are also interesting from a theoretical point of view. It is sometimes argued that results based on endogenous growth models with learning-by-doing spillovers (Romer 1986) are similar to the results based on R&D-driven growth models (Romer 1990). While it is known that this is not necessarily the case, our contribution shows yet another counterexample that arises when analysing the implications of an increase in the retirement age. Thus from a policy perspective, it is important to take the underlying structure of the economy into account when considering retirement policies.

In a second step, we show that the extent to which economic growth changes in response to rising life expectancy depends on the accompanying pension policies. This is because as long as the boost to savings that is brought about by an increase in longevity is not very strong, the increase in economic dependency that comes with greater longevity *for a given retirement age* will typically lead to a reduction in R&D activity and economic growth. We provide a necessary and sufficient condition for a positive growth impulse, requiring the elasticity of the retirement age with respect to the increase in longevity to be sufficiently large. We show further that an increase in the retirement age in proportion to an increase in life expectancy is sufficient for longevity growth to stimulate economic growth.

Our analysis concludes with a set of numerical examples, studying how the USA, the European Union (EU), Japan, and the average Organisation for Economic Co-operation and Development (OECD) economy have fared in respect to the rise in longevity over the time span 2000–2017. We find that only the USA has clearly benefited in terms of higher economic growth, whereas the growth stimulus from the longevity increase was neutral for the OECD and negative for the EU and Japan. Our numerical results suggest that a longevity-driven boost to savings is much weaker than the effects that run through changes in labour participation. We also find that quantitatively, both channels tend to be dominated by changes to R&D productivity.

To allow for analytical results, our analysis relies on a number of simplifying assumptions, namely: (i) an R&D production function that relies on labour; (ii) constant returns in the production of R&D, and thus the presence of a scale effect; (iii) a stationary population and a mortality rate that is uniform across all ages; (iv) the exogeneity of retirement; and (v) the absence of investments in human capital. We show that our results are robust to (ii)–(v), and argue that while the alternative lab-equipment approach would shut down the impact of ageing on R&D through the savings channel, this seems at odds with evidence on the role of financial markets for R&D-driven growth.

The paper is organized as follows. In Section I, we set up an overlapping generations version of the R&D-based endogenous growth model of Romer (1990) with a fixed retirement age. In Section II, we derive our main results and discuss their relevance for actual retirement policies carried out in different countries. In Section IV, we assess how our results change depending on the underlying economic assumptions. Finally, in Section V, we draw our conclusions. A number of mathematial derivations have been relegated to an Appendix.

I. The Model

Household side Consider an economy in which individuals enter the labour market as adults at time t_0 and maximize the discounted stream of their remaining lifetime utility as given by

$$U = \int_{t_0}^{\infty} \log(c)\ \mathrm{e}^{-(\rho+\mu)(t-t_0)}\ \mathrm{d}t, \tag{1}$$

where c is instantaneous consumption, ρ is the pure rate of time preference, and μ represents the mortality rate. The latter augments the rate of time preference because the risk of death constitutes a further reason to consume earlier in life rather than later. Individuals earn non-capital income w (wages and lump-sum redistributions of profits from intermediate goods producers) as long as they are not retired. Suppressing time arguments and following Yaari (1965) in assuming that individuals save in terms of fair annuities that insure against the risk of dying with positive capital holdings, the flow budget constraint reads

$$\dot{k} = \chi w + (\mu + r)k - c, \tag{2}$$

where k denotes the individual capital stock, and χ is an indicator function taking value 1 when working, and 0 when retired (Bloom *et al*. 2007; Prettner and Canning 2014). The first term in the flow budget constraint relates to income earned on the labour market and from receiving the lump-sum redistributions of profits (Kuhn and Prettner 2016). This term becomes zero once an individual retires. The second term refers to the interest earnings on capital holdings (rk), which are augmented by the redistribution of capital from people who die to those who survive, via the annuity market (μk). If individuals have income higher than their consumption expenditures at a given instant, then their capital stock accumulates ($\dot{k} > 0$).

Solving the intertemporal maximization problem as represented by equations (1) and (2) leads to the standard Euler equation (see Optimal consumption in Appendix)

$$\dot{c} = (r - \rho)c, \tag{3}$$

stating that consumption expenditure growth depends positively on the difference between the interest rate and the rate of time preference.

The lifetime budget constraint is

$$\int_{t_0}^{\infty} \mathrm{e}^{-(\mu+r)(t-t_0)}\ c(t_0, t)\ \mathrm{d}t = \int_{t_0}^{t_0+R} \mathrm{e}^{-(\mu+r)(t-t_0)}\ w(t_0, t)\ \mathrm{d}t, \tag{4}$$

where lifetime consumption expenditures (the left-hand side) have to equal lifetime income (the right-hand side). In this expression, the working lifespan is denoted by R, such that the age at retirement is given by $t_0 + R$ in the upper bound of the integral on the right-hand-side that represents lifetime income. For a constant age at labour market entry t_0, an increase in R is tantamount to an increase in the retirement age.

Denoting the aggregate capital stock by K and aggregate consumption expenditures by C, we have the following definitions to derive the corresponding variables (see, for example, Blanchard 1985; Prettner 2013; Heijdra 2017, ch. 15):

$$K(t) \equiv \int_{-\infty}^{t} k(t_0, t) N(t_0, t)\ \mathrm{d}t_0,$$

$$C(t) \equiv \int_{-\infty}^{t} c(t_0, t) N(t_0, t)\ \mathrm{d}t_0,$$

where $N(t_0, t)$ denotes the size of the cohort entering the labour market at time t_0 as of date t, while $k(t_0, t)$ and $c(t_0, t)$ are the capital holdings and consumption levels of the members of this cohort at time t, respectively. To rule out the counterfactual prediction of hyper-exponential growth, we follow the crucial assumption in the underlying framework of Romer (1990) that the population is stationary. In our setting, this implies that the birth rate equals the death rate. In this case, the flow of labour market entrants is $N(t, t) = \mu\, N(t)$, where $N(t) = \int_{-\infty}^{t} N(t_0, t)\ \mathrm{d}t_0 \equiv N$ represents the adult population size, and $L(t) = \int_{t-R}^{t} N(t_0, t)\ \mathrm{d}t_0$ is the labour force. Note that in our setting: (i) each adult cohort is of size $\mu N\ \mathrm{e}^{\mu(t_0 - t)}$ at a certain date $t > t_0$; (ii) the cohort fertility rate stays constant for a changing mortality rate μ, such that the fertility decisions of individuals do not change for changing parameters; and (iii) a change in the mortality rate does not change the population size, such that the scale effect in the Romer (1990) framework is neutralized with respect to the overall population size. Assuming a growing population, as in Buiter (1988), together with a semi-endogenous growth framework, as in Jones (1995), would imply effects similar to the ones that we find during the transition to the long-run balanced growth path (BGP; see Remark 3 and Derivation of the BGP in the Jones (1995) model in Appendix).

Taking into account our demographic structure and using the stated aggregation rules leads to the following dynamic equations for the aggregate capital stock and aggregate consumption (see Aggregating over cohorts in Appendix):

(5) $$\dot{K} = rK + W - C,$$

(6) $$\frac{\dot{C}}{C} = r - \rho - \mu(\rho + \mu)\ \frac{K}{C},$$

where W refers to aggregate non-capital income (see Remark 4 and the Accounting for realistic demography in Appendix for a more general demographic structure).

The resource constraint states that aggregate production Y is either consumed or invested in physical capital such that the goods market clearing condition

(7) $$\dot{K} = Y - C$$

is fulfilled. Next, we turn to the description of the production side of the economy. Since the production side closely follows the R&D-based endogenous growth literature, we state only the key equations required for the further analysis of changing the retirement age and changing life expectancy.

Production side Labour is either employed in the final goods sector (L_Y) to assemble the consumption aggregate, or in the R&D sector (L_A) to develop the new technologies ('blueprints' for machines or simply 'ideas') that drive productivity growth in knowledge-based economies (see, for example, Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992). The representative firm in the final goods sector combines workers and machines according to the production technology

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha \, \mathrm{d}i, \tag{8}$$

where A is the stock of technology in the country, x_i is the quantity of a specific machine i used in final goods production, and $\alpha \in (0,1)$ is the elasticity of final output with respect to machines. Taking the final good as the numeraire, profit-maximization of final goods producing firms together with the assumption of perfect competition in the final goods market implies that the wage rate for workers w_Y, and the price of machines p_i, are given by

$$w_Y = (1-\alpha)\,\frac{Y}{L_Y}, \quad p_i = \alpha L_Y^{1-\alpha} x_i^{\alpha-1}.$$

Each intermediate firm produces one of the differentiated machines such that there is monopolistic competition in the vein of Dixit and Stiglitz (1977). After a firm has purchased a blueprint from the R&D sector, it has access to a production technology that allows it to convert one unit of capital k into one machine x such that $k_i = x_i$ for all firms i. Thus operating profits can be written as

$$\pi_i = p_i k_i - r k_i = \alpha L_Y^{1-\alpha} k_i^\alpha - r k_i. \tag{9}$$

Profit-maximization of firm i yields the optimal price $p_i = r/\alpha$, where $1/\alpha$ is the markup over marginal cost (Dixit and Stiglitz 1977). As a consequence, the aggregate capital stock is equal to the total quantity of intermediates, that is, $K = Ax$. Using this, the aggregate production function (equation (8)) becomes $Y = (AL_Y)^{1-\alpha} K^\alpha$, and production per capital unit can be written as a function of the interest rate and the elasticity of final output with respect to machines $r = \alpha p = \alpha^2 Y/K$, which gives $Y/K = r/\alpha^2$.

The R&D sector employs scientists to discover new technologies (Romer 1990). Depending on the productivity of scientists λ, and their employment level L_A, the stock of blueprints evolves according to

$$\dot{A} = \lambda A L_A. \tag{10}$$

R&D firms maximize their profits $\pi_A = p_A \lambda A L_A - w_A L_A$, with p_A representing the price of a blueprint, by choosing the employment level L_A. The first-order condition of this profit-maximization problem pins down wages in the research sector as $w_A = p_A \lambda A$. Due to perfect labour mobility, wages of workers in the final goods sector and wages of scientists equalize at the labour market equilibrium such that

$$w_A = p_A \lambda A = (1-\alpha)\,\frac{Y}{L_Y} = w_Y. \tag{11}$$

Firms in the R&D sector charge a price for the blueprint that is equal to the present value of operating profits in the intermediate goods sector. This is because there is always a potential entrant willing to bid a lower price. Consequently,

$$p_A = \int_t^\infty \mathrm{e}^{-\Omega(\tau)}\, \pi \, \mathrm{d}\tau$$

holds in equilibrium, where $\Omega(\tau)=\int_t^\tau r(s)\,\mathrm{d}s$ denotes the compound interest between t and τ. Via the Leibniz rule and the fact that prices of blueprints do not change along a BGP, we obtain the long-run equilibrium price of a blueprint as

$$p_A=\frac{\pi}{r}.$$

Next, by using equation (9), we get operating profits as $\pi=(1-\alpha)\alpha Y/A$ such that the price of blueprints becomes $p_A=(1-\alpha)\alpha Y/(rA)$. Using the labour market clearing condition $L=L_A+L_Y$, we can then determine the quantity of labour employed in the final goods sector and in the R&D sector by using equation (11), as

$$L_Y=\frac{r}{\alpha\lambda},\quad L_A=L-\frac{r}{\alpha\lambda}. \tag{12}$$

This endogenous division of labour determines the flow of new technologies in the R&D sector. Inserting equation (12) into equation (10) leads to the evolution of technology as

$$\dot{A}=\max\left\{\lambda AL-\frac{rA}{\alpha},0\right\}. \tag{13}$$

Now we have all the necessary ingredients to solve for the long-run BGP and to assess the effects of changing life expectancy and a changing retirement age on economic growth.

II. The Impact of Rising Longevity and Changing Retirement Policies on Long-run Growth

Balanced growth impact of changes in the retirement age and in mortality Along a BGP, we know that the growth rates of technology, capital and consumption coincide such that $\dot{A}/A=\dot{C}/C=\dot{K}/K=g$. Collecting equations (6), (7) and (13), recalling that labour supply is given by $L(t)=\int_{t-R}^t N(t_0,t)\,\mathrm{d}t_0$, and utilizing the definition $C/K\equiv\xi$, we derive the following three-dimensional system describing our model economy along the BGP:

$$g=\frac{r}{\alpha^2}-\xi, \tag{14}$$

$$g=r-\rho-\mu(\rho+\mu)\,\frac{1}{\xi}, \tag{15}$$

$$g=\lambda\int_{t-R}^{t}N(t_0,t)\,\mathrm{d}t_0-\frac{r}{\alpha}. \tag{16}$$

In this system, the endogenous variables are the interest rate r, the consumption-to-capital ratio ξ, and the long-run economic growth rate g. We turn to implicit comparative statics to prove the analytical results in Propositions 1 and 2, and generate two sets of insights. In a first step, we consider the balanced-growth impact of an isolated change in the retirement age. In a second step, we consider the balanced-growth impact of a change in longevity, and study how this depends on potential adjustments in the retirement age.

Proposition 1. In the endogenous growth framework of Romer (1990) with overlapping generations and retirement, an increase in the retirement age (a rise in R) leads to:

(i) an increase in the interest rate (r);
(ii) an increase in the long-run economic growth rate (g).

Proof. Noting that $\int_{t-R}^{t} N(t_0, t)\,\mathrm{d}t_0 = N\left(1 - \mathrm{e}^{-\mu R}\right)$, we rewrite the system (14)–(16) as

$$W(\xi, g, r) := \frac{r}{\alpha^2} - \xi - g = 0, \tag{17}$$

$$X(\xi, g, r) := r - \rho - \mu(\rho + \mu)\,\frac{1}{\xi} - g = 0, \tag{18}$$

$$Y(\xi, g, r) := \lambda N\left(1 - \mathrm{e}^{-\mu R}\right) - \frac{r}{\alpha} - g = 0. \tag{19}$$

Applying the implicit function theorem and Cramer's rule, we obtain the following comparative statics:

$$\frac{\mathrm{d}g}{\mathrm{d}R} = \frac{\lambda\mu N\ \mathrm{e}^{-\mu R}[\alpha^2\xi^2 + \mu(\mu + \rho)]}{(1 + \alpha)[\alpha\xi^2 + \mu(\mu + \rho)]} > 0, \tag{20}$$

$$\frac{\mathrm{d}r}{\mathrm{d}R} = \frac{\alpha^2\lambda\mu N\ \mathrm{e}^{-\mu R}[\mu(\mu + \rho) + \xi^2]}{(1 + \alpha)[\alpha\xi^2 + \mu(\mu + \rho)]} > 0. \tag{21}$$

□

Hence, in contrast to Futagami and Nakajima (2001) and Heijdra and Mierau (2011), who base their analysis on a Romer (1986) framework in which growth is driven by physical capital accumulation via learning-by-doing spillovers, an increase in the retirement age unambiguously raises economic growth in a Romer (1990) setting. The intuition is that a rise in the retirement age implies an increase in the labour force and therefore raises the number of scientists that are available for the production of blueprints in the R&D sector. While there is also a reduction in individual savings due to the longer working life (as in Futagami and Nakajima 2001; Heijdra and Mierau 2011), the associated negative growth effect is overcompensated by the positive effect of the larger labour force.

The difference in the results suggests that in economies in which growth is driven mainly by purposeful R&D investments (such as in Germany, Japan and the USA), an increase in the retirement age will indeed lead to a rise in the long-run economic growth rate. However, in economies in which growth is driven mainly by physical capital accumulation coupled with learning-by-doing spillovers (predominantly small economies that are not advancing the worldwide research frontier), a rise in the retirement age could lead to a reduction in the growth rate. Consequently, any adjustment of the retirement age should be considered in light of the underlying structure of the economy.

Proposition 1 shows that a rise in the retirement age may be an appropriate policy response to the phenomenon of secular stagnation as described in detail by Eggertsson *et al*. (2019, 2020). According to these contributions, we were facing a prolonged period of stagnation in many countries with sluggish economic growth and an equilibrium interest rate stuck below zero after the global financial crisis. Our results show that raising the retirement age increases the workforce and—at the same time—reduces savings. Both of these effects put upward pressure on the interest rate and, overall, lead to faster economic growth.

The effects of increasing life expectancy are more subtle because there are opposing channels: (i) a reduction of the generational turnover leading to higher aggregate savings and thus a lower interest rate, which in turn encourages investment in R&D and thereby fosters economic growth; (ii) a reduction of the labour supply for a given retirement age because a lower mortality rate then implies a lower support ratio for a stationary population.

Proposition 2. In the endogenous growth framework of Romer (1990) with overlapping generations and a constant retirement age, an increase in life expectancy (a decrease in μ) leads to a decrease in the interest rate and an ambiguous response of the long-run economic growth rate.

Proof. Applying the implicit function theorem and Cramer's rule to the system of equations (17)–(19), we obtain the following comparative statics:

$$\frac{\mathrm{d}g}{\mathrm{d}\mu} = \frac{\mathrm{e}^{-\mu R}\left\{\lambda NR[\alpha^2\xi^2 + \mu(\mu+\rho)] - \alpha\xi(2\mu+\rho)\,\mathrm{e}^{\mu R}\right\}}{(\alpha+1)[\alpha\xi^2 + \mu(\mu+\rho)]} \gtreqless 0, \tag{22}$$

$$\frac{\mathrm{d}r}{\mathrm{d}\mu} = \frac{\alpha^2\left\{\xi(2\mu+\rho) + \lambda NR\,\mathrm{e}^{-\mu R}\,[\mu(\mu+\rho)+\xi^2]\right\}}{(\alpha+1)[\alpha\xi^2 + \mu(\mu+\rho)]} > 0. \tag{23}$$

This establishes the claim in the proposition. □

These results constitute an analytical underpinning for Bloom *et al*. (2007) and Aksoy *et al*. (2019), who provide evidence that, indeed, an increase in longevity triggers an increase in aggregate saving and a decline in the interest rate. Our result is also consistent with the finding of Aksoy *et al*. (2019) that ageing has led to a reduction in R&D activity and in economic growth over the time frame 1970–2014 in a set of OECD countries.

Growth-preserving retirement response to longevity change Against this backdrop, the next proposition spells out a set of conditions on the response of the retirement age (however brought about) to mortality change, under which a mortality decline translates into a lower interest rate and an increase in economic growth.

Proposition 3. In the endogenous growth framework of Romer (1990) with overlapping generations and an exogenous policy response of the retirement age, a decrease in mortality μ (and thus an increase in life expectancy) leads to:

(i) a decrease in the interest rate (r) if and only if the retirement response satisfies

$$\frac{\mathrm{d}R}{\mathrm{d}\mu}\,\frac{\mu}{R} > -1 - \frac{\xi(\rho+2\mu)}{R\lambda N\ e^{-\mu R}\,[\mu(\rho+\mu)+\xi^2]} := \Psi_r; \tag{24}$$

(ii) an increase in the long-run economic growth rate (g) if and only if the retirement response satisfies

$$\frac{\mathrm{d}R}{\mathrm{d}\mu}\,\frac{\mu}{R} < -1 + \frac{\alpha\xi(\rho+2\mu)}{R\lambda N\ e^{-\mu R}\,[\mu(\rho+\mu)+\alpha^2\xi^2]} := \Psi_g. \tag{25}$$

Proof. Combining (22) and (23) with (20) and (21), we obtain the forms

$$\frac{\mathrm{d}g}{\mathrm{d}\mu} + \frac{\mathrm{d}g}{\mathrm{d}R}\,\frac{\mathrm{d}R}{\mathrm{d}\mu}, \quad \frac{\mathrm{d}r}{\mathrm{d}\mu} + \frac{\mathrm{d}r}{\mathrm{d}R}\,\frac{\mathrm{d}R}{\mathrm{d}\mu}.$$

Rearranging these expressions provides the conditions reported in the proposition. □

Proposition 3 provides a set of conditions on the elasticity of the retirement response with respect to mortality, for a decline in mortality to have a negative impact on the interest rate and a positive impact on economic growth. The following remark provides an equivalent formulation in terms of a change in longevity.

Remark 1. We have cast our analysis directly in terms of the mortality rate μ. In the Blanchard (1985) setting, life expectancy is given by the identity $LE \equiv \mu^{-1}$. Thus we obtain the relationship

$$\frac{\mathrm{d}\mu}{\mathrm{d}LE} = -\frac{\mu}{LE}$$

by which to multiply all relevant derivatives to obtain the corresponding expressions in terms of life expectancy. We can then express the elasticity of the retirement age with respect to longevity as

$$\varepsilon_{R,LE} = \frac{\mathrm{d}R}{\mathrm{d}LE}\,\frac{LE}{R} = \frac{\mathrm{d}R}{\mathrm{d}\mu}\,\frac{\mathrm{d}\mu}{\mathrm{d}LE}\,\frac{R}{LE} = -\frac{\mathrm{d}R}{\mathrm{d}\mu}\,\frac{\mu}{R}, \tag{26}$$

such that the two conditions in (24) and (25) can now be written in terms of life expectancy:

$$\varepsilon_{R,LE} < -\Psi_r \tag{27}$$

and

$$\varepsilon_{R,LE} > -\Psi_g. \tag{28}$$

To establish a benchmark, consider that the retirement age is not adjusted to a change in longevity, that is, $\varepsilon_{R,LE} \equiv 0$. It is immediately verified for this case that $\mathrm{d}r/\mathrm{d}LE < 0$, implying that the interest rate decreases in response to an increase in life expectancy, reflecting a boost in savings. By contrast, we have $\mathrm{d}g/\mathrm{d}LE \gtreqless 0$, implying an ambiguous effect of longevity on economic growth. This is because it is not clear whether the shift of resources into the R&D sector that is triggered by the decline in the interest rate (Prettner 2013; Kuhn and Prettner 2016) compensates for the reduction in labour force participation, as measured by $R\mu$, or equivalently, by R/LE.

Equation (28) provides a condition on the elasticity of the retirement response to an increase in longevity for the overall impact on economic growth to be positive. Specifically, an increase in the retirement age in response to an increase in longevity is required for a positive impact on economic growth whenever $-\Psi_g > 0$, which is true if the savings response is relatively weak. Notably, however, the retirement age may be lowered, that is, $\varepsilon_{R,LE} < 0$, if the increase in longevity translates into a strong savings response (as may be true for low levels of the retirement age), such that $-\Psi_g < 0$. Furthermore, the condition shows that by guaranteeing a constant labour force, a proportional adaptation of the retirement age to longevity, as implied by the unit elasticity $\varepsilon_{R,LE} = 1$, ensures a positive impact of a longevity increase on balanced growth. Finally, we see from (27) that longevity improvements lead to a decline in the interest rate as long as retirement is not boosted by too much. It follows immediately from $-\Psi_g \leq -\Psi_r$ that as long as $\varepsilon_{R,LE} \in [-\Psi_g, -\Psi_r]$, there is a joint decline in the interest rate and a boost to economic growth. Indeed, this is always true for a proportional adaptation.

Numerical illustration of the longevity–retirement–growth nexus The ambiguous effect of an increase in life expectancy on economic growth in case of a fixed retirement age begs the questions as to (i) by how much different economies need to adjust their retirement age to secure a positive impact of rising longevity on economic growth, and (ii) whether actual changes to the retirement age are adequate in this regard. In the following, we provide a numerical assessment of how four economies—the USA, the EU, Japan and the OECD (average) economy—have fared with respect to these questions. To obtain a long-term assessment and to balance out short-term fluctuations, we first calibrate the economies to reflect the average growth rate over the time frame 2000–2017, dropping the years 2008 and 2009 during which the financial crisis was creating strong distortions. On this basis, we calculate the threshold Ψ_g, as defined in equation (25), and the elasticity of the change in the retirement age in response to a 1% increase in life expectancy, $\varepsilon_{R,LE}$. This allows us to assess whether the elasticity exceeds the threshold (recall equation (28)) and thus whether an increase in longevity should have translated into a growth stimulus for the economies under consideration.

Substituting successively for r and ξ in the system (14)–(16), and subsequently reinserting, we can solve for the closed-form solution:

$$g^* = \frac{1}{2(1+\alpha)}\left((1+\alpha)\lambda N\left(1-e^{-\mu R}\right)-\rho - \sqrt{\left[(1-\alpha)\lambda N(1-e^{-\mu R})+\rho\right]^2+4\alpha\mu(\mu+\rho)}\right), \tag{29}$$

$$r^* = \alpha\lambda N\left(1-e^{-\mu R}\right)-\alpha g^*, \tag{30}$$

$$\xi^* = \frac{r^*}{\alpha^2}-g^* = \frac{\lambda N\left(1-e^{-\mu R}\right)}{\alpha}-\frac{1+\alpha}{\alpha}\,g^*. \tag{31}$$

For all countries, we set $\alpha = 0.33$ in line with Jones (1995) and Acemoglu (2009), and a time preference rate $\rho = 0.05$ that is within a range of plausible values (cf. Warner and Pleeter 2001; Grossmann *et al*. 2013a,b). We take the economy-specific growth rate g and life expectancy $LE = \mu^{-1}$ from the World Development Indicators (World Bank 2019), and calculate the average effective retirement age R from OECD (2019) data.[1] Based on this, we employ equation (29) to calibrate the value of λN. Using this value in equation (31) to determine ξ^*, we are able to derive the threshold Ψ_g. Finally, we compute the elasticity that corresponds to the percentage change in the retirement age from 2010–2017 to 2000–2007 for a 1% increase in life expectancy across these two time frames:

$$\varepsilon_{R,LE} = \frac{(R_{2010-2017}-R_{2000-2007})/R_{2000-2007}}{(LE_{2010-2017}-LE_{2000-2007})/LE_{2000-2007}}.$$

When calculating the elasticity from the data, we average the values of retirement and life expectancy over the time spans 2000–2007 and 2010–2017. Table 1 summarizes our findings.

The four economies exhibit relatively little variation across growth rates, with the EU experiencing an average growth rate of around 1.75% over the time frame 2000–2017 as opposed to some 1.5% only in Japan. By contrast, Japan leads in terms of life expectancy with around 83.5 years, while the USA is lagging with some 78 years. With 68.5 years, the Japanese retire late and experience only around 15 years in retirement, whereas EU citizens

TABLE 1
COMPARISON BETWEEN THE THRESHOLD (Ψ_g) AND THE ELASTICITY ($\varepsilon_{R,LE}$) ACROSS COUNTRIES

Variable	USA	EU	Japan	OECD
Average growth rate 2000–2017 (in %)	1.56	1.75	1.48	1.60
Average life expectancy 2000–2017	77.96	79.28	82.62	78.90
Average retirement age 2000–2017	64.84	61.55	68.53	63.10
Threshold (Ψ_g)	0.966	0.969	0.967	0.967
Elasticity ($\varepsilon_{R,LE}$)	1.260	0.582	0.484	0.900

Source: Authors' own calculations based on World Bank (2019) (growth rates, life expectancy, gender shares in total labour force) and OECD (2019) (average effective retirement age).

live more than 17.5 years in retirement. The threshold value for the growth-preserving increase in the retirement age is close to 1 for all economies, which reflects a modest marginal savings response to an increase in longevity. Hence there is not much leeway for any of the economies to remain below an increase in the retirement age that is in lockstep with longevity growth unless they are willing to forgo economic growth.

As it turns out, there is considerable variation in the elasticities of the actual retirement age with respect to longevity change, ranging from 1.26 in the USA, implying an overcompensation of the increase in longevity, to 0.48 in Japan. According to our results, only the USA would have experienced a positive growth stimulus from the increase in longevity, whereas Japan and the EU economy would have suffered a loss. Notably, the OECD average economy lies close to the threshold, implying that the longevity increase was almost neutral.

We consider the quantitative implications as part of a second numerical exercise. Here, we distinguish the time range 2000–2007 as opposed to 2010–2017 not only according to longevity and retirement, but also with respect to the value of λN as a shifter of the growth rate of R&D. Evidently, such shifts embrace size effects (via N) and genuine productivity effects (via λ), where the latter may be related to changes in education but also to changes in the structure and relevance of the innovation process, including policy interventions. We thus recalibrate our model to obtain time-specific values of λN, reflecting the 2000–2007 and 2010–2017 growth rates in the data, which complement the time-specific values of longevity and retirement. We can then decompose the change in growth rates across the time frames 2000–2007 and 2010–2017, $\Delta_g := g_{2010-2017} - g_{2000-2007}$, according to its drivers: productivity change ($\Delta_{\lambda N} := \lambda N_{2010-2017} - \lambda N_{2000-2007}$), changing longevity ($\Delta_{LE} := LE_{2010-2017} - LE_{2000-2007}$) and changing retirement age ($\Delta_R := R_{2010-2017} - R_{2000-2007}$). The findings are summarized in Table 2.

While the USA and the OECD have experienced only a moderate decline in economic growth, the EU has experienced close to a percentage point loss in growth, albeit starting from a high growth rate in 2000–2007. Japan, in contrast, has experienced a modest gain. Our analysis shows that for all economies, changes in R&D productivity are the key driver: negative for the USA, the EU and the OECD, and positive for Japan. Longevity changes and changes in the retirement age generate small effects in comparison, the largest being the −0.14 and −0.15 percentage point growth drags from longevity increases in Europe and across the OECD, respectively. At a positive impact of 0.09 percentage points, the countervailing effect of an increase in the retirement age is largest in the OECD. In line with our predictions from the elasticity rule, we find that the increase in the retirement age is sufficient to overturn the longevity increase in the USA, and insufficient in the EU, Japan and the OECD. For the OECD, the decomposition also shows that the increase in the retirement age falls short of what is required for a compensation of longevity growth by a larger margin than might be

TABLE 2
DECOMPOSITION OF CHANGE IN GROWTH RATE BETWEEN 2000–2007 AND 2010–2017

Variable	USA	EU	Japan	OECD
Average growth rate 2010–2017 (in %)	1.44	1.33	1.62	1.40
Average growth rate 2000–2007 (in %)	1.68	2.16	1.34	1.81
Percentage point change in growth rate[a]	−0.23	−0.83	0.29	−0.41
Driven by (in percentage points):				
Productivity change	−0.25	−0.77	0.32	−0.46
Longevity change	−0.06	−0.14	−0.06	−0.15
Change in retirement age	0.08	0.07	0.03	0.09
Complementarity across channels	0.0	0.01	0.0	0.12

Notes [a] Entries for the USA, Japan and the OECD exhibit minor rounding errors.
Source: See Table 1.

TABLE 3
INCREASE IN THE RETIREMENT AGE THAT WOULD HAVE NEUTRALIZED THE 2000–2007 TO 2010–2017 INCREASE IN LONGEVITY (GIVEN 2010–2017 PRODUCTIVITY)

Variable	USA	EU	Japan	OECD
Average growth rate 2010–2017 (in %)	1.44	1.33	1.62	1.40
Counterfactual growth rate 2010–2017 (in %)[a]	1.43	1.39	1.65	1.40
Compensating change in retirement age (years)	1.12	2.22	1.28	1.55
Actual change in retirement age (years)	1.46	1.14	0.64	1.45

Notes
[a] With productivity at 2010–2017; retirement and longevity at 2000–2007 levels.
Source: See Table 1.

TABLE 4
COMPENSATING (GROWTH NEUTRAL) INCREASE IN PRODUCTIVITY

Variable	USA	EU	Japan	OECD
Average growth rate 2000–2007 (in %)	1.68	2.16	1.34	1.81
Average growth rate 2010–2017 (in %)	1.44	1.33	1.62	1.40
Compensating change in productivity (in %)	−0.33	1.17	0.59	0.11
Actual change in productivity (in %)	−4.46	−12.67	6.07	−7.05

Source: See Table 1.

expected from the elasticity rule in equation (28). The discrepancy can be understood with respect to the *ceteris paribus* nature of this rule. Notably, for the OECD, complementarity across the three drivers of the growth rate seems to matter, but this may also reflect variations across the different OECD economies.

We conclude the numerical analysis with two counterfactual policy experiments, as portrayed in Tables 3 and 4. Specifically, we ask: (i) which increase in the retirement age would have neutralized the 2000–2007 to 2010–2017 increase in longevity when holding the productivity measure at its 2010–2017 level (see Table 3); and (ii) which increase in productivity would have neutralized the joint 2000–2007 to 2010–2017 increase in longevity and the retirement age (see Table 4).

A comparison of the actual 2010–2017 growth rates to the counterfactual rates, in which longevity and retirement remain fixed at their 2000–2007 levels, shows the failure of the EU and Japan to compensate for the increase in longevity, which has led to a loss in economic growth (see Table 3). In the EU, in particular, the sizeable increase in longevity would have called for a significant increase in the retirement age in excess of 2 years, with the actual increase falling short by a little more than 1 year. In Japan, the compensating retirement adjustment would have been more minor, but the actual increase was lowest, at a little more than 6 months. In contrast, the USA has been overadjusting, leading to an increase in the economic growth rate, whereas the OECD's compensation turned out to be appropriate to maintain the level of growth. Notably, however, the drag on growth of a failure to adjust retirement in line with the longevity change is rather minor, and the same applies to the gain in case of overcompensation.

Table 4 presents the increase in productivity that would be necessary to compensate for the increase in longevity and retirement in scenario (ii) above, and thereby maintain the 2000–2007 economic growth rate. As it turns out, the compensatory productivity adjustments, which could come through greater productivity in the R&D process or through improvements in education, are modest. At a required increase in productivity of 1.17% over a 10-year period, they are largest for the EU, much smaller for Japan and the OECD, and even negative for the USA, the latter reflecting slow longevity growth and the overcompensating increase in the retirement age. For all economies, the compensatory productivity change is swamped by the actual change (as determined from the model), which constitutes a drag on growth in all countries but Japan.

III. Alternative Underlying Economic Structures

So far, we have discussed the mechanisms in a prototype model economy of the Romer (1990) type in which (i) R&D is based on the employment of scientists, (ii) the strong scale effect is present, (iii) the population is stationary and mortality is age-independent, (iv) the retirement age is varied exogenously (e.g. by a policymaker), and (v) human capital accumulation does not play a role.

In this section, we discuss the robustness of our results to different modelling assumptions and describe the qualitative implications of considering changes in these assumptions on our results.

Robustness 1: Lab-equipment approach We first discuss the implications of considering a lab-equipment specification of R&D (see also Our results in the context of the lab-equipment approach in Appendix).

Remark 2. If we replace equation (10) by a lab-equipment specification as in Rivera-Batiz and Romer (1991) such that

$$\dot{A} = \hat{\lambda} Z,$$

where Z is a fraction of final output interpreted as lab equipment, and λ is a scaling factor reflecting the productivity of lab equipment in generating new ideas, then the scale effect is still present but the interest rate channel drops out of the model (see Our results in the context of the lab-equipment approach in Appendix and Rivera-Batiz and Romer 1991, eq. (10)). Since the financing of R&D seems, however, to matter in light of the empirical evidence (cf. Brown *et al*. 2009; Ang 2010; Howell 2016), and because R&D is a very labour-intensive activity to date, we deem the Romer (1990) framework to be better suited to capture the relevant aspects of reality than the lab-equipment approach.

Robustness 2: Role of scale effect Readers who are familiar with R&D-driven growth theory will correctly anticipate that the underlying mechanism of the effects of a changing retirement age rests on the scale effect. This effect has been criticized in the literature (cf. Jones 1995; Peretto 1998; Segerström 1998) because it does not conform with data on the relation between productivity growth and employment in R&D. We therefore analyse the implications of using a semi-endogenous growth model in the vein of Jones (1995) instead of the endogenous growth model of Romer (1990) as the underlying framework, while allowing for a non-stationary population (see Derivation of the BGP in the Jones (1995) model in Appendix). In this case, population growth is given by the difference between the fertility rate β and the mortality rate μ as $n = \beta - \mu$. Workforce growth differs from population growth because of the exogenous retirement age at which all surviving workers leave the workforce instantaneously. Following Prettner (2013) and Prettner and Trimborn (2017) in modifying the R&D productivity equation to

$$\dot{A} = \lambda A^{\phi} L_A, \tag{32}$$

where $\phi < 1$ measures the strength of intertemporal knowledge spillovers, leads to a long-run balanced growth rate

$$g^{Jones} = \frac{\beta - \mu^*}{1 - \phi}, \tag{33}$$

where μ^* is the cumulative exit rate from the workforce determined by mortality and retirement. Thus the long-run growth rate of GDP is pinned down by the long-run growth rate of the workforce, $\beta - \mu^*$. We summarize the consequences in the following remark.

Remark 3. Changing the retirement age or changing life expectancy in the semi-endogenous growth models of Jones (1995), Kortum (1997) or Segerström (1998), or in Schumpeterian scale-free growth models such as those of Peretto (1998) and Young (1998), affect long-run economic growth only if they bear on the growth rate of the workforce. All other effects described above for the Romer (1990) setting apply only during the transition towards the balanced growth rate. However, the transition in these growth models is usually rather slow (see, for example, Prettner and Trimborn 2017), such that even transient growth effects span a substantial time period and thus imply substantial level effects on per capita GDP.

Robustness 3: Realistic demography Next, we turn to the case of a fully realistic demography, involving both a possibly non-stationary population and an age-dependent mortality schedule. In this case, our findings on the impact of variation in the retirement age continue to apply. The findings on the impact of changing life expectancy continue to hold whenever the decline in the aggregate death rate leads to a decline in the 'generational turnover' that acts as a drag on consumption growth. Intuitively, mortality reductions tend to boost demand in settings where consumption increases with age such that more individuals tend to live through ages with high consumption spending (cf. Kuhn and Prettner 2018). This effect is present in the baseline model (see the last term in equation (15) of the growth system (14)–(16)) and can be shown to hold under plausible circumstances in a more general model with age-specific mortality (see the Accounting for realistic demography in Appendix). This channel is complemented by a decline in the interest rate, as triggered by the increase in aggregate saving that is necessary to accommodate the higher levels of consumption spending in old age. We can summarize as follows.

Remark 4. If mortality is age-dependent, then any change in the mortality pattern that leads to a reduction in the aggregate death rate, or equivalently, to an increase in life expectancy, will typically stimulate economic growth in economies in which consumption is increasing with age. Evidence in Kuhn and Prettner (2018) suggests that this channel is indeed relevant for a number of important industrialized countries, including the USA, Japan and a number of EU economies.

The final two aspects for consideration relate to the exogeneity of retirement and education within our model, both of which have been shown to be important, and indeed related sets of choices from an individual lifecycle perspective (see, for example, Hazan 2009; Hansen and Lønstrup 2012; Cervellati and Sunde 2013; Sánchez-Romero *et al*. 2016).

Robustness 4: Endogenous retirement Turning to the possible endogeneity of retirement first, we follow Prettner and Canning (2014) in embedding an optimal choice of the retirement age within general equilibrium. We can thus show the following (see Accounting for endogenous retirement in Appendix).

Remark 5. Consider an extension of our model where the representative individual decides on an optimal duration of the working life. The following then hold along a BGP. (i) The impact of longevity on the optimal retirement age is positive if the retirement age is sufficiently high to begin with, and the impact of a longevity increase on the economic growth rate is bounded from above. (ii) The impact of longevity on the economic growth rate is positive if the response of the (optimal) retirement age is sufficiently strong. It is always positive if the elasticity of longevity with respect to retirement, as defined in equation (26), satisfies $\varepsilon_{R,LE} \geq 1$.

Finding (i) with respect to the impact of longevity on the optimal retirement age generalizes Prettner and Canning (2014), who consider the general equilibrium of a stationary economy and find that the retirement age increases with longevity if it is sufficiently high to begin with. Prettner and Canning (2014) show this to be true numerically for a number of OECD economies. Our finding confirms this for a Romer (1990) balanced growth economy with the additional condition that a possible positive impact of longevity on economic growth is not excessive. Finding (ii) generalizes our Proposition 3 to the case of optimal retirement. Indeed, the condition for a longevity increase to bear positive on the economic growth rate can again be expressed in terms of the elasticity of the retirement age to longevity around the optimum retirement age. As before, the growth impact of a longevity extension is positive if the retirement age increases by a sufficient amount, depending on the impact of longevity on the generational turnover. A positive impact is guaranteed if the elasticity is equal to or exceeds 1.

While this shows that our findings are robust to endogenizing the retirement age, we should also point out that—from a practical point of view—it is often sufficient to focus on the result in Proposition 3 based on an exogenous variation in the retirement age. This is true in particular if we are interested in the change in the retirement age that is required to ensure that longevity growth translates into economic growth. Whether the variation in retirement then arises through an exogenous variation in a statutory retirement age or through a variation in an optimally chosen retirement age is of secondary importance. This is not least when taking into account the evidence that individual retirement decisions tend to cluster around the statutory retirement age and other focal points, for example, relating to age thresholds determining the eligibility to certain pension schemes or retirement incentives (see, for example, Gruber and Wise 1998; Gustman and Steinmeier 2005; Seibold 2021).

Robustness 5: Endogenous human capital Finally, the accumulation of human capital acts as an important complementary force to R&D in a Romer (1990) economy (cf. Strulik *et al*. 2013). While, for reasons of tractability, we do not capture this process explicitly, it is relatively straightforward to infer its role. At the macroeconomic level, an increase in human capital would amount to an increase in the effective labour force, $\widehat{L}(t) = h(t)\ L(t)$, with $h(t)$ being a measure of human capital. It is easily seen from equation (16) that this would amount to an increase in economic growth, while the growth effects of increasing the retirement age and of increasing life expectancy would remain qualitatively unaffected. In our numerical exercise, variations in human capital across economies and over time are reflected in the respective variations of the productivity measure λN (see Table 2). Naturally, it is impossible at this level to disentangle the macro-level impacts of human capital from any other drivers of productivity.

When considering endogenous human capital accumulation, the relationship between the retirement age and educational investments becomes more intricate. While an increase in the retirement age, and thus the duration of the working life, will increase educational investments, education may still be boosted by increasing longevity even if the retirement age declines. This is true if decreases in mortality benefit in particular the working-age population (Strulik and Werner 2012; Cervellati and Sunde 2013; Sánchez-Romero *et al*. 2016). We summarize this in the final remark.

Remark 6. Considering exogenous human capital accumulation would leave our qualitative results unaffected, while endogenous human capital accumulation would typically strengthen our results with respect to the growth effects of increasing the retirement age and increasing longevity.

IV. Conclusions

We showed that a rise in the retirement age implies faster long-run economic growth in modern knowledge-based economies, and that the growth effects of increasing life expectancy depend on the underlying retirement policies. If the retirement age is left constant, then an increase in life expectancy is likely to reduce economic growth. By contrast, if the retirement age rises in lockstep with life expectancy, then this is sufficient for economic growth to be boosted by an increase in life expectancy. We provide a more specific threshold requirement for the increase in the retirement age that is necessary to preserve a positive growth stimulus, and show numerically that this criterion is overachieved by the USA, (nearly) met by the OECD average economy, and not attained by the EU and Japan. The growth impact is of relatively modest magnitude, however.

Overall, our results differ from those based on models in which growth is driven mainly by physical capital accumulation coupled with learning-by-doing spillovers. This modelling reflects small economies that are not advancing the worldwide research frontier and adopt technologies developed abroad. In these economies, a rise in the retirement age is prone to depress the long-run economic growth rate (Futagami and Nakajima 2001; Heijdra and Mierau 2011). Overall, our results therefore suggest that keeping the underlying structure of the economy in mind is particularly important when conducting pension policies.

The policy recommendations emanating from our R&D-based endogenous growth model with demography and retirement would be: (i) to raise the retirement age in the face of secular stagnation because this boosts economic growth and puts upward pressure on the interest rate; and (ii) to couple 'on average' the retirement age to life expectancy.

However, we are well aware that crucial differences between different types of labour are present in reality. For employees in the R&D sector, it may easily be possible and even desirable to extend the working age, whereas workers in the production sector may struggle, for example, due to health issues. Consequently, retirement may well need to be designed in a flexible way such that an increase in the retirement age is possible for those who are still able and willing to work, while there are options for earlier retirement in physically demanding occupations and for those with health problems. More generally, studying the design of pension schemes within an R&D-driven economy remains a task for future research.

A second qualifier is that we are currently observing breathtaking advances in automation technologies that are replacing workers (Acemoglu and Restrepo 2018; Prettner and Strulik 2020; Prettner 2019). These advances are particularly pronounced in countries that are subject to rapid population ageing (Abeliansky and Prettner 2017; Acemoglu and Restrepo 2017, 2022). While it remains to be seen whether automation could help to avert the negative effects of a declining workforce, this possibility is worth being considered in the context of future social security and pension policies.

APPENDIX

Optimal consumption

The control variable is consumption c. The current-value Hamiltonian is given by

$$H = \log(c) + \phi[\chi w + (\mu + r)k - c].$$

The first-order conditions (FOCs) are

$$\frac{1}{c} = \phi,$$
$$(\mu + r)\phi = (\rho + \mu)\phi - \dot{\phi}.$$

From the first FOC we get $-\dot{c}/c^2 = \dot{\phi}$ such that the second FOC implies the consumption Euler equation (3):

$$\frac{\dot{c}}{c} = (r - \rho)c. \tag{A1}$$

Aggregating over cohorts

For our demographic structure, the aggregation rules to calculate aggregate consumption and aggregate capital are given by

$$C(t) = \mu N \int_{-\infty}^{t} c(t_0, t)\ \mathrm{e}^{\mu(t_0 - t)}\ \mathrm{d}t_0, \tag{A2}$$

$$K(t) = \mu N \int_{-\infty}^{t} k(t_0, t)\ \mathrm{e}^{\mu(t_0 - t)}\ \mathrm{d}t_0. \tag{A3}$$

Differentiating equations (A2) and (A3) with respect to time yields

$$\text{(A4)}\qquad \dot{C}(t) = \mu N\left[\int_{-\infty}^{t} \dot{c}(t_0,t)\, \mathrm{e}^{\mu(t_0-t)}\, \mathrm{d}t_0 - \mu\int_{-\infty}^{t} c(t_0,t)\, \mathrm{e}^{\mu(t_0-t)}\, \mathrm{d}t_0\right] + \mu N\ c(t,t)$$
$$= \mu N\ c(t,t) - \mu\ C(t) + \mu N\int_{-\infty}^{t} \dot{c}(t_0,t)\, \mathrm{e}^{-\mu(t-t_0)}\, \mathrm{d}t_0,$$

$$\text{(A5)}\qquad \dot{K}(t) = \mu N\left[\int_{-\infty}^{t} \dot{k}(t_0,t)\, \mathrm{e}^{\mu(t_0-t)}\, \mathrm{d}t_0 - \mu\int_{-\infty}^{t} k(t_0,t)\, \mathrm{e}^{\mu(t_0-t)}\, \mathrm{d}t_0\right] + \mu N\ k(t,t)$$
$$= \mu N\ k(t,t) - \mu\ K(t) + \mu N\int_{-\infty}^{t} \dot{k}(t_0,t)\, \mathrm{e}^{-\mu(t-t_0)}\, \mathrm{d}t_0.$$

Newborns do not own any capital because we abstract from bequests, that is, $k(t,t)=0$. From equation (2), it then follows that

$$\dot{K}(t) = -\mu\ K(t) + \mu N\int_{-\infty}^{t} [\chi(t_0,t)\ w(t) + (\mu+r)\ k(t_0,t) - c(t_0,t)]\, \mathrm{e}^{-\mu(t-t_0)}\, \mathrm{d}t_0$$
$$= r\ K(t) - C(t) + W(t),$$

which is the law of motion for aggregate capital, with W being aggregate non-capital income defined as $\left(\mu N\int_{-\infty}^{t}\chi(t_0,t)\ w(t)\ \mathrm{d}t_0\right)\mathrm{e}^{-\mu(t-t_0)}$.

Reformulating an agent's optimization problem subject to the lifetime budget constraint, as in Prettner and Canning (2014), we have

$$\text{(A6)}\qquad \max_{c(t_0,\tau)} U = \int_{t}^{\infty} \mathrm{e}^{(\rho+\mu)(t-\tau)} \log[c(t_0,\tau)]\ \mathrm{d}\tau$$
$$\text{subject to}\quad k(t_0,t) + \int_{t}^{R+t} w(\tau)\ \mathrm{e}^{-D^A(t,\tau)}\mathrm{d}\tau = \int_{t}^{\infty} c(t_0,\tau)\ \mathrm{e}^{-D^A(t,\tau)}\ \mathrm{d}\tau,$$

where the discount factor is $D^A(t,\tau) = \int_t^{\tau}[r(s)+\mu]\ \mathrm{d}s$. The FOC is

$$\frac{1}{c(t_0,\tau)}\ \mathrm{e}^{(\rho+\mu)(t-\tau)} = \mu(t)\ \mathrm{e}^{-D^A(t,\tau)}.$$

For the period $\tau = t$, this implies that

$$c(t_0,t) = \frac{1}{\mu(t)}.$$

Therefore we can write

$$\frac{1}{c(t_0,\tau)}\ \mathrm{e}^{(\rho+\mu)(t-\tau)} = \frac{1}{c(t_0,t)}\ \mathrm{e}^{-D^A(t,\tau)},$$

which gives

$$c(t_0,t)\ \mathrm{e}^{(\rho+\mu)(t-\tau)} = c(t_0,\tau)\ \mathrm{e}^{-D^A(t,\tau)}.$$

Integrating over time and using (A6) provides

$$\int_{t}^{\infty} c(t_0,t)\ \mathrm{e}^{(\rho+\mu)(t-\tau)}\ \mathrm{d}\tau = \int_{t}^{\infty} c(t_0,\tau)\ \mathrm{e}^{-D^A(t,\tau)}\ \mathrm{d}\tau,$$
$$\frac{c(t_0,t)}{\rho+\mu}\left[-\mathrm{e}^{(\rho+\mu)(t-\tau)}\right]_t^{\infty} = k(t_0,t) + \int_{t}^{T} w(\tau)\ \mathrm{e}^{-D^A(t,\tau)}\ \mathrm{d}\tau$$
$$\Rightarrow c(t_0,t) = (\rho+\mu)\left[k(t_0,t) + h(t)\right],$$

where $h = \int_t^{R+t} w(\tau)\, \mathrm{e}^{-D^A(t,\tau)}\, \mathrm{d}\tau$ refers to non-capital wealth, that is, wage income plus lump-sum transfers of profits. These calculations show that optimal consumption is proportional to total wealth with a marginal propensity to consume of $\rho + \mu$ (Heijdra and van der Ploeg 2002; Grafeneder-Weissteiner and Prettner 2013; Prettner and Canning 2014; Heijdra 2017, ch. 15). Aggregate consumption is then given by

$$\text{(A7)} \qquad \begin{aligned} C(t) &\equiv \mu N \int_{-\infty}^{t} c(t_0, t)\, \mathrm{e}^{\mu(t_0 - t)}\, \mathrm{d}t_0 \\ &= \mu N \int_{-\infty}^{t} \mathrm{e}^{\mu(t_0 - t)} (\rho + \mu)\, [k(t_0, t) + h(t)]\, \mathrm{d}t_0 \\ &= (\rho + \mu)\, [K(t) + H(t)], \end{aligned}$$

where H refers to aggregate non-capital income. Since newborns do not own capital because there are no bequests, their consumption is given by

$$\text{(A8)} \qquad c(t, t) = (\rho + \mu) h(t).$$

Using equations (A1), (A4), (A7) and (A8), we finally get

$$\begin{aligned} \dot{C}(t) &= \mu(\rho + \mu)\, H(t) - \mu(\rho + \mu)\, [K(t) + H(t)] + \mu N \int_{-\infty}^{t} (r - \rho)\, c(t_0, t)\, \mathrm{e}^{-\mu(t - t_0)}\, \mathrm{d}t_0 \\ &= \mu(\rho + \mu)\, H(t) - \mu(\rho + \mu)\, [K(t) + H(t)] + (r - \rho)\, C(t), \end{aligned}$$

and thus

$$\begin{aligned} \frac{\dot{C}(t)}{C(t)} &= r - \rho + \frac{\mu(\rho + \mu)\, H(t) - \mu(\rho + \mu)\, [K(t) + H(t)]}{C(t)} \\ &= r - \rho - \mu(\rho + \mu) \frac{K(t)}{C(t)}, \end{aligned}$$

which is the Euler equation for aggregate consumption.

Our results in the context of the lab-equipment approach

The no-arbitrage condition for investment is given by

$$r P_A = \pi + \dot{P}_A,$$

where P_A is the price/value of a patent (or the firm that is founded after the purchase of the patent). This equation states that investing the amount P in a standard interest-bearing saving vehicle would lead to a return of $r P_A$ and that this return has to be equal to the return of investing in a patent to establish a firm, which would yield operating profits π and a valuation gain $\dot{P}_A$. Further, we know that the profit of an intermediate goods producer is

$$\pi = (1 - \alpha)\alpha\, \frac{Y}{A}.$$

Along the BGP, the price of a patent is given as the discounted stream of all operating profits because otherwise there would be either entry or exit into intermediate goods production. Thus we have

$$P_A^* = \frac{\pi}{r} = \frac{(1 - \alpha)\alpha Y}{rA}.$$

Following Acemoglu (2009, ch. 13), the research production function in the lab-equipment approach is

$$\dot{A} = \hat{\lambda} Z,$$

where Z is the amount of final goods invested for the production of new ideas, and $\hat{\lambda}$ is the productivity of Z in generating new ideas. Then technological progress would be given by

$$\frac{\dot{A}}{A} = \hat{\lambda}\,\frac{Z}{A}.$$

Overall, free entry into R&D requires

$$\hat{\lambda} P_A^* = 1$$

because one unit of final output invested in R&D must be able to exactly recoup its cost in a perfectly competitive environment. Now we can plug in the above expressions to get

$$\hat{\lambda}\,\frac{(1-\alpha)\alpha Y}{rA} = 1 \quad \Leftrightarrow \quad r = \frac{\hat{\lambda}(1-\alpha)\alpha Y}{A}.$$

In the lab-equipment version of Acemoglu (2009, ch. 13) we have that the profit-maximizing output by the producers of intermediate goods equals $x_i = L$ such that we can write final goods output as

$$Y = L^{1-\alpha}\int_0^A x_i^\alpha \,\mathrm{d}i = AL,$$

and therefore $Y/A = L$. Then we get the following system of equations in equilibrium:

$$\begin{aligned} r &= \lambda(1-\alpha)\alpha L = \lambda(1-\alpha)\alpha L, \\ g_A^{Lab} &= \lambda(1-\alpha)\alpha L - \rho - \mu\,\frac{\rho+\mu}{\xi}, \\ g_A^{Lab} &= \frac{r}{\alpha^2} - \xi = \lambda\left[\frac{1-\alpha}{\alpha}\right]L - \xi, \end{aligned}$$

where g_A^{Lab} is the rate of technological progress in the lab-equipment approach. The growth rate and the value of ξ are both fully determined by the second and third equations. It is important to note that the interest rate (together with the interest rate channel of the effect of demographic changes on economic growth) drops out. Overall, the economic growth effects of increasing L, and thus of increasing the retirement age, are then unambiguously positive.

Derivation of the BGP in the Jones (1995) model

In the Jones (1995) model of semi-endogenous growth, a BGP requires that the rates of technological progress and per capita GDP growth are constant and equal to one another. From equation (32), we can derive the constant long-run growth rate of A by calculating the growth rate of the growth rate of A and setting the result equal to zero:

$$\begin{aligned} g_A^{Jones} &= \frac{\dot{A}}{A} = \frac{\lambda L_A}{A^{1-\phi}} \\ &\Rightarrow \log(g_A^{Jones}) = \log(\lambda) + \log(L_A) - (1-\phi)\log(A), \\ g_{g_A^{Jones}} &\overset{!}{=} 0 = \beta - \mu^* - (1-\phi) g_A^{Jones} \\ &\Rightarrow g_A^{Jones} = \frac{\beta - \mu^*}{1-\phi}. \end{aligned}$$

Going from the second to the third line, we used the fact that λ is constant and that the growth rate of the labour force in the long run is the difference between the birth rate (β) and the combined rates of death and labour force exit due to retirement (μ^*). Note that in this context, an increase in the retirement age would lead to a level shift in the size of the workforce but to no shift in its long-run growth rate.

Accounting for realistic demography

In this subsection, we study the properties of the model and likely results when considering a non-stationary population—that is, a setting where the gross birth rate β differs from the death rate μ—and when allowing for age-specific mortality. Following Kuhn and Prettner (2018), consider a lifecycle utility function

$$U = \int_{t_0}^{D} \log(c)\, \mathrm{e}^{-\rho(t-t_0)-M(t-t_0)}\, \mathrm{d}t,$$

where $M(t-t_0) = \int_0^{t-t_0} \mu(s)\, \mathrm{d}s$ denotes cumulative mortality up to age $t-t_0$, $\mu(s)$ denotes the age-specific mortality at age s, and D denotes the maximum age of survival. Assume the same budget dynamics as in the main body of the paper, $\dot{k}(t) = \chi(t)\, w + [\mu(t) + r]\, k(t) - c(t)$, where the annuity return is now age-specific, $\mu = \mu(t)$. Deriving the utility-maximizing consumption stream in the presence of an annuity market continues to yield the individual-level Euler equation (3).

Assume now that the birth rate β may deviate from the aggregate death rate $\overline{\mu} := \int_{t-D}^{t} n(t_0, t)\, \mu(t - t_0)\, \mathrm{d}t_0$, where $n(t_0, t) = N(t_0, t)/N(t)$ denotes the population share of birth cohort t_0 at time t. The flow of labour market entrants is $N(t, t) = \beta\, N(t)$, where $N(t) = \int_{-\infty}^{t} N(t_0, t)\, \mathrm{d}t_0$ represents the adult population size, and $L(t) = \int_{t-R}^{t} N(t_0, t)\, \mathrm{d}t_0$ is the labour force. Note that each adult cohort is of size $N(t_0, t) = \beta\, N(t)\, \mathrm{e}^{\mu(t_0-t)}$, and thus $n(t_0, t) = \beta\, \mathrm{e}^{\mu(t_0-t)}$ at a certain date $t > t_0$.

Following the derivations in Kuhn and Prettner (2018) (while setting $b = 0$, $\sigma = \theta = 1$), we can show that the aggregate Euler equation as one component (see equation (15)) of the growth system (14)–(16) is given by

(A9) $$g(t) = \frac{\dot{C}(t)}{C(t)} = r - \rho + \Omega(t),$$

where

(A10) $$\Omega(t) := -\overline{\mu} \left[\frac{c^{\dagger}(t)}{c(t)} - \frac{\beta}{\overline{\mu}}\, \frac{c(t,t)}{c(t)} \right]$$

represents the generational turnover effect on aggregate consumption growth. In this expression, $c(t) = C(t)/N(t)$ is per capita consumption,

$$c^{\dagger}(t) = \frac{1}{\overline{\mu}} \int_{t-D}^{t} \mu(t-t_0)\, c(t_0, t)\, n(t_0, t)\, \mathrm{d}t_0$$

is average consumption of the deceased, and $c(t, t)$ is consumption of the newborn cohort.[2] We then find that the impact of the generational turnover effect on aggregate consumption growth is negative, that is, that $\Omega(t) < 0$ if and only if the ratio between consumption of the deceased and consumption of the newborns exceeds the ratio between births and deaths by a sufficient amount, that is, if and only if

$$\frac{c^{\dagger}(t)}{c(t,t)} > \frac{\beta}{\overline{\mu}}.$$

This condition is likely satisfied for economies that feature a relatively steep age profile of consumption up to those ages in which the majority of deaths occurs, and for populations that are ageing due to low birth rates β. Such populations are typically also characterized by higher death rates $\overline{\mu}$, because the population is concentrated within older age classes that are subject to higher mortality rates. Furthermore, it is readily verified from equation (A10) that the direct effect of a change in the aggregate death rate $\overline{\mu}$ on the generational turnover is negative, implying that a mortality reduction across the board will tend to lower it, as is the case considered in the main body of the paper. From equation (A9), this implies that a decline in the aggregate death rate will typically tend to provide a demand-side boost to economic growth, as the process at which individuals with high levels of late-life consumption spending tend to be replaced at a lower rate. This direct impact is complemented by a reduction in the interest rate, as savings tend to increase for the funding of higher consumption spending at high ages. Kuhn and Prettner (2018) show that for a set of important industrialized countries, including the USA, Japan and a number of EU countries, the generational turnover effect is negative. This suggests that the 'generational turnover channel' is indeed relevant.

Accounting for endogenous retirement

Following Bloom *et al*. (2007) and Prettner and Canning (2014), individuals enter the labour market as adults at time t_0 and then maximize their discounted stream of lifetime utility

$$U = \int_{t_0}^{\infty} \left[\log(c) - \chi\ \nu(\mu,t)\right] \mathrm{e}^{-(\rho+\mu)(t-t_0)}\ \mathrm{d}t,$$

where $\nu(\mu,t)$ describes instantaneous disutility of work given a remaining life expectancy of $1/\mu$ at time t, and χ is an indicator function taking value 1 when working, and 0 when retired. With all other variables and the consumption-saving choice remaining the same as in the benchmark model, we focus on the retirement decision only.

For individuals to be willing to work at time t, the instantaneous marginal utility of doing so, which amounts to the marginal utility of the consumption afforded by an additional year's worth of earnings (including dividend payments), must exceed the marginal disutility of work, and we have

$$\chi = 1 \quad \Leftrightarrow \quad u'(c)\ w \geq \nu(\mu,t).$$

To get analytical solutions, we follow Bloom *et al*. (2007) and Prettner and Canning (2014), and assume that the disutility of work increases exponentially with the mortality rate, that is, $\nu(\mu,t) = d\ \mathrm{e}^{\mu(t-t_0)}$, where d is a scaling parameter measuring the unwillingness of individuals to work.

Integrating the lifetime budget constraint (4) and using $c(t_0,t) = c(t_0,t_0)\ \mathrm{e}^{(r-\rho)(t-t_0)}$, which follows from the individual Euler equation, and $w(t_0,t) = w(t_0,t_0)\ \mathrm{e}^{g(t-t_0)}$, denoting earnings growth by g, we arrive at an expression for the fraction of consumption expenditures to earnings at the beginning of the working life depending on the retirement age R:

$$\text{(A11)} \qquad \frac{c(t_0,t_0)}{w(t_0,t_0)} = \frac{\mu+\rho}{\mu+r-g}\left[1-\mathrm{e}^{-(\mu+r-g)R}\right].$$

Here, we note that $\mu+r-g > \rho \geq 0$ holds for all death rates along a BGP. From writing out the switching condition $u'(c)\ w = \nu(\mu,t)$, one can express the optimal retirement age R^* implicitly as a function of the fraction of earnings to consumption expenditures at the beginning of the working life:

$$\text{(A12)} \qquad \frac{c(t_0,t_0)}{w(t_0,t_0)} = \frac{\mathrm{e}^{(g+\rho-r-\mu)R^*}}{d}.$$

Intuitively, this expression tells us that if individuals wish to consume more in relation to initial income, then they have to retire later. Combining equations (A11) and (A12) yields

$$(\mu+\rho)d = (\mu+r-g)\ \mathrm{e}^{(g+\rho-r-\mu)R^*} + d(\mu+\rho)\ \mathrm{e}^{(g-r-\mu)\ R^*},$$

being an implicit relationship between the optimal retirement age R^*, the mortality rate μ, the discount rate ρ, the measure of the unwillingness to work d, the pace of earnings growth g, and the interest rate r.

Noting that $\int_{t-R^*}^{t} N(t_0,t)\ \mathrm{d}t_0 = N(1-\mathrm{e}^{-\mu R^*})$, we can now write the general equilibrium system as

$$\text{(A13)} \qquad W(\xi,g,r,R^*) := \frac{r}{\alpha} - \xi - g = 0,$$

$$\text{(A14)} \qquad X(\xi,g,r,R^*) := r - \rho - \mu(\rho+\mu)\ \frac{1}{\xi} - g = 0,$$

$$\text{(A15)} \qquad Y(\xi,g,r,R^*) := \lambda N\left(1-\mathrm{e}^{-\mu R^*}\right) - \frac{r-\delta}{\alpha} - g = 0,$$

$$\text{(A16)} \qquad Z(\xi,g,r,R^*) := (\mu+r-g)\ \mathrm{e}^{(g+\rho-r-\mu)R^*} + d(\mu+\rho)\left(\mathrm{e}^{(g-r-\mu)R^*}-1\right) = 0.$$

Applying the implicit function theorem and Cramer's Rule, we obtain after some manipulation the following comparative statics:

(A17)
$$\frac{\mathrm{d}g}{\mathrm{d}\mu} = \frac{1}{\Delta_4}\left\{\Delta_3 Z_{R^*}\frac{\mathrm{d}g}{\mathrm{d}\mu}\bigg|_{R^*} + \frac{\mu\lambda N\ \mathrm{e}^{-\mu R^*}}{\alpha^2}\left[-\alpha^2 X_\mu Z_r + Z_\mu(X_\xi+\alpha^2)\right]\right\}$$
$$= \frac{1}{\Delta_4}\left\{\Delta_3 Z_{R^*}\left(\frac{\mathrm{d}g}{\mathrm{d}\mu}\bigg|_{R^*} - \frac{Z_\mu}{Z_{R^*}}\frac{\mathrm{d}g}{\mathrm{d}R^*}\right) - \mu\lambda N\ \mathrm{e}^{-\mu R^*}\ X_\mu Z_r\right\},$$

(A18)
$$\frac{\mathrm{d}R^*}{\mathrm{d}\mu} = \frac{-1}{\Delta_4}\left\langle \Delta_3 Z_\mu - \frac{1}{\alpha^2}\left\{-X_\mu\alpha(\alpha Z_r - Z_g)\right.\right.$$
$$\left.\left. + R^*\lambda N\ \mathrm{e}^{-\mu R^*}\left[\alpha^2(Z_r+Z_g) + X_\xi(\alpha^2 Z_r + Z_g)\right]\right\}\right\rangle$$
$$= \frac{-1}{\Delta_4}\left\{\Delta_3 Z_\mu\left(1+\frac{\mathrm{d}g}{\mathrm{d}\mu}\bigg|_{R^*}\right) - \left[\frac{\Delta_3}{\alpha}\frac{\mathrm{d}r}{\mathrm{d}\mu}\bigg|_{R^*} + \frac{1-\alpha^2}{\alpha^2}X_\mu\right]Z_r\right\},$$

where

$$\Delta_4 = \Delta_3 Z_{R^*} - \mu\lambda N\ \mathrm{e}^{-\mu R^*}\left[Z_g(X_\xi+\alpha^2)\alpha^{-2} + Z_r(X_\xi+1)\right]$$
$$= \Delta_3\left[Z_{R^*} - X_\xi Z_r\frac{\mathrm{d}\xi}{\mathrm{d}R^*} - d\left(1-\mathrm{e}^{(g-r-\mu)R^*}\right)\frac{\mathrm{d}g}{\mathrm{d}R^*}\right] > 0$$

is the determinant of the full system (A13)–(A16), where

$$\Delta_3 = -(1+\alpha)(X_\xi+\alpha)\alpha^{-2} < 0$$

is the determinant of the subsystem (A13)–(A15), where

$$\begin{aligned}
X_\xi &= \mu(\rho+\mu)\xi^{-2} > 0,\\
X_\mu &= -(\rho+2\mu)\xi^{-1} < 0,\\
Z_{R^*} &= (\mu+r-g)\left[(g+\rho-r-\mu)\ \mathrm{e}^{(g+\rho-r-\mu)R^*} - d(\mu+g)\ \mathrm{e}^{(g-r-\mu)R^*}\right] < 0,\\
Z_r &= \mathrm{e}^{(g+\rho-r-\mu)R^*} - R^* d(\mu+g),\\
Z_g &= -Z_r - d(1-\mathrm{e}^{(g-r-\mu)R^*}),\\
Z_\mu &= Z_r - d(1-\mathrm{e}^{(g-r-\mu)R^*}),
\end{aligned}$$

and where $1-\mathrm{e}^{(g-r-\mu)R^*} > 0$ follows from the equilibrium condition $Z(\cdot)=0$.

Here, the derivatives $\mathrm{d}\xi/\mathrm{d}R^* > 0$ and $\mathrm{d}g/\mathrm{d}R^* > 0$ correspond to the derivatives that would hold for an exogenous variation of the retirement age within the original system (A13)–(A15), and the derivatives $\mathrm{d}g/\mathrm{d}\mu\ |_{R^*}$ and $\mathrm{d}r/\mathrm{d}\mu\ |_{R^*} > 0$ correspond to the derivatives that would hold for a variation in mortality within the original system (A13)–(A15) with a fixed retirement age.

Considering the impact of a variation in the mortality rate on the optimal retirement age first, we find from (A18) that $\mathrm{d}R^*/\mathrm{d}\mu < 0$ if

(i) $Z_\mu\left(1+\frac{\mathrm{d}g}{\mathrm{d}\mu}\bigg|_{R^*}\right) < 0,$

(ii) $Z_r = \mathrm{e}^{(g+\rho-r-\mu)R^*} - R^* d(\mu+g) = \left[1-R^*(\mu+r-g)\right]\mathrm{e}^{(g+\rho-r-\mu)R^*} - R^* d(\mu+g)\ \mathrm{e}^{(g-r-\mu)R^*} \le 0,$

where (ii) is satisfied if $R^* \ge (\mu+r-g)^{-1}$. Thus for condition (ii) to be satisfied, the retirement age has to be sufficiently large to begin with. Noting that $Z_\mu < Z_r \le 0$ in this case, it follows that condition (i) is satisfied if $1 + (\mathrm{d}g/\mathrm{d}\mu)\,|_{R^*} > 0$, implying that the impact of mortality (longevity) on the economic growth rate has to be bounded from below (above).

Finally, consider the impact of a variation in mortality on the economic growth rate, $\mathrm{d}g/\mathrm{d}\mu$. Noting that

$$\left.\frac{\mathrm{d}g}{\mathrm{d}\mu}\right|_{R^*} = \frac{R^*}{\mu}\,\frac{\mathrm{d}g}{\mathrm{d}R^*} - \frac{X_\mu}{\alpha\Delta_3},$$

and $-Z_\mu/Z_{R^*} = \partial R^*/\partial\mu$ in the sense of a partial response of optimal retirement to a variation in mortality, we can rewrite equation (A17) as

$$\frac{\mathrm{d}g}{\mathrm{d}\mu} = \frac{1}{\Delta_4}\left\{\Delta_3 Z_{R^*}\left(1 + \frac{\mu}{R^*}\,\frac{\partial R^*}{\partial\mu}\right)\frac{R^*}{\mu}\,\frac{\mathrm{d}g}{\mathrm{d}R^*} - \left(\frac{Z_{R^*}}{\alpha} + \mu\lambda N\ \mathrm{e}^{-\mu R^*}\ Z_r\right)X_\mu\right\}.$$

From this, we find that $\mathrm{d}g/\mathrm{d}\mu > 0$ if and only if

$$\frac{\mu}{R^*}\,\frac{\partial R^*}{\partial\mu} = -1 + \frac{\left((Z_{R^*}/\alpha) + \mu\lambda N\ \mathrm{e}^{-\mu R^*}\ Z_r\right)X_\mu}{\Delta_3 Z_{R^*}(R^*/\mu)(\mathrm{d}g/\mathrm{d}R^*)}.$$

Recalling that

$$-\left(\frac{Z_{R^*}}{\alpha} + \mu\lambda N\ \mathrm{e}^{-\mu R^*}\ Z_r\right)X_\mu > 0 \quad \text{and} \quad \Delta_3 Z_{R^*}\,\frac{R^*}{\mu}\,\frac{\mathrm{d}g}{\mathrm{d}R^*} > 0,$$

we see that this condition is analogous to the one reported in Proposition. Specifically, it holds when

$$\varepsilon_{R,LE} = -\frac{\mu}{R^*}\,\frac{\partial R^*}{\partial\mu} \geq 1.$$

NOTES

1. According to OECD (2019): 'the average effective age of retirement is calculated as a weighted average of (net) withdrawals from the labour market at different ages over a 5-year period for workers initially aged 40 and over. In order to abstract from compositional effects in the age structure of the population, labour force withdrawals are estimated based on changes in labour force participation rates rather than labour force levels.' OECD (2019) reports the average effective retirement ages by gender. (The original 2019 data have been removed from the OECD website, but are available from the authors on request.) To arrive at a general population average, we weight the gender-specific retirement ages with the gender shares in the total labour force, as reported in World Bank (2019).
2. For the setting considered in the main body of the paper, we have $\mu(t) \equiv \mu = \overline{\mu} = \beta$, and thus $c^\dagger(t) = \int_{t-D}^{t} c(t_0, t)\ n(t_0, t)\ \mathrm{d}t_0 = c(t)$. Furthermore, recalling that $c(t,t) = (\rho + \mu)\ H(t)/N(t)$, we can write

$$\Omega(t) = -\mu\left[1 - \frac{c(t,t)}{c(t)}\right] = -\mu\left[1 - \frac{(\rho+\mu)\ H(t)}{C(t)}\right] = -\mu(\rho+\mu)\ \frac{K(t)}{C(t)},$$

where the last equality follows because $C(t) = (\rho + \mu)[H(t) + K(t)]$. Thus, the case with age-invariant mortality and a stationary population can indeed be viewed as a special case.

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