A REFERENCE POINT APPROACH TO NONLINEAR MACROECONOMIC MULTIOBJECTIVE MODELS

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ABSTRACT

Programming-type multisectoral macroeconomic planning models are almost exclusively linear. Also, they often rely on traditional approaches such as sensitivity analysis and aggregated social welfare functions in their treatment of multiple conflicting objectives. In this paper the traditional linear programming framework is extended to handle nonlinear models and combined with an adaptive interactive decision support system to deal with multiple objectives. The decision support system is based on the reference point method.

Results obtained from a simplified model of the Hungarian economy provide a numerical illustration of the approach, and an appendix containing an analysis of the shadow prices derived from the linear and nonlinear planning models is also given.

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1. INTRODUCTION

In most of the socialist (or centrally planned) countries, linear multisectoral models are used in various stages of the planning process. In some countries (Hungary, for example), these models are already an integral part of national economic planning, where they provide an additional source of information for traditional (nonmathematically oriented) planners.

The effective use of these models has, however, been hindered by several factors. One of these is clearly the lack of appropriate channels of communication between traditional planners (or decision makers) and modelers. We believe that interactive adaptive decision support systems could provide a means of facilitating communication between planners and modelers, and we describe one such system (DIDASS) developed at IIASA.

Another factor hindering the use of planning models is their linear formulation, which has provoked criticism for several reasons. Most of the relationships between economic variables are obviously nonlinear, and can be reflected only poorly or not at all in linear programming models. For example, the mutual dependence of real and price variables cannot properly be taken into account by linear models. Some symptoms of this syndrome, such as overspecialized solutions, can be eliminated only by adding individual bounds to the models; these distort the shadow price system, introduce some *ad hoc* elements into the model, and make the model less transparent.

For the above reasons, this paper will investigate the possibility of extending the usual linear programming framework to include nonlinearities. This extension is based on experience gained in studying recently developed nonlinear multisectoral models of the general equilibrium type.

The solution of the resulting nonlinear multisectoral and multiobjective model is illustrated by a simple numerical example based on 1976 data for the Hungarian economy.

The paper is organized as follows. Section 2 contains a discussion of the nonlinear, multisectoral planning model framework, in which a typical linear model is taken and extended to include nonlinearities. Section 3 gives a description of the special multiple-criteria (reference optimization) approach we have adopted, together with an outline of its computer implementation and a numerical example. The paper concludes with a few final remarks.

2. A NONLINEAR MACROECONOMIC PLANNING MODEL FRAMEWORK

2.1. Background

Linear multisectoral programming models have become more or less integrated into the complex process of national economic planning in most of the socialist countries. Similar models have also been used for development planning purposes in several Western and developing countries. The relative simplicity of the underlying techniques has concealed many of the conceptual differences between modeling in East and West.

However, the recent development of more sophisticated, nonlinear models, under the general title of computable general equilibrium models, has apparently enhanced these differences to the extent that these new models are regarded as appropriate only for Western economies. Taking the models used in plan coordination as an example, we will show that this is not in fact the case.

The use and philosophy of macroeconomic models in coordinating a central plan can be summarized in the following way. Suppose that at some stage in the planning process the coordinating unit decides to summarize the calculations made so far, and as a result some provisional values of the sectoral outputs, inputs, consumption, etc., are made available. The coordinating unit wishes to know whether these more or less separately planned figures represent a consistent and balanced picture, and, if not, how this could be rectified. The unit also wishes to check how certain changes in one part of the plan would affect other parts of the provisional plan and its overall efficiency. In Hungary, formal models are used to help in checking the consistency, reasonableness and efficiency of a draft plan.

Economy-wide planning models built into and upon the traditional planning methodology of a socialist country differ from their Western counterparts, especially from recent computable general equilibrium models, in several respects. First, they almost exclusively contain "real" variables and relations reflecting physical constraints on allocation. Second, because the prices used in a planning model are either constant or planned, being predicted more or less regardless of "real" processes, the interdependence of the real and value (prices, taxes, rate of return requirements, etc.) variables is not considered explicitly in the model. Third, most mathematical planning models are closely related to and rely upon traditional or nonmathematical planning. This means, among other things, that the values of the exogenous variables and parameters and also certain upper and/or lower target values for some of the endogenous variables would not be derived directly from statistical observations, but would be based on calculations provided by traditional planners. (This is not to say, however, that more or less sophisticated statistical estimation techniques would not be combined with experts' "guesstimates" in traditional planning.) Finally, planning modelers in socialist countries tend to concentrate more on the problem of how to fit their models into the actual process of planning and make them practically applicable and useful than Western modelers. Therefore, applied planning models tend to be both theoretically and methodologically simpler than those in the development planning literature.

This section is intended to give a brief description of how certain techniques and certain types of models developed in the general equilibrium tradition can be viewed as natural extensions of the linear planning techniques developed to date (for more details see Zalai, 1980, 1982a, b). To this end, we introduce the nonlinear macroeconomic planning model as a variant of a typical linear programming model.

2.2. A Linear Macroeconomic Planning Model

To make our argument as clear as possible, we adopt rather simple model-building rules. We treat a large part of household and government *consumption* as fixed, both in level and in structure ($\bar{b}_{id}$, $\bar{b}_{im}$). These data are supposed to come from traditional plan calculations, although in our example they were determined from actual 1976 Hungarian data, taking 95 percent of the final consumption as fixed. We could employ a similar assumption for *investments*. We disregard the investment allocation problem for the sake of simplicity, taking only the level of investment as a decision variable. This is achieved by assuming the same (average) capital formation coefficients ($\bar{b}_{ij} = \bar{b}_i, \forall_j$) in each sector. *Gross investment* is determined as the sum of *replacement* (assumed to be identical with amortization) and new (*net*) *investment*. Capital allocation is variable, and therefore both components of gross investment are variable. In order to avoid overconsumption, an exogenously given policy variable ($\bar{\sigma}$) limits the consumption/net accumulation ratio from above.

The *foreign trade* part of the model is based on the following assumptions. World market prices for exports and imports are fixed ($\bar{P}_i^{WE}$, $\bar{P}_i^{WI}$), as is the target surplus (deficit) on the balance of foreign trade ($\bar{d}$). To avoid an overspecialized solution, individual bounds limit both export and import activities. In the case of exports we use upper bounds to reflect the capacities of the foreign markets to absorb exports (these bounds are assumed to be estimated by experts). In the case of imports we specify limits not on total volumes, but rather on the ratio of imports to domestically produced goods.

The *production* part of the model is assumed to follow closely the input-output modeling tradition. Thus, we assume a knowledge of the average input coefficients for both intermediate ($\bar{a}_{ij}$) commodities and primary factors of production, i.e., labor ($\bar{l}_i$) and capital ($\bar{k}_i$) in our case. For simplicity we disregard sectoral differences and bounds on allocations, or, in other words, we assume that decisions on allocations are still quite flexible at the given stage of planning for, say, five years ahead. Thus, we have only two overall constraints on labor and capital use in the model.

The above assumptions more or less specify the structure of the model as a system of linear inequalities. If the data are in any way consistent, we will have a large number of possible alternatives, and we must then consider how to further reduce the freedom of choice in a way that guarantees that feasible plans still exist. It is well known (see, for example, Kornai, 1974) that, in practice, models for plan coordination in socialist countries usually employ alternative objective functions in combination with parametrically varying constraints to determine several efficient variants of the plan. This approach can also be seen as a pragmatic method for analyzing conflicting objectives. The simple fact is that in a programming model it is not only the objective function but also the constraints which reflect the "objectives" of economic policy makers and/or traditional planners. The choice of which objective to incorporate in the objective function and which to regard as constraints with various *aspiration levels* can be regarded as purely arbitrary.

Since this issue is one of the main themes of this paper we will come back and discuss it in more detail in a later section. Our aim here is first to develop a macroeconomic planning model of the linear programming type, and next to show how it can be naturally extended to form a nonlinear model. A possible interactive method for handling the multiple-objective problem is then presented for the nonlinear model. We begin by simply assuming that there is only one objective function considered in the model. We also assume that we wish to maximize that part of total consumption that can be varied and, moreover, that the sectoral composition of this consumption is specified exogenously (this is the so-called Kantorovich type of objective function).

If we follow the rules outlined above, we end up with a linear programming model such as that given below. The model can be specified in a number of different ways, depending on the circumstances: we have chosen neither the shortest nor the most transparent form, but rather the one which is most convenient for our purpose. We begin with a list of variables and parameters (some of which will be used only in later specifications).

Variables

U_i *total variable domestic use of commodity $i = 1, 2, \ldots, n$

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*In the model each sector produces only one kind of commodity and each commodity is produced by only one sector, i.e., an input-output framework is adopted. Therefore, there is a one-to-one correspondence between sectors and products.

X_i total production of commodity i = 1,2,...,n

I_g gross investment

I_n net investment

C, C_a, C_i total variable consumption, average variable consumption, and its sectoral composition (i = 1,2,...,n)

U_{id}, U_{im} share of domestically produced and imported goods in the *total* variable domestic use of commodity i = 1, 2,...,n

M_i import of commodity i = 1,2,...,n

Z_i export of commodity i = 1,2,...,n

K_j, k_j capital used and capital/output ratio in sector j = 1,2,...,n

L_j, l_j labor employed and labor/output ratio in sector j = 1,2,...,n

Parameters

$\bar{a}_{ij}$ input-output coefficients (i = 1,2,...,n; j = 1,2,...,n)

$\bar{b}_{ij}$ capital formation coefficients (i = 1,2,...,n; j = 1,2,...,n)

$\bar{c}_i$ relative weight of commodity i = 1,2,...,n in the variable part of consumption

$\bar{b}_{id}, \bar{b}_{im}$ fixed (committed) part of consumption of commodity i = 1,2,...,n produced domestically (d) and imported (m)

$\bar{m}_i^-, \bar{m}_i^+$ lower and upper limits for the imported/domestically produced goods ratio in the total variable use of commodity i = 1,2,...,n

$\bar{h}_{im}, \bar{h}_{id}, \bar{\eta}_i$ parameters in the domestic-foreign goods substitution function for commodity i = 1,2,...,n

$\bar{\delta}_j$ depreciation rate in sector j = 1,2,...,n

$\bar{\sigma}$ consumption/net investment ratio

$\bar{P}_i^{WE}, \bar{P}_i^{WI}$ world market prices for export and import of commodity i = 1,2,...,n

$\bar{d}$ target surplus (deficit) in the foreign trade balance

$\bar{k}_j, \bar{l}_j$ fixed capital/output and labor/output ratios in sector j = 1,2,...,n

$\bar{A}_j, \bar{\alpha}_j$ coefficients in the output capacity (production) function for sector j = 1,2,...,n

$\bar{Z}_i$ upper bound on export of commodity i = 1,2,...,n

$\bar{Z}_i^o, \bar{\varepsilon}_i$ parameters in the export price-quantity (demand) function for commodity i = 1,2,...,n

$\bar{K}$ total amount of available capital

$\bar{L}$ total amount of available labor

Linear Programming Version of the Macroeconomic Planning Model

*Constraints**

1. Total variable domestic use (U_i)

(P_{ia}) $$\sum_j \bar{a}_{ij} X_j + \bar{b}_i I_g + C_i \leqq U_i \tag{1}$$

2. Balance of use and domestic products

(P_{id}) $$U_{id} \leqq X_i - Z_i - \bar{b}_{id} \tag{2}$$

*The symbol(s) in brackets in front of each constraint denote the dual variable(s) associated with that constraint. The dual variables are considered in the Appendix.

3. Balance of use and imports

(P_{im}) $$U_{im} \leqq M_i - \bar{b}_{im} \qquad (3)$$

4. Constraints on the domestically produced/imported goods share

(P_{iu}) $$U_{id} + U_{im} \geqq U_i \qquad (4.1)$$

(τ_i^-, τ_i^+) $$\bar{m}_i^- U_{id} \leqq U_{im} \leqq \bar{m}_i^+ U_{id} \qquad (4.2)$$

(4)

5. Gross investment identity

(P_v) $$\sum_j \bar{\delta}_j K_j + I_n \leqq I_g \qquad (5)$$

6. Constraint on the consumption/investment ratio

(λ) $$\sum_i \bar{b}_{id} + \sum_i \bar{b}_{im} + \sum_i C_i \leqq \bar{\sigma} I_n \qquad (6)$$

7. Trade constraints

7.1. Balance of foreign trade

(V) $$- \sum_i \bar{P}_i^{WE} Z_i + \sum_i \bar{P}_i^{WI} M_i \leqq \bar{d} \qquad (7.1)$$

(7)

7.2. Absorptive capacity limitation on exports

(ψ_i) $$Z_i \leqq \bar{Z}_i \qquad (7.2)$$

8. Balance of labor

(W) $$\sum_j L_j \leqq \bar{L} \qquad (8)$$

9. Balance of capital assets

$$(\rho) \qquad \sum_j K_j \leqq \bar{K} \qquad (9)$$

10. Labor and capital input requirement

$$(W_j') \qquad L_j \geqq \bar{l}_j X_j \qquad (10.1)$$

$$(Q_j) \qquad K_j \geqq \bar{k}_j X_j \qquad (10.2)$$

(10)

11. Objective

$$(P_{ic}) \qquad C_i \geqq \bar{c}_i C \qquad (11.1)$$

$$C \rightarrow \max \qquad (11.2)$$

(11)

2.3. A Nonlinear Extension of the Planning Model

The use of individual bounds in development planning models is not universally advocated. One of the main criticisms is that these are *ad hoc*, arbitrary restrictions, which can also distort the shadow prices (see, for example, Taylor, 1975).

This criticism is, however, only partially justified. If, for example, one looks at the models used to assist in plan co-ordination in centrally planned economies, one finds that individual bounds are based on detailed (traditional) plan calculations. In this case the degree of arbitrariness brought into the model by the individual bounds is probably much smaller than that introduced by any other method of handling the overspecialization problem, which is common to most macroeconomic models. It is true, however, that the longer the period covered by the plan, the higher the degree of arbitrariness introduced by individual bounds. And, of course, when development planning models do not have sufficient support from a traditional planning system or statistical system, then the degree of arbitrariness of the whole model is considerably greater.

In our view, the second part of the above criticism is more important and more valid than the first. It is quite common when using applied linear development programming models to find that the dual solution is unstable and distorted to such an extent that it cannot be used for any practical purpose. In Hungary, for example, where different types of linear programming models have been in use in plan coordination for almost 20 years, there has been practically no attempt to use the shadow prices for economic analysis or price planning. (In fact, there were attempts to develop separate linear programming models for price planning purposes; the stability of the "dual" part of the model was achieved at the cost of making the "primal" side useless.)

Thus, in our opinion, alternative methods for handling the problem of overspecialization are particularly interesting from the point of view of their effect on shadow prices. As will be seen, one such alternative is to introduce *flexible* rather than *rigid* bounds by means of nonlinear relationships. This approach* is based on experience gained recently in computable general equilibrium modeling**. The conceptual background to this approach is described in more detail elsewhere (Zalai 1980, 1982a) -- these papers also show that in many cases it is not only feasible but positively advantageous to completely abandon the programming framework and use computable general equilibrium models instead. Here we try to introduce the nonlinear forms as briefly as possible before turning our attention to the nonlinear multiobjective optimization problem.

There are four sets of conditions (4, 7, 10, and 11) in which we want to replace the linear terms by appropriate nonlinear forms. Condition 4 defines bounds on the substitutability of domestically produced and imported commodities. Our implicit assumption was that they are *perfect substitutes* ($U_{id} + U_{im} = U_i$).

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*Similar solutions have also recently been suggested by Ginsburgh and Waelbroeck (1981) in a somewhat different context.

**See, for example, Adelman and Robinson (1978), Dervis and Robinson (1978), Dixon et al. (1977) and Johansen (1959). Models of this type developed at IIASA are discussed in Bergman and Pör (1980), Karlström (1980), Kelley and Williamson (1980) and Shishido (1981).

This implies that whenever their relative shadow prices differ, the logic of the optimizing model will suggest that only the "cheaper" commodity should be used. This extreme behavior is limited by lower and upper bounds given by $\bar{m}_i^-$ and $\bar{m}_i^+$. Instead of this, however, we could assume that the domestically produced and imported commodities are less than perfect substitutes. Suppose that their substitutability can be described by a CES-type function:

$$(P'_{iu}) \qquad \left(\bar{h}_{id}\, U_{id}^{-\bar{\eta}_i} + \bar{h}_{im}\, U_{im}^{-\bar{\eta}_i}\right)^{-1/\bar{\eta}_i} \geqq U_i \qquad (4')$$

where the parameters do not necessarily have to be estimated econometrically. In a central planning context, for example, we might choose the size of $\bar{\eta}_i$ such that it would reflect expert judgements concerning the possibility of departing from the planned (or observed) relative shares ($\bar{m}_i^o$). Thus $\bar{\eta}_i$ plays a role similar to those of $\bar{m}_i^-$ and $\bar{m}_i^+$ earlier. The distribution parameters can then be calculated by assuming that the planned relative shares will not change if the relative efficiency (shadow) prices are equal. It can be shown* that the above assumption leads to:

$$\bar{h}_{id} = \left(\frac{1}{1 + \bar{m}_i^o}\right)^{1+\bar{\eta}_i} \quad ; \quad \bar{h}_{im} = \left(\frac{\bar{m}_i^o}{1 + \bar{m}_i^o}\right)^{1+\bar{\eta}_i}$$

Figures 1 and 2 illustrate the differences between the two solutions. In the linear programming case the substitution possibilities are represented by the piecewise-linear curve; the nonlinear formulation results in a smooth curve. The special advantage of the nonlinear form is that the deviation from the planned ratio ($\bar{m}_i^o$) is an increasing function of the relative difference in shadow prices (see Figure 2).

The difference between the two solutions can best be explained by the following analogy. In the linear case the modeler puts up rigid "fences" around the planned share so that the

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*See, for example, Zalai (1982b).

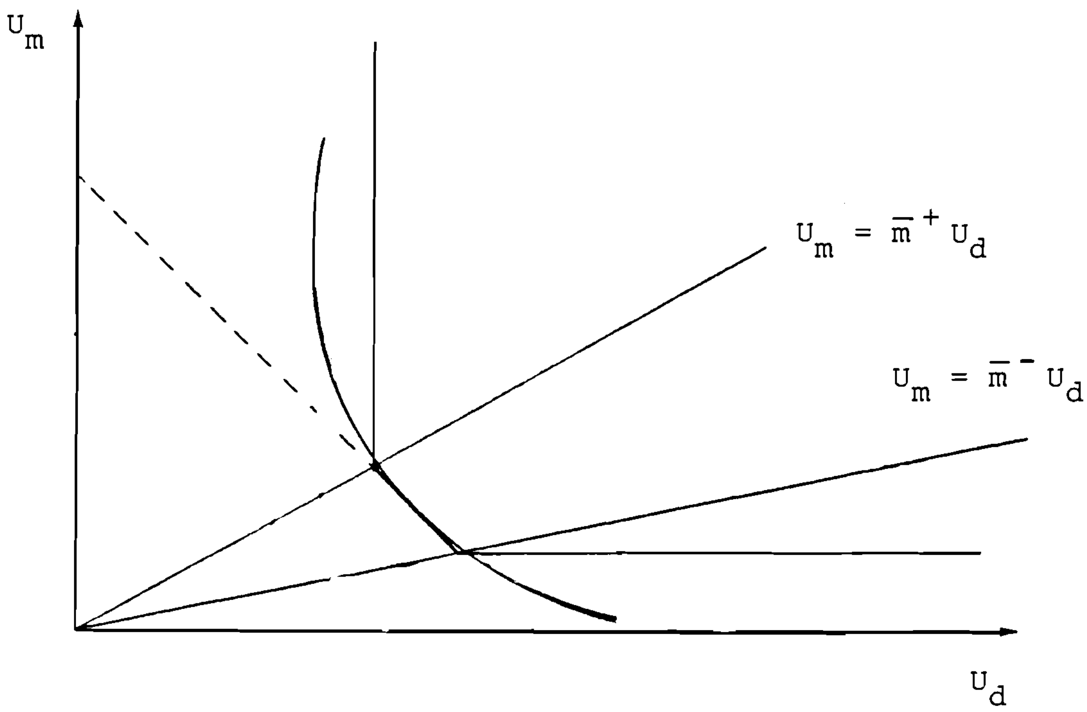

Figure 1. Assumed substitutability in linear and nonlinear models.

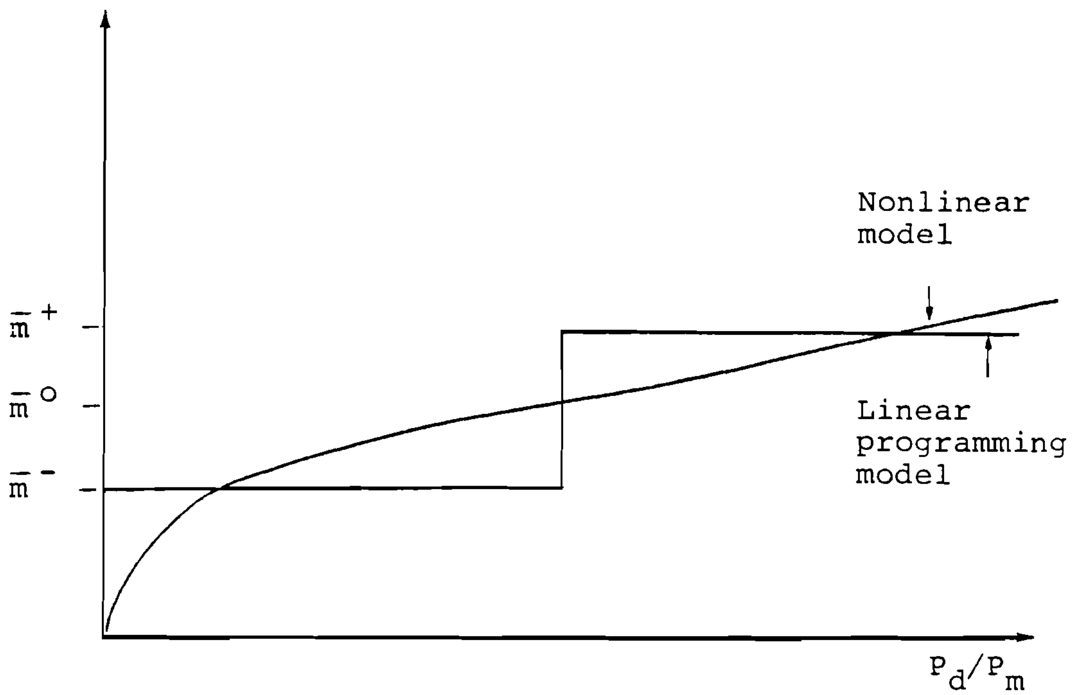

Figure 2. The shadow-price dependent import share functions implied by the linear and nonlinear models.

calculated share cannot escape from its immediate neighborhood. In the nonlinear case, however, he links the share calculated by the model to the planned (base) share, with a flexible "rope" to stop it from straying too far from the planned value.

In most cases one would <u>not</u> expect the primal solutions obtained from the linear and nonlinear formulations to differ greatly. The only possible source of concern might be the arbitrary substitution effect introduced by $\bar{\eta}_i$. If this is a real concern then it is possible to retain the assumption of perfect substitutability, using the CES form only to determine the relative shares (m) of the two sources.

With regard to the export bound ($\bar{Z}_i$), we may argue that it depends, among other things, on the expected unit export earnings, represented by $\bar{P}_i^{WE}$ in the model. Or, reversing the argument, and making use of the central planning context once again, we could reason in the following way. $\bar{P}_i^{WE}$ reflects the planned unit export price associated with the planned amount of exports ($\bar{Z}_i^o$). If this latter changes, the unit earnings will change too. Efficiency considerations will then automatically set limits to the increase in the amount of exports. Expressing this relationship in a simple mathematical form, we may define the unit export price as follows:

$$P_i^E = \left(\frac{Z_i}{\bar{Z}_i^o}\right)^{\bar{\lambda}_i} \bar{P}_i^{WE}$$

where $\bar{\lambda}_i$ reflects the speed of assumed price deterioration following the increase in export volume. ($\bar{\lambda}_i$ is in fact the reciprocal of the price elasticity $\bar{\varepsilon}_i$ in an implied export demand function, and thus, in principle, should have a value between -1 and 0.)

We can replace the constant unit export price by the above function in the balance of trade constraint, thus getting rid of the individual export bound. In other words, we can replace linear constraints (7) by the following nonlinear inequality:

$$(V) \qquad -\sum_i \bar{Z}_i^{o\,-1/\bar{\varepsilon}_i}\, \bar{P}_i^{WE}\, Z_i^{(1+\bar{\varepsilon}_i)/\bar{\varepsilon}_i} + \sum_i \bar{P}_i^{WE} M_i \leqq \bar{d} \qquad (7')$$

where $\bar{\varepsilon}_i$ is the price elasticity of the export demand.

The next nonlinear form does not need much explanation. Instead of fixed labor and capital input coefficients, we want to use variable ones. That is, we want to allow for different degrees of capital (labor) intensive technological development in various sectors. We assume that this substitution possibility does not affect the other input coefficients. Thus, we follow Johansen (1959) in defining a technology as a combination of the input-output framework and smooth production functions. In our numerical example we assume that the substitutability of the two factors is given by Cobb-Douglas functions. Thus, we replace constraints (10) by

$$(S_i) \qquad X_j - \bar{A}_j L_j^{\bar{\alpha}_j} K_j^{1-\bar{\alpha}_j} \leqq 0 \qquad (10')$$

Our last modification concerns the objective function, i.e., the determination of the variable (excess) part of consumption. We will replace conditions (11) by

$$C_a = C_1^{\bar{c}_1} C_2^{\bar{c}_2} \cdots C_n^{\bar{c}_n} \rightarrow \max \quad . \qquad (11')$$

It has been shown elsewhere (see Zalai, 1980, 1982a) that this replacement implies the possibility of substitution between the components of the excess consumption. If (see Figure 3) the shadow prices of the various commodities turned out to be equal to the planned prices (i.e., the base prices in our planning model), then the model would come up with the exact structure required by the preferences of the planners $(\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_n)$. If, however, the shadow prices differed from the planned prices, the model would look for some more efficient structure by substituting some of the relatively more expensive commodities by less expensive ones (in terms of shadow prices). It should be noted that it is possible to use alternative specifications, reflecting different assumptions. The particular

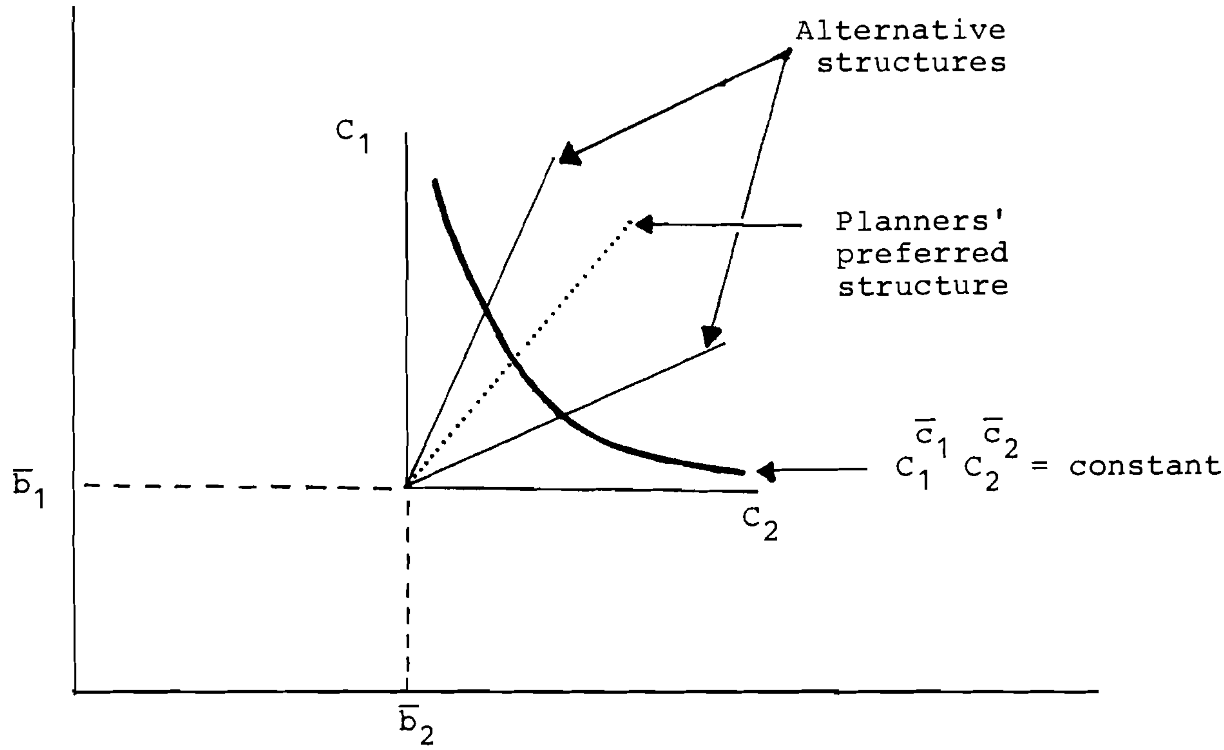

Figure 3. Replacing a fixed excess consumption structure by a variable one.

specification adopted here leads to an implied, shadow-price dependent demand system of the Linear Expenditure type (Zalai, 1980, 1982a).

The nonlinear planning model analyzed in the next section therefore consists of constraints 1, 2, 3, 4', 5, 6, 7', 8, 9, 10', and 11'. In the multiobjective analysis we will treat the balance of trade target as a variable and consider three possible objectives: maximizing excess consumption, maximizing net investment and maximizing the surplus on the balance of trade.

3. THE MULTIOBJECTIVE ANALYSIS

3.1. The Reference Point Method

As pointed out in Section 2.2 above, economic planning must by its very nature involve the consideration of multiple objectives. The traditional approach is to assume that it is possible to construct a social welfare function which somehow includes all of these (possibly conflicting) objectives. However, this idea of a single aggregated objective function has rather limited usefulness. Wierzbicki (1982) makes the same point in connection with another aggregated objective function -- the utility function. He shows that expressing preferences by utility functions is syntactically important, but that the semantic usefulness of the approach is limited because empirical tests have shown that the behavior of the decision maker is not always consistent with the assumptions of utility theory.

We believe that it would be more appropriate to use an adaptive framework to express the economic planner's preferences in matters which are, after all, highly political and sensitive. An adaptive framework capable of handling conflicting objectives may be constructed using Simon's concept of *satisficing decision making*, which has recently been reformulated by Wierzbicki. A solution technique (the reference point approach) based on this concept has been shown to work successfully in a number of practical applications (Kallio et al., 1980 and Grauer et al., 1982). This approach combines the advantages of the well-known goal programming method (Ignizio, 1978) and the method of displaced ideals (Zeleny, 1974). The basic idea is as follows:

(I) The *decision maker* (DM) works with aspiration levels, i.e., he specifies acceptable values for each of his objectives. This is consistent with Simon's statement (Simon, 1957, p. 141) that: "most human decision making, whether individual or organizational, is concerned with the discovery and selection of satisfactory alternatives; only in exceptional cases is it concerned with the selection of optimal alternatives".

(II) The decision maker works with the modeler and the computer in an interactive adaptive framework designed in such a way that the computer solves problems using information on aspirations, etc., supplied directly by the DM. This means that the policy maker is involved in the evaluation of alternatives and can use unquantifiable information (such as personal judgment) in doing so.

The basic idea of the reference point approach is to rank multidimensional decision alternatives q, defined as points in the R^p $(p \geq 2)$, relative to a reference point $\bar{q}$ which reflects a development corresponding to the preferences of the decision maker.

The ranking of the decision alternatives is based on a partial ordering of the R^p:

$$q^1 \leq q^2 \quad ; \quad q_i^1 \leq q_i^2 \quad ; \quad \forall i = 1,2,\ldots,p \; ; \; q^1;q^2 \in R^p \tag{12}$$

The decision problem is to determine an n-vector x of decision variables satisficing all given constraints while taking into account the p-vector of objectives. We will assume that each component of q should be as large as possible.

As mentioned above, a *reference point* is a suggestion $\bar{q}$ by the DM which reflects in some sense the "desired levels" of the various objectives. An achievement scalarizing function $s(q-\bar{q})$ defined over the set of objective vectors q may be associated with reference point $\bar{q}$. The general forms of functions s for which Pareto optimal (or weakly Pareto optimal) points minimize s over the attainable points q are given in Wierzbicki (1981).

If we regard the functions $s(q-\bar{q})$ as the "distance" between the points q and $\bar{q}$, then, intuitively, the problem of finding such a minimum may be interpreted as the problem of finding from within the Pareto set the point $\hat{q}$ "nearest" to the reference point $\bar{q}$. (However, the function s is not necessarily related to the usual notion of distance.) With this interpretation in mind, reference point optimization may be viewed as a way of guiding a sequence $\{\hat{q}^k\}$ of Pareto points generated from a sequence $\{\bar{q}^k\}$ of reference objectives. These sequences are generated in an interactive procedure and this should result in a set of attainable noninferior points $\{\hat{q}^k\}$ of interest to the decision maker. If the sequence $\{\hat{q}^k\}$ converges, the limit may be seen as the solution to the decision problem.

Let us assume that the nonlinear planning model described in Section 2 can be expressed as a nonlinear constrained multiple-objective programming problem in the following standard form:

$$\max_{x_{n\ell}, x_\ell} \begin{bmatrix} f_1(x_{n\ell}) + c_1^T x_{n\ell} + d_1^T x_\ell = q_1 \\ f_2(x_{n\ell}) + c_2^T x_{n\ell} + d_2^T x_\ell = q_2 \\ \dots \quad \dots \quad \dots \\ f_p(x_{n\ell}) + c_p^T x_{n\ell} + d_p^T x_\ell = q_p \end{bmatrix} \tag{13}$$

subject to:

$$g(x_{n\ell}) + A_1 x_\ell \le b_1 \tag{14}$$

$$A_2 x_{n\ell} + A_3 x_\ell \le b_2 \tag{15}$$

$$\ell \le \begin{bmatrix} x_{n\ell} \\ x_\ell \end{bmatrix} \le u \tag{16}$$

where $g(x_{n\ell}) = (g_1(x_{n\ell}), g_2(x_{n\ell}), \dots, g_m(x_{n\ell}))^T$ is the vector of nonlinear constraints and $f_1(x_{n\ell})$, $f_2(x_{n\ell}), \dots, f_p(x_{n\ell})$ in (13) represents the nonlinear parts of the performance criteria. The decision variables are divided into two subsets: a vector of "nonlinear" variables $(x_{n\ell})$ and a vector of "linear" variables (x_ℓ).

In the multiobjective analysis that follows, f_1 represents the excess consumption, f_2 the foreign trade account, and f_3 the net investment. The CES-type imported/domestic goods substitution functions (4') and the production functions (10') are examples of constraints of type (14); the balance of labor (8) and the balance of capital (9) are linear constraints of type (15).

As mentioned above, this type of approach to multiobjective analysis has so far been applied only to linear models. Therefore, it is worth describing in some detail the basic features of the computer model developed at IIASA for the nonlinear case (Section 3.2). After introducing the decision support system we will give a numerical illustration based on a three-sector model of the Hungarian economy (Section 3.3).

3.2. The Computer Implementation of the Approach

The computer implementation of the multiple-objective decision analysis and support system is based on a two-stage model of the decision-making process. In the first stage -- the exploratory stage -- the DM is informed about the range of his alternatives, giving him an overview of the problem. In the second stage -- the search stage -- the DM uses the system in an interactive way to analyze possible efficient alternatives $\{\hat{q}^k\}$ guided by his reference objectives $\{\bar{q}^k\}$. The initial information for the exploratory stage is provided by maximizing all of the objectives in (13) separately. A matrix D_s which yields information on the range of numerical values of each objective is then constructed. We shall call this the *decision support matrix*.

$$D_s = \begin{bmatrix} q_1^* & q_2^1 & \cdots & q_i^1 & \cdots & q_p^1 \\ q_1^2 & q_2^* & \cdots & q_i^2 & \cdots & q_p^2 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ q_1^j & q_2^j & \cdots & q_i^* & \cdots & q_p^j \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ q_1^p & q_2^p & \cdots & q_i^p & \cdots & q_p^* \end{bmatrix} \qquad (17)$$

Row j corresponds to the solution vector x_j which maximizes objective q_j. The vector with elements $q_i^i = q_i^*$, i.e., the diagonal of D_s, represents the *utopia (ideal) point*. This point is not attainable (if it were, it would be the solution of the proposed planning problem), but it is presented to the decision maker as a guideline from above to the sequence $\{\bar{q}^k\}$ of reference objectives. Let us consider column i of the matrix D_s. The maximum value in the column is q_i^*. Let q_i^n be the minimum value, where

$$\min_{1 \leq k \leq p} \{q_i^k\} = q_i^n$$

We shall call this the *nadir* value. The vector with elements $q_1^n, q_2^n, \ldots, q_p^n$ represents the *nadir point*, and may be seen as a guideline from below to the values of the decision maker's objectives. This was first presented for the linear case in Benayoun et al. (1971).

The general structure of the multiple-criteria package is presented in Figure 4. The linear part of the problem is input in MPS format and the nonlinear constraints and objectives as FORTRAN statements. The processor "Utopia" automatically compiles, links, and prepares the input for the p separate maximizations of the q_j, initiates the optimization process, and extracts information for the numerical and graphical presentation of the decision support matrix (17) to the DM.

The search stage of the decision analysis is supported by software consisting of three parts. These are (see Figure 4):

- The interactive "editor" for manipulating the reference point and the objectives (nlpmod)
- The preprocessor, which converts the input file containing the model description in standard format (see (13)-(16)) into its single-criterion equivalent (nlpmulti)
- The postprocessor, which extracts the information from the system output file, computes the values of the

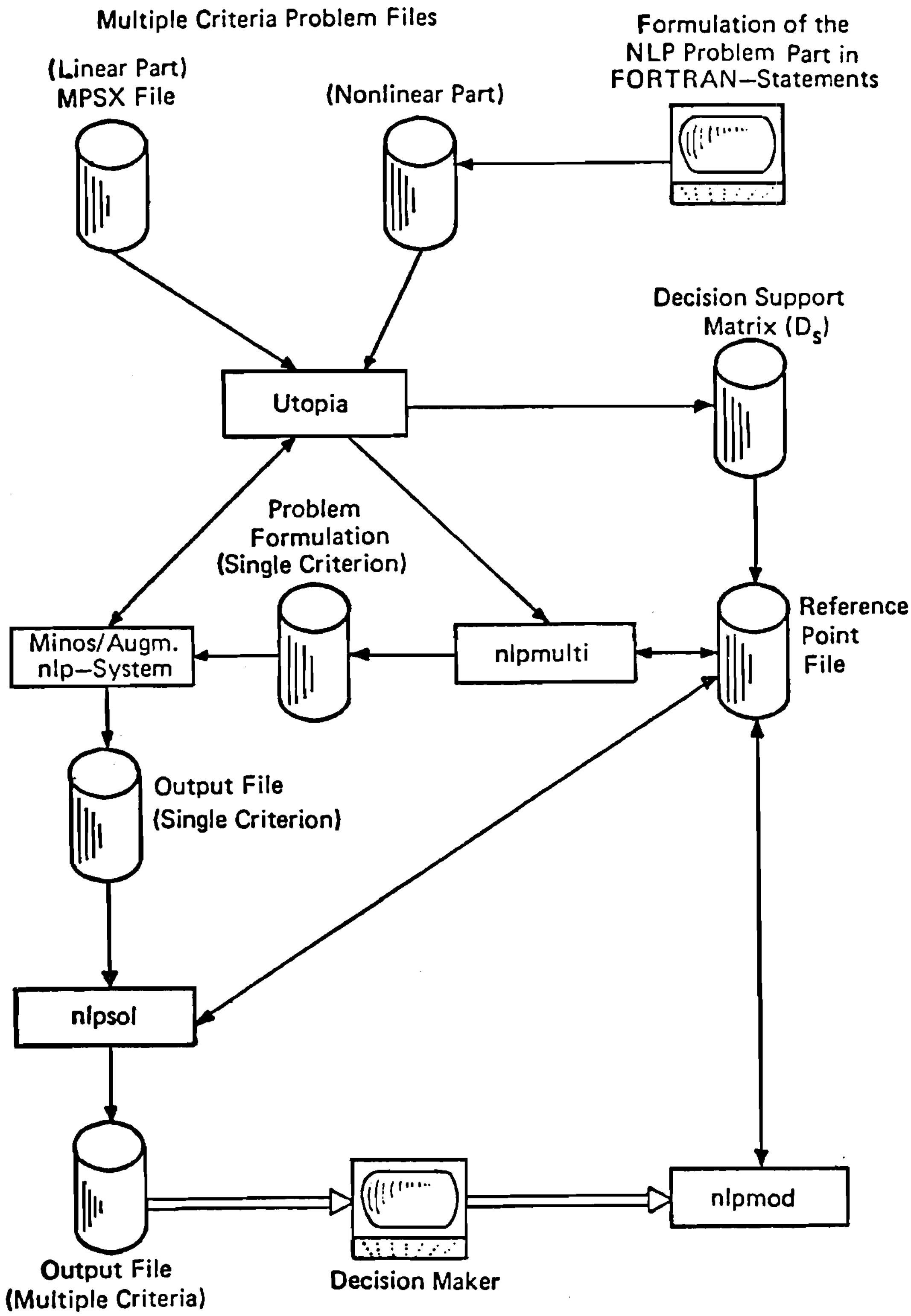

Figure 4. The structure of the nonlinear multiple-criteria package.

objectives, and displays the necessary information to the decision maker (nlpsol).

We used the following achievement scalarizing function:

$$s(w) = \sum_{i=1}^{p} (\gamma_i w_i)^2 \tag{18}$$

where γ_i is a scaling factor and $w_i = (\bar{q}_i - q_i)/\bar{q}_i$.

The single-criterion nonlinear programming problem obtained using (18) is then solved using the NLP-system MINOS/AUGMENTED (Murtagh and Saunders, 1980).

3.3. Numerical Illustration

Before launching into our example, we should perhaps warn the reader not to attach too much importance to the numbers on which it is based -- they are intended only to illustrate the use of the method. In fact, the example includes many observed data, but the model is simplified and aggregated to such an extent that its results would be of little use to an economist interested in real-life problems. The model contains only three sectors, which correspond roughly to the usual primary, secondary and tertiary sectors.

As already mentioned, the analysis was based on 1976 data for the Hungarian economy. Our main intention was to check that the nonlinear multiobjective solution algorithm worked properly, but we also wanted to compare its performance with that of an algorithm based on a solution technique for general equilibrium models*. We shall concentrate on the first aspect of the analysis in this discussion.

*This second algorithm was designed in the Hungarian Planning Office by A. Pór and A. Tihanyi for a model developed by one of the authors.

Table 1. Comparison of primal activities for maximum consumption.

Activity	Variable	Base case	Equilibrium approximation	Programming approximation
Production of commodity i	X_i	286216.4 695183.3 304003.8	270973.4 731076.8 306343.3	270786.7 731219.8 306348.4
Import of commodity i	M_i	38422.5 179448.0 6480.9	37102.4 143984.3 6472.2	37124.8 144120.4 6426.6
Net investment	TI	165998.5	172403.5	172406.1
Gross invest.	$\sum GI_i$	187171.9	194220.2	194164.6
Variable consumption	C_i	2786.6 7074.5 8370.8	4850.1 13422.9 14049.6	4849.6 13427.8 14051.1
Export of commodity i	Z_i	29329.9 161716.3 10484.4	7079.2 131647.0 3967.0	6946.6 132017.9 3924.4
Domestic source share (per cent)	S_i^d	86.9891 74.8290 97.8397	87.8765 82.9228 97.9788	87.8642 82.8893 98.0104
Imported source share (per cent)	S_i^m	13.0109 25.1710 2.1603	12.1880 18.3071 2.0268	12.1583 18.3307 1.9962
Labor used	L_i	1799.2 1964.1 2003.0	1640.8 1980.9 2144.6	1646.4 1981.3 2138.6
Capital used	K_i	393191.3 368387.3 1292359.2	405735.7 435225.2 1212976.2	401656.7 435299.4 1216982.0
Objective function		6628.3	11732.1	11730.4

Table 1 contains the base solution (actual 1976 data) and the consumption maximizing solutions calculated by the two algorithms. The algorithms give practically the same solution, as they should do, but we have nevertheless found it extremely useful to have such a checking device in the early phases of model development and calibration. A comparison of the base solution with the others shows what kind of 'optimal' adjustments our very simple model suggests.

Table 2 contains the decision support matrix and a compromise solution obtained from the multiobjective analysis, while Table 3 compares the shadow prices associated with the three individual maxima. The shadow prices were scaled in order to make them comparable: the scaling criterion was that the shadow value of the fixed consumption (the sum of $\bar{b}_i$'s) should be the same for each solution. Thus, the shadow prices of the domestically produced and imported commodities can be interpreted directly as percentage changes in the corresponding prices. The table shows that our model yields shadow prices which exhibit very stable behavior and can be interpreted in a very straightforward way. This interpretation is again left for interested readers.

Table 2. Decision support matrix and compromise solution.

	Average consumption Obj(1)	Foreign trade deficit Obj(2)	Investment Obj(3)
Obj(1) → max	11 730.4	-22 822.1	172 404.4
Obj(2) → max	0.1	14 998.8	157 911.4
Obj(3) → max	0.1	-22 822.1	207 157.6
Compromise solution	1 486.8	- 1 019.3	159 632.3

Table 3. Comparison of shadow prices.

		Base	Consumption maximization	Trade surplus maximization	Net investment maximization
Domestic goods	P_{1d}		99.6	98.6	99.5
	P_{2d}	100	82.7	82.6	82.8
	P_{3d}		106.8	104.5	106.4
Imported goods (Exchange rate)	$P_{im}=V$	100	115.9	125.2	117.2
Investment	P_v	100	92.0	93.2	92.2
Value added	s_1	45.8	44.3	43.0	44.2
	s_2	19.1	18.8	18.2	18.7
	s_3	64.0	66.2	63.9	65.8
Labor	ω	5.1	5.1	5.0	5.1
Capital	ρ	5.0	7.1	6.8	7.0

4. CONCLUDING REMARKS

In this paper we presented some preliminary results of research directed toward the incorporation of multiobjective decision analysis into various types of macroeconomic planning models. We concentrated our attention on a static nonlinear macroeconomic model and the reference point method.

This should be seen primarily as a methodological paper: our model and its results have to be refined considerably before they can be applied to any real planning process. Nevertheless, these first results seem encouraging and we believe that the method described here can easily be applied to the programming-type macroeconomic models currently in use.

In future research we shall try to extend our method to cover *multi-period* planning models. In this case the decision makers would be asked to give their aspirations in terms of trajectories rather than single points. We also intend to extend our method to include *computable general equilibrium* models. At present, models of this type do not explicitly incorporate multiple-objective analysis, partly because of the apparent lack of effective nonlinear solution algorithms. It was not our aim in this paper to discuss the special advantages of the computable general equilibrium framework, nor to show how one could proceed from a nonlinear model to a computable general equilibrium model. Some of these issues have been discussed in other papers by one of the authors (Zalai, 1980, 1982a,b) and others left for future research. We hope, nevertheless, that effective algorithms for this latter type of multisectoral macroeconomic models can be developed along the lines presented in this paper.

APPENDIX: COMPARATIVE DISCUSSION OF THE TWO REGIMES OF SHADOW PRICES

Here we make a detailed derivation and comparison of the shadow prices resulting from linear and nonlinear models. There are only a few important differences, some of which can be seen as alternative hypotheses, while others may be viewed simply as various means of smoothing out the roughness of the linear model.

A detailed analysis of the shadow prices will hopefully also help the reader to get some idea of the corresponding general equilibrium formulation. Only a few steps separate the Kuhn-Tucker necessary conditions for optimality in the nonlinear case from a set of equations more common in the general equilibrium tradition. Since this exercise is rather simple and we have done it elsewhere, the above few steps will be left as an exercise for the interested reader.

The *cost of capital* is made up of amortization ($\overline{\delta}_j$) and rent (ρ) Amortization is calculated on the basis of the reevaluated capital stock ($P_v\overline{k}_j$), whereas the rent for capital is calculated on the basis of the base value ($\overline{k}_j$). Introducing R for ρ/P_v makes it possible to transform everything to a uniform basis so that we can rewrite the unit cost of capital (Q_j) in sector j in the following form:

$$(K_j) \qquad Q_j = \overline{\delta}_j P_v + \rho = (\overline{\delta}_j + R) P_v \qquad \text{(A1)}$$

The reader familiar with computable general equilibrium theory should recognize this formulation -- it is quite commonly used in this field. Following Johansen (1959), Q_j is normally referred to as the user's cost of capital.

In an earlier paper by one of the present authors (Zalai, 1980) it was shown that the introduction of sectorally differentiated rental rates (e.g., with $\overline{r}_j R$ instead of R in equation (A1)) in a general equilibrium model would have a similar effect to the use of upper and lower limits on the sectoral allocation of capital (i.e., additional individual bounds such as $K_j^- \leqq \overline{k}_j X_j \leqq K_j^+$).

The two solutions are, however, not completely identical in that the sectoral differences in the rates (values of $\bar{r}_j$) are exogenous in the equilibrium model, but endogenous in the programming one.

The sectoral shadow *price of labor* (W_j) in our model is identical with its global shadow price (W). This can be seen from the dual constraint associated with L_j:

$$(L_j) \qquad W_j = W \tag{A2}$$

Here again, introducing upper and lower limits on the sectoral allocation of labor would result in differing sectoral shadow prices. However, the endogenously determined sectoral wage differentials may be quite different from their actual values. Replacing the labor constraint by a wage constraint would resolve this problem but at the cost of excluding the labor constraint. Without elaborating on this issue, we wish to indicate that the general equilibrium formulation can again handle this problem more flexibly than the programming model. Thus, we may have the labor constraint in the 'primal' part and exogenously determined wage differentials in the 'dual' part (say, as $W_j = \bar{w}_j W$).

It should be noted that the specific features of the linear programming model discussed earlier may result in a zero shadow price either for capital or for labor. This is a common feature of linear programming models which do not have enough substitutability built into them, and can be handled in the linear model by introducing a sufficient number and variety of technological alternatives. This would, however, significantly increase the size of the model, and so is usually avoided in macromodels. Nonlinear models allow for a more 'size-conscious' treatment of this problem.

A special advantage of the general equilibrium formulation should also be mentioned here. This is connected with the treatment of amortization and replacement, two factors which it is recognized can differ significantly. The replacement rate is usually smaller than the amortization rate. One is important in the 'primal' part of the problem (replacement is part of gross investments), the other in the 'dual' formulation (cost of capital). The *strict duality* properties of programming models do not make such a distinction possible. Once, however, we relax the strict mechanistic duality of the physical and value phenomena (in the form of an equation system similar to the Kuhn-Tucker necessary conditions for optimality) the above distinction can be made.

Coming back to our dual equations, we see that the shadow *prices of domestically produced goods* are given by the following equation:

$$(X_j) \qquad P_{jd} = \sum_i P_{ia}\bar{a}_{ij} + Q_j\bar{k}_j + W_j\bar{l}_j \qquad (A3)$$

where P_{ia} is the average shadow price of all the goods used, as will be seen later.

The dual constraints in the *nonlinear case* are only slightly different from their linear counterparts, although at first glance they seem to be completely different. The partial derivatives of the Lagrangean yield the following conditions:

$$(K_j) \qquad S_j \frac{\partial X_j}{\partial K_j} = \bar{\delta}_j P_V + \rho \qquad (A1')$$

$$(L_j) \qquad S_j \frac{\partial X_j}{\partial L_j} = W \qquad (A2')$$

$$(X_j) \qquad P_{jd} = \sum_i P_{ia}\bar{a}_{ij} + S_j \qquad (A3')$$

It can be shown that, due to Euler's theorem on homogeneous functions, conditions (A1') and (A2') imply that

$$S_j = (\bar{\delta}_j P_V + \rho)k_j + Wl_j$$

Thus the domestic price (P_{jd}) formation rule (equation (A3')) is, in fact, the same as before (equation (A3)), except for the fact that the labor and capital input coefficients (l_j and k_j) are now variables, with optimal values dependent on (A1') and (A2'). These conditions therefore assume a 'functional' role instead of the simple 'definitional' one played in the linear case.

We would like to draw attention to the close similarity of the determination of the shadow prices for domestic commodities outlined above to the usual input-output price calculations. We should also point out that, unlike the programming formulation, a general equilibrium model can take into account several types of price distortions, including profits and taxes.

The dual constraint associated with gross investment determines the *price of new capital goods* as an average of the input prices:

$$(I_g) \qquad P_v = \sum_i P_{ia}\bar{b}_i \qquad \text{(A4)}$$

We could have distinguished the investment input requirements for each sector ($\bar{b}_{ij}$) and thus define price indices for capital goods destined for individual sectors (P_{jv}). This type of distinction becomes especially crucial in a multiperiod model. It is interesting to note that the price formation rule for capital goods in computable general equilibrium models is the same as that given above.

The dual constraints corresponding to the import activities in the primal case determine the *price indices of imports*:

$$(M_j) \qquad P_{mj} = V\bar{P}_j^{WI} \qquad \text{(A5)}$$

where V is the exchange rate, i.e., the shadow price associated with the balance of trade constraint. Here again, computable general equilibrium models allow for exogenously introduced tariffs and subsidies, and for simulation of their possible effect on other variables. Programming models, on the other hand, can more readily accommodate import quotas in the form of individual bounds. These, in turn, will lead to endogenously determined price distortions in the form of import taxes.

The dual constraints corresponding to the *shares of domestically produced and imported commodities* in the total variable use are as follows:

$$(U_{id}) \qquad P_{iu} = P_{id} + (\bar{m}_i^- \tau_i^- - \bar{m}_i^+ \tau_i^+) = (1 + \tau_{id})P_{id} \qquad \text{(A6)}$$

$$(U_{im}) \qquad P_{iu} = P_{im} - (\tau_i^- - \tau_i^+) = (1 + \tau_{im})P_{im} \qquad \text{(A7)}$$

where τ_{id} and τ_{im} are rates which are explained in more detail below.

Before trying to interpret the above pricing rules we should note that, due to the complementary slackness, the products of the difference terms $(\bar{m}_i^- \tau_i^- - \bar{m}_i^+ \tau_i^+)$ and $(\tau_i^- - \tau_i^+)$ with U_{id} and U_{im}, respectively, will be equal. It is also easy to see that the dual constraint associated with the variable U_i simply states the equality of P_{ia} and P_{iu}. From these two observations it follows that

$$P_{iu}U_i = P_{id}U_{id} + P_{im}U_{im} = P_{ia}U_i \qquad \text{(A8)}$$

Thus, P_{ia} is really the *average price of goods from the available sources*. This may be expressed more clearly as follows:

$$P_{ia} = s_{id}P_{id} + s_{im}P_{im} \qquad \text{(A9)}$$

where

$$s_{id} = U_{id}/U_i \quad \text{and} \quad s_{im} = U_{im}/U_i$$

Returning to equations (A6) and (A7), it is now clear that they reflect a simple average price setting rule in a situation of *perfect substitutability*. The two goods (domestically produced and imported) are treated as perfect substitutes, with unit prices of P_{id} and P_{im}, and an average price of $P_{ia} = P_{iu}$. Since these goods are perfect substitutes, the users have to be charged the same price (P_{iu}) for them. This means that appropriate taxes and/or subsidies have to be introduced to compensate for the individual price differences. τ_{id} and τ_{im} represent the necessary tax or subsidy rates.

In the nonlinear programming case we assume that the two kinds of goods are *less than perfect substitutes*. In this case, therefore, the price differences are assumed to guide the users' decision about the optimal mix of goods from the two sources, and there is no need to homogenize prices through taxes and subsidies. Thus, in the nonlinear case we replace equations (A6) and (A7) by the following equations:

$$(U_{id}) \qquad \frac{\partial U_i}{\partial U_{id}} P_{iu} = P_{id} \qquad \text{(A6')}$$

$$(U_{im}) \qquad \frac{\partial U_i}{\partial U_{im}} P_{iu} = P_{im} \qquad \text{(A7')}$$

It is interesting to note that, due to Euler's theorem on homogeneous functions, (A6') and (A7') also lead to (A8) as above. After some analytical manipulation these equations also yield

$$m_i = \bar{m}_{io} \left(\frac{P_{id}}{P_{im}}\right)^{\bar{\mu}_i} \qquad \text{(A10)}$$

where $m_i = U_{im}/U_{id}$. This is an import demand function commonly used in computable general equilibrium models (see also Figure 2).

Next we consider the dual constraints associated with exports in the linear case

$$(Z_i) \qquad P_{id} = V\bar{P}_i^{WE} - \Psi_i \qquad \text{(A11)}$$

If the individual upper bound is not binding then the domestic price (P_{id}) and the export price (VP_i^{WE}) are equal. If the bound is binding then the above pricing rule has a simple interpretation in terms of perfectly elastic supply. If $V\bar{P}_i^{WE}$ were larger than P_{id}, then suppliers would try to sell everything on foreign markets. To limit exports to $\bar{Z}_i$ would require a tariff (Ψ_i) which would take away the incentive to increase exports beyond this value. In fact, the question of how to divide production between domestic and foreign markets then becomes meaningless, because all decisions provide the same amount of income for the producers.

The nonlinear case is very similar. There we have the following condition:

$$(Z_i) \qquad P_{id} = V\bar{P}_i^{WE}\,\bar{Z}_i^{0\;-1/\varepsilon_i}\,Z_i^{1/\varepsilon_i}\left(\frac{1+\varepsilon_i}{\varepsilon_i}\right) \qquad (A11')$$

which at first glance looks quite different to the corresponding equation in the linear case. However, observe that in this case

$$\bar{P}_i^{WE}\left(\frac{Z_i}{\bar{Z}_i^0}\right)^{1/\varepsilon_i} = P_i^E$$

is nothing but the variable export price P_i^E (see p. 14). Therefore (A11') reduces to

$$P_{id} = VP_i^E\left(\frac{1+\varepsilon_i}{\varepsilon_i}\right) = VP_i^E + \frac{1}{\varepsilon_i}\,VP_i^E \qquad (A11'')$$

This is already closer to equation (A11). The other important difference apart from the variable export price is that the size of the tariff is determined explicitly by the size of the export price elasticity (see Zalai (1982) for a more detailed analysis of this issue, which is known as the optimum tariff problem in the international trade literature).

Finally, we will examine the dual constraints corresponding to the two elements of final use: net investments and variable consumption. In the linear case we have the following:

$$(I_n) \qquad \bar{\sigma}\lambda = P_v \qquad (A12)$$

$$\left.\begin{aligned} &(C_i) \qquad P_{ic} = P_{ia} + \lambda \qquad (A13.1)\\ &(C) \qquad 1 = \textstyle\sum \bar{c}_i P_{ic} \qquad (A13.2)\end{aligned}\right\} (A13)$$

In the nonlinear case (A13) is replaced by

$$(C_i) \qquad \frac{\partial C_a}{\partial C_i} = P_{ia} + \lambda \qquad (A13')$$

A brief analysis again reveals the essential similarities and differences between the two systems. Equation (A13.1) can be viewed as a simple definitional equation giving the shadow price of one unit of the variable consumption of commodity i as the sum of its shadow price (P_{ia}) plus λ. This latter can be viewed as a special turn-over tax: each unit of the variable consumption has to 'earn' the price of $1/\bar{\sigma}$ unit net investment associated with it ($\lambda = P_v/\bar{\sigma}$). The same expression also appears on the right-hand side of (A13').

Equation (A13.2) is simply a price-scaling condition. Multiplying both sides by C yields

$$C = \sum P_{ic}C_i \qquad \text{(A14)}$$

i.e., the sum of the values of variable consumption is equal to its general level (C).

Denote the right-hand side of (A13') by P_{ic}, multiply both sides by C_i and sum over i. Once again making use of Euler's theorem, we obtain

$$C_a = \sum P_{ic}C_i \qquad \text{(A14')}$$

Thus, we can see that the nonlinear case conceals the identical price normalization rule found in the linear situation. The crucial difference lies in the fact that in the linear case the consumption structure is fixed, while in the nonlinear one it is variable. Condition (A13') therefore actually has a role in guiding decisions in addition to the more formal (definitional) role shared by conditions (A13).

It is shown elsewhere (Zalai, 1980) that both conditions yield special demand systems that can be used in computable general equilibrium models. The nonlinear case yields the familiar Linear Expenditure System, the linear case one which is formally very similar. In the nonlinear case the planning model modifies the initial (planners' preferred) consumption structure to produce a more efficient (less expensive) variant.

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