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TIME-SERIES ANALYSIS OF NUTRIENT
LOADINGS IN THE ZALA RIVER, LAKE BALATON

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PREFACE

One of the principal research themes of the Resources and Environment (REN) Area's studies in water quality modelling and management (1977-1982) has been the Lake Balaton Case Study (see, for example, WP-82-79). The case study was a collaborative project involving scientists from a number of Hungarian research institutions, IIASA, and other research organizations. The project's objective was to examine the problems of controlling eutrophication in general and, more specifically, the problems of eutrophication control in large, shallow lakes.

This paper reports the results of an analysis of the dynamic (daily, seasonal, and annual) variations in the nutrient loadings discharged to Lake Balaton from the Zala River, the principal tributary of the Lake (see also CP-81-21). As with the majority of the research undertaken during the case study, the emphasis in the analysis of this report is on phosphorus as the critically important rate-limiting nutrient. The focus of the analysis on the Zala River also reflects the overriding significance of this particular tributary as a target for implementing facilities for the control of eutrophication in Lake Balaton (see also WP-82-10).

ACKNOWLEDGEMENTS

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1. INTRODUCTION

The distinctions between point and nonpoint sources of watershed nutrient loadings and between the physical and chemical forms of these loadings, i.e. between forms being available or unavailable for metabolism in phytoplankton growth, are important factors for the control of lake eutrophication. It is not a trivial problem, however, to make accurate quantitative estimates of the relative distributions of nutrient loadings among different types from different sources (see, for example, Jolankai and Somlyódy, 1981). More generally, there is considerable uncertainty associated with the description and understanding of the processes affecting the erosion, transport, deposition, and transformation of the nutrient material as it passes from the watershed to the receiving lake. The frequency of sampling for the observations of stream nutrient concentrations is normally much slower than the fast dynamic characteristics of storm events, during which periods a major portion of the nutrient load is considered to be transported. This, quite apart from the additional difficulties of describing the mechanisms by which nutrients reach the stream from, for example, agricultural nonpoint sources (Haith and Tubbs, 1981), accounts for the significant uncertainty in computing estimates of the nutrient loads and in understanding their dynamic variations.

The River Zala is the principal tributary of Lake Balaton, a large shallow lake in the western part of Hungary (see Figure 1), and the subject of a major case study in eutrophication control (van Straten et al, 1979; van Straten and Somlyódy, 1980; Somlyódy, 1982). As indicated in Figure 1, the Zala watershed (with an area of 2622 km²) occupies over half of the total lake watershed (4522 km²). According to the estimates of Jolankai and Somlyódy (1981) an average load of about 225 kg day⁻¹ total phosphorus (TP), of which some 104 kg day⁻¹ is orthophosphate-P and 128 kg day⁻¹ available-P, enters the western end of the lake from the Zala discharge. This makes the Zala River the single most important contributor of nutrients to the lake; in addition, for example, it contributes an average load of approximately 2500 kg day⁻¹ of total nitrogen (TN). The role of the nutrient loads delivered from the Zala watershed must clearly be considered of primary significance in the persistent longitudinal gradient in the observed state of eutrophication (see van Straten et al, 1979), with basin I in Figure 1 being the most polluted sector of the lake. Hughes' (1982) study of optimal allocations of investments for the control of eutrophication in Lake Balaton indicates that the installation of phosphorus removal facilities at the two major sewage treatment plants in the Zala catchment would tend to dominate the most effective control options. Approximately equally effective in terms of control would be the construction of a shallow reed pond, the Kis-Balaton (small Balaton), at the mouth of the Zala where formerly marshland existed before drainage in the last century (see van Straten et al, 1979; see also Figure 1).

The undoubted significance of the River Zala was therefore instrumental in the initiation of a daily sampling programme for monitoring the discharge of sediment and nutrient loads to the western end of Lake Balaton (Joó, 1980). These measurements were begun in July, 1975, at the Fenekpuszta site at the mouth of the Zala and subsequently, in January, 1977, at the Zalaapati site 25 km upstream from Fenekpuszta (Figure 1). The resulting record of observations provides a unique basis for assessing the magnitudes and variations of water-borne sediment and nutrient

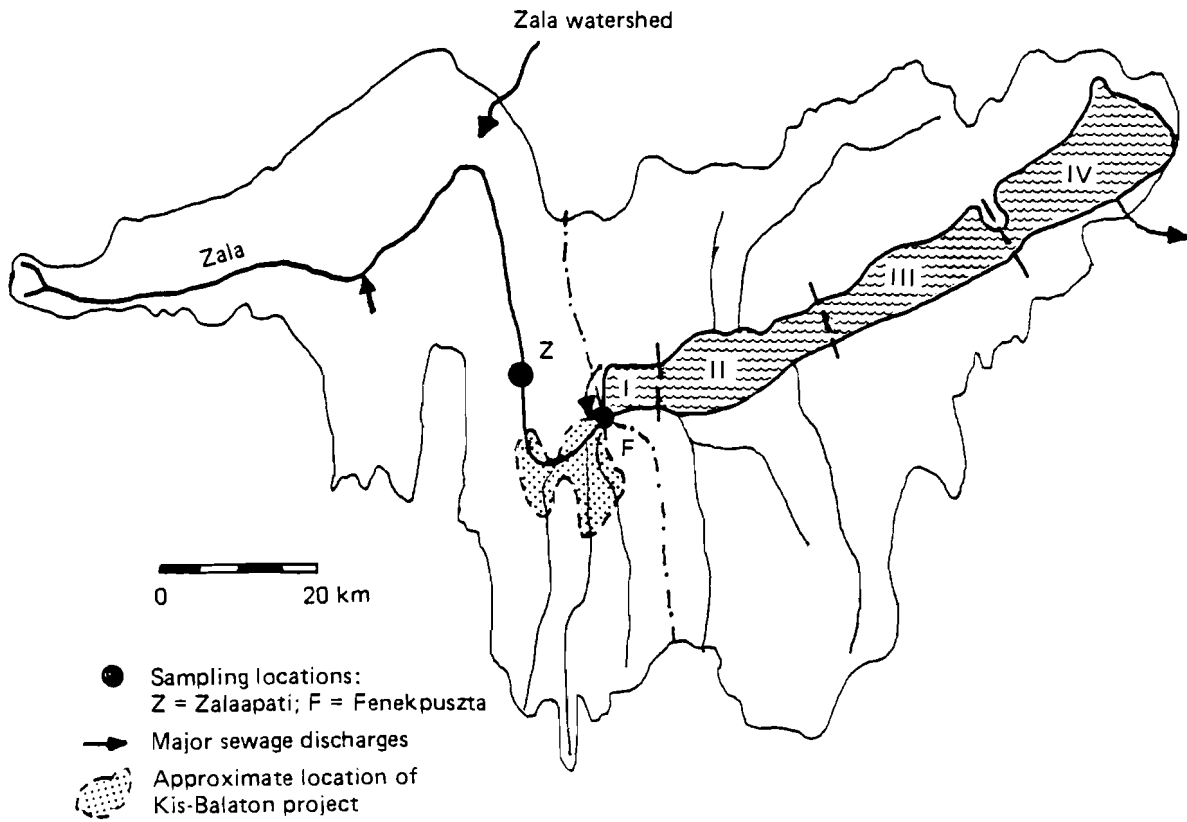


Figure 1. Map of Lake Balaton and its surrounding watershed, showing the location of the river Zala and the division of the Lake into four component basins (I-IV)

loadings from a predominantly agricultural watershed. The purpose of this study was therefore to use methods of time-series analysis--in particular, a recursive instrumental variable (IV) estimator (Young, 1974)--to determine the portions of the loadings deriving from point and nonpoint sources, the possible distribution of the phosphorus fractions among dissolved, particulate, available, and unavailable forms, and the dynamic relationships between these fractions (as outputs) and meteorological variations and other disturbances (as inputs) occurring in the watershed. The input/output models on which this study is based are essentially vehicles in the process of acquiring understanding. This does not debar the models from being applied subsequently for generating scenarios in the evaluation of management strategies, for example in generating input sequences to assess the performance of the Kis-Balaton project. However, understanding, albeit of a preliminary kind, is the key objective of this discussion.

The organization of the paper is as follows. Section 2 outlines the procedure of the analysis, without going into the details of the algorithm used for model identification and parameter estimation. Section 3 presents the main body of the results for the analysis of discharge, suspended solids (SS) load, total phosphorus (TP) load, and total nitrogen (TN) load variations for the year 1978 and for both the Zalaapati and Fenekpuszta locations. In section 4 the same models are used to estimate the corresponding load variations (at Fenekpuszta only) for the years 1975-1977; this can be considered in part as an exercise in model validation (or invalidation). There is also in section 4 a more extensive discussion of the various problems of seasonal factors affecting the in-stream phosphorus cycle, and the significance of marshland drainage and snowmelt in determining the patterns of TP load variations. Section 5 brings together the essence of what has been learned of the behaviour of the watershed as a source of nutrients delivered to Lake Balaton; it also suggests possible directions for further experimental work and for the use of the models in evaluating the performance of the Kis-Balaton project.

2. PROCEDURE

The procedure of the analysis is separated into four phases:

- (i) a preliminary correlation analysis;
- (ii) the identification of model structure;
- (iii) analysis of the trajectories of the recursive parameter estimates;
- (iv) evaluation of the given model structures and parameter estimates in the context of validation/invalidation.

The preliminary correlation analysis is intended to indicate broadly promising types of model structure to be assessed in more detail in the second phase of the procedure. This is conventional procedure, although here conducted with less sophistication than, for instance, in Box and Jenkins (1970). The important point, in fact with respect to all phases of the analysis, is that it is not well known a priori how the input disturbances, for example, precipitation, are related to the observed output responses of the SS, TP, and TN loadings at Fenekpuszta and Zalaapati.

The second phase of the procedure, identification of an appropriate model structure, is conducted for the following class of discrete-time multiple input/single output (MISO) models,

$$A(q^{-1})y(t_k) = \sum_{\ell=1}^m B_{\ell}(q^{-1})u_{\ell}(t_k) + \xi(t_k) \quad (1)$$

in which $u_{\ell}(t_k)$, $\ell = 1, 2, \dots, m$ and $y(t_k)$ are respectively observations of the multiple (m) system inputs and the system output at the k th sampling instant; $\xi(t_k)$ is a sequence of zero-mean, random errors or disturbances. q^{-1} is the backward shift operator, defined as

$$q^{-1}\{y(t_k)\} = y(t_{k-1}), \quad \text{etc.}, \quad (2)$$

$A(q^{-1})$ and $B_{\ell}(q^{-1})$ are polynomials of q^{-1} , of orders n and p_{ℓ} respectively, with parameters a_j and b_{ij} to be estimated,

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_n q^{-n} \quad , \quad (3a)$$

$$B_\ell(q^{-1}) = b_{\ell 0} + b_{\ell 1} q^{-1} + \dots + b_{\ell p_\ell} q^{-p_\ell}; \ell = 1, 2, \dots, m. \quad (3b)$$

Provided the estimator can operate reasonably effectively in the presence of high levels of noise ($\xi(t_k)$) which is usually the case for the IV estimator, further discussion or identification of the properties of ξ will not be of concern. Again, the important point is that the primary purpose of the analysis is to identify the structure of the relationships among the various combinations of input and output variables. Accuracy of the parameter estimates, but not necessarily their unbiasedness, is of secondary importance. The speeds and magnitudes of the responses in an output variable to changes in an input variable are the kind of macroscopic model properties of greater interest in this analysis.

A basic assumption for the estimation of the parameters a_i and b_{ij} in the $A(q^{-1})$ and $B_\ell(q^{-1})$ polynomials of equation (1) is that these parameters are constant. This assumption may be relaxed in order to analyze the trajectories of the recursive parameter estimates, as in the third phase of the procedure. Let us suppose that we have the following general "model" of the variations with time for the vector of parameters $\underline{\alpha}$, (containing elements a_j and b_{ij}),

$$\underline{\alpha}(t_k) = \underline{\alpha}(t_{k-1}) + \underline{\zeta}(t_{k-1}) \quad (4)$$

where $\underline{\zeta}(t_k)$ is a vector of zero-mean, white-noise sequences with covariance matrix

$$D = E\{\underline{\zeta}(t_k) \underline{\zeta}^T(t_k)\} \quad (5)$$

in which $E\{\cdot\}$ is the expectation operation and superscript T denotes the transpose of a vector or matrix. According to this model the parameters are assumed to vary in an unknown random walk fashion with $D = 0$ representing the special case in which the parameters are truly constant. Let us now suppose that a set of estimates $\hat{\underline{\alpha}}$ has been obtained for the identified model

structure, under the assumption that $D = 0$, and that the a posteriori covariance matrix of associated parameter estimation errors is given by P , i.e.

$$P = E\{\tilde{\alpha}\tilde{\alpha}^T\} \quad (6)$$

where $\tilde{\alpha}$ is a vector of estimation errors, and where P in this case can be approximately derived from the IV estimation algorithm (Young, 1974). A further pass through the set of observations $y(t_k)$, $u(t_k)$, for $k = 1, 2, \dots, N$, could be made for which the a priori parameter estimates are chosen as $\hat{\alpha}$ and where D is specified as being proportional to P in some way. Those parameters that are poorly estimated and thus associated with the larger elements of P --in part, possibly because the previous assumption of constant parameters is invalid--would have a higher probability of having estimates for this type of analysis that exhibit considerable variation with time. The opposite would apply to those parameters in the model that are well estimated, under the assumption that the intensity of variations in the parameter estimates are likely to be proportional to the magnitudes of the elements in D . Operation of the estimation algorithm in this manner is tantamount to an evaluation of the appropriateness of the identified model structure. Significant variations in the recursive parameter estimates can be indicative both of the failure, or inadequacy, of the model structure and of possible modifications that might be made to improve it, especially if the parameter variations are apparently correlated with the variations in some of the observed variables (for further discussion of the use of recursive estimation algorithms for this purpose, see, for instance, Young, 1978; Beck, 1979, 1982).

The fourth phase of the analysis is the more familiar matter of validation. The principal aspect of such analysis in the present case is to establish discrepancies between the behaviour of the Zala watershed as identified for 1978 and the observed behaviour for 1975-1977. In other words we shall make the confident assumption that behaviour over 1975-1978 is entirely consistent in order to emphasize, and hence to interpret, the more obvious inconsistencies.

3. ANALYSIS FOR 1978

The reasons for choosing to begin the analysis with the year 1978 are twofold. First, there are complete records at both the Fenekpuszta and Zalaapati sites for the years 1977 and 1978 only (observations for subsequent years have been made, but the data were not available at the time of starting this study). Second, of these two years the records for 1977 suggested initially an apparently irregular behaviour in the observed nutrient loadings during the winter of that year. Beginning the analysis with 1978 was therefore thought to be somewhat simpler and less likely to lead to ambiguous results.

3.1 Correlation Analysis

Figure 2 shows the cross-correlation functions between precipitation (P), as input, and the stream discharge (Q), SS, TP, and TN loadings at Fenekpuszta and Zalaapati, as outputs. Figure 3 shows the cross-correlation functions between the upstream (input) and downstream (output) discharges and loadings. The significantly distinguishing features of these functions are few. Treating the correlation functions as approximate impulse responses, however, it is notable that the downstream discharge at Fenekpuszta, in response to a precipitation event, is more attenuated and more widely distributed than the upstream discharge, as would be expected (Figure 2(a)). The Fenekpuszta SS load response to precipitation is apparently delayed by one day relative to the Zalaapati load (Figure 2(b)). The upstream and downstream TP load response to precipitation are notably simultaneous (Figure 2(c); also Figure 3(c)) and only the SS load exhibits any strong tendency towards the existence of an identifiable stream transport delay between Zalaapati and Fenekpuszta (Figure 3).

Most of the conclusions to be drawn from this analysis are of a weakly negative character. The reasonably high correlations between precipitation and the outputs of Figure 2 are superficially promising for the model identification and parameter estimation analysis that follows. The difficulty, however, is that the results of Figures 2 and 3 are very probably dominated by only

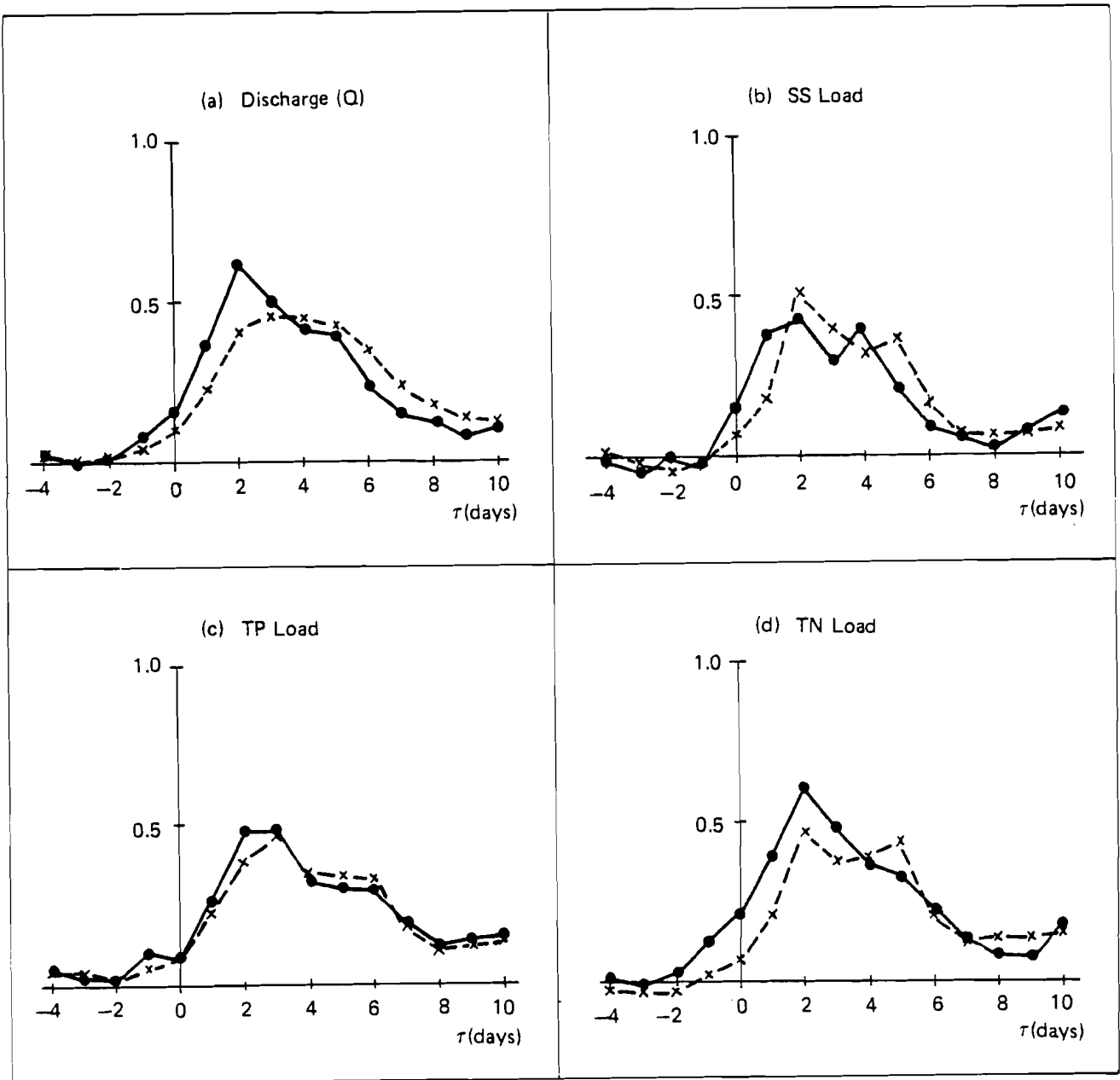


Figure 2. Cross-correlation functions between precipitation as the input time-series and (a) stream discharge (Q), (b) SS load, (c) TP load, and (d) TN load as output time series. The continuous lines represent the respective outputs at Zalaapati; the dashed lines represent the respective outputs at Fenekpuszta

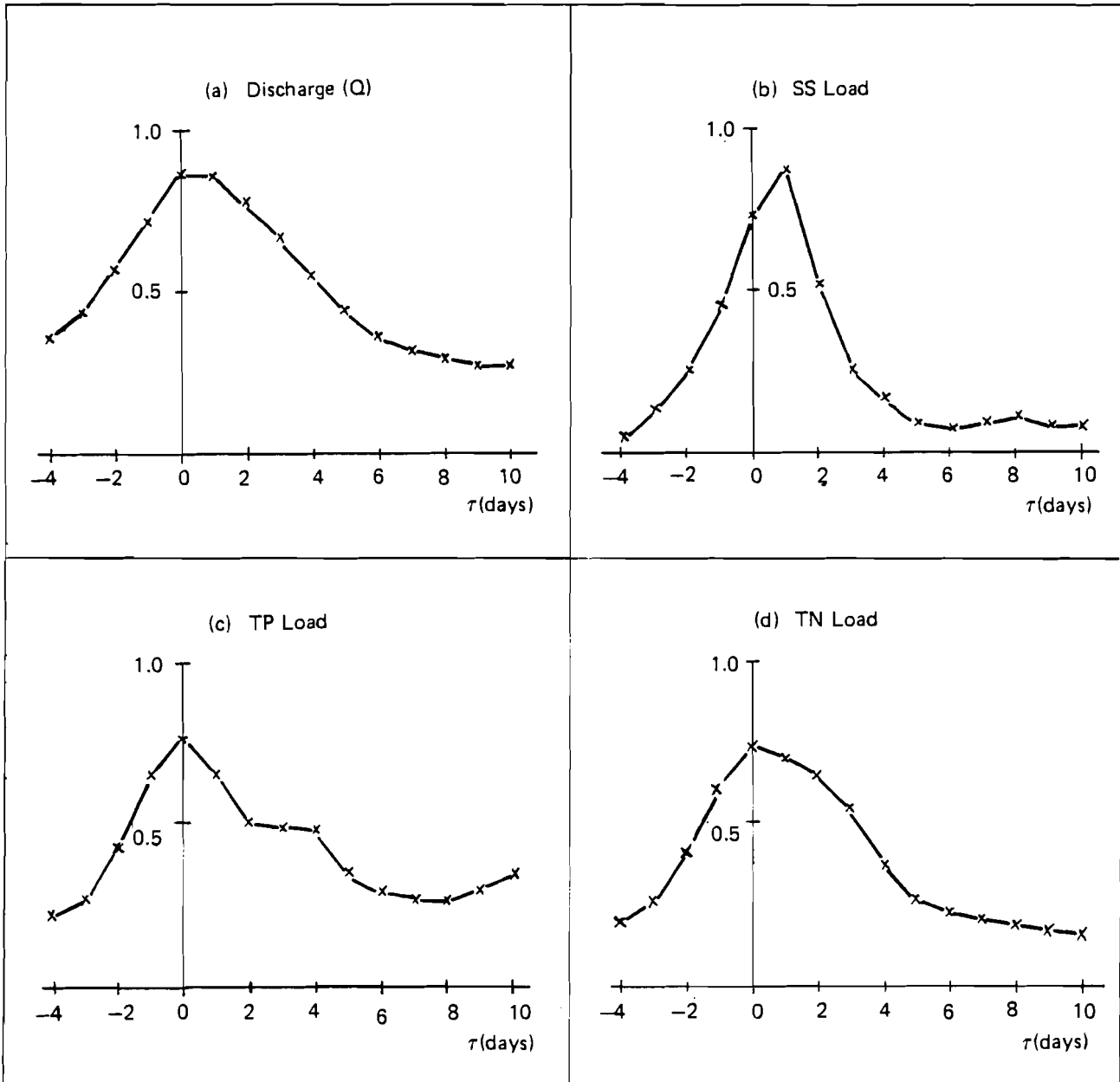


Figure 3. Cross-correlation functions between upstream (Zalaapati) input observations and downstream (Fenekpuszta) output observations for (a) stream discharge (Q), (b) SS load, (c) TP load, and (d) TN load

two, and at most four, major flood events, which, although they are clearly of critical importance to the problem at hand, are likely to distort the performance of any estimation algorithm. Second, the closely similar characteristics of the relationships between precipitation and loadings at the two spatial locations precludes the possibility of examining models for the relationships between precipitation and the net increase in loadings between Zalaapati and Fenekpuszta. The difference in the watershed areas at these two points is quite significant (about 1000 km²) and one would have expected nonpoint source contributions to be equally significant and identifiable. However, differencing the load time-series for the two locations, especially for TP, merely has the common effect of amplifying the apparently stochastic component of the resulting sequences. Third, the velocity of the stream discharge during the important flood events can be easily greater than 0.5 ms⁻¹, in which case the travel-time between the two locations is less than one half of the sampling interval of the observations (one day). This too has the effect of obscuring any possible distinctions between the nature of the load variations at the two points.

3.2 Model Structure Identification

The results from the analysis of model structure identification are summarized in Tables 1 and 2, these being the "better" model structures for each category indicated. The following notational conventions apply. The model denoted pf.1 in Table 1, for example, refers to a model of total phosphorus load variations at the Fenekpuszta location and would be written according to equation (1) as,

$$y(t_k) = a_1 y(t_{k-1}) + b_{11} u_1(t_{k-1}) + b_{12} u_1(t_{k-2}) + b_{13} u_1(t_{k-3}) + b_{20} u_2(t_k) + b_{40} \quad , \quad (7)$$

where u_1 is the observed precipitation input (in mm day⁻¹), u_2 is the observed SS load at Fenekpuszta (in kg day⁻¹), and b_{40} represents, in effect, a constant "base" load of total phosphorus (in kg day⁻¹). The b_{1j} coefficients therefore have units of [kg day⁻¹][mm day⁻¹]⁻¹, and the b_{2j} coefficients [kg TP][kg SS]⁻¹. u_1 is a mean areal value, based on observations at five locations

Table 1. Summary of parameter estimates and error statistics for model structure identification (1978 data)

Model Code*	Auto-regressive term a ₁	Precipitation input (u ₁) [†] coefficients					SS load input(u ₂) coefficients		Respective upstream load input (u ₃)		Bias term b ₄₀	Standard deviation of errors σ	Coefficient of determination R _T ²
		b ₁₁	b ₁₂	b ₁₃	b ₁₄	b ₁₅	b ₂₀	b ₂₁	b ₃₀	b ₃₁			
<u>Discharge:</u>													
qf.1	-0.907	0.078	0.153	0.023								2.862	0.78
qf.2	-0.811	0.067	0.152	0.040						0.73		2.274	0.89
qz.1	-0.747	0.118	0.228	-0.065						0.53		1.685	0.88
<u>SS load:</u>													
sf.1	-0.079		7091.0	2864.0	1623.0	3709.0						57.85(10 ³)	0.43
sf.2	-0.105		2586.0					0.232	0.433			28.26(10 ³)	0.86
sz.1	-0.155	5275.0	5587.0	586.0	5932.0							78.40(10 ³)	0.37
<u>TP load:</u>													
pf.1	-0.524	4.78	0.07	3.92			0.00102				68.9	102.0	0.88
pf.2	-0.730	4.95	7.05	5.64							30.6	142.0	0.76
pf.3	-0.602	4.55	3.19	2.95					0.245			122.0	0.79
pz.1	-0.512	8.28	12.76				0.00097				42.3	170.0	0.76
pz.2	-0.678	8.79	17.13	4.67							15.8	202.0	0.66
<u>TN load:</u>													
nf.1				35.9	56.3	78.6	0.01444				1485.0	1179.0	0.82
nf.2	-0.234	26.2	143.8	39.6	64.0	111.3					956.0	1378.0	0.75
nf.3	-0.712	11.0	89.6	-61.0					0.300			1128.0	0.83
nz.1	-0.545	42.2	106.7				0.00395				389.0	911.0	0.84
nz.2	-0.692	64.7	125.7	-23.3							218.0	1057.0	0.78

* q denotes discharge (in m³ s⁻¹); s denotes SS load (in kg day⁻¹); p denotes TP load (in kg day⁻¹); n denotes TN load (in kg day⁻¹); f denotes Fenekpuszta location; z denotes Zalaapati location.

[†] precipitation is measured in mm day⁻¹.

Table 2. Estimated average rates of daily loadings from different model source terms for the year 1978; all loads given in kg day⁻¹.

Model Code	Average load from precipitation (u ₁)	Average load from SS (u ₂)	Average load from upstream (u ₃)	Average bias load (base) (b ₄₀)	Average total load
<u>SS load:</u>					
sf.1	27.7(10 ³)	-	-	-	27.7(10 ³)*
sf.2	4.8(10 ³)	-	18.3(10 ³)	-	23.1(10 ³)
sz.1	34.3(10 ³)	-	-	-	34.3(10 ³)*
<u>TP load:</u>					
pf.1	30.8	46.0	-	144.7	221.5
pf.2	109.1	-	-	113.3	222.4
pf.3	44.9	-	127.7	-	172.6*
pz.1	72.0	48.8	-	86.7	207.5
pz.2	158.7	-	-	49.1	207.8
<u>TN load:</u>					
nf.1	285.0	310.0	-	1485.0	2080.0
nf.2	839.0	-	-	1248.0	2087.0
nf.3	230.0	-	1680.0	-	2010.0
nz.1	546.0	213.0	-	855.0	1614.0
nz.2	906.0	-	-	708.0	1614.0

* These estimated total loads differ substantially from the observed values (see Table 3).

across the watershed; where an SS load input (u_2) is indicated in Table 1, this refers to the SS load at the same location as the corresponding output (TP or TN load). The respective upstream load input (u_3) refers to the SS load (or TP or TN load) at Zalaapati in a model for the corresponding output load at Fenekpuszta. The units of the b_{3j} coefficients, like the a_j coefficients, are therefore dimensionless. The coefficient of determination (or total correlation function) indicated in Table 1 as R_T^2 is defined as

$$R_T^2 = 1 - \frac{\left[\sum_{k=1}^N \varepsilon^2(t_k) \right]}{\left[\sum_{k=1}^N Y^2(t_k) \right]} \quad , \quad (8)$$

where $\varepsilon(t_k)$ is the deterministic model response error defined by

$$\varepsilon(t_k) = y(t_k) - \hat{x}(t_k) \quad , \quad (9)$$

with \hat{x} , the deterministic model response, given by

$$\hat{A}(q^{-1}) \hat{x}(t_k) = \sum_{\ell=1}^m \hat{B}_{\ell}(q^{-1}) u_{\ell}(t_k); \quad \hat{x}(t_0) = y(t_0) \quad . \quad (10)$$

In table 2 the estimated average rates of daily loadings from the various input (source) terms are computed from

$$\bar{Y}_{\ell} = \kappa_{\ell} \bar{u}_{\ell} \quad , \quad (11)$$

in which \bar{u}_{ℓ} is the average observed value of input ℓ for 1978 and κ_{ℓ} is the corresponding steady-state gain constant for input ℓ derived from the model as

$$\kappa_{\ell} = (b_{\ell 0} + b_{\ell 1} + \dots + b_{\ell p_{\ell}}) / (1 + a_1) \quad . \quad (12)$$

In the case of the base load, \bar{Y}_4 is simply

$$\bar{Y}_4 = b_{40} / (1 + a_1) \quad . \quad (13)$$

It is important to clarify at the outset what possible interpretations can be attached to these four separate input-source terms (u_1, u_2, u_3, b_{40}). It is easiest to assign a conceptual interpretation to the last of these terms, namely the bias, or base load,

b_{40} : let us suppose that it reflects the load carried by the stream under conditions of zero precipitation and zero SS load. One would expect this load in practice to be dominated both by nutrients derived from point-source sewage discharges and by non-particulate forms of nutrients. The upstream load (u_3) is perhaps most usefully interpreted by its complement: all loads estimated other than those deriving from u_3 are probably dominated by lateral inflows to the stream (tributaries, surface runoff) that reflect loads not transported downstream from Zalaapati to Fenekpuszta. The SS load (u_2) refers to a load (conceptually) dominated by particulate forms of nutrients. The precipitation-induced load (u_1), when estimated jointly with an SS load (i.e. in models pf.1, pz.1, nf.1, and nz.1), is assumed to be indicative of a nutrient load associated with surface runoff that is essentially not dominated by particulate forms of nutrients. Otherwise, this source term reflects loads that are either not the base load or not a transported load, depending upon the particular model.

The main point to be borne in mind during the following discussion is that although the analysis is cast within a framework of the relatively microscopic (short-term) dynamic behaviour of the watershed, the key objective is to reconstruct an estimate of relatively macroscopic nutrient loading characteristics. The objective is to infer the relative distributions of average loadings between point and nonpoint sources and between available and unavailable forms. However, these latter distributions obviously cover intersections and unions among the sets of source terms defined above. This complication in turn will be exacerbated during the estimation procedure by a lack of complete statistical independence among the input variables (for example, precipitation (u_1) is undoubtedly correlated with both the SS load (u_2) and the upstream load (u_3)). Clear distinctions in the results for the primary objective of the analysis are therefore not expected. Insights into the microscopic dynamic behaviour of the Zala watershed will be regarded as possible additional dividends of the analysis.

Overall Comments

Figures 4, 5, 6, and 7 show the observed precipitation sequence (u_1) for 1978 and the results of the discharge models (qz.1 and qf.2), two SS load models (sz.1 and sf.1), two TP load models (pz.1 and pf.1) and two TN load models (nz.1 and nf.1) compared with their respective observations. In all of these figures the deterministic model response (\hat{x} from equation (10)) is shown.

Overall the pattern of discharge and load variations during the year are characterized by four major ("conventional") precipitation-runoff events ((1) at $t_{105} \rightarrow t_{110}$; (2) $t_{140} \rightarrow t_{155}$; (3) $t_{160} \rightarrow t_{170}$; and (4) $t_{185} \rightarrow t_{210}$) and by three (or four) minor such events ((5) at $t_{85} \rightarrow t_{90}$; (6) $t_{230} \rightarrow t_{235}$, which is a borderline case; (7) $t_{275} \rightarrow t_{280}$; and (8) $t_{330} \rightarrow t_{335}$). The major events occur during the period April to July and they are all notably characterized by what may be observed in Figure 4(a) to be, in fact, two closely consecutive, but separate, precipitation events. The borderline minor event (6) occurs during August and is a typical summer event where virtually all the precipitation would be either "lost" in evapotranspiration processes or taken up by reducing the soil moisture deficit after a relatively dry period. One major ("unconventional") runoff event, most probably due to the effects of snowmelt, occurs over the period $t_{45} \rightarrow t_{65}$ (February, March). The event is revealed by an excess of observed discharge, TP, and TN loads (but not SS load) over the values estimated by the models and is particularly evident at the Fenekpuszta site.

The patterns of all four output variables (Q, SS, TP, and TN loads) at the Zalaapati station exhibit a more obviously constant base load characteristic, yet higher and narrower responses to the major events than the corresponding patterns at Fenekpuszta. The observed cumulative loads for the two stations (expressed in Table 3 as daily average loads) indicate the notable point that the SS load at the downstream location is less than at the upstream location. This again suggests the greater importance of in-stream mechanisms as factors having a more dominant effect on the observed SS loading variations than, for example, erosion processes in the watershed. The most

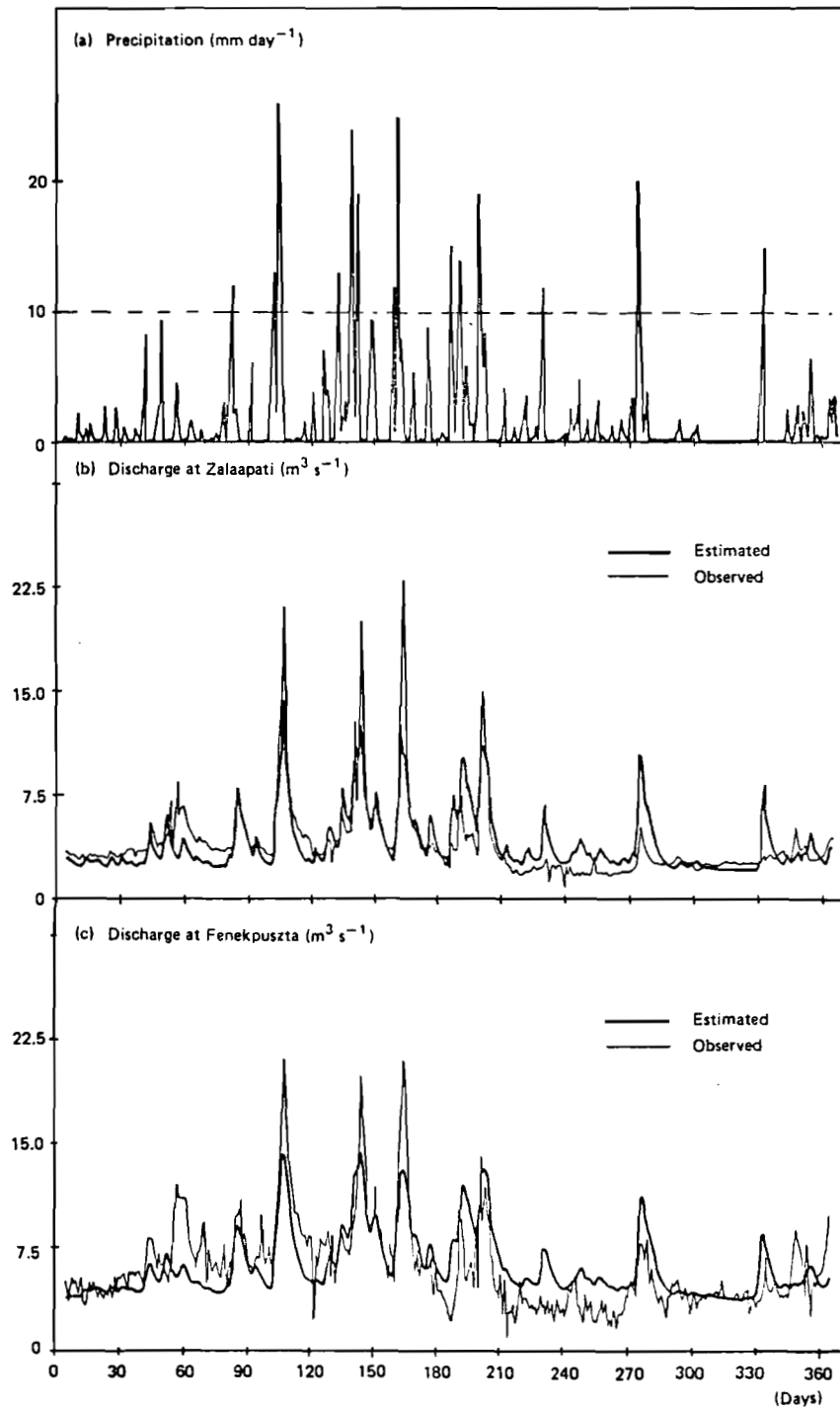


Figure 4. (a) Observed precipitation sequence (u_1) for 1978; observed and estimated stream discharge at (b) Zalaapati and (c) Fenekpuszta

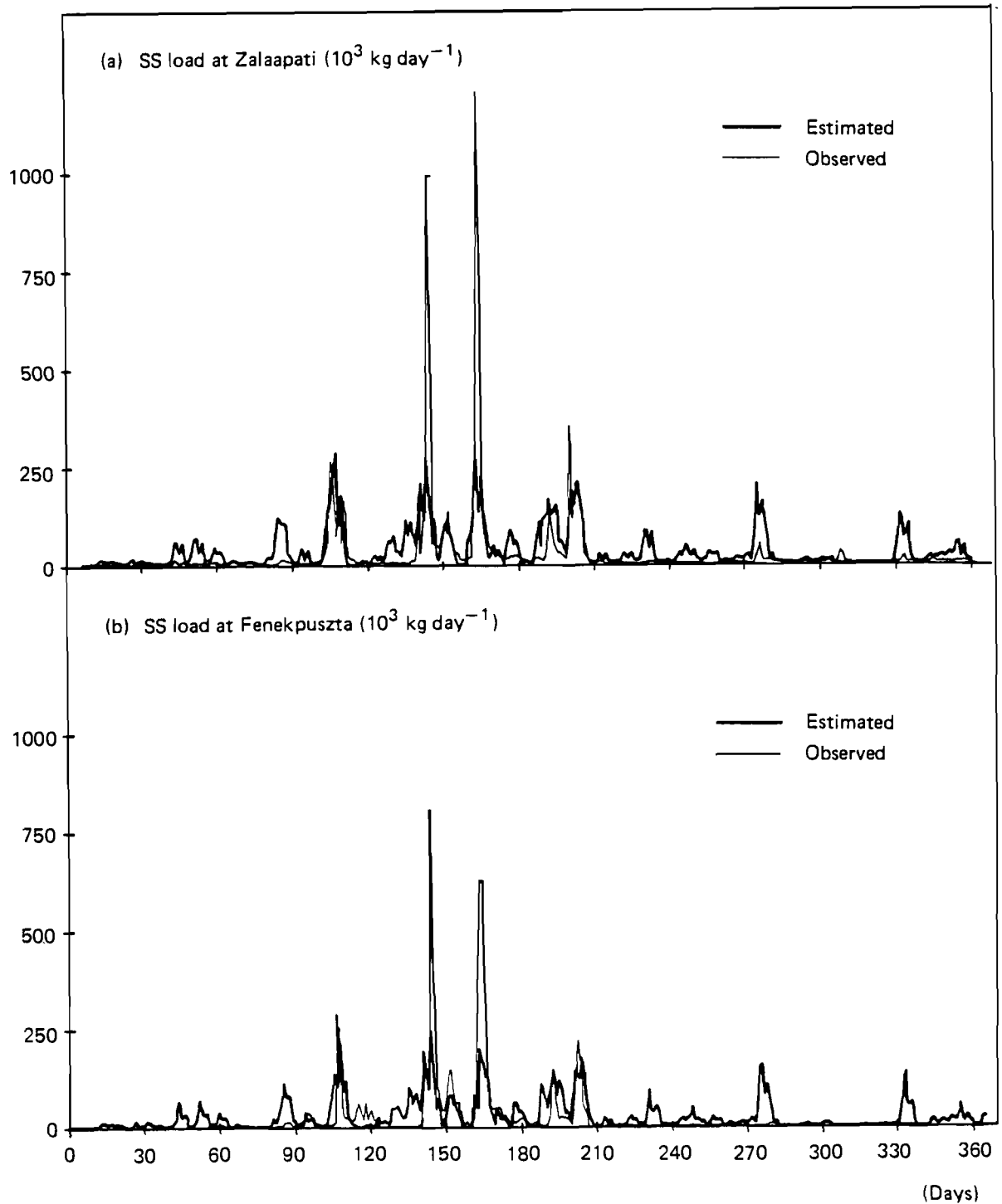


Figure 5. Observed and estimated SS load at (a) Zalaapati and (b) Fenekpuszta

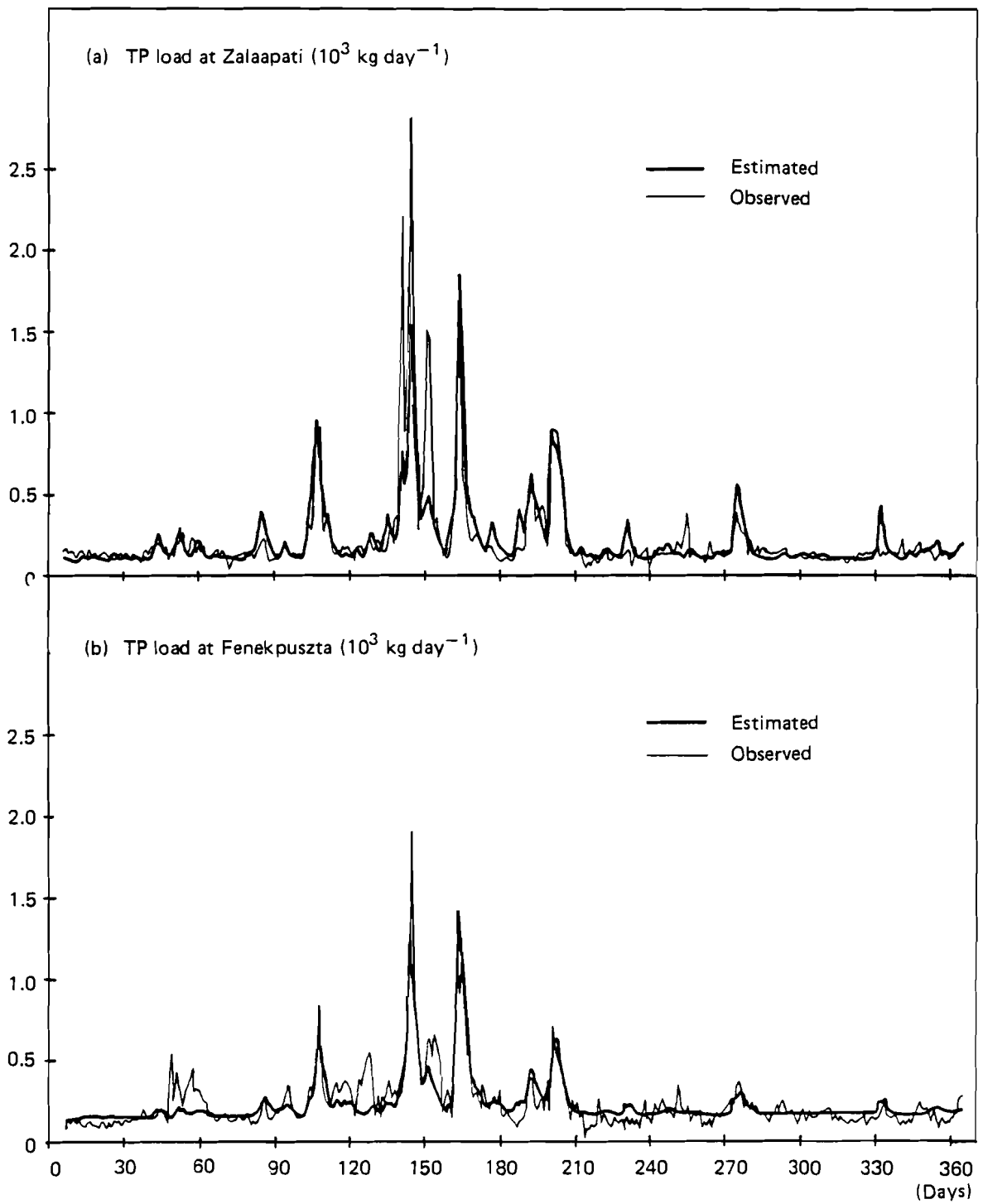


Figure 6. Observed and estimated TP load at (a) Zalaapati and (b) Fenekpuszta

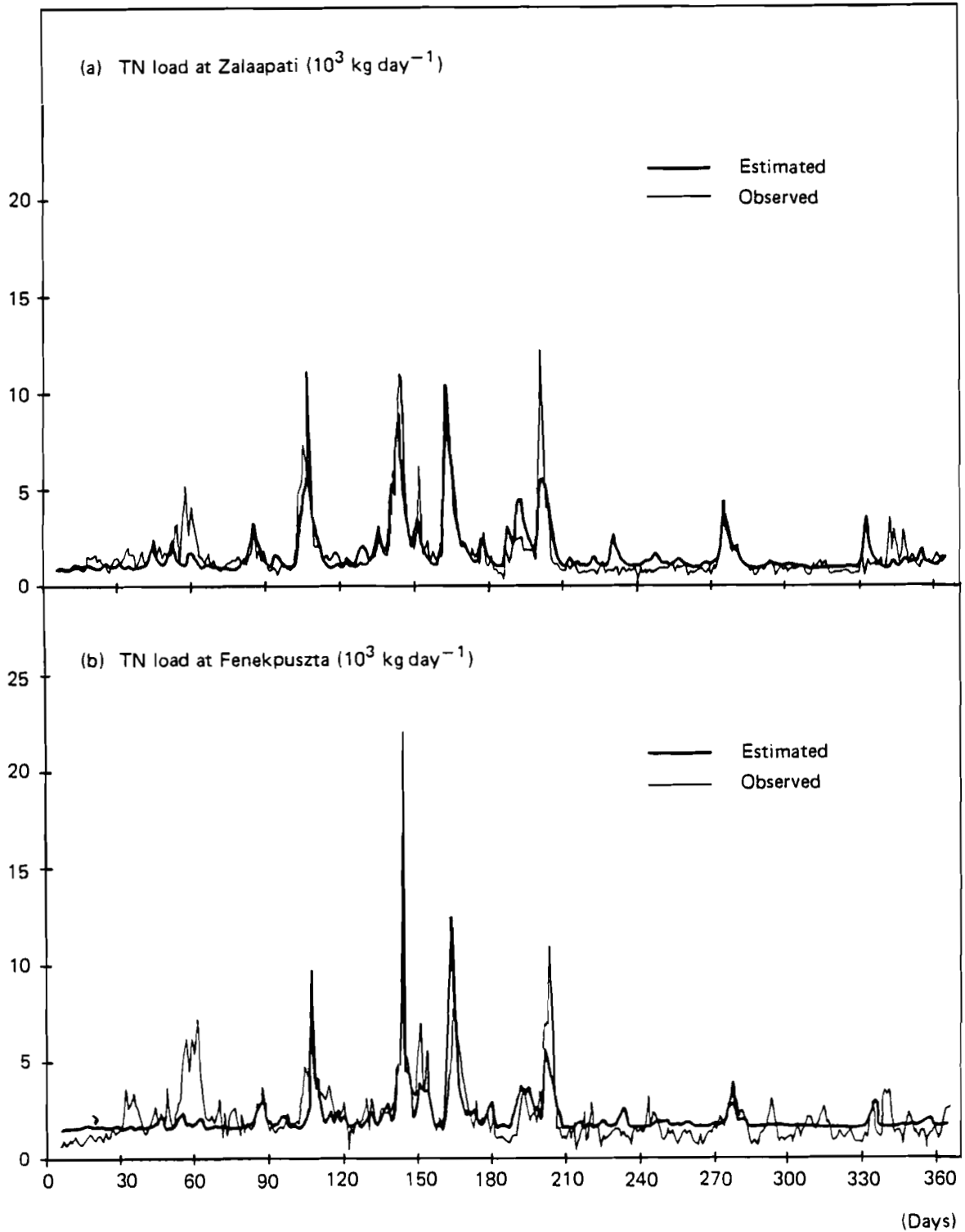


Figure 7. Observed and estimated TN load at (a) Zalaapati and (b) Fenekpuszta

Table 3. Observed average loading rates (in kg day⁻¹) for the years 1975-1978 (1978 only at Zalaapati)

Load and Location	1975	1976	1977	1978
<u>SS load:</u>				
Zalaapati	-	-	-	24.6 (± 96.5) 10 ³
Fenekpuszta	46.5 (± 158.5) 10 ³	27.3 (± 99.4) 10 ³	21.5 (± 52.9) 10 ³	21.5 (± 73.8) 10 ³
<u>TP load:</u>				
Zalaapati	-	-	-	207.4 (± 275.8)
Fenekpuszta	617.7 (± 1138.0)	150.2 (± 237.8)	215.2 (± 201.0)	220.8 (± 186.3)
<u>TN load:</u>				
Zalaapati	-	-	-	1613.0 (± 1600.0)
Fenekpuszta	1885.0 (± 2485.0)	1796.0 (± 2072.0)	2369.0 (± 2328.0)	2076.0 (± 1800.0)

reasonable hypotheses for the lower load at Fenekpuszta are that stream velocity and bed-scouring (or deposition) characteristics, or choice of sampling point in the vertical plane, or the degradation and dissolution of particulate organic matter, are the causes of this effect. The results from the correlation analysis have already been interpreted as being likewise suggestive. The particularly strong distinction between the almost negligible base load and very high flood event responses for the SS loads at both Zalaapati and Fenekpuszta prompts the speculation that the SS load is principally a function of stream-bed erosion operating above a threshold-like value for the stream velocity. The snowmelt event and the minor event(5), at $t_{85} \rightarrow t_{90}$, both led to peak stream discharge responses surpassed only by the four major events, yet neither were associated with any significant SS load response. Threshold values below which erosion in the stream is negligible might therefore be approximately $12 \text{ m}^3\text{s}^{-1}$ for the discharge at Fenekpuszta and $8-9 \text{ m}^3\text{s}^{-1}$ for the discharge at Zalaapati. The corresponding observed stream velocities were about 1.6 ms^{-1} at Fenekpuszta and 0.75 ms^{-1} at Zalaapati.

Given the overriding concern with phosphorus as the rate-limiting nutrient for phytoplankton growth in the case of Lake Balaton (van Straten and Somlyódy, 1980; Jolankai and Somlyódy, 1981), we shall now focus on an interpretation for the Zala TP load variations using interpretations of the SS and TN loads as supporting evidence. This does not mean that the latter are not of interest in their own right, but it does reflect the principal purpose of this analysis as part of the case study as a whole. Our comments are separated according to the four categories of TP loads defined previously.

The Base Load

According to the estimates of Table 2 the base TP load forms about 50% (or more) of the yearly average total TP load at Fenekpuszta and between about 25% and 40% of the total load at Zalaapati. Two questions are important with regard to these estimates:

- (i) do they concur with independent estimates of the loads derived from point-source sewage discharges;
- (ii) can they be said to be largely non-particulate fractions of the total load?

In response to the first question, Jolankai and Somlyódy (1981) give estimates of the sewage loads at Fenekpuszta and Zalaapati respectively as $130.8 \text{ kg day}^{-1}$ (cf. 113.3 or $144.7 \text{ kg day}^{-1}$ from Table 2) and 87.4 kg day^{-1} (c.f. 49.1 or 86.7 kg day^{-1}). The similarities among these estimates, except for the estimate from model pz.2 at Zalaapati, tend toward the conclusion that the base TP loads are predominantly composed of material derived from point-source sewage discharges. Inspection of the results from models pf.1 and pz.1 (Figure 6) and models pf.2 and pz.2 (not shown) indicates that the estimates of $113.3 \text{ kg day}^{-1}$ and 86.7 kg day^{-1} would be the preferred values for the base loads at Fenekpuszta and Zalaapati respectively. For comparison, the corresponding TN base loads (also drawn from Jolankai and Somlyody, 1981) are 841 kg day^{-1} (c.f. 1248 or 1485 kg day^{-1} from Table 2) and 483 (c.f. 708 or 855 kg day^{-1}) at Fenekpuszta and Zalaapati respectively. A significant portion of the TN base load, unlike the TP base load, is thus derived presumably from sources other than sewage discharges. Some of these estimates for the base load characteristics, however, will need to be reappraised in the light of the subsequent analysis for the years 1975-77 (section 4).

An important part of the response to the second of the above questions is the assumption that all particulate phosphorus is observed as part of the SS load and that the ratio of particulate phosphorus to suspended particulate matter is constant. Under this assumption, and given that the observed dry-weather SS load is $5.0(10^3) \text{ kg day}^{-1}$, the results of models pf.1 and pz.1 (in Table 1) indicate that 10 kg day^{-1} of the base TP load could be expected to be of particulate form. The steady-state gain constants (κ_l) for these models define the yield of TP per kg SS to be about 0.002 kg (both at Zalaapati and Fenekpuszta). Such a dry-weather "particulate" TP load is distinct from, and not included in, the base TP load estimate according to our scheme of conceptual differentiation among the components of the overall TP load. This does not imply, however, that there is no distortion resulting from the overlap between the estimates of these

two component base loads that is introduced through the inevitable inadequacies of the estimation procedure. The significance of the relatively low dry-weather particulate TP load is therefore that it supports the probability that the base load as estimated is in fact very largely composed of non-particulate forms of phosphorus.

The Particulate Load

The average TP loads deriving from SS matter are estimated to be about 21% and 24% of the total TP loads at Fenekpuszta and Zalaapati respectively, a perhaps somewhat lower percentage than might have been expected (the corresponding figures for TN are also small, being 15% and 13% respectively, see Table 2). In this case the question to be asked is whether the estimate of the TP load deriving from SS matter ought to be higher. In trying to speculate that the particulate TP load is higher than estimated, we shall in fact be forced to conclude that it is probably as low as suggested here.

Table 4 summarizes an analysis of the aggregate loads observed during the four major flood events (events (1)-(4) as defined above). It is clear that the four major events dominate the total SS load for the whole year and it could thus be asserted that whatever TP load is derived from the SS load it is contributed primarily during these events alone. The model pf.1 estimates the aggregate TP load for the four events as $26.9(10^3)$ kg, which is notably close to the observed value of $26.7(10^3)$ kg; this total aggregate value, according to the model, breaks down into the components of $13.1(10^3)$ kg derived from the SS load, $8.0(10^3)$ kg as a base load, and $5.8(10^3)$ as precipitation-induced non-particulate TP load (as defined previously). The contribution of the base load, even during storm events, is still quite significant.

Let us speculate that the close correspondence between the observed "losses" in the TP and SS (but not TN) loads between Zalaapati and Fenekpuszta (see Table 4) is evidence of a strong relationship between TP and SS load characteristics during flood conditions. If the precipitation induced, non-particulate TP

Table 4. Observed estimates of loads derived from the four major floods, events (1)-(4), in 1978

Load	Zalaapati		Fenekpuszta		Net gain in load between Zalaapati and Fenekpuszta	
	Aggregate load for 4 events (kg)	% of year's total	Aggregate load for 4 events (kg)	% of year's total	(kg)	% of load at Zalaapati
SS	7.63(10 ⁶)	85	6.11(10 ⁶)	78	-1.52(10 ⁶)	-20
TP	34.7(10 ³)	46	26.7(10 ³)	33	-8.0(10 ³)	-23
TN	221.4(10 ⁶)	38	236.8(10 ³)	31	+15.4(10 ³)	+7

load were a false artefact of the estimation procedure and if, therefore, all the non-base TP load for the four major flood events were due solely to the SS load, then κ_2 for the Zalaapati and Fenekpuszta locations would be respectively 0.0025 and 0.003, as opposed to 0.002, kg TP per kg SS. The respective increases in the average TP load deriving from SS would be up to 30% (at Zalaapati) and 31% (at Fenekpuszta) of the total load. If the difference in κ_2 at the locations is significant--and the preferred lower base load estimate at Fenekpuszta, which has not been used in these calculations, would increase the difference--why does it occur? Furthermore, if the dynamic patterns of the SS and TP load responses to precipitation were truly proportional, which is what they should be when superimposed on their respective constant base loads, their time constants (< 0.5 days for all the SS models and > 1.5 days for all the TP models) ought to be more consistent. The lack of simultaneity between the SS and TP load variations is further substantiated by the identified structure of model pz.1, for which the TP load at Zalaapati at time t_k is found to be significantly related to the SS load at Zalaapati at time t_{k-1} , i.e. the previous day. In fact, the peak TP loads that precede and follow the main peak response at Zalaapati during event (2), from t_{140} to t_{155} , are striking discrepancies among the SS and TP load responses in general (Figures 5 and 6). These two peaks are not significantly reflected elsewhere and almost certainly their "absence" from the record at Fenekpuszta would account for the apparent loss of TP load between the two locations. The observed aggregate TP load of the first and

third peaks of event (2) at Zalaapati, less the estimated associated base load for the duration of these peak responses, amounts to $7.0(10^3)$ kg. This, when subtracted, reduces the aggregate load for the four flood events to $27.7(10^3)$ kg at Zalaapati, an estimate close to that of the corresponding downstream load at Fenekpuszta. Thus, assuming that sampling error is not the principal cause of the divergent observations for event (2), the mismatch between the observed SS and TP load patterns points towards a conclusion quite the opposite of that speculated at the introduction of this discussion. The losses in the TP load between Zalaapati and Fenekpuszta during flood conditions do not correspond with the losses in the SS load; these losses in the TP load, if anything, are more probably due to losses in the non-particulate fraction of the overall load.

Additional evidence refuting the speculation that the particulate TP load ought to be larger than estimated can be found in the following argument. The model structures of pf.1 and pz.1 in Table 1 are not well posed for the purposes of identification and estimation. For example, one of the inputs (u_1) is likely to be causally related to both the output (y) and the second input (u_2), and the input (u_2) and output are both functions of the same variable of stream discharge (Q). These violations of good statistical principles, albeit borne of necessity, can nevertheless be turned to some sort of advantage. If the runoff-generated TP load and SS loads were fully proportional, such highly correlated patterns between input and output could be expected to dominate the identified model structure to the virtual exclusion of any identifiable "residual" relationship between input precipitation (u_1) and the output TP load. Since this does not occur, and a significant relationship between u_1 and y persists, we are again led towards the opposite of the conclusion that the TP loads deriving from SS material should be higher than as estimated in Table 2. The more reasonable conclusion is that during flood events a significant portion of the non-base TP load is not derived from particulate matter.

In passing, we may note that the analysis of the relationship between the SS and TP loads reveals another aspect of particular interest. The observation that the Zalaapati discharge during the first of the three TP load peaks for event (2) reached a value of $12.6 \text{ m}^3 \text{ s}^{-1}$, with no significant simultaneous peak SS load being observed, suggests that the previously quoted threshold value for the discharge below which in-stream bed erosion does not occur (i.e. $8-9 \text{ m}^3 \text{ s}^{-1}$) should be revised upwards. This, and the coincidence between the threshold discharge values at both locations (about $12 \text{ m}^3 \text{ s}^{-1}$), fortuitous or not, tend to support the conclusion that in-stream processes, as opposed to watershed processes, are the primary determinants in the observed SS load responses. In this respect Joó (1980) notes that the part of the Zala watershed upstream of Zalaapati is subject to a higher degree of soil erosion than the remaining part of the watershed. In other words, one would expect that the major portion of the particulate material entering the stream from the local land surface enters upstream of the Zalaapati site.

The results of model sf.2 (Tables 1 and 2), which according to the R_T^2 statistic are superior to the other SS-load models, give further indications of the dominant role of transport (in-stream) processes. κ_3 for sf.2 is 0.75--in effect, 75% of the SS-load variations at Zalaapati are reflected in the SS load variations at Fenekpuszta--and only about 20% of the average downstream SS load can be expected to originate from sources between the two measurement locations. The significance of both the b_{30} and b_{31} coefficients (in Table 1) probably covers a mixture of in-stream SS transport processes between the two points and "simultaneous" erosion precipitated by the stream discharges at both points rising simultaneously past their respective threshold levels (the timing of the discharge responses at the two points are roughly concurrent, see Figure (3a) and the b_{1j} coefficients in Table 1).

The Precipitation-induced Non-particulate Load

The third component of the TP load, the precipitation-induced TP load not deriving from the SS load, is certainly not negligible, being about 14% and 35% of the total loads at Fenekpuszta and

Zalaapati respectively. The question is, from what type of source does such a load originate.

It is easy to postulate that if this component load derives primarily from urban surface runoff channeled partly through sewage treatment plants, it should give rise to a relatively fast but short-duration response in the in-stream TP load. A companion hypothesis is that non-particulate agricultural runoff has a slower, longer, and more distributed response in time. The urban-runoff hypothesis implies also the strong assumption that such runoff contains only non-particulate phosphorus fractions. Our analysis in response to the above question reduces to a comparison of the identified dynamic characteristics of the precipitation-induced SS and non-SS components of the TP load. This is revealing in terms of both the theoretical impulse responses of the model structures in Table 1 and the four observed major runoff events. But the analysis cannot accurately resolve the ambiguities inherent in the question it addresses.

Figure 8 defines the numerous ways in which the responses to a unit (1 mm) impulse precipitation event at nominal time t_0 can be computed. Where the output from one model, for example the SS-load at Zalaapati (model sz.1), is an input to another model, for example for estimating the TP-load at Zalaapati from model pz.1, it has been assumed that the overall input/output response is equivalent to the concatenated response of the individual input/output submodels. Figure 9 shows the (precipitation) impulse responses for SS-derived and non-SS TP loads distinguished as components (from models pz.1 and pf.1 at Zalaapati, Figure 9(a), and Fenekpuszta, Figure 9(c), respectively). It also shows the comparison between the linear superposition of these components, i.e. their summation, and the identified aggregate precipitation responses of models pz.2 and pf.2. Three points are notable:

- (i) that the non-SS component clearly precedes the SS-derived component response;
- (ii) that the non-SS component is significantly attenuated, in a relative sense, at the downstream location; and

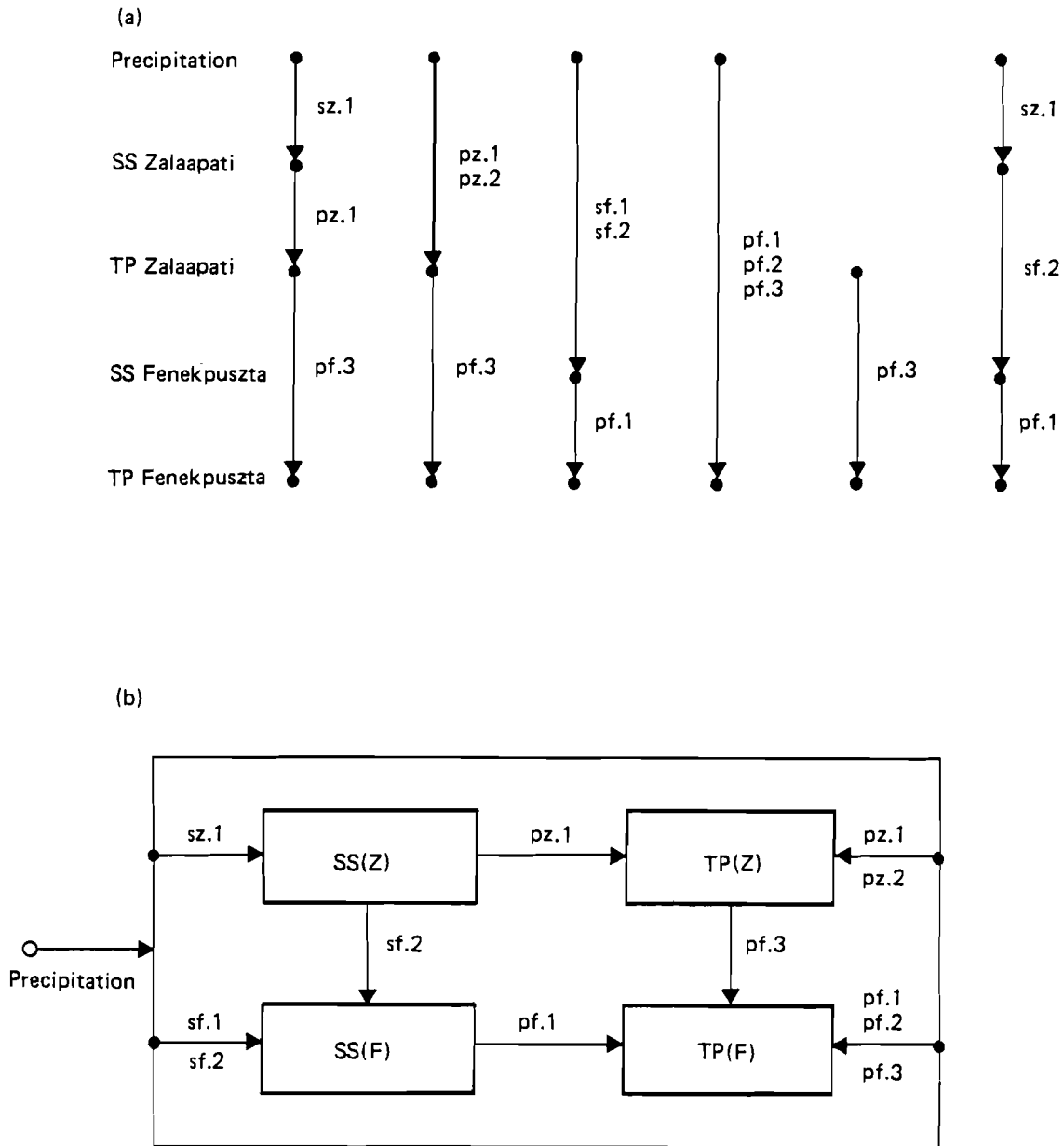


Figure 8. Various combinations of models for computing impulse response characteristics; the abbreviations (Z) and (F) in figure (b) refer to the SS or TP loads at Zalaapati and Fenekpuszta respectively

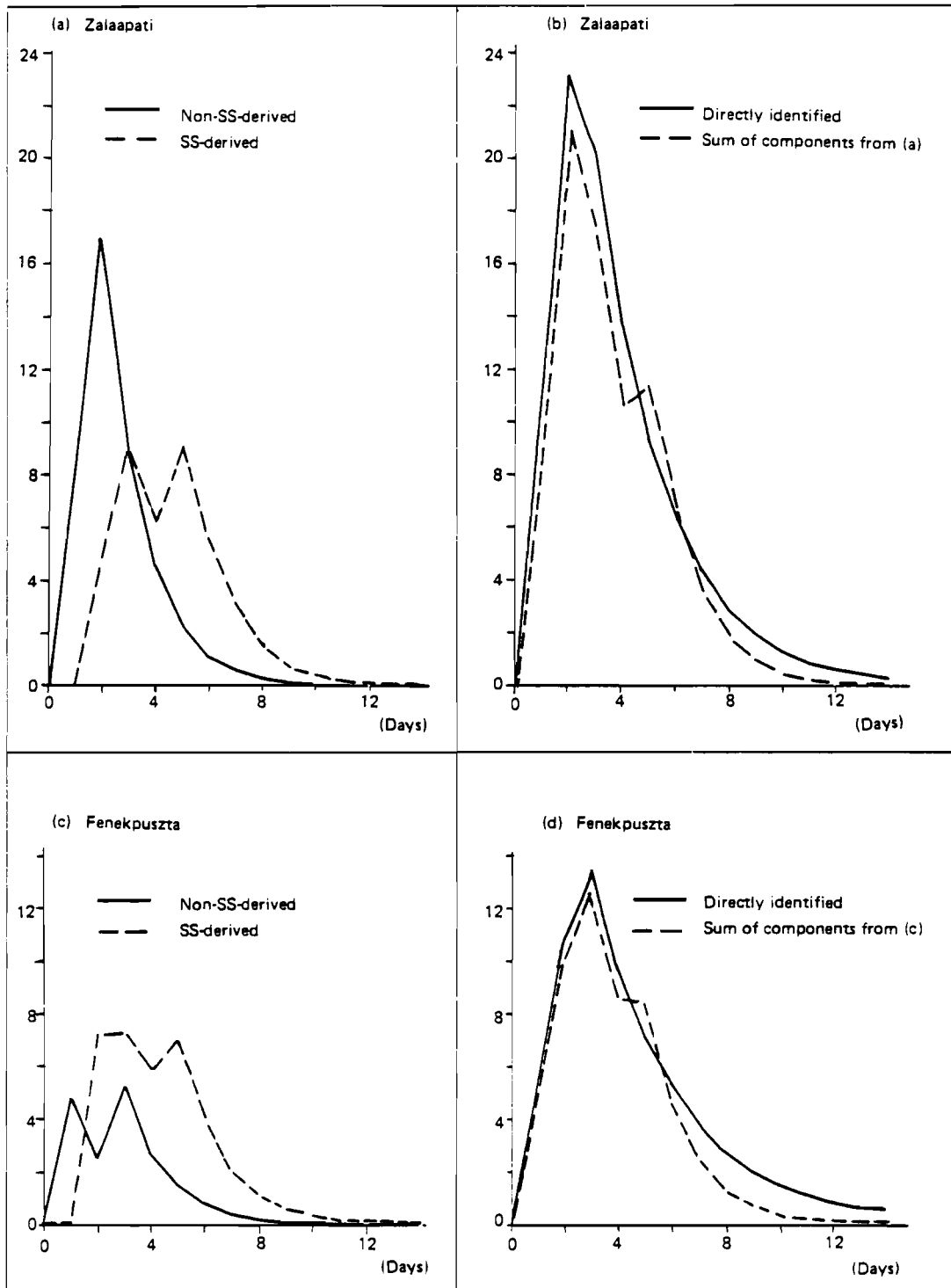


Figure 9. Precipitation-induced unit impulse responses for the TP load (in kg day^{-1})

(iii) that the identification of the components of the precipitation response does not significantly distort the pattern of the estimated aggregate response. The

The double-peak character of the non-SS component responses is a reflection of the observed behaviour in which for all four major event periods two closely succeeding precipitation events led to a single peak SS-load responses (we have already commented on this with respect to Figure 4(a)). The magnitudes of the SS-derived components of the TP load responses, but not their timing and duration, are very probably underestimated. They are based on the inferior models sz.1 and sf.1 (see Table 1), both of which consistently and significantly underestimate the SS-load responses at events (2) and (3), see Figure 5.

The general pattern whereby the non-SS component precedes the SS-load-derived TP component is apparent when the four major precipitation events are reconstructed through models pz.1 and pf.1. This is especially apparent at the Zalaapati site. Figure 10 illustrates the separated components for event (2) and it is evident that the precedence of the non-SS component at Fenekpuszta is relatively ambiguous. From this analysis one could thus conclude that a "first-flush" effect of non-particulate TP loads is likely in the event of a storm, but the evidence does not support any more incisive conclusion about the origin of this load.

The Transported Load and Lateral Influx

The fourth and final category of the component TP loads refers to that identified as being transported downstream from Zalaapati to Fenekpuszta. Such a load component is perhaps more interesting in respect of part of its complement, i.e. the load not transported between the two points yet induced by a precipitation event. An estimate of this latter type of load would allow one to draw inferences about the non-point source loads related to the watershed between Zalaapati and Fenekpuszta. The associated analysis is an indirect means of reconstructing the relationship between precipitation and the dynamic differences in

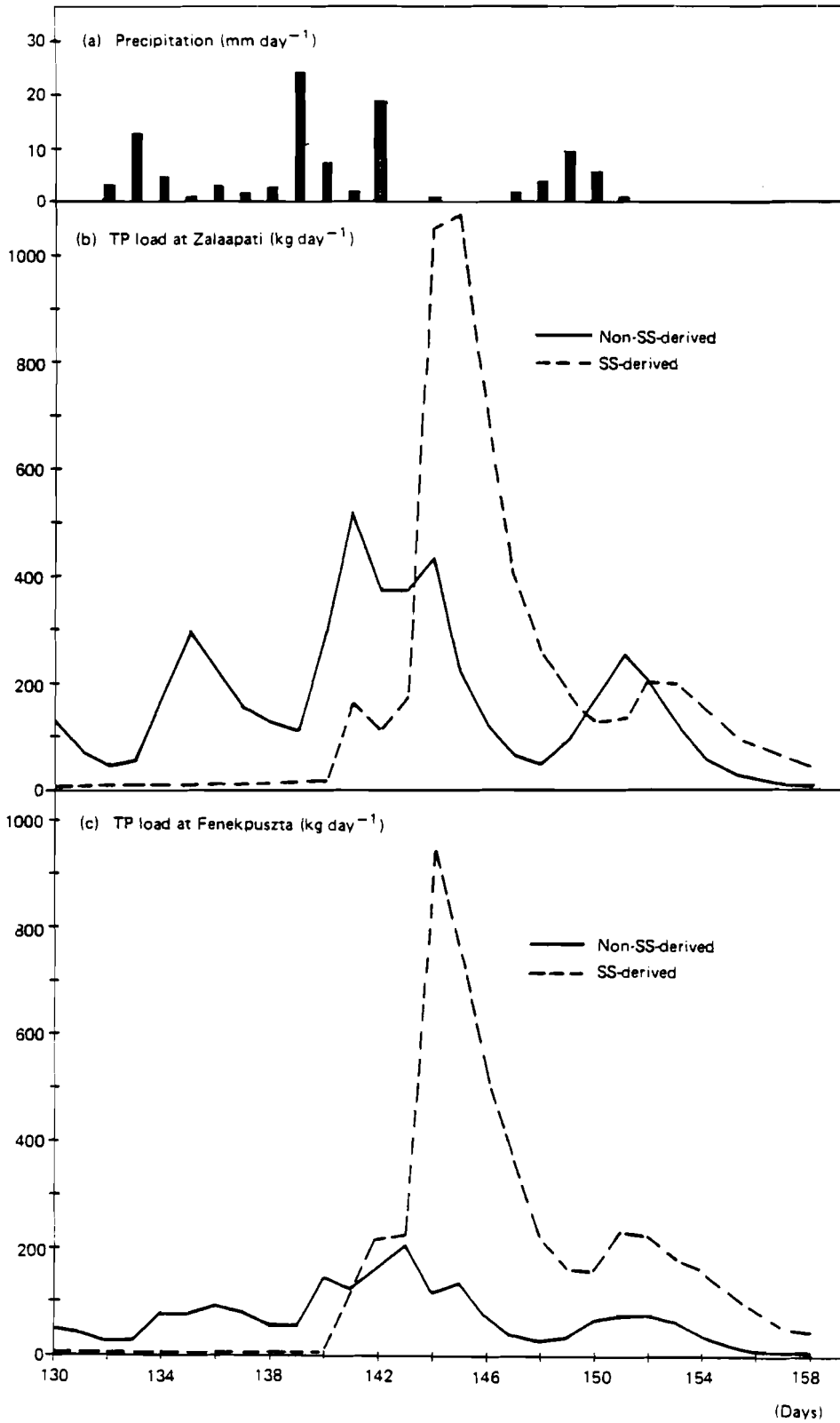


Figure 10. Reconstructed estimate of event (2) for days 130-158 of 1978

the TP loads observed at the two locations (see also the earlier comments on this point at the end of section 3.1). The estimates of Table 2 suggest that of the observed TP load at Fenekpuszta about 57% is on average transported downstream from Zalaapati; about 20% is the result of precipitation-induced runoff entering the river between Zalaapati and Fenekpuszta (c.f. 20% for the SS load and 11% for the TN load); and the remainder, some 23%, is unaccounted for by the model p.f.3. It is apparent from the results of this model (not shown) that the correspondence between the four major flood events at Zalaapati and Fenekpuszta is well identified, yet, because of the higher ratio of peak Zalaapati/Fenekpuszta TP loads during these periods, the base load conditions (lower observed ratio of Zalaapati/Fenekpuszta loads) are consistently underestimated at Fenekpuszta. That is, given a model with parameters assumed to be constant it is not possible to match all aspects of an observed relationship that apparently varies with time. We shall assume, therefore, that the TP load not accounted for by the model is due entirely to base loads entering the stream between the two points under zero precipitation conditions. In fact, it is notable that the lower and upper bounds for the range of differences in the base load estimates at Zalaapati and Fenekpuszta would account for between 12% and 26% of the average total TP load at Fenekpuszta (see Table 2).

The steady-state gains (κ_{ℓ}) computed from Table 1 for the various relationships between precipitation as input and the TP load components as outputs are given in Table 5. Here, three points of significance can be noted, that:

- (i) the identified aggregate, precipitation-induced load agrees roughly with the aggregate sum of the identified individual components, i.e. there is a reasonable degree of consistency among the model results;
- (ii) the lateral influx of precipitation-induced TP loads identified at Fenekpuszta is just under half of the precipitation-induced load identified at Zalaapati and subsequently transported to Fenekpuszta;

Table 5. Steady-state gains (κ_l) computed from Table 1 for the various relationships between precipitation as input and the TP load components as outputs; the model sequences for each gain are indicated in parentheses (refer also to Figure 8). All gains have units $[\text{kg day}^{-1}][\text{mm}]^{-1}$

Location	Aggregate load identified	SS-load derived component (1)	Non-SS component (2)	Aggregate of components (1)+(2)
Zalaapati (direct)*	95.00 (pz.2)	40.93 (sz.1;pz.1)	43.11 (pz.1)	84.04
Fenekpuszta (direct)*	65.33 (pf.2)	34.86 (sf.1;pf.1)	18.42 (pf.1)	53.28
Fenekpuszta (indirect)**	58.90 (pz.2;pf.3)	25.38 (sz.1;pz.1; pf.3)	26.54 (pz.1; pf.3)	51.92
Fenekpuszta (indirect)+	(58.90)	32.00 (sz.1;sf.2; pf.1)	(26.54)	(58.54)
Fenekpuszta (lateral)++	26.86 (pf.3)	6.19 (sf.2)	-	-

* A direct relationship between precipitation and TP load at the given point.

** A relationship between precipitation and the TP load thereby generated at Zalaapati and subsequently transported downstream to Fenekpuszta.

+ A relationship between precipitation and the SS load thereby generated at Zalaapati and subsequently transported downstream to Fenekpuszta and there "converted" to a TP load.

++ A relationship between precipitation and TP loads entering the stream between Zalaapati and Fenekpuszta.

- (iii) whereas the SS-load-derived component is between 43-49% and 43-55% of the aggregate precipitation-induced TP load at Zalaapati and Fenekpuszta respectively (depending on the estimates assumed for the aggregate), it represents only 23% of the lateral influx of TP loads between the two stations.

The dynamic characteristics of the impulse response functions for these combinations of the identified model structures are not as distinctively revealing as for the preceding analysis. Only the identified aggregate lateral influx response and its SS-load component are compared in Figure 11. The important conclusion that thus emerges is that if the precipitation-induced sources of TP originating from the watershed between Zalaapati and Fenekpuszta are predominantly agricultural they are composed largely of non-particulate fractions. There appears to be a significantly smaller aggregate TP load deriving from runoff from the subwatershed below Zalaapati (per unit area) than from that above Zalaapati, and this runoff from the lower subwatershed contains a significantly smaller portion of particulate-based TP load. Both of these latter conclusions are consistent with Joó's (1980) statement that the lower subwatershed is less subject to erosion (see also details given by van Straten et al, 1979).

3.3 Trajectories of the Recursive Parameter Estimates

In the present analysis the trajectories of the recursive parameter estimates are not especially illuminating, at least not in the sense demonstrated for a similar problem by Whitehead et al (1979). According to the purposes outlined in section 2 for this step of the (general) procedure for analysis, it could be concluded that neither are any of the identified model structures demonstrably inadequate nor are any useful modifications of these structures suggested. Let us take this conclusion, therefore, as evidence that there is no reason for the tentative conclusions already made in the preceding analysis to be amended. A more fruitful interpretation of the recursive estimation trajectories, given the fact that they reflect the continuous adaptation of the model structure to the changing observed dynamics of the stream discharge and nutrient loadings, is to view them as a means for reconstructing smoothed snapshots of the principal precipitation events.

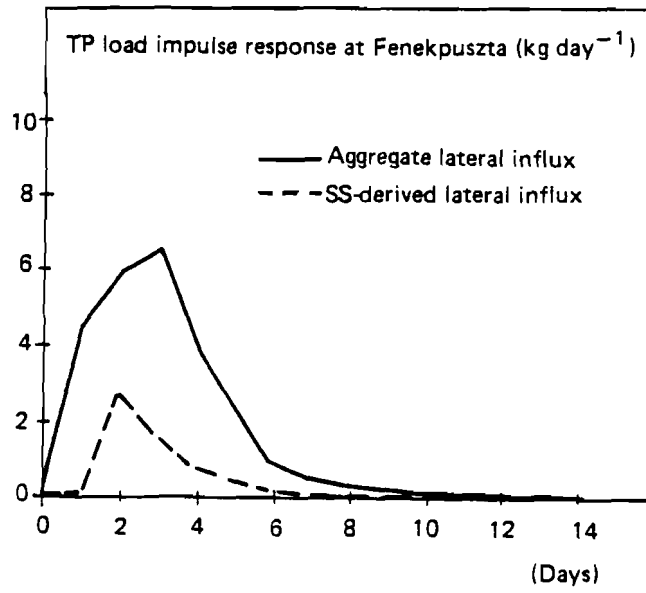


Figure 11. Precipitation-induced unit (1 mm) impulse responses for the lateral influx of TP load (in kg day⁻¹) at Fenekpuszta

The recursive parameter estimation trajectories for model pf.2 in Table 1, under the assumption that the parameter b_{40} is constant, are shown in Figure 12. They are typical of the results in general. The estimate \hat{a}_1 exhibits a clear seasonal variation, albeit subject to quite severe transient fluctuations coincident with the major events, the midsummer recession time-constant (~ 1.5 days) being about twice as fast as that during winter (~ 3.0 days). The estimates \hat{b}_{1j} , for coefficients of the precipitation input, are obviously adapted significantly to the individual major precipitation events but show no tendency towards longer-term seasonal variability. It is probable that there is such seasonal variability, which would be due to the effects of temperature and soil moisture deficit in modulating the amount of runoff available as stream discharge (as we shall see in the next section). However, we can merely surmise that it is not apparent in the results for 1978, for which year all four major events occurred within roughly a single quarter of the year.

The range of dynamic characteristics implied by the recursive estimates of Figure 12 can be gauged by the associated model impulse responses for two of the precipitation events, as shown in Figure 13. These impulse responses are in fact computed using the average parameter estimates for the duration of each event. Event (1), which occurred in early April, 1978, has a remarkably low peak response. Over twice as much TP loading is "yielded" at Fenekpuszta from an identical amount of precipitation during event (3), which occurred in mid-May. The essential difference in the two responses, which is similarly apparent from the analysis of the data at Zalaapati, is due to the relatively low mean concentration of TP in the runoff associated with event (1). The mean concentrations of TP* in the precipitation-induced in-stream TP load for events (1)-(4) are respectively 0.31 gm^{-3} , 0.66 gm^{-3} , 0.62 gm^{-3} and 0.62 gm^{-3} at Fenekpuszta, and at Zalaapati, 0.46 gm^{-3} , 1.49 gm^{-3} , 0.85 gm^{-3} and 0.96 gm^{-3} . For comparison the mean base load

* The concentration is computed from the steady-state gain for the precipitation to TP load response divided by the steady-state gain for the precipitation to stream discharge response.

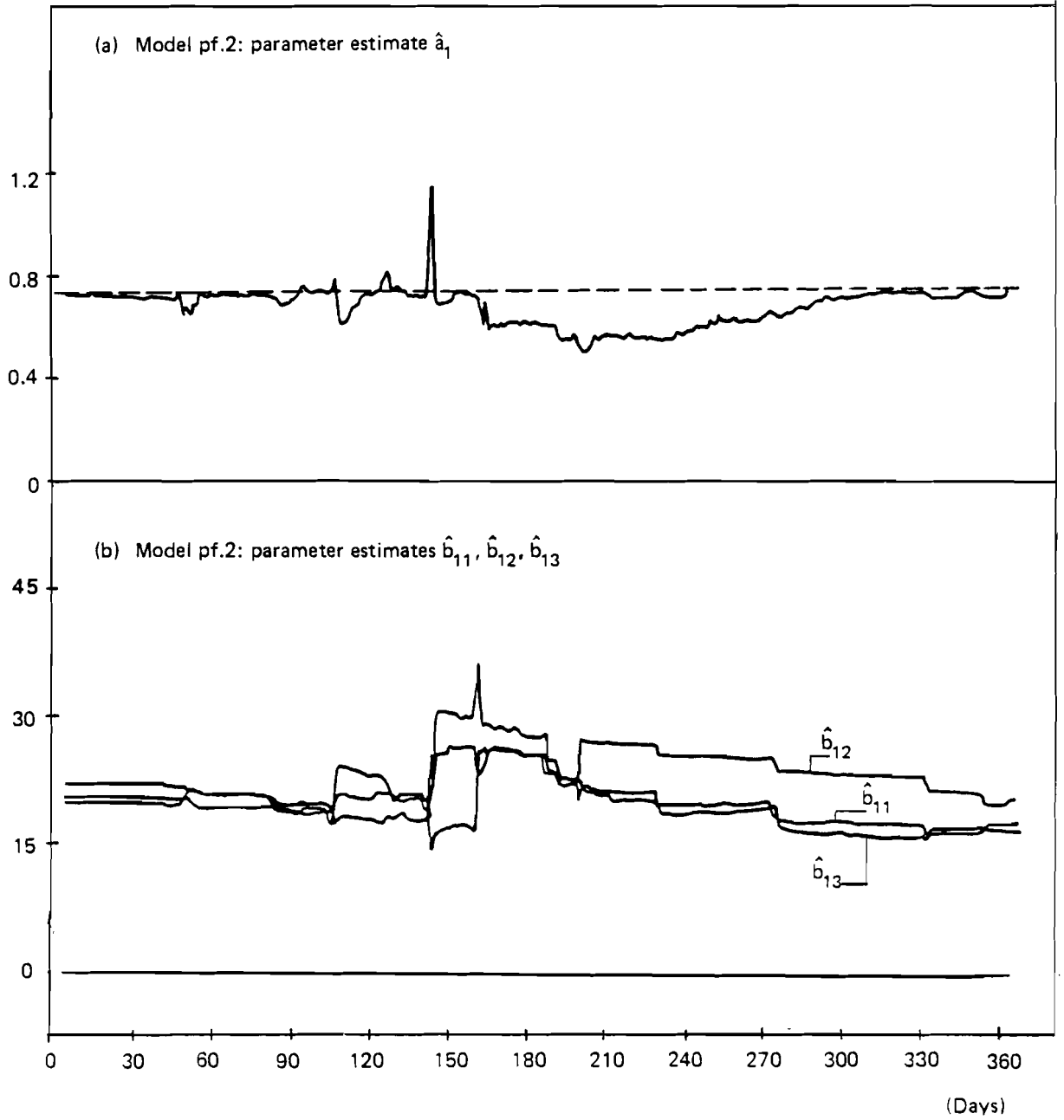


Figure 12. Recursive parameter estimates for model pf.2 for the year 1978

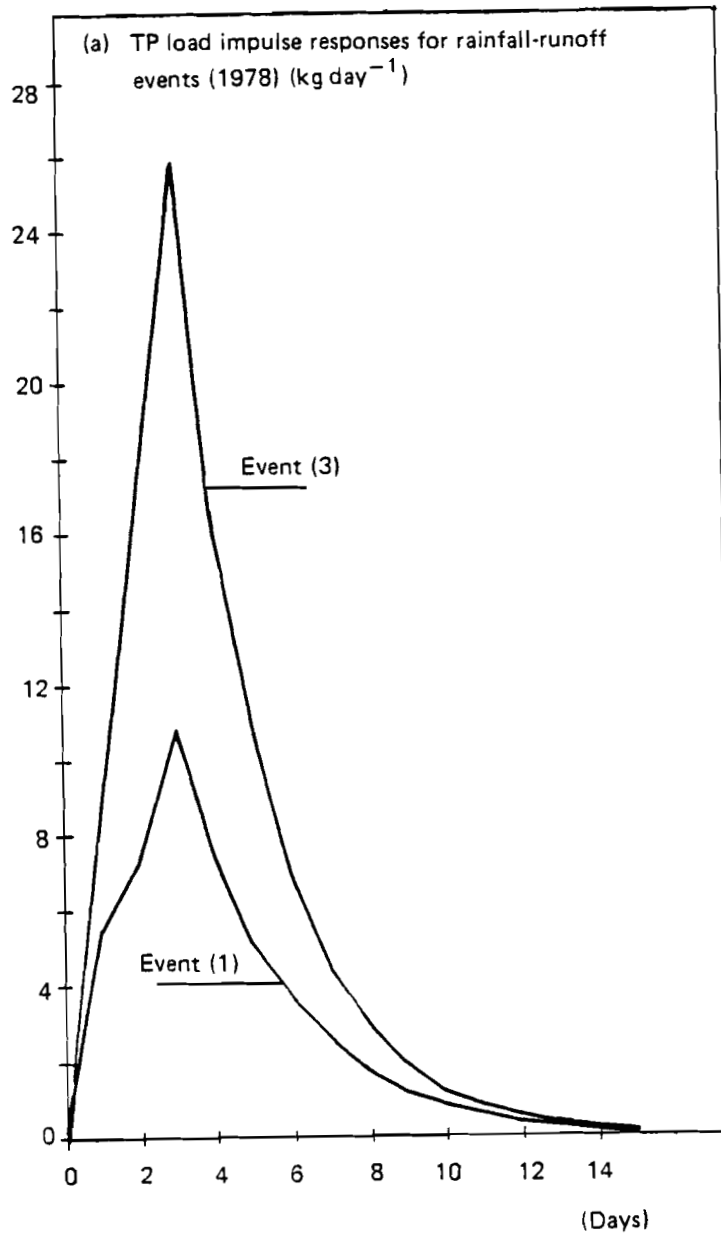


Figure 13. Precipitation-induced unit (1 mm) impulse responses for the TP load during events (1) and (3) of 1978

concentrations of TP are 0.34 gm^{-3} and 0.27 gm^{-3} at Fenekpuszta and Zalaapati respectively. The generally higher concentrations in the precipitation-induced loads, relative to the base loads, and the consistently higher concentrations in the urnoff at Zalaapati are notable.

To summarise, it is difficult to obtain a clear picture of any seasonal load variations from such a small sample of events occurring over a relatively short period of time. In particular, despite the variability of the impulse responses, it is not possible to identify those times of the year at which the potential of the watershed to generate nutrient loads is more or less sensitive to a major precipitation event.

4. ANALYSIS FOR 1975-1977

The essential assumption in using the model structures identified for the year 1978 in order to estimate the corresponding loads for 1975-1977 is that 1978 is typical of a yearly behaviour pattern for the Zala watershed. From the point of view of model validation or invalidation the analysis is of interest both for the regularities and irregularities of behaviour that are revealed. From the results of Figure 14, where the TP load observations at Fenekpuszta for 1975-1977 are compared with estimates derived from model pf.1 with the parameter values of Table 1, it is apparent that many of the hypotheses made for interpretation of the 1978 observations should be dismantled as invalid. The model is a grossly inadequate estimator of the loading patterns for the second half of 1975; it does, however, reflect well the two major storm events of 1976, but notably and significantly over-estimates the base load for this year; over-estimation of the base load for the summer of 1977 is also evident, although to a lesser extent than for 1976, and the model is clearly inadequate for the persistently high loads that occurred during the first quarter of 1977.

In spite of the advantages of using the 1978 "template", as it were, as a means of emphasizing the irregularities between the different years, there now follows the extreme difficulty of assessing why these irregularities are observed. It has to be

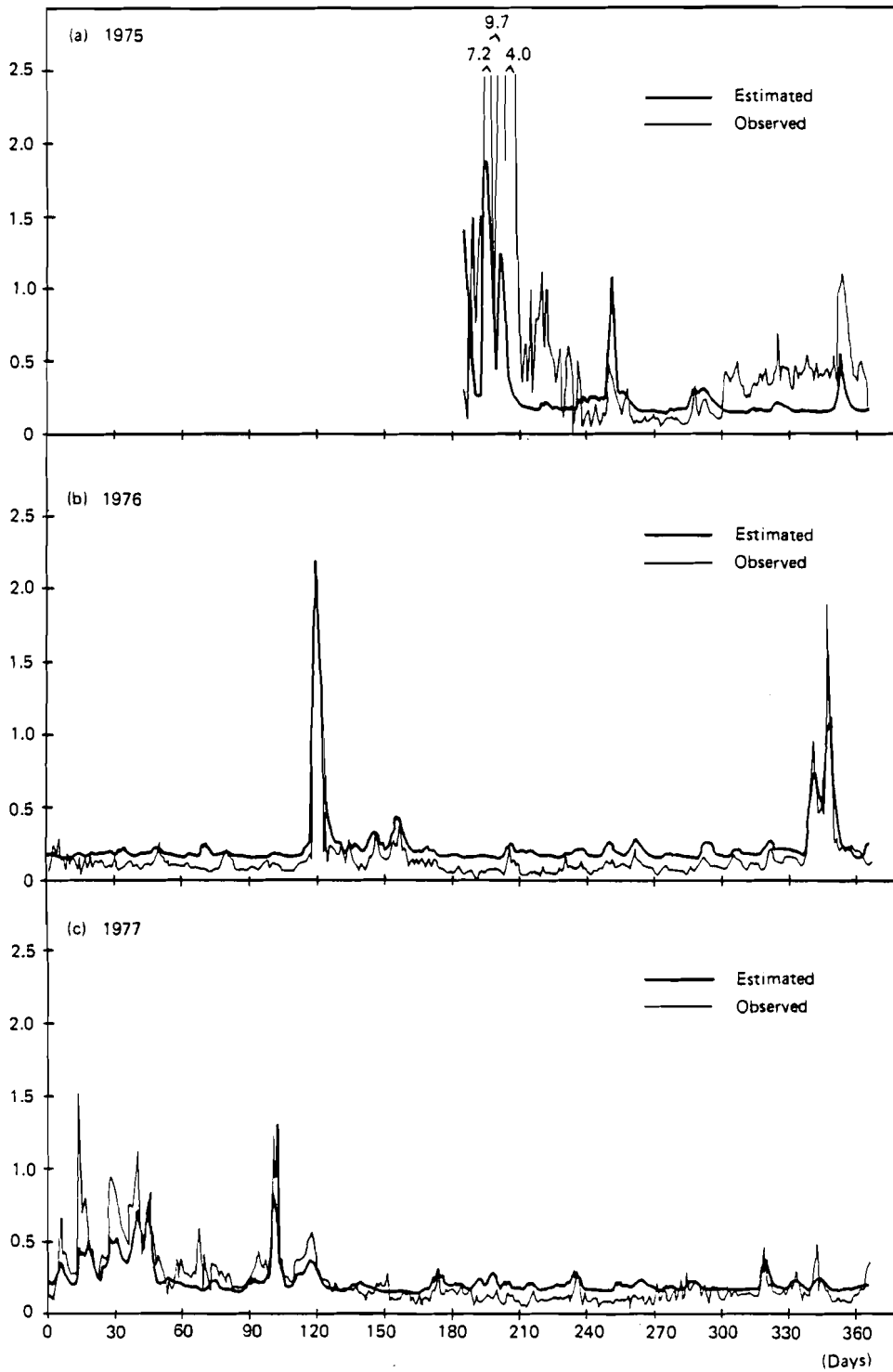


Figure 14. Observed and estimated TP loads (in 10^3 kg day^{-1}) at Fenekpuszta for the years 1975-77.

accepted that parts of the analysis for 1978, particularly the quantity and characteristics of the base load, are not valid for other years. This would arguably negate the results of the analysis for 1978, if it could be demonstrated that behaviour for the years 1976 and 1977 exhibited a greater degree of regularity. It is much more evident, however, that the contrary applies: that the behaviour of the Zala watershed for the years 1975 to 1978 is irregular, with virtually only seven or eight individual, short-term storm events having a consistent, reproducible character. The results of the 1978 analysis remain conditionally valid for 1978 alone, but with the important caveat that the assumed constant base load clearly indicates an average yearly estimate that significantly over-estimates a seasonal summer variation. This base load figure may equally well underestimate a seasonal winter variation, but this is not readily apparent. The underlying causes of the summer base load discrepancy, together with the other longer-term irregularities not principally related to short-term precipitation events, will form the basis for the discussion of this section.

The irregularities that pervade the observed nutrient load characteristics for the years 1975-1978 are not due to variations in cumulative yearly precipitation but due to large variations in seasonal (quarter-year) rainfall patterns, see Table 6. (We may note, however, that Joó (1980) indicates that the observed precipitation over the whole of this period was 10-15% less than average.) The oddity of the year 1978 was that it had the wettest second quarter of the 3½ year record, and three of the four major conventional rainfall-runoff events of that year occurred in this quarter. In fact the second quarter of 1978 was the wettest quarter of the recorded period.

The extremely high TP loads of the summer of 1975 are also obviously but not exclusively related to the especially high precipitation of that quarter. Other factors are potentially even more significant. For example, the SS and TN Loads (not shown) are most at variance with the TP load variations during the last two quarters of 1975. The same is apparent from Table 3, where the average ratio of SS load to TP load is roughly

Table 6. Aggregate quarterly precipitation in the Zala watershed for the years 1975-1978 (in mm)

Year	Days 1-90	Days 91-180	Days 181-270	Days 271-365(6)	Total
1975	-	-	250	116	(366)
1976	65	183	143	191	582
1977	152	129	195	113	589
1978	84	266	166	93	609

constant at 100:1 for 1977 and 1978, is 182:1 for 1976, and only 75.1 for 1975. The average daily TN load for all four years is by contrast relatively invariant with time. A relative consistency of variation among the three loads is otherwise evident. Both SS and TN load patterns for 1975 match their respective 1978 templates much better than the TP loads shown in Figure 14(a). The persistent discrepancies between the model response and observations during the two quarters of 1975 are indicators that errors in the estimated short-term precipitation-induced runoff are not responsible for such irregularities. The observed TP load over the period $t_{195} \rightarrow t_{230}$ appears to be dominated by a much slower response to a precipitation event, as if subsurface drainage (as opposed to overland surface flow), and possibly after extensive flooding, is a governing factor. The intensity of the mid-July (1975) event is, however, matched by the late April event of 1976 at about t_{120} (see Figure 14(b)), but the load responses are entirely different. The comparison underlines what appears to be a distinct "vulnerability" of the watershed's potential for generating high TP loads to a midsummer storm event.

The abrupt increase in the TP load of the last quarter of 1975 at t_{300} , and its equally abrupt decline at t_{365} , is due entirely to a corresponding increase (and decline) of the observed TP concentration (from a preceding average of roughly 0.3 gm^{-3} to a "plateau" of $0.8-0.9 \text{ gm}^{-3}$ and back to a succeeding average of 0.3 gm^{-3}). An obvious choice of factor responsible for this high load is the drainage of marshland waters with an exceptionally high TP concentration. The observed stream discharge is not notably

increased by the supposed addition of the marshland waters, hence the postulated high concentrations of TP in these waters. Such high concentrations are exceptional in that, if marshland drainage was practiced in the fourth quarters of each of the four years of the record, the observed features of the 1975 TP load are not repeated with anything like the same significance in subsequent years. Since neither this "excess" load of the fourth quarter of 1975 nor the "excess" loads of the preceding third quarter are reflected in the observed or estimated SS loads, it can be assumed that these high loads are composed predominantly of non-particulate TP matter. It is tempting to speculate further that the postulated high concentration marshland waters are a function of the earlier summer storm events.

Apart from the extreme character of the summer precipitation and TP loads of 1975 the third quarters of the other three years have roughly equivalent cumulative precipitation figures. Why, then, is the 1976 summer TP load so low? In fact, if it is assumed that the precipitation-induced TP load for 1976 is well estimated by the model, an estimate of the yearly average base load can be made by subtracting the estimate of the precipitation-induced TP load from the total observed load. The resulting estimate is a value of 62.3 kg day^{-1} for the base load, i.e. between only 43% and 57% of the base load for 1978 (depending upon the different figures derived from models pf.1 of pf.2). However, this base load estimate is still about 42% of the total TP load for 1976 (cf. between 51% and 65% for 1978). The corresponding estimate for the particulate TP load component for 1976 is about 39% of the total load (cf. 21% for 1978).

There could be several explanations for the particularly low summer TP load in 1976 drawn from various combinations of the following four reasons:

- (i) the field observations were persistently in error;
- (ii) the point-source sewage discharges of TP were significantly lower in that summer;
- (iii) higher temperatures caused a greater loss of precipitation through higher rates of evapotranspiration;

- (iv) higher temperatures and lower stream discharges (longer residence times) increased the rate of apparent loss of in-stream TP by, for example, the growth of attached plants and by deposition to the bed sediments.

There is no evidence to suspect the presence of the first of these reasons. The second reason also appears to be improbable, although it would require a much more detailed understanding of the operating characteristics of the relevant wastewater treatment plants than is possible here. Reason (iii) is probable, although the point under discussion concerns the observed base load and not the TP load due to precipitation-induced runoff. The most pertinent and probable candidate reason (or rather hypothesis) is thus the fourth. If, however, the phosphorus cycle in the stream system were assumed to be essentially closed (the corresponding nitrogen cycle would be open if significant denitrification were to occur), then one would expect to observe subsequently a corresponding apparent gain of TP as the attached plants die and decay or as the sediments are scoured by high flows.

Our apparent loss and gain speculation for the TP load looks attractive when now considering the persistent underestimation of the observed loads for the winter 1976/7 (Figure 14(c)). During the first 120 days of 1977 the total observed TP load is underestimated by an aggregate amount of $11.1(10^3)$ kg TP. Taking an average, low figure for the apparent rate of loss of the base load in 1976 with regard to the estimate for 1978 from model pf.2, i.e. a rate of loss of 51 kg day^{-1} , it would have taken a period of about 220 days for the "excess" load of early 1977 to have accumulated in the stream system in 1976. The length of this period is notably close to the period between the two major storm events of 1976. This, of course, proves nothing, and a counter argument can readily be constructed. For example, the first 120 days of 1977 are characterized by a persistently high stream discharge, nearly always above $10 \text{ m}^3 \text{ s}^{-1}$ and often greater than $20 \text{ m}^3 \text{ s}^{-1}$. The autumn and winter periods of 1976 and 1977 were relatively wet periods (see Table

6). One can assume that from early on in the winter period the soil moisture deficit and evapotranspiration rate would both be low such that a much greater proportion of the precipitation would be observed as runoff to stream discharge. If it is a continually higher rate of runoff that is responsible for the high TP loads this would suggest that the precipitation → watershed → load system has a high "throughflow", as opposed to a low "throughflow" with high release rates of previously accumulated in-stream material. Again, if the long-term seasonal accumulation-release hypothesis is to be corroborated by other evidence, why apparently does the same mechanism not recur during the 1977/8 winter?

The occurrence of snowmelt, signified both by an obviously asynchronous observed and estimated response to a precipitation event and by peak TN and TP loads in the absence of corresponding SS loads, might also be suspected for the winter of 1976/7. The absence of a discernible SS load response is, however, conditional upon stream discharge remaining below the threshold at which scouring of the streambed would take place, for example, below $10-12 \text{ m}^3 \text{ s}^{-1}$ as already noted. The simultaneous estimated and observed TP load peaks between t_1 and t_{55} in 1977 (Figure 14c), which correspond closely with SS load peak responses, is evidence pointing away from the significance of snowmelt. Inspection of Figures 6(b) and 7(b), relating to 1978, shows in comparison a much more probable occurrence of snowmelt over the period $t_{45} \rightarrow t_{65}$ for that year. Snowmelt might also be of minor significance in the winter of 1976 (Figure 14(b)), where an estimated TP load response is given at about $t_{65} \rightarrow t_{70}$ but an observed response occurs at $t_{75} \rightarrow t_{80}$. This event for 1976 is more distinct in the TN load variations (not shown) and generally it appears that snowmelt generates a greater TP or TN load response than a conventional rainfall-runoff episode.

5. A SCENARIO FOR INTERPRETATION AND FURTHER EXPERIMENTATION

It is important to step back from the detail of the preceding two sections in order to summarize the interpretation of the observed past behaviour of nutrient loadings in the Zala watershed. Since many questions about this behaviour remain open,

let us call the interpretation a scenario. The scenario can also be applied to the definition of further analyses, principally, for example, on additional experimental work in relation to some of the open questions, and on the use of the results of the present analysis for evaluation of the Kis-Balaton project.

During 1978 the base load of total phosphorus, under zero precipitation conditions, was about 87 kg day^{-1} at Zalaapati and 113 kg day^{-1} at Fenekpuszta. The latter is approximately 50% of the total TP load, and our results suggest that this is a low estimate for the base load for that year. The estimates are in good agreement with the estimates quoted previously by Jolankai and Somlyódy (1981). This base load is composed predominantly of non-particulate phosphorus material deriving from point-source discharges and in this sense can be considered to be largely a form of phosphorus available for algal growth. The base load estimate for 1976 at Fenekpuszta is, however, considerably lower, roughly half that for 1978, although it still comprises some 42% of the total TP load for 1976. The particulate fraction of the TP load, being respectively 39% and 21% of the overall loads at Fenekpuszta for the years 1976 and 1978 respectively, is very strongly associated with the major precipitation events. For example, 85% and 78% respectively of the (year's) particulate TP loads at Zalaapati and Fenekpuszta for 1978 were transported during four major precipitation events. A non-negligible portion of the precipitation-induced TP load is, however, of a non-particulate form. Approximately 14% of the overall load at Fenekpuszta in 1978 is estimated to be from this "source", which (dynamically) can be likened to a "first-flush" effect of a precipitation event.

It is especially difficult to distinguish conclusively the origins of the precipitation-induced TP loads. This is partly due to the mechanisms identified as governing the entry and transport of nutrients in the river. The scenario developed from our analysis is as follows. Both particulate and non-particulate fractions of the load can be assumed to enter the river simultaneously during a storm event. If the stream discharge is below a certain threshold value (estimated to be about $10\text{-}12 \text{ m}^3 \text{ s}^{-1}$ at both Zalaapati and Fenekpuszta), the

particulate fraction settles to the streambed. Only subsequently, when the stream discharge becomes sufficiently high for scouring of the bed to occur, is a significant SS load observed at the two measuring sites. The dominance of the in-stream deposition and erosion processes for particulate matter distorts thus the coupling between watershed erosion and observed in-stream TP load. A first-flush effect is equally so a complement of this distortion, such that it is not possible to conclude whether this non-particulate load derives from urban runoff channeled partly through treatment plants or from agricultural runoff.

There is greater clarity about the probable areal distribution of nonpoint source TP loads. The major portion of observed particulate material carried by the stream enters upstream of Zalaapati. Loads generated by runoff entering the stream between Zalaapati and Fenekpuszta are significantly smaller (per unit area) and contain a smaller fraction of particulate material. These conclusions are consistent with the areal distribution of the watershed's susceptibility to erosion processes (see Joó, 1980; van Straten et al, 1979). The average concentration of TP in runoff-generated stream discharge is generally considerably higher than that in the base stream discharge.

Some of the most important unresolved questions about the behaviour of the Zala watershed arise from the analysis of the longer-term loading variations over the period 1975-78. Nevertheless, it is apparent from this analysis that the short-term dynamic response of the loadings to a major, but not extreme, precipitation event is relatively well characterized and should not be a priority for further experiment and analysis. The seasonal variability of wastewater treatment plant performance and more so the seasonal variations in in-stream factors affecting the phosphorus cycle, for example, the uptake of phosphorus by attached stream plants, require more detailed examination. That these factors might be responsible for an average reduction of up to 50% (50 kg day^{-1} or more) of the base load delivered from the Zala River to the lake makes them significant. They are equally significant if, for the assumption of a closed phosphorus cycle, the same amount of TP load, albeit possibly in a less readily available form, is released from the river system during

autumn and winter periods by, for example, the death and decay of organic material. That they may be associated with the correlation between the low Zala base TP load of 1976 and the absence of an observed chlorophyll-a peak in the Keszthely basin of the lake for that year is an additionally suggestive coincidence of potentially considerable significance. Within the stream phosphorus cycle bed sediment processes may also be important for the longer-term transformation of phosphorus forms. However, the observed loss of SS-load between Zalaapati and Fenekpuszta (for the year 1978) is unlikely to result from processes of biochemical transformation of particulate-bound phosphorus. A greater amount of SS-load is observed to be "lost" during the major precipitation events alone, when residence times are smallest, than the loss for the whole of the year (i.e. there is a net "gain" of solids during non-storm conditions).

The capacity of the watershed to generate extremely high TP loads in response to very high precipitation events occurring during the summer period (a load of nearly $10,000 \text{ kg day}^{-1}$ was observed on one day in 1975) is important because of the sheer scale of the event. It is also important because of other poorly understood factors that influence the medium-term response of the TP load to such an event, in particular the possibility of slow drainage of flooded land. A careful retrospective analysis of that particular event in 1975--the areas that were flooded and whether the event influenced the concentration of TP in the subsequent (deliberate) drainage of marshland the following autumn--might be quite revealing and significant, especially with respect to the performance of the Kis-Balaton project.

The computational load of all the models discussed in this paper is trivially small. It is true that such models can not be used to assess the changes in nutrient load variations that would result from changes of watershed management practices. They are nevertheless quite suitable for the generation of input disturbances, i.e. forcing functions, for evaluating the dynamic behaviour of, and control strategies for the reed-lake system to be located at Kis-Balaton. The time-scale of the models is admittedly relatively small with respect to the possible time-constants of the reed-lake system. However, the cost of working

with a daily time-step for these models is small when compared, at least initially, with the potential gain of preserving a wide range of dynamic characteristics for the test input sequences to the reed-lake system. Whether the models derived for Zalaapati are more appropriate than those derived for Fenekpuszta depends upon the precise location of the project. For either case three interacting models would be necessary, for example, qz.1, sz.1, and pz.1, in order to simulate discharge variations and to distinguish between particulate and non-particulate fractions of the TP load. The model structures of Table 1 might further be prudently adapted in order to introduce a degree of seasonal variation, especially in view of the discussion of section 4, and to compensate for the shortcomings of the SS-load models in characterizing the nonlinear, threshold-like behaviour of in-stream bed scouring. These modifications could easily be incorporated in a model of the following form (from qf.2, sf.1, and pf.1):

$$\text{Discharge (Q): } x_1(t_k) = a_1^1 x_1(t_k) + \sum_{j=1}^3 b_{1j}^1(t_{k-j}) u_1(t_{k-j}) + b_4^1, \quad (14a)$$

$$\begin{aligned} \text{SS-load: } x_2(t_k) &= a_1^2 x_2(t_k) + \sum_{j=1}^4 b_{1j}^2(t_{k-1-j}) u_1(t_{k-1-j}) + \\ &+ b_4^2(t_{k-1}), \end{aligned} \quad (14b)$$

$$\begin{aligned} \text{TP-load: } x_3(t_k) &= a_1^3 x_3(t_k) + \sum_{j=1}^3 b_{1j}^3(t_{k-j}) u_1(t_{k-j}) + \\ &+ b_{20}^3 x_2(t_k) + b_4^3(t_{k-1}), \end{aligned} \quad (14c)$$

where x_1 , x_2 , and x_3 are respectively the stream discharge, SS-load, and TP load, and where, for example,

$$\begin{aligned} b_{1j}^1(t_k) &= f_1\{\theta(t_k)\}; \text{ and similarly for } b_{1j}^2(t_k) \text{ and} \\ &b_{1j}^3(t_k) \end{aligned} \quad (15a)$$

$$b_4^2(t_k) = f_2\{x_1(t_k)\} \quad (15b)$$

$$b_4^3(t_k) = f_3\{x_1(t_k), \theta(t_k)\} \quad (15c)$$

in which $\theta(t_k)$ is an observation of air temperature. The expression f_1 would therefore be intended to account for the seasonal variations in soil-moisture deficit and evapotranspiration; f_2 would account for the nonlinear influence of a high stream discharge ($>10 \text{ m}^3 \text{ s}^{-1}$, for instance) on the SS-load; and f_3 would account for the effects of large in-stream residence times and high temperatures on the seasonal uptake and release of TP. The observed records of u_1 , precipitation, and θ would presumably provide a suitable set of input sequences for the above model under the assumption that they would reflect significant seasonal variability in the clustering of precipitation events. All of the analysis sketched in this discussion could be conducted within the framework of Monte Carlo simulation using the distributions of parameter estimates implied by the variance-covariance matrices P of equation (6). Care would be required, however, in ensuring that intuitively appropriate correlations among the parameters of equations (14) are preserved.

6. CONCLUSIONS

This analysis of nutrient loadings in the Zala River has been formulated at the relatively microscopic level of day-to-day variations in behaviour, although the objectives of the study have been relatively macroscopic, i.e. to distinguish the aggregate, or average, distribution of the phosphorus loadings between point and nonpoint sources and between forms available and not available for phytoplankton growth. There has also been an implicit objective of determining the quantity of nutrients transported to the lake during storm events, a factor strongly influencing the estimates of aggregate loads but usually not easily identified due to the slow sampling frequency of the available field data. With reference to these objectives a partial success has been achieved in separating the estimates of particulate and non-particulate fractions of the TP load, which approximately corresponds with the division between unavailable and available forms of phosphorus. However, it has been surprisingly difficult to identify a consistent estimate among different years of the point-source TP load during dry-weather conditions and, rather

more as expected, equally difficult to distinguish unambiguously between loads deriving from point and nonpoint sources during precipitation events.

The results emphasise the observation that the variations of TP loadings in the Zala River for the years 1975-78 exhibit few features that are regular. This variability of behaviour is due primarily to considerable variations in the aggregate quarterly precipitation patterns of these years. For the year 1978, to which most of the analysis refers, the average dry-weather, point-source TP load has been estimated to range between 110-145 kg day⁻¹ at the mouth of the Zala River, i.e., about 50-60% of the total aggregate TP load for that year. In 1976, however, this load could have been as low as 60 kg day⁻¹ (about 40% of the average daily total load) and one can only speculate that this low value reflects significant in-stream uptake of TP between the point-source discharges and the mouth of the Zala at Fenekpuszta.

The TP loads arising from particulate (SS) material are strongly dependent upon the major precipitation events. In 1978 the SS-derived TP load was on average 45 kg day⁻¹ (20% of the total) and for 1976 it was about 60 kg day⁻¹ (40% of the total for that year). The surprisingly low figure for this type of load for 1978 is, however, unlikely to be seriously in error. The major factor affecting the SS-load variations, and hence that component of the TP load derived from SS material, is the stream discharge, which determines the rate of deposition or scouring of particles to and from the bed sediments. The threshold value for the discharge above which scouring occurs is roughly 10-12 m³s⁻¹. In general, and for all the years 1975-78, the overall characterisation of the short-term (day-to-day) TP load response to a precipitation event is one of the few regular features in the observed behaviour of the watershed.

Only a part of the TP load arising from a precipitation event is due to a particulate fraction. There is also a non-particulate fraction, whose dynamics resemble a first-flush effect, and which amounted to an average TP load of 30 kg day⁻¹

in both 1976 and 1978. This type of load could well be of an available phosphorus form, although it is not possible to infer whether it originates from a point or a nonpoint source. In view of the equality of this load for the two years 1976 and 1978, which reflects merely an equality of yearly aggregate precipitation, the maximum difference in the potentially available phosphorus delivered to Lake Balaton is simply the difference in the above two estimates of the base, dry-weather TP load, i.e., some 85 kg day^{-1} on average. If phosphorus were to behave as a conservative substance, and there being no sedimentation of this non-particulate fraction, the difference in the resulting steady-state TP concentrations in the Keszthely bay of Lake Balaton might be about 0.1 gm^{-3} (which is actually in absolute terms about the maximum value of TP concentration observed in this part of the lake over the period 1976-1978, see van Straten et al., 1979). Such a variation in average dry-weather loading rates and its causes are clearly of considerable significance. Further study in this direction would be desirable.

Finally, there are two other inter-related results of the analysis that likewise deserve further study. In the summer of 1975 extremely high TP loads were observed following a storm event, and over and above the regular short-term dynamic response of the load a longer-term, slower response in the TP load at Fenekpuszta appeared to have been stimulated. This latter part of the response could have been due to the flooding and subsequent slow drainage of marshland; the potential of the watershed to generate such high and relatively persistent loads is again a feature of considerable significance and worthy of further research. The significance of this feature is indeed enhanced given the location of the Kis-Balaton reed-pond project. The simple models developed from the present analysis would provide a useful starting point for the generation of typical TP load time-series as input disturbance sequences for evaluating the possible performance of this project as a control option for lake eutrophication management.

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