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METHODS FOR GENERATING PREFERABLE TECHNOLOGY ACTIVITIES FOR A LP MODEL

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FOREWORD

The IIASA's Food and Agriculture Program focused its Task 2 on the long term aspects of technological transformation with respect to resource limitations and environmental consequences. In developing the methodology and modelling techniques for such an analysis one of the problems has been to genrate and handle a large number of alternative techniques while at the same time considering their technical feasibility.

This paper describes one method of solving this problem.

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METHOD FOR GENERATING PREFERABLE TECHNOLOGICAL ACTIVITIES IN THE LP MATRIX OF THE MODEL

V.N. Iakimets and D.R. Reneau

1. Introduction

When the general methodology for the FAP Task 2 case study models (IIASA WP-81-15) was being developed, an area of particular concern was how to define the various agricultural production processes that occur or could be used within each case study region. This was a problem of defining the agricultural technology presently used in the region and also the technology that could be used given at least the more likely possible futures. In essence this required that the technology definition be complete in those aspects in which we were interested. This requirement immediately created the twin problems that such a definition would likely lead to a data set that was too time-consuming and expensive to compile, and too large to manipulate computationally. Thus the problem became one of finding a way to define technology that would result in a data set that would still be in some sense complete but that could be created expiditiously and be in a sufficiently limited and manipulatable form.

The way that technology would be defined in our study was further constrained by our decision to use a linear programming algorithm as the static optimal decision calculator. Each activity in a LP matrix consists of a vector of fixed coefficients. Set amounts of quantifiable variables that are either limited in total availability or of accounting interest make up the elements in this vector. It was natural to think of each of these activity vectors as a separate technique or part of a technique. Thus it seemed logical to define technology, at least at the final LP stage of the model, as a set of techniques where each technique would be represented by one or more LP activity vectors.

2. The Model Structure for the Task 2 Case Studies

The "Limits and Consequences of Agricultural Food Production" task case studies are primarily focused on optimization of agricultural production in a region both at present and over a time horizon of 20 to 25 years. Of particular concern are the effects on the environment of that production both internal; how production affects the process itself through feedback effects on land and water resources, and external; the effect of agriculture on nonagricultural activities in the same area. The conceptualization of the present agricultural system will be done with a static linear programming model which can be optimized given one of several possible goal functions. In order to view the process of change within the region over time a recursive component is to be added to the LP model. This recursive component will consist of a series of modules that replicate the physical processes within the agricultural system as well as the economic decisions that are made at other than the farm level. (Farm level decisions being defined by the LP model solution). It is important that the complete model be able to show the path of change in regional agricultural production as the system is confronted by shifts in the availability and relative value of inputs and changes in the level or composition of demand for agricultural commodities. The steps in the path of change will be the single period LP solutions. Each solution will indicate a preferred level of each input and output commodity and also a preferred set of techniques (activities) to transform these inputs into outputs. Thus the amount and direction of change from period to period will be dependent not only on shifts in input availability and prices and output demand but also on the techniques available that define the transformation process. Therefore which techniques are available at each decision point, and how the available set changes between decision points are essential aspects in delineating the path of change for the agriculture system. It is the function of the technology matrix generator to create a matrix of techniques in such a way that an activity set for the initial LP matrix can be chosen from them that closely reflects the techniques presently used in the region and new activity sets for each subsequent LP matrix are available that flow logically from the solution at all past decision points and are consistent with expectations about the future.

3. The Technology Matrix Generation Module

The technology matrix generator as outlined in Figure 1 is designed to take the large but fragmented knowledge available, both about the region of interest and about agricultural production processes in general, and through a consistent manual and computer algorithm develop a limited, precise matrix of agricultural production techniques. Each technique, which will act as the basis for a LP production activity, will consist of a vector of coefficents that quantify the amount of each input and each joint output connected with the production of a given unit of the major output commodity of interest. Each technique chosen for inclusion in this limited set must be demonstratably possible, complete (all important inputs and outputs) and efficient (not dominated by another vector in the set). As a whole, the set of techniques must also be complete in the sense that if there is a technique in existence that best fits a particular set of input-output prices that technique will have at least as high a probability of being in the final limited technology matrix as the particular input-output price vector has of occuring.

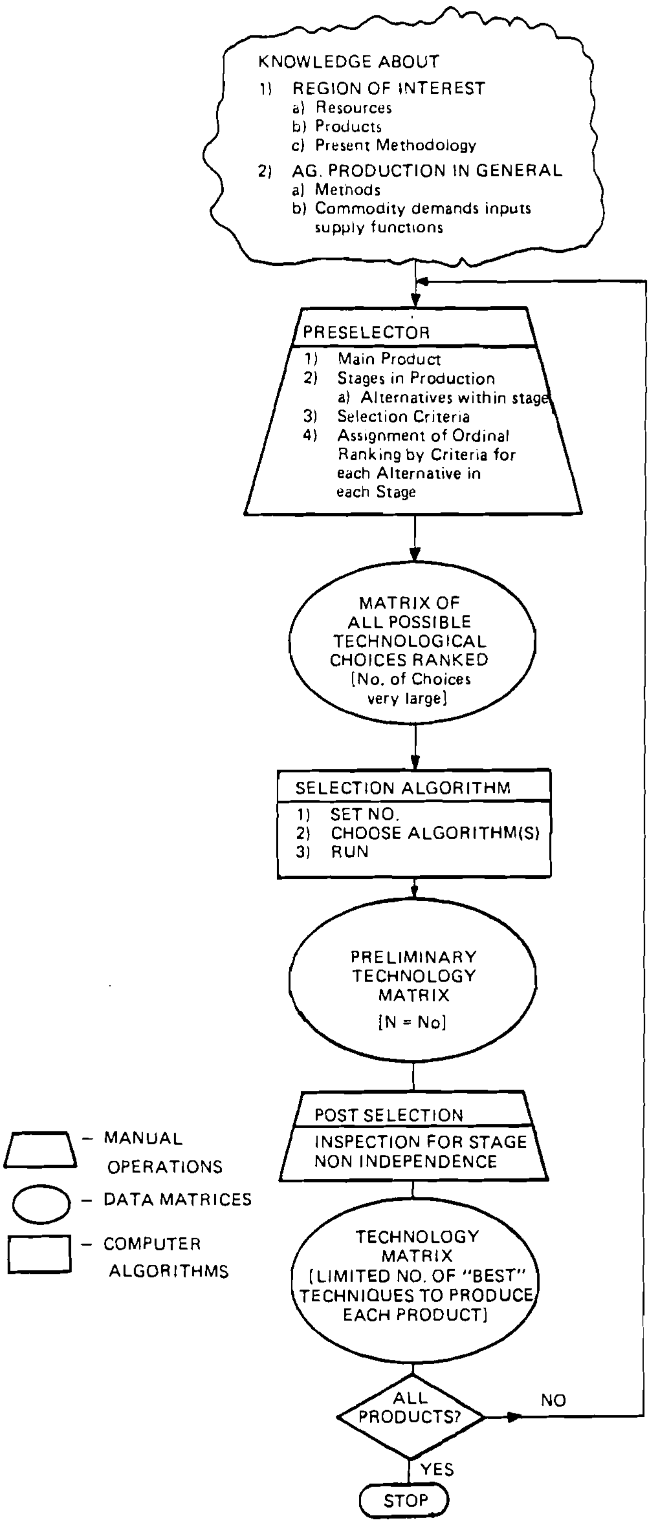

Figure 1. Technology Matrix Generator

The general purpose of the module is to take a vast amount of knowledge, break it into small chunks that can be handled, classify the chunks in a consistent manner, evaluate each chunk by a series of criteria and then assemble a limited number of production techniques that are the best combination of the chunks.

In order to accomplish these tasks, a specific sequence of manual and computer steps has been outlined. The heart of the module is a set of computer algorithms that choose a given number of efficient production techniques according to the specified criteria to produce a unit of desired output from the large number of possible techniques. Setting up the matrix of choices, selecting the criteria and inspecting the results are all important steps in the process.

The preselector stage is a manual operation carried out by a group of experts who have a wide range of knowledge concerning the region of interest and agricultural production in general. At this point a list of the important commodities that are, or could be, produced in the region is compiled. Then for each commodity the principal stages in its production process is delineated. For each stage the alternative ways to accomplish it are listed including the inputs needed and the joint products produced. It is at this point that a crucial step in the whole technology generation process must be completed. This crucial step is the gathering and ordering of selection criteria by which the elements of the alternative ways to complete each production stage will be judged. The selection and application of these criteria, which must be done by the subjective opinion of experts, allows the subsequent selection algorithm to consistently and mechanically reduce the massive but disorderly information available into a limited set of techniques which can be handled by the LP model. Because of the importance of these criteria a bit more needs to be said about them.

The selection criteria should allow evaluation of the internal structure of various ways to produce each product. Thus, besides the obvious first criteria that the less of each input used, or the less, negative joint product produced, the better, other physical relationships should be judged. The final matrix will consist of vectors where each vector will be a specific technique consisting of elements that give the fixed physical units of each input and output. As such the vector will represent an average of input levels for the amount of output. Thus a criteria that might prove useful would evaluate the variance of expected output for each alternative within the production stages. Such a criteria would allow weighting of not only the importance of the exact input mix but also the dependence of the production alternative on timing of operations and the occurance of particular weather patterns. Furthermore, the final vectors will not carry any information about the general knowledge and experience in the region with the various production alternatives. Thus another helpful criteria might judge the ability of farmers within the region to successfully use each alternative in each production stage. This judgement could be based on the complexity of the task, the length of time the operation has been used in the region, how widespread its use has been and how difficult the operation is to learn.

Other criteria might include consideration of special cultural or social conditions or other constraints that would not appear in a simple listing of inputs used and outputs produced but still would have a bearing on the agricultural production capacity of the region.

The output of the pre-selector phase is a matrix that lists all the possible technical choices by each stage of production and their ordering by each of the selection criteria. This matrix is created for each single final product in turn. By combining an element or alternative from each stage of production a complete production technique can be formulated. However, since for most products there are likely to be several production stages and many alternatives in each stage, the total number of possible combinations is very large.

It is the function of the selection algorithms to choose a limited number (N_o) of techniques from these possible combinations. The operation of these algorithms and their theoretical foundation will be discussed in the next section of this paper. The number of combinations which the algorithm is to select (N_o) must be specified by the user. Its value will indicate how many production activities for each product the LP matrix will contain and as such should be set with the tradeoff between the advantages of more choice versus ease of computation in mind. It could be set at the same level for each product or varied by the importance of the product or the variability of inputs.

The selection algorithms will generate a set of techniques, N_o in number. The final manual operation, the post selection phase, is designed to review these techniques to assure that they are possible. When the production process is broken into stages and then reassembled from these stages, an assumption of stage independence is implied. This may not be true for all alternatives in each stage and thus this inspection for unlikely or impossible combinations is necessary.

The final output from the Technology Matrix Generator is a data file consisting of sets of production techniques for each agricultural product. This data file will be used by the interface module when building and revising the LP matrix.

4. Description of the Lexicographic Morphological Method

First it is necessary to introduce several terms, notations and definitions. Let us denote: $E = \{ e_i, i \in \overline{1,n} \}$ is a finite set of system elements;

$\sigma : E \to \overline{1,L}$ is the partioning of the set E on morphological classes $\sigma^{-1}(l), l \in \overline{1,L}, \sigma^{-1}(l) \cap \sigma^{-1}(l^1) = \phi$, if $l \neq l^1$, $\sigma^{-1}(l) \neq \phi$;

L is the number of morphological classes of elements (in the case under consideration L is the number of each technology's stages); $l \in \overline{1,L}$ is l-th morphological class.

Definition 1

Morphological space Λ is called a subset of the set 2^E such that for all $\bar{\lambda} \in \Lambda$, for all $l \in \overline{1,L}$, $\bar{\lambda} \cap \sigma^{-1}(l)$ is a one element set.

It follows from definition 1 that the morphological variant $\bar{\lambda} = (\lambda_1, \dots, \lambda_l, \dots, \lambda_L)$ of a system is the set of the representatives of classes $\sigma^{-1}(l)$, $l \in \overline{1,L}$.

The morphological space Λ can be represented by the form of the L-dimensional integral lattice: $\Lambda = \prod_{l=1}^{L} \Lambda_l$, $\Lambda_l \equiv \sigma^{-1}(l)$ (Iakimets, 1980a; Iakimets, 1980b;). $x_j : E \to X_j$ is the j-th criteria by which the contribution of each system's elements to the attainment of the goal is evaluated, X_j is the scale of j-th criteria, $X_j \in R^1$, $j \in \overline{1,m}$. The set $\{\bar{x}(\lambda_l) \in X = \prod_{j=1}^{m} X_j, l \in \overline{1,L}]$ is put in correspondence to each of the variants $\bar{\lambda} = (\lambda_1,\ldots,\lambda_l,\ldots,\lambda_L) \in \prod_{l=1}^{L} \Lambda_l$.

Let us denote $\bar{z}(\bar{\lambda}) = (z_1(\bar{\lambda}), z_2(\bar{\lambda}),\ldots,z_j(\bar{\lambda}),\ldots,z_m(\bar{\lambda}))$, as the vector evaluation of variant $\bar{\lambda}$, where $z_j(\bar{\lambda})$ is a sum function from $\bar{x}_j(\lambda_l)$, i.e. $z_j(\bar{\lambda}) = z(x_j(\lambda_l))$. If the criteria are strictly ordered : $x_1 \succ x_2 \succ \cdots \succ x_m$, then the relation of the lexicographic ordering is given on the variant's evaluations.

Definition 2

Variant $\bar{\lambda}$ is lexicographically preferable to $\bar{\lambda}^1$ ($\bar{\lambda} \overset{lex}{\succ} \bar{\lambda}^1$) if and only if one of the following conditions is realized:

(1) $z_1(\bar{\lambda}) > z_1(\bar{\lambda}^1)$,

(2) $z_1(\bar{\lambda}) = z_1(\bar{\lambda}^1)$, $z_2(\bar{\lambda}) > z_2(\bar{\lambda}^1)$,

. . .

(μ) $z_j(\bar{\lambda}) = z_j(\bar{\lambda}^1)$, $j = 1,2,\ldots,\mu-1$; $z_\mu(\bar{\lambda}) > z_\mu(\bar{\lambda}^1)$,

. . .

(m) $z_j(\bar{\lambda}) = z_j(\bar{\lambda}^1)$, $j = 1,2,\ldots,m-1$; $z_m(\bar{\lambda}) > z_m(\bar{\lambda}^1)$.

Definition 3

Variant $\bar{\lambda}$ is equivalent to $\bar{\lambda}^1$ ($\bar{\lambda} \sim \bar{\lambda}^1$) if and only if the following conditions are realized: $\bar{\lambda} \sim \bar{\lambda}^1 \iff$ for all $j \in \overline{1,m}$: $z_j(\bar{\lambda}) = z_j(\bar{\lambda}^1)$.

Definition 4

Variant $\bar{\lambda}$ is no less lexicographically preferable than variant $\bar{\lambda}^1$ ($\bar{\lambda} \overset{lex}{\succeq} \bar{\lambda}^1$) if ($\bar{\lambda} \overset{lex}{\succ} \bar{\lambda}^1$) V ($\bar{\lambda} \sim \bar{\lambda}^1$).

In general the problem of generating the required number of preferable technological activities N_0 for the LP matrix of Task 2 is reduced to the generating of the first N_0 lexicographically preferable morphological variants ordered by its vector values $\bar{z}(\bar{\lambda})$.

The solution to this problem can be found by the lexographic morphological method (Iakimets, 1978; Iakimets, 1980a). This method is based on the morphological principle of generating variants and the lexicographic principle of the element's inclusion in the variant.

The morphological principle is in generating system variants containing exactly one element from each morphological class. The lexicographic principle is that the next element added to each variant is lexicographically preferable to all elements remaining in the morphological classes after the previous element has been chosen.

The lexicographic morphological method has the following main stages:

1. The statement of the problem under study;
2. The determination and ordering of criteria;

3. The determination of morphological classes;
4. The evaluation of elements of each morphological class on each criteria and the construction of the morphological matrix of element values.
5. Generating the required number of lexicographically ordered system variants.

The implementation of the first few stages is specific to the problem to be solved and is discussed in the previous sections of this paper.

The realization of the fifth stage is related to the solution of the following problem:

The vector function $\bar{z}(\bar{\lambda})$ defined in the morphological space $\Lambda = \prod_{l=1}^{L} \Lambda_l$ and with values in the criteria space X is given. It is necessary to generate N_0 lexicographically preferable morphological system variants, i.e. to find $L \subseteq \Lambda$:

1. $|L| = N_0$
2. for all $\bar{\lambda} \in L, \bar{\lambda}^1 \in \Lambda \backslash L => \bar{\lambda} \overset{lex}{\succeq} \bar{\lambda}^1$

Let us consider in detail the solution of this problem in the case when $\bar{z}(\bar{\lambda}) = \sum_{l=1}^{L} \bar{x}(\lambda_l)$.

5. Generating the Morphological Variants Lexicographically Ordered on the Sum of Vector Values of System's Elements.

The case when the vector value of the morphological variant is defined as

$$\bar{z}(\bar{\lambda}) = \sum_{l=1}^{L} \bar{x}(\lambda_l) \qquad (1)$$

will be used to explain the algorithms for generating system variants. It should be noted that various more general relationships between $\bar{z}(\bar{\lambda})$ and $\bar{x}(\lambda_l)$ could be formulated but this formulation best illustrates the principal elements in the construction of algorithms. Thus we shall consider the algorithms for this case.

We need to construct algorithms for generating N_0 element subset $L \subset \Lambda$ such that any variant from L would be lexicographically preferable to any variant from $\Lambda \backslash L$.

A universal algorithm can be constructed for this problem solution. However, the efficiency of the universal algorithm depends significantly upon correlations between the main parameters of the problem under study.

These parameters are:

N_0 - the required number of preferable variants;
n - the number of elements of the set E;
m - the number of criteria;
L - the number of morphological classes;
N - the number of all possible morphological variants of a system.

Furthermore, the efficiency of the universal algorithm depends on criterial values of elements.

The analysis of various correlations between the abovementioned parameters allows us to make conclusions about the usefulness of several algorithms.

Three algorithms were developed (Iakimets, 1980). Depending on the problem structure these algorithms can be used separately or in combination.

5.1. Algorithm "Morphoreset".

This algorithm generates all morphological variants of the system and determines the set of its values. $\{ \bar{z}(\bar{\lambda}) , \bar{\lambda} \in \Lambda \}$. All generated morphological variants are enumerated:

$$N : \overline{1,\prod_{l=1}^{L} k_l} \to \Lambda , \; k_l = | \Lambda_l | .$$

If we denote $\bar{z}(N(n)) = \bar{z}_n$, then

$$\bar{z}(\bar{\lambda}) , \bar{\lambda} \in \Lambda \equiv \left\{ \bar{z}_n , n \in \overline{1,\prod_{l=1}^{L} k_l} \right\} .$$

The standard problem of lexicographic maximization is then solved on the set $\{\bar{z}_n\}$ and N_o -element set L of lexicographically preferable variants of the system is generated. The block scheme of the algorithm is given in Figure 2.

It should be noted that the specific character of the morphological space is not used in this algorithm. Thus this algorithm is inefficient in the case when $N_o \ll N$ and N has a large value for example, $N >> 10^6$.

Using prior knowledge of the size of N_o and N allows construction of more efficient algorithms. One of these algorithms, "Filter", leads to the reduction of the dimension of the morphological space. The information about N_o is used in it to exclude from the morphological matrix all elements which have the worst criterial evaluations. The description of the algorithm "Filter" will not be discussed in this paper. Its description can be found in Iakimets (1980a).

5.2. Algorithm "Sphera".

Definition 5

The lexicographically ordered morphological space is called the space of the kind $\Lambda = \prod_{l=1}^{L} \Lambda_l$ in which each point $\bar{\lambda}$ has corresponding value of the vector function $\bar{z}(\bar{\lambda})$ defined in the criteria space $X = \prod_{j=1}^{m} X_j$ which is ordered by the lexicographic preference relation.

Let us denote $\rho(\bar{x}, \bar{y})$, $\rho : \Lambda^2 \to [0,\infty)$, $\bar{x} , \bar{y} \in \Lambda$ the distance function between two morphological variants $\bar{x}$ and $\bar{y}$.

In accordance with Iakimets (1980 a; 1980 b), the distance between two morphological variants is determined by the following relation;

$$\rho(\bar{x} ,\bar{y}) = \sum_{l=1}^{L} (1 - \delta_{x_l,y_l}) , \qquad (2)$$

$$\delta_{x_l,y_l} , = \begin{cases} 1 , & \text{if } x_l = y_l \\ 0 , & \text{otherwise} , \end{cases}$$

x_l, y_l, are correspondingly l-th elements of $\bar{x}$ and $\bar{y}$.

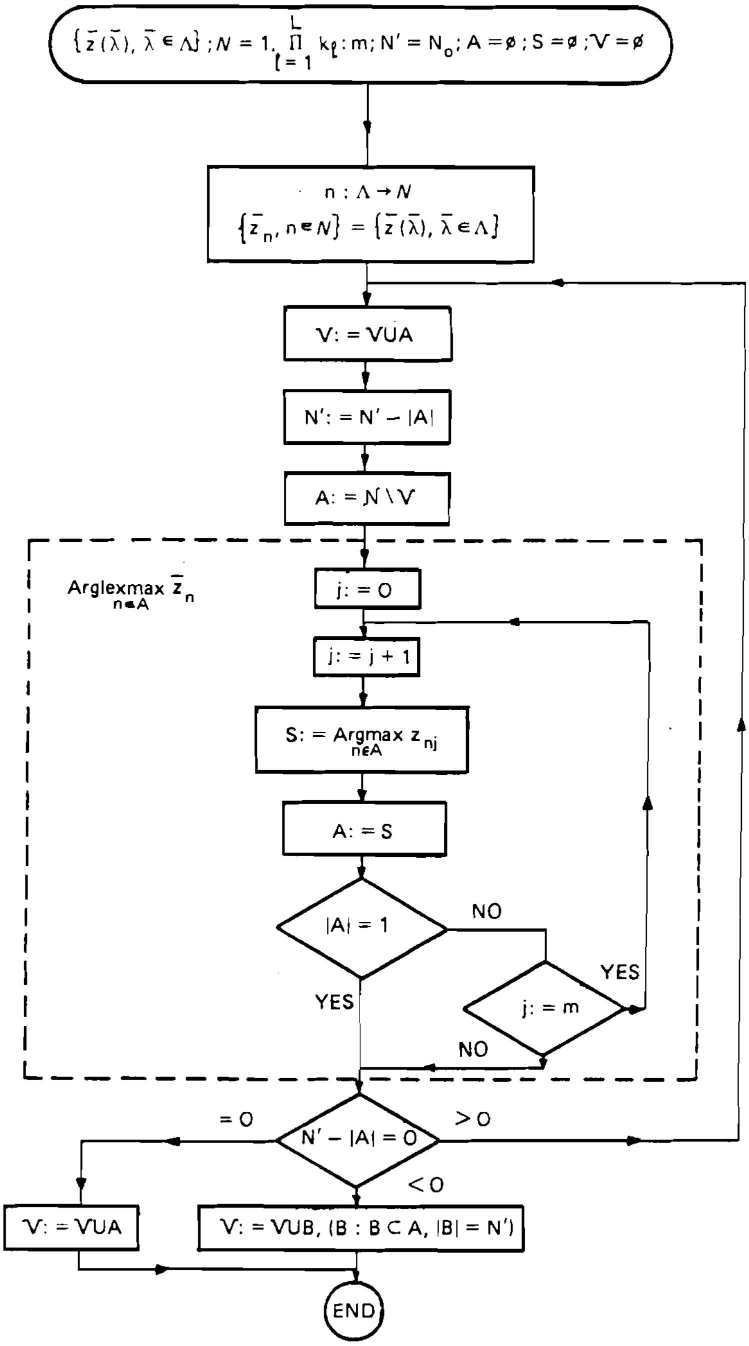

Figure 2. The algorithm "Morphoreset" block-scheme.

Definition 6

The spherical neighborhood $W_{r,L}(\bar{x})$ of radius r (r is an integral non-negative number) about point $\bar{x}, \bar{x} \in \Lambda$ is called the subset of points $\bar{y}, \bar{y}, \in \Lambda$ such that

$$W_{r,L}(\bar{x}) = \{\bar{y} : \rho(\bar{x}, \bar{y}) \leq r\}. \qquad (3)$$

From two lexicographically equivalent morphological variants the first will be one which is morphologically closest (in a sense of the relation (2)) to the lexicographic best variant $\bar{\lambda}_1 = \underset{\lambda \in \Lambda}{\mathrm{arglexmax}}\, \bar{z}(\lambda)$.

Theorem (Iakimets 1980 a)

The first N_o morphological variants lexicographically ordered on vector values are contained in the spherical neighborhood $W_{r,L}(\bar{\lambda}_1)$ of the radius $r \leq \log_2 N_o$ about point $\bar{\lambda}_1$. This estimate cannot be improved, when $\log_2 N_o \leq L$.

The results of the investigation of characteristics of the lexicographically ordered metrized morphological space are the basis of the algorithm "Sphera".

This algorithm has two main blocks. The lexicographic ordering of the elements of each morphological class is realized in the first block. Let us introduce the following notation for the description of the first block:

Λ_l^1 is the domain of the lexicographic maximization;

μ_l is the index of representative of the lexicographically ordered classes of equivalency of elements of the l-th morphological class;

$\bar{x}(\mu_l)$ is the lexicographically maximal element;

A_{μ_l} is the set of solutions of lexicographic maximization problem;

ν_{μ_l} is the capacity of the set A_{μ_l};

The initial conditions are : $l = 0$, $\Lambda_l = \Lambda_l^1$, $L, \mu_l = 0$, $A_{ol} = \phi$, $\nu_{\mu_l} = 0$.

The following actions are realized on the l-th step of the algorithm.

1. $l := l + 1$.
2. Determine a new set Λ_l^1 by exclusion from the previous one all solutions $A_{\mu_l} : \Lambda_l^1 := \Lambda_l^1 \setminus A_{\mu_l}$.
3. If $\Lambda_l^1 \neq \phi$, then go to 4, if $\Lambda_l^1 = \phi$, then go to 8.
4. $\mu_l := \mu_l + 1$.
5. Determine $A_{\mu_l} = \underset{\lambda \in \Lambda_l^1}{\mathrm{Arglexmax}}\, \bar{x}(\lambda)$
6. $\nu_{\mu_l} := | A_{\mu_l} |$.
7. Determine $\bar{x}_{\mu_l} = \underset{\lambda \in \Lambda_l^1}{\mathrm{lexmax}}\, \bar{x}(\lambda)$ and go to 2.
8. If $l \leq L$, then go to 1. If $l > L$, then go to the second block for generating the required number of preferable variants.

In the second block of the algorithm the required number (N_o) of preferable morphological variants which are contained in the spherical neighborhood of radius $\log_2 N_o$ about the lexicographic maximal variant are generated. The feature of the inclusion of the lexicographic preference relation in the relation of partial order (which is defined by the principle of coordinate-wise comparison of vectors) allows construction of the procedure for generating lexicographically preferable system variants by the sequential analysis of the set of Pareto-optimal variants, which are determined at each step of the algorithm. In our case, it can be written as follows:

$$\text{for all } l: \lambda_l \leq \lambda_l^1 \iff \bar{\lambda} \ll \bar{\lambda}^1 \Rightarrow \bar{z}(\bar{\lambda}) \overset{lex}{\succeq} \bar{z}(\bar{\lambda}^1).$$

where $\ll$ is the relation of partial order.

Let us introduce the following notations:

- i is the index of the step ,
- π_i is the set of Pareto-optimal variants on the i-th step ,
- ν is the moving number of variants which have been generated,
- A is the set of lexicographically preferable morphological variants.

The initial conditions are: $i = 0$, $\nu = 0$, $\pi_1 = (1, ..., 1)$.

1. $i := i + 1$
2. Determine $A = \operatorname{Arglexmax}_{\bar{\mu} \in \pi_i} \bar{z}(\bar{\mu})$
3. $\nu := \nu + \sum_{\bar{\mu} \in A} \prod_{l=1}^{L} \nu_{\mu_l}$.
4. If $\nu < N_o$, then go to 5 , if $\nu > N_o$, then go to 10 , if $\nu = N_o$, then go to 11.
5. $l := l + 1$ and for each $\bar{\mu} \in A$ determine by turns $\bar{\zeta} = \bar{\mu} + \bar{e}_l$, $\bar{e}_l = (0,0, \cdots ,0,1,0, \cdots ,0)$ is the unit vector.
6. Check the availability in $\pi_i \backslash A$ element which has even if one of (l - 1) first coordinates which is more than corresponding coordinate in $\bar{\zeta}$. If such an element $\bar{\zeta}^1$ of the set $\pi_i \backslash A$ exists, then go to 7, otherwise go to 8.
7. Exclude $\bar{\zeta}^1$ from again generating set π_i.
8. $\pi_i := (\pi_i \backslash A) \cup \{\bar{\zeta}\}$.
9. if $A \neq \phi$, then go to 5 , if $A = \phi$, then go to 1.
10. Exclude the last ν vectors and go to 11.
11. End

6. Examples

Example 1

To clarify the workings of the algorithm "Morphoreset", a simple numerical example will be used.

Given:

L (the number of morphological classes) = 3

m (the number of criteria) = 2

k_l (the number of elements in l-th morphological class) : $k_1=3, k_2=4, k_3=2$.

There are 24 possible morphological variants in this example,

$$N = \prod_{l=1}^{L} k_l = 24$$

We shall set the required number of variants N_o equal to 4.

Data:

Stages (l)		Criteria (j)	
		K_1	K_2
A	λ_{11}	10	5
	λ_{12}	5	9
	λ_{13}	10	6
B	λ_{21}	7	2
	λ_{22}	6	3
	λ_{23}	8	5
	λ_{24}	5	7
C	λ_{31}	6	1
	λ_{32}	7	5

(1) Generating all morphological variants (N = 24)

1. $(\lambda_{11}, \lambda_{21}, \lambda_{31})$
2. $(\lambda_{11}, \lambda_{21}, \lambda_{32})$
3. $(\lambda_{11}, \lambda_{22}, \lambda_{31})$

. . .

24. $(\lambda_{13}, \lambda_{24}, \lambda_{32})$

(2) Calculation of vector values of each variant

1) (10 + 7 + 6 , 5 + 2 + 1) = (23, 8)
2) (10 + 7 + 7 , 5 + 2 + 5) = (24, 12)
3) (10 + 6 + 6 , 5 + 3 + 1) = (22, 9)

. . .

24) (10 + 5 + 7 , 6 + 7 + 5) = (22, 18)

(3) Order lexicographically and choose the first N_o variants

1) $\bar{\lambda}_{22} = (\lambda_{13}, \lambda_{23}, \lambda_{32}), \bar{z}_1(\bar{\lambda}_{22}) = (25,16)$,
2) $\bar{\lambda}_6 = (\lambda_{11}, \lambda_{23}, \lambda_{32}), \bar{z}_2(\bar{\lambda}_6) = (25,15)$,
3) $v\bar{\lambda}_{18} = (\lambda_{13}, \lambda_{21}, \lambda_{32}), \bar{z}_3(\bar{\lambda}_{18}) = (24,13)$,
4) $\bar{\lambda}_2 = (\lambda_{11}, \lambda_{21}, \lambda_{32}), \bar{z}_4(\bar{\lambda}_2) = (24,12)$.

Example 2.

The procedure of generating lexicographically preferable morphological systems' variants with the help of the algorithm "Sphera" is illustrated in the case of the two dimensional morphological space.

Data

1. $L = 2$; $k_1 = 3$; $k_2 = 4$; $m = 3$; $n = 12$.
2. $K_1 \succ K_2 \succ K_3$
3. Matrix of criterial evaluations of elements.

Stages	Elements	Criteria		
		K_1	K_2	K_3
A	a_1	10	7	6
	a_2	5	10	3
	a_3	7	1	10
B	b_1	6	6	5
	b_2	7	8	1
	b_3	10	5	5
	b_4	10	5	4

(1). Lexicographic Ordering Elements

Stages	Elements	Criteria		
		K_1	K_2	K_3
A	$1 = a_1$	10	7	6
	$2 = a_3$	7	1	10
	$3 = a_2$	5	10	3
B	$1 = b_3$	10	5	5
	$2 = b_4$	10	5	4
	$3 = b_2$	7	8	1
	$4 = b_1$	6	6	5

(2). Generating the required number of lexicographically ordered variants.

If we wish to generate $N_o=2$, then it is necessary to look through the spherical neighborhood with the radius of $r_1 = \log_2 2 = 1$, for $N_o=4$, $r_2=\log_2 4=2$, for $N_o=6$, $r_3=\log_2 6>2$ for $N_o=9$, $r_4=\log_2 9=3$ and so on. In other words, if $N_o>2$, then in the case under consideration we shall look through the whole morphological space.

The sequence of the step-by-step analysis of the whole morphological space with the help of the second block of the algorithm "Sphera" is shown in the following figures (Figs. 3, 4, and 5). The discrete morphological space of system's variants in these figures is illustrated by single points. The sets of Pareto-optimal variants in these diagrams are denoted by Π_i , where i is the step of the procedure. λ_i^* denotes the lexicographically preferable variant which is determined from the set Π_i. All variants marked by circles are those which were selected at the previous step (i - 1) of the procedure.

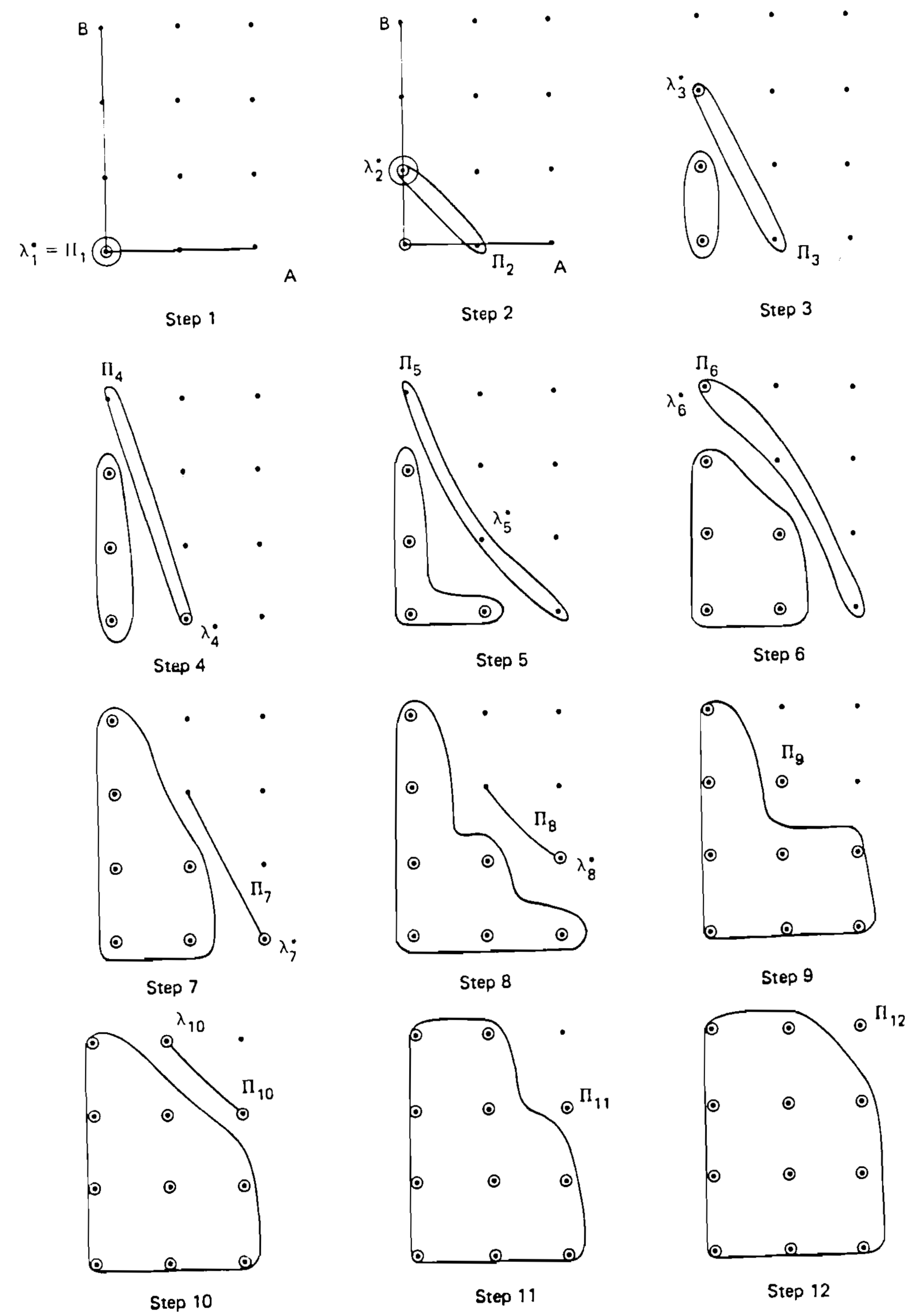

Figure 3. Generating pareto-optimal morphological variants.

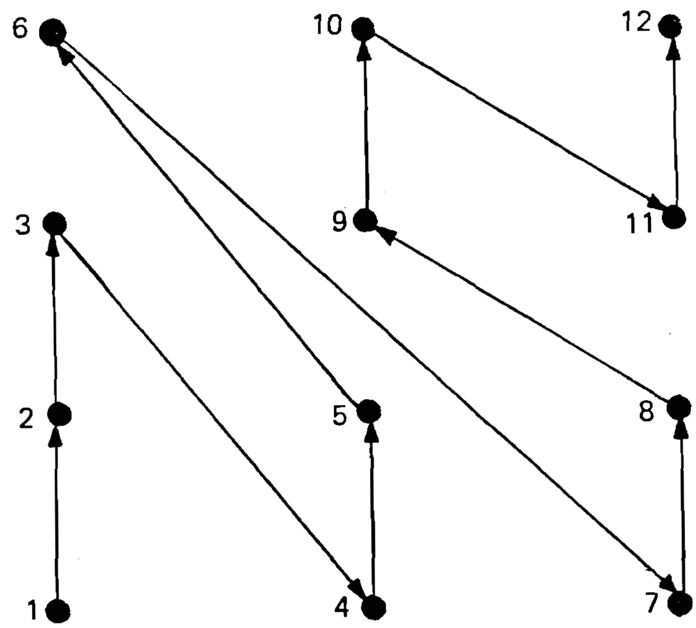

Figure 4. The resultant sequence of generated lexicographically ordered variants (λ_i^*) is shown in this simplified diagram. The vector evaluation of all variants is given in the next figure.

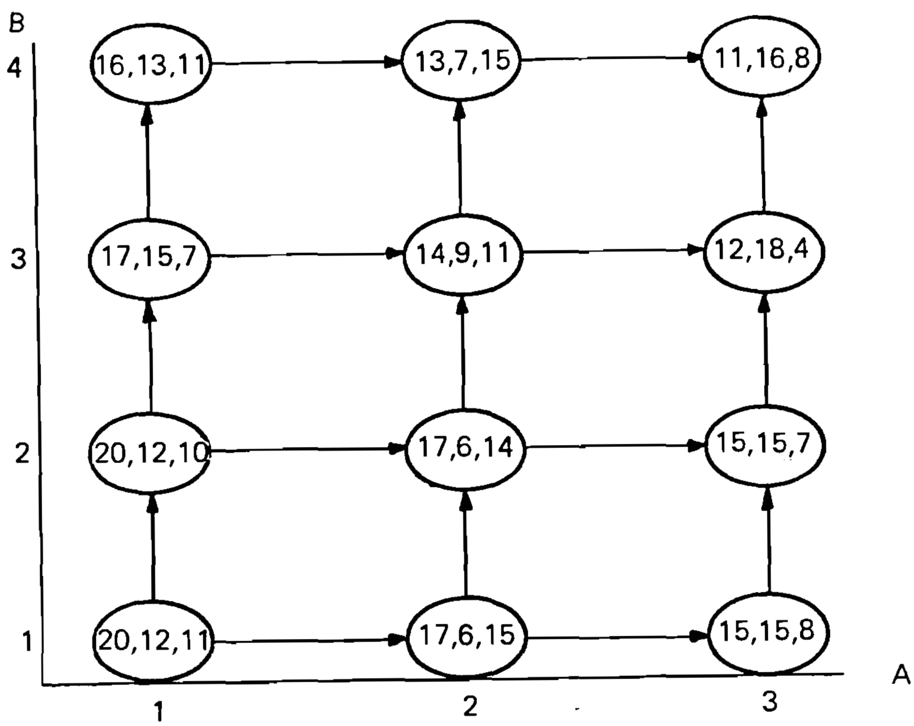

Figure 5. Lexicographically ordered morphological space.

7. Remarks on the Procedure of Data Preparation for the Lexicographic Morphological Method.

The information to be prepared for generating the required number of preferable morphological variants of the system under consideration include :

1. Morphological description of a system
2. List of criteria and its ordering
3. Criterial evaluations of system elements
4. The number of preferable variants to be generated.

The first three blocks of information allow construction of the morphological matrix of evaluations of system elements. The fourth block is determined by the researcher (or decision maker).

The general procedure of data preparation consists of the following stages:

(1) Verbal description of the system,

(2) Determination of main system functions (or technological stages)

(3) Determination of all possible variants of each function (i.e. elements).

(4) Analysis of the goals of the system and determination of the criteria

(5) Strict ordering of the criteria

(6) Evaluation of each element by each criteria.

(7) Determination of the required number of preferable system variants (No).

The first four stages of this procedure are connected with review, analysis and structuring information about the possible technology and goals of its utilization. The implementation of these stages is usually done with the help of experts or on the basis of analysis of patents, books and papers on the technology under consideration. The specific methods of the morphological system description can also be used (Zwicky, 1969; Odrin, Kartavov, 1970;)

The fifth stage can be implemented either by the researcher himself or by a decision-maker. It should be noted that the ordering of the criteria can be changed depending on how many variants of the system we wish to construct and the amount of uncertainty concerning the proper criteria to judge this system.

The sixth stage can be implemented by experts, researchers, or decision-makers. If we have several experts to evaluate each of the systems elements on each criteria, then we need to synthesize a collective opinion. There are several methods for this operation. In some cases we can generate the lists of preferable system variants using the information about criterial evaluations of elements by each expert separately. Thus we will obtain several lists of preferable variants. The determination of the final list of preferable variants is realised by informal discussion of these lists.

The seventh stage consists of the determination of the required number of preferable variants. This is done on the basis of existing limitations of the problem under study.

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