HELSINKI GSE DISCUSSION PAPERS 19 · 2023

Optimal Policy with R&D-based Growth and the Risk of Environmental Disaster

Tapio Palokangas

Helsinki GSE Discussion Papers

Helsinki GSE Discussion Papers 19 · 2023

Tapio Palokangas:
Optimal Policy with R&D-based Growth and the Risk of Environmental Disaster

ISBN 978-952-7543-18-4 (PDF)
ISSN 2954-1492

Helsinki GSE Discussion Papers:
https://www.helsinkigse.fi/discussion-papers

Helsinki Graduate School of Economics
PO BOX 21210
FI-00076 AALTO
FINLAND

Helsinki, November 2023

Optimal Policy with R&D-based Growth and the Risk of Environmental Disaster

Tapio Palokangas*

November 10, 2023

Abstract

The extraction of carbon energy contributes to the global stock of pollution, increasing the risk of welfare-damaging environmental disaster. The governments of the countries educate workers as scientists. Oligopolists produce goods by workers and carbon energy. R&D firms improve efficiency by scientists to supplant incumbent oligopolists through competition, which generates economic growth. In this setup, an international central planner can decentralize the social optimum by setting a precautionary tax on emissions before the occurrence of the disaster. That tax hampers pollution, but speeds up economic growth. The socially optimal level of the tax is derived.

*Journal of Economic Literature: H*21*; O*32*; O*44*; Q*54*; Q*58

Keywords: emissions, pollution, R&D, endogenous growth, environmental disaster, precautionary policy

*Tapio Palokangas, University of of Helsinki, Helsinki GSE, IIASA (Laxenburg, Austria). Address: P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland. Email: **tapio.palokangas@helsinki.fi**. Homepage: **https://blogs.helsinki.fi/palokang**. The author thanks IIASA for the opportunity to visit in Summer 2012 and to present earlier versions of this document in its seminars. He is also grateful to the participants of the 15th Viennese Conference on Optimal Control and Dynamic Games (July 2022, Wien, Austria) for constructive comments.

1 Introduction

The European Union (EU) attempts to control global pollution by taxing CO_2 emissions and subsidizing R&D for technologies that use little fossil fuels. Still, very little attention has been paid to the comprehensive analysis of optimal policy in the case where growth is generated by R&D and education, but carbon emissions threaten to trigger an environmental disaster. This document aims to fill this gap in the literature.

There has been two approaches to examining environmental effects of global pollution. One assumes that the stock of pollution (e.g., atmospheric carbon) affects welfare *incrementally* at each moment of time.[1] This document, however, focuses on the alternative approach in which environmental degradation is examined as a low-frequency shock:[2] the disaster is a random regime shift that occurs only once, with the post-event regime holding indefinitely. Recurrent events, where several shifts occur at random times with independent intervals, can be analyzed using the same methodology.[3]

Polasky et al. (2011) analyze how the threat of future regime shift affects the optimal management of natural resources. They focus on harvesting a renewable resource (e.g., fishery), whose growth rate depends on the regime and whose stock can trigger a regime shift. They show that the possibility of the regime shift encourages *precautionary* policy, i.e., the policy maker's willingness to maintain a large stock of the resource. In many dynamic models of pollution, the damage function is assumed to be *smooth*.[4] Because the policy maker can then immediately respond at the moment pollution occurs, there is no need for precautionary policy. In this document, the damage function is assumed to be *discrete* due to the regime shift, so that precautionary policy is needed before the occurrence of the disaster.

Haurie and Moresino (2006) examine the central planner's optimal pol-

[1]Cf., Acemoglu et al. (2016) motivate this approach by the possibility to calibrate the parameters of the model by US microdata. It would be possible to add incremental effects also into the model of this document by inserting the stock of pollution, P, directly into the utility or production function. Because that extension would excessively complicate the model, it is not pursued in the study.

[2]Cf., Tsur and Zemel (2008, 2009), Polasky et al. (2011), and de Zeeuw and Zemel (2012).

[3]Cf., de Zeeuw and Zemel (2012).

[4]E.g., van der Ploeg and de Zeeuw (1992), and Dockner and Long (1993).

icy in an economy with two different physical capital stocks: one being the general productive physical capital; and the other being an equipment that will alleviate the social cost of the catastrophe when it occurs. In that setup, the optimal precautionary policy is to maintain a large stock of the alleviating capital. As shown in Golosov et al. (2014), physical capital could be introduced as an additional factor of production into a model of endogenous growth. Because this document focuses on endogenous innovation, and because the coexistence of physical capital and a random shock would excessively complicate the analysis, that extension is not pursued in the study.

Without physical capital, this document does not consider the *green paradox*: if investment in capital is irreversible in several sectors and the implementation of corrective taxes is delayed, then the response of the emitting sectors before the implementation can undo some of their responses following the implementation.[5] In the model of this document, the precautionary taxes are immediately implemented and kept until the expected shock occurs.

Tsur and Zemel (2008, 2009) and de Zeeuw and Zemel (2012) consider the management of a system that is subject to the risk of an abrupt and random jump in pollution damage. They examine a market economy where firms employ labor, capital and two energy inputs that are perfect substitutes: green input that does not emit, and brown input whose emissions accumulate the"hazardous" stock that threatens to trigger the damaging change. As a result, they obtain a Pigouvian tax on the "hazardous" input. In the model of this document, there is a market economy where oligopolists employ labor and carbon energy, R&D employs scientists and the governments educate workers as scientists. R&D and education generate economic growth and the extraction of carbon energy accumulates the "hazardous" stock of pollution.

Palokangas (2021) introduces optimal taxation into a market economy where families determine fertility, investment in capital and mortality-decreasing health care, while capital accumulation and population growth threatens to trigger an environmental shock with a discrete increase in mortality. In that case, capital and population are the "hazardous" stocks. Consequently, the optimal precautionary policy is to tax capital income and health care. In the model of this document, the externality of pollution is

[5] Cf., Sinn (2008), Valente (2011), Tsur and Zemel (2011), Smulders et al. (2012) and Afonso et al. (2021).

the same as in Palokangas (2021), but instead of population growth there is endogenous growth that is generated by R&D and education.

In the model, there is a number of countries. In each country, economic growth is based on *vertical R&D* that improves the efficiency of production with time. If the countries were of different size, then they would tend to grow at the different rates in the steady state. This *scale effect* can be eliminated by introducing *horizontal R&D*, where the firms adjust so that they all, and consequently all countries, will in the end grow at the same rate (cf., Peretto 1998, and Dinopoulos and Thompson 1999), into the model. Because this document focuses on endogenous vertical innovation, and because horizontal R&D would unnecessarily complicate the model, the scale effect is eliminated here by the assumption that the countries are of equal size.

In the model, the countries educate workers as scientists, but private education is ignored, for simplicity. Because the countries are small, they ignore the effect of their emissions on the risk of global disaster. Consequently, there must be a central planner that controls pollution by international taxes.

The remainder of this document is organized as follows. The structure of the economy is defined in Section 2. Utility and the externality through emissions is introduced in Section 3. Production is modeled in Section 4. R&D and education are defined as sources of economic growth in Section 5. The government's behavior is examined in Section 6 and the central planner's optimal policy in Section 7. The results are summarized in Section 8.

2 Structure of the economy

The economy contains a large number of identical countries and a benevolent central planner. In each country, there is a benevolent government that can independently exercise public policy. To make the supply of workers exogenous, for simplicity, it is assumed that the fertility rate is equal to the exogenous mortality rate α.[6] Local governments train some of the workers

[6] In the models of population growth (cf., Becker 1981 and Palokangas 2021), the fertility rate is made endogenous by the assumption that the children are a kind of consumption good in their parents' preferences. Even in that case, the fertility rate is equal to the mortality rate in the steady state of the model. Because endogenous fertility would excessively complicate the model without new relevant results, it is not pursued here.

as scientists for R&D, leaving the remainder of them to be employed in production. Aggregate emissions due to the extraction of carbon energy for production contribute to the aggregate stock of pollution, increasing the risk of welfare-damaging disaster.

The model is organized as an extended form game where the decisions are exercised in the following order:

(i) The *central planner* recognizes the link from the extraction of carbon energy to the risk of the disaster. It sets international taxes.

(ii) Each local *government* recognizes the risk of the disaster being unable to affect it, trains labor as scientists and sets taxes in their jurisdictions.

(iii) The wages and the interest rate clear the labor and capital markets.

(iv) R&D firms employ scientists to innovate to supplant oligopolists.

(v) Oligopolists produce intermediate goods by workers and carbon energy.

(vi) Competitive firms extract carbon energy from the nature by workers and produce the final good from the intermediate goods.

(vii) Households allocate their consumption over time.

By the principle of dynamic programming, this game is solved backwards.

3 Utility and pollution

In the model, time t is continuous. The households are risk averse, i.e., their *constant rate of relative risk aversion* (CRRA) is $\sigma < 1$. Their *rate of time preference*, $\rho > 0$, is constant.[7] At time T, the representative household derives utility from its consumption c and the *state of nature*, q, over the foreseeable future $t \in [T, \infty)$ as follows:

$$\int_T^\infty qc^{1-\sigma} e^{\rho(T-t)} dt, \quad 0 < \sigma < 1, \quad \rho > 0. \tag{1}$$

[7] The exogenous mortality rate α increases the effective rate of time preference, but this increase is included in the constant ρ, for convenience.

Aggregate emissions m contribute to the stock of pollution, P, but the nature absorbs a constant proportion β of that stock:

$$\dot{P} \doteq \frac{dP}{dt} = m - \beta P, \quad \beta > 0, \quad P(T) = P_T, \tag{2}$$

where P_T is the initial value of P at time T. The environmental disaster decreases the state of nature, q, discontinuously from 1 to constant $\varphi \in (0,1)$. An increase in pollution P increases the *probability* of that disaster, π. Thus, the state of nature is determined as follows:

$$q = \begin{cases} \varphi \in (0,1) & \text{with probability } \pi(P) \in (0,1), \\ 1 & \text{with probalility } 1 - \pi(P), \end{cases} \qquad \pi' > 0. \tag{3}$$

The household maximizes its utility (1) by its consumption c, given the interest rate r and the state of nature, q. In this case, the standard analysis shows that consumption c evolves according to the *Euler equation*

$$g \doteq \frac{\dot{c}}{c} = \frac{r-\rho}{\sigma} \quad \Leftrightarrow \quad r = \rho + \sigma g,$$

where g is the growth rate of consumption c. Then, the *effective rate of time preference* in a growing economy is the interest rate minus the growth rate: $r - g = \rho + (\sigma - 1)g$. The expectation on the disaster (i.e., the fall of q from 1 to φ) causes the relative loss $1 - \varphi$ in periodic utility $qc^{1-\sigma}$ with probability π [cf., (1) and (3)]. Consequently, before the disaster, $q = 1$, the expected relative loss $(1-\varphi)\pi$ increases the effective rate of time preference. Thus, the *effective precautionary rate of time preference* can be defined as follows:

$$\gamma \doteq \rho + (\sigma - 1)g + (1 - \varphi)\pi. \tag{4}$$

4 Production

4.1 Extraction of carbon energy

In the model, the unit of carbon energy is defined so that the extraction of it generates one unit of emissions m. Competitive firms extract carbon energy m with increasing and convex costs in terms of workers' labor:

$$v(m), \quad v(0) = 0, \quad v' > 0, \quad v'' > 0. \tag{5}$$

The government sets the *ad valorem* tax x on carbon energy. Then, competitive firms set the price for carbon energy, μ, equal to $1+x$ times the workers' wage w times the marginal extraction costs in terms of labor, v' [cf., (5)]:

$$\mu = (1+x)wv'(m) \text{ with } v'' > 0. \tag{6}$$

4.2 The final good

There is one final good that is chosen as the *numeraire* in the model. Competitive firms produce the output of that good, y, from a continuum of intermediate goods $j \in [0,1]$ according to the CES function

$$y = \left(\int_0^1 y_j^{1-1/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)} \text{ with } \varepsilon > 1, \tag{7}$$

where y_j is the quantity of intermediate good j and ε is the constant *elasticity of substitution* between any pair of the inputs $j \in [0,1]$. From this it follows that in equilibrium the price of each intermediate good j, p_j, is equal to the marginal product that good, $\frac{\partial y}{\partial y_j}$ [cf., (7)]:

$$p_j = \frac{\partial y}{\partial y_j} = \left(\frac{y}{y_j} \right)^{1/\varepsilon} \text{ with } \frac{y_j}{p_j}\frac{\partial p_j}{\partial y_j} = -\frac{1}{\varepsilon} \text{ for } j \in [0,1]. \tag{8}$$

4.3 Oligopolistic competition

Each intermediate good $j \in [0,1]$ is produced by a different oligopolist with the same label j. Oligopolist j produces its output y_j from labor l_j and carbon energy m_j with efficiency a by constant-returns-to-scale technology

$$\begin{aligned} &y_j = af(l_j, m_j), \ \ f_l \doteq \frac{\partial f}{\partial l_j} > 0, \ \ f_m \doteq \frac{\partial f}{\partial m_j} > 0, \ \ f_{ll} \doteq \frac{\partial^2 f}{\partial l_j^2} < 0, \\ &f_{mm} \doteq \frac{\partial^2 f}{\partial m_j^2} < 0, \ \ f_{lm} \doteq \frac{\partial^2 f}{\partial l_j \partial m_j} > 0, \ \ f \text{ linearly homogeneous.} \end{aligned} \tag{9}$$

Oligopolist j maximizes its operating profit

$$\Pi_j \doteq p_j y_j - w l_j - \mu m_j \tag{10}$$

by its inputs (l_j, m_j) subject to its production technology (9) and the demand for its output, (8), given its efficiency a, the wage w for labor l_j, the price μ

for carbon energy m_j and the aggregate quantity of the final good, y. This yields the first-order conditions

$$w = \left(p_j + y_j \frac{\partial p_j}{\partial y_j}\right)\frac{\partial y_j}{\partial l_j} = \left(1 - \frac{1}{\varepsilon}\right)p_j a f_l(l_j, m_j), \tag{11}$$

$$\mu = \left(p_j + y_j \frac{\partial p_j}{\partial y_j}\right)\frac{\partial y_j}{\partial l_j} = \left(1 - \frac{1}{\varepsilon}\right)p_j a f_m(l_j, m_j). \tag{12}$$

Inserting (11) and (12) into (10) and noting the linear homogeneity of the function f in (9), one obtains the equilibrium profit as follows:

$$\Pi_j = p_j a[f - (1 - 1/\varepsilon)(\underbrace{f_l l_j + f_m m_j}_{f})] = p_j a f(l_j, m_j)/\varepsilon \text{ for } j \in [0,1]. \tag{13}$$

4.4 The equilibrium of the production sector

The demand for carbon energy m_j by the oligopolists $j \in [0,1]$ is equal to the supply of carbon energy, m: $m = \int_0^1 m_j dj$. In the system (5)-(13) and $m = \int_0^1 m_j dj$, there is perfect symmetry over $j \in [0,1]$. Thus, in the equilibrium, it holds true that $l_j = l$, $m_j = m$, $p_j = 1$, $y_j = y$, $\Pi_j = \Pi$ and

$$y = af(l,m), \quad \Pi = \frac{af}{\varepsilon}, \quad w = \left(1 - \frac{1}{\varepsilon}\right)af_l, \quad \frac{f_m}{f_l} = \frac{\mu}{w} = (1+x)v'. \tag{14}$$

5 Sources of economic growth

5.1 Quality ladders

Efficiency a has quality ladders: the previously accumulated knowledge in R&D improves the productivity of present scientists who work on R&D, i.e., they "stand on the shoulder of past giants" (Acemoglu 2009, p. 444). Thus, the *state-of-the-art efficiency* is given by

$$a \doteq \max_{\kappa \in [0,1]} a_\kappa. \tag{15}$$

A R&D firm employs scientists s_j in an attempt to displace incumbent oligopolist j as the producer of good j. In a short period dt, it succeeds in increasing its efficiency a_j over and above the state-of-the-art efficiency a

with probability $\delta s_j dt$, but fails to do so with probability $1 - \delta s_j dt$, where δ is a *scientist's exogenous productivity.* This can be modeled as follows:

$$\frac{da_j}{a} = \delta s_j dt. \tag{16}$$

By (16), efficiency in the production of good j evolves according to

$$\dot{a}_j \doteq \frac{da_j}{dt} = \delta s_j a, \quad s_j \geq 0, \quad a(T) = a_T, \tag{17}$$

where a_T is the initial of efficiency a at time T.

In the short period dt, the expected revenue of a R&D firm as the producer of good j is $\Pi \delta s_j dt$, where $\delta s_j dt$ is its probability of success [cf., (16)] and Π [cf., (14)] its operative profit in the case of success. The expenditures of that firm in the period dt are $\varpi s_j dt$, where ϖ is the scientists' wage. Thus, in the period dt, the expected profit of that R&D firm is $\Theta_j \doteq \delta \Pi s_j dt - \varpi s_j dt = (\delta \Pi - \varpi) s_j dt$. With free entry, the profit Θ_j is in equilibrium equal to zero and the scientists' wage ϖ is determined as follows [cf., (14)]:

$$\Theta_j = 0 \quad \Leftrightarrow \quad \varpi = \delta \Pi = \delta a f(l, m)/\varepsilon. \tag{18}$$

5.2 Technological change

Because, in the short period dt, the probability of the R&D firm to succeed in innovation, (16), is small, then the probability that two (or more) R&D firms would sumultaneously innovate in that small period dt is approximately zero relative to the length of the period dt. Thus, by (16), the probability that any *R&D* firm innovates in that period dt is

$$\begin{aligned}\frac{da}{a} &= \lim_{dt \to 0} \left[\int_0^1 \frac{da_j}{a} dj + \int_{j \in [0,1]} \int_{\kappa \in [0,1]} \frac{da_j}{a} \frac{da_\kappa}{a} dj\, d\kappa \right] \\ &= \delta\, dt \lim_{dt \to 0} \Bigg[\underbrace{\int_0^1 s_j dj}_{=s} + \underbrace{\left(\int_{j \in [0,1]} s_j dj \int_{\kappa \in [0,1]} s_\kappa d\kappa \right) dt}_{\to 0} \Bigg] = \delta s\, dt,\end{aligned}$$

where $s \doteq \int_0^1 s_j dj$ is the total mass of scientists. This is equivalent to

$$\dot{a} \doteq \frac{da}{dt} = \delta s a, \quad a(T) = a_T. \tag{19}$$

5.3 Education

In each country, the government educates new scientists n from workers with increasing and convex costs:

$$z(n), \quad z' > 0, \quad z'' > 0, \quad z(0) = 0. \tag{20}$$

Because, with the exogenous mortality rate α, the proportion α of scientists dies at each time t, the mass of scientists, s, evolves according to

$$\dot{s} \doteq \frac{ds}{dt} = n - \alpha s, \quad 0 < \alpha < 1, \quad s(T) = s_T, \tag{21}$$

where n is education and s_T the initial mass of scientists s at time T.

5.4 The goods market

Because the fertility rate is equal to the mortality rate, the mass of the labor force is constant. It is normalized at unity, for convenience. In equilibrium, it is equal to labor in production, l, labor in the extraction of carbon energy, v, and labor being trained as scientists, z: $1 = l + v + z$. By this, (5) and (20), labor input l is a function of carbon energy m and education n:

$$l(m,n) \doteq 1 - v(m) - z(n) \text{ with } \frac{\partial l}{\partial n} = -z' \text{ and } \frac{\partial l}{\partial m} = -v'. \tag{22}$$

By (14) and (22), the marginal costs of carbon energy m and education n can be defined in terms of output y as follows:

$$\zeta_m \doteq -\frac{\partial y}{\partial l}\frac{\partial l}{\partial m} = af_l v' > 0, \quad \zeta_n \doteq -\frac{\partial y}{\partial l}\frac{\partial l}{\partial n} = af_l z' > 0. \tag{23}$$

6 The countries

6.1 The representative country

Because the countries are small, their governments take aggregate pollution P as given and observes the state of nature as follows [cf., (3)]:

$$q = \begin{cases} \varphi \in (0,1) & \text{with probability } \pi \in (0,1), \\ 1 & \text{with probalility } 1 - \pi, \end{cases} \tag{24}$$

where the risk of the environmental disaster, $\pi(P)$, is given. Because the countries are identical, it is possible to focus on the behavior of the representative country (called hereafter as the *country*) and its government.

The central planner sets the international tax τ on emissions m and balances its budget by paying the transfer R per person to the labor force 1 in each country. The government of the country includes the revenue from its *ad valorem* tax x [cf., (6)], its costs of education, $wz(n)$, and its net payments to the central planner, $\tau m - R$, in is its budget, balancing this by a non-distorting tax on the labor force 1.

In the country, consumption c is equal to output y minus the net payments to the central planner, $\tau m - R$. By this, (14) and (22), it holds true that

$$c = y - (\tau m - R) = af\big(l(m,n), m\big) - \tau m + R. \tag{25}$$

6.2 The effect of the emission tax

The government of the country controls the extraction of carbon energy (= emissions), m, by its *ad valorem* tax x through the extractor's equilibrium condition (6). Thus, it maximizes the household's utility (1) by emissions m and education n subject to the state of nature (24), consumption (25), the evolution of efficiency, (19), and the evolution of the mass of scientists, (21), given the risk of the environmental disaster, π, and the central planner's tax τ and transfer R. This yields the functions (cf., A)

$$m(a,s,\pi,\tau,R), \ \ \frac{\partial m}{\partial \tau} < 0, \quad n(a,s,\pi,\tau,R), \ \ \left.\frac{\partial n}{\partial \tau}\right|_{\tau \geq 0} > 0, \tag{26}$$

as well as the equilibrium conditions (cf., A)

$$\tau = a(f_m - f_l v'), \quad f_l z' = \frac{\delta f}{(\alpha+\gamma)\gamma}. \tag{27}$$

By (21), the supply of scientists in the steady state is $s_{\dot{s}=0} = n/\alpha$. Noting this, the results (26) lead to the following conclusion:

Proposition 1 *An increase in the central planner's emission tax τ decreases emissions m. It increases education n, research and development (R&D) and the steady-state growth rate $g_{\dot{s}=0} = \delta s_{\dot{s}=0} = (\delta/\alpha)n$, at least when the initial level of the emission tax τ is non-negative.*

The tax τ discourages the extraction of carbon energy and emissions m. Because education and extraction compete for the same labor resources (22), a decrease in extraction promotes education, increasing the supply of scientists for R&D and the steady-state growth rate of the economy.

6.3 The government's policy

From (14), (23) and the left-hand equation in (27) it follows that

$$f_m = (1+x)v' f_l \quad \Leftrightarrow \quad x = \frac{f_m - f_l v'}{f_l v'} = \frac{\tau}{a f_l v'} = \frac{\tau}{\zeta_m}.$$

This result can be rephrased as follows:

Proposition 2 *The government of the country can run optimal policy directly by setting its* ad valorem *tax on carbon energy, x, equal to the central planner's emission tax τ divided by the marginal cost of carbon energy, ζ_m.*

The central planner's tax τ on emissions causes the wedge $af_m > af_l v'$ between the marginal product af_m and the marginal cost $af_l v'$ of carbon energy. Because the country ignores the externality through emissions, it faces that wedge as a distortion. The government of the country eliminates that distortion by its own tax x on carbon energy.

From (14), (23) and the right-hand equation in (27) it follows that

$$\xi_n = a f_l z' = \frac{\delta f a}{(\alpha+\gamma)\gamma} = \frac{\delta y}{(\alpha+\gamma)\gamma}.$$

This result can be rephrased as follows:

Proposition 3 *The government of the country must educate scientists n up to the level at which the marginal cost of education, ζ_n, is equal to the marginal revenue of education for the society, $\frac{\delta y}{(\alpha+\gamma)\gamma}$, where δ is a scientist's productivity, y output, γ the effective precautionary rate of time preference and α the mortality rate.*

Because the relative increase of income y is proportional to a scientist's productivity δ, the product δy is a scientist's marginal product. Thus, Proposition 3 can be interpreted as follows. The marginal revenue of education,

$\frac{\delta y}{(\alpha+\gamma)\gamma}$, increases, if the scientists' mortality rate α falls, the scientists' marginal product δy rises, or the households become more patient (i.e., the time preference γ falls) and more willing to invest in future. In these cases, the government can increase the level of education, n, until the marginal cost of education, ζ_n, is equal to the marginal revenue of education.

7 The central planner

7.1 First-best policy

At the level of the whole economy, consumption c is equal to output y. The central planner maximizes the household's utility (1) by education z and emissions m subject to aggregate output $c = y = f\big(l(m,n),m\big)$ [cf., (14) and (22)], the evolution of efficiency, the mass of scientists and the stock of pollution [(19), (21) and (2)], and the environmental shock (3). This maximization yields the following equilibrium conditions (cf., B):

$$(f_m - f_l v')_{q=1} = \frac{f}{1-\sigma}\frac{1}{\gamma}\frac{1-\varphi}{\gamma+\beta}\pi' > 0, \quad (f_m - f_l v')_{q=\varphi} = 0, \tag{28}$$

$$f_l z' = \frac{\delta f}{(\alpha+\gamma)\gamma}. \tag{29}$$

7.2 The optimal tax of emissions

Comparing the first of the government's conditions (27) with the central planner's condition (28) for emissions m and noting (3) and (14) yield

$$\tau = a(f_m - f_l v'), \quad \tau|_{q=\varphi} = a(f_m - f_l v')_{q=\varphi} = 0,$$
$$\tau|_{q=1} = a(f_m - f_l v')_{q=1} = \frac{af}{1-\sigma}\frac{1}{\gamma}\frac{1-\varphi}{\gamma+\beta}\pi' = \frac{1}{1-\sigma}\frac{(1-\varphi)y\pi'}{(\gamma+\beta)\gamma} > 0. \tag{30}$$

The expected marginal cost of pollution $(1-\varphi)y\pi'$ is equal to the expected relative loss of the disaster, $1-\varphi$, times income y times the marginal effect of pollution P on the risk of the disaster, π' [cf., (3)]. Thus, the results (30) can be rephrased as follows:

Proposition 4 *The central planner must tax emissions m before the occurrence of the disaster, $q = 1$, but not thereafter, $q = \varphi$. Its precautionary* *optimal emission tax* $\tau|_{q=1}$*, is equal to* $\frac{1}{1-\sigma}\frac{(1-\varphi)y\pi'}{(\gamma+\beta)\gamma}$*, where* σ *is the rate of relative risk aversion,* $(1-\varphi)y\pi'$ *the expected marginal cost of pollution,* γ *the effective precautionary rate of time preference [cf.,* (4)*] and* β *the depreciation rate of the stock of pollution.*

Before the disaster, the precautionary tax τ on emissions m must be proportional to the expected marginal cost of pollution, $(1-\kappa)y\pi'$. It must be the higher, the more the households avert any risk (i.e., a higher σ), including the risk of the disaster. The tax τ must be the higher, the more patient the households are (i.e., the lower their effective rate of time preference, γ). The faster the nature absorbs pollution P (i.e., the greater β), the smaller tax τ is needed to control emissions.

7.3 Eduction policy

The condition (29) holds true by the government's second condition in (27). Noting (30) and Proposition 3, this result can be rephrased as follows:

Proposition 5 *The central planner should not intervene in the government's education policy. Then, an increase in the central planner's emission tax* τ *increases education* n*, research and development (R&D) and the steady-state growth rate* $g_{\dot{s}=0} = \delta s_{\dot{s}=0} = (\delta/\alpha)n$*.*

8 Concluding remarks

In this document, the interaction of aggregate pollution and technological change is examined in a union of several countries. Oligopolists produce intermediate goods by labor and carbon energy. R&D firms attempt to deprive the markets from the incumbent oligopolists by improving their efficiency by scientists. The governments of the countries train ordinary workers as scientists. The extraction of carbon energy from the nature contributes to the stock of pollution, which increases the risk of environmental disaster and harms welfare. This document examines how, in this setup, the central planner of the union could improve welfare by international taxation.

While a great deal of caution should be exercised when a stylized stochastic growth model is used to derive results on environmental policy, the following general conclusions seem to be justified. With two layers of policy makers, the central planner should focus on the control the global distortion, while the local governments should in their jurisdictions control R&D by education. When pollution affects welfare incrementally, the policy could be exercised at the same time. In contrast, when pollution affects welfare through the risk of a discrete shock, the *precautionary principle* must be applied: the policy must be exercised before the shock comes true.

Because the key externality in the model is created by carbon, the tax on carbon emissions is necessary for decreasing emissions. In the absence of that externality, there is no reason for the central planner to intervene. The governments of small countries cannot alone influence the total stock of pollution, but they can nevertheless observe the risk of disaster and take it into account in their education policy. If the scientists' mortality rate falls or their marginal product rises, or if the households become more patient, then the social marginal revenue of education increases. Then, each government must increase education, until the marginal cost gets equal to the social marginal revenue of education.

The central planner's precautionary tax on carbon energy decreases the extraction of carbon energy, emissions and pollution, but speeds up economic growth, because resources freed from extraction will be employed in education and R&D. That tax must be the higher, the more the households avert risk, the more patient they are or the slower the nature absorbs pollution. The local governments can directly implement optimal policy by setting their *ad valorem* tax on carbon energy equal to the central planner's emission tax divided by the marginal cost of carbon energy.

A The country's optimum (eqs 26 and 27)

A1. The government's problem

The government maximizes (1) by (m, n) s.t. (19), (21), (24) and (25), given (π, τ, R). The value function of this problem is

$$\Psi(a, s, \pi, \tau, R) \doteq \max_{(m,n) \text{ s.t. } (19),(21),(24),(25)} \int_T^\infty qc^{1-\sigma} e^{\rho(T-t)} dt. \tag{31}$$

The *Bellman equation* for the problem (31) is

$$\rho\Psi(a,s,\pi,\tau,R)=\max_{m,n}\Omega(m,n,a,s,\pi,\tau,R)\ \text{ with} \tag{32}$$

$$\begin{aligned}\Omega(m,n,a,s,\pi,\tau,T)&\doteq qc^{1-\sigma}+\frac{\partial\Psi}{\partial a}\dot a+\frac{\partial\Psi}{\partial s}\dot s+\pi\big(\Psi\big|_{q=\varphi}-\Psi\big)\\&=\big[af\big(l(m,n),m\big)-\tau m-\theta s+R\big]^{1-\sigma}+\frac{\partial\Psi}{\partial a}\delta sa+\frac{\partial\Psi}{\partial s}(n-\alpha s)\\&\quad+\pi\big(\Psi\big|_{q=\varphi}-\Psi\big),\end{aligned}\tag{33}$$

where, at the moment of the environmental shock, q jumps from 1 down to φ and $\pi\big(\Psi\big|_{q=\varphi}-\Psi\big)$ vanishes. The first-order conditions for the maximization of the function (33) are [cf., (22)]

$$0=\frac{\partial\Omega}{\partial m}=(1-\sigma)c^{-\sigma}q\big[a(f_m-f_lv')-\tau\big]\ \Leftrightarrow\ \tau=a(f_m-f_lv'),\tag{34}$$

$$0=\frac{\partial\Omega}{\partial n}=\frac{\partial\Psi}{\partial s}-(1-\sigma)c^{-\sigma}qaf_lz'\ \Leftrightarrow\ af_lz'=\frac{c^\sigma/q}{1-\sigma}\frac{\partial\Psi}{\partial s}.\tag{35}$$

Because $c^{1-\sigma}$ is strictly concave in (m,n), the equilibrium (34) and (35) is unique.

A2. The direct effects of the tax

The first-order conditions (34) and (35) define the controls (m,n) as functions of the state and exogenous variables (a,s,π,τ,R). Differentiating them totally yields the matrix equation

$$0=\begin{bmatrix}\frac{\partial^2\Omega}{\partial m^2} & \frac{\partial^2\Omega}{\partial m\partial n}\\ \frac{\partial\Omega}{\partial m\partial n} & \frac{\partial^2\Omega}{\partial n^2}\end{bmatrix}\begin{bmatrix}dm\\ dn\end{bmatrix}+\begin{bmatrix}(\sigma-1)c^{-\sigma}q\\ 0\end{bmatrix}d\tau,\tag{36}$$

where, by the strict concavity of the function (33), it holds true that

$$\frac{\partial^2\Omega}{\partial m^2}<0,\quad \frac{\partial^2\Omega}{\partial n^2}<0,\quad \mathcal{J}\doteq\begin{vmatrix}\frac{\partial^2\Omega}{\partial m^2} & \frac{\partial^2\Omega}{\partial m\partial n}\\ \frac{\partial\Omega}{\partial m\partial n} & \frac{\partial^2\Omega}{\partial n^2}\end{vmatrix}>0.\tag{37}$$

By (9), (20) and (22), one obtains

$$\begin{aligned}&\frac{\partial}{\partial m}\ln[\underbrace{c^{-\sigma}f_l\big(l(m,n),m\big)}_{+}]_{\tau\geq0}=\frac{\partial}{\partial m}\big[-\sigma\ln c+\ln f_l\big(l(m,n),m\big)\big]_{\tau\geq0}\\&=\left(-\frac{\sigma}{c}\frac{\partial c}{\partial m}+\frac{f_{ll}}{f_l}\frac{\partial l}{\partial m}\right)_{\tau\geq0}=\left(\frac{\sigma}{c}\tau-\underbrace{\frac{f_{ll}}{f_l}}_{-}\underbrace{z'}_{+}\right)_{\tau\geq0}>0.\end{aligned}$$

Because a logarithm is an increasing transformation, from this it follows that $\frac{\partial}{\partial m}[c^{-\sigma} f_l]_{\tau \geq 0} > 0$. This, (20) and (35) yields

$$\left.\frac{\partial^2 \Omega}{\partial m \partial n}\right|_{\tau \geq 0} = -\underbrace{(1-\sigma)}_{+} \underbrace{qaz'(n)}_{+} \underbrace{\frac{\partial}{\partial m}[c^{-\sigma} f_l]_{\tau \geq 0}}_{+} < 0. \tag{38}$$

By (36)-(38), one obtains the partial derivatives in (26):

$$\frac{\partial n}{\partial \tau} \doteq -\frac{1}{\mathcal{J}} \begin{vmatrix} \frac{\partial^2 \Omega}{\partial m^2} & (\sigma - 1) c^{-\sigma} q \\ \frac{\partial \Omega}{\partial m \partial n} & 0 \end{vmatrix} = \underbrace{\frac{1}{\mathcal{J}}}_{+} \underbrace{\frac{\partial \Omega}{\partial m \partial n}}_{-} \underbrace{(\sigma - 1)}_{-} \underbrace{c^{-\sigma} q}_{+} > 0 \text{ for } \tau \geq 0,$$

$$\frac{\partial m}{\partial \tau} \doteq -\frac{1}{\mathcal{J}} \begin{vmatrix} (\sigma - 1) c^{-\sigma} q & \frac{\partial^2 \Omega}{\partial m \partial n} \\ 0 & \frac{\partial^2 \Omega}{\partial n^2} \end{vmatrix} = \underbrace{\frac{1}{\mathcal{J}}}_{+} \underbrace{(1-\sigma)}_{+} \underbrace{c^{-\sigma} q}_{+} \underbrace{\frac{\partial^2 \Omega}{\partial n^2}}_{-} < 0.$$

These yield the results (26).

A3. The steady state

Because the countries are identical, the central planner's budget constraint $\tau m + \theta s = R$ holds in equilibrium. Then, there exists a steady state in the system (19), (21), (25), (34) and (35). In that steady state, consumption c and efficiency a grow at the same rate, while the control variables (m, n) and the mass of scientists s are constants. The steady-state path is

$$(m, n) \text{ constant}, \quad c = af(l(m,n), m), \quad \frac{\dot{c}}{c} = \frac{\dot{a}}{a} = \delta s, \quad \dot{s} = 0 \;\Leftrightarrow\; n = \alpha s,$$
$$\tau m + \theta s = R. \tag{39}$$

A4. The value function

The value function (31) must satisfy the Bellman equation (32) with (33) in the steady state (39). To obtain a solution, let's assume for a while that that function is the periodic utility $qc^{1-\sigma}$ in the steady state (39) divided by a positive and piecewise differentiable function $\gamma(s)$ [cf., (22)]:

$$\Psi(a, s, \pi, \tau, T) \doteq \left.\frac{qc^{1-\sigma}}{\gamma(s,q)}\right|_{(39)} \quad \text{with}$$

$$\frac{1}{\Psi}\frac{\partial \Psi}{\partial a} = \frac{\partial \ln \Psi}{\partial a} = (1-\sigma)\frac{\partial \ln c}{\partial a} = (1-\sigma)\frac{1}{c}\frac{\partial c}{\partial a} = (1-\sigma)\left.\frac{f}{c}\right|_{(39)} = \frac{1-\sigma}{a} \quad \text{and}$$

$$\frac{1}{\Psi}\frac{\partial \Psi}{\partial s} = (1-\sigma)\frac{\partial \ln c}{\partial s} - \frac{\partial \ln \gamma}{\partial s} = (1-\sigma)\frac{1}{c}\frac{\partial c}{\partial s} - \frac{1}{\gamma}\frac{\partial \gamma}{\partial q}$$
$$= (1-\sigma)\frac{1}{c}\left[af_l \underbrace{\underbrace{l_n}_{-z'} \left.\frac{dn}{ds}\right|_{(39)}}_{=\alpha} + b\right] - \frac{1}{\gamma}\frac{\partial \gamma}{\partial q} = (1-\sigma)\frac{1}{c}(b - af_l z'\alpha) - \frac{1}{\gamma}\frac{\partial \gamma}{\partial q}. \tag{40}$$

A5. The jump in the value function

At the occurrence of the environmental shock, q falls down from 1 to φ, but the other state variables (a, s) do not change. Therefore, by (40), the *relative damage* of that shock in terms of utility Φ is defined by

$$\frac{\Psi - \Psi\big|_{q=\varphi}}{\Psi} = q - \varphi \geq 0. \tag{41}$$

Inserting (40) and (41) into the Bellman equation (32) with (33) in the steady state (39) and dividing by Ψ, one obtains

$$\rho = \frac{qc^{1-\sigma}}{\Psi} + \delta \frac{a}{\Psi}\frac{\partial \Psi}{\partial a} s + \pi(\varphi - q) = \gamma + (1-\sigma)\delta s + \pi(\varphi - q) \quad \Leftrightarrow$$
$$\gamma = \rho + (\sigma - 1)\delta s + \pi(q - \varphi).$$

By this result, one can generalize the differentiable function γ as follows:

$$\gamma(s, q) \doteq \rho + (\sigma - 1)\delta s + \pi(q - \varphi) \text{ with } \frac{\partial \gamma}{\partial q} = (\sigma - 1)\delta, \tag{42}$$

With the specification (42), the function (40) satisfies the Bellman equation (32) with (33) in the steady state (39).

A6. The equilibrium of the country

Finally, noting $c = af$ [cf., (39)] and plugging (40) and (42) into the condition (35) yields

$$af_l z' = \frac{c^\sigma/q}{1-\sigma}\frac{\partial \Psi}{\partial s} = \frac{\Psi c^\sigma/q}{1-\sigma}\left[(1-\sigma)\frac{1}{c}(-af_l z'\alpha) - \frac{1}{\gamma}\frac{\partial \gamma}{\partial q}\right]$$
$$= \frac{\Psi}{q}c^{\sigma-1}\left(-af_l z'\alpha - \frac{c}{1-\sigma}\frac{1}{\gamma}\frac{\partial \gamma}{\partial q}\right) = \frac{1}{\gamma}\left(b - af_l z'\alpha + \frac{\delta c}{\gamma}\right) \quad \Leftrightarrow$$
$$(\alpha + \gamma)af_l z' = \frac{\delta c}{\gamma} = \frac{\delta af}{\gamma} \quad \Leftrightarrow \quad f_l z' = \frac{\delta f}{(\alpha+\gamma)\gamma}. \tag{43}$$

The results (27) are given by (34) and (43).

B The social optimum (eqs 28 and 29)

B1. The central planner's problem

The central planner maximizes utility (1) with $c = f\big(l(m,n),m\big)$ by (z,m) s.t. (19), (21), (2) and (3). The value function of this problem is

$$\Phi\big(a(T),s(T),P(T),q(T),T\big) \doteq \max_{\substack{(m,n)\text{ s.t.}\\(19),(21),(2),(3)}} \int_T^\infty qc^{1-\sigma}e^{\rho(T-t)}dt$$
$$\text{with } c = af\big(l(m,n),m\big). \tag{44}$$

The *Bellman equation* for the problem (44) is

$$\rho\Phi\big(a,s,P,q,T\big) = \max_{m,n}\Lambda(m,n,a,s,P,T) \text{ with} \tag{45}$$

$$\begin{aligned}\Lambda(m,n,a,s,P,T) &\doteq qc^{1-\sigma} + \frac{\partial\Phi}{\partial a}\dot a + \frac{\partial\Phi}{\partial s}\dot s + \frac{\partial\Phi}{\partial P}\dot P + \pi(P)\big(\Phi\big|_{q=\varphi}-\Phi\big)\\ &= qa^{1-\sigma}f\big(l(m,n),m\big)^{1-\sigma} + \frac{\partial\Phi}{\partial a}\delta sa + \frac{\partial\Phi}{\partial s}(n-\alpha s) + \frac{\partial\Phi}{\partial P}(m-\beta P)\\ &\quad + \pi(P)\big(\Phi\big|_{q=\varphi}-\Phi\big),\end{aligned} \tag{46}$$

where, at the moment of the environmental shock, q jumps from 1 down to φ and $\pi(P)\big(\Phi\big|_{q=\varphi}-\Phi\big)$ vanishes. The first-order conditions for the maximization of the function (46) are [cf., (22) and (44)]

$$0 = \frac{\partial\Lambda}{\partial m} = (1-\sigma)qa^{1-\sigma}f^{-\sigma}(f_m - f_l v') + \frac{\partial\Phi}{\partial P}, \tag{47}$$
$$0 = \frac{\partial\Lambda}{\partial n} = \frac{\partial\Phi}{\partial s} - (1-\sigma)qa^{1-\sigma}f^{-\sigma}f_l z'. \tag{48}$$

B2. The value function

In the steady state of the system (21), (19), (22), (2), (3), (47) and (48), consumption c and efficiency a grow at the same rate, while the control and state variables (m,n,s,P,q) are constants. This steady-state path is

$$(m,n)\text{ constant},\quad \frac{\dot c}{c} = \frac{\dot a}{a} = \delta s,\quad \dot s = 0 \;\Leftrightarrow\; n = \alpha s,\quad \dot P = 0 \;\Leftrightarrow\; m = \beta P. \tag{49}$$

The value function (44) must satisfy the Bellman equation (45) with (46) in the steady state (49). To obtain a solution, let's assume for a while that

Φ is the periodic utility $qc^{1-\sigma}$ in the steady state (49) divided by a piecewise differentiable function $\gamma(s, P, q)$ [cf., (22)]:

$$\Phi(a, s, P, q, T) \doteq \frac{1}{\gamma(s, P, q)} qa^{1-\sigma} f\big(l(z, m), m\big)^{1-\sigma}\big|_{(49)} \text{ with } \frac{1}{\Phi}\frac{\partial \Phi}{\partial a} = \frac{1-\sigma}{a},$$
$$\frac{1}{\Phi}\frac{\partial \Phi}{\partial P} = \frac{\partial \ln \Phi}{\partial P} = (1-\sigma)\frac{\partial \ln f}{\partial m}\frac{dm}{dP}\bigg|_{(49)} - \frac{\partial \ln \gamma}{\partial P} = (1-\sigma)\frac{\partial \ln f}{\partial m}\beta - \frac{\partial \ln \gamma}{\partial P}$$
$$= (1-\sigma)\frac{\beta}{f}(f_m - f_l v') - \frac{1}{\gamma}\frac{\partial \gamma}{\partial P} \text{ and}$$
$$\frac{1}{\Phi}\frac{\partial \Phi}{\partial s} = \frac{\partial \ln \Phi}{\partial l}\frac{\partial l}{\partial n}\frac{dn}{ds}\bigg|_{(49)} - \frac{\partial \ln \gamma}{\partial s} = (1-\sigma)\frac{\partial \ln f}{\partial l}(-z')\alpha - \frac{1}{\gamma}\frac{\partial \gamma}{\partial s}$$
$$= -(1-\sigma)\frac{f_l}{f}z'\alpha - \frac{1}{\gamma}\frac{\partial \gamma}{\partial s}. \tag{50}$$

B3. The jump in the value function

At the occurrence of the environmental shock, q falls down from 1 to φ, but the other state variables (a, s, P) do not change. Therefore, by (50), the *relative damage* of that shock in terms of utility Φ is defined by

$$\frac{\Phi - \Phi\big|_{q=\varphi}}{\Phi} = q - \varphi \geq 0. \tag{51}$$

Plugging (50) and (51) into the Bellman equation (45) with (46) in the steady state (49) and dividing it by Φ solves for γ:

$$\rho = \frac{1}{\Phi}\Lambda\big|_{(49)} = \frac{qy^{1-\sigma}}{\Phi} + \frac{1}{\Phi}\frac{\partial \Phi}{\partial a}\delta as + \pi(P)(\varphi - q)$$
$$= \gamma + (1-\sigma)\delta s + \pi(P)(\varphi - q) \iff \gamma = \rho + (\sigma - 1)\delta s + \pi(P)(q - \varphi).$$

By this result, the function $\gamma(P)$ can be generalized as follows:

$$\gamma(s, P, q) \doteq \rho + (\sigma - 1)\delta s + \pi(P)(q - \varphi), \quad \frac{\partial \gamma}{\partial s} = (\sigma - 1)\delta, \quad \frac{\partial \gamma}{\partial P} = (q - \varphi)\pi'. \tag{52}$$

By $\varphi < 1$ and $q \in \{\varphi, 1\}$, the multiplier (52) is a piecewise differentiable function. Thus, the specification of value function, (50) with (52), satisfies the Bellman equation (45) with (46) in the steady state (49).

B4. Optimal emissions and education

Plugging (3), (50) and (52) into the first-order conditions (47) and (48) and noting (49), one obtains

$$
\begin{aligned}
f_m - f_l v' &= -\frac{1}{(1-\sigma)qa^{1-\sigma}f^{-\sigma}}\frac{\partial\Phi}{\partial P} \\
&= -\frac{f\Phi}{(1-\sigma)qa^{1-\sigma}f^{1-\sigma}}\left[(1-\sigma)\frac{\beta}{f}(f_m - f_l v') - \frac{1}{\gamma}\frac{\partial\gamma}{\partial P}\right] \\
&= -\frac{f/\gamma}{1-\sigma}\left[(1-\sigma)\frac{\beta}{f}(f_m - f_l v') - \frac{1}{\gamma}\frac{\partial\gamma}{\partial P}\right] \\
&= -\frac{1}{\gamma}\left[\beta(f_m - f_l v') - \frac{f}{1-\sigma}\frac{1}{\gamma}\frac{\partial\gamma}{\partial P}\right] \quad \Leftrightarrow \\
(\gamma+\beta)(f_m - f_l v') &= \frac{f}{1-\sigma}\frac{1}{\gamma}\frac{\partial\gamma}{\partial P} = \frac{f}{1-\sigma}\frac{1}{\gamma}(q-\varphi)\pi',
\end{aligned}
$$

$$
(f_m - f_l v')_{q=1} = \frac{f}{1-\sigma}\frac{1}{\gamma}\frac{1-\varphi}{\gamma+\beta}\pi' > 0, \quad (f_m - f_l v')_{q=\varphi} = 0, \tag{53}
$$

$$
\begin{aligned}
f_l z' &= \frac{1}{(1-\sigma)qa^{1-\sigma}f^{-\sigma}}\frac{\partial\Phi}{\partial s} = \frac{f\Phi}{(1-\sigma)qa^{1-\sigma}f^{1-\sigma}}\left[-(1-\sigma)\frac{f_l}{f}z'\alpha - \frac{1}{\gamma}\frac{\partial\gamma}{\partial s}\right] \\
&= \frac{f/\gamma}{1-\sigma}\left[-(1-\sigma)\frac{f_l}{f}z'\alpha - (\sigma-1)\frac{\delta}{\gamma}\right] = \frac{1}{\gamma}\left(-f_l z'\alpha + \frac{\delta f}{\gamma}\right) \quad \Leftrightarrow
\end{aligned}
$$

$$
(\gamma+\alpha)f_l z' = \frac{\delta f}{\gamma} \quad \Leftrightarrow \quad f_l z' = \frac{\delta f}{(\alpha+\gamma)\gamma}. \tag{54}
$$

The results (53) and (54) yield (28) and (29).

References:

Acemoglu, D. (2009) *Modern Economic Growth.* Oxford: Princeton University Press.

Acemoglu, D., Akcigit, U., Hanley, D., Kerr, W. (2016) "Transition to Green Technology." *Journal of Political Economy* 124: 52–104.

Afonso, O., Fonseca, L., Magalhaes, M., Vasconcelos, PB. (2021) "Directed Technical Change and Environmental Quality." *Portuguese Economic Journal* 20: 71–97.

Becker, G.S., 1981. A Treatise on the Family. Cambridge (Mass.): Harvard University Press.

de Zeeuw, A., Zemel, A. (2012) "Regime Shifts and Uncertainty in Pollution Control." *Journal of Economic Dynamics and Control* 36: 939–950.

Dinopoulos, E., Thompson, P. (1999) Scale Effects in Schumpeterian Models of Economic Growth. *Journal of Evolutionary Economics* 9: 157û185. https://doi.org/10.1007/s001910050079

Dockner, E.J., Long, N.V. (1993) "International Pollution Control: Cooperative versus Noncooperative Strategies." *Journal of Environmental Economics and Management* 24: 13–29.

Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A. (2014) "Optimal Taxes on Fossil Fuel in General Equilibrium." *Econometrica* 82: 41–88.

Palokangas, T. (2021) "Optimal Taxation with Endogenous Population Growth and the Risk of Environmental Disaster." In: *Dynamic Economic Problems with Regime Switches.* Series "Dynamic Modeling and Econometrics in Economics and Finance" No. 25. Edited by J. Haunschmied, R. Kovacevic, W. Semmler and V.M. Veliov. Springer Nature Switzerland AG.

Peretto, P.F. (1998) "Technological Change and Population Growth." *Journal of Economic Growth* 3: 283–311.

Polasky, A., de Zeeuw, A., Wagener, F. (2011) "Optimal Management with Potential Regime Shifts." *Journal of Environmental Economics and Management* 62: 229–240.

Sinn, H.-W. (2008) "Public Policies against Global Warming: a Supply Side Approach." *International Tax and Public Finance* 15: 360–394.

Smulders, S., Tsur, Y., Zemel, A. (2012) "Announcing Climate Policy: Can a Green Paradox Arise without Scarity?" *Journal of Environmental Economics and Management* 64: 364–376.

Tsur, Y., Zemel, A. (2008) "Regulating Environmental Threats." *Environmental and Resource Economics* 39: 297–310.

Tsur, Y., Zemel, A. (2009) "Endogenous Discounting and Climate Policy." *Environmental and Resource Economics* 44: 507–520.

Tsur, Y., Zemel, A. (2011) "On the Dynamics of Competing Energy Sources." *Automatica* 47: 1357–1365.

Valente, S. (2011) "Endogenous Growth, Backstop Technology Adoption, and Optimal Jumps." *Macroeconomic Dynamics* 15: 293–325.