ESTIMATION OF CAPITAL MATRICES FOR MULTISECTORAL MODELS: AN APPLICATION TO ITALY AND TUSCANY

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FOREWORD

This paper refers to the Tuscany case study which constitutes a systems analysis of integrated regional development in the Tuscany region. A core of this study is the development of applied models and methods undertaken by the Regional Development Group at IIASA, in collaboration with the Regional Institute for Economic Planning of Tuscany (IRPET). A bi-regional input-output model has a central part in the system of model development. In order to capture the dynamic process of capacity creation and removal, the capital formation has to be included into the input-output framework in a systematic way. This presupposes an estimation of capacity change and of capital coefficient matrices.

This paper presents a systematic approach to obtain these estimates, also in the case where only a limited set of data is available. In summary, the method combines a vintage type production theory and an estimation technique based on information theory.

Boris Issaev
Leader
Regional Development
Group

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1. INTRODUCTION: CAPITAL FORMATION IN THE MULTISECTORAL MODELS INTIMO AND TIM

Recently, two models of input-output type have been developed for the Italian economy. The INTIMO model covers the economy as a whole, while TIM is a biregional model confronting the region of Tuscany with the rest of Italy. In order to introduce endogenously determined investments into these models—both for short- and medium-term—capital coefficient matrices are calculated in this paper.

The paper also has some general interest in the sense that it presents an attempt to reconstruct data which have not been directly observable. One important starting point for this is a small set of assumptions based on a vintage type of production theory. To illustrate the theoretical background, empirical observations from the Swedish economy are presented in Section 3. These results also provide empirical support to the approach utilized in this study. Moreover, they indicate how the Italian models could gain in further precision and usefulness if more data of this kind were supplied from the Statistical Bureau of Italy.

Section 2 presents the basic structure of the multisectoral models. Section 4 applies the assumptions introduced in Section 3 by describing methods to calculate the change of capacity and productivity in different sectors of the Italian and Tuscany economies. It also presents estimates and calculations as regards these change processes. Section 5 presents a general method to estimate capital coefficient matrices, and applies it to the data available for the Italian economy from 1970-1980. Estimation results are presented in Section 6.

2. INVESTMENT REQUIREMENTS AND CAPACITY CHANGE IN A MEDIUM-TERM INPUT-OUTPUT MODEL

The Tuscany case study involves two multisectoral models of the input-output type. One is a nation-wide model and the other is a regional model with Tuscany and the remaining part of Italy as regions. In their medium-term versions both these models may be represented by this comprised formulation:

$$x = Ax + h + c \tag{2.1}$$

where

$x = \{x_i\}$ is a vector in which x_i represents the production of sector i

$A = \{a_{ij}\}$ is a matrix in which a_{ij} denotes deliveries from sector i per output of sector j; for the biregional case this matrix has to distinguish between deliveries both with regard to sectors and the two regions

$h = \{h_i\}$ is a vector in which h_i represents the output from sector i used for investment in the production system

$c = \{c_i\}$ is a vector in which c_i represents the final demand of the model (exports, import, consumption, etc.).

The input-output framework was introduced into applied economic analysis as an instrument to ensure that a solution to a model is internally consistent. The system described in formula (2.1) fails to satisfy such a consistency requirement as regards the development of production capacities and capital formation.

2.1. Two Dimensions of Capacity Change

The change of the capacity in a sector consists of two interlinked processes: new capacities are created and old capacities are removed because of economic and/or technical obsolescence. New capacities may be added to existing production units or may appear in the form of new production establishments. The removal of capacities occur both as shut down of entire plants and removal of equipment and parts of a plant. In the sequel we attempt to give a coherent description of these processes.

Let t denote a year and let $x_j(t)$ be the realized production at the same point in our time scale. We may then introduce the following fundamental constraint:

$$\bar{x}_j(t) \geq x_j(t) \tag{2.2}$$

where $\bar{x}_j(t)$ denotes the available capacity at time t. Suppose that we can observe how the capacity is changing over time so that we can calculate

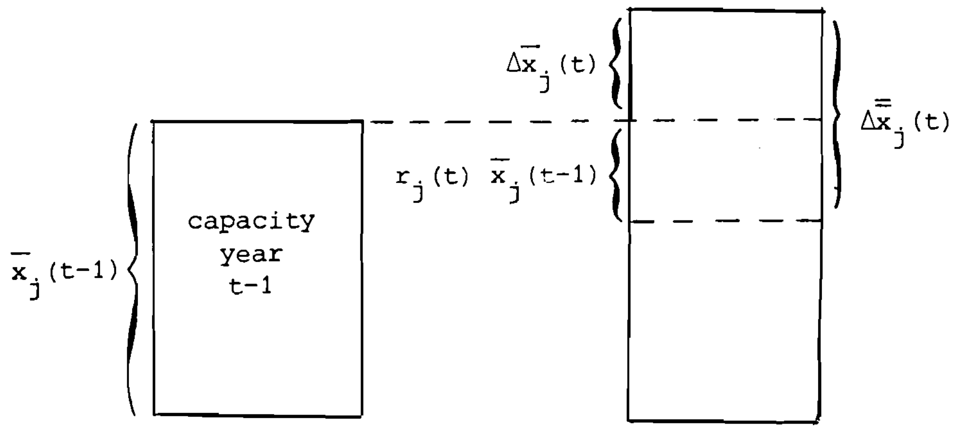
$$\Delta\bar{x}_j(t) = \bar{x}_j(t) - \bar{x}_j(t-1) \tag{2.3}$$

where $\Delta\bar{x}_j(t)$ represents the net change of sector j's capacity (see Figure 2.1). Consider next the removal coefficient $r_j(t)$ which shows the fraction of the capacity $\bar{x}_j(t-1)$ which has been removed between year t-1 and t. The total removal is then $r_j(t)\bar{x}_j(t-1)$. Hence, the gross change of capacity, $\Delta\bar{\bar{x}}_j(t)$, becomes

$$\Delta\bar{\bar{x}}_j(t) = \Delta\bar{x}_j(t) + r_j(t)\bar{x}_j(t-1) \tag{2.4}$$

Formula (2.4) shows that the system may require investments which are creating new capacity also in sectors which experience a decreasing capacity, i.e., a negative net change of capacity.

Consider now a medium-term sequence of years from t = 0 to t = T. Let $\bar{x}_j(0),\ldots,\bar{x}_j(T)$ denote the path showing how the capacity level of sector j is developing over this sequence. Suppose next that the expected removal during the time period is $\bar{r}_j\bar{x}_j(0)$ satisfying



$\bar{x}_j(t)$ = capacity

$r_j(t)$ = rate of removal

$\Delta\bar{\bar{x}}_j(t)$ = gross change of capacity

$\Delta\bar{x}_j(t)$ = net change of capacity

Figure 2.1. Capacity change, removal, and gross capacity change.

$$\bar{r}_j\bar{x}_j(0) = \sum_{t=1}^{T} r_j(t)\bar{x}_j(t-1)$$

The total demand for gross capacity change then becomes

$$\Delta\bar{\bar{x}}_j = \bar{x}_j(t) - (1 - \bar{r}_j)\ \bar{x}_j(0) \tag{2.5}$$

This gross capacity change will require deliveries of investment goods, h_{ij}, from different sectors i to sector j so that

$$h_{ij} = k_{ij}\ \Delta\bar{\bar{x}}_j \tag{2.6}$$

where k_{ij} is an investment coefficient showing the amount of deliveries from sector i which is needed in order to produce one unit of capacity in sector j. Assuming that the capacity is increasing with a constant amount each period, t, we have for $x(t) = Ax(t) + h(t) + c(t)$ that

$$h_i(t) = \sum_j k_{ij}\ \Delta\bar{\bar{x}}_j/T \quad , \qquad t = 1,\ldots,T \tag{2.7}$$

With this formulation the consistency gap in formula (2.1) has been filled.

2.2. Directly and Indirectly Observed Variables

In terms of the variables introduced in Section 2.1, this paper has the following aims:

— estimation of $\bar{x}_j(t)$, $\Delta\bar{\bar{x}}_j(t)$, and $r_j(t)$, 1970-1980, for Italy (and partly for Tuscany), and

— estimation of investment matrices $K = \{k_{ij}\}$ for Italy and its two regions.

The information system available has only made it possible to observe the following variables:

$x_j(t)$ = current production

$L_j(t)$ = current employment

$I_j(t) = \sum_i p_i(t)\ k_{ij}\ \Delta\bar{\bar{x}}_j(t+1)$ = current purchase of capital equipment installed in sector j (2.8)

$H_i(t) = \sum_j p_i(t)\ k_{ij}\ \Delta\bar{\bar{x}}_j(t+1)$ = current value of deliveries of investment goods from sector i.

where $p_i(t)$ denotes the price level in sector i. For some of these variables, information has only been available with regard to aggregates of sectors, certain years and regions during the period 1970-1980.

The relation between current production and available capacity may be specified as follows:

$$x_j(t) = u_j(t)\ \bar{x}_j(t) \quad , \qquad 1 \geq u_j(t) \geq 0 \tag{2.9}$$

where $u_j(t)$ denotes the degree of capacity utilization.

Let us now assume that we can observe the creation of new capacity each year. Using a fixed price system such that $\tilde{p}_i(t) = 1$ for all i, we may form an aggregate marginal capital output ratio, k_j, such that

$$k_j = \sum \tilde{p}_i(t)\ k_{ij} = I_j(t)/\Delta\bar{\bar{x}}_j(t+1) \tag{2.10}$$

This formula puts a constraint on the estimations we shall make. Given a path $\{\bar{x}_j(t)\}$, the problem may be posed as a search for two other paths, $s_j(t)$ and $r_j(t)$, such that

$$s_j(t)\ \bar{x}_j(t-1) = r_j(t)\ \bar{x}_j(t-1) + [\bar{x}_j(t) - \bar{x}_j(t-1)] \qquad (2.11)$$

where $s_j(t)\ \bar{x}_j(t-1) = \Delta\bar{\bar{x}}_j(t)$. In order to fulfill this task we have to make use of certain elements from production theory, which are introduced in the next section.

3. PRODUCTION THEORY: CHANGES IN CAPACITY AND PRODUCTIVITY

A production unit may be characterized by its different types of durable resources such as (i) buildings and constructions, (ii) machinery, equipment and production techniques, (iii) skill of the labor force (including management), and (iv) output mix, etc. The composition of such resources will generally vary between units in the same sector. With a vintage production theory adhering to the putty-clay tradition (Salter 1960, Johansen 1972), one may capture some basic features distinguishing different units from each other. In particular, one should emphasize that each production unit usually has (i) a fixed location, (ii) an upper capacity bound which is given in the short-run and which can only be changed by means of investments, and (iii) a given production technique implying approximately fixed input-output relations. We shall illustrate these properties with the help of production data referring to the Swedish industry.

3.1. Productivity Pattern in a Sector

Consider a production unit k in sector j. Let $\bar{x}_j^k(t)$ denote its capacity and let the vector $\{a_{ij}^k(t)\}$ denote its input requirements and $l_j^k(t)$ its labor input requirement per unit of output. This means that

$$x_j^k(t) \leq \min\ \{\bar{x}_j^k(t),\ L_j^k(t)/l_j^k(t)\} \qquad (3.1)$$

Hence, production cannot exceed the capacity given by the technical design of and the equipment installed in the unit. Moreover, if the number of employed, $L_j^k(t)$ is less than $\bar{L}_j^k(t) = l_j^k(t)\ \bar{x}_j^t(t)$, the labor constraint becomes active.

The coefficients $a_{ij}^{k}(t)$ and $l_j^k(t)$ may be changed gradually due to learning by doing effects. Such processes tend to be slow relative to the changes which are caused by investments. The latter bring new capacity into the unit and renew the production technique of the unit. In the absence of investments, the input coefficients are almost fixed in a medium term perspective. This also implies that the distribution of productivity in a sector does not change. The labor productivity μ_j^k, of unit k in sector j is

$$\mu_j^k = 1/l_j^k \tag{3.2}$$

The observed average productivity at time t has the form

$$\mu_j(t) = \sum_k \mu_j^k \, x_j^k(t)/\sum_k x_j^k(t) \tag{3.3}$$

Let

$$\sigma_j^k(t) = L_j^k(t)/\sum_k L_j^k(t)$$

and let k = 1,2,..., be an ordered sequence such that $\mu_j^k > \mu_j^{k+1}$ for all k. Then the following sequence of pairs

$$[\mu_j^k, \ \sigma_j^1(t) + \cdots + \sigma_j^k(t)] \quad , \qquad k = 1,2,\ldots$$

define a normed productivity curve. Figure 3.1 illustrates such curves for three different years. The curves describe the observed productivity pattern of the Swedish chemical industry. The productivity measure used is value added per person employed, which is defined by

$$\tilde{\mu}_j^k = (p_j - \sum_i p_i \, a_{ij}^k)/l_j^k \tag{3.4}$$

Figure 3.2 contains essentially the same information as Figure 3.1. It describes two productivity curves for the chemical industry (1978 and 1985) based on a simultaneous time series, cross sectional estimation (1968-1978) of the following continuous productivity function

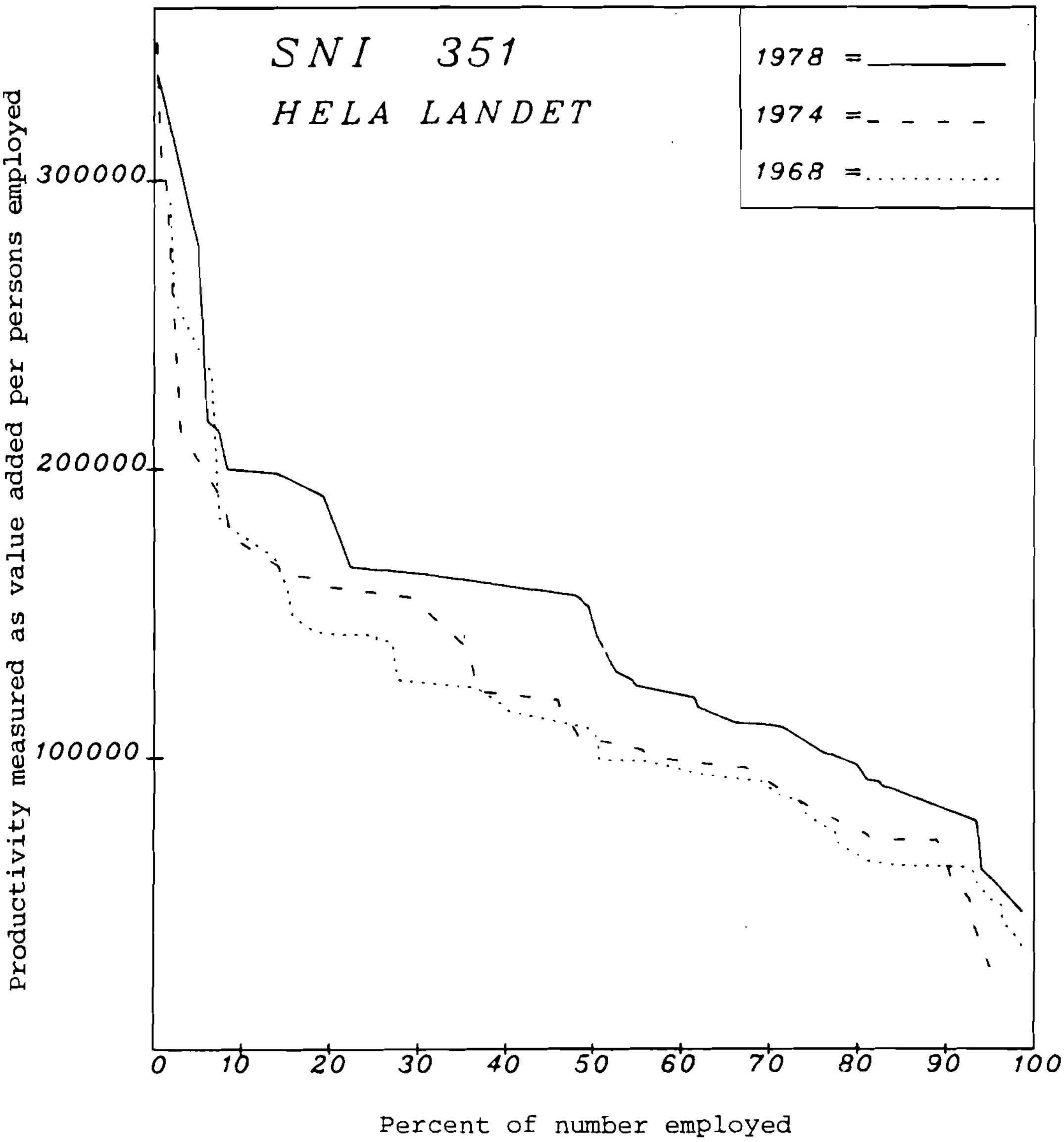
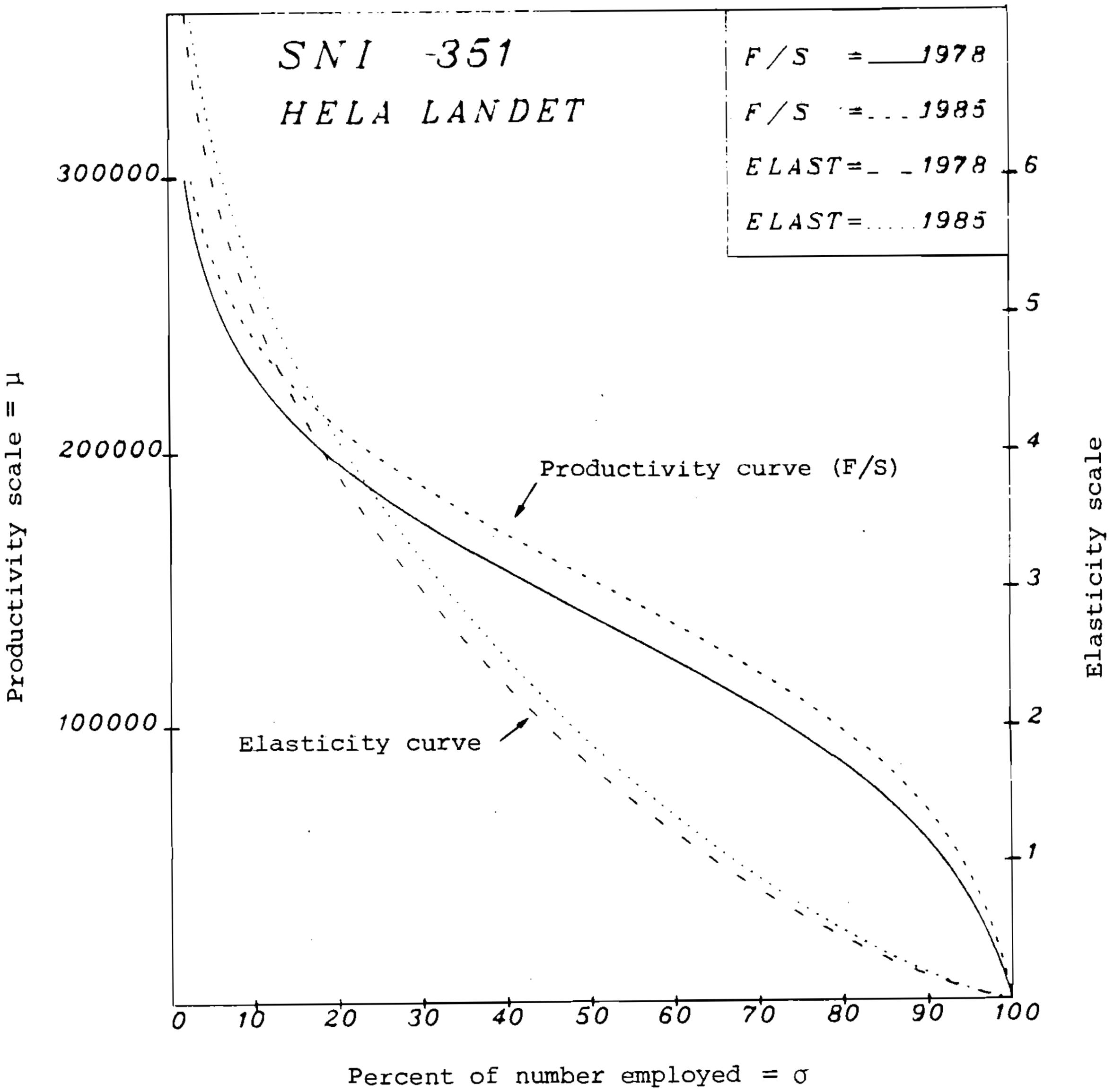


Figure 3.1. Observed productivity curves of the chemical industry in Sweden (1968, 1974, and 1978), fixed prices (1975). (Source: Industrial Statistics of Sweden, SCB, unpublished.)



Note: The elasticity function is $[d\sigma/d\tilde{\mu}]\,[\tilde{\mu}/\sigma]$.

Figure 3.2. Continuous productivity and elasticity functions of the chemical industry, Sweden 1968-1978.

$$\tilde{\mu}_j = (1/a_1)[\ln\sigma - \ln(1-\sigma) - a_2 t - a_o] \qquad (3.5)$$

which may also be written as

$$\sigma = \exp[A(\tilde{\mu}_j,t)]/\{1 + \exp[A(\tilde{\mu}_j,t)]\}$$

where $A(\tilde{\mu}_j,t) = a_0 + a_1\tilde{\mu}_j + a_2 t$, t denotes time and σ is the proportion of persons employed in units with a productivity which is higher or equal to $\tilde{\mu}_j$. The estimated parameters satisfy $a_0 > 0$, $a_1 < 0$, $a_2 > 0$. When the extreme values $\sigma \geq 0.999$ and $\tilde{\mu} \geq 300$ are eliminated, the R^2-value equals 0.89 in the case de-described by Figure 3.2.

Estimates based on Swedish data for a variety of sectors and geographical disaggregations exhibit a similar degree of invariance.. This indicates a structural constancy of the productivity pattern in a sector.

3.2. The Capacity Removal Process

In the analysis of production units the gross profit per unit output, β_j^k, plays a central role

$$\beta_j^k = p_j - \sum p_i \, a_{ij}^k - w_j \, l_j^k \qquad (3.6)$$

In stylized versions of vintage theory it is assumed that (i) the production is continued in a unit with technique (vintage) k as long as β_j^k remains positive, and (ii) the unit is shut down or scrapped when β_j^k becomes negative.

Empirical analyses of Swedish industrial establishments suggest the following description of the capacity removal process. For a given production unit or group of units with the same production technique the probability of removal is (i) positive also for units with positive profits, and (ii) increasing as the profit is decreasing. However, also among the set of units with negative profits the ratio between annually removed and remaining capacities is less than unity.

In order to describe the frequency of removal, let

$$\omega_j^k = w_j \, l_j^k/(p_j - \sum_i p_i \, a_{ij}^k) \tag{3.7}$$

denote the wage share of value added for units with technique k. Obviously, ω_j^k will be increasing as gross profits per unit output β_j^k is decreasing. Moreover, $\omega_j^k > 1$ implies $\beta_j^k < 0$. Figure 3.3 illustrates how the removal $r_j(\omega_j)$ increases exponentially (in an interval around $\omega_j = 1$) as the wage share increases. Functions of the type illustrated in the two figures have been estimated for 20 Swedish industry sectors. These functions have the following form

$$r_j(\omega_j) = \delta_j^0 \exp [\delta_j^1(\omega_j - \bar{\omega}_j)] \tag{3.8}$$[1]

where δ_j^0, δ_j^1, and $\bar{\omega}_j$ are positive parameters, and where ω_j is the realised wage share of a production unit.

3.3. The Capacity Increasing Process

New capacities may enter into a sector in two different ways. A new capacity may enter in the form of a new production unit. It may also enter as the result of the following composite process. A new capacity is added to the capacity of one already existing production unit; simultaneously some old capacity may be removed from the unit.

Before continuing, let us illustrate a production technique with the help of Figure 3.4. The figure utilizes a normalized price system $p_i = 1$ for all i. Such a price system can always be obtained by selecting a suitable scale for measuring the quantity of each sector's output. It is then obvious that the profit per unit output, β_j^k, is increased by means of a technical change which reduces the inputs, Σa_{ij}^k, and the labor input coefficient l_j^k, for a given wage level, w_j, which is measured relative to the selected price system.

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[1]Alternatively, we may write

$$r_j^k = \delta_j^0 \exp [\delta_j^1(\omega_j^k - \bar{\omega}_j)]$$

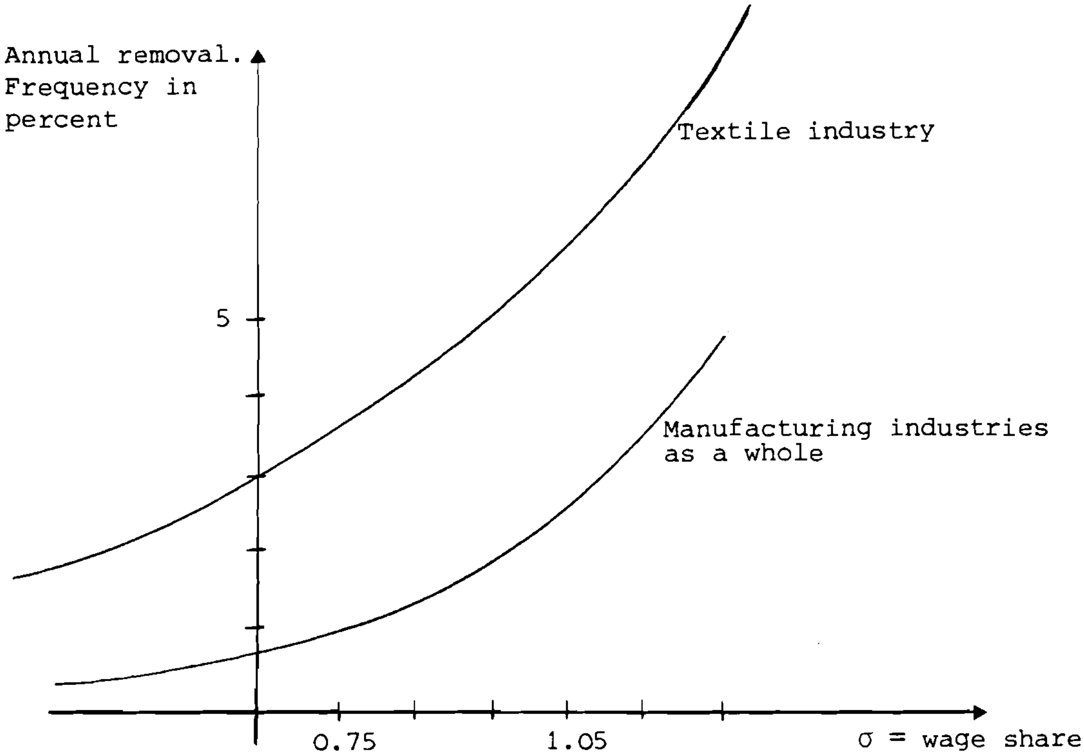
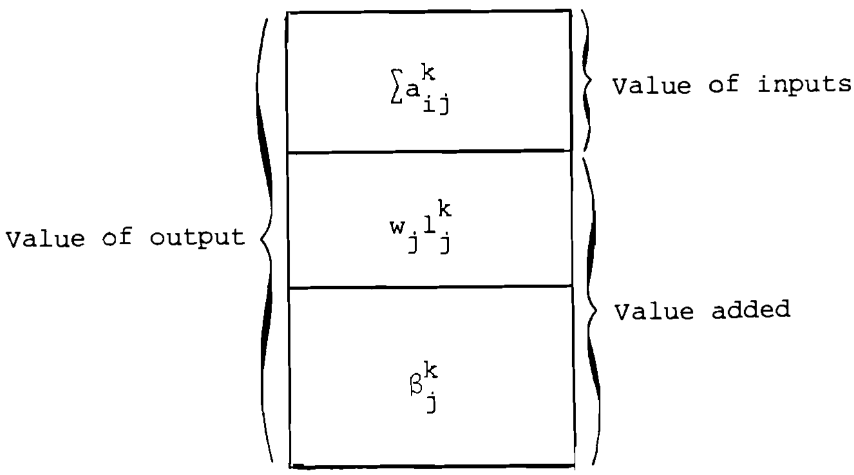


Figure 3.3. Removal functions for two industrial sectors, Sweden 1969-1977.



p_i = 1 for all i

a_{ij}^k = input coefficient

l_j^k = labor input coefficient

w_j = wage level

β_j^k = gross profit per unit output

Figure 3.4. Illustration of a production technique.

Given the assumptions behind Figure 3.4 one may conclude that a reduction of $\Sigma\, a^k_{ij}$ and/or of l^k_j will increase the value added per person employed. For a given wage level, this will also increase the profit per person employed.

Consider the capital coefficient, k_j, which is introduced in formula (2.10). With our normed price system and with the fixed coefficient k_j, the ratio between gross profits and investment costs becomes

$$\beta^*_j/k_j = (1 - \sum a^*_{ij} - \omega_j\, l^*_j)/k_j$$

where * denotes a technique associated with a new capacity. Best practice may in this context be interpreted as the combination of l^*_j and a^*_{ij} coefficients which makes β^*_j/k_j as large as possible[1]. For a market economy in which the wage level is increasing relative to prices, technical development tends to generate best practice solutions such that l^*_j is decreasing over time.

Let Figure 3.5 describe the productivity structure over different capacities in either a sector or a single production unit. For both these cases, the figure illustrates how the introduction of a capacity with the productivity $\mu^* = 1/l^*$ and the removal of an old capacity $\mu^o = 1/l^o$ simultaneously increase the average productivity of the sector or the production establishment. The only thing we have to assume is that μ^* is higher and μ^o lower than the average productivity.

3.4. Composite Effects of Investments and Capacity Change

Let the average gross profit of sector j, β_j, be defined from the formula (3.6) as follows

$$\tilde{\beta}_j = \sum_k \beta^k_j\, x^k_j / \sum_k x^k_j \qquad (3.9)$$

and let the associated artificial production technique of sector j be $\tilde{a}_{1j},\ldots,\tilde{a}_{nj};\ \tilde{l}_j$. Next let, from formula (2.10), $k_j = \Sigma\, p_i\, k_{ij}$.

[1] Observe that we implicitly assume invariant pay-back profiles (and durabilities) for different potential techniques.

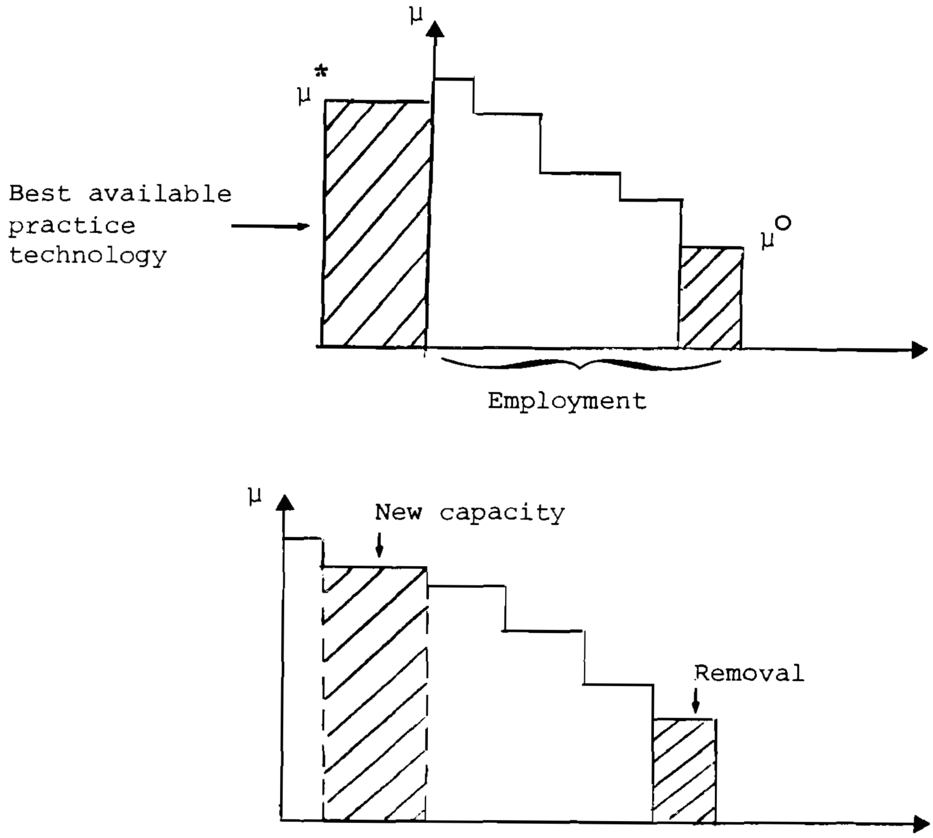


Figure 3.5. Capacity changes in a sector or a production unit.

In the framework introduced in Section 2 we have assumed that at a given time $k_j = k_j(p)$, $p = (p_1,\ldots,p_n)$, describes the costs of a unit of new capacity with best practice technology. Suppose that one may also invest in the average technique at the cost $\tilde{k}_j = \tilde{k}_j(p) \leq k_j(p)$. Now, let β_j^* be the profit per unit output using best practice. Then, profit-maximizing behavior implies selecting the best practice if[1]

$$\beta_j^*/k_j > \tilde{\beta}_j/\tilde{k}_j \tag{3.10}$$

which implies that $\beta_j^* > \tilde{\beta}_j$. We may also express this condition as

$$\tilde{l}_j/(p_j - \sum_i p_i \tilde{a}_{ij}) > l_j^*/(p_j - \sum p_i a_{ij}^*) \tag{3.11}$$

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[1]This assumes no differences in terms of durability, etc.

which means that investment in new capacity embodies an increase in the value added per person. Formula (3.11) implies that $\tilde{l}_j$ can be less than l_j^* only if $\Sigma\, p_i(\bar{a}_{ij} - a_{ij}^*) > 0$. Hence, without further assumptions we cannot make any definite conclusions as regards the relation between $\tilde{l}_j$ and l_j^*. Therefore, we have to make use of the observation that technological change of a sector has a bias such that l_j^* is decreasing over time while $\Sigma\, p_i(\tilde{a}_{ij} - a_{ij}^*)$ remains close to zero, usually with a negative sign.

Figure 3.6 illustrates the change pattern of the technology of a sector for which the productivity pattern has been divided into four equal segments, I,...,IV, of the total value added in the sector. Capacity changes in each such segment generates the following change pattern as the normal case:

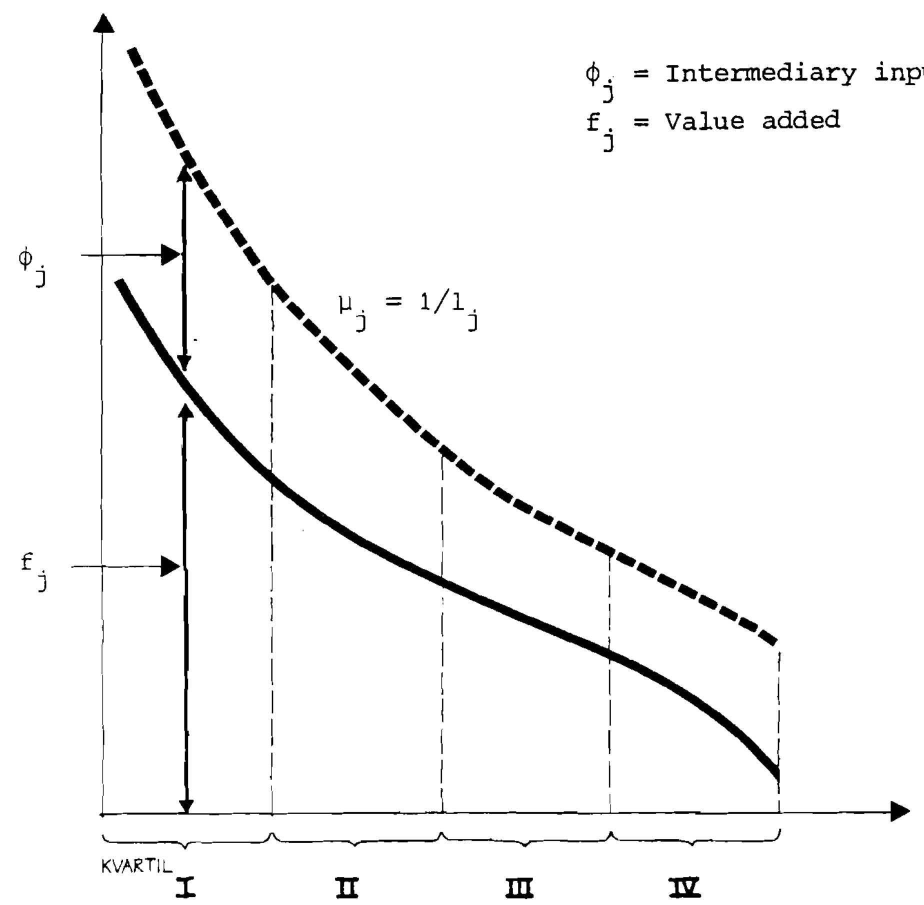


Figure 3.6. Illustration of the productivity pattern in a sector.

— $\phi_j = \sum p_i a_{ij}/l_j$ increases

— $f_j = (p_j - \phi_j)/l_j$ increases

Since this implies that p_j/l_j increases for p_j kept constant, it also implies that $1/l_j$ is increasing. Table 3.1 illustrates such a change process in the Swedish manufacturing industry.

A similar pattern as shown in Figure 3.6 may also be verified for disaggregated sectors, for which the computation of fixed prices becomes more relaible. The productivity change in the lower segment of the curve IV in Figure 3.6 is, to a large extent, caused by removal of capacities with low productivity.

Table 3.1. Productivity changes in the Swedish manufacturing industry 1968-1979.

Segments of the productivity curve	Annual increase in percent of		
	$\mu_j = 1/l_j$	ϕ_j	f_j
I	0.7	0.7	1.8
II	2.6	2.5	2.5
III	2.6	3.0	2.4
IV	2.7	3.0	2.8

Source: Johansson (1982).

3.5. Basic Assumption About Capacity Change

Let the following three labor input coefficients of capacities in sector j have the following meaning

l_j^* = best practice technique

$l_j = \sum_k l_j^k x_j^k / \sum_k x_j^k$ which refers to the average technique

l_j^o = technique in removed capacities

Based on the observations presented in Sections 3.1 - 3.4, we shall assume that the following relationship holds over time

$$l_j^o \geq l_j \geq l_j^* \qquad (3.12)$$

In the following sections we shall use this assumption to estimate capacity changes from observations on current production and employment over time. To illustrate the structure, consider the following notations:

L_j = current employment

$\bar{L}_j = \sum l_j^k \bar{x}_j^k$ denotes employment at full capacity utilization (3.13)

μ_j = current average productivity

$\bar{\mu}_j = \sum \bar{x}_j^k / \bar{L}_j$

$x_j(t)$ = current production

Using (3.13) one may state that

$$r_j(t) > 0 \Rightarrow \bar{\mu}_j(t) > \bar{\mu}_j(t-1)$$

$$x_j(t) > \bar{x}_j(t-1) \Rightarrow \bar{\mu}_j(t) > \bar{\mu}_j(t-1) \qquad (3.14)$$

$$\bar{\mu}_j(t) \geq \bar{\mu}_j(t-1)$$

Moreover, the capacity utilization $u_j(t)$ introduced in formula (2.9) must satisfy the following inequality which follows directly from (3.14):

$$u_j(t) \leq x_j(t)/[\bar{\mu}_j(t)\ L_j(t)] \tag{3.15}$$

4. CALCULATIONS OF CAPACITY CHANGE IN SECTORS

In this section the basic assumptions and conclusions from the preceding sections are utilized to formulate methods by which removal rates, capacity increase, and capacity utilization levels can be calculated. The methods are selected in order to make full use of the available data on the Italian economy during the period 1970-1980. The utilized procedures represent a systematic way to combine available data (observable variables) with constraints derived from production theory in order to indirectly determine unobserved variables.

4.1. Rigidities in the Adjustment of Employment

Capital equipment in a production unit may be regarded as a fixed factor in the sense that short-term variations in the output level does not result in decisions to change the equipment. In many respects also the employment/labor force of an establishment displays such a property. This type of rigidity was observed early by Solow among others with regard to the American economy during the 1950s.

This property may be analyzed or understood in the following way. A change in market demand affects the production level with a comparatively short delay or lag. This response to market variations takes the form of a cyclic pattern of current production. The adjustment of the employment level is a much slower process. When the demand is falling the labor force is usually reduced so slowly that the demand has started to rise again before the initial effect on employment has become significant. Therefore, the employment variations exhibit much smaller amplitudes than prodcution and capacity utilization.

Figure 4.1 illustrates the difference between production and employment with regard to annual variations. The two curves have been calculated by means of the following formula:

$$\hat{y}(t) = |y(t) - y^*(t)| / y^*(t) \qquad (4.1)$$

where $\hat{y}(t)$ represents the calculated curve, where $y(t)$ is the actually observed variable and where $y^*(t)$ is the estimated linear trend.

Figure 4.1 refers to the economy as a whole. However, the same property as that illustrated has also been verified by observations for the individual sectors of the economy. The relatively seen small variation in employment suggests the following approach. Given information about $\bar{\mu}_j(t)$ as specified in formula (3.13), the current capacity is calculated as follows:

$$\bar{x}_j(t) \geq \bar{\mu}_j(t)\ L_j(t) \qquad (4.2)$$

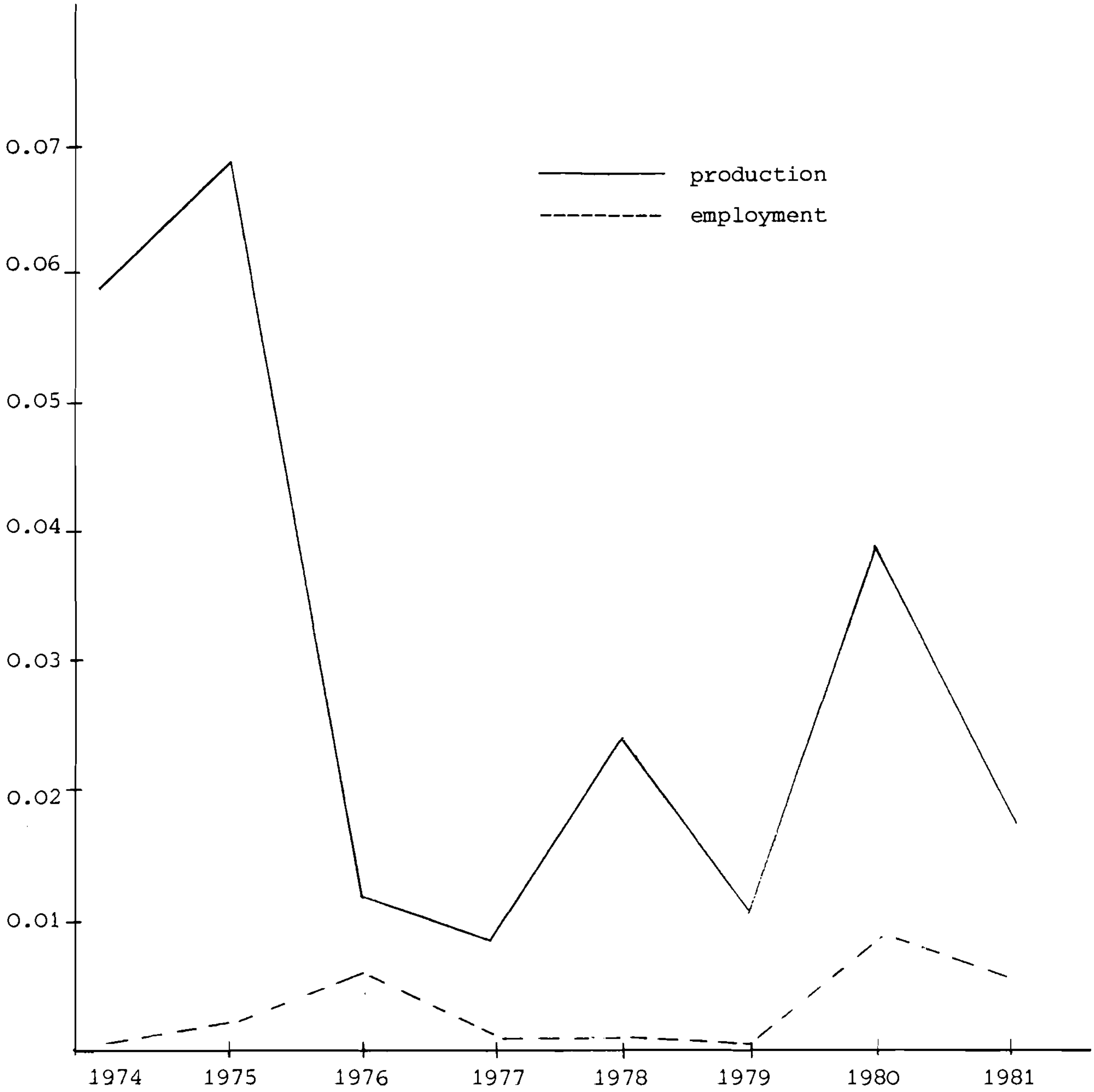
4.2. Production, Employment, and Productivity

The full capacity indicator of productivity, $\bar{\mu}_j(t)$, plays a fundamental role in our estimation procedure. The observable variables are $x_j(t)$, $L_j(t)$, and $\mu_j(t) = x_j(t)/L_j(t)$. The observations with regard to the Italian economy as a whole are made for the period 1970-1980 and covers 41 sectors. With regard to Tuscany the number of sectors is 31 and the period is 1974-1978.

Using the assumption that $\bar{\mu}_j(t) \geq \bar{\mu}_j(t-1)$ and formula (4.2) one may construct an upper envelope with regard to the full capacity productivity indicator in the following way:

$$\bar{\mu}_j(t) = \max\ [\bar{\mu}_j(t-1), \mu_j(t)] \qquad (4.3)$$

Such an envelope referring to a single sector is depicted in Figure 4.2.



Note: The residuals are measured as $\frac{|x(t) - \overset{*}{x}(t)|}{\overset{*}{x}(t)}$, where x(t) is the observed value in year (t) and $\overset{*}{x}(t)$ is the estimated trend value the same year.

Figure 4.1. Employment and production in the Italian economy: standardized, absolute residuals from the trend 1974 1974-1981.

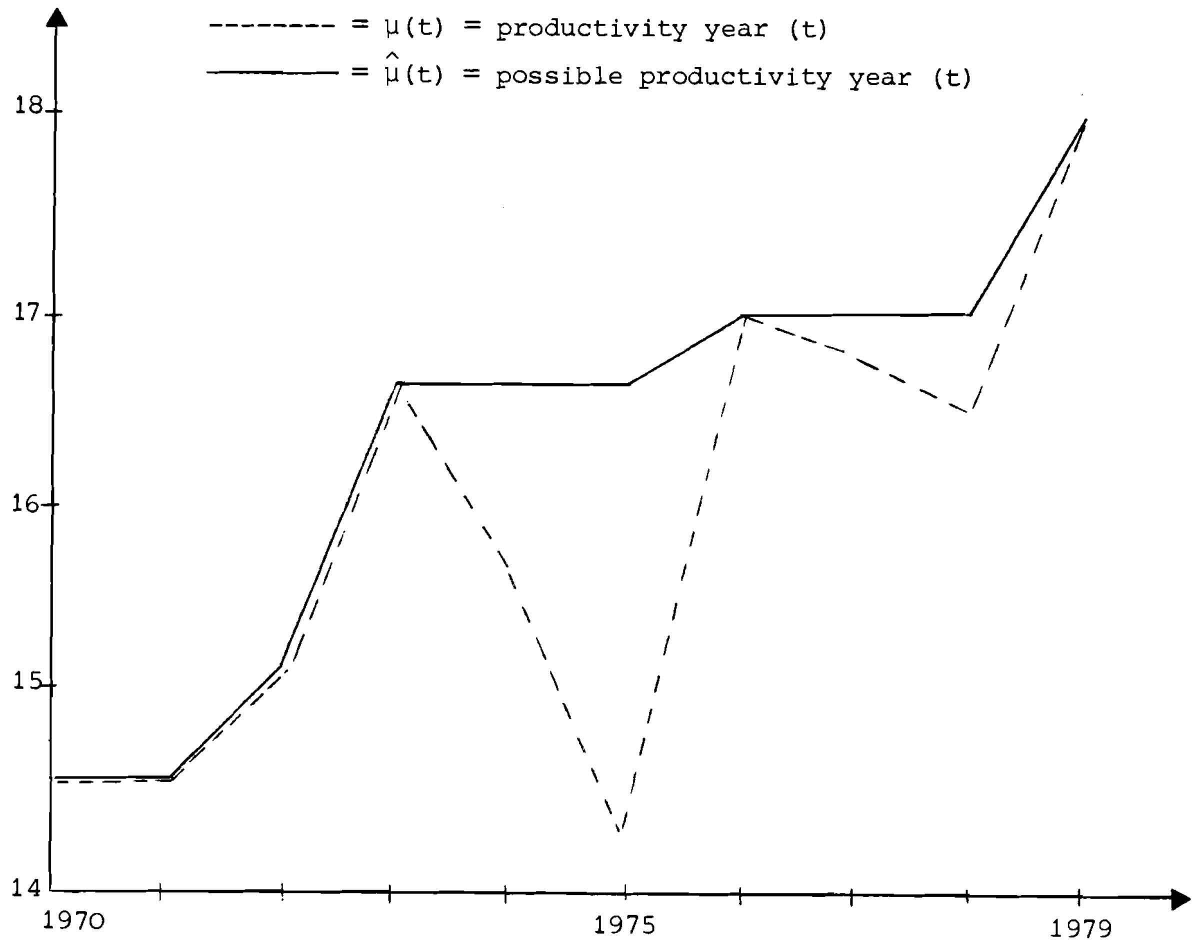


Figure 4.2. The relation between maximum oroductivity and observed productivity in the sector Other Transport Equipment in Italy.

4.3. The Development of Capacity Constraints in Sectors

Utilizing formula 4.3 the development of production capacities in each sector has been calculated for 41 sectors of the Italian economy. For one sector, coal products (number 2), a smoothing procedure has been used. In particular during the first half of the period 1970-1980, this sector shows a very rapid fall in productivity. Therefore, the calculation of $\bar{\mu}_j(t)$ for this sector has been made in the following way:

$$\bar{\mu}_j(t) = \begin{cases} \max\ [\bar{\mu}_j(t-1), \mu_j(t)] \text{ if } \Delta\mu_j(t) \geq 0 \\ \max\ \{\max\ [\bar{\mu}_j(t-1), \ldots, \bar{\mu}_j(t-5)];\ \mu_j(t)\} \text{ if } \Delta\mu_j(t) < 0 \end{cases} \quad (4.4)$$

This means that if the productivity continues to fall for more than five years, it is recorded as a fall in the indicator of the full capacity productivity. In fact, in this very case we have rejected the hypothesis that $\Delta\bar{\mu}_j(t) \geq 0$. However, the procedure applied in (4.4) requires quite "strong evidence," before an observed fall of productivity is recorded as a definite fall. The choice of the lag structure in this case is based on ad hoc considerations.

The calculation method specified in (4.3) represents a primitive but systematic way of detecting existing capacity constraints at each point in time, given the available data. The method requires that the time series is not too short, since formula (4.3) determines the envelope sequentially over time. One should observe that the capacity utilization generally varies within a year as indicated by Figure 4.3. Therefore, the calculated capacity value, $\bar{x}_j(t)$, does not represent the maximum production level from a technical point of view. It tends instead to represent the production at the "normal capacity utilization level". Hence, the variable h_j in Figure 4.3 signifies the necessary "overcapacity" which must exist due to seasonal variations.

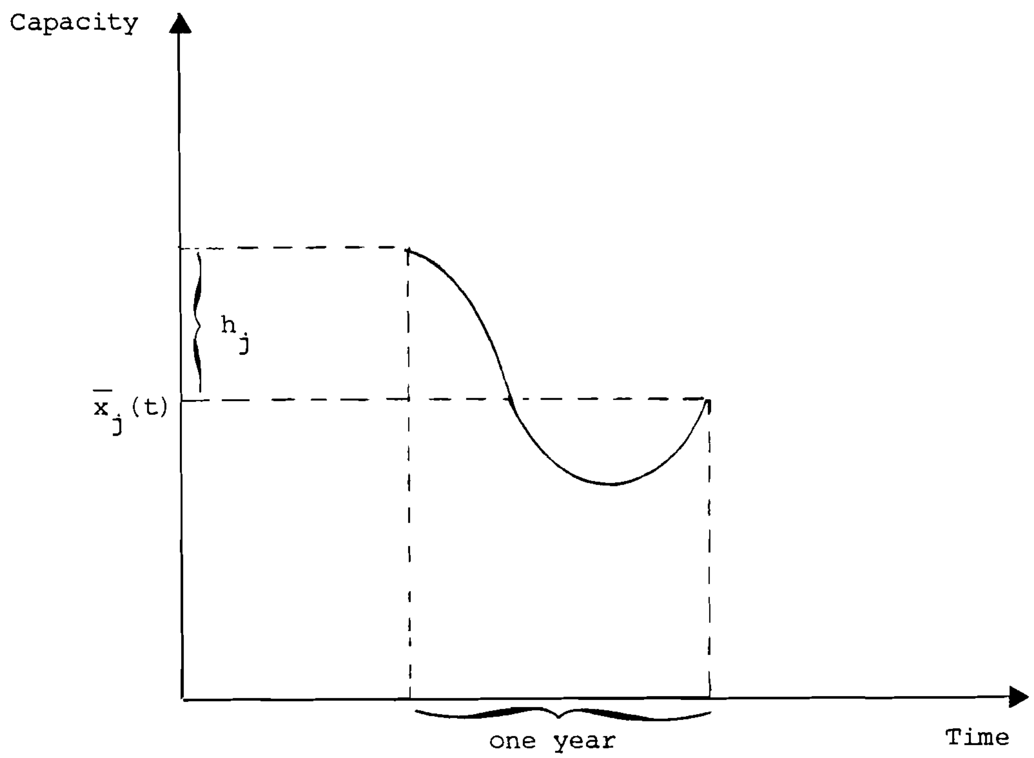


Figure 4.3. Normal capacity level during a year.

In Appendix 4 we present capacity estimates for 1980 in Italy. They are calibrated with aggregate information recorded by the Statistical Bureau of Italy in order to reflect the maximum production capacity, including the "overcapacity" illustrated by Figure 4.3. Table 4.1 describes for three periods the average annual change of the capacity level of each sector in the economy of Italy. As indicated by the estimates, the change process has been deviating considerably from a balanced growth path. The capacity development is calculated in relative terms according to the formula

$$\Delta\bar{x}_j/\bar{x}_j = \sum_{\tau=1}^{t} \Delta\bar{x}_j(\tau) \Big/ \sum_{\tau=1}^{t} \bar{x}_j(\tau)$$

4.4. Removal Rates and Capacity Changes

The rate of removal in a single sector is specified in formulas (2.4) and (2.5). Given a calculated capacity path, $\{\bar{x}_j(t)\}$, one may introduce a supplementing source of information, the sequence of investments $\{I_j(t)\}$, at fixed prices. As indicated by formula (2.10), such a sequence determines the sequence $\{\Delta\bar{\bar{x}}_j(t)\}$ for a given investment coefficient k_j[1]. Rearranging formula (2.4) one obtains

$$r_j(t) = \Delta\bar{\bar{x}}_j(t) - \Delta\bar{x}_j(t) \,/\, \bar{x}_j(t-1) \qquad (4.5)$$

In order to avoid a detailed inquiry into the problem of lags between investments and installation of new capacities, the rate of removal has been calculated for periods of several years in the following way:

$$r_j = \left[\sum_{\tau=1}^{t} \Delta\bar{\bar{x}}_j(\tau) - \sum_{\tau=1}^{t} \Delta\bar{x}_j(\tau)\right] \Big/ \sum_{\tau=1}^{t} \bar{x}_j(\tau-1) \qquad (4.6)$$

[1] Ciaschini (1981) contains estimates of investment coefficients of this type.

Table 4.1. Annual relative change of capacity for sectors of the Italian economy. Average values for three periods.

Sector	1970-1980	1970-1975	1975-1980
1	0.021	0.016	0.025
2	-0.100	-0.044	-0.230
3	0.018	0.045	-0.005
4	0.021	0.033	0.011
5	0.045	0.058	0.034
6	0.0	0.0	0.0
7	0.036	0.061	0.015
8	0.031	0.033	0.029
9	0.053	0.056	0.050
10	0.006	0.010	0.002
11	0.021	0.026	0.016
12	0.111	0.078	0.133
13	0.032	0.036	0.028
14	0.023	0.008	0.038
15	0.073	0.061	0.083
16	0.045	0.047	0.043
17	0.043	0.046	0.040
18	0.038	0.036	0.040
19	0.034	0.056	0.015
20	0.023	-0.014	0.057
21	0.021	0.009	0.032
22	0.035	0.052	0.020
23	0.055	0.059	0.052
24	0.030	0.039	0.022
25	0.039	0.067	0.015
26	0.066	0.101	0.040
27	0.000	0.000	0.000
28	0.036	0.035	0.037
29	0.036	0.041	0.032
30	0.034	0.036	0.032
31	0.036	0.036	0.036
32	0.044	0.038	0.048
33	0.039	0.033	0.045
34	0.058	0.066	0.052
35	0.055	0.064	0.049
36	0.042	0.032	0.049
37	0.039	0.042	0.037
38	0.062	0.062	0.062
39	0.039	0.043	0.036
40	0.031	0.044	0.021
41	0.018	0.031	0.006

The removal rates specified in formula (4.6) have been calculated for three different time periods, I = 1970-1979, II = 1970-1974, and III = 1975-1979. Table 4.2 describes the calculated values $r_j(I)$, $r_j(II)$, $r_j(III)$, and $r_j(III) - r_j(II)$ with regard to Italy. For the main part of sectors the removal rate was higher during the first period. The average rate for the period 1970-1979 is close to nine percent.

5. ESTIMATION OF CAPITAL COEFFICIENT MATRICES FOR ITALY AND TUSCANY

This section describes a method for estimating capital coefficient matrices (K-matrices) containing marginal capital coefficients or, in other terms, investment coefficients, k_{ij}, of the kind introduced in formula (2.6). The calculations described in Section 4 provide one set of data input. Here we shall combine this set with available information about investment flows and *a priori* given information about aggregate capital coefficients to obtain a "least biased" estimate of K-matrices for Italy and Tuscany. The section contains a description of the estimation technique which may be compared with suggestions in Batten (1981).

5.1. Information Sources for the Estimation

The information available for the estimation of K-matrices referring to Italy as a whole differs considerably from the information directly related to Tuscany. As a consequence of this, we first estimate a matrix for Italy. In a second step one may adjust this matrix to reflect any additional information available for Tuscany.

Different types of information has been available at different levels of aggregation. For Italy the objective is to construct a K-matrix with 41 sectors and for Tuscany a matrix with 31 sectors. The degree of aggregation will, in the sequel, be signified by the term "n-sector level".

Table 4.2. Calculations of the rate of removal in different sectors of the Italian economy, 1970-1979, percent.

Sector	r_j(I) 1970/79	r_j(II) 1970/74	r_j(III) 1975/79	r_j(III) - r_j(II)
1	9	8	9	1
2	17	10	35	24
3	12	10	13	3
4	9	9	9	0
5	8	9	8	- 1
6	—	—	—	—
7	10	13	8	- 5
8	9	11	8	- 3
9	7	9	5	- 4
10	12	12	12	0
11	11	12	11	- 1
12	5	13	0	-13
13	10	10	11	1
14	10	13	7	- 6
15	6	10	3	- 7
16	8	10	6	- 4
17	8	9	6	- 3
18	8	10	6	- 4
19	8	7	8	1
20	9	14	5	- 9
21	10	13	8	- 5
22	10	11	10	- 1
23	6	9	5	- 4
24	9	10	9	- 1
25	12	13	10	- 3
26	5	4	5	1
27	10	10	11	1
28	10	13	8	- 5
29	8	8	7	- 1
30	9	10	8	- 2
31	9	9	8	- 1
32	8	10	7	- 3
33	8	10	7	- 3
34	8	6	9	3
35	6	8	5	- 3
36	7	10	5	- 5
37	8	9	6	- 3
38	5	7	3	- 4
39	8	9	7	- 2
40	7	8	7	- 1
41	10	10	11	1

With regard to Italy, the following information has been available through direct observations or through indirect reconstructions of the type described in Section 4:

H_i = investment goods delivered from sector i in 1975; 41-sector level (2.8)

I_j = the value of investment goods received by sector j; 23-sector level 1970-1979 (2.8)

I_{ij} = investment goods delivered from sector i to sector j in 1975; i = 1,...,41; j = 1,...,23 (2.6)

$\Delta\bar{x}_j$ = estimated capacity increment in sector j; 41-sector level, 1970-1980 (2.3)

r_j = rate of removal in sector j, 41-sector level, 1970-1980 (2.4)

k_j = calculations of the aggregate capital coefficient of sector j based on information from 1970-1978, 23-sector level (2.10).

All values are recorded at fixed prices (1975).

5.2. Estimation of the K-Matrix for Italy

Using the I_{ij} information, a matrix of α_{ij}-coefficients may be calculated as follows

$$\alpha_{ij} = I_{ij}/I_j \tag{5.1}$$

where α_{ij} denotes the investment deliveries from sector i as a share of total investment deliveries per unit of capacity created in sector j. Using available k_j-estimates we can form the following coefficients

$$k'_{ij} = k_j\ \alpha_{ij} \tag{5.2}$$

These coefficients are then obtained at the 23-sector level. Since these sectors are aggregates of the 41 sectors, we may for each sector j define a subindex j(h) such that j(1),j(2),..., signifies the disaggregation of sector j into subsectors on the

41-sector level[1]. By setting $\alpha_{im(h)} = \alpha_{im}$ we obtain the expanded k_{ij}-matrix

$$k_{ij} = k'_{im(h)} \ \alpha_{im(h)} \qquad \begin{aligned} i,j &= 1,\ldots,41 \\ m &= 1,\ldots,23 \\ m(h) &= 1,\ldots,41 \end{aligned} \qquad (5.3)$$

The matrix $K = (k_{ij})$ is based on information from 1975 only. It represents our *a priori* estimate. Using the additional information available we shall form a new matrix $K^* = (k^*_{ij})$ by utilizing the "minimum information principle" (Snickars and Weibull 1977).

The procedure requires the following steps. First, we select a time period consisting of five or 10 years. Given the selected time period the following values are determined:

$$\Delta\bar{\bar{x}}_j = \Delta\bar{x}_j + r_j\bar{x}_j \qquad j = 1,\ldots,41 \qquad (5.4)$$

where

$$\Delta\bar{x}_j = \sum_{\tau=1}^{t+1} \Delta\bar{x}_j(\tau)/t$$

$$\bar{x}_j = \sum_{\tau=0}^{t} \bar{x}_j(\tau)/t$$

and

$$v_{ij} = k_{ij} \ \Delta\bar{\bar{x}}_j \qquad j = 1,\ldots,41 \qquad (5.5)$$

$$I_j = \sum_{\tau=0}^{t} I_j(\tau)/t \qquad j = 1,\ldots,41 \qquad (5.6)$$

$$H_i = H(t) \qquad \begin{aligned} t &= 1975 \\ i &= 1,\ldots,41 \end{aligned} \qquad (5.7)$$

where k_{ij} is given by (5.3), H_i is given by direct observations for 1975, and where the I_j's are based on observations on the

[1]The sector classification key is described in Appendix 1.

23-sector level. These observations are transformed to the 41-sector level by means of a bridge vector[1].

Given the calculations in (5.4)-(5.7), the following optimization problem is formulated:

$$\max = \sum_i \sum_j \omega_{ij} \ln \omega_{ij}/v_{ij}$$

so that

$$\sum_i \omega_{ij} = I_j \qquad \text{for all } j \tag{5.8}$$

$$\sum_i \omega_{ij} = H_i \qquad \text{for all } i$$

where the I_j's and H_i's are calibrated so that $\Sigma H_i = \Sigma I_j$. The associated Lagrange function is

$$L = - \sum\sum \omega_{ij} \ln \omega_{ij}/v_{ij} + \sum \beta_i (\sum \omega_{ij} - H_i)$$

$$+ \sum \gamma_j \ (\sum \omega_{ij} - I_j)$$

The solution is obtained in explicit form by differentiating L with respect to ω_{ij}, which yields

$$\omega_{ij} = v_{ij} \exp \ (- \beta_i - \gamma_j - 1) \tag{5.9}$$

With specific assumptions about the underlying probability structure, one may interpret the results in (5.9) as maximum likelihood estimates (see Snickars and Weibull 1977). From the solution in (5.9) one obtains the k^*_{ij} coefficients as

$$k^*_{ij} = \omega_{ij}/\Delta\bar{\bar{x}}_j \tag{5.10}$$

The data set currently available for Tuscany is very meager with regard to investment flows. Therefore an aggregated 31-sector version of the K-matrix, referring to Italy, has been calculated as proxy for a Tuscany-matrix proper.

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[1]See Appendix 1.

6. REMARKS ON THE ESTIMATION RESULTS

6.1. The 41-Sector K-Matrix

The estimated 41-sector K-matrix is described in Appendix 5.[1] The estimation result depends critically on the *a priori* given information about the aggregate investment coefficients k_j. The calculation of removal rates in Table 4.2 may be regarded as a way to check the reliability of the *a priori* coefficients. By combining (2.10) and (4.6) one can see that the removal rates, r_j, are determined by the chosen values k_j as follows:

$$r_j = [I_j/k_j - \Delta\bar{x}_j] / \bar{x}_j \tag{6.1}$$

Table 4.2 shows that the r_j-values are not remarkably high. At the same time one must admit that both the *a priori* values k_j and the final values $k_j^* = \Sigma\, k_{ij}^*$ are comparatively low.

In Table 6.1 the coefficients k_j^* are compared with similar estimates from Swedish data.[2] Moreover, the table contains a calculation of the investment coefficients which obtains if the removal is zero in each sector. In that case, formula (6.1) yields aggregate coefficients $\hat{k}_j = I_j/\Delta\bar{x}_j$. These values represent the maximum level which the coefficients can reach, given that they shall be consistent with observed investment flows I_j.

Table 6.2 illustrates the aggregate investment coefficients of the 31-sector K-matrix which refers to Tuscany and the rest of Italy.

6.2. Accelerator Relations Between Sectors

The relation between a sector receiving investment goods and the sectors delivering these goods is an accelerator connection. The receiving sector accelerates the growth process by demanding investment goods from the capital goods producing

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[1] The algorithm utilized for solving the estimation model in (5.8) is developed by Håkan Persson and is described in Andersson and Persson (1982).

[2] The Swedish coefficient matrix has the dimension 28 x 28. This means that in several cases the Swedish sector has to represent several sectors in the Italian matrix.

Table 6.1. Aggregate capital coefficients.

Sector	Estimation outcome k_j^*	Maximum coefficient value	Swedish coefficient
1	1.148	6.3	2.7
2	1.000	2.1	2.1
3	0.873	7.2	2.1
4	0.878	4.9	0.4
5	0.877	2.8	9.4
6	0.0	0.0	9.4
7	0.678	2.9	2.1
8	0.611	2.6	2.1
9	0.783	1.9	1.2
10	0.317	6.7	1.3
11	0.309	2.1	1.3
12	0.343	0.5	1.1
13	0.343	1.6	0.9
14	0.624	3.6	1.0
15	0.623	1.2	1.0
16	0.168	0.5	0.9
17	0.168	0.5	0.9
18	0.168	0.6	0.7
19	0.167	0.6	1.1
20	0.169	0.9	1.1
21	0.201	1.3	1.1
22	0.201	0.9	1.1
23	0.103	0.3	0.9
24	0.279	1.3	3.2
25	0.372	1.6	1.2
26	0.104	0.2	1.3
27	0.141	55.0	0.4
28	0.103	0.4	2.0
29	0.586	2.0	7.2
30	0.410	1.7	1.2
31	1.463	5.5	7.2
32	1.462	4.6	7.2
33	1.463	5.0	7.2
34	3.018	7.7	7.2
35	0.396	0.4	2.0
36	3.177	9.3	18.5
37	3.167	10.3	2.0
38	3.175	6.1	2.0
39	3.178	10.2	2.0
40	1.395	5.1	—
41	3.200	22.1	2.0

Table 6.2. Aggregate capital coefficients of the 31-sector K-matrix.

Sector	Coefficient = k_j^*
1	1.148
2	0.877
3	0.877
4	0.678
5	0.610
6	0.783
7	0.316
8	0.309
9	0.343
10	0.343
11	0.624
12	0.168
13	0.167
14	0.168
15	0.166
16	0.169
17	0.201
18	0.200
19	0.102
20	0.278
21	0.371
22	0.104
23	0.141
24	0.486
25	0.410
26	1.462
27	3.017
28	0.396
29	3.176
30	3.176
31	1.454

sectors. When the demand for capacity varies over time, this type of sectoral connection represents a strongly destabilizing factor. Table 6.3 depicts the accelerator couplings which exhibit a strong connection.

Table 6.3. Accelerator connections between sectors receiving and sectors delivering investment goods.

Delivering sectors	Receiving sectors: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
8																						
10	O						O	O	O			O	O	O	O							
11	⊕	⊕	⊕	⊕	⊕		⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	O	O	O	O	O	O	O
12							O	O	O			O		O	O							
13	O	O	O	O	O		⊕	O	⊕	O	O	O	O	⊕	⊕						O	O
14								O														
15	O																					
22																						
23	O																					
27	⊕	⊕	⊕	⊕	⊕		⊕	⊕	⊕	O	O	O	O	⊕	⊕	O	O	O	O	O	O	O
29	O	O	O	O	O				O													

Delivering sectors	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
8												O							
10			O									⊕							
11	O	O	O	O	O	O	O	O				⊕	O	O	O	O	O		O
12			O									⊕							
13		O	O				O					⊕		O	O	O	O		O
14					O		⊕	⊕	⊕	⊕	⊕			O	O	O	O		O
15								⊕	⊕	⊕	⊕								
22																			O
23							⊕	⊕				O	O	O	O	O	O		O
27	O	O	O		O	O	O	O	O	O	O	O	O	⊕	⊕	⊕	⊕	⊕	⊕
29									O	O	O	⊕		⊕	⊕	⊕	⊕	O	⊕

Note: O = Strong connection
⊕ = Very strong connection

6.3. The Bi-Regional K-Matrices

Let K be the estimated 31-sector K-matrix intended to be applied to both Tuscany and the rest of Italy. Moreover, let x^1 and x^2 denote the production in the two regions, respectively. The associated gross capacity change is denoted by $\Delta\bar{\bar{x}}^1$ and $\Delta\bar{\bar{x}}^2$. Before proceeding we have to observe that only a fraction α_i, $1 \geq \alpha_i \geq 0$, of the investment goods delivered from sector i to sectors in region 1 have their origin in the same region. The remaining fraction $(1 - \alpha_i)$ has its origin in region 2. Hence, we may define

α_i = the share of investment goods of type i delivered to sectors in region 1 which are also produced in the same sector (6.2)

β_i = the share of investment goods of type i delivered to sectors in region 2 which are also produced in the same region.

Naturally, $\alpha_i, \beta_i \geq 0$ and $\alpha_i, \beta_i \leq 1$. By $\langle\alpha\rangle$, $\langle\beta\rangle$, $\langle 1-\alpha\rangle$, and $\langle 1-\beta\rangle$ we denote diagonal matrices with α_i, β_i, $1-\alpha_i$, and $1-\beta_i$ as elements.

According to the assumption in (6.2) a gross capacity increase, $\Delta\bar{\bar{x}}^1$, will generate the following demand in region 1 for investment deliveries from region 1 and 2 respectively.

$$\begin{aligned} \langle\alpha\rangle K\Delta\bar{\bar{x}}^1 &= \text{demand in region 1} \\ \langle 1-\alpha\rangle K\Delta\bar{\bar{x}}^1 &= \text{demand in region 2} \end{aligned} \qquad (6.3)$$

The demand generated by $\Delta\bar{\bar{x}}^2$ can be specified analogously. The full bi-regional investment matrix must therefore have the following form

$$\begin{bmatrix} \langle\alpha\rangle K & , \langle 1-\beta\rangle K \\ \langle 1-\alpha\rangle K , & \langle\beta\rangle K \end{bmatrix} \qquad (6.4)$$

The aggregate coefficients of the K-matrix in (6.4) are presented in Table 6.2.

Finally observe that if we are able to specify matrices [α] and [β] with off-diagonal elements different from zero, then this presupposes the availability of information detailed enough to estimate K-matrices of the type K^{rs}, containing investment coefficients for deliveries between regions r and s. In that case (6.4) becomes

$$\begin{bmatrix} K^{11} & K^{12} \\ K^{21} & K^{22} \end{bmatrix}$$

If no distinction is made between bi-regional trade of goods for (i) intermediary use, (ii) investment, and (iii) consumption, one may use an overall trade matrix. This is the approach followed in the Tuscany case study and therefore the bi-regional capital coefficient matrix becomes

$$\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$$

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APPENDIX 1: SECTOR CLASSIFICATIONS

The 41-sector classification for Italy has the following structure:

1. Agriculture, forestry fishing
2. Coal
3. Coke
4. Petroleum, gas, refining
5. Electricity, gas, water
6. Nuclear fuels
7. Ferrous, non-ferrous ores
8. Non-metal minerals, mineral products
9. Chemical products
10. Metal products
11. Agricultural and industrial machinery
12. Office, precision, and optical instruments
13. Electrical goods
14. Motor vehicles
15. Other transport equipment
16. Meat
17. Milk
18. Other foods
19. Non-alcoholic and alcoholic beverages
20. Tobacco
21. Textiles
22. Leather and shoes
23. Wood and furniture
24. Paper and printing products
25. Rubber and rubber products
26. Other manufacturing products
27. Construction
28. Recovery and repairs services
29. Trade
30. Hotels and restaurants
31. Inland transport
32. Sea and air transport
33. Transport services
34. Communication
35. Banking and insurance
36. Other private services, real estate
37. Private education services
38. Private health services
39. Recreation and culture
40. Public services
41. Domestic servants

The 31-sector classification for the bi-regional model (Tuscany and the rest of Italy) has the following structure:

1. Agriculture
2. Coal and oil
3. Other energy forms and water
4. Minerals
5. Minerals, non-metal
6. Chemicals
7. Metal products
8. Machinery for industry, agricul.
9. Other machinery
10. Electrical equipment
11. Transport equipment
12. Meat
13. Milk
14. Other food products
15. Beverages
16. Tobacco
17. Textiles
18. Footwear
19. Wood and furniture
20. Paper and paper products
21. Rubber and rubber products
22. Other manufactures
23. Construction
24. Commerce
25. Hotels
26. Transport
27. Communication
28. Credit and insurance
29. Housing
30. Other marketable services
31. Non-marketable services

The bridge vector which rearranges 41-sectors to 31 aggregated sectors has the following structure:

Classification

41-sector	31-sector		41-sector	31-sector	
1	1.000	1	21	0.789	12
2	0.001	2	22	0.211	12
3	0.033	2	23	0.464	15
4	0.618	2	24	1.000	13
5	0.348	2	25	1.000	14
6	0.0	2	26	0.124	15
7	1.000	3	27	1.000	16
8	1.000	4	28	0.412	15
9	1.000	5	29	1.000	17
10	1.000	6	30	1,000	18
11	1.000	7	31	0.575	19
12	1.000	8	32	0.223	19
13	1.000	9	33	0.202	19
14	0.721	10	34	1.000	20
15	0.279	10	35	1.000	21
16	0.265	11	36	0.626	22
17	0.101	11	37	0.027	22
18	0.500	11	38	0.111	22
19	0.062	11	39	0.215	22
20	0.072	11	40	1.000	23
			41	0,021	22

APPENDIX 2: PRODUCTIVITY OF SECTORS IN TUSCANY
MILL LIRE/THOUSANDS EMPLOYMED CONSTANT
PRICES (1975)

Sector	1974	1975	1976	1977	1978
1	4.94	4.72	4.85	4.29	5.17
2	214.99	131.82	189.84	206.40	152.35
3	41.09	36.62	36.18	37.67	37.28
4	40.67	34.37	36.56	37.90	30.14
5	13.58	11.45	12.72	14.38	14.49
6	38.23	31.92	36.87	36.82	36.61
7	16.25	14.40	16.96	18.25	19.39
8	17.43	16.36	18.68	23.13	23.23
9	13.20	13.18	13.45	15.44	14.60
10	14.22	15.84	15.04	18.26	21.54
11	17.87	17.24	20.73	20.67	21.07
12	65.01	60.04	66.52	67.52	56.39
13	34.93	31.15	24.14	31.00	29.59
14	45.79	47.17	45.65	41.98	38.33
15	21.90	21.76	21.90	22.25	19.01
16	106.03	105.39	88.69	74.21	69.48
17	12.37	11.64	14.75	15.63	15.53
18	14.61	12.23	16.88	17.97	21.04
19	9.14	11,17	10.32	14.31	14.73
20	23.54	18.64	21.86	20.14	24.30
21	22.48	13.21	24.22	18.12	16.48
22	23.31	13.85	26.92	20.96	21.56
23	10.98	10.07	10.98	12.09	11.58
24	8.82	8.34	8.74	9.09	9.05
25	14.19	12.97	14.27	14.82	15.58
26	8.83	8.91	9.16	9.92	11.14
27	10.58	11.46	11.20	12.00	10.87
28	31.76	32.29	31.02	28.57	29.51
30	10.96	10.54	9.84	9.59	10.21
Average	6.42	6.36	6.45	6.34	7.58

APPENDIX 3: PRODUCTIVITY OF SECTORS IN ITALY, MILL LIRE/ THOUSANDS EMPLOYED, CONSTANT PRICES (1975)

Sectors	1975	1976	1977	1978	1979	1980	1981
1	4.72	4.67	4.84	5.11	5.50	5.89	6.25
2	4.44	4.62	4.40	4.40	5.20	4.62	4.62
3	146.22	144.86	142.50	135.56	139.17	152.22	138.46
4	333.31	350.86	327.85	342.85	349.40	289.07	296.17
5	38.30	41.57	42.39	44.28	45.50	45.99	44.54
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	30.57	33.43	32.72	34.89	34.05	36.34	35.70
8	12.60	13.72	14.17	14.35	15.40	16.88	16.48
9	32.59	37.45	37.71	41.77	44.68	46.98	46.89
10	14.35	15.64	15.90	15.11	15.36	16.72	15.98
11	19.13	19.67	20.69	20.05	20.04	22.05	21.24
12	14.16	16.44	17.15	22.62	29.35	32.48	41.50
13	14.13	15.20	15.96	16.32	16.73	18.42	17.45
14	21.83	22.89	24.13	25.69	24.73	25.09	24.87
15	15.24	16.66	16.91	17.17	17.74	23.15	28.44
16	86.37	90.44	92.09	94.90	97.61	104.48	92.05
17	40.82	42.76	45.47	47.68	49.11	50.18	52.16
18	34.20	38.45	36.71	39.33	42.28	44.23	44.88
19	25.34	26.91	26.00	28.01	31.90	31.22	32.16
20	107.16	112.70	116.23	102.84	97.00	92.95	92.93
21	9.74	11.74	11.48	10.72	12.26	12.71	12.26
22	12.64	13.54	13.38	13.01	14.30	13.93	13.30
23	10.08	12.58	13.64	13.04	14.66	15.69	14.52
24	19.65	23.53	22.90	25.29	26.82	27.24	27.00
25	14.31	17.02	16.83	16.58	18.15	17.68	16.95
26	17.96	20.83	24.29	27.03	27.65	27.40	23.77
27	10.25	10.36	10.45	10.42	10.71	11.02	10.96
28	8.60	8.90	8.89	8.82	8.40	8.95	8.44
29	8.78	9.21	9.37	9.67	10.10	10.28	10.06
30	12.33	12.60	12.71	12.83	13.23	13.41	13.14
31	7.55	7.69	7.94	8.14	8.43	8.69	8.51
32	32.14	35.12	38.50	39.66	39.97	39.50	28.76
33	13.28	13.88	14.50	15.09	15.53	15.87	15.52
34	10.19	10.23	10.36	10.77	11.73	12.15	12.51
35	32.52	32.46	32.14	32.52	33.27	33.57	33.66
36	33.05	32.08	31.20	30.24	28.90	28.91	28.21
37	4.89	4.69	4.85	4.83	4.91	5.08	5.10
38	15.03	15.12	14.89	14.34	14.00	13.42	12.99
39	10.62	10.74	10.83	10.98	11.29	11.32	11.04
40	5.57	5.59	5.55	5.46	5.38	5.40	5.36
41	0.97	1.00	1.00	1.01	1.01	1.05	1.06

APPENDIX 4: CAPACITY AND CAPACITY UTILIZATION

The base year for the scenarios in the Tuscany studies is 1980. To make such scenarios possible, maximal capacity and actual capacity utilization have been calculated for 1980 in Table A4.1. Normal capacity level for 1980 is obtained by multiplying the maximum level by factor 0.9.

Table A4.2 describes the capacity utilization as a share of maximum capacity for the period 1975-1980 in Italy. Table A4.3 contains the same information with regard to Tuscany for the period 1975-1978. One should observe that 1975 is characterized by a low degree of capacity utilization both in Italy as a whole and in Tuscany. This is important to note, since the input-output structure with regard to Tuscany and the rest of Italy has been estimated with data from this year which is characterized by a low activity level and a high level of idle capacity in many sectors.

Table A4.1. Production capacity and idle capacity by sectors, Italy, 1980. 1975 values, millions of Lire.

Sector	Production	Maximal capacity	Idle capacity	Production divided by maximal capacity
1	16256.	17882.	1626.	91
2	12.	15.	3.	81
3	548.	622.	74.	80
4	9337.	14765.	5428.	65
5	7197.	7917.	720.	88
6	0.	0.	0.	--
7	10656.	11722.	1066.	89
8	6810.	7491.	681.	89
9	13811.	15192.	1381.	91
10	7394.	3296.	902.	85
11	3928.	9821.	893.	88
12	2595.	2595.	0.	100
13	7323.	8055.	732.	86
14	6616.	7453.	837.	88
15	3200.	3520.	320.	91
16	6593.	7525.	659.	80
17	2564.	2820.	256.	91
18	12823.	14105.	1282.	91
19	1561.	1755.	194.	91
20	1543.	2122.	579.	73
21	14760.	16236.	1476.	88
22	3446.	3891.	445.	85
23	7942.	8736.	794.	84
24	7070.	7777.	707.	90
25	3766.	4253.	487.	85
26	2121.	2355.	234.	78
27	19157.	21073.	1916.	90
28	5500.	6050.	550.	86
29	26530.	29183.	2653.	89
30	7936.	8785.	799.	89
31	6520.	7172.	652.	89
32	2691.	2884.	293.	65
33	2339.	2573.	234.	89
34	2994.	3293.	299.	91
35	11327.	12753	1426.	89
36	16075.	20310.	4235.	77
37	745.	820.	75.	91
38	3060.	4113.	1053.	72
39	5928.	6521.	593.	89
40	15960.	19018.	3058.	83
41	580.	559.	51.	91
Total	292093.	321302.	29209.	91

Table A4.2. Capacity utilization in different sectors of the Italian economy.

Sector	1975	1976	1977	1978	1979	1980
1	0.91	0.90	0.91	0.91	0.91	0.91
2	0.31	0.57	0.87	0.87	0.91	0.81
3	0.85	0.84	0.83	0.79	0.81	0.88
4	0.73	0.77	0.72	0.75	0.76	0.63
5	0.86	0.91	0.91	0.91	0.91	0.91
6	0.0	0.0	0.0	0.0	0.0	0.0
7	0.79	0.86	0.85	0.90	0.88	0.91
8	0.80	0.87	0.90	0.91	0.91	0.91
9	0.85	0.91	0.91	0.91	0.91	0.91
10	0.77	0.83	0.85	0.81	0.82	0.89
11	0.83	0.85	0.89	0.86	0.86	0.91
12	0.83	0.96	1.00	1.00	1.00	1.00
13	0.80	0.86	0.90	0.91	0.91	0.91
14	0.83	0.88	0.91	0.91	0.88	0.89
15	0.81	0.89	0.90	0.91	0.91	0.91
16	0.91	0.91	0.91	0.91	0.91	0.91
17	0.89	0.91	0.91	0.91	0.91	0.91
18	0.87	0.91	0.87	0.91	0.91	0.91
19	0.77	0.82	0.79	0.85	0.91	0.89
20	0.89	0.91	0.91	0.80	0.76	0.73
21	0.84	0.91	0.89	0.83	0.91	0.91
22	0.88	0.91	0.90	0.87	0.91	0.89
23	0.74	0.91	0.91	0.87	0.91	0.91
24	0.73	0.88	0.85	0.91	0.91	0.91
25	0.78	0.91	0.90	0.89	0.91	0.89
26	0.69	0.81	0.91	0.91	0.91	0.90
27	0.85	0.86	0.87	0.86	0.89	0.91
28	0.88	0.91	0.91	0.90	0.86	0.91
29	0.86	0.90	0.91	0.91	0.91	0.91
30	0.91	0.91	0.91	0.91	0.91	0.91
31	0.88	0.90	0.91	0.91	0.91	0.91
32	0.91	0.91	0.91	0.91	0.91	0.90
33	0.91	0.91	0.91	0.91	0.91	0.91
34	0.90	0.90	0.91	0.91	0.91	0.91
35	0.86	0.86	0.85	0.86	0.88	0.89
36	0.90	0.88	0.85	0.83	0.79	0.79
37	0.91	0.87	0.90	0.90	0.91	0.91
38	0.83	0.84	0.83	0.80	0.78	0.74
39	0.91	0.91	0.91	0.91	0.91	0.91
40	0.86	0.87	0.86	0.85	0.84	0.84
41	0.91	0.91	0.91	0.91	0.91	0.91

Table A4.3. Capacity utilization in different sectors of the economy in Tuscany.

Sector	1975	1976	1977	1978
1	0.87	0.89	0.79	0.91
2	0.56	0.80	0.87	0.64
3	0.81	0.80	0.83	0.82
4	0.77	0.82	0.85	0.67
5	0.77	0.85	0.91	0.91
6	0.76	0.88	0.88	0.87
7	0.81	0.91	0.91	0.91
8	0.85	0.91	0.91	0.91
9	0.91	0.91	0.91	0.86
10	0.91	0.86	0.91	0.91
11	0.88	0.91	0.91	0.91
12	0.84	0.91	0.91	0.76
13	0.81	0.89	0.81	0.77
14	0.91	0.88	0.81	0.74
15	0.90	0.91	0.91	0.78
16	0.90	0.76	0.64	0.60
17	0.86	0.91	0.91	0.90
18	0.82	0.91	0.91	0.91
19	0.91	0.84	0.91	0.91
20	0.72	0.84	0.78	0.91
21	0.53	0.91	0.68	0.62
22	0.54	0.91	0.71	0.73
23	0.83	0.91	0.91	0.87
24	0.86	0.90	0.91	0.91
25	0.83	0.91	0.91	0.91
26	0.91	0.91	0.91	0.91
27	0.91	0.89	0.91	0.82
28	0.91	0.87	0.91	0.83
29	1.00	1.00	1.00	1.00
30	0.87	0.82	0.80	0.85
31	0.90	0.91	0.89	0.91
Total	0.84	0.88	0.87	0.86

APPENDIX 5: THE 41-SECTOR CAPITAL COEFFICIENT MATRIX OF ITALY

Table A5.1. Capital coefficient matrix, Italy (41 x 41 sectors) in percent.

From sector	To sector 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	14.7	0.4	0.3	0.3	0.3	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	7.3	4.2	3.6	3.7	3.7	0.0	9.0	7.1	10.7	4.0	3.7	4.7	4.4	8.3	8.3
10	43.2	24.6	21.5	21.6	21.6	0.0	53.4	41.6	63.2	23.5	21.6	27.8	26.2	48.9	48.8
11	216.8	123.4	107.8	108.3	108.3	0.0	267.7	208.8	316.9	117.8	108.3	139.5	131.2	245.3	244.9
12	40.5	23.0	20.1	20.2	20.2	0.0	50.0	39.0	59.1	22.0	20.2	26.0	24.5	45.8	45.7
13	92.4	52.6	45.9	46.2	46.2	0.0	114.1	89.0	135.1	50.2	46.2	59.5	55.9	104.6	104.4
14	16.0	15.6	13.6	13.6	13.6	0.0	15.7	37.6	7.2	10.8	11.2	6.5	6.7	15.7	15.7
15	24.1	14.9	13.0	13.1	13.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.5	0.1	0.1	0.1	0.1	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0
23	33.5	9.3	8.1	8.1	8.1	0.0	4.0	4.8	8.8	1.7	2.0	1.8	1.6	5.4	5.9
25	5.0	1.4	1.2	1.2	1.2	0.0	0.6	0.7	1.3	0.2	0.3	0.3	0.2	0.9	0.9
26	0.6	0.6	0.2	0.1	0.2	0.0	0.1	0.1	0.1	0.2	0.0	0.0	0.0	0.0	0.0
27	604.8	688.2	601.1	604.0	603.9	0.0	132.7	154.9	145.6	72.4	81.8	61.9	77.0	120.9	120.7
29	40.2	34.7	30.3	30.5	30.5	0.0	24.9	22.4	28.8	11.6	11.3	12.6	12.6	23.0	22.9
31	8.5	7.3	6.4	6.4	6.4	0.0	5.3	4.7	6.1	2.5	2.4	2.7	2.7	4.9	4.9
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A5.1 continued.

From sector	To sector 16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	1.5	1.5	1.5	1.5	1.5	2.4	2.4	1.2	3.7	5.1	1.2	1.4	1.2	3.1	1.3
10	9.0	9.0	9.0	8.9	9.1	14.4	14.4	7.0	21.6	30.0	7.1	8.5	7.0	18.4	7.4
11	45.1	45.0	45.1	44.7	45.4	72.1	72.0	35.2	108.4	150.5	35.8	42.6	35.3	92.2	37.3
12	8.4	8.4	8.4	8.3	8.5	13.5	13.4	6.6	20.2	28.1	6.7	8.0	6.6	17.2	7.0
13	19.2	19.2	19.2	19.1	19.4	30.7	30.7	15.0	46.2	64.2	15.2	18.2	15.0	39.3	15.9
14	7.7	7.7	7.7	7.6	7.8	6.4	6.4	6.0	7.0	11.5	6.1	29.7	6.0	133.7	58.2
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.3	0.3	0.3	0.3	0.3	0.1	0.1	0.0	0.1	0.1	0.0	0.1	0.0	1.7	1.9
23	19.9	19.8	19.8	19.7	20.0	6.3	6.3	0.5	3.9	3.9	0.5	7.7	0.5	116.0	94.1
25	3.0	3.0	3.0	2.9	3.0	0.9	0.9	0.1	0.6	0.6	0.1	1.2	0.0	17.3	14.1
26	0.4	0.4	0.4	0.4	0.4	0.1	0.1	0.0	0.1	0.1	0.0	0.1	0.0	2.1	1.7
27	46.0	45.9	46.0	45.6	46.3	45.2	45.2	26.3	54.1	61.3	26.9	13.9	26.6	116.7	152.3
29	6.4	6.4	6.4	6.3	6.4	7.4	7.4	3.8	10.3	13.7	3.8	5.5	3.8	23.4	16.1
31	1.3	1.3	1.3	1.3	1.4	1.6	1.6	0.8	2.2	2.9	0.8	1.2	0.8	4.9	3.4
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table A5.1 continued.

From	To sector										
sector	31	32	33	34	35	36	37	38	39	40	41
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.3	0.3	0.3	35.1	1.5	2.8	2.8	2.8	2.8	2.7	2.8
10	2.0	2.0	2.0	207.0	0.9	16.3	16.3	16.3	16.3	3.9	16.4
11	10.0	10.0	10.0	1038.4	45.3	81.9	81.6	81.8	81.9	19.8	82.5
12	1.9	1.9	1.9	193.8	8.5	15.3	15.2	15.3	15.3	3.7	15.4
13	4.3	4.3	4.3	442.7	19.3	34.9	34.8	34.9	34.9	8.4	35.2
14	406.2	405.9	406.3	16.8	4.8	76.8	76.6	76.8	76.9	10.3	77.4
15	531.3	538.9	539.6	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0
22	0.1	0.1	0.1	0.6	0.5	0.6	0.6	0.6	0.6	0.3	40.1
23	7.1	7.1	7.1	37.3	34.7	39.8	39.7	39.8	39.8	18.5	40.1
25	1.1	1.1	1.1	5.6	5.2	5.9	5.9	5.9	5.9	2.8	6.0
26	0.1	0.1	0.1	0.7	0.6	0.7	0.7	0.7	0.7	0.3	0.7
27	427.3	427.0	427.5	907.1	249.5	2770.2	2761.3	2768.5	2770.9	1268.6	2790.3
29	52.0	51.9	52.0	109.6	14.2	108.6	108.2	108.5	108.6	47.4	109.4
31	11.0	11.0	11.0	23.2	3.0	23.0	22.9	22.9	23.0	10.0	23.1
32	0.1	0.1	0.1	0.1	0.0	0.1	0.1	0.1	0.1	0.0	0.1