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A SYSTEMS APPROACH TO EUTROPHICATION MANAGEMENT WITH APPLICATION TO LAKE BALATON

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PREFACE

One of the main research efforts of the Resource and Environment Area of IIASA during the past few years was the Lake Balaton Eutrophication Study, undertaken in cooperation with the Hungarian Academy of Sciences and other Hungarian institutions. The study was initiated because of the recognition that our understanding on eutrophication was much less satisfactory for shallow lakes than for deep lakes. Lake Balaton was selected for several reasons, one of them being the pronounced economic interest in solving the real-life problems of the lake and the surrounding region. The study covered various scientific issues (coupled biochemical-hydrophysical modeling of a lake, sediment-water interaction, the derivation of nutrient loads, uncertainties and stochastic influences, etc.) on one side, and the management of the lake's water quality--a much more macroscopic issue--on the other.

This paper offers a summary, albeit brief, of the entire effort, reflecting a stage shortly before the close of the study. More detailed results on the application of the water quality management model will be set out in the forthcoming book on Eutrophication Management for Lake-region Systems, and the proceedings of the conference on the Eutrophication of Shallow Lakes: Modeling, Monitoring and Management (The Lake Balaton Case Study), 29 August to 3 September, 1982, Veszprem, Hungary, both publications to appear in fall 1983.

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A SYSTEMS APPROACH TO EUTROPHICATION MANAGEMENT

WITH APPLICATION TO LAKE BALATON[1,2]

László Somlyódy[3]

Abstract.--The problem of the lake-region system studied is characterized by complexity, uncertainty, and by the need to incorporate different levels of analysis such as scientific understanding and policy making. The approach adopted is based on decomposition and aggregation. Detailed studies were made on subprocesses (sediment-water interaction, biochemical and hydrophysical phenomena, nutrient loads, etc.). The essential features of these processes were preserved at the level of water quality management and incorporated in an optimization model accounting for uncertainties.

INTRODUCTION

The artificial eutrophication of lakes and reservoirs is recognized worldwide as a serious problem; it encompasses both the physico-chemical changes of water quality and the economic and social processes that underlie these changes. Excessive algal blooms, provoked by augmented nutrient loads of agricultural, municipal, and industrial origin, reduce the recreational potential of the water bodies and have serious adverse effects on their suitability as sources of drinking water. Furthermore, eutrophication entails undesired changes in the ecology of the system, making it generally less stable and more vulnerable.

The major feature of man-caused eutrophication is that, although the consequences appear within the lake, the cause--the gradual increase of nutrients (various phosphorous and nitrogen compounds) reaching the lake--lies in the region. Consequently, eutrophication *management* requires analysis of the complex interactions between *the water body* vulnerable to eutrophication and its *surrounding region*. In the lake, different biological, chemical and hydrophysical processes are important, while in the region there are human activities generating nutrient residuals that are ultimately washed to the lakes and reservoirs.

To control eutrophication is not an easy task. Although in most cases a solution would include reducing nutrient loads in one form or another--in itself a difficult task--various *uncertainties* inevitably exist regarding the extent of the reductions required to achieve desirable improvements in the lake's water quality. Thus, the effectiveness of alternative solutions measured by their responses in the lake (and consequently with respect to its desired functions) calls for the explicit treatment of the problem-related uncertainties (ranging from data uncertainty to uncertainty due to a limited understanding of the processes involved).

Thus, using a *systems approach* to analyze and evaluate *alternative management strategies* offers particular advantages. In fact, this is perhaps the only way to perform an integrated study encompassing the lake, the region, and the related physical, biological, chemical, economic, and social processes. This approach structures information in a format appropriate for both the *research phase* and the subsequent phase of *policy implementation*. At the core of the approach, *models* play an important role and are effective tools. Model results can be fed back to field workers to focus their attention on areas where our knowledge of the real world is still insufficient. Model results also enable managers to visualize the effects of possible management options.

The relation of cause and effect of eutrophication is similar both for deep and shallow lakes. Still the understanding of processes in shallow lakes is much less satisfactory than for deep lakes. In contrast to deep lakes, the relatively fast and irregular dynamics influenced by the nearly complete absence of temperature stratification, and climatic and hydrologic factors being strongly stochastic in nature should be mentioned. Because the lake is shallow, wind-induced sediment/water interaction and spatial mass exchange play important roles. The light conditions in the water fluctuate a great deal more, owing to changes in the concentration of suspended solids. As a special issue, the gap between hydrophysical and ecological modeling should also be noted, the reason being that biological, chemical, and physical processes of quite different time and spatial scales are involved.

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[1]Paper presented at the Third International Conference on State-of-the-Art in Ecological Modelling, Colorado State University, May 24-28, 1982.

[2]This study was initiated by IIASA's Resources and Environment Area and carried out in cooperation with the Hungarian Academy of Sciences and other Hungarian institutions.

[3]Dr. L. Somlyódy (on leave from the Research Center for Water Resources Development (VITUKI), Budapest, Hungary) is at present leader of the Balaton Case Study of the Resources and Environment Area at the International Institute for Applied Systems Analysis, Laxenburg, Austria.

To analyze all these issues, Lake Balaton, the largest lake in Central Europe, was selected as the subject of a case study. This lake is the most important recreational area in Hungary and has exhibited the unfavorable signs of artificial eutrophication. There is not only a scientific, but also a strong economic interest in the study (roughly 40% of the income from tourism in Hungary stems from the Balaton region). This paper gives a summary of the study, which is now nearing completion. For further details on the case study, the reader is referred to van Straten et al., 1979, van Straten and Somlyódy 1980, Somlyódy 1981a, and Somlyódy and van Straten, forthcoming.

MAJOR CHARACTERISTICS OF THE SYSTEM

The lake and its watershed are illustrated in figure 1. The length of the lake is 78 km, the average width around 8 km (surface area nearly 600 km^2) and the average depth 3.1 m. The major inflow of the lake is the River Zala at the southwestern end of the lake which drains half of the total catchment area (~ 5800 km^2). There is a single outflow at the other end of the lake, Siófok, through a control gate. The mean residence time of water is about 2 years.

The fluctuation in the water's temperature is high. There is a relatively long ice-covered period (around two months), while the temperature in summer may exceed 25° C. Concerning the chemical composition of the water, the high calcium carbonate content and pH value (8.3 to 8.7) should be mentioned. Wind action is important resulting in favorable oxygen conditions and a permanent back and forth motion (seiche) along the lake and a complicated circulation pattern. Wind strongly influences sedimentation and resuspension of the sediment (its organic material content is low) which is associated with an effective adsorption-desorption process. The yearly net desorption as internal load has the same order of magnitude as the external one (see Gelencsér et al., 1982).

In recent years, remarkable changes have been observed in the water quality due to the rapid increase in tourism, sewage discharges, fertilizer use, and other factors. The algal biomass (algae is the most important primary producer in this case) increased by a

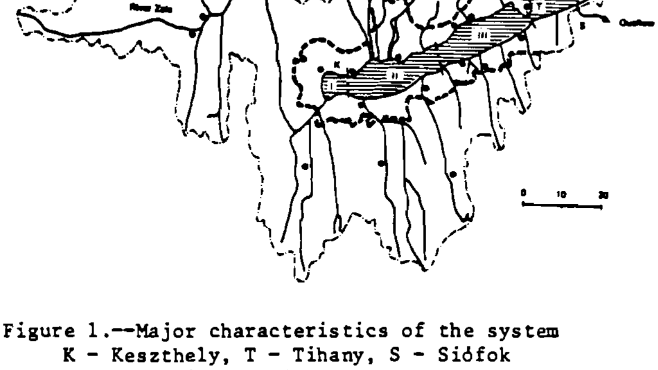


Figure 1.—Major characteristics of the system
K - Keszthely, T - Tihany, S - Siófok
I...IV typical basins of the lake
-.- boundary of the catchment
--- boundary of the recreational area
· sewage discharges in the region

factor of 10 when compared with the past 15-20 years. The trend in primary production is similar and at the most polluted western basin, peaks of up to 13.6 gC/m^2d were observed, a hypertrophic value (Herodek and Tamás, 1975). In short, the average lake conditions moved from mesotrophic to eutrophic, thus endangering the use of the lake for recreational purposes, the prime water use in this case.

Phosphorus plays a dominant role in the eutrophication of the lake. Thus, both from the point of view of understanding and managing the system, tracing the phosphorus compounds in the lake and on the watershed is of primary interest. The total phosphorus load of the lake is around 1000 kg/d (0.52 mg/m^2d in a lake-wide average) (Jolánkai and Somlyódy, 1981), half of which is estimated to be available for algal uptake. The load has many components: 33% is derived from sewage, 27% from diffuse sources, 22% is related to runoff processes in the direct vicinity of the lake, while the contribution of atmospheric pollution is 18%. The ratio of sewage discharges in the available load is higher; only the sewage released in the recreational area (figure 1) accounts for 36% of the available load. This direct load varies quite a lot in time, following the fluctuations in population due to tourism, and has a 2-4 times higher value in summer than during the off-season. The load distribution along the lake is approximately uniform (the tributary load is higher for the Western end of the lake, while the sewage load is quite the opposite); however, the volume related value is twelve times higher at the Keszthely Bay (fig. 1) than at the other end of the lake, due to differences in the volume of the four main basins. This fact is also reflected in the pronounced longitudinal gradient of various water quality parameters, e.g., for Chlorophyll-a the ratio of the maximum and minimum values ranges between 4 and 20 (van Straten et al., 1979). The gradient observed at the same time indicates that the strong wind action and the mixing associated with it are still not sufficient for leveling out the spatial nonuniformities.

From an analysis of the data it is clear that there is not only a critical state of the water quality at Keszthely Bay, but also a spreading deterioration process which extends towards other areas of the lake where the water quality is still good. Thus action is urgently required from the view of the entire lake.

Based on hydrologic and water quality considerations, the lake was divided into four basins, as indicated in figure 1. The application of the principle of segmentation proved to be a useful tool for modeling, data collection, and data handling.

Concerning data, extensive records are available on hydrology and meteorology. Regular water quality monitoring started ten years ago, in two network systems consisting of 9 and 16 spatial sampling points, respectively (10-20 measurements per year), but irregular data are also available dating back to the early sixties. Several other in situ and laboratory measurements were also taken (primary production, extinction, sediment-water interaction, velocity, etc.). A survey was done on the nutrient load between 1975-1979, which involved 20 tributaries and 27 sewage discharge points (Jolánkai and Somlyódy, 1981) (indicated in fig. 1). On the major tributary, daily observations were made during this period.

THE APPROACH

As mentioned in Section 1 the study should cover several interrelated but still quite diverse issues and processes such as scientific understanding and policy making, the lake and its region, watershed processes, biochemistry, hydrodynamics, etc. Thus the dilemma is which approach can be used for the research and within this, for modeling. Two extremes are as follows:

(i) to base it on intuition and to drastically simplify the problem a priori;

(ii) to establish one large, detailed model which accounts for all the subprocesses and the different levels (understanding and management).

As the first approach is not acceptable and the second is unrealistic, a different approach was worked out which is based on the idea of *decomposition and aggregation* (Somlyódy, 1981a).

The approach begins by decomposing the system into smaller, tractable units forming a hierarchy of issues subject to study. One can make detailed investigations of each of these issues, using in situ and laboratory observations, models and other available information. This step is followed by aggregation, the aim of which is to preserve and integrate only the issues that are essential for the higher strata of the hierarchy (at the uppermost level, ultimately for answering questions of management), ruling out the unnecessary details. From a structural viewpoint, this is an *off-line approach*, which allows application of different techniques and principles at various levels as desired, accounts for uncertainties, and leads to a realistic, yet simple, model at the highest level of the hierarchy, where evaluating management alternatives is the objective.

It is noted that in ecological modeling, there is a certain gap between "larger" models and "smaller" models (see Beck, 1982). In the first case there is nearly no hope for a precise calibration, but "smaller" models can also be just as unrealistic for complex problems because of their simplicity. The approach outlined offers a reasonable alternative for such cases.

The application of this approach for the Lake Balaton problem is explained with the help of figure 2. The first decomposition that directly comes to mind is the distinction between lake and watershed, since, as

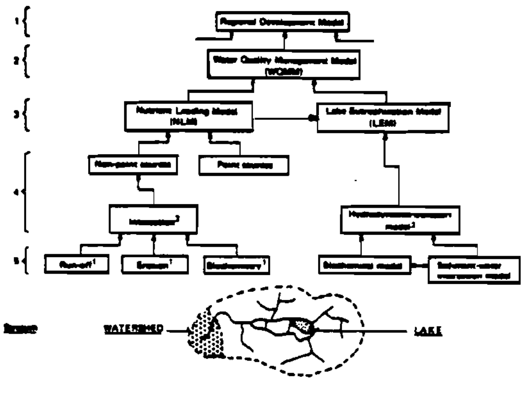


Figure 2.--The method of decomposition: hierarchy of models
(1) submodels for uniform segments (dotted areas)
(2) coupling of the submodels.

mentioned before, the water quality problem lies in the lake, but the causes, and practically all control possibilities, are to be found in the watershed.

The procedure, involving five strata, will be discussed in greater detail for the Lake Eutrophication Model (LEM), with reference to the models which have been elaborated. The parallels in the Nutrient Loading Model (NLM) can be seen in figure 2.

Stratum 5

First, those segments of the lake should be isolated which can be considered approximately uniform from the viewpoint of water quality (complete mixing inside each unit) and from the factors influencing them. The objective of the models on this stratum is to describe the algal dynamics and nutrient cycling for all the segments, involving both the water body and the sediment, since the latter is a sink and source of various materials and their interaction plays an important role in shallow lakes. These kinds of models based on the mass conservation principle and formulated through a set of nonlinear ordinary differential equations (ODEs) are well-known in the literature (Scavia and Robertson, 1979). In the frame of the present study, three submodels, BEM, BALSECT, and SIMBAL were developed (Herodek et al., 1980; Leonov, 1980; and van Straten, 1980), with respect to their comparison (van Straten and Somlyódy, 1980) which differs basically in the number of state variables (between 4 and 7) and essential parameters (10-17), as well as in the mathematical formulation of various processes and in the parameter estimation technique adopted. It is noted here that some of the parameters can be derived from further isolation up to a lower level with appropriately designed experiments. As examples, the estimation of algal growth parameters from vertical primary production measurements (van Straten and Herodek, 1981) and the study of wind induced sediment-water interaction (see the section on wind induced sediment water interaction) may be mentioned.

Stratum 4

On the next level the segment-oriented biochemical and sediment models are coupled by involving mass in- and outflows at the boundaries of the units. For this purpose, a hydrodynamic-transport model can be used. In light of the experiences gained from the study of the Great Lakes (Boyce et al., 1979), it was decided not to use a coupled multi-dimensional hydrodynamic-transport model incorporating the submodels of the lower stratum: the gain in information is not proportional to the increase in complexity. Here again, an off-line technique is applied. The basic assumption is that it is sufficient to subdivide the lake in a longitudinal direction only. This is supported by the riverine shape of the lake and the presumably extensive transversal mixing, since the prevailing wind direction is nearly perpendicular to the lake's axis (the description of the shoreline effects is not the objective here). Consequently, the parallel development of an unsteady three-, two-, and one-dimensional hydrodynamic model was decided on (Shanahan et al., 1981; Shanahan, 1981; Somlyódy, 1982; Somlyódy and Virtanen, 1982). The first two can be used to derive convection and the longitudinal dispersion coefficient (either directly or indirectly as an "empirical" function of the major wind parameters), while the 1-D model could describe convection only (its advantage lies in its simplicity and short execution time on the computer). Subsequently, the submodels of stratum 5 will be incorporated in a straightforward way into a set of longitudinal dispersion equations on stratum 4.

At this level, the 1-D model was aggregated from the 3-D version. A further aggregation can be arrived at through the use of the coupled dispersion-biochemical model (see the section on application of hydrodynamic models). Originally, in all three biochemical models (see "Major Characteristics of the System") four segments (or mixed reactors, see fig. 1) are assumed; their coupling is based on hydrologic throughflow and a wind influenced mass exchange process described globally. Since the model structure based on ODEs has many advantages, one of the objectives of the study on the 1-D coupled model is whether the four basins concept can be maintained or not.

Stratum 3

The involvement of mass exchange among segments as described before will result in the Lake Eutrophication Model (LEM) (fig. 2) which has several forcing functions, such as solar radiation, water temperature, wind, etc., (natural or uncontrollable factors) and the nutrient load. Since the latter is the only factor which can be controlled, it plays a distinguished role. A thorough data collection and the derivation of a nutrient balance for the whole lake gave a solid background (for details see Jolánkai and Somlyódy, 1981). The main conclusions have been summarized in "Major Characteristics of the System". Because of the high contribution of the sewage load and the insufficient amount of watershed data only limited effort was expended on non-point source modeling (Bogárdi and Duckstein, 1979). Rather, the analysis of historical river load data was preferred, which then satisfactorily resulted in uncertainties in the load (see the section on nutrient load under uncertainty and stochasticity) due to the stochastic character of the hydrologic regime and data scarcity. The research also allowed the derivation of the temporal and spatial pattern of the load components, both for LEM and the Water Quality Management Model (WQMM) on Stratum 2.

Stratum 2

The objective of WQMM is to generate alternative management options and strategies (the effect of these being expressed through NLM which should be used here in a planning mode) and to select from among these alternatives, on the basis of one or more objectives. Both water quality and costs can be used as objective functions or constraints, and quite often their weighting is required. Frequently the load can replace the lake's water quality in the optimization, in which case LEM is used merely to check the reaction of the lake and WQMM may have a simpler structure. Admittedly however, the inclusion of water quality is more obvious because of the nature of the problem. This formulation however leads to the dilemma: how should a complex model be incorporated into the optimization framework?

At this step aggregation is also needed. This starts with the selection of certain water quality indicators characterizing the large scale and long-term behavior of the system serving as a basis for decision making. Different parameters (yearly peak, different averages, duration of critical concentrations, frequency distributions, etc.) of typical water quality components (primary production, algal biomass, Chlorophyll-a, etc.) can be employed as indicators. Subsequently the dynamic model LEM can be used in terms of indicators established, I, under reduced loading conditions or in another way under several loading scenarios, L. Since the definition of indicators introduces temporal averaging, it is expected that the lake's response will be less complex compared to the dynamic simulation and a simple, direct I(L) type relationship can be found for the new equilibrium. If such a solution has already been attained, LEM could be replaced by I(L) in WQMM; an essential aggregation (see the section on the water quality management model).

Among the management alternatives, only the two most important options are mentioned here: (i) tertiary treatment (point source load reduction), (ii) establishing reservoirs (consisting of two segments serving for the removal of both particulate and dissolved nutrient forms, respectively (van Straten et al., 1979) at the mouth of rivers which are the recipients of point and non-point source pollutants. The optimization should then be based on the trade-off between the two basic alternatives, with respect to their locations and the spatial variation of the lake's water quality.

Stratum 1

For the sake of completeness it has to be mentioned that WQMM could be thought of as being a part of a regional development policy model forming the top of the pyramid, a field which is beyond the scope of this study.

ILLUSTRATION OF THE DIFFERENT STEPS OF THE APPROACH

Wind Induced Sediment Water Interaction (Stratum 5)

For studying the sediment-water interaction in lakes, several approaches are possible (Sheng and Lick, 1979). In this study, yet another method was chosen (Somlyódy, 1980), in recognition that when eutrophication is considered, more than just the physical processes should be examined. Daily measurements were taken for 6 months, at the mid-point of the Szemes basin (Basin 2, fig. 1). The measurements included Secchi depth, temperature, suspended solids (SS), Chlorophyll-a, and phosphorus fractions at different vertical locations. Wind velocity and direction were recorded continuously, from which hourly averages were calculated. The objective of the first part of the analysis was to describe the dynamics of the suspended solids as a function of wind. This then allowed for a characterization of the temporal changes in the light conditions, the deposition, and resuspension of particulate material and the associated sorption phenomenon. Here the behavior of SS will be discussed.

The analysis started from a simplified transport equation for describing the temporal and vertical changes of the average SS concentration for the basin, neglecting inflow and outflow (Somlyódy, 1980).

Afterward the equation was integrated along the depth leading to an ordinary differential equation incorporating the unknown *fluxes* of *deposition*, ϕ_d, and *resuspension*, ϕ_e, on the right hand side. The objective is to estimate these fluxes from the observations. In order to do this, hypotheses were made based on the literature: ϕ_d is proportional to the depth integrated SS concentration $\tilde{c}$, while ϕ_e to some power of the wind speed, W^n (for details, see Somlyódy, 1980 and 1981b). This procedure led to the equation for SS concentration

$$\frac{d\tilde{c}}{dt} = -K_1 \tilde{c} + K_2 W^n \qquad (1)$$

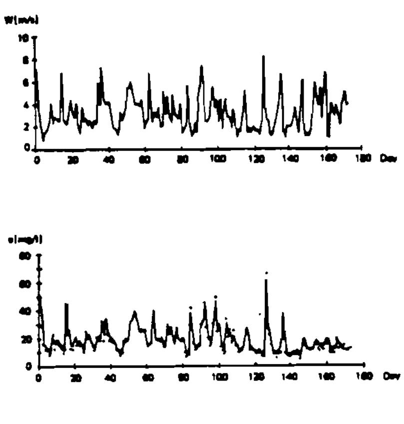
where K_1 and K_2 comprise the unknown coefficients, derived from the hypotheses. Consequently, the structure of the model should be identified and the parameter values K_1, K_2, and n, estimated from measurements. The feasibility of Equation (1) can be appreciated from figure 3a, which clearly shows the influence of wind velocity on the concentration.

First a non-recursive deterministic estimation technique was adopted to derive the unknown coefficients which resulted in realistic values, but without proving the correctness of the hypotheses (a posteriori model structure identification, see Beck, 1982).

For this purpose, as a second step, the Extended Kalman Filter (EKF) method was applied (Beck and Somlyódy, 1982). For the power n a value near to 1 was derived which corresponded to the small Richardson number (Somlyódy, 1981b). Subsequently n was fixed to 1 since in this case the physical interpretation of the results is more obvious. The recursive estimation started from the estimates of the deterministic technique. The results are illustrated in figure 3a. As is apparent, the agreement between observations and model calculation is reasonably good, and the parameters become approximately constant after the first 40-50 days (fig. 3b), proving that the model structure is adequate and the data do not contain more information than described by the model. Some slight parameter changes can be observed at the end of the period; this may be caused, e.g., by the exclusion of inflow-outflow processes (or by other phenomena such as algal blooms). This suggests that the isolation of subprocesses is generally not complete. From the analysis, a realistic order of magnitudes follows for all the essential physical quantities; in this connection see Somlyódy, 1981b.

As can be observed in figure 3, for one month in the middle of the total period, no measurements were available, so the model was used for prediction. The appropriateness of the model is also illustrated by the fact that after getting new data, the parameter values did not change. This second period served for validation, following the identification and calibration procedure.

The advantage of the simple interaction model arrived at is that it can be easily incorporated into the modeling approach (fig. 2), for (i) characterizing transparency conditions in the water and (ii) for describing the release of the sediment layer as the internal P source. As was shown in the report by Gelencsér et al. (1982) the desorption of resuspended particles is the primary cause of the sediment phosphorus



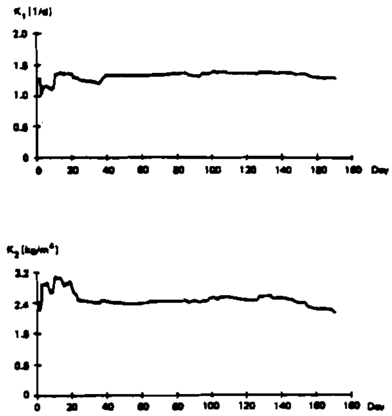


Figure 3a.—Identification and parameter estimation of a model for wind induced sediment-water interaction for Lake Balaton: recursive estimate of the suspended solids concentration; W - daily average wind speed, c - suspended solids concentration, * - observations.

Figure 3b.—Recursive parameter estimates for the sediment-water interaction model.

release (diffusion and convection of pore water contribute to a lesser extent); thus, wind-induced interaction is really of importance.

Application of Hydrodynamic Models (Stratum 4)

The results gained from the transient 3-D, 2-D (horizontal), and 1-D models (Shanahan et al., 1981; Somlyódy and Virtanen, 1982; and Somlyódy, 1982) showed that the models could be equally calibrated against the dynamic water level data. For an example of the application of the 1-D model, see figure 4. The model was already used in the validation phase. The storm was characterized by a long-lasting longitudinal wind followed by smaller shocks from different directions (fig. 4a). The agreement between measured and simulated water levels at the two ends of the lake is excellent (fig. 4b). The discharge at the Tihany peninsula shows a striking oscillation between -2000 and 3000 m^3/s (fig. 4c) associated with the seiche phenomenon. This back and forth motion is higher by two orders of magnitude than the hydrologic throughflow.

As mentioned previously, the 1-D model alone does not give satisfactory information for a water quality study as it serves the longitudinal convection term only, but not dispersion. Still this model version, the simplest, is extremely useful. Two examples discussed subsequently illustrate this statement:

(i) To find a more satisfactory agreement between model simulation and observation than that given in figure 4 is often impossible. The reason is quite simple: a small error in the wind direction may cause a drastic change in the wind force, if the direction is far from the longitudinal one. In fact, there are many kinds of uncertainties in the wind direction, such as random fluctuation (turbulence), the influence of hills on the northern side of the lake, which cause nonuniformities in the wind field, measurement errors, etc. Figure 5 illustrates the case (transversal wind conditions). A deterministic simulation did not prove acceptable. Bearing in mind the possible role of uncertainties, a random component was subsequently added to the wind direction (Gaussian distribution, zero mean,

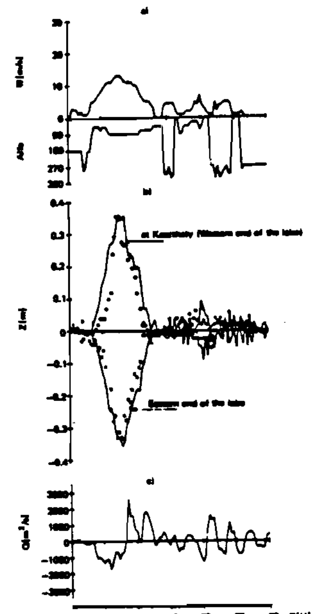


Figure 4.—Simulation of a historical event: longitudinal wind conditions. (a) wind record (Muszkalay, 1979), W speed, ALFA angle (North = 0°); (b) comparison of simulated and observed water levels (T = 0 corresponds to 16/11/1966, 8 a.m.). Dots indicate measurements (Muszkalay, 1979); (c) computed streamflow rate at Tihany.

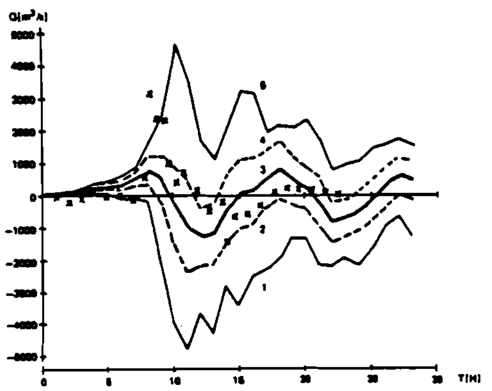


Figure 5.—The influence of wind data uncertainty on the discharge at the Tihany peninsula (T = 0 corresponds to 8/7/1963, 8 a.m.);
· discharge derived from measurements (Muszkalay, 1979).

17° standard deviation: a modest value) and a *Monte Carlo simulation* was performed (for details, see Somlyódy, 1982). Figure 5, which summarizes the results of 100 runs, does not require detailed discussion: it stresses the *extreme senstivity to input data uncertainty* (compared to this, the parameter sensitivity is negligible) and illustrates how difficult it is to validate a deterministic model (the stiuation is similar for a two-dimensional model, Somlyódy 1982).

(ii) At a given location in the lake the intensive back and forth motion causes an oscillation of various constituents within a day, which also strongly depends on the longitudinal gradient. This may result in quite a large error in the concentration determined through instantaneous sampling. For such a short time scale as a day, biological reactions can be neglected and the concentration fluctuation can be analysed through a coupled 1-D hydrodynamic-transport model assuming conservative material. Through this model an uncertainty range of historical observations can be specified. Simulations showed that for this particular lake the sampling strategy for Basin II (fig. 1) is of major importance; the error range of a single sample at a fixed location can reach ±30%, depending on the actual gradient and streamflow pattern.

The calibration of the two-dimensional hydrodynamic model (Shanahan, 1981) resulted in the same wind drag coefficient and bottom friction parameter as the 1-D model version. The simulations showed a pronounced circulation pattern observed also on some satellite photographs and a physical model (Györke, 1975). It is noted that the transient 3-D model (Shanahan et al., 1981) and also other steady state models tested, (van Straten and Somlyódy, 1980) reflected much less circulation in the lake, clearly showing that our *understanding* of the *three-dimensional water motion in shallow lakes* (and within this, of the vertical eddy viscosity) is *far from complete*.

In order to couple the horizontally 2-D hydrodynamic model to a phosphorus cycle model through a set of longitudinal dispersion equations (see "The Approach" and "The Lake Eutrophication Model (Stratum 3)"), Shanahan (1981) computed the dispersion coefficient (as a function of time and space) from the velocity field through extending the method of Fischer (1979). Dispersion is the highest near the two ends of the lake and at the vicinity of the peninsula, due to strong changes in geometry and the associated secondary currents and gyres. The large dispersion coefficient (up to 40 m^2/s) is due to strong winds. As a temporal and spatial average, 1 m^2/s was found by Shanahan to be appropriate enough for water quality simulations.

The cross sectionally averaged streamflow (or velocity) for the 1-D dispersion model was derived from the 2-D hydrodynamic model after integration. The same pattern and magnitudes were arrived at as suggested in figures 4 and 5. The fast dynamics lead to a small seiche excursion (less than 1-2 km). This means that there is an oscillatory translation of water particles of a short time scale, within which biochemical reactions can be practically neglected and thus the concentration averaged over a seiche type event is unchanged (see item (ii)). From this feature it follows (Shanahan, 1981) that convection with its uncertainties (see item (i)) can be neglected and only dispersion and hydrologic throughflow should be accounted for in the coupled lake eutrophication model (Section 4.4).

The Nutrient Load under Uncertainty and Stochasticity (Stratum 3)

The deterministic load estimate and the spatial distribution for a specific historical year (1975-79) is derived on the basis of the daily observations on the Zala River's draining 50% of the total watershed, the survey on data for other rivers and sewage treatment plants, on pilot zone studies, watershed characteristics, etc. (Jolánkai and Somlyódy, 1981). The temporal pattern is derived from the dynamics of the Zala River load, observations made in treatment plants during the off-season and summer period, respectively, and population fluctuation related to tourism. Such a load estimate is acceptable for the descriptive use of the lake model, LEM, but certainly not for planning purposes.

For *management* of the system, the *stochastic character* of the load and other existing *uncertainties* should be accounted for. In order to develop a *load scenario generator*, first the allowable integration period of the load input was tested through the dynamic lake model. The analysis showed that *monthly averages* for all the *forcing functions* can be *satisfactorily used*; an important finding, as it allows generation of the load on a monthly basis. This can be reasonably derived from the data available, while the procedure for a shorter time scale would be unrealistic.

With this conclusion, the Zala River data were a priori aggregated to monthly averages and a simple regression analysis was done between phosphorus loads (TP and PO4-P) and streamflow rate. Acceptable expressions were arrived at (of course, with error terms). Deriving the statistics of the monthly average streamflow from long-term observations (Baranyi, 1979), the load can be calculated in a stochastic fashion. Figure 6 shows the characteristics of the load pattern for 1976-79 (from observations) and the 90% confidence levels derived for the long-term load. For illustrating the influence of the hydrologic regime an event of low probability in July, 1975, is likewise indicated. The stochastic influence of the hydrologic regime for other subwatersheds was derived from the analysis outlined and available data for these catchments. The daily observations at the mouth section of the Zala River were also used to study the *implication of infrequent observations* typical for most of the tributaries (one or two samples per month). Due to scarcity of data, the contribution of floods to the load are partially unobserved. As the "accurate load" for a certain period (e.g., long-term monthly or yearly averages) for the Zala River can be gained from the original data, it allows one to study the error caused by scanty observations. The procedure is a straightforward Monte Carlo type technique which starts with a random selection on the detailed data set following the sampling strategy of the other tributaries and calculates the load of the period in question. After making a sufficient number of random selections the statistical parameters of the load can be determined. The results for the long-term monthly average load (on the basis of a four year long observation period) are illustrated in figure 7. As can be seen from figure 7, which shows the mean and extreme values, as well as the domain of ± standard deviation, the error is quite high and its fluctuation follows the change in the mean value. On the basis of this study, a random component was added to the monthly average load component (Somlyódy and Eloranta, 1982).

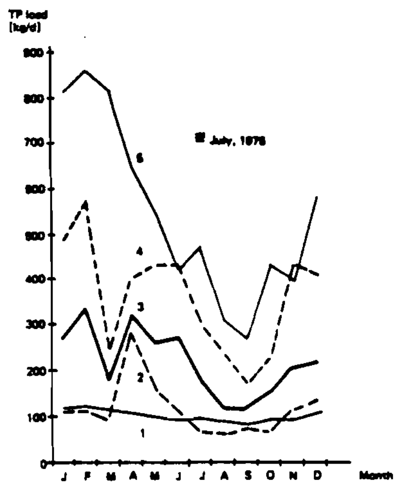


Figure 6.--Influence of the hydrologic regime on the monthly average load, Zala River: 3 - average load (1976-79), 4 and 2 - minimum and maximum values (observed), 5 and 1 - 90% confidence levels.

From figure 7, the question automatically arises: how can the uncertainty domain be reduced? In a different way, *what sampling strategy should be followed?* This issue was also studied. Besides, first order analysis (Cochran, 1963), different sampling strategies (regular, random, stratified, etc.), were realized in a Monte Carlo type fashion. Also, various kinds of estimates (simple, ratio, etc., see e.g., Dolan et al., 1981) were tested in order to reduce bias and variance of the load estimate. Without going into detail, (the reader is referred to Somlyódy and van Straten, forthcoming), it should be noted that, with proper stratified sampling (few samples when the variance is small--low-flow conditions--and frequent sampling for floods characterized by large variance (Cochran, 1965)), the total amount of samples can be reduced to one fourth or one fifth. An important conclusion of the study is that the variance of the load can be replaced by that of the discharge, Q. As Q is an easily measureable quantity, a realistic stratified sampling strategy can be worked out in practice, on this basis.

Returning to the development of the load generator, for sewage load, the same pattern is used as in the descriptive fashion, but in addition, an uncertainty component is introduced, which expresses the overload in the treatment plants due to the population increase in the main tourist season.

As a final output of the research outlined in this section, a load scenario generator was developed for the whole lake, which accounted for both uncertainty and stochasticity, discussed above. For further details see Somlyódy and Eloranta (1982).

It is noted here that using historical data, a similar analysis was made on climatic (uncontrollable) factors, which allowed the water *temperature* and *solar radiation* to be generated in harmony with each other, in a *random fashion*. Thus, future scenarios can be generated for all the essential forcing functions of the lake model--an essential tool for planning purposes (see "The Lake Eutrophication Model (Stratum 3)" and "Water Quality Management Model (Stratum 2)").

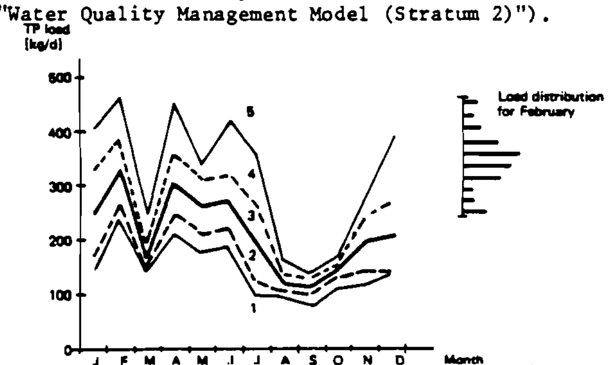


Figure 7.--Monthly average TP load: uncertainty caused by infrequent observations (Zala River, 1976-79): 3 - mean value, 4 and 2 - ± standard deviation, 5 and 1 - extreme values.

The Lake Eutrophication Model (Stratum 3)

Results gained with the simplest model, SIMBAL (van Straten, 1980), developed for Lake Balaton, are given below. The model is a phosphorus cycle model, that is, all the state variables (the essentials are two algal groups, detritus, and dissolved inorganic phosphorus) are expressed in terms of phosphorus, for the four basins indicated in Figure 1. A Monte Carlo simulation is incorporated into the model to find areas in parameter space where the model produces results fully within specified boundaries drawn around the data to account for data uncertainty and thus, is easily applicable for testing various hypotheses (van Straten, 1980; Fedra et al., 1981; Hornberger and Spear, 1980).

Among the calibration runs, results for the phytoplankton phosphorus, PPP, for the four basins, are given in figure 8 (as 1977 forcings data was used) together with the corresponding observation variable, Chlorophyll-a (basin average values). It is pointed out that Chlorophyll-a and PPP cannot be directly compared to each other; however, since a more or less linear measurement equation is expected among them, PPP should follow the pattern of Chlorophyll-a: a trend which can be generally observed. For illustration, the standard deviation around the trajectory for Basin 2 estimated through the Monte Carlo simulation is also indicated (parameter uncertainty). Further discussion on the calibration and model improvement required canbe found in van Straten (1980).

The results presented here were from the four box model (fig. 1 and "The Approach"). Whether the concept of the four box model can be preserved or not, was tested through the *coupled hydrodynamic-dispersion-P cycle model* (see "Application of Hydrodynamic Models (Stratum 4)"). In the linked model, the parameter values of the original model were maintained. From the comparison of the simulation results of the "continuous" and four box model (see Shanahan, 1981 and Shanahan and Harleman in these proceedings), we may conclude as follows:

(i) the coupled transport-water quality model obviously better reflects the spatial details and local influences;

(ii) the four box model underestimates the various phosphorus concentrations for one of the basins, while the basin wide averages are satisfactory for the rest of the lake;

(iii) the return flow velocity cannot be used since the four box formulation introduces a priori artificial dispersion, which is higher than the wind induced dispersion.

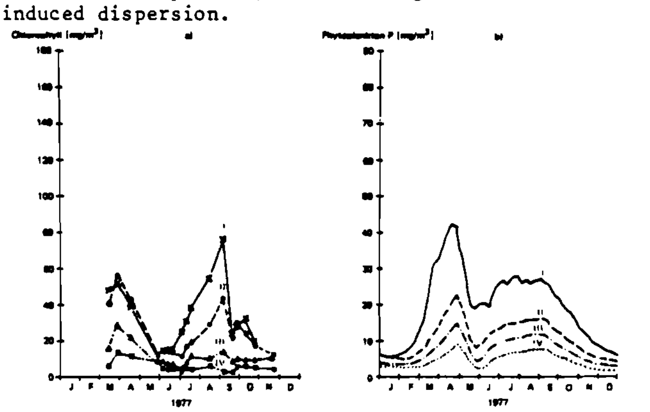


Figure 8.--Results from SIMBAL. Comparison of field data for four basins (left) and average model for runs satisfying the behavior definition (right), (1)...(4). Basins 1...4. Adopted from van Straten (1980).

(iv) *the four box model* with its ODE structure and all the advantages associated with this *can be reasonably maintained for practical purposes* and subsequent analysis.

For management purposes the simulation of historical events cannot be used. Either some critical, unfavorable environmental conditions should be introduced or the model should be considered stochastic through input data. Here the latter approach was adopted and the generators outlined in the previous section coupled to the lake model. Two essential results for Basin I are presented in figures 9 and 10.

In the first case, *uncertainties* caused by *natural factors* were considered and the 1977 load was maintained. The summary of 100 Monte Carlo runs (mean, ± standard deviation, and the extremes of PPP) suggests the relatively large sensitivity of the lake's water quality to meteorological factors and explains the essential year to year changes observed in the behavior of the lake even when the load remained unaffected. The second case (fig. 10) involved the *random generation of both natural and controllable factors*. While for the previous example the specific 1977 load was adopted, here the mean load of the input generator was derived from data for the period 1975-1979 ("The Nutrient Load under

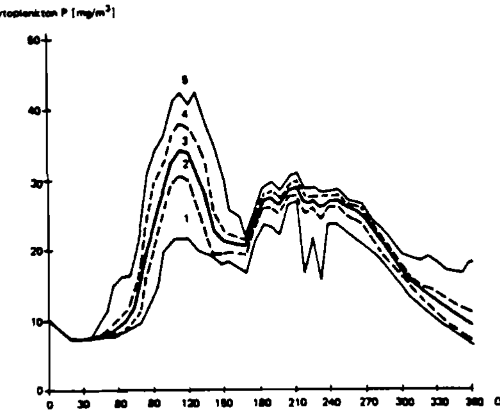


Figure 9.--The influence of meteorologic factors on the water quality. Basin 1: 3 - mean value, 4 and 2 - ± standard deviations, 5 and 1 - extreme values.

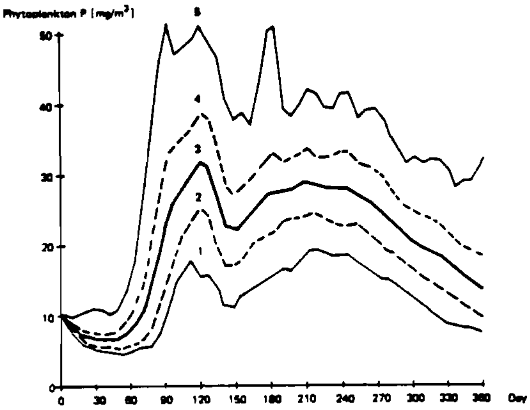


Figure 10.--The combined influence of uncertainty and stochasticity in the meteorology and loading, respectively, on the water quality. Basin 1: 3 - mean value, 4 and 2 - ± standard deviations, 5 and 1 - extreme values.

Uncertainty and Stochasticity (Stratum 3)"). This is the reason why the average trajectory shown in figure 10 differs from that shown in figure 9. The inclusion of load randomness had an obvious influence: the range of uncertainty of the water quality simulation results became much wider.

Compared to the role of parameter uncertainty (fig. 8), the meteorological factors represent the same order of magnitude, while the contribution of load to the model uncertainty is twice as high. It is stressed that in the frame of the present example, no load reduction was employed. Thus figure 10 shows in which domain the water quality may range under the present conditions, since the changes in trend are already relatively small from year to year. Control alternatives - depending on their character - can modify not only the mean load, but also the related uncertainties (see "Water Quality Management Model (Stratum 2)".

At the end of this section it is stressed that the present version of SIMBAL has no sediment compartment and thus no memory (the two other models are more complex in this respect). The internal load is represented by an equilibrium dissolved inorganic P concentration in the lake water (calibrated in the course of the Monte Carlo procedure) which controls sorption. This load is approximately half of the external one - a realistic value - but since no feedback exists it does not change from year to year. Accordingly, the model, when used under reduced load conditions gives the "immediate" response of the lake but no conclusions can be drawn for the final equilibrium and the renewal time.

In order to improve the model in both short- and long-term reactions the number of state variables was essentially increased from four to ten for each basin (Somlyódy and van Straten, forthcoming). Among the six additional variables, three are associated with the sediment (particulate inorganic-, dissolved inorganic- and detritus P), one is suspended solids (see Eq. (1)), and the rest are the particulate inorganic and dissolved organic P concentrations, respectively, in water. The establishment of such a complex model also describing resuspension, sorption, diffusion, and the changes in the active sediment layer was felt to be realistic, as recently our a priori knowledge of the behavior of the sediment has greatly improved (Gelencsér et al., 1982). Our preliminary results allow two important conclusions to be drawn:

(i) through the inclusion of *resuspension* and *sorption*, the short-term behavior became more realistic. The influence of the wind on *algal dynamics* is *apparent* and it is in harmony with observations. This phenomenon is expected to be *generally important for shallow lakes*;

(ii) the magnitude of simulation results is *quite sensitive* on the *parameters of sorption isotherms* (within the range suggested by the observations). In addition, it became clear that the long-term behavior of the lake depends on some crucial sediment parameters (initial conditions, loss from the active layer to deeper zones, etc.) for which only rough guesses are available. All these suggest that (i) the inclusion of more state variables (and parameters) still requires more quantitative knowledge in order to improve the practical use of the model, in contrast to the simpler version; and (ii) for problems where *sediment plays a role,* at present, we *can hardly say anything* about the *renewal process* of the lake.

Water Quality Management Model (Stratum 2)

First we are dealing with the issue of how to incorporate the lake model (LEM) into the management framework, a question raised in "The Approach". In the course of the analysis outlined subsequently, the different parameters of the PPP(t) distribution were selected as water quality indicators characterizing the algal behavior, and deterministic simulations were performed with the dynamic lake model, SIMBAL, under reduced loading conditions. It turned out from the study that the lake's response expressed in terms of the yearly average or peak of PPP is quite linear on the load in a wide range (Somlyódy and Eloranta, 1982)--there was the same experience with the other two models. It is noted that a similar linearity is expressed for total phosphorus, TP, by several empirical models. However, for shallow lakes, TP may not properly characterize the process of eutrophication since this component is strongly influenced by wind induced interaction at the bottom (see "Wind Induced Sediment Water Interaction (Stratum 5)").

The recognition of the linearity leads to an important aggregation: the *dynamic lake model* can be replaced at the *level of* WQMM (see fig. 2) by a simple *linear equation* (fig. 11):

$$\underline{c} = \underline{c}_0 + \underline{\underline{A}}\,(\underline{L} - \underline{L}_0) \qquad (2)$$

or

$$\Delta\underline{c} = \underline{\underline{A}}\;\Delta\underline{L} \qquad (3)$$

Here $\underline{L}_0$ and $\underline{c}_0$ are the initial volume related load (yearly mean) and concentration "vectors" respectively, defined by the number of uniform segments assumed in LEM (at present N = 4, fig. 1); $\Delta\underline{L}$ represents the reduction of volume related load achieved by various control alternatives, while $\Delta\underline{c}$ is the corresponding response of the lake.

The elements of the $\underline{\underline{A}}$ matrix maintain the essence of LEM: a transition from a "large" model to a "smaller" on a higher level of the hierarchy of figure 2. The first remarkable feature of equations (2) and (3) is that they clearly preserve the influence of subprocesses on the lower strata and show the subsequent steps of aggregation. The elements of the main diagonal are determined primarily by biochemical processes and sediment-water interaction (stratum 5), while the other elements mainly express the influence of hydrodynamics and associated mass exchange (stratum 4), showing that a management action at the region of the i^th segment will

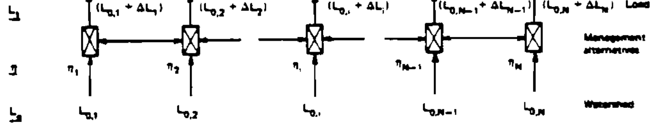


Figure 11.--Incorporation of the lake model in WQMM:
- i - uniform segments in the lake
- ⇆ - interaction among basins through mass exchange and hydrologic throughflow
- ↔ - interaction among load components through control activities.

affect the water quality of other segments as well (fig. 11). It is noted that through Monte Carlo simulations with LEM (figs. 9 and 10), it was possible *to include the uncertainties* in Equation (2). Three terms appear: two correspond to uncertainties in load and meteorology, while the third is due to their combined effect.

The second remarkable feature of Equation (2), and this is of primary importance in stratum 2, is that its *linear structure* allows its direct involvement in an *optimization framework*: a solution that we were looking for.

Optimization of this kind is not an easy task as each $L_{0,i}$ load element (fig. 11) is itself a vector, which can be controlled in many different ways. In addition, the various kinds of uncertainties discussed before should be accounted for, an issue which has not yet been explored satisfactorily for water quality problems.

Two approaches are under elaboration in this respect. Both use the same load equations with control variables for the four basins and the stochastic version of Equation (3) established for the yearly peak concentration. Control variables are the removal efficiency of treatment plants (fig. 1, $0 \leq x_k \leq 1$) and that of the reservoir projects ("The Approach", Stratum 2) assumed for the two largest rivers at the Western end of the lake ($0 \leq y_j \leq 1$). Cost functions (total annual cost) are highly nonlinear for tertiary treatment. It is stressed that x_i influences the expected value of load only, while y_i influences both expectation and variance. For those rivers which are recipients of sewage discharges and for which reservoirs can be constructed at the mouths, the product of the corresponding x_k and y_j variables will appear, leading to further nonlinearities and clearly showing the tradeoff between the two alternatives. The two approaches differ slightly in formulating the objective function and in the optimization technique adopted. For the first version, the weighted sum of the expectations of the square of the actual concentration minus the goal, for all the four basins, are minimized, subject to budget and other constraints. The technique is a stochastic quasigradient method (see e.g., Ermoliev, 1981). For the second version, the load is further aggregated from monthly to yearly average and the expectation and standard deviation of Equation (3), $E(\Delta C_i)$ and $\sigma(\Delta C_i)$ respectively, are derived analytically. Subsequently

$$\sum_{i=1}^{4} w_i \left[E\,(\Delta c_i) + t\sigma(\Delta c_i)\right] \qquad (4)$$

is maximized, subject to the same constraints as before (w_i and t serve various kinds of weightings). In other words, the stochastic problem is replaced by a deterministic one, which captures the major features of the original problem formulation. The cost functions and x_k, y_i terms are linearized (see Loucks et al., 1981), thus allowing the application of linear programming for the optimization. Important parameters of the model not mentioned until now are effluent standards and river detention coefficients.

It has to be noted, that the final solution of the sewage P removal will be to divert it from the watershed. Thus the present model serves as a decision making tool to find a provisional, feasible solution for the next 15-20 years. It was, however, recognized (Benedek and Szabó, 1981) that biological treatment has to be intensified in many plants before beginning on phosphorus removal (that is, a fixed cost exists, not related directly to the eutrophication problem). This feature was accounted for by (0,1) decision variables. Among the conclusions gained from the second model, three are listed below (Somlyódy and van Straten, forthcoming):

(i) in harmony with the findings of other approaches (Hughes, 1982; Dávid and Telegdi, 1982), as much P should be removed at the Western end of the lake as possible, under the given constraints;

(ii) if uncertainties are neglected tertiary treatment is chosen, but not reservoir projects (see Hughes, 1982);

(iii) however, if *uncertainties* are accounted for as given in Equation (4), ($w_i = t = 1$) results in a certain *combination of tertiary treatement plants* and *reservoir projects* for the Zala River catchment, clearly indicating that sewage treatment is a more effective technique for reducing the expected value of the load, but does not influence the variance in the tributary load and that of the in-lake concentrations. This shows that it is extremely *important* to include *stochastic features* at the level of *water quality management*.

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