AN APPROACH TO THE SIMULATION OF INTERNATIONAL OIL TRADE

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PREFACE

Modeling of energy systems has been an important part of IIASA's research program since the formation of the Energy Systems Program in 1973. Many important results have been obtained during this time, most of which have been summarized in the report "Energy in a Finite World" published by Ballinger in 1981. However, the energy modeling activities are continuing and this paper reports previously unpublished work on a model (SMIOT) that simulates international oil trade. This model was developed in collaboration with the System and Decision Sciences (SDS) Area at IIASA: in this paper, Yuri Ermoliev of SDS describes the mathematical basis of the gaming algorithm used to simulate the process of trade, while Alexandre Papin of the Energy Systems Program discusses the structure of the model and the philosophical background to the general approach.

ABSTRACT

This paper describes the Simulation Model for International Oil Trade (SMIOT) developed at IIASA in 1979. The model is designed to calculate balanced states for the oil market taking into account the conflicts of interests between exporters and between importers. One of the main objectives of this report is to discuss the philosophy behind this approach; particular attention is also paid to the gaming algorithm used to simulate the process of trade.

INTRODUCTION

One of the most important features of the energy analysis completed recently by the IIASA Energy Group is that meeting the demand for liquid fuels will continue to be a major world problem for at least the next 50 years. This problem has many aspects, one of the most important of which is the evolution of the international oil trade. The future of international oil trading is hedged about with uncertainties: what will be the long-term policies of oil exporters, given their desire to stretch out the life-time of their resources while maintaining the level of oil revenues? How will the increasing use of oil in the developing countries of Asia and Africa influence the availability of oil for the developed economies? Should industrialized countries reconsider their import policies in favor of the developing countries, and if so, how?

The results of the IIASA energy study throw some light on these uncertainties. However, the resources of the fairly small research group at IIASA were insufficient to pay great attention to this particular problem, not least because of the lack of an appropriate model to assist in such an analysis.

This situation has now been rectified: the so-called Simulation Model for International Oil Trade (SMIOT) has been developed at IIASA, and is the subject of this paper. The model is designed to provide an aggregate assessment of long-term trends in international oil prices and flows under varying conditions of economic and energy development. It should be noted, however, that interest in this field grew rapidly during the 1970s, with the result that a large number of models of the world oil market are now available (see, for example, Choucri (1981), Gately (1981), Kilgore (1977), Salant (1976), and Chichilnisky (1981)). SMIOT has a number of features in common with some of these approaches.

The main purpose of this paper is not to give a detailed description of particular features of SMIOT, but rather, as it is the first publication on the subject, to discuss the philosophy of this approach to international oil trade. Special attention is given to the gaming-type algorithm used to simulate the process of trade. This algorithm was designed to take into consideration the conflicts of interest among exporters and importers of oil, and the impacts of these conflicts on global oil trade patterns.

1. BASIC FEATURES OF THE APPROACH TO OIL TRADE

1.1. The scope of the approach

The prices and international flows of oil over time may be taken to characterize the evolution of the world oil trade over the long term. These parameters are highly interdependent, and are also affected by various national and international factors. The factors that influence the long-term development of international oil trade *directly* are the possibility and cost of producing, substituting, and conserving liquid fuels at the national or supranational (regional) levels (see Figure 1). An example is the possibility of developing unconventional sources of liquid fuel in certain parts of the world--under certain circumstances, these sources would be capable of halting the rise in international oil prices over the long term (Häfele, 1981, pp. 541-545). These factors represent technological and economic interfaces in the liquid fuel production and consumption sectors of the national/regional energy systems, and as such, they reflect the competitive background of a country (region) in world oil trade. We shall therefore describe them as *direct competitive factors* (or relationships) in the oil market.

Another group of *direct* factors shown in Figure 1 is concerned with the *non-competitive behavior* of oil trade partners. This occurs when a number of traders form a coalition which dictates all or part of their behavior in the oil market. It is clear that non-competitive behavior of this type can exert a strong influence on the world oil market.

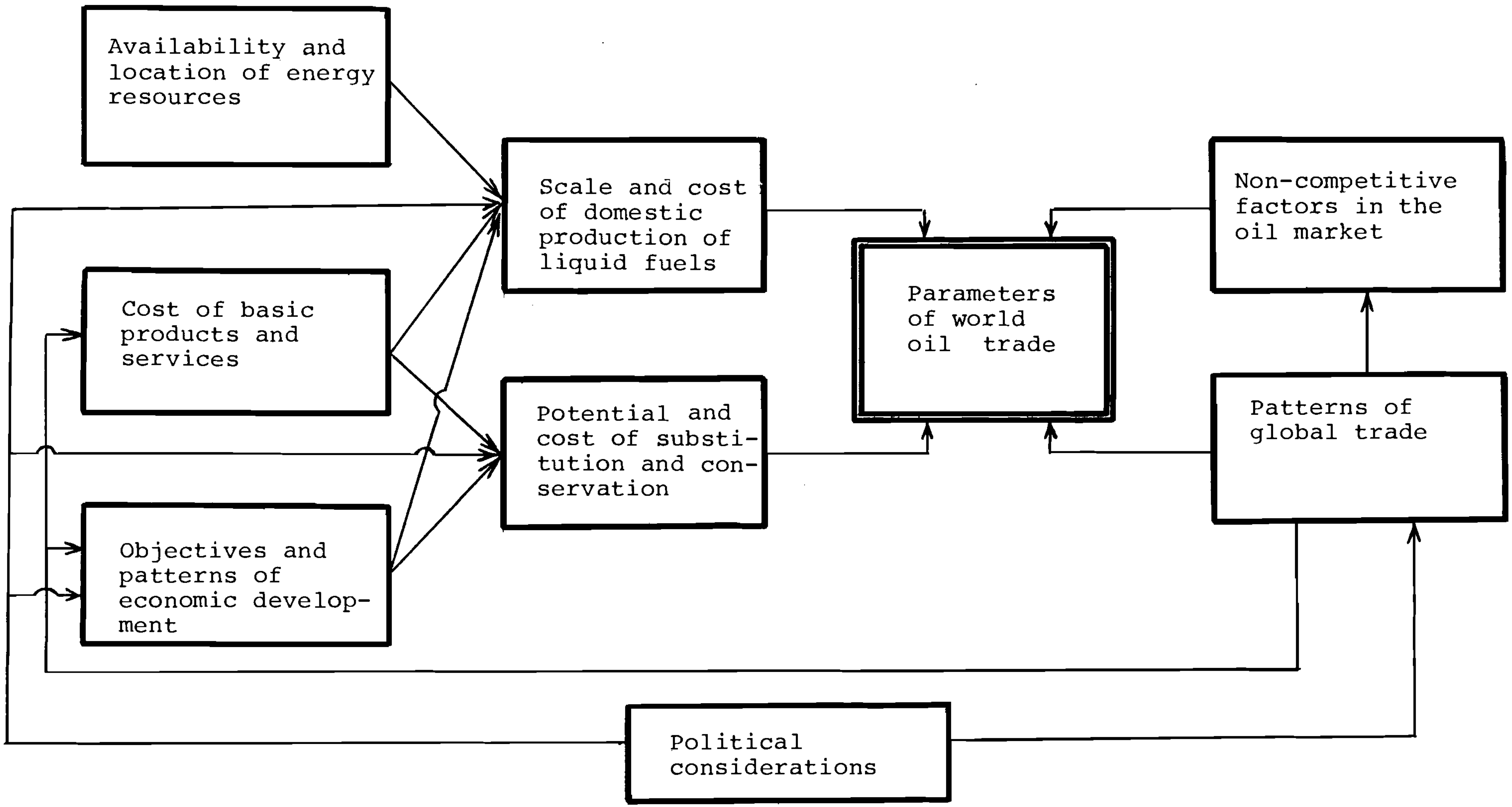

Figure 1. Factors and relationships affecting long-term world oil trade.

There is one more group of factors of, perhaps, even greater importance, which acts *indirectly* through the other groups. This includes factors of both a competitive and a non-competitive nature, such as objectives and patterns of national economic development, the cost of basic economic products and services, the availability and location of energy resources, patterns of overall world trade, and the political environment (see Figure 1). These factors may act in a number of ways, which can be classified as follows:

1. Through interaction of the oil market with other global and local markets for energy and non-energy products.
2. Through intentional subordination by an oil trader of its competitive behavior to macroeconomic and political goals.

For example, any changes in the cost of capital, labor, or basic materials will undoubtedly have a strong influence on the international price of oil. These changes are themselves believed to be caused by changes in the patterns of world trade and national economic development.

But even this is a simplification. Figure 1 does not show the feedbacks of the patterns of international oil trade to national economies and their energy sectors, or to the evolution of world trade. However, these links *do* exist and they make the picture still more complex.

Any study of the long-term future of international oil trade should take all of these factors and feedbacks into consideration. However, since the problem is very complex it must be decomposed in some way before it can be modeled. Figure 2 shows one possible decomposition - it is by no means the only possibility, and is not necessarily the best. However, one part of the decomposition seems obvious, i.e., separating the national and international aspects of energy and economic development. We must then consider whether it is necessary (or desirable) to decompose the problem still further.

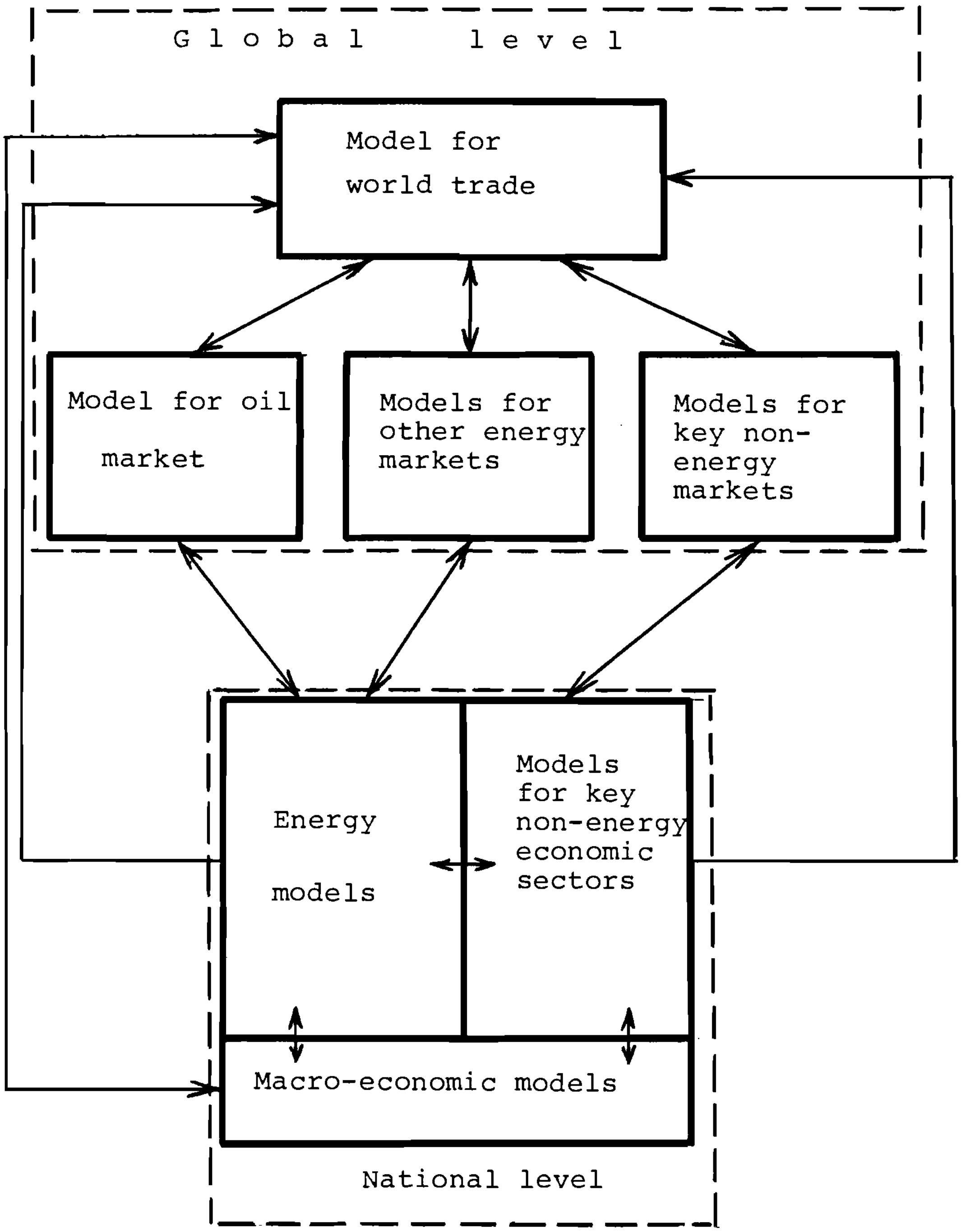

Figure 2. Possible decomposition of the oil trade problem.

In our opinion, further decomposition is both necessary and desirable. It is unlikely that a single model for world trade would be able to analyze both the specific problems of international oil trade and world trade in other products and services. Such a model would either be too rough to take into account all of the links and feedbacks in the markets for particular goods, or it would be too detailed to be feasible in terms of computerization and analysis.

It seems much more promising to link an aggregated world trade model with a set of models dealing with markets for certain essential energy and non-energy products and services (as shown in Figure 2). In this type of system the international oil trade model should:

- describe direct competitive factors in the oil market;
- simulate the non-competitive behavior of oil traders;
- take indirect factors into account.

The Simulation Model for International Oil Trade (SMIOT) has been designed to meet these requirements. However, it should be noted that the rest of the ideal model system illustrated in Figure 2 is purely hypothetical - the other models do not actually exist. Under these circumstances, it seems reasonable to adopt a scenario approach to world oil trade, in which SMIOT would act as a simulator, forecasting the consequences of various possible policies, situations, or developments at the national or regional level. An analyst would then generalize the responses of the model to the various assumptions made in the scenarios.

1.2. Introduction of direct competitive factors

We shall divide the world into a number of groups of countries, which we shall call regions. Let each region be characterized by a set of functions (one for each time point considered) reflecting the rates of domestic production of liquid fuels from all available sources at varying production costs. These are generally increasing convex functions, as illustrated in Figure 3(a) (increasing curve). We shall call this type of function the production-cost function(PCF).

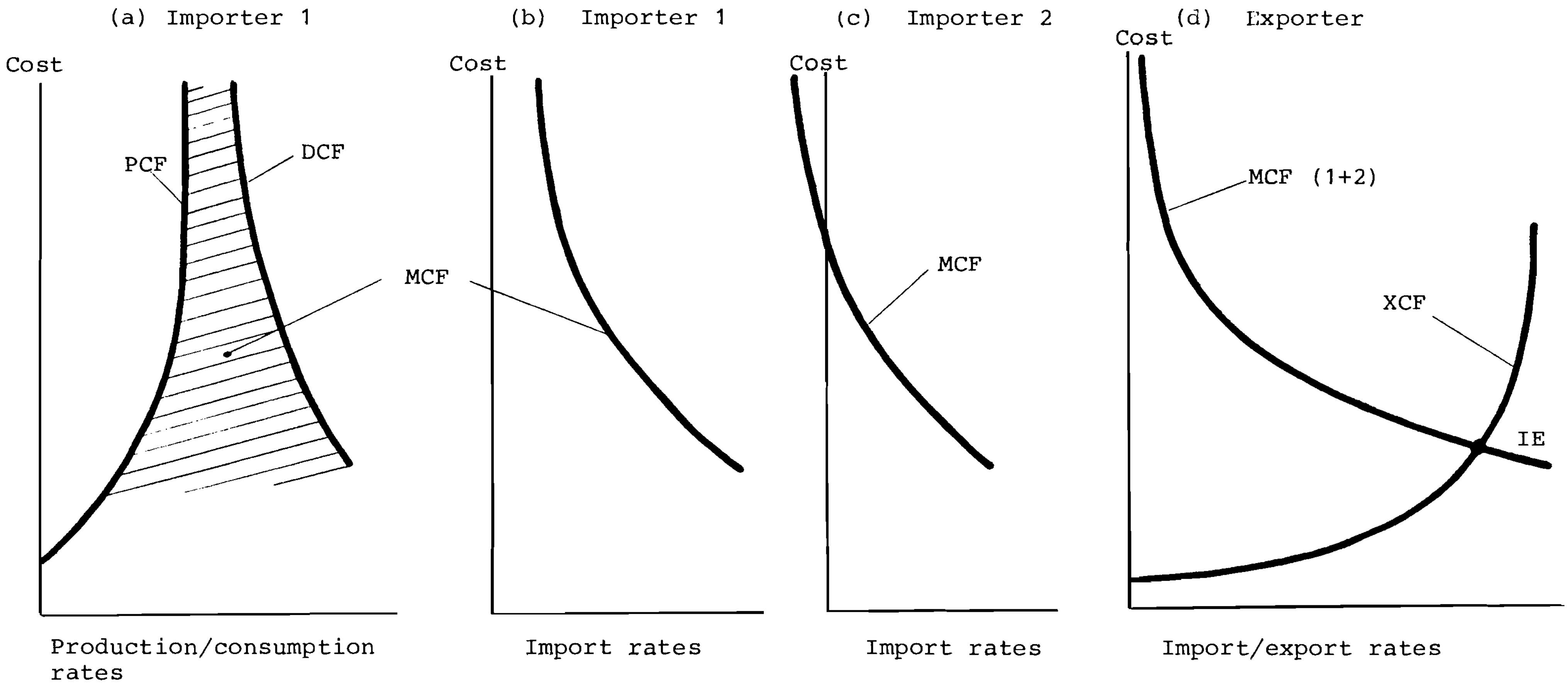

Figure 3. Regional production, demand, import, and export functions (PCF, DCF, MCF, and XCF, respectively).

We shall also assume the existence of another set of regional functions describing the demand for liquid fuels, and call these the demand-cost functions (DCFs). These are generally decreasing convex functions (decreasing curve in Figure 3(a)). The DCFs show how much the demand for liquid fuels can be reduced through substitution and conservation, at the various costs that such measures entail.

The shaded area between the DCF and the PCF in Figure 3(a) represents economic imports to a given region at varying market prices for liquid fuel. By analogy, we will call this function the *import-cost function* (MCF) - it is illustrated in Figure 3(b).

The region characterized by Figure 3(a) and 3(b) (Importer 1) has a high dependence on oil imports which, however, decreases with increasing costs. Many parts of the world are believed to be tending to this type of situation in the long term. However, some regions may well be able to attain self-sufficiency in liquid fuels at some level of cost and may even become exporters. This type of MCF is illustrated in Figure 3(c) (Importer 2). Still other regions export large quantities of oil and are characterized by their export-cost functions (XCFs) rather than their MCF. This case is shown in Figure 3(d) (increasing curve).

It is assumed that these import/export cost functions are sufficient to characterize the positions of different world regions in the oil market. Taken together, these functions represent the competitive determinants of the world oil market.

If the evolution of the oil market were modeled using only the relationships described above, it would lead to an *ideal equilibrium* (IE) point of the type illustrated in Figure 3(d). Here the IE solution corresponds to the point where the XCF of the exporter and the generalized MCF for the two importers intersect. An idealistic solution of this type is clearly not likely to arise in practice, but it is thought to offer a basis around which other, more realistic, situations can be simulated. In other words, it is believed that any realistic situation would represent not a full contrast to the IE but a deviation from it.

1.3. Indirect factors

The indirect factors are treated as exogenous constraints and values by using them to transform the regional import/export cost functions discussed above.

For instance, any changes in the cost of capital, labor, or basic materials other than liquid fuels would result ultimately in changes in the costs of liquid fuel production and/or use technologies, thus affecting the slope of the PCFs and/or DCFs. Further, all kinds of prohibitive domestic policies concerning oil (for instance, policies designed to save oil, policies attempting to reduce oil consumption and/or oil imports, and policies restricting spending on oil imports) can be interpreted as limits to the quantity of oil consumed, produced, or imported.

If the evolution of the oil market is modeled on the basis of direct competitive relationships, taking account of indirect factors, we obtain a solution somewhat different from the previous one which we shall call a *constrained equilibrium* (CE) point. The difference between the CE and IE situations is illustrated in Figure 4 (full and dashed lines, respectively).

1.4. Simulation of non-competitive behavior

The CE solution assumes that each oil trader acts individually, and that his behavior is based on micro and macroeconomic domestic considerations. In this sense, therefore, it can be considered as an extreme situation--potentially feasible but not very likely to occur in the long term. Another feasible extreme is the dominance of oil importers over exporters, which could occur if the importers were to act in unison. In this case, the driving force in the oil market would be the maximization of the economic benefits from oil trade for the whole group of oil importers. Some point MM (importers' monopoly) along the generalized XCF would be the outcome in this situation (see Figure 4). Finally, if the exporters were to act as a unit, they would dominate the importers; the driving force in the oil market would be the maximization of the total oil revenues received by the exporters, and the situation would be represented by XM (exporters' monopoly) on the generalized MCF (see Figure 4).

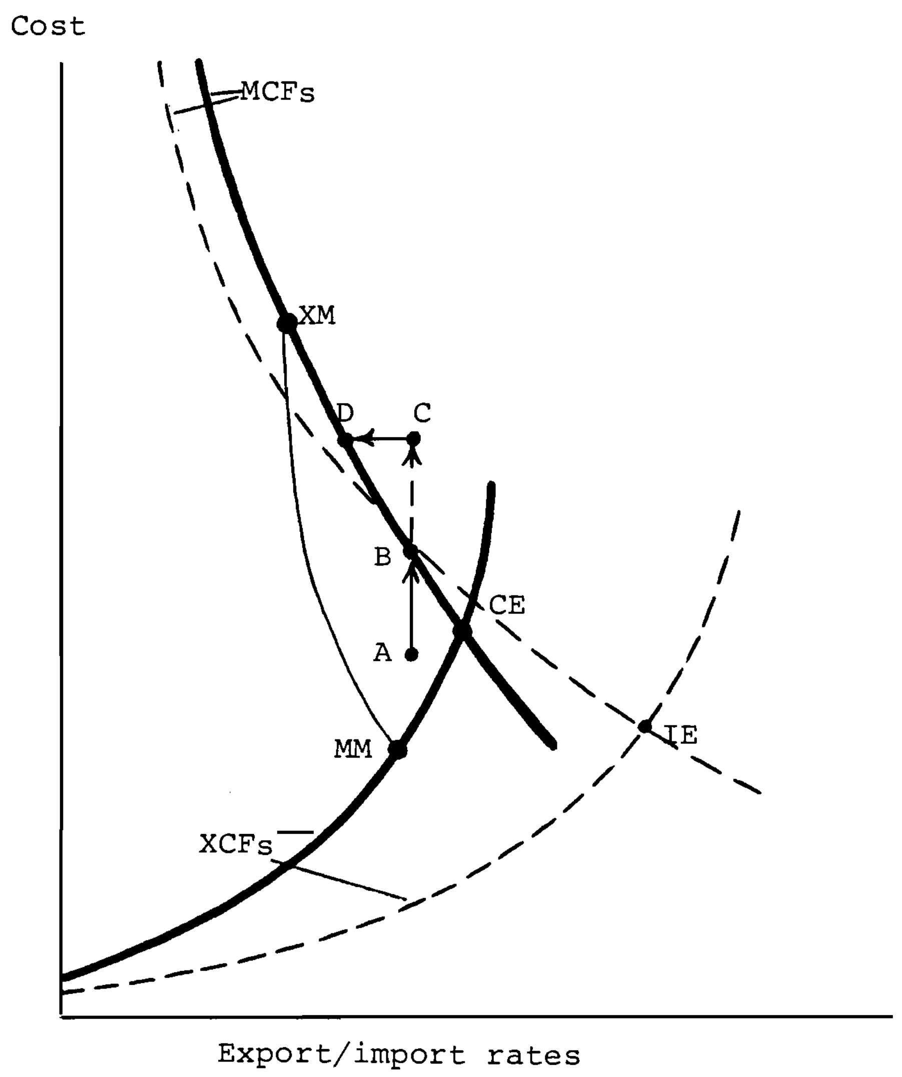

Figure 4. Possible market situations.

The three extreme situations described above define the limits to all possible trade agreements among oil exporters and importers, represented by the region CE-MM-XM-CE in Figure 4. Any point in this region implies a certain level of non-competitive behavior on the part of the traders. For example, although the points along the MCF in the range XM-CE are all based on the assumption that the exporters are dominant, each of these points corresponds to a different level of competitiveness among the exporters, ranging from no coalitions at all to a complete monopoly. The points in the range MM-CE along the XCF correspond to analogous levels of competitiveness among importers in an importer-dominated market. It is easy to see that the points on the line MM-XM represent various compromises between the two monopolies. Finally, the points inside the region defined by these three lines represent all possible combinations of coalitions in a nondominated market.

When modeling the oil market, it would be useful to be able to simulate the situations corresponding to all of the points in the region CE-MM-XM-CE discussed above. However, SMIOT does not do this, since we have limited the range of situations studied to those believed to be more likely. It has basically been assumed that the present economic and political power of the oil exporters will be maintained in the future because:

- the dependence of the developed economies on oil imports is expected to decrease quite slowly despite vigorous attempts by the importers to reduce this dependence as rapidly as possible; and
- the demand of the developing countries for world oil resources is expected to grow very rapidly.

This assumption means that the oil trade alternatives simulated in the model are limited to those corresponding to points on the line XM-CE in Figure 4. The simulation algorithm used in SMIOT identifies certain points in this range with specified cartel relations among oil exporters. By varying the values of a few simulation parameters, it is possible to move from one of these points to other points which imply different relations among exporters. It is thus possible to examine points throughout the XM-CE range.

2. OIL TRADE SIMULATION ALGORITHM

2.1. Core assumptions

The basic assumptions of our world oil trade model are as follows:

1. The oil market will remain an exporters' market.
2. Importers can prevent excessive growth in oil prices by reducing their imports.
3. Allocation of given oil imports among the importers will obey the constrained equilibrium conditions defined above.
4. The driving force in the market is the maximization of oil revenues by each exporter, under certain conditions reflecting the interdependence of their export policies.

The first assumption has been discussed above. It basically states that exporters are free to choose their trade agreements from those represented by the region CE-XM-MM-CE (see Figure 4). Since the best feasible exporters' alternative to any point A within this region is its vertical projection B on the MCF, the exporters can reasonably limit themselves to the alternatives lying along the MCF.

The second assumption implies that the alternatives above the MCF are not feasible for the exporters. If the exporters try to impose an agreement C that is not economic for the importers, the latter may respond by reducing their imports by an amount equivalent to projecting point C horizontally onto the MCF (point D in Figure 4).

The third assumption implies that the whole group of importing regions may be regarded as one entity, and is obviously a simplification. This assumption is adopted for the simple reason that the model can then be run on a small computer. Other kinds of competition between importers can be *simulated* by performing multiple runs of the model under a number of fixed assumptions concerning the availability and cost of oil exports.

The fourth assumption is that the general intention of any exporter is to maximize its own revenues. However, if each of the exporters followed this policy, the result would be a CE situation in the market. The outcome for any individual exporter would be better if it partially waived its own interests and compromised with other exporters. The character of the compromise between the exporters and the way in which it is reached are described in the next section.

2.2. General description of the algorithm

In order to understand how the algorithm works, it is necessary to answer the following questions:

1. What is meant by "maximization of oil revenues by an exporter" and how is this modeled?
2. How is the interdependence of the exporters' market policies interpreted?
3. How is the compromise between exporters defined and reached?

To answer these questions, let us first consider the simple case of a market consisting of one exporter and one importer, as illustrated in Figure 5. The exporter is represented by its DCF, a series of PCFs (Figure 5(a)), and a series of XCFs (Figure 5(b), increasing curves). The importer is characterized only by its MCF (Figure 5(b), decreasing curve).

The PCFs described in Section 1.2. reflect oil production rates which should be attainable at a particular time given the availability of resources at varying production costs. This function is shown by the dashed line in Figure 5(a). The XCF corresponding to this curve is given by the dashed line in Figure 5(b); this gives rise to an IE situation in the market. However, economic and political considerations in the exporting regions may well reduce maximum production and export rates to the PCF and XCF labeled v_{max} and x_{max} in Figure 5, thus defining a CE situation in the market.

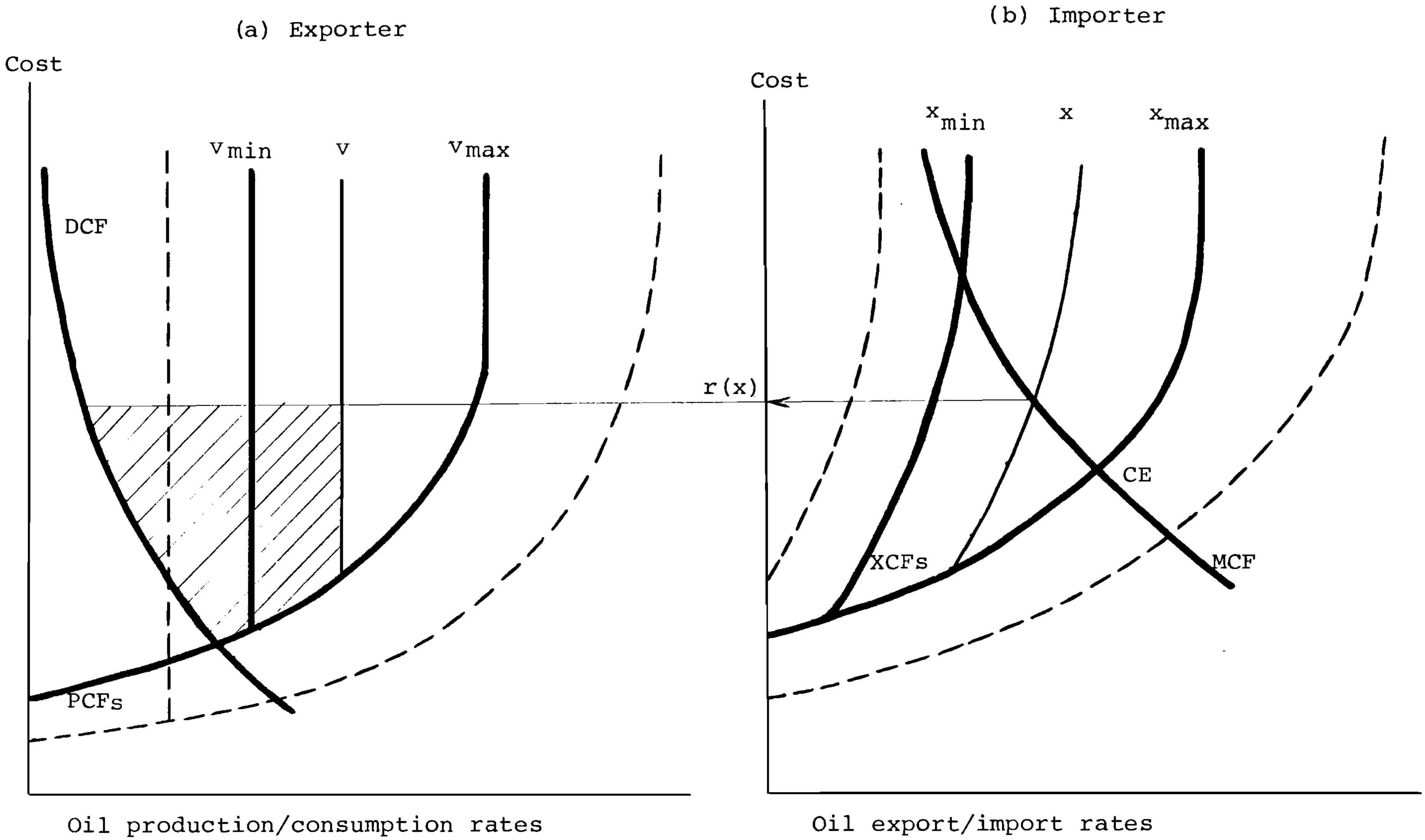

Figure 5. A market consisting of one exporter and one importer.

There are also technological limits to minimum oil production and export rates (the dashed alternatives to the PCF and XCF on the far left of each part of Figure 5) which are generally determined by the production rates achieved in previous years. Economic and political considerations may move these lower bounds to the right to give the bold continuous lines labeled v_{min} and x_{min} in Figure 5.

In the case of a single exporter, the competitive considerations of the exporter are the principal driving force behind his choice of oil production and export rates (which must however lie in the admissible range). Assuming that the exporter dominates the market, and given the character of the importer's MCF, any oil production rate v and oil export strategy x chosen by the exporter within the admissible range will define the market price of oil r(x).

The amount of oil exported in accordance with any export strategy x multiplied by the market price r(x) gives the gross revenue received by the exporter. The net revenue is calculated by deducting the sum of oil production, substitution , and conservation costs from the gross revenue, and may be represented by the shaded area in Figure 5(a). The model assumes that an exporter bases its preferences on the sum of net revenues, and that this sum is maximized by choosing between oil production rates within the admissible range.

Given the assumptions of this approach, it is only possible to maximize net oil revenues in the case of a market with a single exporter. Let us now consider the case in which the market consists of two exporters and one importer.

This case is illustrated in Figure 6. For simplicity, each of the exporters is represented by a couple of XCFs from the x_{min}-x_{max} range. The importer is represented by its MCF. Four possible combinations of exporters' strategies lead to four different market prices, as shown in Figure 6(c). The most important conclusion that can be drawn from this diagram is that the value of net oil revenues associated with a fixed strategy chosen by one exporter depends on the strategy adopted by the other exporter. Indeed,

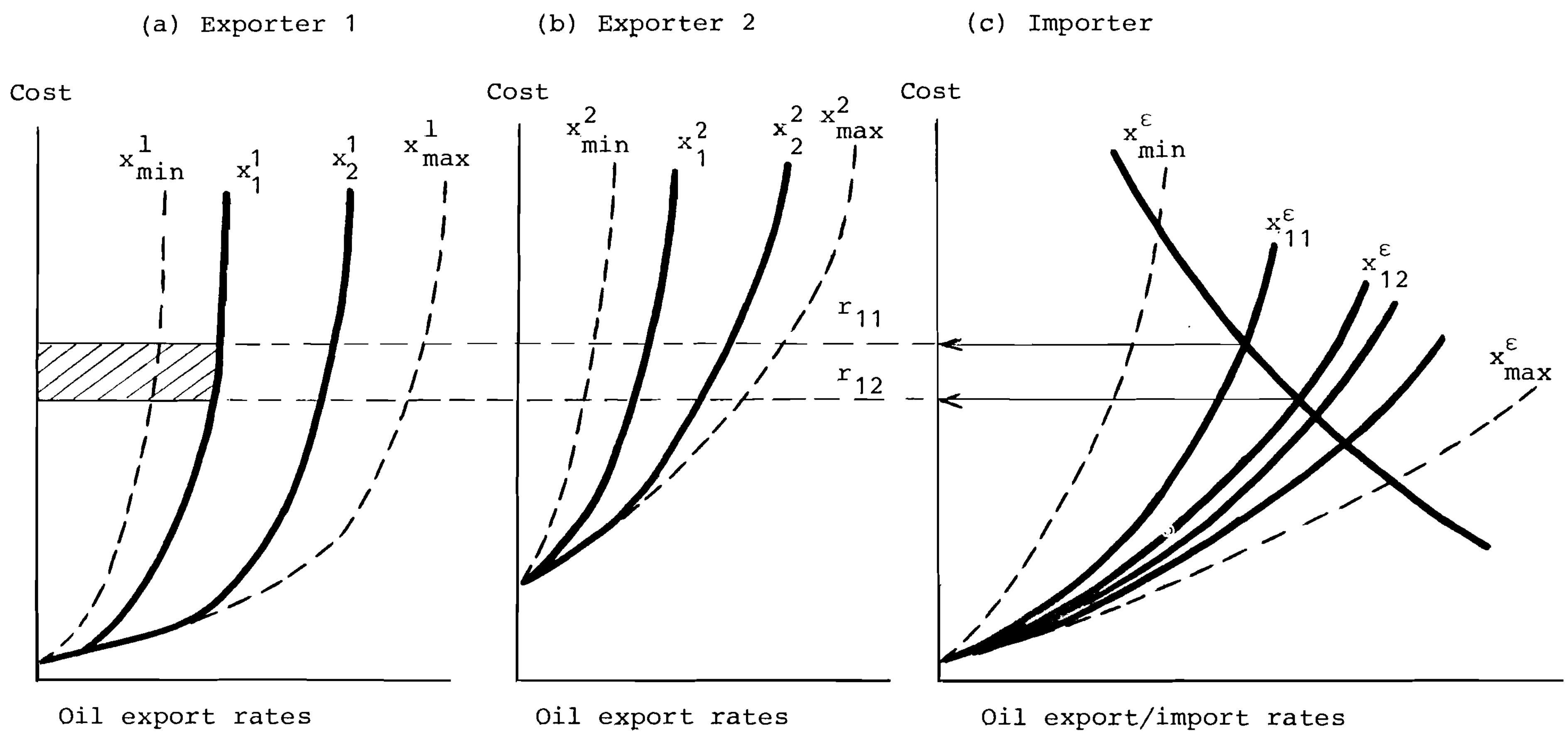

Figure 6. A market consisting of two exporters and one importer.

Figure 6 shows that the sum of net revenues received by Exporter 1 using strategy x_1^1 depends on the strategy (x_1^2 or x_2^2) chosen by Exporter 2, and the difference between the two levels of revenue can be quite large (shaded area in Figure 6(a)).

Thus, the oil export strategies of different regions are interdependent. There are clearly a number of possible compromises between the exporters, which depend upon their willingness to form a coalition and how much they know about the position of other exporters in the market (i.e., about the probable export strategies of possible competitors). The approach adopted in SMIOT is to define the compromise between the exporters in a general form, from which more specific cases can be derived by varying certain parameters.

In this approach it is assumed that each exporter is trying to maximize its own oil revenues, and, with this aim, enters open multistep negotiations with other exporters as equals. The aim of the exporter in these negotiations is to respond to the export strategies of other exporters in the coalition so that its own profit is maximized. Under these conditions, a single equilibrium point--a certain combination of exporters' strategies--seems to exist. This point has the following characteristics:

1. Each partner maximizes its objective function within the set of its own admissible export strategies and those of other exporters.
2. Any unilateral deviation from this point by an exporter is followed by a response from other exporters which decreases the revenues of the first exporter.

We shall now show how this negotiation process is simulated, using the above example of two exporters and one importer. Figure 7 shows the oil revenue gained by each of the two exporters as a function of its own exports, on the assumption that the other exporter adopts certain fixed export strategies.

Let Exporter 1 take the first step in the negotiations (the result is independent of this assumption). Since it has no idea of the strategy adopted by Exporter 2, Exporter 1 can choose any initial strategy with the same degree of confidence. The

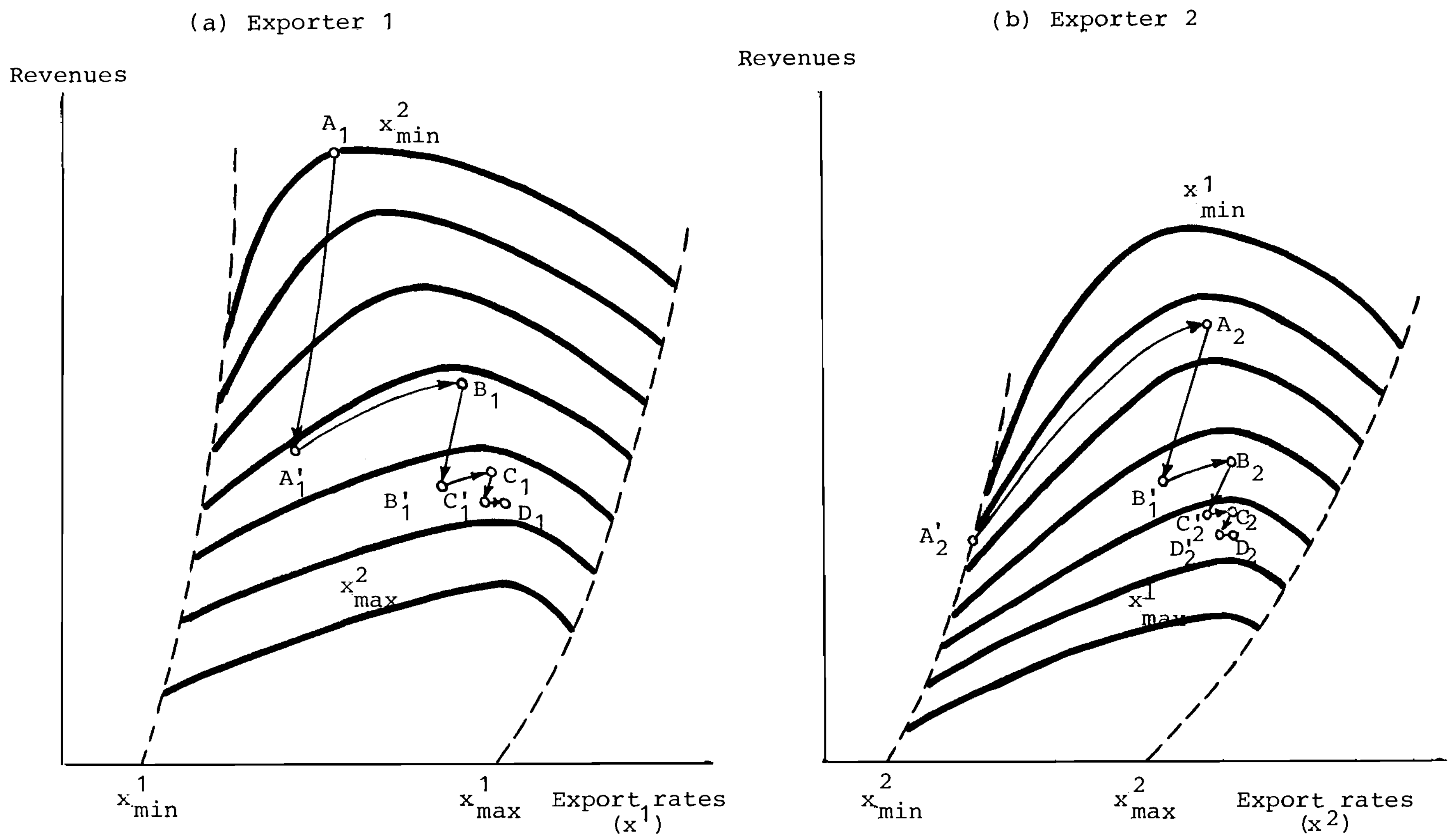

Figure 7. Graphical interpretation of the negotiation process.

model assumes that the initiator chooses the strategy that maximizes its objective function--point A_1 in Figure 7(a). It then informs the other exporter about this choice. This allows Exporter 2 to identify its own position as seen by Exporter 1--point A_2' in our particular example. By optimizing its exports given Exporter 1's strategy, Exporter 2 finds point A_2 to be the best alternative to point A_2', and communicates its decision to Exporter 1. In turn, Exporter 1 identifies its new best position based on the choice made by Exporter 2--point A_1'.

Each of these steps can be considered as one iteration in the negotiation process. The discrepancy between the interests of the exporters decreases with successive iterations until they come to an agreed point D, any deviation from which is profitable for neither of them.

However, this description does not cover some extremely important points concerning the mathematical convergence of the algorithm, without which the overall formal validity of the approach may be questioned.

These points may be listed as follows:

1. Does the equilibrium point defined above actually exist?

2. Is it a unique point?

3. Is it reached by the algorithm adopted?

These questions are examined mathematically in the next section.

The algorithm described below represents only one possible method of simulating the behavior of exporters in the oil market. There are obviously other approaches which differ in their definition of the equilibrium point, the ways in which they reach this point, and so on. One of these alternative algorithms is discussed in Section 2.4.

2.3. Mathematical basis of the algorithm

From the mathematical point of view, the simulation of international oil trade can be regarded as a problem in game theory. Different concepts of market equilibrium and different negotiation rules give rise to different formulations of the problem, which, when combined with appropriate iterative methods of solution, may be viewed as dynamic models of oil trade processes. These iterative procedures may vary in their complexity, their information requirements, and the speed with which they attain equilibrium.

We shall now consider the market for a single homogeneous product, which we shall assume to consist of a single importer and a number of exporters. We also assume that the exporters are not allowed to enter coalitions which would influence the minimum price paid by the importer.

Let $i = 1,2,\ldots,n$ represent the exporters, $f^i(z)$ the marginal cost at which any exporter i produces an amount z of the product for sale, and $r(z)$ the maximum price at which the importer would agree to buy an amount z of the product.* If x_i denotes the amount of the product actually sold by exporter i, then the revenue, ψ^i, of exporter i can be expressed as follows:

$$\psi^i(x_1,\ldots,x_i,\ldots,x_n) =$$

$$r(x_1 + \ldots + x_i + \ldots + x_n)x_i - \int_0^{x_i} f^i(z)dz \quad . \qquad (1)$$

In choosing his strategy x_i, each exporter wishes at least to fulfill the condition

$$\psi^i(x_1,\ldots,x_i,\ldots,x_n) \geq 0 \quad .$$

Note that the terms of the problem are such that $r(z) \geq 0$, $f^i(z) \geq 0$,

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*For simplicity, it is assumed that the functions $f^i(z)$ and $r(z)$ do not vary over time.

$i = \overline{1,n}$, and therefore exporter i can always meet the above condition by choosing x_i subject to the constraints

$$x_i \geq 0, \; i = \overline{1,n} \quad . \tag{2}$$

If we suppose that each exporter chooses the amount x_i that maximizes his revenue in any market situation [characterized by a vector $x = (x_1, \ldots, x_i, \ldots, x_n)$ with non-negative components], then we have the following problem: find an equilibrium situation $x^* = (x_1^*, \ldots, x_i^*, \ldots, x_n^*)$ such that

$$\psi^i(x_1^*, \ldots, x_i^*, \ldots, x_n^*) = \max_{x_i \geq 0} \psi^i(x_1^*, \ldots, x_i, \ldots, x_n^*), \; i = \overline{1,n} \quad . \tag{3}$$

We shall call point x^* the optimal solution - it is also known as the Nash equilibrium point (see Luce and Raiffa, 1957). It should be noted that upper bounds of the type $x_i \leq \delta_i$ can be included by constructing functions $f^i(z)$, introducing "barriers" or "penalties" for $x_i > \delta_i$ (see Figure 8).

The simulation approach described in the preceding section represents one of the simplest iterative methods of solving problem (3). This procedure requires only sufficient information for each exporter to develop a new strategy: exporter i needs to know only the functions $f^i(z)$, $r(z)$ and in general the amount produced by other exporters ($\sum_{k \neq i} x_k$). We shall now give a formal description of this scheme.

The process starts by identifying the point $x^0 = (0, \ldots, 0)$ as an initial approximation of the optimal solution. Each successive approximation $x^s (s = 1,2,\ldots)$ is defined according to the relationship

$$x_i^{s+1} = \arg \max_{x_i \geq 0} [r(x_1^s + \ldots + x_i + \ldots + x_n^s) x_i - \int_0^{x_i} f^i(z) dz]$$

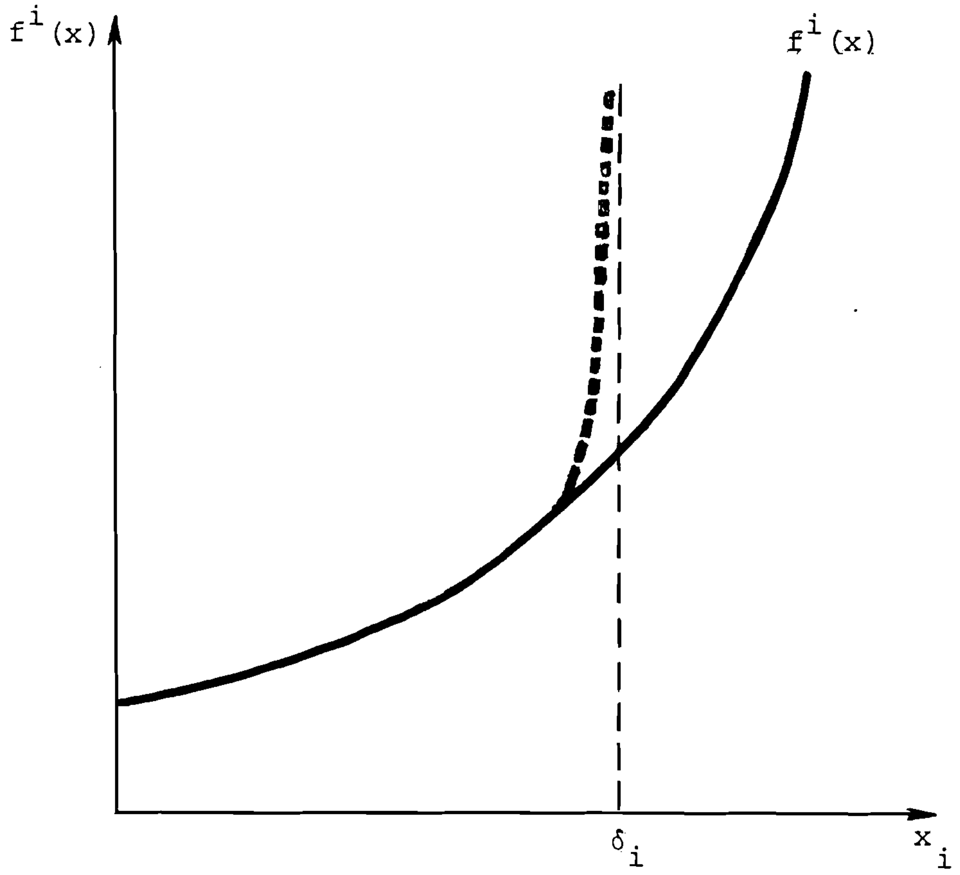

Figure 8. The class of functions $f^i(x)$ considered.

or

$$x_i^{s+1} = \arg\max_{x \geq 0} [r(y_i^s + x)x - \int_0^x f^i(z)dz] \quad , \tag{4}$$

where

$$y_i^s = \sum_{k \neq i} x_k^s \quad .$$

Convergence of procedure (4), i.e., convergence of the sequence $\{x^s\}_{s=0}^{\infty}$, $x^s = (x_1^s, \ldots, x_n^s)$, to the optimal solution x^* follows from some additional assumptions concerning the functions $f^i(\cdot)$ and $r(\cdot)$.

From procedure (4), it is evident that

$$x_i^2 \geq x_i^0 = 0, \quad i = \overline{1,n} \quad .$$

We shall now show that, under certain assumptions, the following chain of inequalities is true:

$$x_i^2 \geq x_i^0 \Rightarrow x_i^1 \geq x_i^3 \Rightarrow x_i^4 \geq x_i^2 \Rightarrow x_i^3 \geq x_i^5 \Rightarrow \ldots$$

Let $x_i(y)$ be given by

$$x_i(y) = \arg\max_{x \geq 0} [r(y + x)x - \int_0^x f^i(z)dz]$$

where $y \geq 0$.

<u>*LEMMA 1*</u> *Assume that*

(a) $r(z) \geq 0$, $f^i(z) \geq 0$, $i = \overline{1,n}$;

(b) $r(z)$ *is a differentiable function and* $r'(z) < 0$;

(c) $r'(y) \geq r'(z)$, $f^i(y) \leq f^i(z)$ *at any non-negative* y *and* z *such that* $y \leq z$.

Then

$$x_i(y) \geq x_i(z), \quad i = \overline{1,n} \quad .$$

This lemma is illustrated in Figure 9.

<u>*Proof*</u>. There are two possible cases: $x_i(y) > 0$ or $x_i(y) = 0$.

1. Let $x_i(y) > 0$. Then from the optimality conditions for $x_i(y)$ we obtain:

$$r'(y + x_i(y))x_i(y) + r(y + x_i(y)) - f^i(x_i(y)) = 0 \quad . \tag{5}$$

Arguing by contradiction, we suppose that $x_i(y) < x_i(z)$. Then, from the same kind of optimality condition for $x_i(z)$, we have:

$$x_i(z) = \frac{f^i(x_i(z)) - r(z + x_i(z))}{r'(z + x_i(z))} \quad .$$

Since $y \leq z$ and $x_i(y) < x_i(z)$, then $y + x_i(y) < z + x_i(z)$. Under the assumptions of the lemma:

$$r(y + x_i(y)) \geq r(z + x_i(z)) \quad ,$$

$$f^i(x_i(y)) \leq f^i(x_i(z)) \quad ,$$

$$r'(y + x_i(y)) \geq r'(z + x_i(z)) \quad .$$

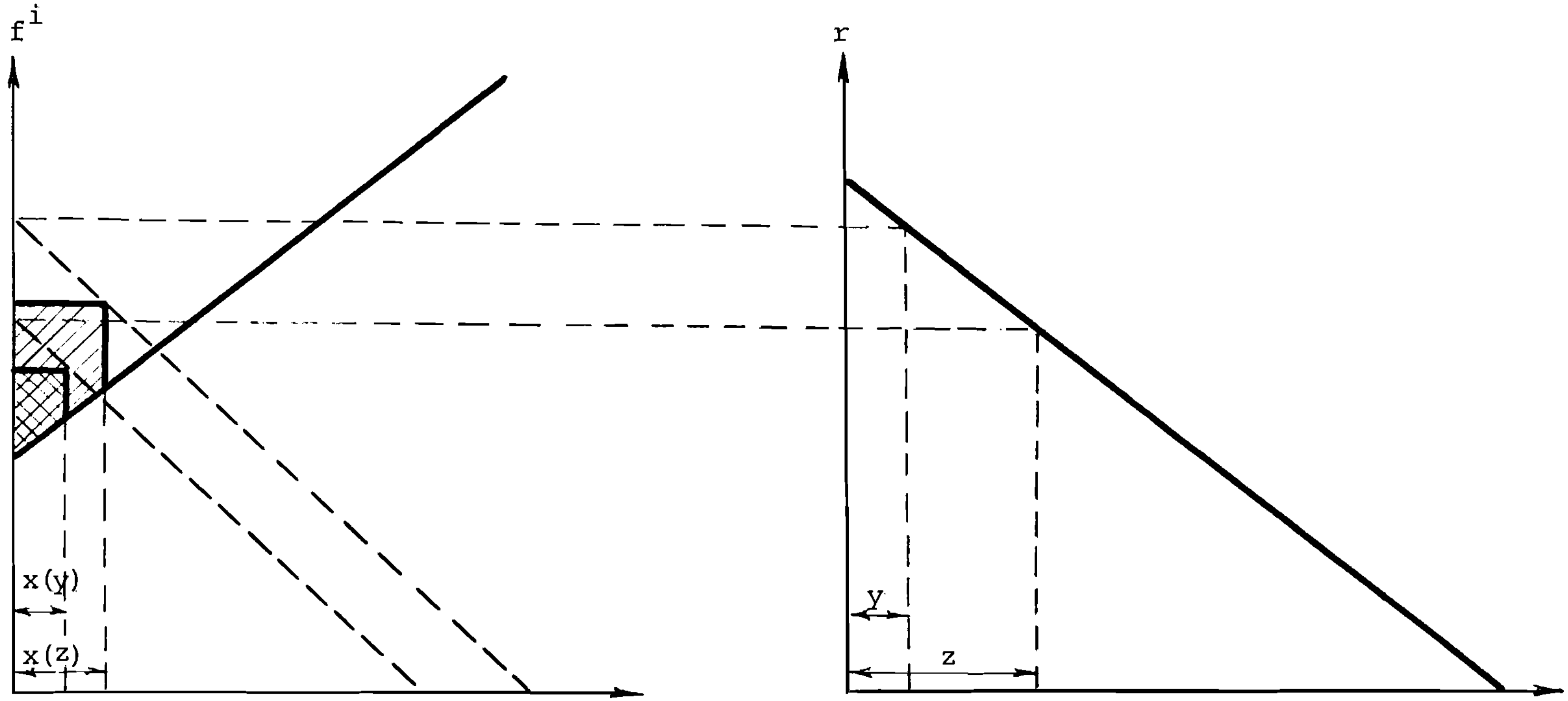

Figure 9. A graphical illustration of Lemma 1.

It therefore follows from equation (5) that

$$r'(z + x_i(z))x_i(y) + r(z + x_i(z)) - f^i(x_i(z)) \leq 0$$

$$x_i(y) \geq \frac{f^i(x_i(z)) - r(z + x_i(z))}{r'(z + x_i(z))} = x_i(z) \quad .$$

This contradicts the fact that $x_i(y) < x_i(z)$. Thus Lemma 1 is valid for $x_i(y) > 0$.

2. If $x_i(y) = 0$, then $x_i(z) = 0$, remembering that $r(y + x) \geq r(z + x)$. This completes the proof.

<u>LEMMA 2</u> *Let the assumptions of Lemma 1 be valid. Then for all* $i = \overline{1,n}$

$$x_i^1 \geq x_i^2,\ x_i^3 \geq x_i^4,\ x_i^5 \geq x_i^6, \ldots$$

$$x_i^1 \geq x_i^3,\ x_i^4 \geq x_i^2,\ x_i^3 \geq x_i^5,\ x_i^6 \geq x_i^4, \ldots \quad .$$

<u>Proof</u>. The proof of this statement follows from Lemma 1. It is evident that the relations $x_i^2 \geq x_i^0$, $x_i^1 \geq x_i^0$, $i = \overline{1,n}$, must hold. Assume also that for some $k > 1$ the relations

$$x_i^{2k} \geq x_i^{2(k-1)},\ x_i^{2k-1} \geq x_i^{2(k-1)},\ i = \overline{1,n}$$

are obeyed. We shall now show that

$$x_i^{2(k+1)} \geq x_i^{2k},\ x_i^{2k+1} \geq x_i^{2k} \ ,$$

$$x_i^{2k-1} \geq x_i^{2k},\ x_i^{2k-1} \geq x_i^{2k+1} \quad .$$

We have

$$x_i^{2k-1} = \arg\max_{x \geq 0} \left[r(y_i^{2(k-1)} + x)x - \int_0^x f^i(z)dz \right] ,$$

$$x_i^{2k} = \arg\max_{x \geq 0} \left[r(y_i^{2k-1} + x)x - \int_0^x f^i(z)dz \right] ,$$

$$x_i^{2k+1} = \arg\max_{x \geq 0} \left[r(y_i^{2k} + x)x - \int_0^x f^i(z)dz \right] ,$$

$$x_i^{2(k+1)} = \arg\max_{x \geq 0} \left[r(y_i^{2k+1} + x)x - \int_0^x f^i(z)dz \right] ,$$

Since it is assumed that $y_i^{2k} \geq y_i^{2(k-1)}$, $y_i^{2k-1} \geq y_i^{2(k-1)}$, then from Lemma 1 we may deduce that $x_i^{2k-1} \geq x_i^{2k}$, $x^{2k-1} \geq x_i^{2k+1}$. From this, $y_i^{2k-1} \geq y_i^{2k}$, $y_i^{2k-1} \geq y_i^{2k+1}$. Therefore, from Lemma 1 we have $x_i^{2(k+1)} \geq x_i^{2k}$ and $x_i^{2k+1} \geq x_i^{2k}$ and the proof is complete.

Thus, according to Lemma 2: $x_i^1 \geq x_i^3 \geq x_i^5$ ..., $x_i^2 \leq x_i^4 \leq x_i^6$... and $x_i^3 \geq x_i^2$, $x_i^5 \geq x_i^4$,..., $x_i^{2k+1} \geq x_i^{2k}$ for $k = 1,2,\ldots$, $i = \overline{1,n}$. If

$$x_i' = \lim x_i^{2k+1} = x_i'' = \lim x_i^{2k}, \quad i = \overline{1,n}, \quad k \to \infty ,$$

then $X^* = (x_1', \ldots, x_n')$ will be the solution of problem (3).

We shall now show that under certain conditions

$$x_i^{2k+1} - x_i^{2k} \to 0 \text{ for } k \to \infty .$$

THEOREM 1. *Assume that*

(a) *The assumptions of Lemma 1 hold;*

(b) $\min\limits_{1 \le i \le n} x_i^2 > 0$;

(c) *There exist numbers* $a_i > 0$, $i = \overline{1,n}$ *which satisfy the conditions*

$$f^i(z) - f^i(y) \ge a_i(z - y) \text{ for } y \le z ;$$

(d) *There exist non-positive numbers* c_1, c_2, c_3, c_4 *and* $k^* \ge 1$ *such that* $c_1 \le r'(z) \le c_2$, $c_3 \le r''(z) \le c_4$ *for* $y^2 \le z \le y^1$, *where*

$$y^1 = \Sigma_{i=1}^n x_i^1, \; y^2 = \Sigma_{i=1}^n x_i^2$$

$$\alpha = \frac{(n-1) \min\limits_{1 \le i \le n} (c_1 + c_3 x_i^{2k^*-1})}{\max\limits_{1 \le i \le n} (2c_2 + c_4 x_i^{2k^*} - a_i)} < 1 .$$

Then $x_i^{2k+1} - x_i^{2k} \to 0$ for $k \to \infty$.

Note that requirement (b) is essential for convergence. Figure 10 demonstrates this point for $n = 4$ and linear f^1, f^2, f^3, f^4 and r. It is clear from this diagram that $x_i^1 > 0$ $(i = 1,2,3,4)$; $x_i^2 = 0$ $(i = 1,2,3,4)$; $x_i^3 = x_i^1$ $(i = 1,2,3,4)$; and so on

Note also that for the linear functions $f^i(z) = a_i z + b_i$ and $r(z) = cz + d$ we have $c_1 = c_2 = c$, $c_3 = c_4 = 0$. Therefore

$$\alpha = \frac{(n-1)c}{\max\limits_i (2c - a_i)}$$

and if $n = 2$ and $c > \max a_i$, then $\alpha < 1$.

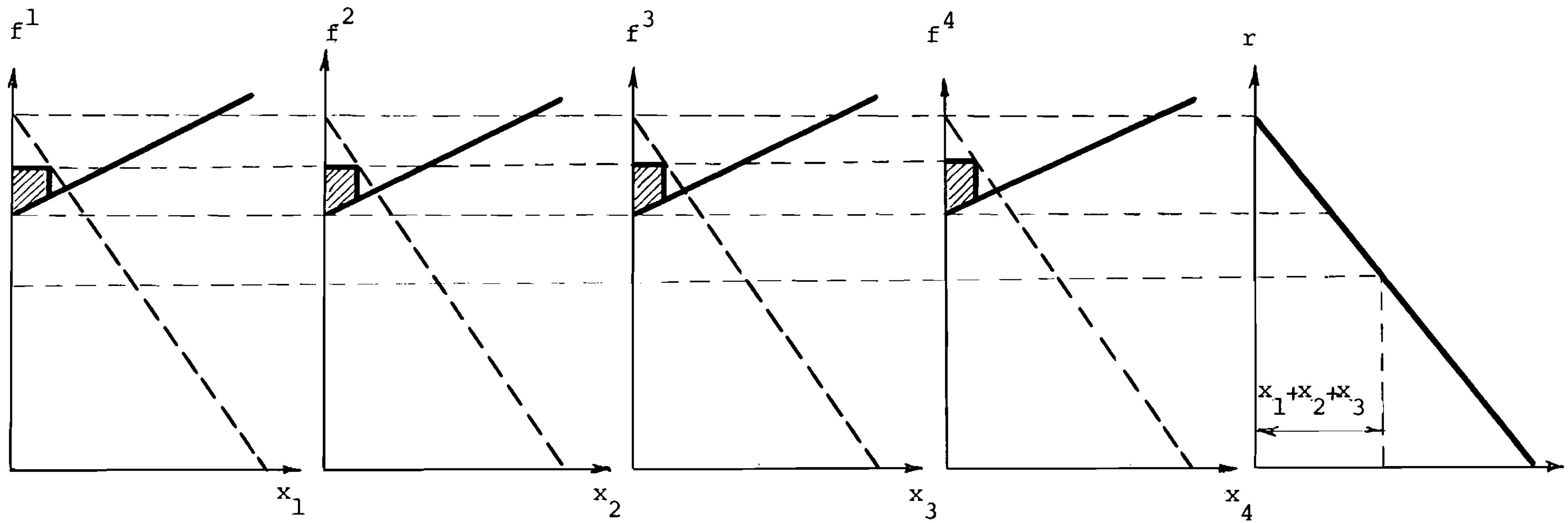

Figure 10. A graphical illustration of Theorem 1.

Proof. Since $x_i^2 > 0$ for $i = \overline{1,n}$, it follows from Lemma 2 that $x_i^s > 0$, $i = \overline{1,n}$, $s \geq 1$. Therefore, from the optimality conditions (5) we have

$$r'(y_i^{2k} + x_i^{2k+1})x_i^{2k+1} + r(y_i^{2k} + x_i^{2k+1}) - f^i(x_i^{2k+1}) = 0$$

$$r'(y_i^{2k-1} + x_i^{2k})x_i^{2k} + r(y_i^{2k-1} + x_i^{2k}) - f^i(x_i^{2k}) = 0$$

Then

$$r'(y_i^{2k} + x_i^{2k+1})x_i^{2k+1} - r'(y_i^{2k-1} + x_i^{2k})x_i^{2k} =$$

$$r(y_i^{2k-1} + x_i^{2k}) - r(y_i^{2k} + x_i^{2k+1}) + f^i(x_i^{2k+1}) - f^i(x_i^{2k}) \quad .$$

Evaluating both sides of this relationship, we obtain:

(a) $$r'(y_i^{2k} + x_i^{2k+1})x_i^{2k+1} - r'(y_i^{2k-1} + x_i^{2k})x_i^{2k}$$

$$= (x_i^{2k+1} - x_i^{2k})r'(y_i^{2k} + x_i^{2k+1}) + x_i^{2k}[r'(y_i^{2k} + x_i^{2k+1})$$

$$- r'(y_i^{2k-1} + x_i^{2k})] \quad .$$

$$\leq (x_i^{2k+1} - x_i^{2k})c_2 + x_i^{2k} \cdot r''(\cdot)(y_i^{2k} + x_i^{2k+1} - y_i^{2k-1} - x_i^{2k})$$

$$\leq (x_i^{2k+1} - x_i^{2k})c_2 + x_i^{2k}c_3(y_i^{2k} - y_i^{2k-1}) + x_i^{2k} \cdot c_4(x_i^{2k+1} - x_i^{2k})$$

(b) $$f^i(x_i^{2k+1}) - f^i(x_i^{2k}) \geq a_i(x_i^{2k+1} - x_i^{2k})$$

(c) $$r(y_i^{2k-1} + x_i^{2k}) - r(y_i^{2k} + x_i^{2k+1}) = r'(\cdot)[y_i^{2k-1} + x_i^{2k} - y_i^{2k} - x_i^{2k+1}]$$

$$\geq c_1(y_i^{2k-1} - y_i^{2k}) + c_2(x_i^{2k} - x_i^{2k+1})$$

(a)-(c) lead to the inequality

$$c_2(x_i^{2k+1} - x_i^{2k}) + c_3x_i^{2k}(y_i^{2k} - y_i^{2k-1}) + c_4x_i^{2k}(x_i^{2k+1} - x_i^{2k})$$

$$\geq a_i(x_i^{2k+1} - x_i^{2k}) + c_1(y_i^{2k-1} - y_i^{2k}) + c_2(x_i^{2k} - x_i^{2k+1}) \quad ,$$

or

$$(2c_2 + c_4x_i^{2k} - a_i)(x_i^{2k+1} - x_i^{2k}) \geq (c_1 + c_3x_i^{2k})(y_i^{2k-1} - y_i^{2k}) \quad .$$

That is,

$$\Sigma_{i=1}^n (2c_2 + c_4x_i^{2k} - a_i)(x_i^{2k+1} - x_i^{2k}) \geq (n-1) \min_{1\leq i\leq n} (c_1 + c_3x_i^{2k})(y^{2k-1} - y^{2k})$$

where $y^k = \Sigma_{i=1}^n x_i^k$. Then

$$\max_{1\leq i\leq n} (2c_2 + c_4x_i^{2k} - a_i)(y^{2k+1} - y^{2k}) \geq (n-1)\min_{1\leq i\leq n}(c_1+c_3x_i^{2k})(y^{2k-1} - y^{2k}) \quad .$$

Since for any k, k^*, where $k > k^*$, we have $x_i^{2k^*-1} \geq x_i^{2k} \geq x_i^{2k^*}$, we obtain (for $k \geq k^*$):

$$y^{2k+1} - y^{2k} \leq \frac{(n-1)\ \min\ (c_1 + c_3x_i^{2k})}{\max(2c_2 + c_4x_i^{2k} - a_i)}\ (y^{2k-1} - y^{2k})$$

$$\leq \frac{(n-1)\ \min\ (c_1 + c_3x_i^{2k^* - 1})}{\max\ (2c_2 + c_4x_i^{2k^*} - a_i)}\ (y^{2k-1} - y^{2k-2})$$

or

$$y^{2k+1} - y^{2k} \leq \alpha^{k-k*}(y^{2k^*-1} - y^{2k^*-2})$$

Since $\alpha < 1$, it follows that $y^{2k+1} - y^{2k} \to 0$ or indeed that $x_i^{2k+1} - x_i^{2k} \to 0$ for $k \to \infty$.

2.4. An alternative to the algorithm

As noted above, it is assumed that each exporter i, at time s, acts only on the basis of knowledge about the functions $f^i(z)$, $r(z)$, and y_i^s. However, it is possible for the exporters to choose their export policies x_i^s using broader information, obtained, for instance, by analyzing the derivatives of the functions $f^i(z)$ and $r(z)$. With this additional assumption, convergence takes place under less severe conditions. One possible procedure using this additional information is based upon the fact that the optimality conditions of problem (1)-(3) resemble those of a nonlinear programming problem.

Let us consider the problem in more detail. Assume that we have a decision vector $x = (x_1,\dots,x_i,\dots,x_n)$, $x_i \in R^1$, a pay-off function $\psi_i(x)$ for each player (exporter), and a joint constraint $x \in X$. Suppose that the $\psi_i(x)$ are concave continuous functions and X is a convex compact set. We wish to find a point $x^* = (x_1^*,\dots,x_i^*,\dots,x_n^*)$ which would satisfy the condition

$$x_i^* = \arg\max \{\psi_i(x_1^*,\dots,x_i,\dots,x_n^*) \,|\, x_i,\ (x_1^*,\dots,x_i,\dots,x_n^*) \in X\}.$$

There are a number of ways of solving this problem if the functions $\psi^i(x)$ are differentiable. In particular, the method proposed by Rosen (1960, 1961) is closely related to the well-known gradient projection method. However, we shall consider the problems which arise when the functions $\psi^i(x)$ are nondifferentiable - this is generally the case in complex trade problems with uncertainties. Let us consider the function

$$\Phi(y,x) = \Sigma_{i=1}^{n}\psi^{i}(y_1,\ldots,x_i,\ldots,y_n)$$

and the point $x^* \in X$ at which

$$\Phi(x^*,x^*) = \max\{\Phi(x^*,x) \mid x \in X\} \quad . \tag{6}$$

The point x^* is generally known as the normalized equilibrium point. It is not difficult to prove that each normalized equilibrium point is also a "normal" equilibrium point, although the converse is not true. Thus, the problem of identifying a normalized equilibrium point is identical to the standard problem:

$$\max\{\Phi(x^*,x) \mid x \in X\} \quad ,$$

from which it is possible to obtain optimality conditions for x^* in a nonlinear programming form.

Consider the vector

$$g(x) = \hat{\Phi}(x,z)\,|\,z = x \quad ,$$

where $\hat{\Phi}_z(x,\cdot)$ denotes a generalized gradient of the function $\Phi(x,z)$ with respect to z. Let $\partial g(x)$ be the set of $g(x)$. The general conditions of optimality for nonlinear programming problems then yield the relation

$$\min_{g \in \partial g(x^*)} \max_{x \in X} \langle g, x - x^* \rangle = 0 \tag{7}$$

as the necessary and sufficient condition for a normalized equilibrium point. Expression (7) suggests the existence of a vector $g(x^*)$ such that

$$\max_{x \in X} \langle g(x^*),\ x - x^* \rangle = 0 \quad . \tag{7'}$$

If the function Φ is differentiable then the vector g is its gradient:

$$g(x^*) = \Phi_z(x,z)\,|\,z = x^* \ .$$

Then, if

$$X = \{x\,|\,Ax = b\}$$

it can be deduced from condition (7) that the vector x^* is the solution of the system of equations

$$g(x) + uA = 0 \quad . \tag{8}$$

This means that, under certain conditions concerning the uniqueness of the normalized equilibrium point, this point can be identified by using an iterative procedure to solve system (8). For the initial problem (1)-(3), we can write down:

$$\Phi(y,x) = \Sigma_{i=1}^{n} [r(y_1 + \ldots + x_i + \ldots + y_n) x_i - \int_0^{x_i} f^i(z)dz] \quad .$$

Then, assuming that the function g(z) is differentiable, we have,

$$g(x) = (g_1(x), \ldots, g_i(x), \ldots, g_n(x)) \quad ;$$

$$g_i(x) = r(x_1 + \ldots + x_i + \ldots + x_n) + r'(x_1 + \ldots + x_i + \ldots + x_n) x_i - f^i(x_i) \quad . \tag{9}$$

For the nondifferentiable function Φ, it is natural to make use of some of the general ideas of nondifferentiable optimization. The specific character of the problem considered is such that even if Φ were differentiable it would be difficult to construct a function which would change monotonically during the iterative process. It therefore seems reasonable to use nonmonotonic optimization procedures to deal with the problems under consideration (see, for instance, Ermoliev, 1981).

We shall consider the following iterative process (which is analogous to the well-known generalized gradient method):

$$x^{s+1} = \Pi_X(x^s + \rho_s g(x^s)), \quad s = 0,1,\ldots, \tag{10}$$

where $\Pi_X(y)$ denotes the projection of the point y onto the set X and ρ_s is the step-size multiplier. For instance, if

$$X = \{x = (x_1,\ldots,x_i,\ldots,x_n) \mid x_i \geq 0, \; i = \overline{1,n}\} \quad ,$$

and the vector $g(x^s)$ is defined by expression (9), then procedure (10) will assume the following form:

$$x_i^{s+1} = \max\{0, x_i^s + \rho_s g_i(x^s)\}, \; i = \overline{1,n}, \; s = 0,1,\ldots \; . \tag{10'}$$

The resulting dynamic trade process (10') implies that each of the exporters will change its current strategy (at time s) so as to increase its revenue function (1) by step ρ_s.

Before studying the convergence of process (10), it is necessary to make some assumptions concerning g(x). Note first that where $\Phi(x^*,x)$ is concave with respect to x, we have

$$\Phi(x^*,x) - \Phi(x^*,x^*) \leq \langle g(x^*), \; x - x^* \rangle \quad .$$

Unfortunately, this does not lead to any conclusions concerning either the sign of $\langle g(x), x^* - x \rangle$ or the convergence of procedure (10).

Rosen (1960, 1961) studied the existence of normalized equilibrium points for differentiable functions $\Phi(x,x)$ and made the following assumption:

$$\langle g(x) - g(y), y - x \rangle > 0, \quad x,y \in X, \quad x \neq y \tag{11}$$

i.e., $g(x)$ is strictly decreasing. This assumption is equivalent to the following:

$$\langle g(x), y - x \rangle > 0, \; x \neq y \quad ,$$

because

$$\langle g(x), y - x \rangle = \langle g(x) - g(y), y - x \rangle + \langle g(y), y - x \rangle \quad .$$

We shall therefore assume that for all x^s, $s = 0,1,\ldots,$ generated by procedure (10),

$$\langle g(x^s), x^* - x^s \rangle > 0 \text{ for } x^s \neq x^* \tag{12}$$

THEOREM 2. Let condition (12) *be fulfilled, and let* $\| g(x^s) \| \leq$ const., $\rho_s \geq 0$, $\rho_s \to 0$, and $\Sigma_{s=0}^{\infty} \rho_s = \infty$. Then:

(1) a subsequence $\{z^{s_k}\}_{k=0}^{\infty}$ *of a sequence* $\{x^s\}_{s=0}^{\infty}$ *exists such that* $z^{s_k} \to x^*$;

(2) if, in addition, $\Sigma_{s=0}^{\infty} \rho_s^2 < \infty$, *then* $x^s \to x^*$ *for* $s \to \infty$.

Proof. Let x^* be the normalized equilibrium point. Then, from the properties of the projection operation, we have:

$$\| x^* - x^{s+1} \|^2 \leq \| x^* - x^s \|^2 - 2\rho_s \langle g(x^s), x^* - x^s \rangle + \rho_s^2 \| g(x^s) \|^2 \tag{13}$$

Let c denote any constants. Then

$$\|x^* - x^{s+1}\|^2 \le \|x^* - x^s\|^2 + \rho_s[-2\langle g(x^s), x^* - x^s\rangle + c\rho_s] \quad .$$

Choose some $\delta > 0$. There are two cases possible for any s, namely:

$$2\langle g(x^s), x^* - x^s\rangle + c\rho_s \le -\delta \quad ;$$

or

$$2\langle g(x^s), x^* - x^s\rangle + c\rho_s > -\delta \quad .$$

We shall now show that no N exists such that

$$2\langle g(x^s), x^* - x^s\rangle + c\rho_s \le -\delta$$

for $s \ge N$.

If such a value of N did exist, then for $s \ge N$ the relation

$$\|x^* - x^{s+1}\|^2 \le \|x^* - x^s\|^2 - \delta\rho_s \le \|x^* - x^n\|^2 - \delta\Sigma_{k=n}^{s}\rho_k$$

would hold with the right-hand side decreasing as $s \to \infty$. This contradicts the non-negativity of the left-hand side. Thus, a sequence s_k, $k = 0,1,\ldots$, $s_k \to \infty$ exists such that

$$2\langle g(x^{s_k}), x^* - x^{s_k}\rangle + c\rho_{s_k} > -\delta \quad .$$

Since $\rho_s \to 0$, it follows that for any $\varepsilon_k \downarrow 0$ a subsequence x^{τ_k} of $\{x^{s_k}\}$ can be found such that

$$\langle g(x^{\tau_k}),\ x^* - x^{\tau_k}\rangle \leq \varepsilon_k \quad .$$

Thus,

$$\langle g(x^{\tau_k}),\ x^* - x^{\tau_k}\rangle \to 0$$

and the proof of the first part of the theorem follows immediately.

Now let $\Sigma_{s=0}^{\infty}\rho_s^2 < \infty$. Using (12), it may be deduced from (13) that

$$\|x^* - x^{s+1}\|^2 \leq \|x^* - x^s\|^2 + c\rho_s^{\ 2} \quad .$$

From this, we obtain the relations

$$\gamma_s \geq 0 \ ; \qquad \gamma_{s+1} \leq \gamma_s$$

where

$$\gamma_s = \|x^* - x^s\|^2 + c\Sigma_{k=s}^{\infty}\rho_s^2 \quad .$$

This means that the sequence of values γ_s and, therefore, the sequence of values $\{\|x^* - x^s\|^2\}$ converges. Since there is a subsequence of the sequence $\{x^s\}$ that converges to an optimal solution, it follows that

$$\|x^* - x^s\| \to 0$$

and the theorem is proved.

To conclude this section, we shall comment on condition (11) for vector (9) in the initial problem (1)-(3). For simplicity, we shall consider once again the linear functions

$$r(z) = cz + d \ ; \qquad f^i(z) = a_i z + b_i \quad .$$

In this case we can write down:

$$\langle g(x) - g(y), y - x \rangle = \Sigma_{i=1}^{n} [c(\Sigma_{k=1}^{n} x_i) + cx_i - a_i x_i$$

$$- c(\Sigma_{k=1}^{n} x_k) - cy_i + a_i y_i](y_i - x_i)$$

$$= - c[\Sigma_{i=1}^{n}(x_i - y_i)]^2 - c\Sigma_{i=1}^{n}(x_i - y_i)^2 + a_i \Sigma_{i=1}^{n}(x_i - y_i)^2 \quad .$$

Thus condition (11) holds if $c < 0$ and $a_i > 0$.

2.5. Particular uses of the algorithm

So far we have considered only one application of the algorithm - to oil exporters. However, SMIOT is not limited to estimating the impacts of inter-exporter compromises on oil trade patterns under one fixed set of assumptions. The model is designed to be used in a scenario-writing process, and it is therefore very important to be able to vary the assumptions governing behavior in the oil market.

This can be achieved in two ways:

- by varying the simulation parameters. (This can be done either before or during a model run, since the model operates in an interactive mode.)
- by multiple iterative model runs.

The simulation parameters provided in the model include:

x_i^{min} - minimum admissible oil export/import strategy of trader i $(i = 1,2,...,n)$

x_i^{max} - maximum admissible oil export/import strategy of trader i $(i = 1,2,...,n)$

$i_k = (k = 1,...1)$ - monopoly-like trading coalitions (importers or exporters)

The first two parameters can be used to narrow or extend the admissible range of oil exports/imports for any region or to check a fixed strategy. The third type of parameter makes it possible to examine the effect of complete unity among certain traders--in the model, this type of coalition implies the aggregation of the regional functions into a single set and the allocation of oil exports/imports (and profits) among members according to IE criteria.

The simulation parameters can also be varied between iterative runs of the model - this is a very powerful means of analysis but very difficult to generalize in an introductory paper of this nature.

Some examples of the possible use of the algorithm are outlined in the following sections. Note that any combination of these uses is also possible.

2.5.1. Different relations among exporters/importers

(i) Monopoly-like coalition

There may be subgroups of exporters or importers with similar PCFs and DCFs and similar macroeconomic and political goals. In this case, the relations among the regions forming the subgroup can be approximated by those inherent in a monopoly-like coalition, i.e.,

- united efforts to set the price that yields maximum total revenues/economic effect for the coalition;
- rationing of oil exports/imports and profits on some proportional basis.

This case is simulated in SMIOT simply by introducing an additional trader--a monopoly--and referring to the participating regions as the members of this coalition. In doing this, SMIOT generalizes the functions of these regions into a single set (that is, it considers the coalition as a single trader) before making interregional calculations, and allocates oil exports/imports among the members on some proportional basis after these calculations have been carried out. There can be up to three non-intersecting monopolies in the model at the same time.

(ii) Individual actions

The model can also analyze individual actions other than negotiating with other traders or entering a coalition. These actions can be classified according to the information held by one trader concerning the market positions of other traders. The following cases can be simulated:

1. No information is available. In this case, the outcome is given by the CE solution for the individual trader. (This is obtained by assuming $x_i^{min} = x_i^{max} = x_i^{max}$ for all individual traders.)

2. Information on the admissible ranges of other traders' strategies is available. This situation leads to a minimax solution for a given trader, and is simulated by a two-step use of the model. In the first step, the trader searches for its best strategy x_k^* (that is, the strategy that maximizes its profit) under the most unfavorable assumptions concerning other traders' strategies--in this case, under the assumption that $x_i^{min} = x_i^{max} = x_i^{max}$ for all $i \neq k$, where k represents the trader under consideration. In the second step, the strategy x_k^* is fixed, the assumption $x_i^{min} = x_i^{max} = x_i^{max}$ $(i \neq k)$ is replaced by other assumptions (which depend on the objectives of the researcher), and the model is rerun.

3. Information on the likely subrange $\hat{x}_i^{min} - \hat{x}_i^{max}$ or strategy $\hat{x}_i$ of other traders (within the admissible range) is available. This situation is simulated as described in the preceding paragraph.

Note that these cases can be analyzed either for exporters or for importers but not for both at the same time.

2.5.2 Evaluation of fixed oil import/export strategies

One of SMIOT's major uses will lie in the evaluation of the internal and external impacts of given regional oil import/export strategies. Under a given set of assumptions concerning oil market conditions, the model can evaluate a strategy x_k^* for region k, assuming $x_k^{min} = x_k^{max} = x_k^*$, and compare the solution with those obtained using other assumptions.

3. BRIEF OUTLINE OF SMIOT AND ITS TEST RESULTS

3.1. Basic features of SMIOT

The basic features of SMIOT are as follows:

1. SMIOT is an aggregated one-product model that makes all its physical evaluations in terms of standard barrels of crude oil equivalent. The barrel value of different kinds of liquid fuel is calculated by taking comparable input cost figures on crude oil production from different sources.
2. Underlying the model is a belief in the power of market forces to determine the international price of oil over the long term - this is consistent with the anticipated increasing elasticities of liquid fuel demand and domestic supplies in the oil-importing countries. As a consequence, the model yields the same price for oil in all parts of the world.
3. SMIOT simulates the process of trading in the world oil market as a game involving a number of countries or groups of countries (regions). The number of players is limited to seven (in line with the general IIASA approach to the energy problem described in Häfele, 1981). However, the composition of these regions may vary depending on the problem considered.

4. SMIOT is a time-step model. It provides for a maximum of eleven equal time intervals, the size of which can be selected by the researcher. We are currently using ten five-year periods spanning the period from 1980 to 2030. The solution (in terms of oil prices and regional oil production and consumption levels) for each period depends on the results obtained in preceding periods and influences those of later periods. The time relationships introduced into the model describe the depletion and changing costs of natural oil resources, the growth and price elasticities of the demand for liquid fuels, the dynamics of existing and new liquid fuel production capacities, and the changing production costs of unconventional liquid fuels.

5. All cost-versus-quantity characteristics used in the model are described in piecewise linear form as a means of taking into account the non-linearities in the development of liquid fuel demand and supply.

6. In its interactive mode, SMIOT allows variations in certain critical factors in regional and interregional oil trade - the constraints limiting the amounts of oil imported or exported by each trader, the extent of the traders' uncertainty about each other's policies, and the number, membership, and character of coalitions. In this way it is possible to check a spectrum of hypotheses concerning regional positions in the oil market and their effects on patterns of world oil trade.

7. SMIOT is represented schematically in Figure 11. At each time step the model first calculates curves of regional liquid fuel demand and supply versus cost, which reflect the changing domestic potential of production, substitution, and conservation of liquid fuels as costs increase. Taken together, these curves define the regional demand for imported oil or the availability of oil for export at varying costs. These characteristics serve as immediate input data for interregional oil-balancing calculations.

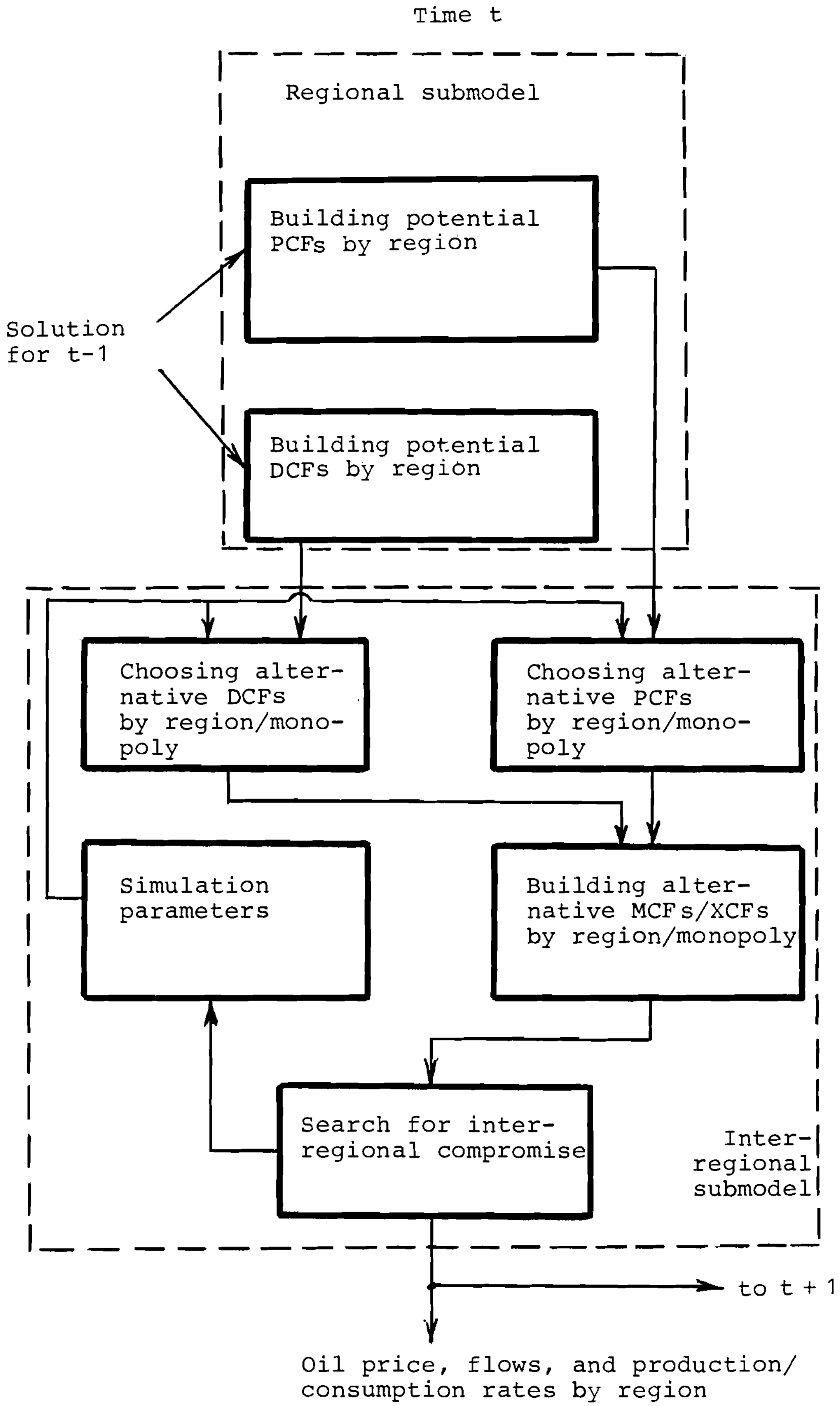

Figure 11. Schematic diagram of SMIOT.

8. SMIOT considers the regional primary demand for liquids in up to 6 consuming sectors. The input information required for each sector includes:

   - a "reference" estimate of liquid fuel demand over time, showing demand at a reference price;
   - parameters describing the price elasticity of demand.

   These data illustrate the dependence of sectoral demand for liquid fuels on cost, for a certain range of demand variations over time. However, the size of the demand range considered at each step depends on the demand for oil obtained in previous time intervals. Using this information, SMIOT then defines total regional demand as a function of cost. Having carried out the interregional calculations, the model produces a price-consistent set of regional demands for liquid fuels, both by consuming sectors and for the region as a whole.

9. SMIOT distinguishes between 7 different sources of liquid fuel for each region. The model takes into consideration:

   - primary oil resources classified by type available at varying costs;
   - probable rates of addition to existing reserves of conventional oil, heavy oils, deep offshore oil, and oil made available through use of enhanced recovery techniques;
   - constraints on the rate of development of unconventional liquid fuel supplies (including coal liquefaction) as a function of the rates attained in previous periods;
   - resource-to-production ratios for the development of conventional and deep offshore oil resources, oil available through the use of enhanced recovery techniques, and heavy oils accessible by conventional methods; also rates of production from sources developed in previous periods;
   - changes in the cost of producing oil from tar sands, shales, and coal over time.

10. Two other features of the model are particularly noteworthy:

- SMIOT can take into account the scarcity cost (defined endogenously) of depleting the oil resources in each region;
- the time-lag between price rise and regional demand and/or supply response (up to 15 years) is included in the model.

3.2. Illustration of the results obtained with SMIOT

The purpose of this final section is to illustrate the way in which the gaming algorithm works in practice, by giving the results of a few SMIOT runs. To do this, we shall consider one of the several demonstration scenarios built with the help of SMIOT. All of the assumptions made in this scenario are consistent with those adopted in the oil analysis carried out by the IIASA Energy Group; these are outlined on pp. 533-546 of the IIASA global energy study (Häfele, 1981). The major features of this scenario are as follows:

1. The IIASA Region VI (the Middle East and North Africa) was subdivided into two regions (VIA and VIB) in order to analyze the possible global consequences of different types of coalitions between exporters.
2. The IIASA High scenario was taken as the reference case; in addition, an oil demand-to-price elasticity of 0.4 was introduced for all regions.*
3. The estimates of ultimately recoverable resources of conventional oil, and of the maximum build-up rates of production from unconventional sources were somewhat less optimistic than in the IIASA study. At the same time, the costs related to liquid fuel production were assumed to be higher than in the IIASA study (in line with the latest published estimates).

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*This means that a 1% increase in the price of oil would result in a 0.4% decrease in oil consumption.

Figures 12, 13, and 14 illustrate the SMIOT estimates of interregional oil prices, price-consistent total oil exports from Regions VIA and VIB, and total oil revenues earned by these regions, for the period 1980-2030. These estimates were obtained under four different hypotheses concerning the behavior of the oil-exporting regions, namely:

- ideal market equilibrium;
- dominance of exporters over importers through a monopoly-like coalition;
- dominance of exporters over importers through a cartel-like coalition;
- exporter-dominance through a cartel-like coalition, as above,but with the additional assumption that Regions VIA and VIB have a total production ceiling of 30 million barrels per day through 2030.

Although these estimates are only the results of demonstration runs and do not permit any far-reaching practical observations concerning long-term trends in world oil trade, they do suggest certain methodological conclusions. As can be seen from Figures 12-14, market equilibrium leads to a low price in the short run which results, first, in accelerated depletion of conventional oil resources in Regions VIA and VIB and, second, in slower development of unconventional liquid fuel production technologies. This forces the price upward in the long run.

In the case of a monopoly, the price is high from the very beginning and serves as an incentive for the rapid introduction of new technologies in importing regions, thus preparing for a shift away from oil imported from regions VIA and VIB in the long run. This results in the price of oil being much lower in the long term than in the case of market equilibrium.

As can be seen from the diagrams, the cartel situation leads to a price between the two extremes. It entails more moderate use of oil from the exporting regions, as well as development of unconventional technologies.

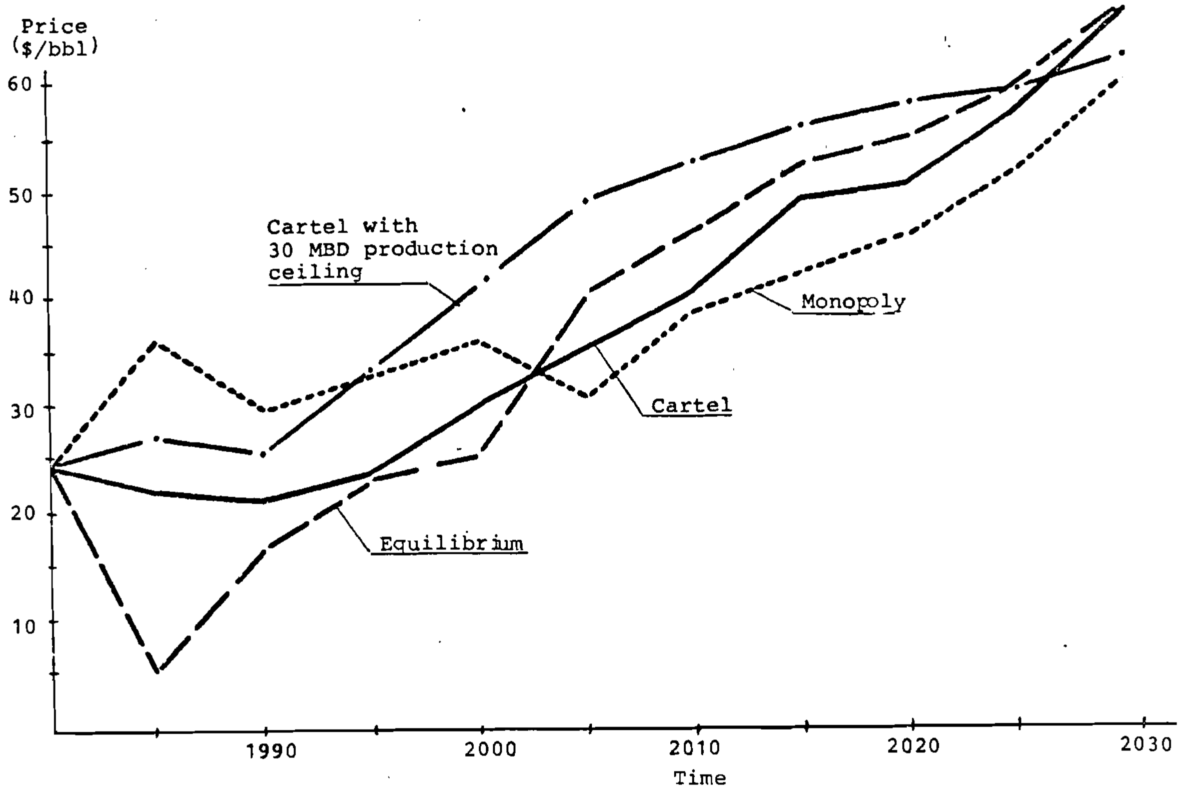

Figure 12. Oil price estimates, 1980-2030, under different hypotheses concerning the behavior of exporters.

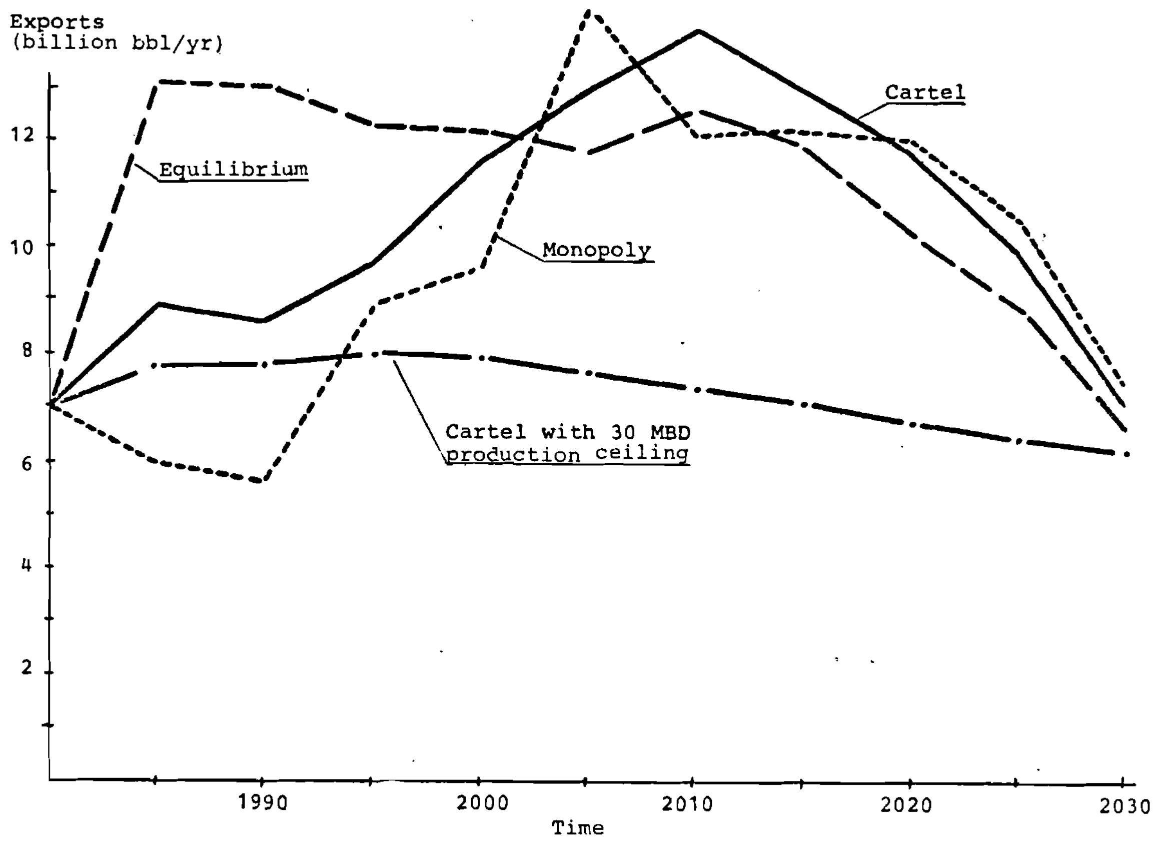

Figure 13. Price-consistent export policies, 1980-2030, under different hypotheses concerning the behavior of exporters.

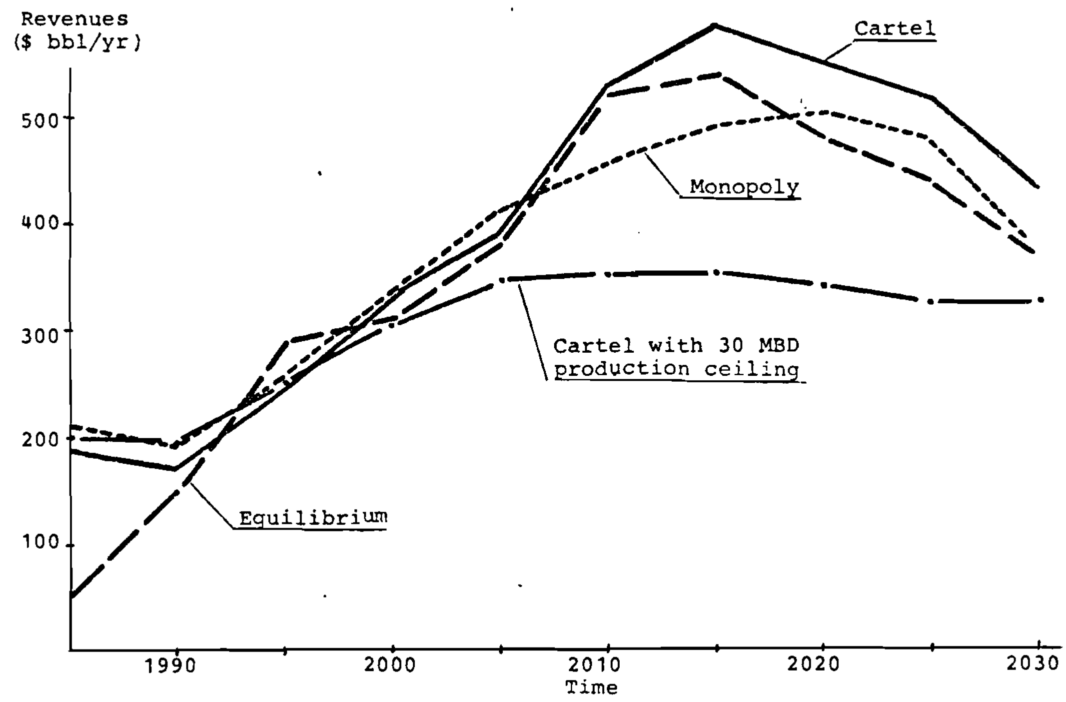

Figure 14. Revenues of oil exporting regions, 1995-2030, under different hypotheses concerning the behavior of exporters.

From the viewpoint of the major oil producers, the cases described above are not very realistic since they do not take into account the resource-saving policy pursued by the exporters and result in oil revenues which may be difficult to absorb. Introduction of an oil production ceiling for these regions changes the picture considerably. This assumption yields the highest price in the short term and allows regions VIA and VIB to keep revenues at a reasonable level while exporting much less oil. At the same time, it is found that the rate of development of unconventional oil technologies in importing regions is even higher than in the case of a monopoly - this decreases the rates of price increase over the long term.

4. CONCLUDING REMARKS

A model of a system is always less detailed than the system itself. This is especially true if the system, like the world oil market, contains informal relationships which are difficult to quantify. Thus, because models are only poor reflections of reality, they should not be expected to produce results which may be used immediately by decision-makers. Rather, they should be viewed as an auxiliary, though powerful, means of increasing the ability of researchers to analyze and generalize the behavior of systems under different external and internal conditions, and, therefore, as a tool providing a more substantive basis for making decisions. Our model is no exception to this general principle.

This paper describes only the most recent version of the model, which has a number of shortcomings. These shortcomings are recognized and work on the final version of the model is continuing.

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