SOME APPLICATIONS OF STOCHASTIC METHODS IN INVESTIGATIONS ON THE DYNAMICS OF ECOSYSTEMS

V.A. Svetlosanov

May 1982
WP-82-38

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

Deterministic models are traditional in the investigations conducted in ecological studies. In some cases, they are sufficient for dealing with the problems which arise, but in others they cannot even describe certain phenomena which occur in natural systems. The application of pure stochastic methods leads to extraordinary mathematical difficulties and in many cases is almost impossible. A compromise is needed, especially for asymptotic cases. In this paper, some possible methods are put forward for describing the phenomena, which are elusive when a purely deterministic approach is used.

ABSTRACT

This paper describes some possible applications of stochastic methods which may be used in ecological studies. The role of stochastic methods in investigations of the dynamics of ecosystems is gaining in importance. It is a new trend which has arisen in ecological studies, related to the development of methods for the control of the environment. Stochastic methods are very useful for investigating the stability of ecosystems and the criteria of stability of natural systems, especially where the influence of permanent small-scale random disturbances have been noticed. Some criteria have been suggested and examples of use of these criteria are given in this paper. It must also be noted that the complex problems of predicting and controlling processes in natural systems must be solved by mathematical tools which permit analysis of anthropogenic factors without in-situ experimentation.

TABLE OF CONTENTS

SOME APPLICATIONS OF STOCHASTIC METHODS IN INVESTIGATIONS ON THE DYNAMICS OF ECOSYSTEMS

V.A. Svetlosanov

INTRODUCTION

The problem of interaction between man and the environment has gained in importance among the present scientific and technological problems. The depletion of natural resources, pollution of the atmosphere, soil, seas, and oceans, upsetting the biological balance--these and other factors create problems which have to be urgently solved. It is evident that the problems arising are complex, and their solution calls for joint efforts by specialists from many countries and scientists in different fields of research. The main task when studying the dynamic processes of the environment is to predict the effect of today's factors on its future state. Quantitative prediction necessitates dynamic mathematical modelling. This paper considers the use of stochastic models of ecosystems, especially in cases where different variants of the system's development depend on many parameters, including man's activity.

DETERMINISTIC AND STOCHASTIC APPROACHES

All currently used models of environmental processes can be classified into four groups: (1) static deterministic, (2) static stochastic, (3) dynamic deterministic, and (4) dynamic stochastic.

The majority of mathematical models are static and belong to the first two groups. Many processes in ecology are dynamic and the development of the third and fourth group of models is becoming useful. Since most of the ecological processes are stochastic, the fourth group of models--dynamic stochastic models--is of particular importance. The first three groups may be regarded as auxiliary in the development of models for studying ecological processes.

Any regular dynamic process is characterized by random deviations. Each process differs from the other. However, in certain cases, one can ignore fortuitous elements, leaving only the major factors affecting a process, i.e., a deterministic dynamic model of a phenomenon is provided for its analysis. Such models reveal the basic regularity inherent in the phenomenon and permit prediction (on the average) of the system's development, proceeding from the initial conditions. The progress of science makes it possible to increase the number of factors, enabling a more accurate prognosis. Theoretically, prediction in each problem can be made more accurate by the gradual introduction of new groups of factors: from the essential to the insignificant stage. Practical experience rules out such an approach, for it unduly complicates a problem and renders analysis of the effect produced by the factors involved in prediction more difficult. By applying systems analysis for solving this problem it is possible to distinguish between primary factors which determine the dynamics of a process on the average and secondary factors which are regarded as "disturbances". When a given process is examined in depth, there is always a moment when the investigator must not only identify its basic regularities, but also analyze possible deviations from them. This is where dynamic stochastic methods must play a decisive role.

STABILITY OF ECOSYSTEMS

In recent years, a great deal of attention has been given to the problem of stability of ecosystems. Holling (1973) advanced the idea, that natural ecosystems possess two characteristics, resilience and stability and gave them qualitative definitions. He pointed out that ecosystems can have several equilibrium positions and under disturbances can go from one position to another, and gave some concrete ecological examples of such situations.

A special question arises here: how can one calculate the system's transition from one position of equilibrium to another? The answer to this question is connected with the stability ecosystem. Stability may be quantitatively defined by introducing the criteria of ecosystem stability. Stability is one of the fundamental concepts as regards the development of complex natural systems. This has become a topical problem in view of the tremendous impact of man on ecosystems. Research in this direction enables one to set forth stability criteria for ecosystems affected by man and to determine the maximum permissible loads on them. The use of stochastic dynamic models must contribute to the correct solution of a given problem.

Studies on the stability of natural systems may be divided into two main categories. The first category includes modeling of natural systems and determination of the stability of such model systems to various disturbances. The second category involves attempts to find a characteristic in an ecosystem that would be responsible for the stability of the system as a whole. Such a characteristic is generally assumed to be a function of variables that can be measured easily. Measurements of the characteristics of various natural systems produce a number of comparisons which reveal their relative stability.

Perhaps Mac-Arthur (1975) was the first who tried to confront the stability of natural system with the number of relationships inside the system. In order to describe the stability s of the association, he suggested the following eutrophy formula:

$$s = - \sum_{i=1}^{n} p_i \log p_i \quad , \qquad (1)$$

where p_i is the relative number of the species.

The general idea of this formula is that the more relationship there is inside the system, the more stable it becomes. This formula gives a chance to calculate very quickly the measure of stability of the associations. But this is a simplified approach; it does not consider the structure of the systems, the type of disturbances and their quantitative values. All these characteristics are however responsible for the stability of the associations.

Another approach is possible, which is a synthesis of the first two. It consists of constructing a mathematical model and studying the effects of disturbances on the dynamics of development of a given system. As a result, in some cases, the investigator knows which factors and which functional relationships are responsible for the stability of the system under consideration. This approach has been developed to assess the effect of small random disturbances on the stability of natural ecosystems.

The above mentioned first category presupposes the existence of a mathematical model describing the dynamics of natural systems. The models may be deterministic and stochastic, but the use of purely stochastic models involves serious mathematical difficulties. In some cases, the effect of influences can be determined only by means of deterministic models. Let us consider such a case as in the classical studies concerning the stability of solutions of ordinary differential equations applied to small step unit disturbances. Liapunov's (1950) method permits examination of the stability without solving the equations describing a model, by resorting only to the coefficients of the model equations. Please note that a step unit disturbance is nothing but a particular case of the possible disturbances affecting a natural system.

POSSIBLE DISTURBANCES IN NATURAL SYSTEMS AND SOME ESTIMATION OF THEIR INFLUENCES ON THE STABILITY OF THE LOGISTIC CURVE

It was mentioned earlier that especially the stability of the ecosystems depends on the disturbances affecting them. Among all the possible disturbances, we pay attention to the following six groups:

1. Step unit short-duration low-amplitude pulses
2. Step unit short-duration high-amplitude pulses
3. Periodic and nonperiodic pulses of different amplitudes
4. Permanent small-scale random disturbances
5. Permanent large-scale random disturbances
6. Disturbances affecting the parameters of natural systems (structural changes in the system).

Take an ecological example with the consideration of the disturbances influencing the system. The logistic curve (or the Ferchulst curve), which describes the growth of the number of population species N, is well known in ecology. The logistic curve is very convenient for analysis because it is simple and the results are obtained in an analytic form.

This logistic curve (Figure 1):

$$N = \frac{\alpha/\beta}{1 + [\alpha/\beta N_o - 1]e^{-\alpha t}} , \qquad (2)$$

represents the solution of the differential equation:

$$\dot{N} = \alpha N - \beta N^2 , \qquad (3)$$

where α is a coefficient characterizing the difference between birth and death of the species and β is a coefficient of intraspecific competition.

This system has two stable positions of equilibrium-- $N_1 = \alpha/\beta$ and $N_2 = 0$/biologically stable/. As can be seen from Figure 1, in the course of time, the curve approaches the asymptotic stable equilibrium position, $N_1 = \alpha/\beta$.

Suppose the number of species N is near to the position of equilibrium $N_1 = \alpha/\beta$, it is easy to see that this population system is stable to the first group of disturbances. The step unit short-duration low-amplitude pulse causes the system to deviate from the equilibrium position, but the system will return to the position of equilibrium (Figure 2). It is obvious that the disturbance of the second group can destroy the system

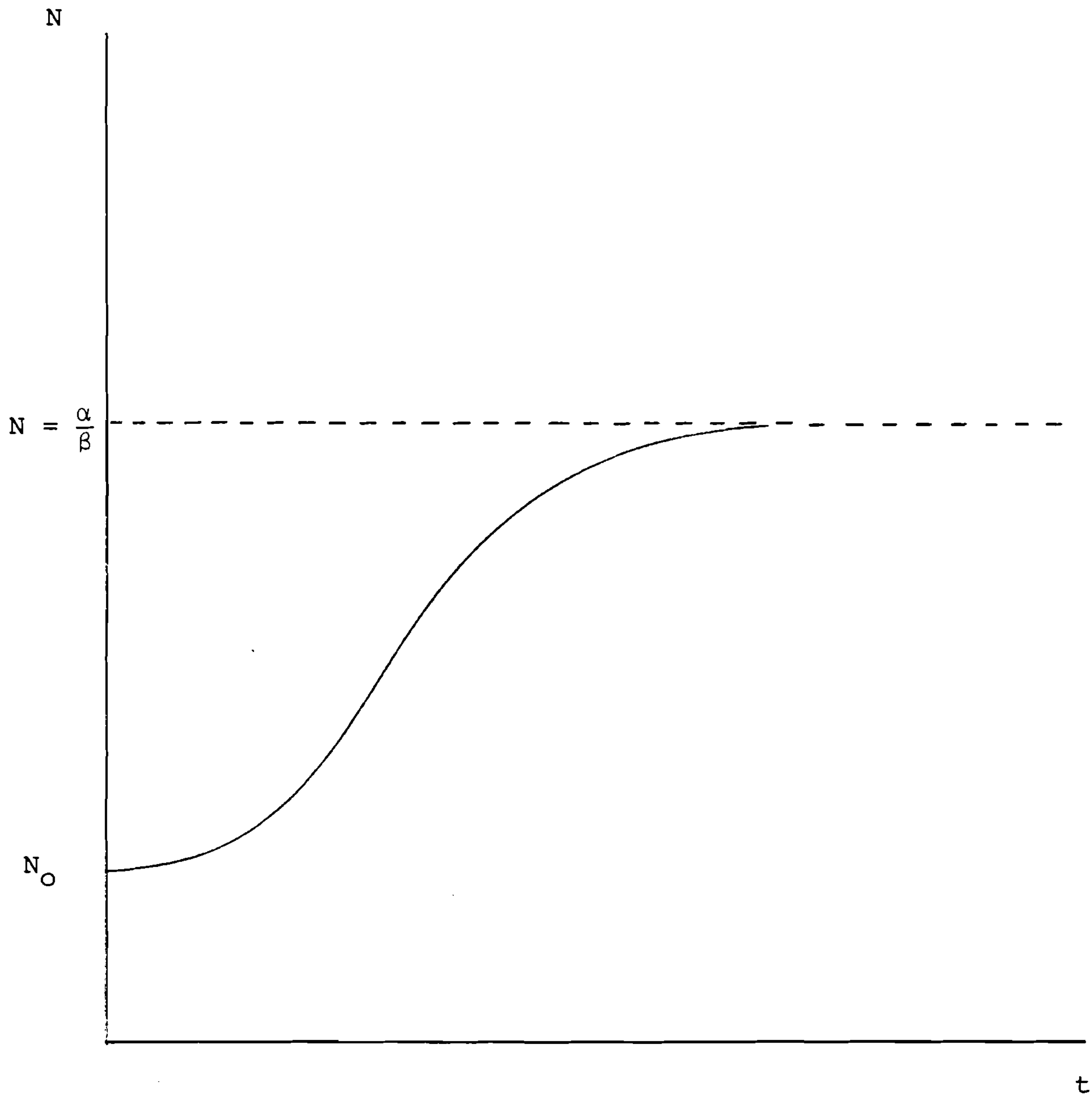

Figure 1. Logistic Curve of Population Growth (the Ferchulst curve)

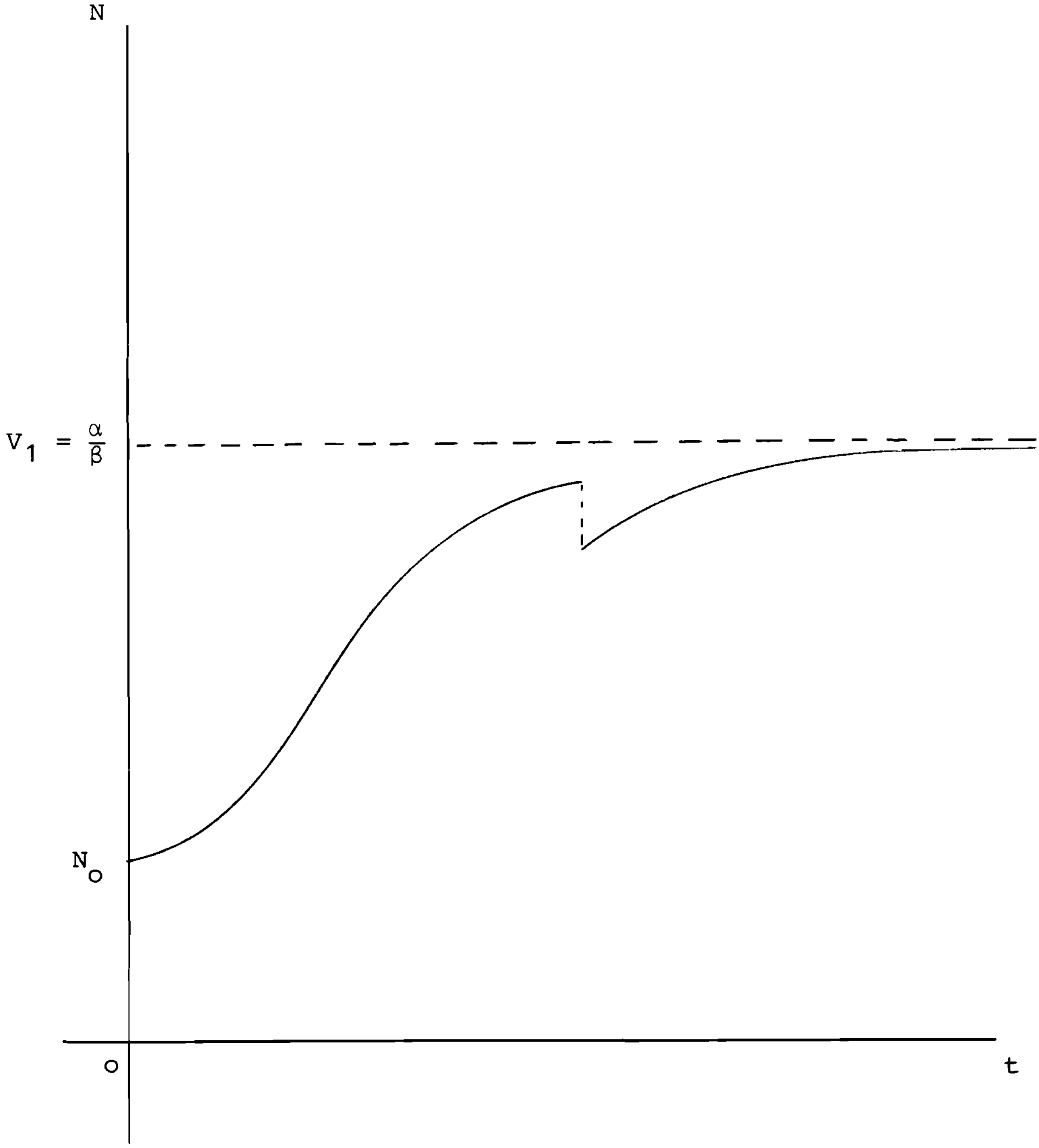

Figure 2. The Reaction of the Logistic Curve to the Step Unit Short Duration Low Amplitude Disturbances

which is described by the Ferchulst curve, if the numerical value of the disturbances exceeds α/β. The Monte Carlo method can be used to evaluate the results of disturbances from the third group.

Of special interest is the consideration of the result of permanent small-scale disturbances (Group 4) because this type of disturbances very often exists in real systems. Later on, we consider the mechanism of the evaluation of ecosystems stability to this kind of disturbances, but now we give only the result of the stability calculation of the Ferchulst curve to the disturbances which can be described as "white noise". In this case, the measure of stability of the logistic curve to this type of disturbances depends on the combination of the coefficients α and β and equals $\frac{\alpha^3}{\beta^2}$. Considering different populations with different coefficients α and β one can see that the system is more stable than for the higher combination $\frac{\alpha^3}{\beta^2}$.

MEASURE OF STABILITY FOR THE PERMANENT SMALL-SCALE RANDOM DISTURBANCES OF THE "WHITE NOISE" TYPE

Let us now consider a method for describing the dynamics of natural systems taking due account of permanent small-scale random disturbances (Group 4). We shall assume that the random disturbances are small in a statistical sense, as compared to deterministic components. We believe that in real natural systems, in many cases, there are such disturbances. Let a system have several stable equilibrium positions; if we had applied a deterministic approach, the natural system would, during that time, have come closer to one of the equilibrium positions and stayed there indefinitely. Let the dynamics of the natural system components be described by the equation:

$$\dot{x} = b(x) \quad , \qquad (4)$$

where x is a vector, if the logistic curve $X = N$, $b(x) = b(N) = \alpha N - \beta N^2$.

Suppose the system has different states of equilibrium, and if equation (4) describes the evolution of the natural system precisely, then the system would come to one of the states of equilibrium and be there for an indefinite period of time. But, in fact, the natural system is under the effect of irregular random disturbances. These disturbances may be described by random processes. Assuming that the random process is the "white noise" type (W_t), then the dynamics of the natural system will be described by equation:

$$\dot{x}^{\varepsilon} = b(x^{\varepsilon}) + \varepsilon \dot{W}_t \quad , \tag{5}$$

where ε is a parameter which characterizes the small disturbances as compared with the vector $b(x)$, W_t--Winer process.

Such a stochastic approach gives a chance to observe some phenomena which are natural in the systems and which cannot be described by the deterministic approach. Using the model (5), it is possible to calculate the quantitative characteristics of the above mentioned pheonomena.

Assume the system is near the stable position of equilibrium O_1 and is affected by random disturbances. Note that Π_1 is the sphere of attraction of the point O_1. The stability of the system near the point O_1 may be characterized by the average time which the system needs to leave the sphere of attraction Π_1. For concrete realization, the time it takes for the system to leave the stable position is a random value. Let us note it as τ_1^{ε}. In order to find the expected value at the time of residence of the system $M_x\tau_n^{\varepsilon}$, it is possible to formulate the task for function $n^{\varepsilon}(x) = M_x\tau_n^{\varepsilon}$. This task is very complicated and the analytical decision is very difficult and in many cases impossible, since the stability of the stable position depends on the initial point. It is possible to simplify the task and in the case of a small value ε it is natural to introduce a dominant term of the expected value at the time of residence of the system ($M_x\tau_n^{\varepsilon}$), (Ventzel and Freidlin, 1979) in the n-th surrounded region of the stable equilibrium position:

$$M_x \tau_n^{\varepsilon} \sim \exp\frac{C_n}{2\varepsilon^2} \quad . \tag{6}$$

As we can see from equation (6), the time of residence of the system to be in the n-th region of the stable equilibrium position depends on the funtion C_n. Therefore, it is proposed to use C_n as a measure of stability for the permanent small-scale random disturbances. The numerical values of C_n depend on the type of functions b(x) and on the n-th equilibrium position.

It is easy to find constants C_n when the b(x) field is one-dimensional. According to Ventzel and Freidlin (1979) the function C_n is closely connected with the quasi-potential U(x) of the field b(x)

$$C_n = 4 \times U(x_{n+1}) \tag{7}$$

where x_{n+1} is the position of equilibrium where the system will be after a certain period of time,

where quasi-potential

$$U(x) = - \int_o^x b(x)dx \quad . \tag{8}$$

In the initial position of equilibrium (x_1) this quasi-potential is equal to zero (Ventzel and Freidlin, 1979). This enables calculation of B the constant of integration

$$O = U(x_i) = - \int_o^{x_i} b(x_i)dx + B \quad . \tag{9}$$

In the case of the multidimensional size of the field b(x) the more difficult problem of the calculation of the constants C_n must be solved. In this case, the values C_n are the minimum of the functional τ which is defined as follows:

$$\tau = \int_{T_1}^{T_2} \sum_{i=1}^{n} [x_i - b(x_i)]^2 dt \tag{10}$$

where x_i is the i-th component of the vector x,

(T_2-T_1) - period of time when the system goes from one position of equilibrium to another.

The knowledge of the values of C_n makes it possible to calculate the probabilities of transition from the neighborhood of one stable position to another. Note that the transition from the neighborhood of one stable position to another is impossible to calculate with a purely deterministic approach.

CONCRETE EVALUATION OF STABILITY CRITERIA FOR PERMANENT SMALL-SCALE RANDOM DISTURBANCES

We stated earlier that the value $\frac{\alpha^3}{\beta^2}$ characterizes the criteria of the stability of the permanent small-scale disturbances for the logistic curve. Now we shall give the calculation of this value.

The number of population growth species N which is subject by random process of the "white noise" type (W_t) will be described by equation

$$N^{(\varepsilon)} = \alpha N^{(\varepsilon)} - \beta N^{(\varepsilon)^2} + \dot{W}_t \quad . \tag{11}$$

In this case, the transition from one position of equilibrium ($N_1 = \alpha/\beta$) to another position where the population is zero, is possible. Figure 3 shows one possible realization of the process. Now let us calculate constant C_1--a measure of stability, using the equations (4)-(9).

For this purpose, as mentioned earlier, first of all one must find U(N)--the quasi-potential of the field $\alpha N - \beta N^2$

$$U(N) = \int_0^N (-\alpha N + \beta N^2)\, dN = -\frac{\alpha N^2}{2} + \frac{\beta N^2}{3} + B \quad . \tag{12}$$

In the equilibrium position of the system ($N_1 = \frac{\alpha}{\beta}$), the quasi-potential of the field must be equal to zero (Equation 9)

$$U(\frac{\alpha}{\beta}) = -\frac{\alpha^3}{2\beta^2} + \frac{\alpha^3}{3\beta^2} + B = 0 \quad .$$

From here we can find the meaning of constant B

$$B = \frac{\alpha^3}{6\beta^2} \quad . \tag{13}$$

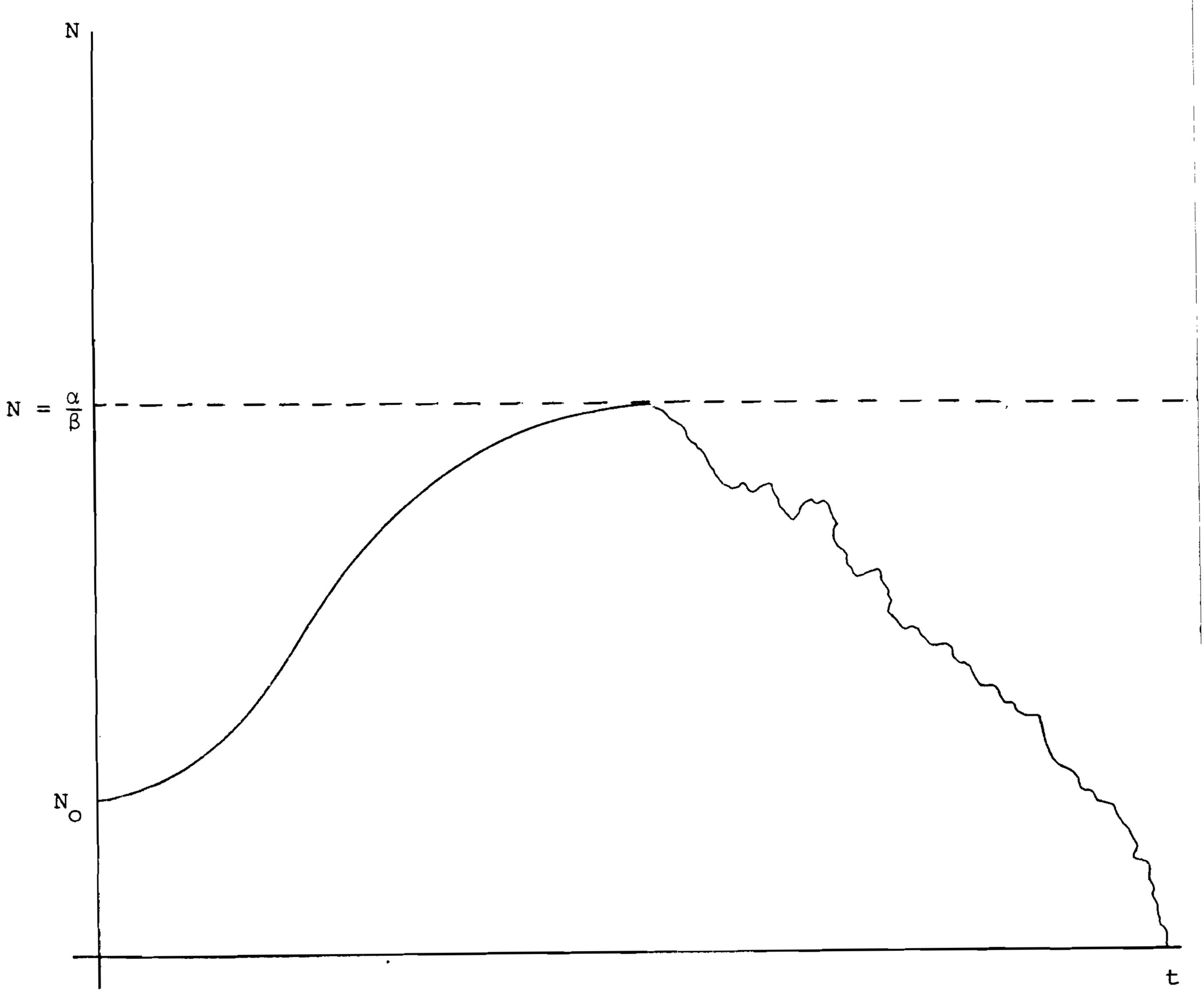

Figure 3. Possible Realization Process of the Death of the Natural System's Component

The above mentioned constant C_1 may be found, when the second position of equilibrium of the system ($N_2 = 0$) is considered (Eq.7):

$$C_1 = 4 \times U(0) = \frac{2}{3} \cdot \frac{\alpha^3}{\beta^2} \quad . \tag{14}$$

Therefore, the dominant term gives the following analytical expression for mathematical expectation (Freidlin and Svetlosanov, 1976),

$$M_x \tau^\varepsilon \sim \exp\frac{C_1}{2\varepsilon^2} = \exp\frac{\alpha^3}{\varepsilon^2 \cdot 3\beta^2} \quad . \tag{15}$$

It is easy to see, when ε is small, the value mentioned below is great:

$$\lim M_x \tau^\varepsilon \to \infty \quad . \tag{16}$$

The measure of stability of the equilibrium position is α^3/β^2, so we can see that the greater the value α^3/β^2, the higher the stability of the system described by the logistic curve (Freidlin and Svetlosanov, 1976). This example shows how the small disturbances can "swing" and even destroy the system which looks very stable for an indefinite time, without consideration of the disturbances. Note that the effect of the "death" of this component of the natural system is impossible to find using the deterministic approach.

Consider now the results of studying the stability of Haefele's (1975) mathematical model under the effect of random disturbances of the "white noise" type (Svetlosanov, 1977). Haefele has proposed a deterministic model representing the relationship between population growth and the energy potential per population unit. The model is described by the following system of differential equations:

$$\frac{de}{dt} = \mu A e^{1/2} - \mu c e^3 - e\delta + \frac{Ke^2}{p} \tag{17}$$

$$\frac{dp}{dt} = \delta p - Ke \quad .$$

Here, p is the population and e is the energy potential per population unit. Figure 4 represents the development of the system in a phase plane. Separatrices divide the phase plane into four parts. If evolution started in one of the four parts, it will always be confined within that part. If it starts in Part II or III, the community will become extinct after a certain period of time. In order to exist, the community must stay in Part I or IV.

Consider the evolution of a given dynamic system, taking into account the effect of small random disturbances of the "white noise" type. The system will be described by the system of differential equations:

$$\frac{de}{dt} = \mu A e^{1/2} - \mu c e^{3} - e\delta + \frac{Ke^{2}}{p} + \varepsilon \dot{W}_{t}$$
$$\frac{dp}{dt} = \delta p - Ke + \varepsilon \dot{W}_{t} \quad . \tag{18}$$

Under the effect of a random disturbance, the system may shift from one point of the phase plane to another along different paths, but there is always curve ξ_t along which the shift is most probable. The curve shape, mathematical expectation, as well as the probability of shift transition from one point of the phase plane to another during time interval $T = T_2 - T_1$, can be derived (according to equation 10) while calculating the minimum of functional:

$$\tau(\xi_t) = \int_{T_1}^{T_2} [(\dot{e} - \mu A e^{1/2} + \mu c e^{3} + e\delta - \frac{Ke^{2}}{p})^{2} + (\dot{p} - \delta p + Ke)^{2}]dt \quad . \tag{19}$$

The main term of probability transition P takes the form:

$$P \sim \exp\{-\frac{\min \tau(\xi_t)}{2\varepsilon^{2}}\} \quad . \tag{20}$$

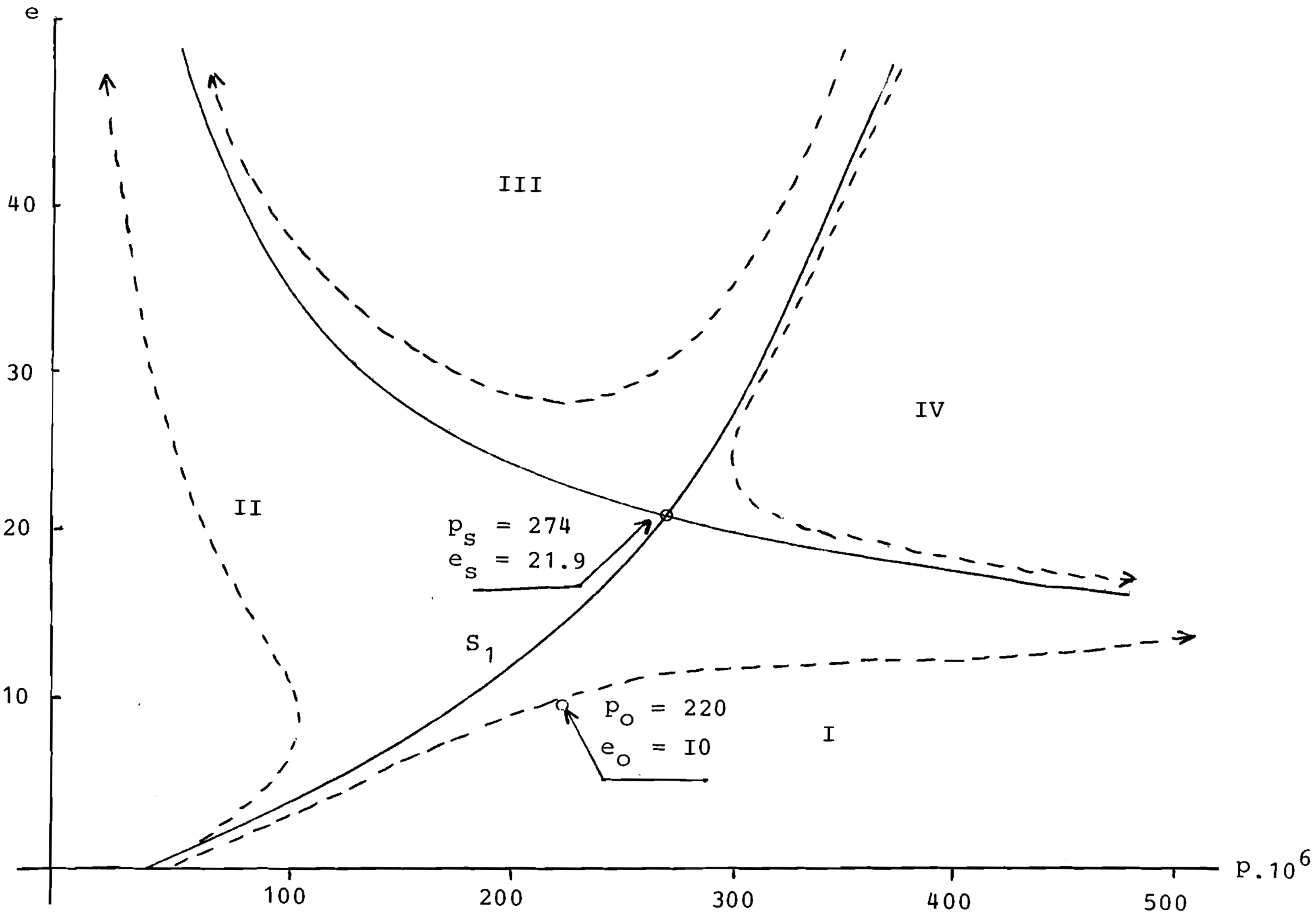

Figure 4. The Growth of Population as a Function of Unit Energy Potential (the Haefele Model, 1975)

Calculation of the minimum of functional $\tau(\xi_t)$ gives the same constant C which is a measure of stability of the system to random disturbances. The calculations of initial values $e_o = 10$, $p_o = 2.2.10^8$ and final values $e_f = 21.9$, $p_f = 2.74.10^8$ have given min $\tau(\xi) = 4.77$ with other initial conditions $e_o = 15$, $p_0 = 3.10^8$ and the same final conditions, $\tau(\xi) = 12.49$. Let us note that the final conditions were the values of the saddle-point. The time horizon was taken equal to 10^3 years.

Let us consider from this point of view the problem of soil degradation. Degradation and instability are often closely correlated. Suppose we have the differential equations which describe the process of soil degradation; first of all, we are to clarify the type of disturbances which influence the soil. If the random process is a "white noise" type, we can use the above mentioned methodology to calculate the criteria of stability of agroecosystems. Knowing this, we can calculate the "life-time" of agroecosystems and evaluate the extent of soil degradation. In the case of other different types of disturbances, we can use (it depends on the situation) Liapunov's method or the Monte Carlo method to calculate whether the system is stable or not. So far, no analytical criteria have been determined for disturbances of Groups 3, 5 and 6. Calculations can be carried out using the Monte Carlo method.

The use of stochastic dynamic methods is not restricted to studies of natural system stability. They were used, for example, to predict the boundary configurations of forests invading the steppes. The interrelations between forest and steppe are quite complex. An analysis in the Streletsky sector of the Kursk steppe has revealed steady encroachment of forests upon the steppes. The statistical probability of emergence of a new tree depends on the distance between trees. The use of this relation in the Monte Carlo method has enabled the boundary configuration of the forest to be predicted fifty years ahead (Andreev et al., 1976).

CONCLUSIONS

Stochastic methods can afford to evaluate some effects in natural systems which are impossible to find by using only a purely deterministic approach. A stochastic approach may be actively used in studying the present actual problem--the stability of natural systems, especially with an idea of obtaining the criteria of system stability.

As the problems of soil degradation and agroecosystem stability are very closely connected to each other, stochastic dynamic models should be used to analyze them.

REFERENCES

Andreev, Yu. B., A.M. Krasnitsky, V.A. Svetlosanov and V.D. Utechin (1976). Prognosis of the Encroachment of Vegetation on the Steppes as a Result of Model Calculations. In: Biota Osnovnich Geosystem of Zentralnoy Lesostepy (in Russian).

Freidlin, M.J., and V.A. Svetlosanov (1976). On the Influence of Small Random Disturbances to the Conditions of Equilibrium of Ecological Systems. Obschaya Biologia N5 (in Russian).

Haefele, W.A. (1975). Objective Functions. Working Paper WP-75-25. International Institute for Applied Systems Analysis, Laxenburg, Austria.

Holling, C.S. (1973). Resilience and Stability of Ecological Systems. Research Report RR-73-3, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Liapunov, A.M. (1950). A General Task of the Stability of Motion. Moscow: Gostechisdat (in Russian).

Mac-Arthur, R.H. (1955). Fluctuation of Annual Populations and a Measure of Community Stability. Ecology, N36.

Svetlosanov, V.A. (1977). The Calculation of System's Stability Measure to Random Perturbations - Isvestia Academia Nauk, Seria Geographiya N 5 (in Russian).

Ventzel, A.D., and M.J. Freidlin (1979). Fluctuations in Dynamic Systems under Small Random Perturbances. Moscow: Nauka (in Russian).