RM - 74 - 5

INTERNATIONAL INSTITUTE FOR IIASA APPLIED SYSTEMS ANALYSIS

RESEARCH MEMORANDUM

A MAXIMIZATION PROBLEM ASSOCIATED WITH DREW'S INSTITUTIONALIZED DIVVY ECONOMY

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March 1974

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Define variables as in Dantzig [1]. Let $U_k(C_{ik})$ be the utility derived by a member of the kth group, and let

$$U = \sum_k \mu_k U_k(C_{ik}) \tag{A}$$

be an aggregate utility function. Equation numbers refer to Dantzig's paper; letters to this paper.

The economy is described by Dantzig's equations (2) - (4)--(2) being subdivided here as (2a) and (2b). The equations can be written in full, with explicit subscripts, as follows:

$$\sum_j L_{ij} - x_j = \sum_k C_{ik}\mu_k \tag{2a}$$

$$\sum_i y_i L_{ij} = \sum_\ell \lambda_\ell R_{\ell j} \tag{2b}$$

$$\sum_j \lambda_\ell R_{\ell j} x_j = \ell \gamma_\ell \tag{3}$$

$$\sum_i y_i C_{ik}\mu_k = \ell \delta_k \ . \tag{4}$$

Equations (2a) and (2b) can be combined to determine $\underline{x}$ and $\underline{y}$:

$$x_i > \sum_j \{L\}^{-1}_{ij} \sum_k C_{jk}\mu_k \tag{2a'}$$

$$y_j > \sum_i \{L\}^{-1}_{ij} \sum_\ell \lambda_\ell R_{\ell i} \;. \tag{2b'}$$

We can substitute for $\underline{x}$ and $\underline{y}$ in (3) and (4), but now write them as equations in λ_ℓ, μ_k and C_{ik}: equations (5) and (6) then become

$$\sum_{ik} \lambda_\ell M_{\ell i}\mu_k \cdot C_{ik} = \ell\gamma_\ell \tag{B}$$

$$\sum_{i\ell} \lambda_\ell M_{\ell i}\mu_k C_{ik} = \ell\delta_k \tag{C}$$

where

$$M_{\ell i} = \sum_j R_{\ell j}\{L\}^{-1}_{ji} \;. \tag{D}$$

The proposed model is

Max U in equation (A)

subject to (B) and (C).

$\underline{x}$ and $\underline{y}$ are given by (2a'), (2b').

Possible Uses of the Model

1) In an existing equilibrium situation, we can assume that $\underline{C}$, $\underline{\lambda}$ and $\underline{\mu}$ exist such that equations (C), (D), (2a'), (2b') are satisfied and U in (A) is a max.

We can then explore the use of the model to investigate the consequences of various types of change.

2) Change in tastes: $U_k \longrightarrow U_k'$. Then since there are fewer constraints (B) and (C) than there are C_{ik}'s, there should exist a set of C_{ik}'s which give the new equilibrium.

3) It may be, however, that imposing conditions (B) and (C) is too strong. An alternative would be to calculate $\underline{C}$ to maximize (A) subject to (C) (which is like a budget constraint), for initially assumed $\underline{\lambda}$; and then to compute $\underline{x}$ from (2a'), $\underline{\lambda}$ from (B), $\underline{y}$ from (2b') and to iterate until a new equilibrium is found.

4) Alternative schemes on the lines of (3) could be investigated for other initial changes--e.g. in $\underline{\lambda}$.

5) So far, we have assumed that the μ_k's remain fixed. A further outer loop could be added to the iteration which solved the LP problem of max U in (A) as a function of μ_k subject to (B) and (C). Alternatively, find μ_k to equalize consumer surplus per capita.

6) A further alternative would be to produce entropy maximizing versions of these models by

$$\text{Max } S = - \sum \log C_{ik}\,! \qquad \text{(E)}$$

subject to, say (B) and (C) and

$$\sum \mu_k U_k(C_{ik}) = \overline{U} \qquad (A')$$

where $\overline{U}$ is assumed given at some suboptimal value.

References

[1] Dantzig, G.B. "Drew's Institutionalized Divvy Economy," Report 73-7, revised, Dept. of Operations Research, Stanford University, 1973.