



Research article

Optimal drilling efforts and industry structure

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Abstract: We consider a resource extraction model with the possibility of extending the resource stock by drilling efforts and an oligopolistic competition of symmetric firms. Assuming a hyperbolic market demand, we adopt the open-loop Nash equilibrium to analyze and compare the outcomes of a private vs a common resource pool. For both cases, the drilling efforts of the individual firms strongly depend on the market structure. For a low competition, the efforts increase the number of firms, and the opposite is true for a high competition. On the other hand, aggregate drilling efforts are different among the two types of pools and are opposite to Schumpeterian's hypothesis (i.e., they are either an inverted U-shaped (private pools) or strictly increasing (single pools)).

Keywords: nonrenewable resources; drilling; cournot nash equilibrium; differential games; oligopoly

1. Introduction

We analyse a model of nonrenewable resource extraction formulated as a differential oligopoly game in which the market demand for the final good or service is hyperbolic and the relevant state equation(s) is (are) nonlinear in the state and control variables. This modeling strategy is intentionally adopted (i) to differentiate our contribution from the bulk of the extant literature on that matter, where the assumption of linearity for such building blocks is commonly adopted to ensure the solvability of the resulting linear-quadratic problem, and (ii) to highlight interesting aspects of the resulting equilibria, although the latter is the outcome of open-loop solutions.

The game is formulated in two alternative ways. The first assumes that the resource pool is parcelised across the population of firms, while, in the second, the whole industry is granted access to the single undivided common pool. In both cases, which are parallel to the extracting activity, the firms perform drilling efforts to expand the pool(s), and the presence of this additional dimension to the strategy space allows us to overlap a relevant and lively discussion belonging to the tradition of industrial organization, namely the bearings of the industry structure (or, the intensity of competition) of the individual and aggregate incentives to invest (in the original literature, either in process or product innovation).

The point of departure of this debate is the diachronic debate between Schumpeter [33] and Arrow [3] concerning the robustness of the Schumpeterian hypothesis, according to which innovation incentives at the industry level should be monotonically increasing in the degree of the market power, being thus maximised in a monopoly. The reason is encapsulated in the so-called strategic effect, whereby a monopolist is always in a position to perform at least as well as any other market form. The objection raised by Arrow [3] instead relies in the replacement effect, whereby an innovative monopolist cannot do more than replacing itself, while patenting a (perfectly) competitive firm innovation may indeed become a monopolist, driving all of its rivals out of business. These positions, both pointing at the emergence of a monotone pattern, have ignited a very large number of theoretical and empirical contributions that alternatively validate either one or the other.* Then, after Aghion *et al.* [2], research has been redirected towards a non-monotone relationship between the degree of the industry concentration and the aggregate innovation efforts, which takes the form of an inverted U-shaped curve.†

To the best of our knowledge, the same approach hasn't been adopted in the area that we are contemplating in the present paper (except for Lambertini [21]), where investments may be meant to expand the size of the pool(s) being exploited in order to extend the residual time horizon before either replacement, exhaustion (if the resource is nonrenewable), or extinction (if it is a renewable or biological one). The literature about nonrenewables has indeed included exploration activities, allowing for the outcome of the exploration process to be affected by uncertainty.‡ The contribution by Boyce and Vojtassak [5] nests itself onto the *theory of 'oil'igopoly* dating back to Salant [32], Loury [22], and Polasky [26, 27], adding investments in exploration to expand the known reserves, showing that firms endowed with lower reserves (and therefore facing shorter residual horizons) should overinvest in exploration as compared to their rivals. A parallel investigation pivoting around the emergence of backstop technologies has focused upon Research and Development (R&D) incentives to deliver substitutes for exhaustible resources (cf., Davidson [9]; Hoel [17]; Dasgupta *et al.* [8]; Olsen [24]; and Harris and Vickers [15]).

In a nutshell, the point of attention raised in this literature is the impact of oil'gopolistic competition (possibly - but not systematically - including uncertainty) on the incentives to explore in order for firms to not drop out of the market having exhausted their resource pools, while leaving the impact of industry structure on exploration aside. In some sense, this aspect was one step away, but remained behind the curtains, probably because the stimulus exerted by the oil crises created other priorities, such as forecasting the pattern of the oil price under scarcity. Understandably, these elements prevailed

*Exhaustive overviews are in Tirole [34], Hoel [18] and Gilbert [14], *inter alia*.

†The ensuing literature is summarised in Gilbert [14] and Delbono and Lambertini [10], *inter alia*.

‡See Peterson [25], Deshmuk and Pliska [11]; Arrow and Chang [4]; Mohr [23]; Quyen [28, 29]; and Cairns and Quyen [6]. For a compact overview, see Lambertini [20].

over other aspects, including the analysis of the impact of the firms' numerosity on the intensity of the exploration efforts.

Our purpose is to fill this specific gap and single out the impact of the industry structure on the individual and collective firms' investment in a dynamic deterministic setting in which the firms are Cournot-Nash players and the matter is not explored under uncertainty to identify additional reserves; however, a well-known ground is drilled to increase the initial size of a resource pool whose exploitation is allocated to firms in the form of either many private pools or a single common pool. We assume a symmetric market endowment (firms face symmetric costs and share the same market price) and a symmetric behavior of the firms (abstracting from equilibria that are asymmetric in their strategic behavior). The open-loop solution of the resulting two games delivers a number of interesting results which do not deliver a vindication of the Schumpeterian hypothesis, but instead illustrate the emergence at the steady state of the strategic equilibrium of a concave and single-peaked aggregate investment curve if the firms are allowed to exploit private pools, and a monotonically increased investment curve in the alternative case where the firms accessed the undivided pool. In both settings, the analysis of the equilibrium behavior of firms is carried out in the parameter range wherein a saddle point stability is ensured.

The assumption of isoelastic demand delivers a richer set of insights as compared to the outcomes of the equivalent games analyzed in Lambertini [21] with linear market demand coupled with a nonlinear state equation, which identified a non-decreasing aggregate effort with an Arrovian or quasi-Arrovian flavor.

The remainder of the paper is structured as follows. We start with the layout of the model in section 2. The game with private pools is investigated in section 3, while that of a single pool is investigated in section 4.[§] Section 5 offers a comparative evaluation of the two equilibria. Section 6 concludes the paper.

2. The model

The setup intentionally departs from the traditional linear-quadratic form, which would facilitate the feedback solution of the game. This modeling choice has been taken on the following grounds: (i) demand functions may not be at least quasi concave - which, in itself, is an assumption usually adopted to ensure the existence of a Nash equilibrium on the relevant market, as we at least know from Dixit [12] - and indeed empirical estimations have often identified hyperbolic ones, as in Hausman [16] and Varian [35]; and (ii) accounting for decreasing returns (in this case, for the extracting technology), which relies on an instantaneous convex cost, may not be the most appropriate choice, as this activity, and probably others, is better described as exhibiting this property *while acting*, hence in the functional form of the state equation. A population of n firms sell a homogeneous good over time $t \in [0, \infty)$ whose instantaneous demand function is hyperbolic (i.e., $p(t) = \alpha/Q(t)$), with $\alpha > 0$ and $Q(t)$ being defined in (2.3b).[¶] Production requires a nonrenewable natural resource $X(t)$ at time t whose initial amount is $X(0) = X_0 > 0$. First, we will consider the scenario in which the resource is symmetrically

[§]Note that analytical derivations are nasty, but follows the way that is sketched in the paper. The explicit expressions have been obtained by Wolfram Mathematica.

[¶]The assumption of a hyperbolic demand function, of course, is the consequence of the implicit assumption of a Cobb-Douglas utility function for the representative consumer. This possibility is usually left aside for technical reasons, while being empirically relevant. For a discussion, see Lambertini [19].

parcelised across the population of firms, where each one holds exclusive exploitation rights on an initial endowment $x(0) = X_0/n = x_0 > 0$.

Departing from a linear-quadratic approach, the second element is the formulation of the drilling technology, which borrows some essential elements from Cellini and Lambertini [7], where a similar state equation is used to study investments in product differentiation. Every firm i ($i = 1, \dots, n$) may independently ‘drill the ground’ to expand its private pool while exploiting it to supply the final good, where the individual state equation is as follows:

$$\dot{x}_i(t) = \frac{vx_i(t)k_i(t)}{1+k_i(t)} - q_i(t). \quad (2.1)$$

In (2.1), $k_i(t) \geq 0$ is the exploration effort, and $v > 0$ is a constant common to all firms. The shape of the exploration function that appears in the above state kinematics says that the exploration activities are affected by decreasing the returns to scale. Additionally, it is appropriate to clarify that, for the moment, each private pool is indexed to ease the exposition, as firms are *a priori* symmetric and the game will indeed deliver a fully symmetric open-loop Nash equilibrium solution. Hence, one could pose $x_0 = X_0/n$ as well as $x(t) = X(t)/n$; however, we are about to show that this modeling choice would have no bearings on the main properties of the game’s solution.

The alternative case is one where an industry-wide common pool is jointly (but still noncooperatively) exploited by the population of firms. In this scenario, the single state is $X(t)$, and its dynamics, which is simultaneously affected by all firms’ control variables, takes the following form:

$$\dot{X}(t) = \frac{vX(t) \sum_{i=1}^n k_i(t)}{1 + \sum_{i=1}^n k_i(t)} - \sum_{i=1}^n q_i(t), \quad (2.2)$$

while the instantaneous profit function of the i -th firm is defined below (see (2.3)). Here, the firms play a differential game with $2n$ controls (outputs \mathbf{q} and efforts \mathbf{k}) and one state (the common pool $X(t)$).

The definition of drilling costs and profits is independent of the assignment of the resource pool(s). Assuming a linear instantaneous cost of exploration efforts, $\Gamma_i(t) = \beta k_i(t)$ with $\beta > 0$ and, for simplicity, a constant returns to a scale technology for the final good with an average and marginal cost normalised to zero, where the instantaneous profits of the firm i are as follows:

$$\pi_i(t) = \frac{\alpha q_i(t)}{q_i(t) + Q_{-i}(t)} - \beta k_i(t), \quad (2.3a)$$

$$Q_{-i}(t) = \sum_{j \neq i} q_j(t), \quad (2.3b)$$

whose flow is discounted at the rate $\rho > 0$, which is common to all firms. Therefore, the firms play a differential game with $2n$ controls (outputs \mathbf{q} and efforts \mathbf{k}) and n states (the vector of private resource pools \mathbf{x}).

Figure 1 summarizes the model and illustrates the interactions between the states, controls, and among the players.

In both scenarios, since there is no obvious candidate for the value function because of the nonlinear nature of the demand function and the state equations (leaving the feedback Nash equilibrium for a numerical investigation), our attention is limited to the open-loop Nash equilibrium. This equilibrium

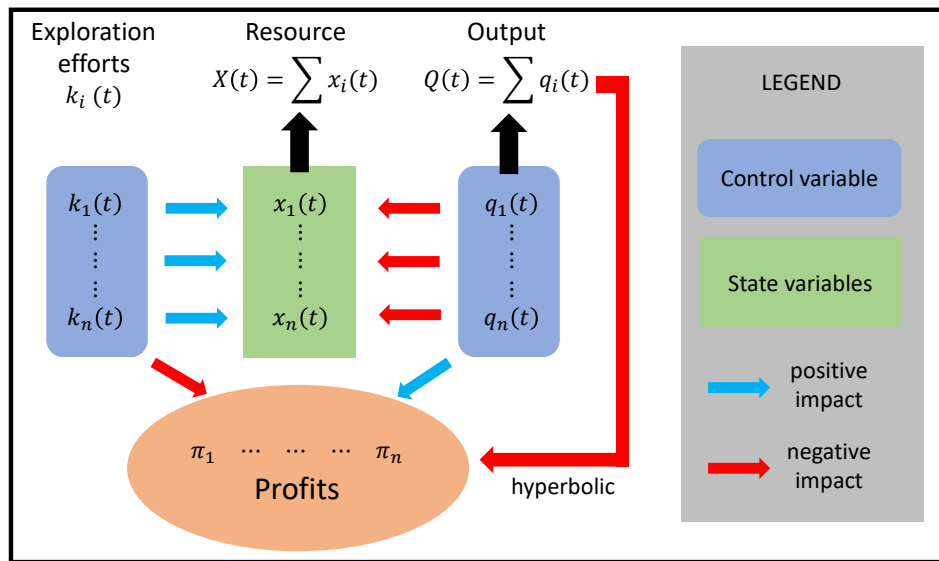


Figure 1. Model overview showing variables and interactions.

concepts assumes that the firms are committed to strategies that are solely functions of time (i.e., the Maximum Principle can be applied).

A general rating of different commitment structures (among others open-loop, closed-loop, feedback) of a Nash equilibrium is not available. However, the majority of papers that considered a comparison found that feedback and closed-loop Nash equilibria implied lower objective functions (in our case profits) due to stronger competition effects in the strategic interaction. For an analytic characterization of this effect see e.g., Reynolds [31] in a classic capital accumulation set up. For a systematic comparison of different commitment/information sets see e.g., Feichtinger et al. [13].

3. Private pools

The firm i must choose $q_i(t)$ and $k_i(t)$ so as to maximise the following

$$\int_0^{\infty} e^{-\rho t} \left[\frac{\alpha q_i(t)}{q_i(t) + Q_{-i}(t)} - \beta k_i(t) \right] dt, \quad (3.1)$$

w.r.t. $q_i(t)$ and $k_i(t)$ (both positive), under the set of constraints posed by the state dynamics (2.1) and the common initial condition $x_0 = x(0)$, $i = 1, 2, \dots, n$.

Therefore, the firm i 's (current value) Hamiltonian is as follows:

$$\mathcal{H}_i(\mathbf{x}, \mathbf{q}, \mathbf{k}) = \frac{\alpha q_i(t)}{q_i(t) + Q_{-i}(t)} - \beta k_i(t) + \lambda_i(t) \left(\frac{v x_i(t) k_i(t)}{1 + k_i(t)} - q_i(t) \right), \quad (3.2)$$

where $\lambda_i(t)$ is the costate variable (evaluated at time t) associated by firm i to its parcel of the stock.

Before proceeding, it is worth noting that it is not necessary to assign a costate variable to the state equation of any rival firm $j \neq i$ because $(\mathbf{q}_{-i}, \mathbf{k}_{-i})$ do not appear in (2.1).^{||}

In order to ensure the existence of an economically meaningful solution (as we will see in the analysis of the problem), we adopt the following:

Assumption A $v > \beta \rho n^2 / [\alpha(n-1)] \equiv v_{PP}^k$.

The necessary conditions (formulated for interior solutions) are the following (henceforth, omitting the time argument for the sake of brevity):

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial q_i} = \frac{\alpha Q_{-i}}{(q_i + Q_{-i})^2} - \lambda_i = 0, \quad (3.3a)$$

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial k_i} = \frac{\lambda_i v x_i - \beta(1+k_i)^2}{(1+k_i)^2} = 0, \quad (3.3b)$$

$$\dot{\lambda}_i = \left(\rho - \frac{v k_i}{1+k_i} \right) \lambda_i \quad (3.3c)$$

and the limiting transversality condition is $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i x_i = 0$ for each firm. It is worth noting that (3.3c) obviously implies that $\lambda_i = 0$ is a viable solution at any time. However, its adoption would imply a solution on the boundary (together with a nonzero derivative of the Hamiltonian): $\lambda_i = 0$ implies $\partial \mathcal{H}_i(\cdot) / \partial q_i > 0$ and $\partial \mathcal{H}_i(\cdot) / \partial k_i < 0$ always. The first implies an immediate complete exploitation as q_i is unbounded, whereas the latter entails the annihilation of all the firms' incentives to invest in drilling forever equivalent to an empty stock. The resulting boundary solution with $q_i = k_i = 0$ ($i = 1, 2, \dots, n$) can be excluded as it is dominated by the inner one discussed in the following.

Consequently, from now on, we restrict the analysis to the interior solutions and consider a non-nil instantaneous solution for the optimal costate variable. From (3.3b), one obtains

$$\lambda_i^* = \frac{\beta(1+k_i)^2}{v x_i} > 0 \quad (3.4)$$

and

$$k_i^* = -1 \pm \sqrt{\frac{\lambda_i v x_i}{\beta}}, \quad (3.5)$$

for all $i = 1, 2, \dots, n$. As k_i is non-negative, when looking at (3.5), one has to select the larger root, which can be differentiated w.r.t. time to generate the control equation that describes the evolution of the optimal research effort over time:

$$\dot{k}_i = \frac{\sqrt{v} (\dot{\lambda}_i x_i + \lambda_i \dot{x}_i)}{2 \sqrt{\beta} \lambda_i x_i}. \quad (3.6)$$

The latter can be simplified using (2.1), (3.3c), and (3.4):

$$\dot{k}_i = \frac{(1+k_i)(\rho x_i - q_i)}{2 x_i}. \quad (3.7)$$

^{||}It is easily checked that, if such $n-1$ costate were introduced to include in (3.2) the state equations of all $x_j(t)$, $j \neq i$, the necessary conditions would yield $\lambda_j(t) = 0$ at all times.

Now, we impose symmetry across the states and controls, and note that the use of (3.4) also makes it redundant to derive a control equation for the Cournot-Nash individual output level, which can be determined on the basis of the corresponding first order condition, whose solution yields the following:

$$q^N(x) = \frac{\alpha(n-1)vx}{\beta(1+k)^2 n^2} \quad (3.8)$$

at any instant and for any given individual stock x . The expression of $q^N(x)$ can be substituted back into (2.1) and (3.7) to obtain the proper representation of the state-control system:

$$\dot{x} = \frac{[\beta k(1+k)n^2 - \alpha(n-1)]vx}{\beta(1+k)^2 n^2}, \quad (3.9a)$$

$$\dot{k} = \frac{(1+k_1)}{2} \left(\rho - \frac{\alpha(n-1)v}{b(1+k)^2 n^2} \right). \quad (3.9b)$$

On the basis of (3.9), we may identify the relevant Jacobian matrix for the private pools' game:

$$J_{PP} = \begin{pmatrix} \frac{[\beta k(1+k)n^2 - \alpha(n-1)]v}{\beta(1+k)^2 n^2} & \frac{[\beta(1+k)n^2 + 2\alpha(n-1)]vx}{\beta(1+k)^3 n^2} \\ 0 & \frac{1}{2} \left[\rho + \frac{\alpha(n-1)v}{\beta(1+k)^2 n^2} \right] \end{pmatrix} \quad (3.10)$$

where the subscript PP mnemonics for *private pools*. With a view to carry out the stability analysis, it is useful to explicitly report its trace \mathcal{T} and determinant Δ :

$$\mathcal{T} = \frac{\beta(1+k)n^2 [\rho + k(r+2v)] - \alpha(n-1)v}{2\beta(1+k)^2 n^2}, \quad (3.11a)$$

$$\Delta = \frac{[\beta k(1+k)n^2 - \alpha(n-1)] [\beta(1+k)^2 n^2 \rho + \alpha(n-1)v] v}{2\beta^2(1+k)^4 n^4}. \quad (3.11b)$$

Now, we may focus our attention onto the steady state equilibrium. Concerning the control variable, $k = 0$ has a single candidate solution,

$$k_{PP}^{SS} = \frac{\sqrt{\alpha\beta\rho(n-1)v} - n\beta\rho}{\beta\rho n}, \quad (3.12)$$

where, superscript SS stands for steady state. Note that the individual drilling effort is positive for all $v > \beta\rho n^2 / [\alpha(n-1)] = v_{PP}^k$, which corresponds to assumption A imposed at the beginning of this section, provided that the industry is at least duopolistic.** Indeed, in the monopoly case, the right hand side of (3.12) is negative, which amounts to saying that a monopolist would not invest at all. The intuitive interpretation of this finding is that (i) a firm enjoying monopoly power is not affected by market competition (which may stimulate investments to expand the size of any private pool), and (ii) the concept of 'private pool' makes no sense itself, as the monopolist would access the entire pool X .

If $k = k_{PP}^{SS}$, and we have $\dot{x} = 0$ in $x_{PP}^{SS} = 0$, then the individual and aggregate output levels, namely $q^N(x_{PP}^{SS}) = q_{PP}^{SS}$ and $Q_{PP}^{SS} = nq_{PP}^{SS}$, are also nil at the steady state, as can be easily ascertained from (3.8). This immediately implies the following.

**The fact that a firm monopolising the market would not invest at all is also illustrated by a key feature of v_{PP}^k , namely $\lim_{n \rightarrow 1} v_{PP}^k = \infty$.

Proposition 1. *At the steady state of the Nash equilibrium, each private pool is exhausted and the market for the resulting final good shuts down.*

As anticipated above, this result does not require the *ex ante* imposition of the full symmetry on the partition of the (common) pool across the population of firms in the industry, being rather a direct consequence of each firm's incentive to not leave any residual portion of the nonrenewable productive asset in the ground when the game is over.

Moreover, substituting x_{PP}^{SS} and k_{PP}^{SS} into (3.11), the trace and determinant of the Jacobian matrix simplify to the following:

$$\mathcal{T}_{PP}^{SS} = v - \frac{\beta\rho nv}{\sqrt{\alpha\beta\rho(n-1)v}}, \quad (3.13a)$$

$$\Delta_{PP}^{SS} = \rho \left(v - \rho - \frac{\beta\rho nv}{\sqrt{\alpha\beta\rho(n-1)v}} \right). \quad (3.13b)$$

To begin with, it is useful to spell out a few key properties of the above trace and determinant. The determinant is positive for all

$$v > \frac{[2\alpha(n-1) + n(b^2 + \sqrt{b(4\alpha(n-1) + bn^2)})]\rho}{2\alpha(n-1)} \equiv v_{PP}^{\Delta} \quad (3.14)$$

and negative for all $v \in (0, v_{PP}^{\Delta})$. In the latter case, the steady state is a saddle point. Outside this range, the steady state would be stable (either a node or a focus) if and only if $\mathcal{T}_{PP}^{SS} < 0$. This, in turn, happens for all

$$v \in (0, v_{PP}^{\mathcal{T}}), \quad \text{where} \quad v_{PP}^{\mathcal{T}} \equiv \frac{\beta\rho n^2}{\alpha(n-1)} = v_{PP}^k. \quad (3.15)$$

This entails that, whenever $k_{PP}^{SS} > 0$, the trace is positive as well. Therefore, the model cannot generate any stable focus or node, and we may only aim for saddle point stability. A quick look at the expression of Δ_{PP}^{SS} reveals the following Lemma.

Lemma 1. *For any $v \in (0, \rho)$ (common to all firms) $\Delta_{PP}^{SS} < 0$ holds, which implies saddle point stability of $(x_{PP}^{SS}, k_{PP}^{SS})$.*

As one may expect, it is easily verified that the sufficient condition $v \in (0, \rho)$ is encompassed by the necessary one appearing in (3.14), as $v_{PP}^{\Delta} > \rho$. With this in mind, accounting for Assumption A and the positivity of (3.12), which requires $v > v_{PP}^k$, the straightforward algebra shows that

$$\rho \geq v_{PP}^k, \quad \text{for } \forall \alpha \geq \frac{\beta n^2}{n-1}. \quad (3.16)$$

Consequently, we may formulate the following Corollary.

Corollary 1. *If $\alpha > \beta n^2 / (n-1)$, then $v \in (v_{PP}^k, \rho)$ suffices to ensure $k_{PP}^{SS} > 0$ and the saddle point stability of the steady state Nash equilibrium $(x_{PP}^{SS}, k_{PP}^{SS})$.*

Now, we may exclude the parameter regions identified by (i) $\alpha > \beta n^2 / (n - 1)$ and $v \in (0, v_{PP}^k]$ and (ii) $\alpha \in (0, \beta n^2 / (n - 1)]$ and $v \in (0, \rho]$, because $k_{PP}^{SS} \leq 0$ in these areas. Hence, we are left with two regions, respectively identified by the following:

- $\alpha > \beta n^2 / (n - 1)$ and $v > \rho$,
- $\alpha \in (0, \beta n^2 / (n - 1)]$ and $v > v_{PP}^k$,

in which $k_{PP}^{SS} > 0$ but the stability properties remain to be assessed.

In the first region, with $v_{PP}^\Delta > \rho > v_{PP}^k$, it holds that $\Delta_{PP}^{SS} < 0$ for all $v \in [\rho, v_{PP}^\Delta)$, whereby we have a saddle point with a positive investment. In the second, with $v_{PP}^\Delta > v_{PP}^k > \rho$, we have $k_{PP}^{SS} > 0$ together with saddle point stability for all $v \in (v_k, v_{PP}^\Delta)$.

To sum up the foregoing discussion, we may formulate the following Proposition.

Proposition 2. *The steady state equilibrium $(x_{PP}^{SS}, k_{PP}^{SS}, q_{PP}^{SS})$ solving the open-loop Nash game with parcelised pools is saddle point stable for all $v \in (\max\{\rho, v_{PP}^k\}, v_{PP}^\Delta)$, with*

$$\max\{\rho, v_{PP}^k\} = \begin{cases} \rho & \forall \alpha > \beta n^2 / (n - 1) \\ v_k & \forall \alpha \in (0, \beta n^2 / (n - 1)] \end{cases} .$$

In correspondence of any $v > v_{PP}^\Delta$, the equilibrium is unstable.

Now, we are ready to investigate the relationship between the intensity of the market competition and the drilling effort at the industry level.

3.1. Drilling investments in private pools

Keeping in mind the opposite positions taken by Schumpeter [33] and Arrow [3] on the interplay between market power and R&D incentives, respectively, and its more recent revival stimulated by Aghion *et al.* [2], we perform an analogous analysis in terms of the subject matter of this section, namely, the effect of a variation in the number of firms on both the individual and collective drilling activities at the open-loop Nash equilibrium. To this aim, we may set out by examining the following partial derivatives of the individual effort:

$$\frac{\partial k_{PP}^{SS}}{\partial n} = -\frac{\alpha(n-2)v}{2n^2 \sqrt{\alpha\beta\rho(n-1)v}}, \quad (3.17a)$$

$$\frac{\partial^2 k_{PP}^{SS}}{\partial n^2} = -\frac{\alpha^2\beta\rho[4(3n-2) - 3n^2]v^2}{4n^3 \sqrt{[\alpha\beta\rho(n-1)v]^3}}. \quad (3.17b)$$

The first is nil at $n_{PP}^k = 2$, while the second —accounting for the integer requirement —is negative for all $n = 1, 2, 3$ and positive thereafter (the inflection point of k_{PP}^{SS} being at $n = 3.155$). However, as we already know, a monopolist would not produce any effort in the drilling activities. Hence, the individual investment curve only exists for $n \geq 2$, and is monotonically decreasing in the industry fragmentation.

Now, we may consider $K_{PP}^{SS} = nk_{PP}^{SS}$, whose essential features are illustrated by the following:

$$\frac{\partial K_{PP}^{SS}}{\partial n} = n \cdot \frac{\partial k_{PP}^{SS}}{\partial n} + k_{PP}^{SS} = \frac{\alpha v}{2\sqrt{\alpha\beta\rho(n-1)v}} - 1, \quad (3.18a)$$

$$\frac{\partial^2 K_{PP}^{SS}}{\partial n^2} = -\frac{\alpha^2\beta\rho v^2}{4\sqrt{[\alpha\beta\rho(n-1)v]^3}}. \quad (3.18b)$$

The second partial derivative is evidently negative everywhere, while the first is nil in correspondence of $n_{PP}^K = 1 + \alpha v / (4\beta\rho) \geq 2$ for all $\alpha \geq 4\beta\rho/v$. Since the critical threshold of α corresponds to 4β when $n = 2$, and

$$\frac{4\beta\rho}{v} > 4\beta, \quad \text{for } \forall \rho > v \quad (3.19)$$

then any $\alpha = 4\beta\rho/v + \varepsilon$, with $\varepsilon > 0$ and is arbitrarily small, proves that the aggregate effort has an inverted U-shape with respect to the number of firms, reaching its maximum at least in a duopoly. Finally, note that $\alpha \geq 4\beta\rho/v$ is a manipulation of the requirement for k_{PP}^{SS} at $n = 2$ (i.e., $v > v_{PP}^k|_{n=2}$).

Accordingly, we may conclude the section by the following statement.

Proposition 3. *The individual open-loop Nash equilibrium effort with private pools is monotonically decreasing in the number of firms for any $n \geq 2$. The aggregate one is instead concave and single-peaked in the number of firms for all $v > v_{PP}^k$.*

4. Common pool

If all firms are granted access to the single pool of an initial size X_0 , then the relevant state equation is (2.2) and the firm i 's Hamiltonian is as follows

$$\mathcal{H}_i(X, \mathbf{q}, \mathbf{k}) = \frac{\alpha q_i(t)}{q_i(t) + Q_{-i}(t)} - \beta k_i(t) + \quad (4.1)$$

$$\lambda_i(t) \left(\frac{vX(t)(k_i(t) + K_{-i}(t))}{1 + k_i(t) + K_{-i}(t)} - q_i(t) - Q_{-i}(t) \right). \quad (4.2)$$

As for the case of private pools, we formulate the following assumption to ensure the non negativity of the drilling efforts.

Assumption B $v > \beta\rho n / [\alpha(n-1)] = v_{PP}^k/n \equiv v_{CP}^k$.

The necessary conditions are as follows:

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial q_i} = \frac{\alpha Q_{-i}}{(q_i + Q_{-i})^2} - \lambda_i = 0, \quad (4.3a)$$

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial k_i} = \frac{\lambda_i v X - \beta(1 + k_i + K_{-i})^2}{(1 + k_i)^2} = 0, \quad (4.3b)$$

$$\dot{\lambda}_i = \left(\rho - \frac{v(k_i + K_{-i})}{1 + k_i + K_{-i}} \right) \lambda_i, \quad (4.3c)$$

and the limiting transversality condition is $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i X = 0$ for each firm.

Imposing symmetry across controls and costates, and proceeding the same way as in the previous case, one obtains the expressions of the optimal drilling effort and costate variable

$$k^* = -\frac{1}{n} + \sqrt{\frac{\lambda v X}{b}} \quad (4.4a)$$

$$\lambda^* = \frac{b(1 + kn)^2}{vX} \quad (4.4b)$$

and the Cournot-Nash output

$$q^N(X) = \frac{\alpha(n-1)}{\lambda n^2} = \frac{\alpha(n-1)vX}{\beta n^2(1+kn)^2}. \quad (4.5)$$

From (4.3c-4.4b), one may obtain the control equation

$$\dot{k} = \frac{\sqrt{\lambda v}(\rho X - nq)}{2n\sqrt{\beta X}} = \frac{(1+kn)(\rho X - nq)}{2nX\sqrt{\beta}} \quad (4.6)$$

and then, from (4.5), posing $q = q^N(X)$,

$$\dot{k} = \frac{\beta \rho n(1+kn)^2 - \alpha(n-1)v}{2n^2\sqrt{\beta^3}(1+kn)} \quad (4.7)$$

while the final form of the state equation is as follows:

$$\dot{X} = \frac{[\beta n^2(1+kn)k - \alpha(n-1)]vX}{\beta n(1+k)^2}. \quad (4.8)$$

On the basis of this state-control system, we may characterise the Jacobian matrix of the common pool game as follows:

$$J_{CP} = \begin{pmatrix} \frac{[\beta k(1+kn)n^2 - \alpha(n-1)]v}{\beta(1+kn)^2 n} & \frac{[\beta(1+kn)n + 2\alpha(n-1)]vX}{\beta(1+kn)^3} \\ 0 & \frac{\alpha(n-1)v + \beta \rho n(1+kn)^2}{2n\sqrt{\beta^3}(1+kn)^2} \end{pmatrix}. \quad (4.9)$$

The coordinates of the steady state equilibrium are as follows:

$$X_{CP}^{SS} = 0 \quad (4.10a)$$

$$k_{CP}^{SS} = \frac{\sqrt{\alpha\beta n(n-1)\rho v} - \beta \rho n}{\beta \rho n^2} \quad (4.10b)$$

with the subscript CP standing for *common pool*. Here, $k_{CP}^{SS} > 0$ follows Assumption B. The straightforward implication of this property is as follows.

Lemma 2. *The region of parameters in which the common pool game delivers a vector of positive drilling efforts at equilibrium is wider than the corresponding region in the private pool game.*

Yet, once again, note that no investment is observed in the presence of a monopoly power; if $n = 1$ then, necessarily $k_{CP}^{SS} = 0$.

Then, using (4.10b), we may write the trace and determinant of (4.9) as follows:

$$\mathcal{T}_{CP}^{SS} = \left(\frac{\sqrt{\beta}}{\beta} - 1 \right) \rho + v - \frac{\beta \rho n v}{\sqrt{\alpha \beta \rho (n-1) n v}}, \quad (4.11a)$$

$$\Delta_{CP}^{SS} = - \frac{\rho \left[\alpha (n-1) (\rho - v) + \sqrt{\alpha \beta \rho (n-1) n v} \right]}{\alpha \sqrt{\beta} (n-1)}. \quad (4.11b)$$

Considering Δ_{CP}^{SS} suffices the validation of the equivalent of Lemma 2 in the common pool game (i.e., the steady state Nash equilibrium (4.10b) is surely a saddle point for any $v \in (0, \rho)$). Yet, this is not necessary, as indeed $\Delta_{CP}^{SS} < 0$ for all $v \in (0, v_{CP}^{\Delta})$, with

$$v_{CP}^{\Delta} \equiv \rho \left[1 + \frac{\beta n + \sqrt{\beta n (4\alpha (n-1) + \beta n)}}{2\alpha (n-1)} \right] > \max \{ \rho, v_{CP}^k \} \quad (4.12)$$

always, while

$$\max \{ \rho, v_{CP}^k \} = \begin{cases} \rho & \forall \alpha > \beta n / (n-1) \\ v_{CP}^k & \forall \alpha \in (0, \beta n / (n-1)] \end{cases}. \quad (4.13)$$

Given that in correspondence of any $v \geq v_{CP}^{\Delta}$ the trace is positive, the following proposition summarizes the stability properties of the steady state.

Proposition 4. *The steady state $(x_{CP}^{SS}, k_{CP}^{SS}, q_{CP}^{SS})$ solving the open-loop Nash common pool game is saddle point stable for all $v \in (\max \{ \rho, v_{CP}^k \}, v_{CP}^{\Delta})$. For values above this interval (i.e., $v \geq v_{CP}^{\Delta}$) the steady state is unstable.*

4.1. Drilling investments in a common pool

As in section 3.1, we will characterise the impact of a variation in the industry structure on the firms' incentives to drill the ground. This case distinguishes from the previous one by the awareness of every single firm that any individual effort to expand the size of the common pool by itself with a spill over to any other firm, that is, each investor knows that the outcome of her/his investment is not entirely privately appropriable.

The following first derivative

$$\frac{\partial k_{CP}^{SS}}{\partial n} = - \frac{\alpha (n-1) v - \sqrt{\alpha \beta \rho (n-1) n v}}{n^2 \sqrt{\alpha \beta \rho (n-1) n v}} \quad (4.14)$$

is nil at

$$n_{CP}^k = \frac{3\alpha v - 2\beta\rho + \sqrt{\beta\rho(3\alpha v + \beta\rho)}}{2(\alpha v - \beta\rho)} \geq 2, \quad \text{for } \forall v > \frac{8\beta\rho}{\alpha}. \quad (4.15)$$

Note that the above threshold exceeds v_{CP}^k for all $n > 1$. Otherwise, (4.14) is positive for all $n \in (1, n_{CP}^k)$ and negative for all $n > n_{CP}^k$. This reveals that, once again, the individual investment is concave and

single-peaked in the number of firms; however, in general, the peak is not located in correspondence of duopoly.

Then, unlike the setting with parcelised pools, this delivers a monotone result as far as the collective incentive to drill the ground is concerned, as implied by the following derivatives:

$$\frac{\partial K_{CP}^{SS}}{\partial n} = \frac{\alpha v}{2\sqrt{\alpha\beta\rho(n-1)nv}} > 0, \quad (4.16a)$$

$$\frac{\partial^2 K_{CP}^{SS}}{\partial n^2} = -\frac{\alpha^2\beta(4n-3)\rho v^2}{4n\sqrt{[\alpha\beta\rho(n-1)nv]^3}} < 0. \quad (4.16b)$$

This amounts to saying that the pattern of individual and industry-wide investments can be summarised in the following Proposition.

Proposition 5. *The individual open-loop Nash equilibrium effort with a common pool is concave and single-peaked in the industry structure, provided that $v > 8\beta\rho/\alpha > v_{CP}^k$. The aggregate one is instead monotonically increasing and concave in the number of firms for all $v > v_{CP}^k$.*

More explicitly, the flavor of the common pool game is definitely Arrovian. This prompts for a brief but relevant last step, which consists of a comparative evaluation of the drilling efforts across the two equilibria.

5. Assessment of drilling effort

Implied by the above analysis, we can formulate the following invariant result.

Lemma 3. *Irrespective of the allocation of the resource pool, a monopolist does not invest at all.*

More explicitly, it takes (at least) two to drill the ground. This tells us that the absence of a market competition erases the monopoly firm's incentives altogether, which in itself is a neat non-Schumpeterian outcome. In turn, it does not make the model necessarily Arrovian, as we observe such a pattern only in the oligopoly game with all firms extracting the resource from the common pool.

When the model is a proper game, we may compare the individual and aggregate efforts to discover some additional details. The first is illustrated in Figure 2, which features the patterns of k_{PP}^{SS} and k_{CP}^{SS} as the number of firms n changes, having set $\alpha = 2$, $\beta = 1$, $\rho = 2/25$, and $v = 1/2$.

The intersection occurs at $n \approx 1.767$ (irrelevant) and $n \approx 4.476$, which, accounting for the integer constraint, means that k_{CP}^{SS} becomes larger than k_{PP}^{SS} at $n = 5$. The expression identifying the intersection between k_{CP}^{SS} and k_{PP}^{SS} is as follows:

$$n_{kk} = \frac{(\alpha v + \sqrt[3]{\Gamma + \Psi})^2 + \beta\rho(\beta\rho + 10\alpha v) + \sqrt[3]{\Gamma + \Psi}}{3\beta\rho\sqrt[3]{\Gamma + \Psi}}, \quad (5.1)$$

in which

$$\Gamma \equiv \alpha^2 v^2 (39\beta\rho - \alpha v) + \beta^2 \rho^2 (15\alpha v + \beta\rho), \quad (5.2a)$$

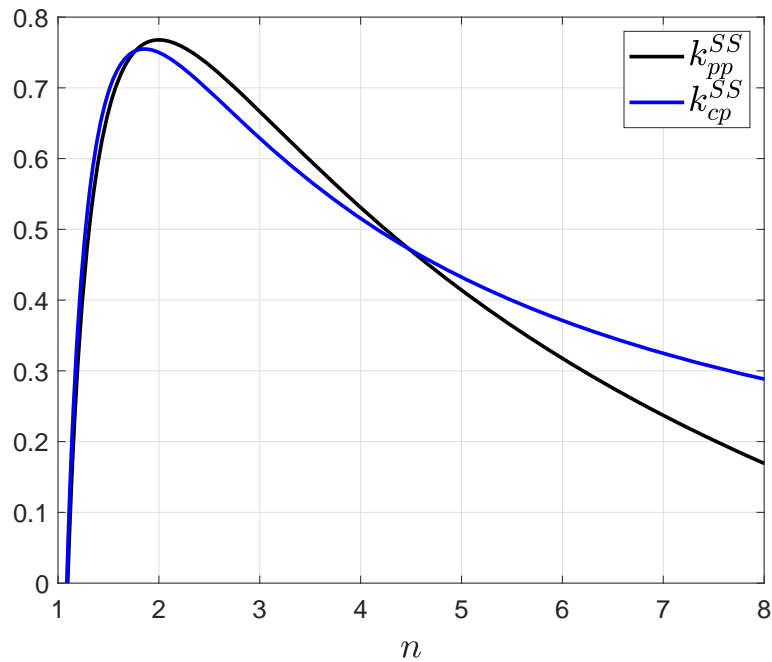


Figure 2. Individual efforts as the intensity of market competition increases.

$$\Psi \equiv 6\alpha v \sqrt{3\alpha\beta\rho [\beta\rho (11\alpha v + \beta\rho) - \alpha^2 v^2]} v. \quad (5.2b)$$

Numerical calculations for various parameter constellations show that $\partial n_{kk}/\partial v > 0$ (i.e., the interval over which $k_{pp}^{SS} > k_{cp}^{SS}$ expands as the drilling technology becomes progressively more effective). To conclude the evaluation of the individual equilibrium efforts, the last aspects of individual efforts that deserve attention are as follows:

$$\lim_{n \rightarrow \infty} k_{CP}^{SS} = 0, \quad (5.3)$$

while $k_{PP}^{SS} = 0$ if

$$n = n_{PP}^{SS} \equiv \frac{\alpha v + \sqrt{\alpha v (\alpha v - 4\beta\rho)}}{2\beta\rho} \geq 2, \quad \text{for } \forall v > \frac{4\beta\rho}{\alpha} = v_{PP}^k|_{n=2} \quad (5.4a)$$

$$\lim_{\rho \rightarrow \infty} \frac{\alpha v + \sqrt{\alpha v (\alpha v - 4\beta\rho)}}{2\beta\rho} = 0. \quad (5.4b)$$

The findings in (5.3-5.4) jointly imply that an increasing competition is bound to annihilate the investments.

Lemma 4. Consider $\rho \in (0, \infty)$ and suppose $v > v_{PP}^k$, in such a way that k_{PP}^{SS}, k_{CP}^{SS} be surely positive at least in a duopoly. While k_{CP}^{SS} becomes nil only under free entry, as the industry becomes infinitely fragmented, k_{PP}^{SS} becomes nil in correspondence to a finite oligopolistic industry structure.

Concerning aggregate drilling efforts in the two games, we may quickly ascertain what follows. Of course, $K_{PP}^{SS} = K_{CP}^{SS}$ if $n = n_{kk}$ and $K_{PP}^{SS} = 0$ if $n = n_{PP}^{SS}$. Therefore, the ranking of aggregate effort replicates that of the individual ones, and K_{PP}^{SS} has the same fate of k_{PP}^{SS} . However, the shape of the

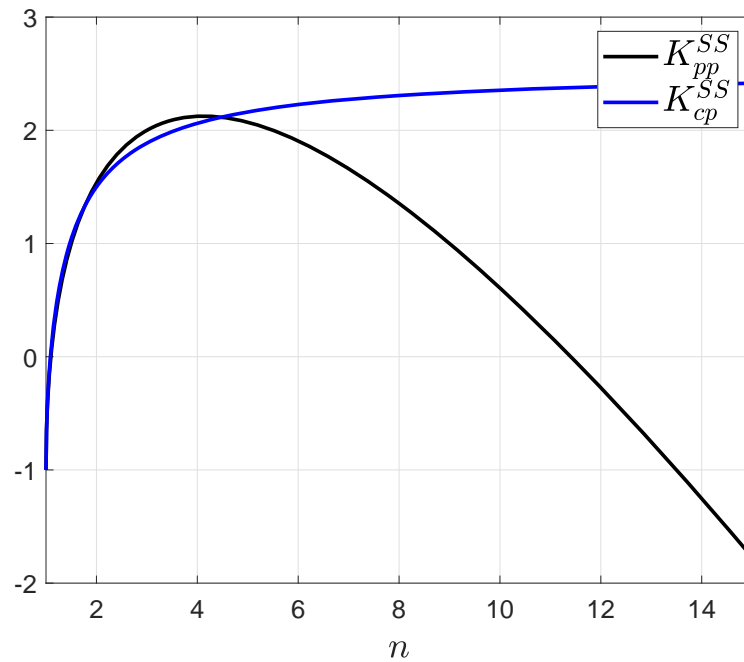


Figure 3. Aggregate efforts as the intensity of market competition increases.

two curves is a bit different, as illustrated in Figure 3, which has been drawn using the same parameter values as for Figure 2. To conclude the discussion, we finish with the following relevant addendum.

Proposition 6. Consider $\rho \in (0, \infty)$ and suppose $v > v_{pp}^k$, in such a way that K_{pp}^{SS} , K_{cp}^{SS} be surely positive at least in a duopoly. Then, there exists a restricted family of concentrated oligopolies investing more if the common pool is parcelised.

The results of this section are robust for parameters α and β covering the price and cost setup. Only if either α is very low (representing low sensitivity of the price on total output) or β is very high (high cost of exploration efforts), the qualitative shape remains; however, the intersections of k_{pp}^{SS} with k_{cp}^{SS} and K_{pp}^{SS} with K_{cp}^{SS} as observed in Figures 2 and 3 disappears. The “possible” range for v has been defined in assumptions A and B for which the results turn out to be robust.

6. Concluding remarks

Firms may invest along several dimensions, including some with an environmental nature, and one should expect the intensity of the market competition to affect their individual and collective behaviors. In line with a well established strand of research belonging to the theory of industrial organization, the games illustrated in the foregoing discussion have tackled the issue of the impact of the industry structure on the incentive of investing to expand the drilling grounds rather than obtaining process or product innovations.

The properties of the resulting steady states do not confirm the Schumpeterian hypothesis, as aggregate efforts are either nonmonotone and single-peaked or monotonically increasing in industry fragmentation. However, this is the response of the open-loop Nash equilibrium solution dictated by

the impossibility of conjecturing a plausible guess on the shape of the value function as necessary for an analytic assessment of a feedback equilibrium. Moreover, the consequences of extraction are not accounted for, and are therefore also unregulated. If the economic exploitation of the nonrenewable resource is responsible for green house gas (GHG) emissions, then the appearance of, say, an emission tax, could modify the results in relevant ways. While the obstacle to the solvability of the game under feedback information is hardly removable, the alternative extension may indeed be practicable, and is left for future research.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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