A MIN-MAX APPROACH TO RESERVOIR MANAGEMENT

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PREFACE

Analysis concerned with problems of the rational use of natural resources almost invariably deals with uncertainties with regard to the future behavior of the system in question and with multiple objectives reflecting conflicting goals of the users of the resources. Uncertainty means that the information available is not sufficient to unambiguously predict the future of the system, and the multiplicity of the objectives, on the other hand, calls for establishing rational trade-offs among them. The rationality of the trade-offs is quite often of subjective nature and cannot be formally incorporated into mathematical models supporting the analysis, and the information with regard to the future may vary with time. Then the challenge to the analyst is to elaborate a mathematical and computer implemented system that can be used to perform the analysis recognizing both the above aspects of real world problems.

These were the issues addressed during the summer study "Real-Time Forecast versus Real-Time Management of Hydrosystems," organized by the Resources and Environment Area of IIASA in 1981. The general line of research was the elaboration of new approaches to analyzing reservoir regulation problems and to estimating the value of the information reducing the uncertainties. Computationally, the research was based on the hydrosystem of Lake Como, Northern Italy.

This paper describes the application of an innovative approach to problems of reservoir management. This approach, which focuses on a risk-adverse regulation of a hydrosystem, takes into account both major aspects of this type of problem: uncertainty with regard to inflows of water into the system, and also multiple objectives which are faced by the manager. The theoretical basis of the approach has been described in another paper of this series of publications. This paper is more application-oriented, and contains also computational results for the regulation problem of Lake Como in Northern Italy.

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A MIN-MAX APPROACH TO RESERVOIR MANAGEMENT

S. Orlovsky, S. Rinaldi,
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1. INTRODUCTION

The problem considered in this paper is the one of real-time management of a multipurpose reservoir. For didactic reasons, it will be assumed that there are only two management goals, namely satisfaction of the water demand of the downstream users and attenuation of the storage peaks in the reservoir. Since these goals are conflicting and incommensurable, the solution of the problem will be a set of efficient operating rules (see Cohon and Marks 1975).

Many different methods for determining such operating rules have been developed so far. They differ in the formal description of the objectives and constraints, in the way they deal with uncertainties, and in the solution algorithms (linear programming, dynamic programming, discrete dynamic programming, optimal control, etc.). One common feature of these methods is the explicit or implicit use of the notion of probability to describe future inflows and evaluate systems performance. For this reason, all these methods could be considered as different formal expressions of the stochastic approach to reservoir management.

Very frequently, managers react in an unfavorable way to the above optimization methods (see Helweg et al. 1982). Reasons for this may be the inadequate description of the physical characteristics of the system, the complexity of the proposed algorithms, and the need for on-line optimization. Other reasons may be the lack of confidence in sophisticated stochastic techniques and the feeling that single-value operating rules are tools too rigid for solving complex but soft decision-making processes. Finally, detailed analyses made in other fields of management science have proved that decision makers very seldom consider long-term expected values of physical and/or economic indicators as representative measures of system performance. Indeed, very often, a manager focuses his attention and effort on avoiding dramatic failures when the system is under stress. In other words, in most cases, decision makers are risk-adverse, even if this entails a worse average performance of the system.

Reservoir managers are no exception. For example, when the results of a detailed optimization study on Lake Como, Italy, (see Guariso et al. 1982a) were presented to the manager, he recognized that he was not completely satisfied by the kind of objectives selected in that study (mean yearly water deficit, average number of days of flood per year, and mean yearly hydropower production). Being risk-adverse, the manager instead showed a pronounced interest toward the possibility of avoiding severe failures of the system during extreme hydrological episodes, like those he had personally experienced in the past.

For all these reasons, we use in this paper a deterministic (min-max) approach to the management problem, which overcomes many of the above criticisms. The theoretical basis of the approach in a more general context of storage control problems is described in Orlovsky et al., 1982. In this approach, the performance of the system is evaluated with reference to a few specific inflow sequences suggested by the manager. These inflow sequences may be some real or synthetic inflow records or some hypothetical sequences of inflows which the manager considers as particularly well-suited for testing the realiability of any operating rule. Of course, the solutions suggested by this approach will be dependent upon the input data, namely the reference inflow sequences. For this reason, one must be particularly careful when selecting these sequences. For example, if there are different seasons of potential droughts and floods, one should consider inflow sequences characterized by extreme values in all these periods.

The efficient solutions suggested by the min-max approach will be shown to have some definite advantage with respect to those obtained by means of the stochastic methods. In fact, they do not require complex algorithms and on-line optimization; they can be visualized in terms of classical storage allocation zones; they make reasonable use of the real-time forecast of the inflows; and they suggest a whole interval of possible releases instead of a single value. The last such property is particularly important, since it introduces some flexibility into the decision-making process. For example, the manager might use this freedom to heuristically accommodate for secondary objectives which were not taken into account in the description of the problem. He might as well satisfy unexpected water demands or other unpredictable needs. Finally, one might even use this freedom to optimize the average long-term performance of the system, thus putting risk-adverse and mean profit-maximizing attitudes in a precise lexicographic order.

The paper is organized in the following way. In the next section, the two objective management problem is formulated, and feasible and efficient operating rules are defined. Sections 3 and 4 show how the problems of satisfaction of demand of downstream users and attenuation of storage peaks in the reservoir can be solved with the min-max approach. Then, the results are used in Sections 5 and 6 to point out feasible and efficient operating rules of the double objective management problem. Finally, Section 7 deals with the so-called "linear case" for which explicit solutions are presented and applied in the next section to solve a lake management problem.

2. PROBLEM STATEMENT

Let us consider a reservoir described on a daily basis by the continuity (mass balance) equation

$$s_{t+1} = s_t + a_t - r_t \quad , \tag{1}$$

where s_t is the storage at the beginning of day t, and a_t and r_t are inflow and release during the same day. The problem we consider is focused on the determination of operating rules of the form

$$r_t = r(t, s_t, a_t) \quad , \tag{2}$$

which can be considered "efficient" in the sense specified below. These operating rules must satisfy the following physical constraint

$$0 \le r(t, s_t, a_t) \le N(s_t) \quad , \tag{3}$$

where $N(s_t)$ is the maximum amount of water that can be released in one day when the storage is equal to s_t at the beginning of that day. In the case where the reservoir is a regulated lake, the function N is the open-gate stage-discharge function associated with the regulation dam.

Let us assume that the daily water demands of the downstream users are given, so that the corresponding reference release r_t^*, t = 0,1,...,364, is known. If the release r_t is greater than r_t^*, there is no benefit surplus. If, on the other hand, there is a deficit, i.e., if r_t is smaller than r_t^*, then there are detectable downstream damages. The minimum yearly value α of the ratio between actual and reference release, i.e.,

$$\alpha = \min_{0 \le t \le 364} \left[\frac{r_t}{r_t^*} \right] \quad ,$$

is considered as a meaningful indicator of yearly damages suffered by the downstream users. The maximization of this indicator is therefore one of the goals of the management.

The second goal, obviously conflicting with the first one, is the attenuation of the storage peaks. To describe this goal in quantitative terms, assume that the maximum storage s_t^*, t = 0,1,...,364, at which there are no damages, is known. This reference storage s_t^* may correspond, for example, to the level of the lowest populated or cultivated area around the lake. The maximum yearly value β of the ratio between actual and reference storage, i.e.,

$$\beta = \max_{0 \leq t \leq 364} \left[\frac{s_t}{s_t^*} \right] \quad ,$$

will be used in the following as the indicator of flood damages. The second goal of the manager is therefore the minimization of this indicator.

In order to compare the performance of different operating rules, reference is made to a set I of n one year-long daily inflow sequences $\{a_t^i\}$, i.e.,

$$I = \{\{a_t^i\}, \; t = 0,\ldots,364; \quad i = 1,\ldots,n\} \quad .$$

In the following, this set is called the reference set. In general, it contains recorded or synthetic sequences of inflows that the manager considers as possible in the future and particularly troublesome. The operating rules which the method will select are those which guarantee satisfactory performance of the system for such reference hydrologic years. For this reason, they look appealing to the manager because they are particularly robust when the system is under stress. In the case when the reservoir is already in operation, one might consider as sequences of the reference set a selection of the most wet and dry years experienced by the manager. In so doing, the proposed operating rules may also be compared with the performance the manager was able to achieve in practice.

Operating rules which minimize flood damages and water shortages in the worst possible case out of the reference set are looked for. In doing that, one must properly constrain the reservoir storage at the end of the year (otherwise, in real operation, good performances in one year could imply very poor performances during the next year). For choosing this constraint it was assumed that the same operating rule when applied during the next year must also guarantee satisfactory values of the deficit and flood indicators for any of the yearly inflow sequences out of the reference set I. In order to quantitatively express these constraints on the terminal conditions, let us indicate with $s_t^i(s_o, r)$ the storage obtained at time t by applying the operating rule r to a reservoir with initial storage s_o and inflows $a_o^i, a_1^i, \ldots, a_{t-1}^i$, where $i = 1, \ldots, n$ is the index of a sequence from the set I. Similarly, let us indicate by $\alpha^i(s_o, r)$ and $\beta^i(s_o, r)$ the corresponding values of the deficit and flood indicators. (In the following, the abbreviated notations s_t^i, α^i, β^i are also used). Thus, the <u>terminal constraints</u> can be given in the following form

$$\alpha^j(s_{365}^k(s_o, r), r) \geq \min_{1 \leq i \leq n} [\alpha^i(s_o, r)] \triangleq \alpha(s_o, r) \quad , \quad (4a)$$

$$\beta^j(s_{365}^k(s_o, r), r) \leq \max_{1 \leq i \leq n} [\beta^i(s_o, r)] \triangleq \beta(s_o, r) \quad , \quad (4b)$$

$$j = 1, \ldots, n; \qquad k = 1, \ldots, n \quad .$$

It is now possible to clearly define feasible and efficient solutions of the double objective management problem (see, for instance, Cohon and Marks 1975). A <u>feasible solution</u> is a pair (s_o, r) of an initial storage and an operating rule which is such that the storages and releases computed by means of Equations (1) and (2) in correspondence to any reference inflow sequence satisfy the physical constraint (3) and the terminal constraints (4). Moreover, a feasible solution (s_o, r) is said to be efficient if there are no other feasible solutions (s_o', r') which can improve one of the two indicators without worsening the other one (see line BC of Figure 1). On the other hand, a feasible solution is called dominated if there are other feasible solutions with better values of both indicators (internal points of the shadowed region of Figure 1). The feasible solutions which are neither efficient nor dominated (see segments AB and CD of Figure 1) will be called semi-efficient. These solutions can obviously be improved by improving one of the two objectives without perturbing the other one. Figure 1 shows all these solutions in the space (α, β) of the indicators and points out the absolute maximum value α_{max} and the absolute minimum value β_{min} of the indicators for which feasible solutions can be found. The point U with coordinates $(\alpha_{max}, \beta_{min})$ is infeasible because the goals are conflicting and is therefore called the utopia point.

In order to find efficient and semi-efficient solutions to the problem, we will first consider two simple problems. The first one (see next section) is called demand satisfaction and consists of determining solutions (s_o, r) which satisfy the physical constraint (3) and the terminal constraint (4a) for a given value, say α, of the indicator $\alpha(s_o, r)$. Similarly, the second problem (see Section 4), called flood protection, consists

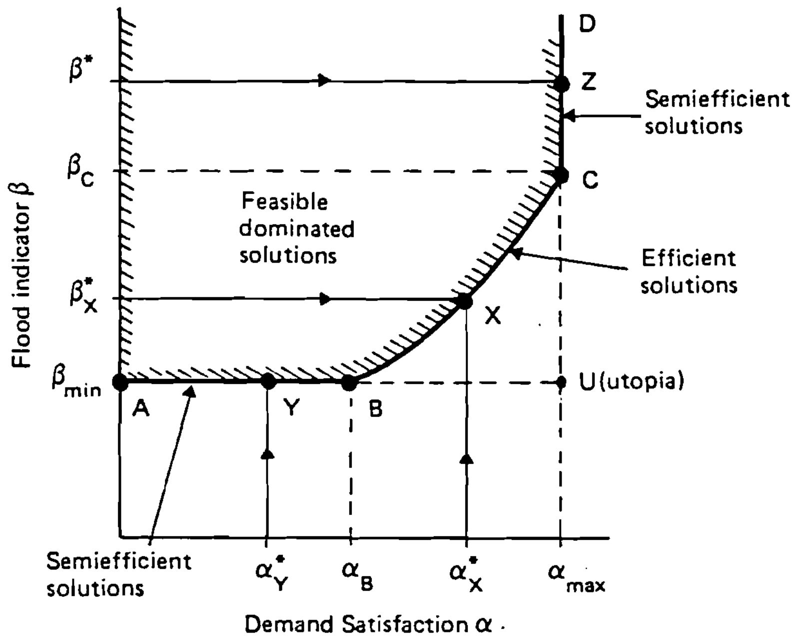

Figure 1. Efficient, semi-efficient, and dominated solutions in the space (α, β) of the indicators.

of determining pairs (s_o, r) which satisfy constraint (3) and the terminal constraint (4b) for $\beta(s_o, r) = \beta$, with β being a given value.

3. DEMAND SATISFACTION

We are now interested in finding initial storage s_o^α and operating rules r^α which can guarantee that the yearly deficit indicator will not be smaller than a prescribed value α for all inflow sequences of the reference set I. This is equivalent to saying that we will find initial storage and operating rules which can guarantee the satisfaction of the <u>reduced water demand</u> αr_t^*.

Of course, a solution to this problem exists, provided the value α of the deficit indicator is sufficiently small. One such solution corresponds to the so-called <u>minimum release policy</u> r_{min}^α given by

$$r_t = r_{min}^\alpha(t, s_t) = \min\{N(s_t), \alpha r_t^*\} \quad . \tag{5}$$

This policy satisfies, by definition, the physical constraint (3). Therefore, a pair (s_o, r_{min}^α) is a solution to the problem, provided the release

$$r_t = r_{min}^\alpha(t, s_t(s_o, r_{min}^\alpha)) \quad ,$$

never drops below the reduced demand αr_t^* and the terminal constraint (4a) is satisfied for all inflow sequences from the reference set. Moreover, if the pair (s_o, r_{min}^α) is a solution to the problem, then any pair (s_o', r_{min}^α) with greater initial storage $(s_o' > s_o)$ is obviously also a solution. Therefore, we are actually interested in finding the minimum initial storage, say $s^{\alpha}_{o\,min}$, which together with the operating rule r^{α}_{min} can guarantee the satisfaction of the reduced water demand αr^{*}_{t}. The <u>minimum storage</u> $s^{\alpha}_{o\,min}$ can be obtained by solving the following simple mathematical programming problem, called Problem O. In this problem, the constraint $s^{i}_{365} \geq s_{o}$ is a surrogate of the terminal constraint (4a).

<u>Problem O</u> (Determination of $s^{\alpha}_{o\,min}$)

$$s^{\alpha}_{o\,min} = \min s_{o} \tag{6a}$$

$$s^{i}_{o} = s_{o} \qquad i = 1,\ldots,n \tag{6b}$$

$$s^{i}_{t+1} = s^{i}_{t} + a^{i}_{t} - r^{\alpha}_{min}(t, s^{i}_{t}) \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \tag{6c}$$

$$r^{\alpha}_{min}(t, s^{i}_{t}) = \alpha r^{*}_{t} \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \tag{6d}$$

$$s^{i}_{365} \geq s_{o} \qquad i = 1,\ldots,n \tag{6e}$$

Problem O can be solved by simulating the reservoir behavior with initial condition s_{o} and operating rule r^{α}_{min} for all inflow sequences $\{a^{i}_{t}\}$ of the reference set I. If constraints (6d) and (6e) are satisfied, then $s^{\alpha}_{o\,min} \leq s_{o}$, otherwise $s^{\alpha}_{o\,min} > s_{o}$. Thus, a very simple one-dimensional searching procedure (e.g., bisection) can be used to determine $s^{\alpha}_{o\,min}$. In the case where the stage-discharge function N is linear, Problem O is actually a linear programming problem. In fact, in Equation (6c), $r^{\alpha}_{min}(t, s^{i}_{t})$ can be replaced by αr^{*}_{t}, (see Equation (6d)), and Equation (6d) can be substituted by

$$\alpha r^{*}_{t} \leq N(s^{i}_{t}) \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \quad ,$$

which is a linear constraint in the case that N is such.

The problem of finding the minimum storage $s^{\alpha}_{\tau min}$ that can guarantee together with the minimum release policy (5) the satisfaction of all the constraints throughout the rest of the year (after day τ), can be formulated in the following way.

Problem τ (Determination of $s^{\alpha}_{\tau min}$, $\tau = 1,\ldots,364$)

$$s^{\alpha}_{\tau min} = \min s_t \qquad (7a)$$

$$s^{i}_{\tau} = s_{\tau} \qquad i = 1,\ldots,n \qquad (7b)$$

$$s^{i}_{t+1} = s^{i}_{t} + a^{i}_{t} - r^{\alpha}_{min}(t, s^{i}_{t}) \qquad i = 1,\ldots,n \quad t = \tau,\ldots,364 \qquad (7c)$$

$$r^{\alpha}_{min}(t, s^{i}_{t}) = \alpha r^{*}_{t} \qquad i = 1,\ldots,n \quad t = \tau,\ldots,364 \qquad (7d)$$

$$s^{i}_{365} \geq s^{\alpha}_{o\,min} \qquad i = 1,\ldots,n \qquad (7e)$$

As in the preceding case, $s^{\alpha}_{\tau min}$ can be obtained by guiding simulations with a one-dimensional searching method, and again the problem reduces to a linear programming problem when the stage-discharge function is linear. It is worthwhile noticing that the solution to Problem O, namely $s^{\alpha}_{o\,min}$, is used in Equation (7e) in order to guarantee the satisfaction of the terminal constraint (4a). This implies that Problem O must be solved first. On the other hand, the solution of Problem τ can be carried out independently for each value of τ.

Now that we have found a solution to the problem of satisfaction of demand, namely $(s^{\alpha}_{o\,min}, r^{\alpha}_{min})$, we can immediately obtain all other solutions $(s^{\alpha}_{o}, r^{\alpha})$. In fact, we only need to notice that a volume of water r^{i}_{t} greater than $r^{\alpha}_{min}(t, s^{i}_{t}) = \alpha r^{*}_{t}$ can be released without any consequence provided that the reservoir storage and/or the inflow are sufficiently high. More precisely, if (in some year i)

$$s_t^i + a_t^i \geq s_{t+1\,min}^{\alpha} + \alpha r_t^* \quad ,$$

any release r_t^i between αr_t^* and the minimum between $(s_t^i + a_t^i - s_{t+1\,min}^{\alpha})$ and $N(s_t^i)$ (see shaded area in Figure 2) will give rise to a storage s_{t+1}^i greater than or equal to $s_{t+1\,min}^{\alpha}$, which is indeed the minimum value of the storage that can guarantee the satisfaction of all the constraints from time t+1 to the end of the year. In conclusion, the solutions to the problem are given by all pairs $(s_o^{\alpha}, r^{\alpha})$ satisfying the following two inequalities

$$s_o^{\alpha} \geq s_{o\,min}^{\alpha} \quad , \tag{8a}$$

$$\min\{N(s_t), \alpha r_t^*\} \leq r^{\alpha}(t, s_t, a_t) \leq \min\{N(s_t), \max\{s_t + a_t - s_{t+1\,min}^{\alpha}, \alpha r_t^*\}\} \tag{8b}$$

Equation (8b) is interpreted in Figure 2, which shows that for sufficiently high values of the storage s_t and/or the inflow a_t, there is a whole interval of possible releases (shaded area). The figure also shows that the storage axis can be divided into four <u>storage allocation zones</u> named I, II, III, and IV. The first one depends only upon α since its upper limit $s_t^{'}$ is given by the storage at which the stage-discharge function N equals the reduced demand αr_t^*. The first zone is never entered if the inflow sequences $\{a_t^i\}$ are those of the reference set. It is therefore a kind of <u>dead zone</u>, which might nevertheless be reached during real operation if a drought more severe than those considered in the reference set occurs. In the second zone, the release equals the reduced demand αr_t^*, while in the third and fourth zones the release can be greater than the reduced demand.

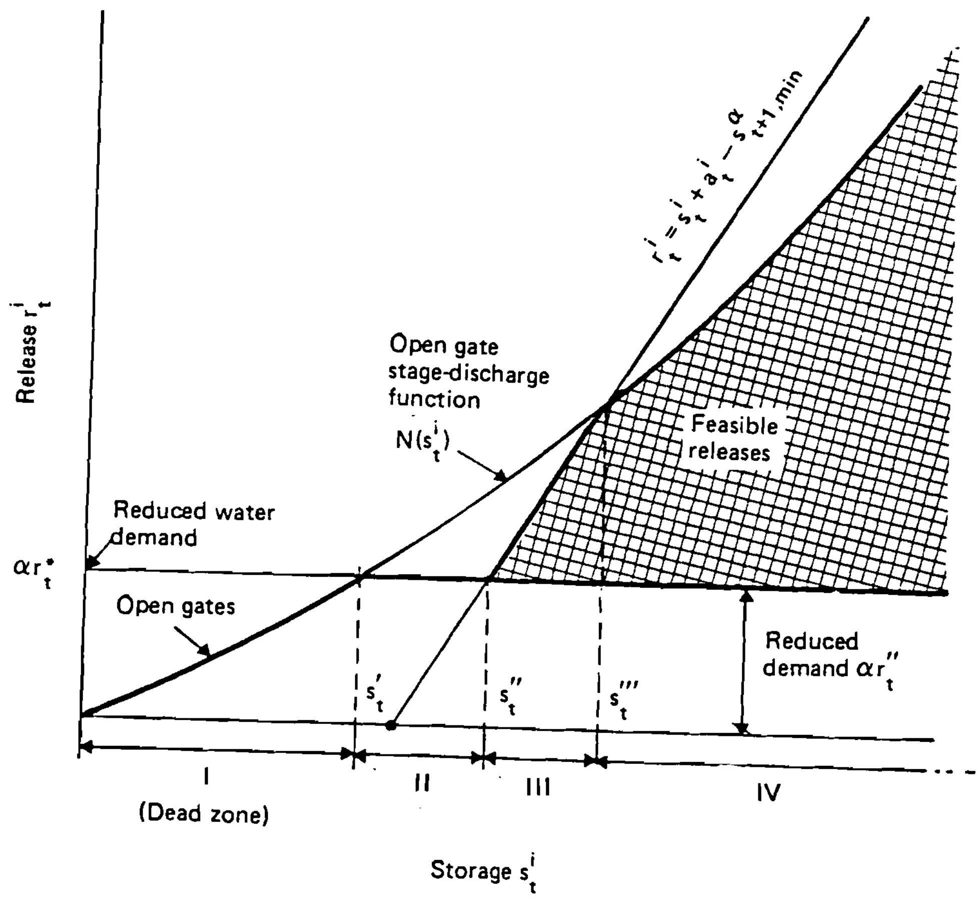

Figure 2. The set of releases which can guarantee the satisfaction of water demand (see Equation 8b).

In particular, in the last zone, the manager might even completely open the gates of the regulation dam without worsening the performance of the system with regard to the demand satisfaction.

It is worthwhile noticing that the values s_t'' and s_t''' dividing these zones depend upon the inflow a_t, since the straight line $r_t = s_t + a_t - s^{\alpha}_{t+1min}$ shifts to the left when a_t increases. This means that Equation (8b) defines a priori only the dead zone, while the others are adapted to the current value of the inflow. In real operation, one must therefore be particularly careful in forecasting the daily inflow a_t.

4. FLOOD PROTECTION

Using the preceding section as a guideline, we now deal with the problem of flood protection. We are interested in finding initial storages s^{β}_o and operating rules r^{β} which can guarantee that for each year i out of the reference set, the storage s^i_t will not be greater than the relaxed reference storage βs^*_t. Of course, solutions to this problem exist, provided the value of β is sufficiently high. Moreover, if a solution (s^{β}_o, r^{β}) exists, then the maximum release policy r_{max} (independent of β) given by

$$r_t = r_{max}(s_t) = N(s_t) \quad , \tag{9}$$

is also a solution for the same initial storage. Finally, any pair (s^{β}_o, r_{max}) will represent a solution provided the storage s^{β}_o is smaller than or equal to the maximum storage $s^{\beta}_{o\,max}$ obtained by solving the following mathematical programming problem.

<u>Problem 0</u> (Determination of $s^{\beta}_{o\,max}$)

$$s^{\beta}_{o\,max} = \max s_o \qquad (10a)$$

$$s^i_o = s_o \qquad i = 1,\ldots,n \qquad (10b)$$

$$s^i_{t+1} = s^i_t + a^i_t - r_{max}(s^i_t) \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \qquad (10c)$$

$$s^i_t \le \beta s^*_t \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \qquad (10d)$$

$$s^i_{365} \le s_o \qquad i = 1,\ldots,n \qquad (10e)$$

As in the problem of demand satisfaction, once the value of $s^{\beta}_{o\,max}$ has been determined, the following problem can be considered.

<u>Problem τ</u> (Determination of $s^{\beta}_{\tau max}$, $\tau = 1,\ldots,364$)

$$s^{\beta}_{\tau max} = \max s_{\tau} \qquad (11a)$$

$$s^i_{\tau} = s_{\tau} \qquad i = 1,\ldots,n \qquad (11b)$$

$$s^i_{t+1} = s^i_t + a^i_t - r_{max}(s^i_t) \qquad i = 1,\ldots,n \quad t = \tau,\ldots,364 \qquad (11c)$$

$$s^i_t \le \beta s^*_t \qquad i = 1,\ldots,n \quad t = \tau,\ldots,364 \qquad (11d)$$

$$s^i_{365} \le s^{\beta}_{o\,max} \qquad i = 1,\ldots,n \qquad (11e)$$

As in Section 3, all these problems can be solved by simulating the reservoir behavior for different (guided) values of the initial storage. Moreover, Equation (9) substituted into Equation (11c) gives rise to a linear programming problem if the storage-discharge function is linear.

All the solutions (s^{β}_o, r^{β}) can immediately be obtained from the solution $(s^{\beta}_{o\,max}, r_{max})$. In fact, one can notice that the performance of the system does not change if the release r_t is smaller than $r_{max}(s_t)$, provided the reservoir is sufficiently empty and/or the inflow is sufficiently low. More precisely, if (in a year i)

$$s_t^i + a_t^i \leq s_{t+1max}^{\beta} + N(s_t^i) \quad ,$$

then any release r_t^i between $\max\{0, s_t^i+a_t^i-s_{t+1max}^{\beta}\}$ and $N(s_t^i)$ (see shaded area in Figure 3) will give rise to a storage s_{t+1}^i smaller than or equal to s_{t+1max}^{β}, which, by definition, is the maximum value of the storage at time t+1 that can guarantee the satisfaction of the constraints from that time up to the end of the year. In conclusion, the solutions of the problem are given by the pairs (s_o^{β}, r^{β}) satisfying the following inequalities

$$s_o^{\beta} \leq s_{o\,max}^{\beta} \tag{12a}$$

$$\min\{N(s_t), \max\{s_t-a_t-s_{t+1max}^{\beta}, 0\}\} \leq r^{\beta}(t, s_t, a_t) \leq N(s_t) \quad . \tag{12b}$$

Equation (12b) is interpreted in Figure 3. The storage axis is divided into three zones, named I, II, and III. In the first one, any decision is possible: the manager might even close the gates of the dam, thus storing all the inflow, without worsening the future performance of the system. In the second zone, different options are still possible, although the manager is forced to become more and more aware of the potential floods when the storage and/or the inflow increase. Finally, in the third zone, which might be properly called the spilling zone, the manager is obliged to release the maximum he can by keeping the gates of the dam permanently open.

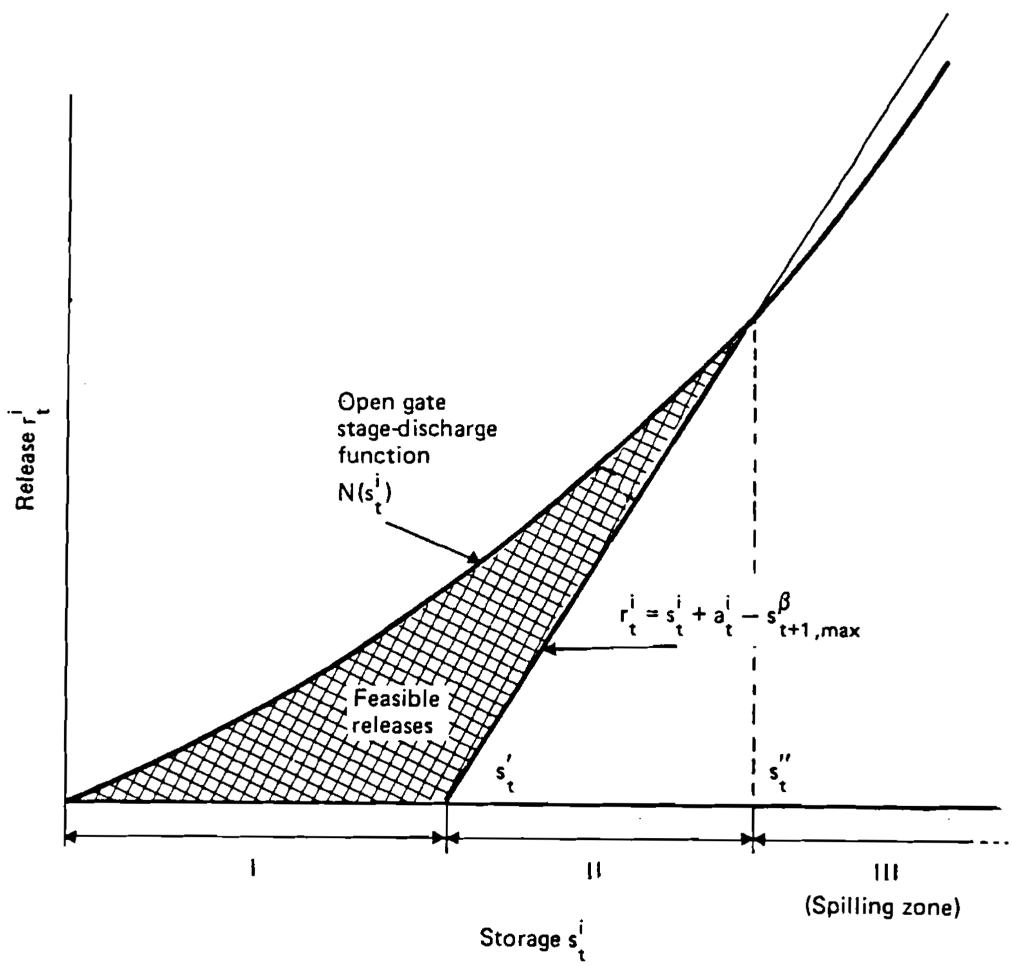

Figure 3. The set of releases which can guarantee flood protection (see Equation 12b).

5. FEASIBLE SOLUTIONS TO THE TWO OBJECTIVE PROBLEM

Feasible solutions $(s_o^{\alpha\beta}, r^{\alpha\beta})$ to the double objective problem formulated in Section 2 can now be found. In fact, by taking the intersection of the intervals defined by Equations (8a), (12a), and by Equations (8b), (12b), and by suitably re-arranging the various terms, one can prove that any pair $(s_o^{\alpha\beta}, r^{\alpha\beta})$ such that

$$s^{\alpha}_{o\,min} \le s_o^{\alpha\beta} \le s^{\beta}_{o\,max} \quad , \tag{13a}$$

$$\min\{N(s_t),\ \max\{s_t+a_t-s^{\beta}_{t+1max},\ \alpha r_t^*\}\} \le r^{\alpha\beta}(t,\ s_t,\ a_t)$$
$$\le \min\{N(s_t),\ \max\{s_t+a_t-s^{\alpha}_{t+1min},\ \alpha r_t^*\}\} \quad , \tag{13b}$$

is a feasible solution of the problem described in Section 2.

Equation (13b) constraining the feasible operating rules is interpreted in Figure 4. The storage axis is divided into six parts. The first (I) and the last (VI) are the dead and spilling zones which have already been discussed. In the second zone--which might be called the <u>buffer zone</u>--the manager has no alternative except to release the reduced water demand αr_t^*. Then, we have the <u>conservation zone</u> $(s_t'' \le s_t^i \le s_t^v)$, which is in turn sub-divided into three subzones (II, III, and IV). In these zones, there is a whole range of possible releases among which the manager can freely choose without any consequence on the system performance. Nevertheless, the three subzones III, IV, and V show the declining importance of one goal (satisfaction of demand) versus the other (flood protection). In fact, in zone III, it is possible to release the reduced water demand, thus saving water to compensate possible future periods of low inflows, while in zone V it is possible to release water from

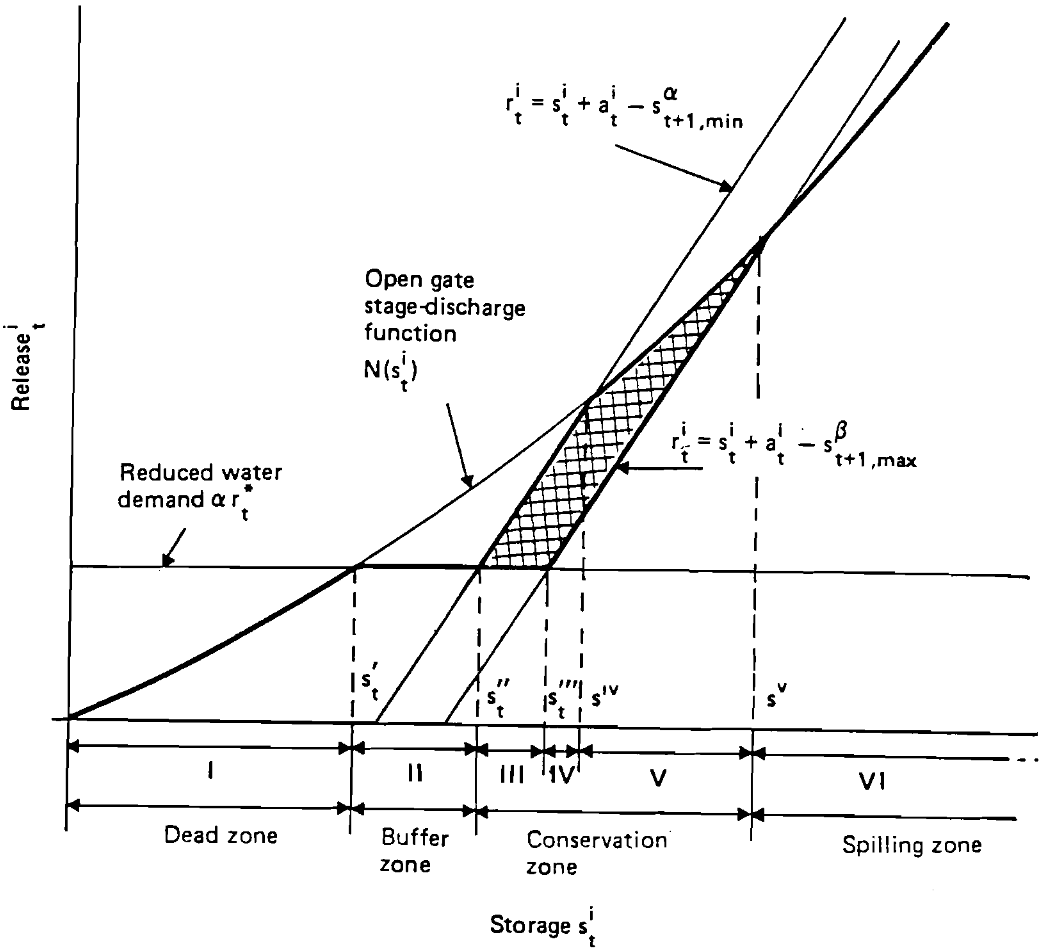

Figure 4. The set of releases which can guarantee satisfaction of demand and flood protection at the same time (see Equation 13b).

the reservoir at the maximum rate, thus avoiding future floods. On the other hand, in zone IV, none of these limit management policies is allowed.

The shaded region in Figure 4 is more or less wide, depending upon the time of the year and upon the values α and β of the two indicators. Actually, the fact that a pair (α,β) cannot be guaranteed at all is simply revealed by the vanishing of this region. This obviously happens when

$$s^{\alpha}_{t\,\min} > s^{\beta}_{t\,\max} \quad ,$$

on some day t. On the other hand, if

$$s^{\alpha}_{t\,\min} \leq s^{\beta}_{t\,\max} \quad , \qquad t = 0,\ldots,364 \tag{14}$$

feasible solutions to the problem always exist. Moreover, if α and β are such that

$$s^{\alpha}_{t\,\min} < s^{\beta}_{t\,\max} \quad , \qquad t = 0,\ldots,364$$

we can say that the corresponding feasible solutions given by Equation (13) are dominated, since we can increase α and reduce β until we obtain Equation (14) with the equality sign holding in at least one constraint. This could actually be a useful test for finding efficient or semi-efficient operating rules. Nevertheless, a much more direct method can be devised, as shown in the next section.

6. EFFICIENT SOLUTIONS

We will now describe a simple method for finding efficient and semi-efficient solutions to the double objective problem described in Section 2. The method includes two steps. First, given a value α^* smaller than or equal to α_{max} (see Figure 1), the corresponding minimum value $\beta^*(\alpha^*)$ of the second indicator is computed. Second, the feasible solutions $(s_o^{\alpha^*\beta^*(\alpha^*)}, r^{\alpha^*\beta^*(\alpha^*)})$ are determined by means of Equation (13). These solutions are either efficient or semi-efficient (see points X and Y in Figure 1).

An analogous procedure starting from a given value β^* of the flood indicator could also be followed. In this case, the corresponding maximum value $\alpha^*(\beta^*)$ of the flood indicator is first obtained, and then the solutions $(s_o^{\alpha^*(\beta^*)\beta^*}, r^{\alpha^*(\beta^*)\beta^*})$ are determined by Equation (13). Again, these solutions are either efficient or semi-efficient (see points X and Z of Figure 1).

The two above procedures can be used sequentially in order to detect if a solution is efficient or semi-efficient. For example, starting from the value α_X^*, point X is obtained, and then the second procedure applied with $\beta^* = \beta_X^*$ (see Figure 1) will again give the same point X, thus confirming that (α_X^*, β_X^*) is an efficient solution. On the other hand, if one starts from α_Y^*, point Y will first be obtained, but then the second procedure will generate point B.

Now, only the first step of the method is described, since the second one has already been discussed in Section 5. For this, assume that a value α^* of the first indicator is given. Equation (8b) can therefore provide operating rules which can guarantee the satisfaction of the reduced water demand $\alpha^* r_t^*$. In particular, consider the operating rule $r_{max}^{\alpha^*}$ which corresponds to the right-hand side of Equation (8b), i.e.,

$$r_{max}^{\alpha^*}(t, s_t, a_t) = \min\{N(s_t), \max\{s_t+a_t-s_{t+1min}^{\alpha^*}, \alpha^* r_t^*\}\} \tag{15}$$

Among all the operating rules which guarantee the value α^* for the first indicator, this is obviously the one which minimizes the flood indicator β. Thus, the following simple mathematical programming problem can be set up for determining $\beta^*(\alpha^*)$.

$$\beta^*(\alpha^*) = \min \beta \tag{16a}$$

$$s_o^i = s_o \qquad i = 1,\ldots,n \tag{16b}$$

$$s_o \geq s_{o\,min}^{\alpha^*} \tag{16c}$$

$$s_{t+1}^i = s_t^i + a_t^i - r_{max}^{\alpha^*}(t,\ s_t^i,\ a_t^i) \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \tag{16d}$$

$$s_t^i \leq \beta s_t^* \qquad i = 1,\ldots,n \quad t = 0,\ldots,364 \tag{16e}$$

$$s_{365}^i \leq s_o \qquad i = 1,\ldots,n \tag{16f}$$

In this paper, constraint (16c) is needed to guarantee Equation (8a), while constraint (16f) ensures the satisfaction of the terminal condition (4b). The problem can be solved by simulating the reservoir behavior with initial storage $s_o \geq s_{o\,min}^{\alpha^*}$ and operating rule $r_{max}^{\alpha^*}$ for all inflow sequences $\{a_t^i\}$ of the reference set. If all constraints (16f) are satisfied with the strict inequality sign, then

$$\beta^*(\alpha^*) < \max_{1 \leq i \leq n} \ \max_{0 \leq t \leq 364} \left[\frac{s_t^i}{s_t^*}\right] ,$$

since one can obviously find better solutions by lowering the initial storage s_o (and hence the maximum flood peak). Therefore, one must simulate the reservoir behavior again for a smaller value of the initial storage and repeat this operation until at least one of the n constraints (16f) is satisfied with the equality sign. The corresponding value of $\max_{1 \le i \le n} \max_{0 \le t \le 364} \left[\frac{s_t^i}{s_t^*} \right]$ is obviously $\beta^*(\alpha^*)$.

7. THE LINEAR CASE

In the case where the stage-discharge function N is linear, i.e.,

$$N(s) = \gamma s + \delta \quad ,$$

Problem O and Problem τ of Sections 3 and 4 become linear programming problems and can be solved explicitly.

Let us first consider the problem of demand satisfaction and define the cumulative water demand R_τ^{*t} in the interval $[\tau, t]$ as

$$R_\tau^{*t} = \sum_{\delta=\tau}^{t} r_\delta^* \quad .$$

Moreover, let us denote by A_τ^t the lowest cumulative inflow of the reference set in the interval $[\tau, t]$, i.e.,

$$A_\tau^t = \min_{1 \le i \le n} \sum_{\delta=\tau}^{t} a_\delta^i \quad .$$

Notice that these data (R_τ^{*t} and A_τ^t) can be pre-computed. Finally, let us indicate by $\tilde{s}_t(\alpha)$ the minimum storage needed to guarantee the reduced water demand αr_t^* at time t, i.e.,

$$\tilde{s}_t(\alpha) = N^{-1}(\alpha r_t^*) = \frac{\alpha r_t^* - s}{\gamma} \quad .$$

Problem O of Section 3 is therefore equivalent to the following problem:

$$s^{\alpha}_{o\,min} = \min s_o \tag{17a}$$

$$s^i_o = s_o \qquad i = 1,\dots,n \tag{17b}$$

$$s^i_{t+1} = s^i_t + a^i_t - \alpha r^*_t \qquad i = 1,\dots,n \quad t = 0,\dots,364 \tag{17c}$$

$$s_o \geq \tilde{s}_o(\alpha) \tag{17d}$$

$$s^i_t \geq \tilde{s}_t(\alpha) \qquad i = 1,\dots,n \quad t = 1,\dots,364 \tag{17e}$$

$$s^i_{365} \geq s_o \qquad i = 1,\dots,n \tag{17f}$$

From Equation (17c) we obtain

$$\min_{1\leq i\leq n} [s^i_t] = s_o + A^{t-1}_o - \alpha R^{*t-1}_o \quad ,$$

so that Equation (17e) can be substituted by

$$s_o \geq \tilde{s}_t\,(\alpha) + \alpha R^{*t-1}_o - A^{t-1}_o \qquad t = 1,\dots,364 \quad .$$

Similarly, constraint (17f) is equivalent to

$$A^{364}_o \geq \alpha R^{*364}_o \quad .$$

Thus, in conclusion, the solution of Problem O is given by

$$s^{\alpha}_{o\,min} = \max\{\tilde{s}_o(\alpha),\ \max_{1\leq t\leq 364} [\tilde{s}_t(\alpha)+\alpha R^{*t-1}_o - A^{t-1}_o]\} \quad , \tag{18}$$

provided α is sufficiently small, namely

$$\alpha \leq \frac{A^{364}_o}{R^{*364}_o} \quad .$$

One must remark that this is a well-known result of mass-curve analysis (see Rippl 1883).

In a similar way we can deal with Problem τ and prove that

$$s^{\alpha}_{\tau\min} = \max\{\tilde{s}_{\tau}(\alpha),\ s^{\alpha}_{o\min} + \alpha R^{*364}_{\tau} - A^{364}_{\tau},\ \max_{\tau+1\le t\le 364} [\tilde{s}_t(\alpha) + \alpha R^{*t-1}_{\tau} - A^{t-1}_{\tau}]\} \quad . \tag{19}$$

Equations (18) and (19) actually also hold if the function N is non-linear. Nevertheless, in the linear case, one can prove that $s^{\alpha}_{o\min}$ and $s^{\alpha}_{\tau\min}$ are piecewise linear, increasing, and convex with respect to α (in fact, all the terms appearing in Equations (18) and (19) are linear functions of α). These properties can be used in an obvious way to save computation time when the operating rules r^{α} must be found for different values of α.

Let us now consider Problem 0 of Section 4. Such a problem can be re-formulated as

$$s^{\beta}_{o\max} = \max s_o \tag{20a}$$

$$s^i_o = s_o \qquad i = 1,\dots,n \tag{20b}$$

$$s^i_{t+1} = s^i_t + a^i_t - \gamma s^i_t - \delta \qquad i = 1,\dots,n \quad t = 0,\dots,364 \tag{20c}$$

$$s_o \le \beta s^*_o \tag{20d}$$

$$s^i_t \le \beta s^*_t \qquad i = 1,\dots,n \quad t = 1,\dots,364 \tag{20e}$$

$$s^i_{365} \le s_o \qquad i = 1,\dots,n \tag{20f}$$

But from Equation (20c) one obtains

$$\max_{1\le i\le n} [s^i_t] = (1-\gamma)^t\, s_o + C^{t-1}_o \quad ,$$

with

$$C^{t-1}_o = \max_{1\le i\le n} \sum_{\delta=o}^{t-1} (1-\gamma)^{\delta} (a^i_{t-1-\delta} - \delta) \quad .$$

which are data that can be pre-computed. Thus, constraints (20e) and (20f) are equivalent to

$$s_o \leq \frac{\beta s_t^* - C_o^{t-1}}{(1-\gamma)^t}$$

$$s_o \geq \frac{C_o^{364}}{1-(1-\gamma)^{365}} \quad .$$

From this, it follows that

$$s^{\beta}_{o\,max} = \min\{\beta s_o^*, \min_{1\leq t\leq 364}\left[\frac{\beta s_t^* - C_o^{t-1}}{(1-\gamma)^t}\right]\} \quad , \qquad (21)$$

if, and only if, the data satisfy the following condition

$$\frac{C_o^{364}}{1-(1-\gamma)^{365}} \geq \min\{\beta s_o^*, \min_{1\leq t\leq 364}\left[\frac{\beta s_t^* - C_o^{t-1}}{(1-\gamma)^t}\right]\} \quad .$$

(Notice that this inequality holds if β is sufficiently high).

Problem τ of Section 4 can be re-formulated and solved in a similar way and the final result is that $s^{\beta}_{\tau max}$ can be given the following explicit expression

$$s^{\beta}_{\tau max} = \min\{\beta s_\tau^*, \frac{s^{\beta}_{o\,max} - C_\tau^{364}}{(1-\gamma)^{364-\tau}} , \min_{\tau+1\leq t\leq 364}\left[\frac{\beta s_t^* - C_\tau^{t-1}}{(1-\gamma)^{t-\tau}}\right]\} \qquad (22)$$

where C_τ^t are the following pre-computed data

$$C_\tau^t = \max_{1\leq i\leq n} \sum_{\delta=o}^{t-\tau} (1-\gamma)^{\delta}(\alpha^i_{t-\delta} - \delta) \quad .$$

Equations (21) and (22) can only be derived by making explicit use of the linearity of the stage-discharge function. Indeed, that property has been used to explicitly integrate the continuing equation with r_t^i linearly related to s_t^i. Moreover, the linearity of N implies that $s^{\beta}_{o\,max}$ and $s^{\beta}_{\tau max}$ are <u>piecewise linear</u>, <u>increasing</u>, and <u>concave</u> with respect to β.

8. EXAMPLE OF APPLICATION

The method described in the previous sections has been applied to the case of Lake Como (Northern Italy). This lake has been regulated on a daily basis since 1946. The main goals of the manager (actually a committee) are the satisfaction of the water requirements of the downstream users and the protection of the lake shores from floods. The water demands of the various users (seven run-of-river hydro-electric power plants with an installed capacity of 92 MW, and six agricultural districts with a total irrigated surface of 114 000 hectares) have been properly combined to generate the desired daily reference release r_t^* which is constant during the winter and obviously attains its peak in summer (see Figure 5). The reference storage s_t^* corresponds to the lake level at which the most sunken part of the town of Como (namely the main square) is flooded. Thus, s_t^* is constant throughout the year. The stage-discharge function has been approximated by a linear function with a very satisfactory fitting (5% maximum deviation in the range of interest). As inflow sequences of the reference set I we have selected the five recorded one year-long daily inflow sequences (over the last 15 years) which were estimated as the most critical ones by the manager. Among them, we have the inflow sequence of 1976, which is characterized by a very dry summer period followed by quite severe floods in early autumn.

On the basis of these data, the efficient and semi-efficient solutions of the double objective management problem have been obtained by using the procedure indicated in Section 6, and the explicit formulae reported in Section 7. The results are shown

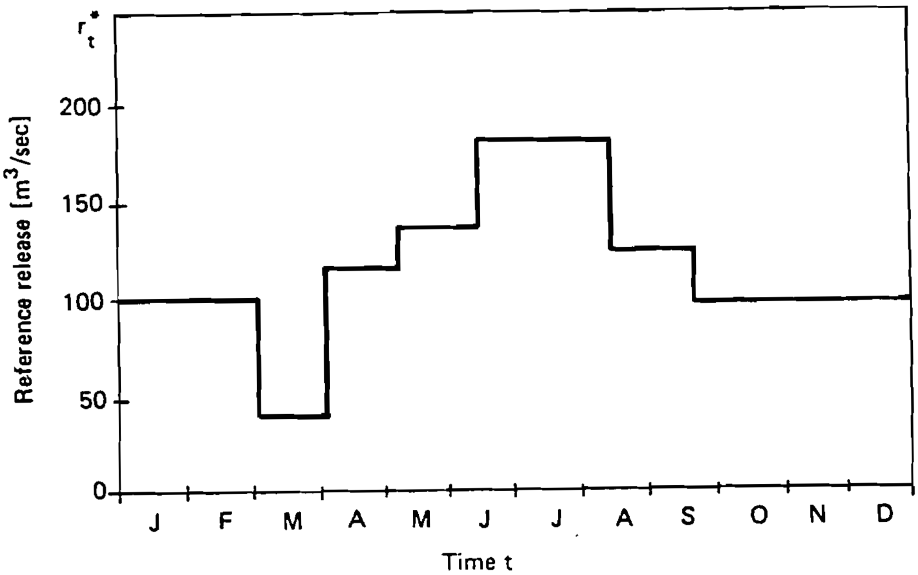

Figure 5. The reference release r_t^* of Lake Como.

in Figure 6 in the space (α, β) of the indicators. In this figure, point H represents the historical solution, namely the real performance of the manager during the years of the reference set. The value $\alpha_H = 0.30$ corresponds to the water shortage of July 1976, while the value $\beta_H = 1.5$ corresponds to the flood of October 1979 (flood peak of 1.36 meters above the main square of Como). The figure shows that the historical solution is "dominated" and can therefore be improved. In fact, all points belonging to the shaded region H P B Q are characterized by better values of the indicators. In particular, point P shows that β could be reduced to 1.35 leaving α unchanged. This would correspond to an attenuation of the maximum flood peak of about 30 cm. Similarly, point Q shows that a substantial improvement in demand satisfaction can be obtained without worsening the maximum flood peak in Como. Obviously, solutions of greater interest are the efficient ones belonging to the line BQ. Among them, point X has been selected and suggested to the manager for implementation. The formulae for the determination of the upper and lower limits of the feasible releases (see Equation (13b)) have been programmed on a microcomputer which also contains software for the real-time forecast of the inflow during the current day. This computer is now used every day by the manager as a tool for his final decision.

The values $s_{t\,min}^{\alpha^*}$ and $s_{t\,max}^{\beta^*}$ of the proposed solution are shown in Figure 7. During the year, $s_{t\,max}^{\beta^*} > s_{t\,min}^{\alpha^*}$ with the exception of one day (August 22), on which $s_{t\,max}^{\beta^*} = s_{t\,min}^{\alpha^*}$ (as shown in Section 6, this indicates that the solution is not dominated). The difference between $s_{t\,max}^{\beta^*}$ and $s_{t\,min}^{\alpha^*}$ is maximal

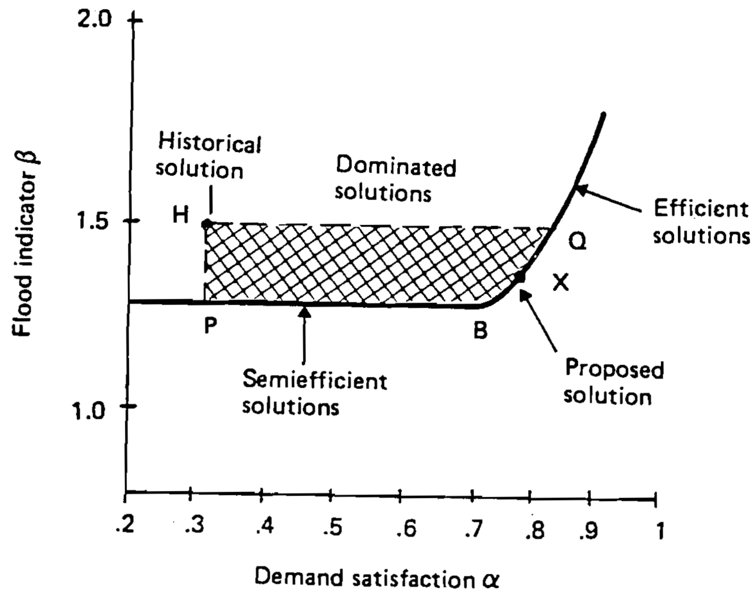

Figure 6. Efficient and semi-efficient solutions for Lake Como.

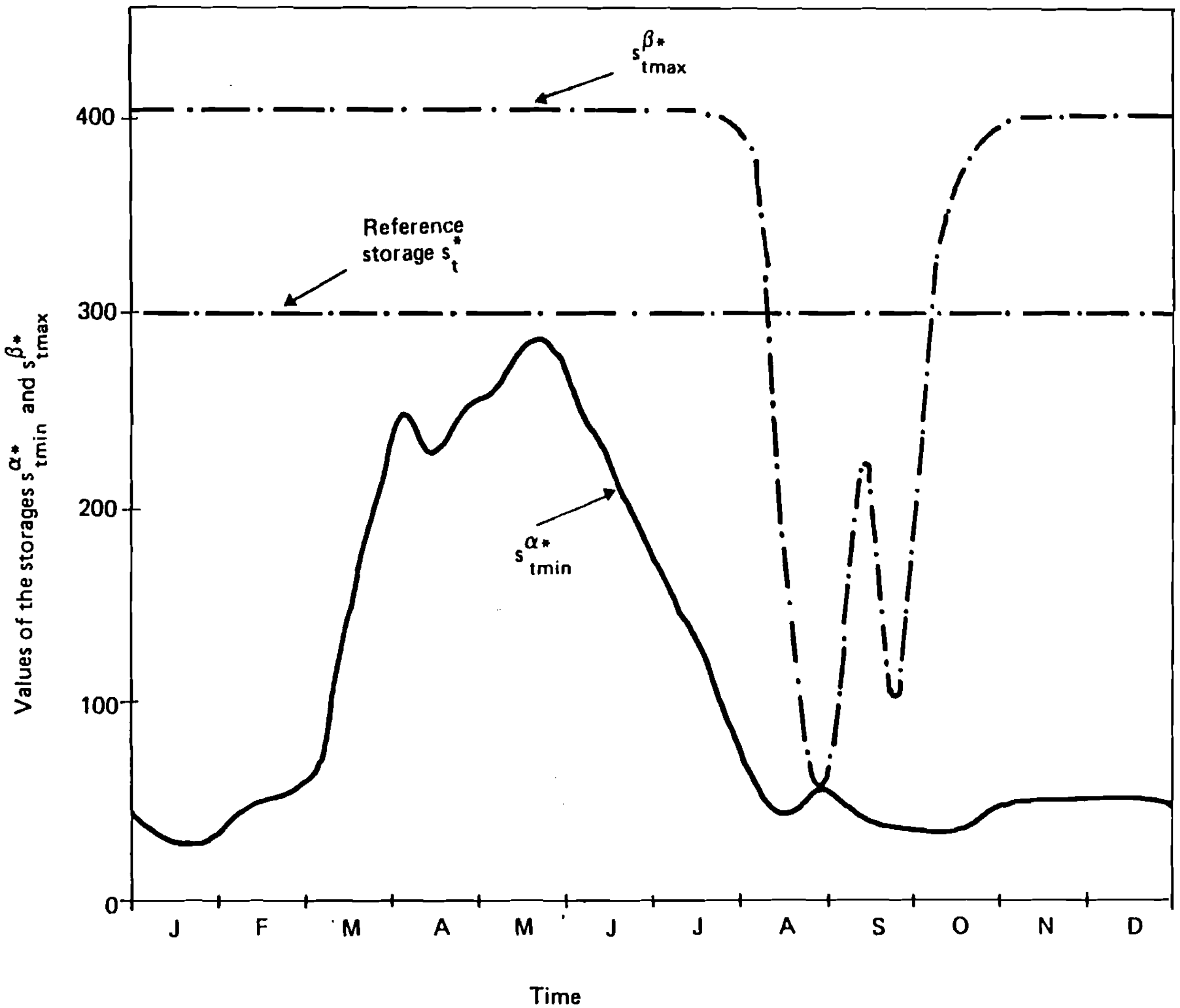

Figure 7. The values of $s^{\alpha^*}_{\tau min}$ and $s^{\beta^*}_{\tau max}$ of the proposed efficient solution.

during the winter (November-February), which means that the conservation zone (see Figure 4) in that period is quite large (in fact, this is perhaps the most relaxed period for the manager). When the snow-melt season approaches, this difference narrows down, and in the middle of May $s_{t\,min}^{\alpha^*}$ almost equals the reference storage s_t^*. This forces the manager to only release the reduced water demand in April and May, until the lake is sufficiently full. Furthermore, this is what the manager of Lake Como does every year (perhaps following a slightly different schedule). Then, the difference between $s_{t\,max}^{\beta^*}$ and $s_{t\,min}^{\alpha^*}$ increases in June and July but shrinks again during the second half of August, a period which is often characterized by sudden heavy rainfall. In particular, $s_{t\,max}^{\beta^*}$ drops dramatically during that period, thus forcing the manager to release the maximum possible amount of water (see spilling zone of Figure 4) just before the potential floods. The two pronounced minima of $s_{t\,max}^{\beta^*}$ in August and September correspond to the two most probable periods of heavy rainfall in the Alps.

This solution to the management problem of Lake Como must only be considered as a realistic numerical example of application of the min-max approach. In reality, the operation of the regulation dam of Lake Como is subject to a certain number of constraints imposed by a formal act of the Ministry of Public Works. These constraints can be taken into account without conceptually modifying the method proposed in this paper. Nevertheless, the ideas behind the method become somehow less transparent. For this reason, the complete analysis of the risk-adverse management of Lake Como is described in another paper (see Guariso et al.

1982), where the reader can find all the details about the hydrologic, economic, and institutional characteristics of the problem. Moreover, in the same paper, an interesting comparison with the solution obtained by means of a stochastic method is also shown.

9. CONCLUDING REMARKS

A deterministic (min-max) approach to a simple but typical multipurpose single-reservoir management problem has been described in this paper. The approach is particularly attractive in the case where the manager is risk-adverse and concentrates his efforts on avoiding substantial failures of the system during severe hydrological episodes. The key data necessary for application of the method is a set of one year-long daily inflow sequences. These inflow sequences should be suggested by the manager as reference for evaluation of the system's performance. For reservoirs already in operation, these sequences may be the recorded daily inflows of the years that the manager considers as particularly critical. In such a case, the performance of the proposed solution can be directly compared with what the manager was able to achieve in practice. For obvious reasons, this might be a real advantage, in particular when the final goal of the study is the implementation of the results.

The efficient min-max solutions have a few interesting properties. First of all, the operating rules can be interpreted in terms of storage allocation zones. In the most general cases, four zones can be identified (dead, buffer, conservation, and spilling zone). The conservation zone is in turn divided into three subzones (see Figure 4). Second, the boundaries between these zones are not fixed a priori, but depend upon the forecast of the daily inflow. This is a second property which recognizes a precise role to real-time inflow predictors. Third, and certainly most important, whenever the storage is in the conservation zone, the manager can select the value of the release within a prescribed set of possible releases. This flexibility is certainly welcomed by the manager who is often interested in satisfying other (hopefully minor) objectives than those considered by the optimization model.

From a computational point of view, the method is very effective. The efficient solutions can be obtained by repetitive simulations of the reservoir behavior for different values of the initial storage. The selection of these values is guided by a simple one-dimensional searching method (e.g., bisection). Moreover, in the case where the stage-discharge function is linear--as in the lake management example considered in Section 8--the determination of the different storage zones and subzones is very simple.

Finally, it is worthwhile noticing that the method avoids on-line optimization. In fact, all the data necessary for determination of the set of possible releases can be computed off-line. If, on the other hand, on-line optimization is allowed, one could introduce some further improvement. In particular, one could solve Problem τ of Sections 3 and 4 at the beginning of each day only with regard to a few reference inflow sequences, namely those that make more sense under the current situation. If, for example, snow-melt is over by May 20, there is no interest after that data, in considering all those reference inflow sequences which have inflow peaks in June.

As a last remark, we might point out that the method seems to interpret in a formal way what practitioners have been doing for a long time. In fact, the solution procedure recalls the mass-curve method, in particular when single-objective management problems are considered (see Sections 3 and 4). However, this is not a surprise, since even stochastic optimization procedures have been proved to have very much in common with this old method.

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