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1. INTRODUCTION

Consider an economy in which agents have different levels of information concerning exogenous random events. How does the pooled information of the agents get revealed in the process of exchange? In particular what is the role played in this by the price system? At least since Hayek this has been a central problem in economics. "My main contention," Hayek wrote in 1937,

> will be that the tautologies, of which formal equilibrium analysis in economics essentially consists, can be turned into propositions which tell us anything about causation in the real world only insofar as we are able to fill those formal propositions with definite statements about how knowledge is acquired and communicated ... The really central problem of economics as a social science, which we pretend to solve is how the spontaneous interaction of a number of people, each possessing only bits of knowledge, brings about a state of affairs in which prices correspond to costs, etc., and which could be brought about by deliberate direction only by somebody who possessed the combined knowledge of all those individuals. Experience shows us that something of this sort does happen, since the empirical observation that prices do tend to correspond to costs was the beginning of our science. The only trouble is that we are still pretty much in the dark about (a) the conditions under which this tendency is supposed to exist and (b) the nature of the process[1] by which individual knowledge is changed.

[1] authors' emphasis

> The "man on the spot" cannot decide solely on the basis of his limited but intimate knowledge of the facts of his immediate surroundings. There still remains the problem of communicating to him such further information as he needs to fit his decisions into the whole pattern of changes of the larger economic system ... We must look at the price system as such a mechanism for communicating information if we want to understand its real function.

One approach to this problem has been taken via the notion of a "Rational Expectations Equilibrium" (R.E.E.). Since our paper is juxtaposed to the R.E.E. model, and meant to be a critique of it, let us first briefly recall what the R.E.E. model* is. Let S be the finite set of states of the world. For each agent $n \in N$ let I^n be a partition of S representing the information of n, and denote by I^* the coarsest partition of S which refines each I^n, $n \in N$. Trade takes place in a finite set L of commodities. Thus the space of state-contingent commodities is $\mathbb{R}_+^{L\times S}$. Each agent n is characterized by a utility $u^n : \mathbb{R}_+^{L\times S} \to \mathbb{R}$ and an endowment $e^n \in \mathbb{R}_+^{L\times S}$. Assume u^n is C^2, strictly concave and monotonic, and that e^n is measurable with respect to I^n.

Consider a price function $p : S \to R_+^L$. Its inverse yields a partition of S which we will denote by $I(p)$. An R.E.E. for this economy is a p, along with allocations $x^n \in \mathbb{R}_+^{L\times S}$ such that, for $n \in N$,

(i) x^n is measurable w.r.t. the coarsest refinement $I^n \vee I(p)$ of I^n and $I(p)$;

(ii) $$x^n \in \operatorname*{argmax}_{x \in \mathbb{R}_+^{L\times S}} \left\{ u^n(x) : x \text{ is measurable w.r.t. } I^n \vee I(p), \text{ and } p_s \cdot x_s \le p_s \cdot e_s^n \text{ for each } s \in S \right\};$$

(iii) $$\sum_{n\in N} x^n = \sum_{n\in N} e^n .$$

(Here, for any vector $x \in R_+^{L\times S}$ and $s \in S$, x_s is the vector in R_+^L obtained by restricting x.) In words this means that each agent n refines his information I^n by what he can deduce from seeing p, then forms his demand x_s^n (subject to the budget constraint

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*We outline here the model described by Radner [7].

$p_s \cdot x_s^n \leq p_s \cdot e_s^n$), and the ensuing total demand $\sum x^n$ can be met by the supply $\sum_n e^n$ at hand. Notice that prices play the dual role of simultaneously determining the budget constraint and revealing information. Radner has shown[1] that "generically" (in the space of utilities) an R.E.E. exists and its prices are fully revealing in that $I(p) = I^*$. Prices are thus shown to convey to each agent all the bits of information held originally in separate minds.

An immediate paradoxical upshot of this was first noted by Grossman and Stiglitz. Since prices reveal all of the collective information at a R.E.E., no agent benefits from his initial superior information! And if this information happens to be costly to acquire then no one will gather any, and there will be none for prices to reveal.

The paradox stems from a grievous omission in the R.E.E. model. It does not even begin to describe how, in the first place, the diverse bits of information of the agents are pooled and "put into" the prices to be revealed. But, as Hayek emphasized, this step in the *market process* is the central issue in an understanding of how information is disseminated through the economy. Our essential criticism of the R.E.E. model is that it throws the baby out with the bath-water because it does not represent any such process at all.

In this paper we consider a model with an explicit process for the flow of information via prices. Roughly it goes as follows. Economic activity takes place in time periods. Agents initially act on the basis of their privately-held information I^n. This results in observable macro-economic outcomes (e.g. prices) through which their information is "betrayed". The extra information so released to everyone is then available for the *next* period of activity. Notice that in the initial period agents with superior information can exploit it and make a "killing". The paradox that information is useless is removed by the simple fact that the process that reveals it takes time (as any process must).

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[1]and this is typical of results obtained in other similar models, e.g. those of Grossman.

This description is, we believe, more realistic than the R.E.E. model (and also more in keeping with what Hayek had in mind). Its very wording invites one to model it as a strategic market game. We shall, for concreteness, choose one such: the Shapley-Shubik model of exchange presented in [8, 9]. But our results seem to be quite robust and not to hinge delicately on this choice (see Remark 1).

Before plunging into the details it might be helpful to describe the contours of our model. S, N, $\{I^n\}_{n\in N}$ are as before. But now there are time periods, for simplicity two. The characteristics of the traders must accordingly be expanded into endowments e^n, $\tilde{e}^n \in \mathbb{R}_+^{L\times S}$ in period 1, 2 ; and utility u^n : $\mathbb{R}_+^{L\times S} \times \mathbb{R}_+^{L\times S} \to \mathbb{R}$.

The game is best viewed in extensive form. Nature moves first to select a state s in S. At each node s all the players in N move simultaneously with information partitions given by I^n. Let $X^n(s)$ be the set of moves available to n at node s (of course, we must then require that $X^n(s)$ is constant for $s \in \gamma \in I^n$). Put $X(s) = \prod_{n\in N} X^n(s)$. There are maps $\varphi_s^n : X(s) \to Z^n$, $\psi_s^n : X(s) \to \mathbb{R}_+^L$. Here Z^n is a space of macro-economic observables for player n (its best to think of it as prices, and set $Z^n = \mathbb{R}_+^L$). For[1] $q_s = (q_s^1,\ldots,q_s^N) \in X(s)$, $\varphi_s^n(q_s)$ is what n observes in Z^n as a consequence of the joint choice q_s of moves by the agents; and $\psi_s^n(q_s)$ is his final holding of commodities in time period 1. Thus the maps ψ_s^n satisfy: $\sum_{n\in N} \psi_s^n(q_s) = \sum_{n\in N} e_s^n$. In time-period 2 the nodes in the game tree are $q_s \in X(s)$. Let $\tilde{X}^n(s)$ be[2] the set of moves of n at q_s. The information partition $\tilde{I}^n$ of n on $\bigcup_{s\in S} X(s)$ is given by I^n refined by what he can observe of others' moves. To make this precise, let $\hat{I}^n$ be the partition of $\bigcup_{s\in S} X(s)$ that is yielded by the equivalence relation: $q_s \sim q_{s'}$ if $\varphi_s^n(q_s) = \varphi_{s'}^n(q_{s'})$. Also

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[1] Without confusion, $N = \{1,\ldots,N\}$; $S = \{1,\ldots,S\}$ etc.

[2] In general we could write $\tilde{X}(q_s)$, but this will not be needed in our model.

extend I^n to a partition I^n_{ext} on $\bigcup_{s\in S} X(s)$ in the obvious way: to each $\gamma \in I^n$ corresponds the set $\bigcup_{s\in\gamma} X(s)$. Then define $\tilde{I}^n = I^n_{ext} \vee \hat{I}^n$. Finally once again there are maps $\tilde{\Psi}^n_s : \tilde{X}(s) \to \mathbb{R}^L_+$, where $\tilde{X}(s) = \prod_{n\in N} \tilde{X}^n(s)$, which specify the transformation of moves to trades in the 2^{nd} period. (Of course, $\sum_{n\in N} \tilde{\Psi}^n_s(\tilde{q}_s) = \sum_{n\in N} \tilde{e}^n_s$ for any $\tilde{q}_s \in \tilde{X}(s)$.)

A strategy of n is to pick a move at each node in $S \cup \{\bigcup_{s\in S} X(s)\}$, subject to the constraint that these be identical at any two nodes that lie in the same information set. Given a choice of strategies by all agents, a play $\Pi(s)$ is determined in the tree for each $s \in S$ in the standard manner. Associated with these are moves $q_s = (q^1_s, \ldots, q^N_s)$ and $\tilde{q}_s = (\tilde{q}^1_s, \ldots, \tilde{q}^N_s)$ in the two time periods. The final holding that accrues to n is then $\Psi^n_s(q_s)$, $\tilde{\Psi}^n_s(\tilde{q}_s)$ in periods 1, 2 in state s. His payoff is simply the utility of his final holding: $u^n\left(\left\{\Psi^n_s(q_s)\right\}_{s\in S}, \left\{\tilde{\Psi}^n_s(\tilde{q}_s)\right\}_{s\in S}\right)$.

We analyze this game for its Nash Equilibria (N.E.) when the X^n, $\tilde{X}^n$, φ^n, Ψ^n, $\tilde{\Psi}^n$ are according to the Shapley-Shubik model (see the next section for details). Our results may be summed-up as follows. If N is non-atomic, then for a generic choice of e^n, $\tilde{e}^n$ and u^n : N.E. exist and are finite in number; they fully reveal S in that $\varphi^n_s(q_s) \neq \varphi^n_{s'}(\tilde{q}_s)$ if $s \neq s'$; and lead to higher utilities for the better-informed agents. If N is finite then generic revelation fails, and N.E. exist robustly (i.e. for an open set of e^n, $\tilde{e}^n$ and u^n) at which some agents do not reveal their information in the first period. This last result depends on the possibility of N.E. which involve "threats". In contrast, when N is non-atomic, "threat-equilibria" can be ruled out. As shown in [ ] the strategies can in this case be taken to depend on history only insofar as that history reveals something about the state of nature. Thus generic revelation by prices is a phenomenon that attaches to perfect competition and is seen to break down in an oligopolistic setting. And -- to reiterate -- in *both* cases agents with superior information benefit from it, so that we steer clear of the R.E.E. paradox.

One might wonder if these results are -- at bottom -- an artifact of the model we have invoked. Could one not concoct an ingenious one-period strategic game whose N.E.'s coincided with the R.E.E.'s of the underlying economy? Such N.E.'s would entail that while strategies are measurable w.r.t. I^n, no one wishes to revise his own even after being informed of the resultant $\tilde{I}^n$. They do not exist in our model, but it is conceivable that in a sufficiently "complex" game they might. Indeed one suggestion is to allow each agent of type n to submit an entire demand function $d_\gamma : \mathbb{R}^L_+ \to \mathbb{R}^L$ for every $\gamma \in I^n$. The market mechanism then performs a complicated fixed point computation to find prices that clear markets for every $s \in S$. Beja has shown that this will not work: N.E.'s will be produced which are not R.E.E.'s. But even if some variant of this game did work, it would be open to the obvious criticism that one cannot imagine real agents who have the kind of capacity of computation needed to play it. We take as a dictum — and this is met by our model — that both the strategy-sets and the outcome map be simple and "playable". In our model strategies are not contingent upon what will happen in the market, only upon the information I^n privately held by agents. We believe that a significant proportion of actual trade takes place this way. A farmer offers to the market his crop of wheat, as a matter of prior commitment, no matter what the price is going to be. At the time of planting there is not necessarily much information for prices to reveal: demand decisions will not be made until much later. Even a system of continuous-trading futures markets could not hope to communicate all the relevant information at the appropriate moment in time. It is from the spot prices that the farmer typically learns the information which would have induced him to plant differently had he known it then, and it is these prices which are his most reliable guide to the future.

Our hypothesis is that in many cases these future markets do not exist anyway. Accordingly we construct a model in which traders learn from past spot prices and undertake simple trading strategies that determine current ones. It is built in the spirit of Cournot. And the results we obtain seem to be robust to variations of the model (Remark 1).

The model is stripped down to concentrate on the flow of information from period 1 to 2. Agents only buy goods and are forced to put everything up for sale. The commodities in the two periods are completely disjoint and there is no inventorying. All this is for simplicity and could easily be rectified (Remark 2). A more subtle condition is on the space of utilities within which our generic result holds. This consists of all functions $u(x,\tilde{x})$ defined on the joint holdings $x,\tilde{x}$ of periods 1,2. One could well ask how important the choice of this space is for our results. If we had restricted ourselves to $u(x,\tilde{x})$ of the form $u(x+\tilde{x})$ then, with inventorying, this would in effect make the two time periods arbitrarily close to each other. One might expect that the agents would trade very little in the first period and simply wait until the second period when they had more information to do most of their trading. The resulting N.E.'s might then look very much like the R.E.E.'s. As we show later in an example, however, this intuition is wrong. Agents will trade in both time periods because the prices will in general be different. And the main result of our paper — that information is of value — would not be violated.

The paper is organized as follows. In Section 2 the basic strategic market game is formulated. In Section 3 the main theorem is proved. In Section 4 a series of examples is presented to "round off" the approach. As we mentioned earlier, if N is finite then information may not be revealed at an N.E. We also model the situation in which information may be bought and sold, and in this case it is possible that no N.E. exists. Finally, an excursion is made into a Betrandian-type of model in which prices can be used as (contingent) strategies. But instead of being functions they are kept very simple, as is the outcome map, in accordance with the dictum stated earlier. We find that again no N.E. may exist; if it does (a) more than one price may prevail for a commodity (b) information is not necessarily revealed if N is finite (c) agents typically benefit from superior information (N finite or continuum).

2. THE STRATEGIC MARKET GAME

Consider the case when the agent-space is non-atomic.[1] For convenience there is a finite number of types of agents: $1,\dots,N$. Type n consists of the continuum $(n-1,n]$ endowed with the Lebesque measure for every $n \in N = \{1,\dots,N\}$. (The triple use of n: as the number n, as the set $(n-1,n]$, as the name of the n^{th} type; as well as the additional fourth use of N as the set of types $\{1,\dots,N\}$; should cause no confusion. The usage will always be clear from the context, and it saves enormously on notation.) To recapitulate from Section 1:

$L \equiv \{1,\dots,L\} \equiv$ set of commodities

$S \equiv \{1,\dots,S\} \equiv$ states of nature

$e^n \in \mathbb{R}_+^{L\times S} \equiv$ endowment of $\alpha \in (n-1,n]$ in period 1

$\tilde{e}^n \in \mathbb{R}_+^{L\times S} \equiv$ endowment of $\alpha \in (n-1,n]$ in period 2

$u^n : \mathbb{R}_+^{L\times S} \times \mathbb{R}_+^{L\times S} \to \mathbb{R} \equiv$ utility function of $\alpha \in (n-1,n]$

$I^n \equiv$ partition of $S \equiv$ information of $\alpha \in (n-1,n]$

A vector in $\mathbb{R}_+^{L\times S} \times \mathbb{R}_+^{L\times S}$ will be broken into $(x,\tilde{x})$ where $x,\tilde{x}$ are each in $\mathbb{R}_+^{L\times S}$. Thus $x(\tilde{x})$ is the vector of state-contingent commodities in period 1 (2). Also for $x \in \mathbb{R}_+^{L\times S}$, $x_{\ell s}$ is its component on the axis $(\ell,s) \in L \times S$; and $x_s(x_\ell)$ is the vector in $\mathbb{R}_+^L(\mathbb{R}_+^S)$ obtained by restricting x.

The L^{th} commodity is singled out as a money to be used for purchases. For $s \in S$, let $\underline{e}_{Ls}^n = \min\left\{e_{Ls'}^n : s' \in I^n(s)\right\}$. (Here $I^n(s)$ is the element of I^n that contains s.) Then $X^\alpha(s)$, the set of moves available to α at s in period 1, is given by:

$$X^\alpha(s) = \left\{ b_s^\alpha \in \mathbb{R}_+^{L-1\times\{s\}} : \sum_{\ell=1}^{L-1} b_{\ell s}^\alpha \leq \underline{e}_{Ls}^{n(\alpha)} \right\} ,$$

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[1] The model for N finite will become clear in the process.

where L-1 is the set $\{1,\ldots,L-1\}$ and $n(\alpha)$ is the type of α. In the interpretation $b^{\alpha}_{\ell s}$ is the amount of money bid by α in period 1 in state s for the purchase of commodity $\ell \in L-1$. A choice of moves $\{b^{\alpha}_{s} : \alpha \in [0,N]\}$ determines prices $p_{\ell s}$ and trades $x^{\alpha}_{s} \in R^{L}_{+}$ by the rules:[1]

$$p_{\ell s} = \frac{\int_0^N b^{\alpha}_{\ell s} d\alpha}{\int_0^N e^{\alpha}_{\ell s} d\alpha} ,$$

$$x^{\alpha}_{\ell s} = \begin{cases} \dfrac{b^{\alpha}_{\ell s}}{p_{\ell s}} & \text{if} \quad p_{\ell s} > 0 \\ 0 & \text{if} \quad p_{\ell s} = 0 \end{cases}$$

for $\ell \in L-1$;

$$x^{\alpha}_{Ls} = e^{\alpha}_{Ls} - \sum_{\ell=1}^{L-1} b^{\alpha}_{\ell s} + \sum_{\ell=1}^{L-1} p_{\ell s} e^{\alpha}_{\ell s} .$$

This completely specifies the maps $\{\Psi^{\alpha}_{s} : \alpha \in [0,N], s \in S\}$. It simply says that all of the goods in L-1 have to be offered for sale and then the goods (money) are disbursed in proportion to the bids (offers). The sets $\tilde{X}^{\alpha}(s)$ and the maps $\tilde{\Psi}^{\alpha}_{s}$ are defined in exactly the same way as $X^{\alpha}(s)$, Ψ^{α}_{s} with $b^{\alpha}_{\ell s}$, $e^{\alpha}_{\ell s}$ replaced by $\tilde{b}^{\alpha}_{\ell s}$, $\tilde{e}^{\alpha}_{\ell s}$. It remains to describe $\tilde{I}^{n}$, equivalently $\hat{I}^{n}$, to complete the definition of the extensive game. Since we are interested in the role played by prices in disseminating information, we shall let prices be observable, i.e., $Z^{n} = \mathbb{R}^{L-1}_{+}$ (the price of the L^{th} commodity being always 1 in our model), and the ℓ^{th} component of $\varphi^{n}_{s}(b_{s}) = \left(\int_0^N b^{\alpha}_{\ell s} d\alpha\right) \Big/ \left(\int_0^N e^{\alpha}_{\ell s} d\alpha\right)$. However, from the

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[1] We consider only the case when the map $\alpha \to b^{\alpha}_{s}$ is integrable. See Remark 2, however.

technical point-of-view of the validity of our theorem, much finer observations can be permitted, as explained in Remark 3.

Let us designate the above game by Γ. We will analyze the *Nash Equilibria* (N.E.) of Γ, i.e., a choice of strategies by the agents in [0,N] at which no one agent can profit by a unilateral deviation.

3. EXISTENCE OF NASH EQUILIBRIUM

Γ has some trivial "inactive" N.E.'s. For instance consider the strategies in which all agents bid zero everywhere. Additional N.E.'s can be constructed which leave any specified subset of the $2 \times L-1 \times S$ trading-posts inactive. Our interest is in pinning down conditions which guarantee the existence of *active N.E.'s*, namely those which produce positive prices in each trading post. Indeed, from now on, we shall always mean an active N.E. when we say N.E.

It turns out that N.E.'s do not always exist for Γ. However if we vary Γ then, for a "generic" choice of Γ, it can be shown that N.E.'s do exist. Let us first make the notion of genericity precise. Let A, B, C, D be positive numbers with $A < B$, $C < D$. Consider the polytope $\mathcal{E}$ in $R_+^{N\times L\times S} \times R^{N\times L\times S}$ consisting of $e^1,\ldots,e^N$, $\tilde{e}^1,\ldots,\tilde{e}^N$ which satisfy:

(i) $A < e^n_{Ls}, \tilde{e}^n_{Ls} < B$, for $n \in N$ and $s \in S$

(ii) $C < \sum_{n\in N} e^n_{\ell s} < D$ for $\ell \in L-1$ and $s \in S$.

Each point in $\mathcal{E}$ represents a choice of endowments $e,\tilde{e}$. Clearly we can find an $E > 0$ such that $\max\left\{\|\sum_{n\in N} e^n\|, \|\sum_{n\in N} \tilde{e}^n\|\right\} < E$ where $\| \ \|$ denotes the maximum norm. Then if $x^1,\ldots,x^N$, $\tilde{x}^1,\ldots,\tilde{x}^N$ is a type-symmetric reallocation of $e^1,\ldots,e^N$, $\tilde{e}^1,\ldots,\tilde{e}^N \in \mathcal{E}$, we automatically have $\|x^n\| < E$, $\|\tilde{x}^n\| < E$. Thus if endowments are to come from $\mathcal{E}$ we can confine ourselves to utilities defined on the cube $\mathcal{C} \subset R_+^{L\times S} \times R_+^{L\times S}$ whose edges have length E. Let $\mathcal{U}$ be the space of all functions defined on a neighborhood η of $\mathcal{C}$ which are C^2, strictly concave, and satisfy (for $0 < \sigma < \sigma'$):

$$\sigma < \frac{\partial u}{\partial x_{\ell s}}, \frac{\partial u}{\partial \tilde{x}_{\ell s}} < \sigma' \quad .$$

With the C^2-topology, $\mathcal{U}$ is a Banach manifold. A point in $\mathcal{U}^N$ represents a choice of utilities for the N types. We will keep all the other data of the game fixed as in Section 2, and vary only the endowments and utilities. $\mathcal{E} \times \mathcal{U}^N$ can then be thought of as the space of games. Our existence theorem is now readily stated.

Theorem. There is an open dense set $\mathcal{D}$ in $\mathcal{E}$, whose complement in $\mathcal{E}$ has zero Lebesgue measure, such that for $(e,\tilde{e}) \in \mathcal{D}$ there exists an open dense set $\mathcal{D}(e,\tilde{e})$ in $\mathcal{U}^N$ with the property:

(i) N.E.'s exist and are finite in number for $\Gamma \in \{(e,\tilde{e})\} \times \mathcal{D}(e,\tilde{e})$

(ii) if $b : [0,N] \to \mathbb{R}_+^{L-1 \times S}$ is the move at any N.E. in (i), then b is fully revealing, i.e.

$$s \neq s' \Rightarrow \varphi_s^n(b_s) \neq \varphi_{s'}^n(b_{s'}) \quad .$$

The proof consists of three steps. We will define a "potential Nash Equilibrium" (p.N.E.) which exists for every $\Gamma = (e,\tilde{e},u) \in \mathcal{E} \times \mathcal{U}^N$ (Section 3.1). Then we define $\mathcal{D}$ and prove that if $(e,\tilde{e}) \in \mathcal{D}$ there is an open dense $\mathcal{D}(e,\tilde{e})$ in $\mathcal{U}^N$ such that every p.N.E. of $\Gamma \in (e,\tilde{e}) \times \mathcal{D}(e,\tilde{e})$ is fully revealing (Section 3.2). From this it is deduced that, for such Γ, the set of p.N.E. = the set of N.E. (Section 3.3).

3.1 Potential Nash Equilibria

Fix $\Gamma = (e,\tilde{e},u)$ in $\mathcal{E} \times \mathcal{U}^N$. The *fictitious game* Γ^* is obtained from Γ by the modifications: (i) the information partition $\tilde{I}^n$ of each type in period 2 is replaced by $I^1 \vee \ldots \vee I^n$ (w.l.o.g. assume that $I^1 \vee \ldots \vee I^N = I^* = (\{1\},\ldots,\{S\})$ from now on); (ii) strategies are restricted to be bids contingent *only* on the information about chance moves and not contingent, beyond this, on others' moves, i.e., $\tilde{b}^\alpha(b(s)) = \tilde{b}^\alpha(b'(s))$ for $b(s), b'(s) \in X(s)$.

For $\Delta > 0$ consider the *Δ-modified fictitious game* Γ_Δ^* in which (in addition to (i) and (ii)) an external agency is imagined to have placed bids of size Δ in each of the $2(L-1)S$ trading-posts. This does not affect the strategy sets of Γ^* but only the strategy-to-outcome map.

A *potential Nash Equilibrium* (p.N.E.) of Γ is simply an N.E. of Γ^*. If $\eta(\Gamma_\Delta)$ denotes the set of N.E. of Γ_Δ, then clearly p.N.E. of $\Gamma = \eta(\Gamma_0^*)$.

Let $\sum^n$ denote the strategy-set of type n in the game Γ_Δ^*, $\Delta \geq 0$. A typical element of $\sum^n$ consists of a pair of vectors b^n, $\tilde{b}^n$ in $\mathbb{R}_+^{L-1 \times S}$ measurable w.r.t. I^n, I^* respectively. Since utilities are *strictly* concave, and the set of agents [0,N] is non-atomic, it is obvious that at any N.E. of Γ_Δ^* agents of a given type use the same strategy. Therefore in our analysis of $\eta(\Gamma_\Delta^*)$ we may restrict ourselves to the set $\sum = \sum^1 \times \ldots \times \sum^N$.

For $\mu > 0$ denote by $\sum_\mu$ the subset of $\sum$ at which all prices $p_{\ell s}$, $\tilde{p}_{\ell s}$ $(\ell \in L-1, s \in S)$ in the two periods are at least μ.

Lemma 1. There is a $\mu > 0$ such that if $\Gamma \in \mathcal{E} \times \mathcal{U}^N$ then $\eta(\Gamma_\Delta^*) \subset \sum_\mu$ for $\Delta \geq 0$.

Proof. First let us show that there is a μ_1 such that if the first period moves at some N.E. of Γ_Δ^* are b, then $p_{\ell s}^b > \mu_1$ for all s,ℓ. (By $p_{\ell s}^b$ we mean the first-period prices that accrue from b in the game Γ_Δ^*.

Case 1

$$\sum_{\ell \in L-1} b_{\ell s}^n < \underline{e}_{Ls}^n$$

for some $n \in N$ and $s \in S$.

If an agent of type n increases his bids $b_{\ell r}^\alpha$ $(r \in I_n(s))$ by $\varepsilon > 0$ then the increase in his payoff, for small ε, is approximately:

$$\varepsilon\left[\sum_{r \in I_n(s)} \left(\frac{\partial u^n}{\partial x_{\ell r}} \Big/ p_{\ell r}^b - \frac{\partial u^n}{\partial x_{Lr}}\right)\right] \geq \varepsilon\left[\sigma \Big/ p_{\ell r}^b - |I_n(s)|\sigma'\right]$$

for any $r \in I_n(s)$. This must be non-positive, therefore

$$p_{\ell r}^b \geq \sigma \Big/ (|I_n(s)|.\sigma') \geq \sigma \Big/ S\sigma' .$$

Case 2

$$\sum_{\ell \in L-1} b^n_{\ell s} = \underline{e}^n_{Ls} \quad \text{for some } n \in N \text{ and } s \in S \quad .$$

Clearly $p^b_{\ell r} \geq \underline{e}^n_{Ls} / \bar{e}_{r\ell}$ for any $r \in I_n(s)$, where $\bar{e}_{\ell r}$ abbreviates $\sum_{n \in N} e^n_{\ell r}$. Consider $q \in I_n(s') \neq I_n(s)$. Put

$$M_\ell = \min \left\{ \underline{e}^{n'}_{Ls'} / \bar{e}_{\ell r} : n' \in N \text{ , } r \in I_{n'}(s') \text{ , } s' \in S \right\}$$

If an agent of type n reduces $b^\alpha_{\ell r}$ by ε and increases $b^\alpha_{\ell r'}$ by $\varepsilon (r \in I_n(s) \text{ , } r' \in I_n(s'))$ then his increase in payoff for small ε is approximately:

$$\varepsilon \left[\left(\sum_{r' \in I_n(s')} \frac{\partial u^n}{\partial x_{\ell r'}} / p^b_{\ell r'} \right) - \left(\sum_{r \in I_n(s)} \frac{\partial u^n}{\partial x_{\ell r}} / p^b_{\ell r} \right) \right]$$

$$\geq \varepsilon \left[\sigma / p^b_{\ell r'} - |I_n(s)| \sigma' / M_\ell \right]$$

for any $r' \in I_n(s')$. This must also be non-positive, thus

$$p^b_{\ell r} \geq M_\ell \sigma / |I_n(s)| \sigma' \geq M_\ell / S\sigma' \quad .$$

Put $M = \min \{M_\ell : \ell \in L-1\}$ and then $\mu_1 = \min \{\sigma / S\sigma' \text{ , } M \text{ , } M / S\sigma'\}$. Combining the two cases, we have shown: $b \in \eta(\Gamma^*_\Delta) \Rightarrow p^b{}_s > \mu_1$ for all ℓ and s. In an exactly analogous manner, one can check that there is a $\mu_2 > 0$ such that if $\tilde{b}$ are the second-period moves at any N.E. of Γ^*_Δ then $p^{\tilde{b}}_{\ell s}(2) > u_2$ for all ℓ and s. Then, with $\mu = \min \{\mu_1, \mu_2\}$, the lemma follows (recall the bounds on endowments in E). □

Lemma 2. If $\Delta > 0$, then $\eta(\Gamma^*_\Delta)$ is non-empty for any $\Gamma \in E \times U^N$.

Proof. If $\Delta > 0$ the strategies-to-outcome map is continuous. (It blows up if $\Delta = 0$, i.e. in the unmodified fictitious game Γ^*_0 at strategies which produce a zero price in any trading-post ...

hence the importance of Lemma 1.) The proof now involves a straightforward use of Kakutani's fixed point theorem. □

Lemma 3. $\eta(\Gamma^*)$ is non-empty for any $\Gamma \in \mathcal{E} \times \mathcal{U}^N$.

Proof. Take a sequence $\{\Delta^m\}$, $\Delta^m \to 0_+$. Let ${}^m b, {}^m\tilde{b} \in \eta(\Gamma_{\Delta^m})$. (By Lemma 2 such ${}^m b, {}^m\tilde{b}$ exist.) Let ${}^*b, {}^*\tilde{b}$ be a cluster-point of the $\{{}^m b, {}^m\tilde{b}\}$. By Lemma 1, $p^{*b}_{\ell s}, \tilde{p}^{*\tilde{b}}_{\ell s} > \mu > 0$ for $\ell \in L-1$ and $s \in S$. Then ${}^*b, {}^*\tilde{b}$ is a point of continuity of the pay-off functions, from which it easily follows that ${}^*b, {}^*\tilde{b} \in \eta(\Gamma^*)$. □

3.2 Generic Full Revelation By Prices

Let us first describe the set $\mathcal{D}$ in $\mathcal{E}$. Though it requires somewhat laboured notation, the idea is simple. For $\gamma \in I^n$ consider the $(L-1)$-dimensional simplex of moves R^n_γ available to n in period 1. Suppose (i) there exist $\gamma_1, \gamma_2 \in I^n$ which only n can distinguish, i.e., $\gamma_1 \cup \gamma_2 \in I^j$ for $j \in N\setminus\{n\}$; (ii) e^n_s is constant over $s \in \gamma_1 \cup \gamma_2$. If at an N.E. of Γ^* it happens that n is at the same "vertex" of $R^n_{\gamma_1}$, $R^n_{\gamma_2}$, e.g. bidding nothing in both γ_1 and γ_2, then irrespective of the strategies used by others only $\gamma_1 \cup \gamma_2$ will be revealed at the start of period 2. There is nothing in the model to stop such N.E.'s from existing robustly (in utilities, i.e., for an open set in $\mathcal{U}^N$). Thus we will require that endowments be in "general position," so that if any subset of players is at vertices then their information is still revealed. To make this precise unfortunately calls for quite cumbersome notation.

For $\gamma \in I^n$ let the zero-vertex of R^n_γ be denoted by $v^n_L(\gamma)$ and the remaining L-1 (corresponding to putting all bids on some $\ell \in L-1$) by $v^n_1(\gamma), \ldots, v^n_{L-1}(\gamma)$. Consider $T^n_\gamma \subset \{v^n_1(\gamma), \ldots, v^n_L(\gamma)\}$, $T^n_\gamma \neq \emptyset$, and define:

$$\dot{T}^n_\gamma = \text{relative interior of the convex hull of vertices in } T^n_\gamma .$$

Let $\tau^n = \{T^n_\gamma : \gamma \in I^n\}$ be a collection of subsets of vertices of R^n_γ, $\gamma \in I^n$. A choice of moves $b^n \in \prod_{\gamma \in I^n} R^n_\gamma$ by type n is *of type* τ^n if $b^n_\gamma \in \dot{T}^n_\gamma$ for all $\gamma \in I^n$.

Given $\tau = (\tau^1,\ldots,\tau^N)$ further define:

(iii) $b = (b^1,\ldots,b^N)$ is of type τ if each b^n is of type τ^n.

(iv) $A(\tau)$ = active players in $\tau = \{n \in N : |T^n_\gamma| > 1$ for some $\gamma \in I^n\}$

(v) For $n \in A(\tau)$, $R^n_a(\tau)$ = active strategies of n in $\tau = \Pi\{\dot{T}^n_\gamma : |T^n_\gamma| > 1\}$

(vi) $\dot{R}^n_a(\tau) = \Pi\{\dot{T}^n_\gamma : |T^n_\gamma| > 1 , v^n_L(\gamma) \in T^n\}$

(vii) $\ddot{R}^n_a(\tau) = \Pi\{\dot{T}^n : |T^n_\gamma| > 1 , v^n_L(\gamma) \notin T^n_\gamma\}$

(Note: $R^n_a(\tau) = \dot{R}^n_a(\tau) \times \ddot{R}^n_a(\tau)$)

(viii) $R_\mu(\tau) = \{b = (b^1,\ldots,b^N) :$ b is a feasible choice of moves in period 1, b is of type τ, $p^b_{\ell s} > \mu$ for $\ell \in L-1$, $s \in S\}$.

By dropping inactive strategies, $R_\mu(\tau)$ can be — and will be — viewed as a subset of $\prod_{n \in A(\tau)} R^n_a(\tau)$.

By Lemma 1 we can confine ourselves to the set $\{(b,\tilde{b}) : b \in \bigcup_\tau R_\mu(\tau)\}$ in the search of N.E. of Γ^*. Observe that $\bigcup_\tau$ is a finite partition. Also since the moves-to-outcome map Ψ is continuous at positive prices, it is uniformly continuous on $\bigcup_\tau R_\mu(\tau)$. Therefore we can find (sufficiently small) neighborhoods $\underline{\dot{T}}^n_\gamma$ of $\dot{T}^n_\gamma$ in {Affine hull of $\dot{T}^n_\gamma$} such that Ψ (defined by the same formulas) is continuous on $\prod_{n \in A(\tau)} \Pi\{\underline{\dot{T}}^n_\gamma : |T^n| > 1\}$ and the image of $\prod_{n \in A(\tau)} \Pi(\ldots)$ under Ψ is contained[1] in $\mathcal{N}$. Now define $\underline{R}^n_a$, $\underline{R}^n_a(\tau)$, $\underline{\dot{R}}^n_a(\tau)$, $\underline{R}_\mu(\tau)$ exactly as before by using $\underline{\dot{T}}^n_\gamma$ in place of $\dot{T}^n_\gamma$. We will consider the τ-*subgame* defined on the players in $A(\tau)$, each of whom has the strategy-set $R^n_a(\tau)$, i.e. all inactive strategies are held

[1] Recall that $\mathcal{N}$ is the neighborhood of C on which utilities are defined.

fixed and Ψ is applied to only the active strategies of the players in $A(\tau)$. These active strategies now vary over $\underline{\dot{T}}^n_\gamma$ instead of $\dot{T}^n_\gamma$ but this causes no problems. The N.E. of the τ-subgame still lie in $\underline{R}_\mu(\tau) \subset \prod_{n\in A(\tau)} \underline{R}^n_a(\tau)$ by Lemma 1, with μ lowered slightly to allow for the extension of the strategic domain from $\dot{T}^n_\gamma$ to $\underline{\dot{T}}^n_\gamma$. Define:

$$M(\tau) \quad = \quad \{b \in \underline{R}_\mu(\tau) : p^b_{s'} = p^b_{s'} \text{ for two distinct s and s' in S}\}.$$

$M(\tau)$ depends on e. We will say that e is in *general position* if $M(\tau)$ is a finite union of submanifolds of codimension at least one in $\underline{R}_\mu(\tau)$, for all τ. Then the set

$$\{e : e \text{ is in general position}\}$$

is obtained from $\mathbb{R}^{N\times L\times S}_+$ by removing a finite number of submanifolds of codimension at least one in $\mathbb{R}^{N\times L\times S}_+$. Let

$$\mathcal{D} = \{(e,\tilde{e}) \in \mathcal{E} : e \text{ is in general position}\} \quad .$$

Clearly $\mathcal{D}$ satisfies all the properties required by the theorem. To prove (ii) of the theorem it will suffice to show that there is an open dense set $O(\tau,e,\tilde{e})$ in $\mathcal{U}^{A(\tau)}$ (for $(e,\tilde{e}) \in \mathcal{D}$) such that: if $u \in O(\tau,e,\tilde{e})$ then at any N.E. of the τ-subgame $p_s \neq p_{s'}$ for $s \neq s'$. For then we can simply set

$$\mathcal{D}(e,\tilde{e}) \quad = \quad \bigcap_\tau \left\{ O(\tau,e,\tilde{e}) \times \prod_{n\notin A(\tau)} \mathcal{U} \right\}$$

to obtain (ii).

The existence of $O(\tau,e,\tilde{e})$ in turn is proved by roughly the following argument. The N.E. of the τ-subgame are generically finite in number and vary continuously. On the other hand, strategies b in $M(\tau)$, at which $p^b_s = p^b_{s'}$ for some pair $s \neq s'$, is made up of submanifolds of codimension ≥ 1. Therefore the N.E. set generically misses $M(\tau)$ and its prices are fully revealing. To change this into a proof requires a routine use of the

Transversal Density and Openness Theorems [1]. Indeed fix $e \in \mathcal{D}$, and consider the map

$$\mathcal{U}^{A(\tau)} \times \underline{R}_\mu(\tau) \overset{D}{\to} \left(\prod_{n \in A(\tau)} \left(\mathbb{R}^{\dot{R}^n_a(\tau)} \times \mathbb{R}^{\ddot{R}^n_a(\tau)} \right) \right) \times \underline{R}_\mu(\tau)$$

where $b \to b$ for $b \in \underline{R}_\mu(\tau)$, and

$$D_{n,x}(u,b) = \frac{\partial u^n}{\partial x}(b)$$

for $x \in \dot{\underline{R}}^n_a(\tau) \times \ddot{\underline{R}}^n_a(\tau)$, i.e., it is the partial derivative of n's payoff w.r.t. his own active strategy x. Let

$$N = \left\{ y \in \prod_{n \in A(\tau)} \left(\mathbb{R}^{\dot{R}^n_a(\tau)} \times \mathbb{R}^{\ddot{R}^n_a(\tau)} \right) : y_{n,x} = 0 \text{ if } x \in \dot{R}^n_a(\tau) \right.$$

$$\left. y_{n,x} = y_{n,z} \text{ if } x,z \text{ are in the same } \dot{T}^n_\gamma \text{ occurring in } \ddot{R}^n_a(\tau) \right\}$$

For b to be a p.N.E. of $u \in \mathcal{U}^{A(\tau)}$, we must have $D(u,b) \in N \times R_\mu(\tau)$; for b to be a p.N.E. at which prices are not fully revealing we must have $D(u,b) \in N \times M(\tau)$ i.e. $D(u,b) \in N \times M_i(\tau)$ for some $i = 1,\ldots,K$ where (since e is in general position) $M(\tau) = M_1(\tau) \cup \ldots \cup M_K(\tau)$ breaks $M(\tau)$ into submanifolds of $\underline{R}_\mu(\tau)$ each of which has codim ≥ 1 in $\underline{R}_\mu(\tau)$. The map D is easily checked to be transverse to every submanifold of its image. The Transversal Density and Openness Theorems now reveal that there is an open dense set $O(\tau,e,\tilde{e})$ such that if $u \in O(\tau,e,\tilde{e})$

(a) $\left.\begin{array}{l} \text{codim } D_u^{-1}(N \times M_i(\tau)) \\ \text{in } \underline{R}_\mu(\tau) \end{array}\right\} = \text{codim } (N \times M_i(\tau))$ for $i = 1,\ldots, K$;

(b) $\left.\begin{array}{l} \text{codim } D_u^{-1}(N \times \underline{R}_\mu(\tau)) \\ \text{in } \underline{R}_\mu(\tau) \end{array}\right\} = \text{codim } (N \times \underline{R}_\mu(\tau))$

Since codim $(N \times M_i(\tau)) > \dim \underline{R}_\mu(\tau)$ the sets $D^{-1}(N \times M_i(\tau))$ are empty. And, since codim $N \times R_\mu(\tau) = \dim \underline{R}_\mu(\tau)$, $D^{-1}(N \times \underline{R}_\mu(\tau))$ has dim 0, i.e. is a discrete set. But recall that $\underline{R}_\mu(\tau)$ is a neighborhood of $R_\mu(\tau)$. Hence the intersection of the discrete set with the closure of $R_\mu(\tau)$ is finite, i.e., the number of N.E. of the τ-subgame is finite.

3.3 Completion of the Proof

We have shown that for $\Gamma = (e,\tilde{e},u)$, if $(e,\tilde{e}) \in \mathcal{D}$ and $u \in \mathcal{D}(e,\tilde{e})$ then:

(i) prices are fully revealing in period 1 at any p.N.E. of Γ (equivalently N.E. of Γ^*)

(ii) the set of 1st period moves in the p.N.E.'s of Γ is finite.

To strengthen (ii) into finiteness of p.N.E.'s repeat the argument used for (ii) with D defined not only on the 1st but also the 2nd period moves, i.e. on strategy sets of Γ^*. We avoided doing this in order not to blow up an already cumbersome notation.

It remains to check that the set of p.N.E. of Γ = N.E. of Γ. This follows from (i) above and Proposition 5 (augmented with Remark 6) of [3].

3.4 Remarks

(1) We forced the agents to put up all of their commodities for sale. This is not essential. In the more general "bid-offer" model [8] the same theorem would hold (by an identical argument but twice the notation). Adding inventorying also does not affect it. In general, for any *smooth* strategic-game which is deterministic in spirit,[1] i.e. has a finite number of N.E.'s in Γ,

---

[1] It turns out that a large class of smooth games do yield this when N is non-atomic (see [4]). The argument in [4] is for a simpler setting than Γ^* but we suspect that it could be carried over.

the theorem will go through with only one extra stipulation: that the moves of period 1 which are *not* fully revealing from a submanifold F of codimension ≥ 1. For then generically the N.E. set would "miss" F. Even when the N.E. set is not finite it is typically a finite union of submanifolds $G_1, \ldots, G_k$ each of which has codim ≥ 1 ([5]). But then $G_i \cap F$ will be lower dimensional than G_i given transversal intersection. Thus "most" N.E.'s (those in $G_i \setminus F$) will still be fully revealing. The smoothness of the game (i.e. of the moves-to-outcome map) and the condition that codim F ≥ 1 both seem likely in any model conceived in the Cournotian spirit. To that extent our results are robust.

(2) We have assumed, in the definition of an N.E., that the strategic choice of the agents lead to jointly measurable moves. This seems to go against the very spirit of a noncooperative game with independent decision-makers. However a model can be described in which measurability is restored after an initial non-measurable choice (see [6]). This in turn makes the assumption more viable.

(3) If we refined $\tilde{I}^n$ by allowing agents to observe (modulo null sets) the entire measurable function b of 1st period moves, this would still leave the set of p.N.E. of Γ^* unaffected [See (3)].

4. SOME EXAMPLES

In this section several examples are supplied in order to illustrate and clarify the distinctions and problems with the market mechanism and to illustrate the nature of information revelation.

We imagine that there is a seller who has offered 20 units of a good for sale (the specifics will be given later). In state 1 these units are of value to all buyers; in state 2 they are valueless.

For specificity let us consider that the utility for any individual i is given by

$$(1) \qquad \phi^i = \frac{1}{2}A \log x_1^i + w^i$$

where x_1^i is the amount of the good obtained by i in state 1 and w^i is his wealth in money.

There is a single market for the good. We must describe the mechanism in detail. Furthermore we observe that the uninformed bidders must submit a state independent bid whereas the informed bidders will submit state dependent bids.

Example 1: The Money Quantity Bid Model

The simplest market clearing mechanism model which may be regarded as "unrealistic," but has the virtue of being well-defined and simple is where individuals bid a fixed amount of money and obtain whatevery quantity alloted by the market price that is formed. The uninformed traders will bid a single amount b each, while the informed traders will bid state dependent amounts b_1 and b_2. As the good is of no value in state 2 we may set $b_2 = 0$. Figures 1a and 1b show market clearance and price formation.

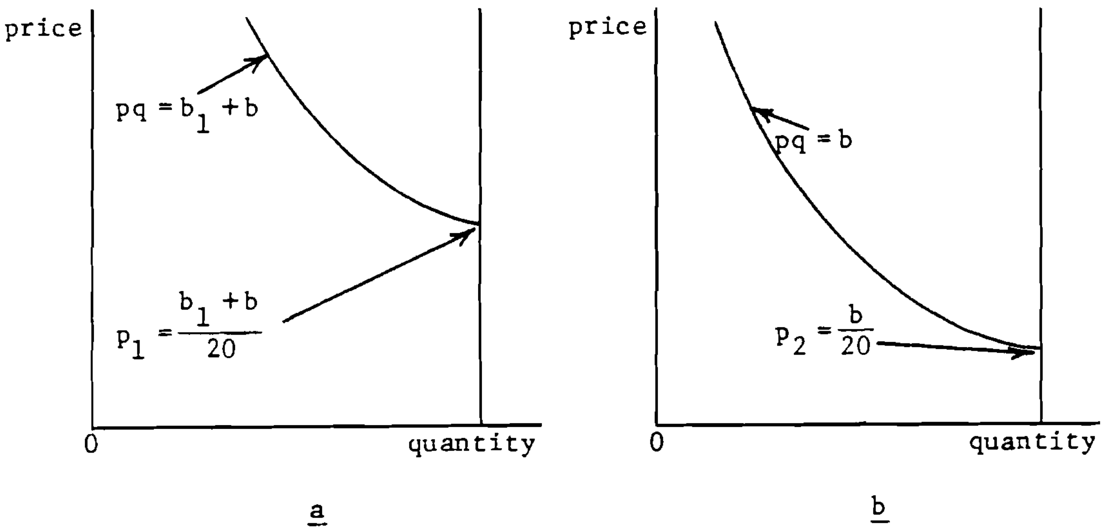

FIGURE 1

Payoffs to informed traders are:

$$(2) \qquad \Pi^1 = \frac{1}{2} A \log b_1/p_1 - b_1 + M$$

where we assume that all traders start with $(0,M)$.

The payoffs to uninformed traders are:

$$(3) \qquad \Pi^2 = \frac{1}{2}A \log b/p_1 - b + M$$

where

$$p_1 = \frac{b + b_1}{20} \quad , \quad p_2 = \frac{b}{20} .$$

Optimization gives us

$$(4) \qquad \frac{A}{b_1} = 1$$

and

$$(5) \qquad \frac{A}{2b} = 1 .$$

Suppose $A = 10$, we have b_{10} , $b = 5$, $p_1 = 3/4$ and $p_2 = 1/4$.

$$x_1^1 = b_1/p_1 = 13\tfrac{1}{3} , \quad x_1^2 = b/p_1 = 6\tfrac{2}{3}$$

$$x_2^1 = 0 \qquad , \quad x_2^2 = 0$$

hence

$$\Pi^1 = 5 \log 13\tfrac{1}{3} - 5 + M \tag{6}$$

$$\Pi^2 = 5 \log 6\tfrac{2}{3} - 5 + M \tag{7}$$

we may leave off the M to see the gains from trade.

Competition with All Uninformed

$$\Pi = \tfrac{1}{2}A \log \frac{b}{p_1} - b + M \quad \text{for all traders} \tag{8}$$

$b = 5$, $p_1 = p_2 = \frac{1}{2}$, $x_s^i = 10$, $\Pi^1 = \Pi^2 = 5 \log 10 - 5 + M$.

Competition with All Informed

$$= \frac{1}{2} A \log \frac{b}{p_1} - b + M \quad \text{for all traders} \tag{9}$$

$b = 10$, $p_1 = 1$, $p_2 = 0$, $\Pi^1 = \Pi^2 = 5 \log 10 - 5 + M$.

The matrix below shows the utility gains at equilibrium trade in four different games when traders of type 1 and 2 are informed or uninformed.

		Type 2 Informed	Uninformed
Type 1	Informed	6.513, 6.513	7.951, 4.486
	Uninformed	4,486, 7.951	6,513, 6.513

TABLE 1

Payoffs at Equilibrium in Four One Stage Games

Observation 1. We have shown by example here that it is easy for a whole class of small traders to gain from extra information even though it is revealed by the prices formed.

Before the other examples are presented, we comment briefly upon other market mechanisms. Several have been suggested. In particular buyers could specify quantities to be bought at any price, or with an upper bound on price; sellers could announce price in advance. Or buyers or sellers could announce whole functions as strategies.

The announcing of whole functions appears to be far less realistic than the money bid we have suggested. But even so it can be considered provided an explicit mechanism for forming price under all circumstances is given.

Figures 2a and 2b illustrate some of the problems in specifying the mechanism for the price-quantity model studied by Shubik [10] and Dubey [2], in the context of differential information. The bid of the informed is p_1^1 , x_1^1 in state 1 and nothing in state 2,

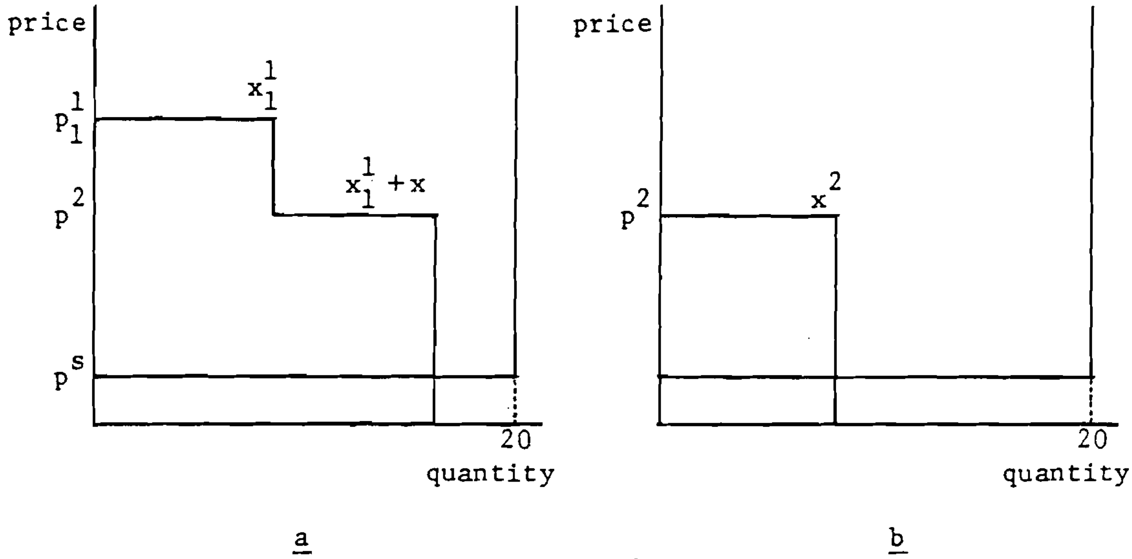

FIGURE 2

the bid of the uninformed is p^2x^2 in either state. The supply is 0 below p^2, 0 to 20 at p^s and above. Using the conventions suggested by Dubey and Shubik prices can be formed in each state and qualitatively the same results concerning the value of information can be established. We expect however to encounter robust sets of games with no equilibria and sales with different prices for the same good.

Example 2: The Purchase of Information

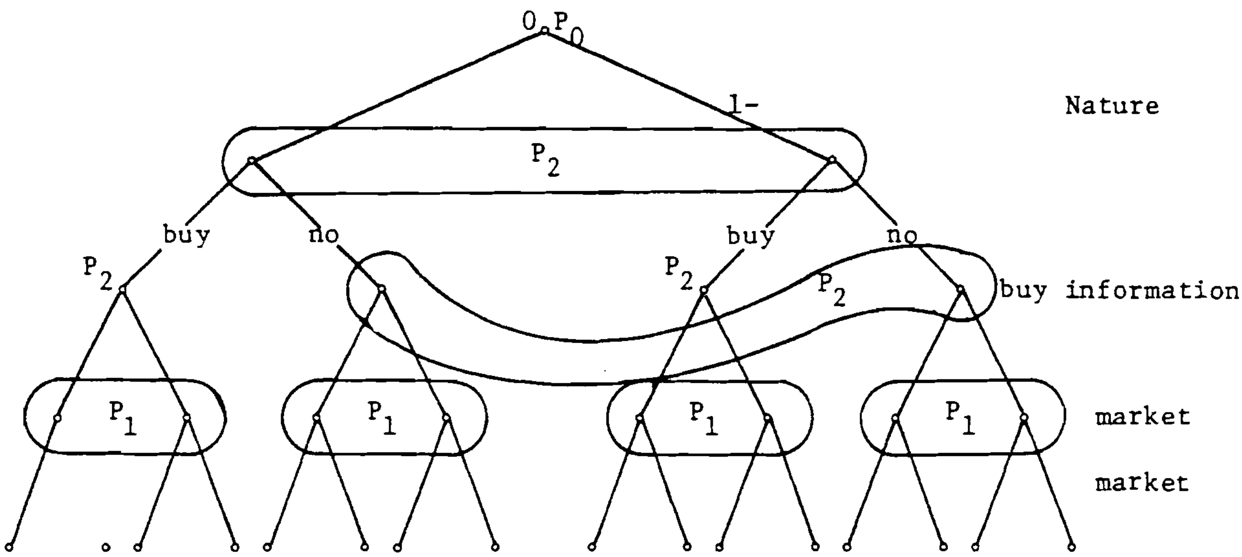

FIGURE 3

In this game tree, after Nature has moved any member of P_2 has the choice to pay Δ to buy information[1] about Nature's move. Then P_2 bids. P_1 is not informed of P_2's bid but he does know if P_2 bought information or not (we could model this the other way; either case is reasonable).

Referring to Table 1 we consider three cases:

(1) $\Delta > 3.465$

(2) $\Delta \leq 3.465$ but $\Delta \geq 2.027$

(3) $\Delta < 2.027$

Given that all traders are small, if a single trader of type 2 were able to be informed costlessly he could change his payoff from 4.486 to 7.951, a gain of 3.465. If $\Delta > 3.465$ it would not pay any member of type 2 to buy information. If $\Delta \leq 3.465$ then any member of type 2 will find it profitable to buy information and earn $7.951 - \Delta$ instead of 4.486. But we now confront a fallacy of composition. If all of type 2 buy information the game changes to one where all types are informed and they can only gain 6.513. Thus if $\Delta \geq 2.027 = 6.513 - 4.486$ it will not have paid all to buy information. The pure strategy equilibrium is destroyed.

If $\Delta < 2.027$ it pays all members of type 2 to buy information.

Observation 2. If information is available for sale it may be bought. Depending upon its price the pure strategy equilibrium may be destroyed.

Example 3: A Two Stage, Two Trader Model

Observation 3. If the number of traders is finite it is possible that they will choose to conceal information in early markets if greater profits are to be made later. If all traders are small this will not be so as

[1]See Shubik [19] for a discussion of the problems in modeling the sale of information.

each, if acting alone could benefit immediately without influencing price, but if all do so they disclose their information and lose future benefits.

Figure 4 shows the game we study in extensive form. Player 1 can

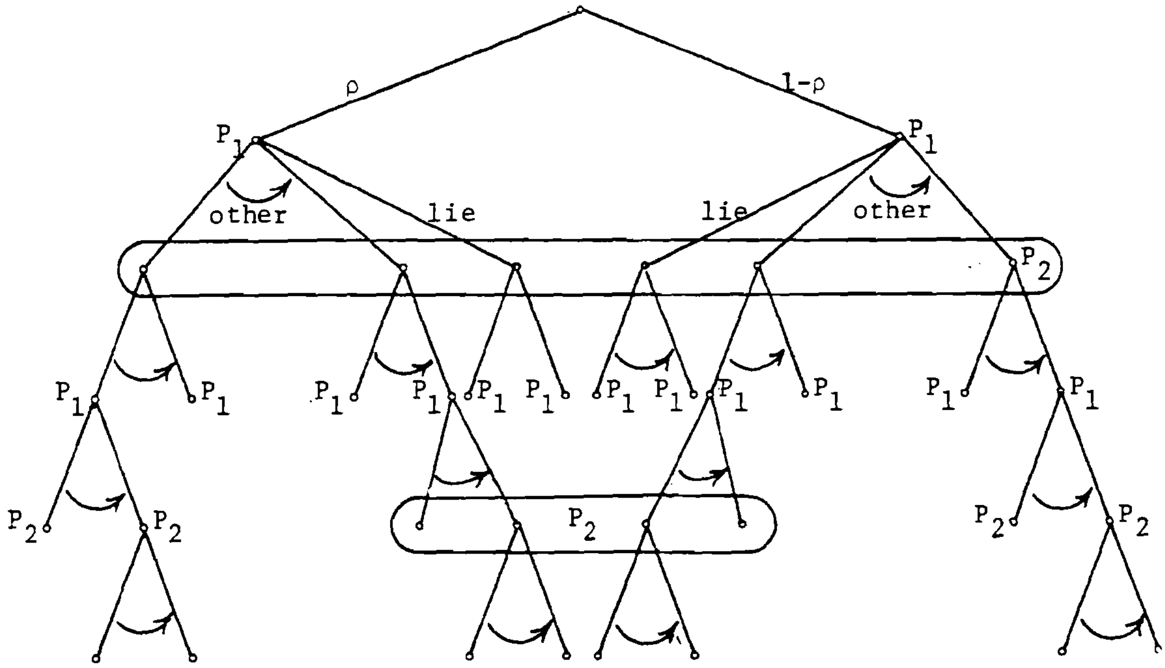

FIGURE 4

pick strategies which will disclose his information, and by doing so may make a higher payoff in the first period. He also has the choice of acting as though he were uninformed. By doing so he earns less in the first period but does not disclose information about the state of Nature.

Suppose for example we assume that there are two periods and the market structure and preferences in the first period are as in example 1. In the second period the commodity for consumption in state 1 is 10 times as valued as in the first period. Furthermore the supply in each period is the same and the good cannot be inventoried by the consumers. The utility function of a trader can be written as:

(10) $$\phi^{i} = \tfrac{1}{2}10 \log x^{i}_{11} + \tfrac{1}{2}100 \log x^{i}_{12} + w^{i}$$

where x^{i}_{st} = the consumption of i in state s at time t.

First suppose there were only one period with one trader with information and one without. The payoffs are as follows:

(11) $$\Pi^{1} = \frac{1}{2}\left\{10 \log 20\frac{b_1}{b + b_1} - b_1\right\}$$

(12) $$\Pi^{2} = 5 \log 20\frac{b}{b + b_1} - b \ .$$

As there are only two traders they each influence price where $p_1 = (b + b_1)/20$ and $p_2 = b/20$.

A little calculation gives

(13) $$b = \frac{10}{\sqrt{2}(1 + \sqrt{2})} = 2.929 \ , \quad b_1 = \frac{10}{(1 + \sqrt{2})} = 4.142$$

$$p_1 = \frac{1}{2\sqrt{2}} = .3536 \qquad p_2 = \frac{1}{2\sqrt{2}(1 + \sqrt{2})} = .1465$$

$$x^{1}_{1} = \frac{20\sqrt{2}}{1 + \sqrt{2}} = 11.716 \qquad x^{2}_{1} = \frac{20}{1 + \sqrt{2}} = 8.284$$

$$\Pi^{1} = 10.234 \qquad \Pi^{2} = 7.643$$

Incomplete-Incomplete Information

If both were uninformed $b = 5/2$ for all $p_1 = p_2 = 1/4$

$$x^{1}_{1} = x^{2}_{1} = x^{1}_{2} = x^{2}_{2} = 10$$

$$\Pi^{1} = 5 \log 10 - 5/2 \qquad \Pi^{2} = 5 \log 10 - 5/2$$

$$= 9.013 \qquad = 9.013$$

Complete-Complete Information

If both were informed $b_1 = 5$, $b_2 = 0$ for all $p_1 = 1/2$

$$p_2 = 0 \qquad x_1^1 = x_1^2 = 10$$

$$\Pi^1 = 5 \log 10 - 5/2 \qquad \Pi^2 = 5 \log 10 - 5/2$$

$$= 9.013 \qquad = 9.013$$

We may construct and interpret Table 2 in the same way as Table 1. It shows the duopsony solutions to the one period strategic market game, in contrast with the competitive solutions shown in Table 1.

		Type 2	
		Informed	Uninformed
Type 1	Informed	9.013, 9.013	10.234, 7.643
	Uninformed	7.643, 10.234	9.013, 9.013

TABLE 2

Four One Stage Games: Duopsony Payoffs

A multiplying of these numbers by 10 yields the payoffs in the second period subgames. It is straightforward to observe that if the informed player choose to earn 10.234 and thereby reveal his information in the first period, he will earn 90.13 in the second period. If, on the other hand, he plays as if he were uninformed in the first period the totals earned are 9.013 and 102.34. The full payoff with disclosure is 100.364 and without is 111.353. It is important to note that it does not matter that the player of type 2 knows that the other player lies; there is nothing he can do about it. He gets no information on Nature.

This result is robust, we could have had k traders of each type, as long as each has influence on price. With a continuum of traders a single individual who is informed is tempted to save money by not buying worthless goods in state 2. All of them would do this, the price would change and the information would be signalled.

Observation 4. We could easily have introduced uncertainty in the knowledge of traders of type 2 concerning whether or not traders of type 1 were informed. The game tree is similar to that in Figure 3 with the information purchase replaced by a move of Nature.

Observation 5. If the payoffs had been identical in each period then traders of type 1 would have been indifferent between revealing information in period 1 or 2. They could obtain 10.234 + 9.013 with immediate revelation or 9.013 + 10.234 with delayed revelation.

Observation 6. If information were for sale in period 1 it would be of considerable value and its availability and not the price mechanism could promote revelation. A reasonable model of this would call for an explicit formulation of the "trustworthiness" of the information and its speed of diffusion, i.e. how fast can people buy it and act upon it. Services such as Disclosure Incorporated indicate the importance of this for obtaining S.E.C. filings.

Observation 7. If the utility of the consumer good were additive across periods for example $\phi(x^i_{s1} + x^i_{s2})$ and if it were possible to inventory the good, depending upon supply conditions in the two periods, for a finite number of traders, revelation will or will not take place in the first period. For a continuum there will be first period revelation.

We suggest that our observations pose no paradoxes. They do indicate that further work may require (1) explicit care in formulating the functioning of market mechanisms, (2) the specification of the relative speeds of information diffusion and market reaction, (3) the modeling of the sale of information and its evaluation, (4) the facing up to the possibility that the assumption of the continuum of traders is merely a mathematical convenience which allows us to obtain some insights of interest, but is of limited application as an approximation to many of the phenomenon of interest.

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