PROCEEDINGS OF TASK FORCE MEETING ON INPUT-OUTPUT MODELING (1981)

A. Smyshlyaev
Editor

June 1982
CP-82-32

Collaborative Papers report work which has not been performed solely at the International Institute for Applied Systems Analysis and which has received only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute, its National Member Organizations, or other organizations supporting the work.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

Work on input-output modeling at IIASA began in 1979 with Clopper Almon and Douglas Nyhus, and the third year of research in this area has now been completed. During this period there has been considerable progress in the construction, linking, and use of input-output models; the programs for the basic IIASA models have been modified so that they can be used on a number of different computer systems and a user's guide to the basic program SLIMFORP has been published by U. Sichra as IIASA Working Paper WP-81-114.

With substantial help from IIASA and the Inter-Industry Forecasting project (INFORUM) at the University of Maryland, a self-organizing network of collaborating institutions has been built up to work on the development and linkage of input-output models. IIASA is particularly suitable for this type of coordination role because of its many contacts in all parts of the world.

A Task Force meeting is held each year to draw together the results obtained by the collaborating groups and to discuss future research. The second of these meetings was held in September 1981 and was attended by 38 participants, representing 20 different institutions from 16 countries. This volume contains the papers presented at the meeting, and is divided into sectors dealing with the specific topics discussed in each session.

The first section deals with results and problems connected with the national models and contains contributions from D. Vanwynsberghe (Belgium); A. Dimitrov (Bulgaria); R. Courbis (France); G. Fink and A. Simon (Hungary); D. Bell (U.K.); and D. Nyhus (U.S.A.).

The second session was devoted to work on modeling of personal consumption expenditure, and both criticism and extensions of the Almon model (which allows a good to be a complement to or a substitute for another good) were presented. Family budget and expenditure data were used to demonstrate methods which estimate income elasticities and which incorporate demographic effects. Contributions from P. Devine (U.S.A.); L. Grassini and M. Grassini (Italy); and G. Kornai (Hungary) are included in this section.

In the third section the participants discussed the problems connected with investment functions and estimation. Papers by C. Ciaschini (Italy); O. Forssell (Finland); and A. Smyshlyaev (U.S.S.R.) deal with the development of improved estimation techniques.

The fourth section contains papers on more general topics: A. Cavalieri (Italy) and A. Lotov (U.S.S.R.) discuss linkage in input-output models, and U. Ludwig (G.D.R.) considers the relationship between final demand and resources.

CONTENTS

PART I

RESULTS AND PROBLEMS CONNECTED WITH NATIONAL MODELS

INFOUK - AN INTERINDUSTRY FORECASTING MODEL FOR THE U.K.

D. Bell

SECTION 1 - INTRODUCTION

One of the greatest deficiencies of conventional macroeconometric models is their almost complete neglect of the process of production. The level of demand is normally carefully modelled and great attention paid to the determination of wage and price levels. However, the supply side of the economy is normally left to fend for itself. The short run supply curve for goods is apparently infinitely elastic; producers are able to meet any level of demand because, in turn, there is always a sufficient supply of factors of production to meet the requirements of the production process.

Now, this approach may be quite reasonable in the short-run, for, of course, the checks and balances implicitly or explicitly embodied in any realistic model of aggregate demand ensure that the productive sector is required to produce any arbitrary level of output. The assumption of infinite elasticity only extends over a fairly narrow band bordered at the lower level by a social and political barrier which is determined by the extent to which society is prepared to tolerate underutilisation of factors and at the upper level by a barrier which reflects the technical and social constraints on production. These endpoints may not be explicitly modelled, but modelbuilders will readily admit that their forecasts are confined within what they believe this range to be.

There are two major drawbacks with this approach: Firstly, the veil which is cast over short-run industrial behaviour implies that such models are incapable of analysing the effects of cyclical fluctuations on particular industries. For example, none of the conventional models are capable of predicting the effect of a fall in the level of investment on the output of the steel industry. Secondly, the long-run behaviour of the economy cannot be reasonably expected to lie within the boundaries either explicitly or implicitly included in conventional models. The boundaries themselves must be modelled for they cannot realistically be regarded as constant in the long-run. Indeed, one can go further and argue that it is the movement of the upper boundary, the boundary on productive capacity, which will determine the long-run growth path of a politically and socially stable economy. Implicit in such a view, of course, is an assumption that markets operate freely in the long-run; wage and price rigidity may determine the level of demand for output and labour in the short-run, but in the long-run market distortions and institutional rigidities can always be circumvented.

The experience of the last two years has led many commentators to the view that the British economy can never return to a position of expanding output and full employment; this would seem to be a doctrine of despair, brought about by a particular combination of policies and events which are unlikely to be repeated. The present severe underutilisation of resources cannot be regarded as a reliable indicator of the long-run performance of the economy. Given that the market mechanism has not been entirely laid to rest, a more balanced view of the long-run performance of the UK economy should incorporate those supply-side factors alluded to above.

This paper has been written to describe a model of the UK economy, known as INFOUK, whose main focus of attention is the productive sector and which eventually, it is hoped, will be able to remedy the twin deficiencies of most existing macro-models* outlined above. This model has been developed at the University of Maryland, USA and at the Fraser of Allander Institute, Glasgow along the lines of a number of national models which share the same basic structure and which will eventually be linked together to provide the

* The most obvious exception to these criticisms amongst existing UK models is the Cambridge Growth Project.

most detailed model of world trade yet constructed. The models derive from the work of Professor C Almon at the University of Maryland who has already completed the construction of a large (200 sector) model of the US economy and has been instrumental in the construction of similar, though smaller models in France, West Germany, Canada, Japan, Belgium, Austria and Italy. These models are known collectively as the INFORUM group of models.

The keystone of each of the models is an input-output (I-O) table which describes the inter-industry transactions which comprise the accounting framework of the productive sector. Thus, for example, the I-O table can be used to determine how the output of the steel industry is distributed as between, say, automobile production, construction and exports; or how the textile industries costs are broken down between, say, the cost of chemicals, energy and labour. Implicit in these transactions is a particular way of making a good; there is a given production technology. As will be apparent subsequently, the structure of the INFORUM models is sufficiently flexible to enable one to revise production technologies in the light of known, or assumed, technical changes.

It is not simply in the modelling of technical change, however, that INFORUM is extremely flexible. The model-user has a wide ability to control the paths of any of the several thousand variables built into the model. Coupled with the advantages of supply-side modelling, the flexibility of INFOUK makes it an extremely powerful and useful tool for economic analysis and forecasting.

The remainder of this paper is concerned to describe the construction and implementation of the model. Thus SECTION 2 describes the major characteristics of INFOUK, while SECTION 3 details the implementation of the model.

SECTION 2 - CHARACTERISTICS OF INFOUK

The claim that INFOUK is able to present a more detailed view of the productive sector than the traditional macromodels is clearly demonstrated by the degree of disaggregation at which it operates. There are 102 separately identified industries in the model, the bulk of these being in the manufacturing sector. These correspond to the sectors in the latest (1974) UK I-O table, which was published in late 1980. The full list of sectors is available in Appendix A-1. Considered as industries (not commodities) these correspond to single or grouped MLH's under the 1968 Standard Industrial Classification and are therefore compatible with a wide range of other government statistics. The square A-matrix, describing intermediate transactions, is supplemented by an extensive array of final demand components and by a breakdown of value added as between sales by final demand, taxes and subsidies, income from employment and other value added.

Final demand is distributed between consumers expenditure, government spending, capital formation, stockbuilding, exports and imports. These are further subdivided, there being 48 categories of consumers expenditure, for example, which are compatible with the Blue Book breakdown of personal spending. There are ten categories of expenditure on food, one for each of the major forms of transportation and so on. Capital spending is broken down between construction and the rest (which encompasses plant, machinery, vehicles, ships and aircraft). There are 36 categories of construction expenditure, one for each of 35 major industrial or commercial sectors and one for residential construction. These same 35 sectors comprise the categories for the remainder of capital formation, separate identification being given to investment in plant and machinery and to investment in ships, vehicles and aircraft. Again, these categories are drawn from classifications used in the Blue Book; in consequence, time series data is readily available to identify investment trends in particular sectors. Government spending is divided into four types; military, national health service, central and local government. While this division is of clear use in assisting the model to assess the impact of particular government expenditure strategies, one should bear in mind that the model is sufficiently flexible to assess policy in a more detailed fashion than the use of only four expenditure categories might, prima facie, suggest. It is possible not only to assess the impact of a broad switch away from local government toward defence, for example, but also to assess the impact of say, a substitution within the defence budget of ships for aircraft.

Exports are accorded a separate sector as are changes in inventories. Imports are considered in considerable detail. A matrix is available describing intermediate import flows, as is information on the import content of each major final demand category; consumption, government spending, capital formation, exports (re-exports), and inventory changes. Not only are these import flows useful in assessing the real impact of changing trade patterns, they also play an important role in the transmission of price changes; a role which will be explicitly modelled when the price side of INFOUK is developed.

The model is set up in producer prices; that is, the cost at which output is sold to intermediate or final demand excluding any taxes or subsidies which the purchaser may have to pay. Further, for consumers expenditure, distribution costs are allocated to distributive trades, transport etc. in order that the data for consumption by sector approximate the prices charged by the ultimate producers of the good.

As implied above, the flows used in the model are also inclusive of imports. The use of total, rather than domestic flows, is of particular value in the analysis of trade and technological change.

To summarise, INFOUK has 102 sectors, 101 of which are active (the 102nd is public administration which has an empty row and column). There are 4 elements of value added and 165 final demand columns (48 for PCE, 111 for investment, 4 for government spending and one each for stock changes, exports and imports). An imports matrix is available and the import content of each of the final demand aggregates (PCE, exports, stock changes, investment and government spending) is known at the 102 sector level.

SECTION 3 - MODEL IMPLEMENTATION

The model was initially developed on the Prime 550 at the University of Maryland over the period May-July 1981. The final "Maryland" version of the model comprises the input-output structure as described in Section 2 but does not include any behavioural equations for the forecasting of final demand. The model can thus run the 'scenario setters' built into the standard INFORUM programs. For example, we can set the level of final demands to what their values have actually been during the period for which such data is available. Forecasts can be developed by using other groups' projections for final demand aggregates and plugging these into the model.

However, this is very much a 'second-best' approach; it is clearly more desirable that INFOUK incorporates its own forecasts of final demands. In order to accomplish this objective, progress has to be made on two fronts. Firstly, time series data must be collected on the components of final demand. With the exception of trade, this can be achieved with little difficulty. The CSO macro-economic databank contains extensive time-series data on PCE, investment, government expenditures and stocks. Work is currently being undertaken which involved setting up programs to interrogate this databank and to perform the usual econometric procedures on the data. The trade data is somewhat more problematic, since commodity-based import and export statistics are not readily available at an appropriate level of disaggregation. A considerable amount of effort will be required to construct the appropriate series.

Secondly, in order to incorporate the estimated behavioural parameters into the system, it is necessary to get the model up and running on a local installation. In central Scotland there are number of computers to which the Fraser of Allander Institute has access which could be used to run INFOUK. The largest of these is the ICL 2980 at the Regional Computing Centre, Edinburgh. The 2980 has 2.5 M byte main store and disc storage capacity of 1800 Mbyte. It communicates to a large number of terminals and terminal control processors (mainly DEC PDP11/10's) via three Front End Processors (DEC PDP 11/34's). This is therefore an extremely large machine which could easily accommodate the INFORUM programs. There is also a FORTRAN 77 compiler mounted on the machine. The particular advantages offered by this compiler are the availability of the PARAMETER statement and the file-handling routines which are comparable to that of the Prime. There should not therefore be much difficulty in ensuring that the models run, though it will be necessary to edit the programs considerably to INSERT those files, such as COMMON and PARAMETER which are used extensively in the system. It is unfortunate that the INSERT statement was not included as part of the FORTRAN 77 standard for this would seem to be its only fundamental difference from PRIME FORTRAN. When our work is completed we hope to have a version of the INFORUM programs which will run on any of the FORTRAN 77 compilers which are now becoming common on a wide range of machines.

As for writing the additional programs necessary to incorporate a more complex and inter-related final demand structure, we intend to follow a somewhat different and certainly novel approach. The Institute possesses a microcomputer which can support two users and has a FORTRAN compiler and RATFOR pre-processor. Running under a UNIX-like operating system with an extremely powerful screen editor, this machine possesses great advantages over the traditional mainframes, few of which presently offer RATFOR. Principal of these is the reduction in program development time which results from the use of a structured language such as RATFOR. Now in terms of sheer computing power, a 64K micro is obviously not in the same league as a 2.5 Mbyte mainframe. The actual running of programs is best left to the latter. Thus we intend to send the debugged versions of the programs into the regional network to be picked up and run on the 2980. We have already had some experience with this technique while constructing a medium term model for Scotland, which shares many features with the INFORUM class of models.

Clearly, there is still a great deal of work to be done. Our major concern at present lies in mounting and running the programs necessary to establish and develop INFOUK. As yet, there has been little time to consider the theoretical aspects of the equations likely to be embodied in the model. Nevertheless, once we have a sound computing base we would hope to make considerable progress and are looking forward to presenting a considerably more sophisticated working model in 12 months time.

THE ANAIS MODEL. AN ANNUAL INPUT-OUTPUT MODEL FOR THE FRENCH ECONOMY

R. Courbis and H. Sok

Le modèle ANAIS - ainsi appelé par abréviation de "modèle ANnuel d'Analyse Inter-Sectorielle) est un modèle de prévision et de simulation annuel de type inter-sectoriel qui permet de prévoir l'évolution de la production et de la demande à un niveau très détaillé : celui de la nomenclature S91 en 90 branches de la Comptabilité Nationale française (sur cette nomenclature voir tableau 1).

Construit par le GAMA, ce modèle concerne seulement l'économie française mais on se propose (voir infra) de l'intégrer au système de modèles I/O nationaux du projet INFORUM ce qui permettrait, pour les simulations de tenir compte des effets de feed-back internationaux(1).

(1) Dans le cadre du projet INFORUM, un modèle I/O de la France a été construit par LEE et ALMON (1978). Cette version a été utilisée récemment pour des simulations "linkées" avec les modèles allemand et belge du projet INFORUM (voir NYHUS et ALMON, 1980). Elle a également fait l'objet d'une réestimation sur les nouvelles séries de la Comptabilité Nationale par les soins de P. HENAFF (1981).

TABLEAU 1 : NOMENCLATURE EN 90 BRANCHES DU MODELE ANAIS

Nomenclature N° d'ordre ANAIS	Nomenclature S91	Intitulé des branches
1	01	Produits de l'agriculture
2	02	Produits de la sylviculture et de l'exploitation forestière
3	03	Produits de la pêche
4	041	Houille, lignite et agglomérés
5	042	Produits de cokéfaction
6	051	Pétrole brut
7	052	Gaz naturel
8	053	Produits pétroliers raffinés
9	06	Electricité distribuée
10	07	Gaz distribué
11	08	Eaux et chauffage urbain
12	09	Minerais de fer
13	10	Produits sidérurgiques
14	11	Produits de la première transformation de l'acier
15	12	Minerais non ferreux
16	13	Métaux non ferreux et demi-produits
17	14	Minéraux divers
18	15	Matériaux de construction
19	16	Produits de l'industrie du verre
20	171	Produits de la chimie minérale
21	172	Produits de la chimie organique
22	18	Produits de la parachimie
23	19	Produits pharmaceutiques
24	20	Produits des fonderies
25	21	Produits du travail des métaux
26	22	Machines agricoles
27	23	Machines outils
28	24	Equipement industriel
29	25	Matériel MTPS
30	26	Matériel d'armement
31	27	Machines de bureau
32	28	Matériel électrique
33	291	Matériel électronique professionnel
34	292	Matériel électronique ménager
35	30	Equipement ménager
36	311	Véhicules automobiles
37	312	Matériel féroviaire roulant
38	32	Produits de la construction navale
39	33	Produits de la construction aéronautique
40	34	Instruments et matériel de précision
41	35	Viandes et conserves de viandes
42	36	Lait et produits laitiers
43	37	Conserves
44	38	Pain et patisserie
45	39	Produits du travail du grain
46	401	Corps gras alimentaires

Nomenclature N° d'ordre ANAIS	Nomenclature N° S91 INSEE	Intitulé des branches
47	402	Sucre
48	403	Autres produits alimentaires
49	41	Boissons et alcools
50	42	Produits à base de tabac
51	43	Fils et fibre artificiels et synthétiques
52	441	Fils et filés
53	442	Produits de la bonneterie
54	443	Ouvrages en filés
55	451	Cuirs et peaux
56	452	Articles en cuir
57	46	Chaussures
58	47	Articles d'habillement
59	48	Produits du travail du bois
60	49	Meubles
61	50	Papier, carton
62	51	Presse, imprimerie, édition
63	52	Pneumatiques
64	53	Produits de la transformation des matières plastiques
65	54	Produits des industries diverses
66	55	Bâtiment et génie civil
67	56	Produits de la récupération
68	57-4	Commerces
69	65	Commerce et réparation automobiles
70	66	Réparations diverses
71	67	Service des hôtels, cafés, restaurants
72	68	Transports ferroviaires
73	691	Transports routiers de marchandises
74	692	Autres transports terrestres
75	70	Navigation intérieure
76	71	Transports maritimes
77	72	Transports aériens
78	73-4	Services auxiliaires de transport
79	75	Services de télécommunications
80	76-9	Services rendus principalement aux entreprises
81	80	Location et crédit-bail mobilier
82	811	Service du logement
83	812	Crédit-bail immobilier
84	82-3	Services d'enseignement (marchands)
85	84	Services de santé (marchands)
86	85-7	Autres services marchands
87	88	Services d'assurances
88	89	Services des organismes financiers
89	90-1	Services d'administration générale
{ 90	92-3	Service d'enseignement (non marchand)
{ 91	94-9	Autres services non marchands

Dans le cadre de cette communication on présentera tout d'abord (au § 1) la structure et les principales relations du modèle ANAIS, et on appréciera ensuite la fiabilité de ce modèle en analysant (au §2) les principaux résultats des simulations rétrospectives dynamiques effectuées sur 1973-1980.

Dans sa première version, telle que présentée dans ce papier, le modèle ANAIS est opérationnel depuis juin 1981 et il a servi dès cette date à effectuer des prévisions année par année à un horizon de 5 ans (1981-1986). On se propose toutefois au cours des deux prochaines années d'améliorer le modèle sur plusieurs points (§3).

1 - STRUCTURE GENERALE ET PRINCIPALES RELATIONS :

Comme indiqué ci-dessus le modèle ANAIS a pour objet de projeter, pour l'économie française, l'ensemble du tableau input-output en 90 branches ; il vise à prévoir pour chacune des 90 branches l'évolution de la demande intérieure (finale ou intermédiaire) ou extérieure (exportations) ainsi que celle de la production.

Dans sa version actuelle, le modèle ANAIS est toutefois un modèle partiel : il part (cf. figure 1) d'un certain nombre de résultats prévisionnels du modèle MOGLI - un modèle économétrique dynamique semi-global de l'économie française construit par le GAMA de 1974 à 1978 (1) - qui constituent des "entrées" du modèle ANAIS : revenu disponible des ménages ; investissements (productifs et non productifs) des différents agents ; évolution des prix à un niveau semi-global. La "demande étrangère" est également une variable exogène du modèle.

La structure générale du modèle ANAIS est, dans ces conditions, donnée figure 1.

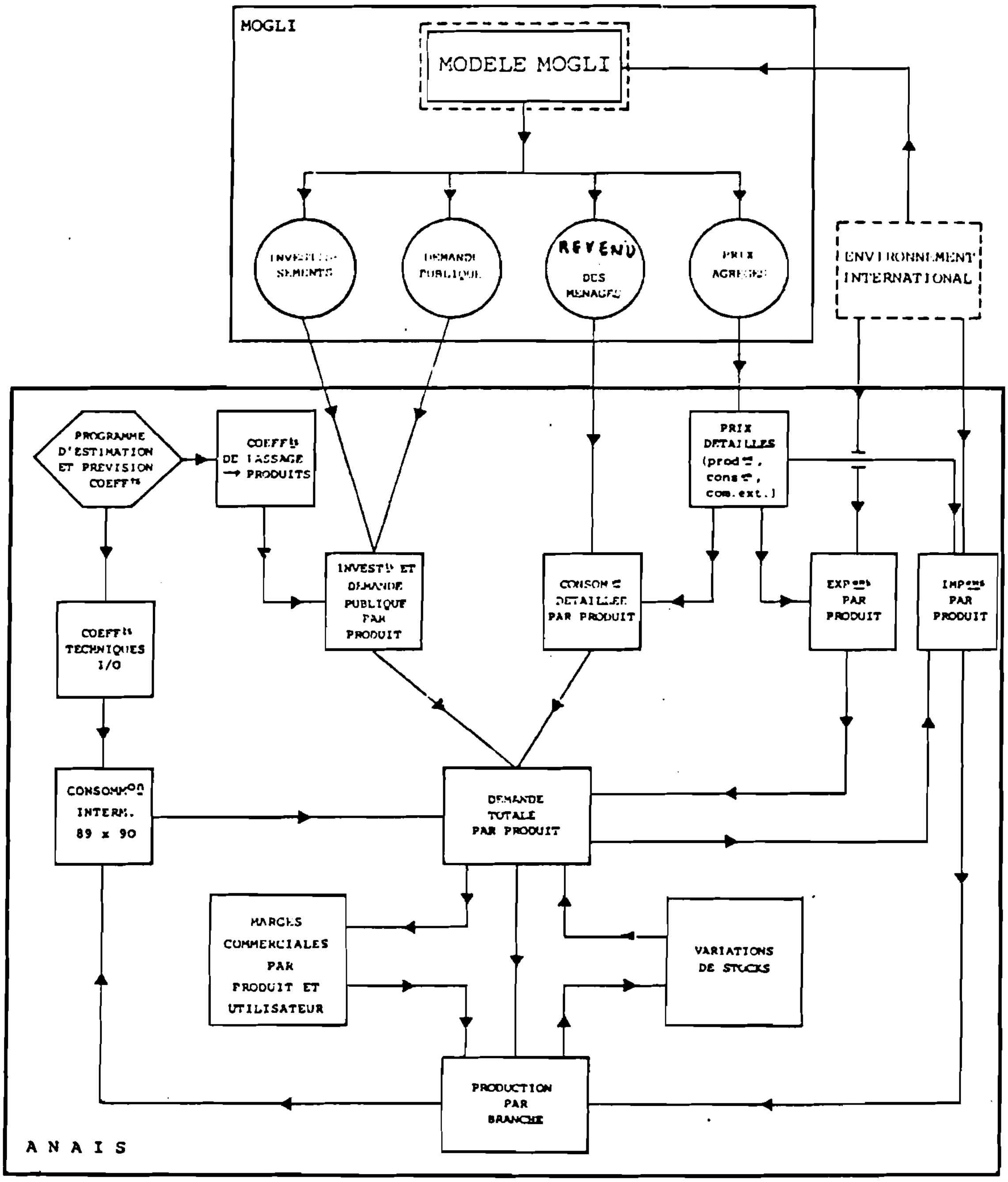
Partant d'une analyse de la demande finale et des échanges extérieurs par produit d'une part, et de la détermination des "coefficients techniques" de demande intermédiaire, le modèle ANAIS permet de calculer la production au niveau de chacune des 90 branches du modèle en tenant compte de l'interdépendance entre la production et la demande qui existe entre les différents marchés.

---

(1) Sur le modèle MOGLI, voir COURBIS et al. (1978,1980).

FIGURE 1

STRUCTURE ET FONCTIONNEMENT DU MODELE ANAIS



En ce qui concerne la demande finale intérieure :

(1) La consommation des ménages par produit (au niveau détaillé du modèle AMAIS) est déterminée en fonction du revenu réel disponible (présent et passé), de l'évolution des prix à la consommation, et de la consommation antérieure du produit considéré.

La formule retenue varie selon les branches :

- pour les biens durables, on a utilisé une approche du type HOUTHAKKER-TAYLOR ;
- pour les biens non durables, on a expliqué en général le coefficient budgétaire correspondant. Celui-ci dépend du prix relatif du produit concerné et des habitudes de consommation ;
- enfin dans quelques cas, on a expliqué directement la consommation en fonction du revenu réel, des habitudes de consommation et des prix.

(2) Les investissements par produit sont calculés en partant de la donnée des investissements par agent (entreprises, ménages, administrations, organismes financiers) - dont l'évolution est prévue à l'aide du modèle MOGLI, Le total des investissements par agent est décomposé par produit sur la base de coefficients structurels projetés en fonction du temps à l'aide de relations économétriques appropriées.

(3) Les variations de stocks par produit sont déterminées sur la base d'une formulation en termes de "stock outil" ;

$$\Delta\ (\Delta Sit) = \beta_1\ \Delta\ (\Delta\ Q_{it}) + \beta_2\ \Delta\ S_{i\ t-1}$$

où: $\Delta\ S_{it}$ = variation de stocks de produits i en volume pour l'année t ;

Q_{it} = production (en volume) de la branche i pour l'année t.

Pour achever de déterminer la demande intérieure totale, on calcule alors la demande intermédiaire par produit et par branche sur la base de "coefficients techniques" qui sont déterminés à l'aide de relations temporelles dont la spécification toutefois varie selon les coefficients.

Par totalisation, on en déduit le total de la demande intermédiaire par produit et par conséquent la demande intérieure totale. Il convient alors de prévoir l'évolution des échanges extérieurs : exportations et importations par produits. Ceux-ci sont déterminés à l'aide de "fonctions d'exportation" et de "fonctions d'importation" :

- les exportations en volume sont reliées à la demande mondiale, à la demande intérieure et au prix relatif à l'exportation (par rapport aux prix étrangers) ;
- les importations par produit sont fonction de la demande intérieure (pondérée selon les contenus d'importations de chaque composante de la demande intérieure) et du prix relatif des produits nationaux et des produits importés.

Les "équilibres emplois-ressources" permettent de déterminer alors la production compte tenu de la demande totale et des échanges extérieurs. Pour certaines branches, la production est toutefois supposée exogène : l'ajustement offre-demande se fait alors soit sur les stocks, soit sur les échanges extérieurs (exportations ou importations).

La demande étant au prix d'utilisation et hors TVA récupérable alors que la production est aux prix de production hors taxe, ceci nécessite toutefois de déterminer également les "marges commerciales" et la "TVA non récupérable" :

- les marges commerciales sont calculées en fonction de la demande et de taux de marge différenciés par catégorie d'utilisateurs ;
- la TVA non récupérable est reliée à la demande des agents qui ne peuvent pas récupérer la TVA (essentiellement les Ménages et les Administrations).

Les "taux de marge" et "les taux de TVA"(1) sont déterminés année par année à l'aide de relations économétriques temporelles spécifiques.

(1) Raisonnant ici en volume, on considère que la législation est constante et que les taux élémentaires sont ceux de l'année de base. Mais compte tenu d'effets de structure, même au niveau de 90 branches, les taux moyens apparents de TVA "en volume" varient néanmoins.

Le calcul des marges commerciales et de la TVA non récupérables permet d'écrire complétement les équilibres emplois-ressources par branche(1), ce qui permet de calculer la production (sauf si celle-ci- cf. supra-est exogène) :

Production H.T. + Importations + Droits de douane sur importation +

TVA non récupérable + Marges commerciales = Demande intérieure totale (prix d'utilisation et hors TVA récupérable)

+ Variations de stocks

+ Exportations.

Il faut tenir compte toutefois de l'interdépendance qui, comme le montre le schéma général du modèle ANAIS (cf. figure 1), existent entre les différentes variables du modèle. Pour résoudre le modèle, on pourrait théoriquement partir de l'inversion de la matrice de Leontief mais compte tenu de la taille de celle-ci (90 branches), on a préféré adopter une résolution par itérations. Celle-ci a en outre l'avantage de ne pas obliger de rendre au préalable "carrée" la matrice des coefficients techniques qui, dans le système français, est une matrice rectangulaire ; elle permet également de tenir compte facilement des autres interdépendances que celles existant au seul niveau des consommations intermédiaires (notamment celles relatives aux importations et aux stocks). Enfin, le choix d'une méthode de résolution par itérations facilitera les améliorations que l'on se propose d'apporter ultérieurement au modèle.

Les ajustements économétriques des relations retenues dans ANAIS ont été effectués sur les données de la Comptabilité Nationale relatives aux années 1970-1980, sauf en ce qui concerne la consommation des ménages pour lesquels les relations ont été estimées sur 1959-1980.

L'estimation et la gestion du modèle ANAIS sont effectuées à l'aide d'un logiciel informatique construit spécialement par le GAMA pour les besoins d'ANAIS. Ce logiciel (le logiciel GERIC(2)) a en particulier l'avantage de permettre une réestimation rapide, de caractère semi-automatique, des équations économétriques d'ANAIS. Mais une réécriture

(1) En fait, il faut également tenir compte de la production par certaines branches de "produits fatals", c'est à dire de produits correspondant à l'activité d'autres branches.

(2) Ce logiciel est dû à Hach SOK.

du programme dans le logiciel SLIMFORP sera nécessaire si on veut intégrer le modèle ANAIS dans le cadre du système mondial de modèles d'INFORUM.

2 - RESULTATS RETROSPECTIFS :

On trouvera au tableau 2 les résultats au niveau agrégé des simulations rétrospectives effectuées de manière dynamique avec le modèle ANAIS sur 1973-1980. Les erreurs sur la consommation des ménages sont faibles (EAM = 0,4 %) mais il faut toutefois tenir compte de ce que l'évolution du revenu disponible réel des Ménages est, dans la version actuelle d'ANAIS, considérée comme exogène (et déterminée par le modèle MOCLI). Par contre l'erreur sur le niveau total des variations de stocks est remarquablement faible par rapport à celle que l'on trouve généralement pour les stocks dans les modèles économétriques agrégés ou semi-globaux. Les erreurs sur les échanges extérieurs, sont toutefois plus élevées, surtout en ce qui concerne les importations.

TABLEAU 2

Simulations dynamiques rétrospectives 1973-1980 avec le modèle ANAIS ;
résultats agrégés.
- Erreur absolue moyenne (EAM)

Variables	E.A.M.
Consommation des ménages (en volume)	0,4 %
Exportations (en volume)	1,5 %
Importations (en volume)	3 %
Variations de stocks (en volume aux prix 1970)	5,1 Milliards de Frs.
Produit intérieur brut (PIB) en volume	0,5 %
PIB marchand	0,6 %

Au total, du fait des compensations, l'erreur absolue moyenne sur le PIB total ou le PIB marchand en volume apparaît limitée : respectivement 0,5 et 0,6 %.

En ce qui concerne les résultats, au niveau détaillé des 90 branches, on trouvera au tableau 3 l'erreur absolue moyenne pour la production en volume par branche pour la simulation rétrospective 1973-1980. Les erreurs sont dans l'ensemble limitées, mais elles sont néanmoins élevées pour plusieurs branches. Pour des branches comme "Minerai de fer" et "sucre" où les erreurs sont particulièrement fortes, ceci s'explique par le fait que la production n'est en fait guère déterminée dans ce cas par la demande ; en conséquence on se propose de considérer la production de ces branches comme exogènes ; on solderait alors sur les importations (minerai de fer) ou sur les exportations et les stocks (sucre).

Une analyse plus détaillée des erreurs reste toutefois à faire : les enseignements qu'elle devrait donner seront susceptibles d'orienter les améliorations à apporter au modèle.

3 - AMELIORATIONS :

Dans sa version actuelle, le modèle ANAIS ne constitue qu'une première version que l'on se propose d'améliorer progressivement, en particulier en ce qui concerne :

- les prix. Ceux-ci sont déterminés actuellement (à partir d'extrapolations de prix relatifs) d'indices de prix agrégés dont l'évolution est prévue avec le modèle MOGLI. On se propose d'endogénéiser véritablement la détermination des prix de production mais ceci demande au préalable que soit analysée dans ANAIS l'évolution des coûts unitaires de production ;

- l'emploi. L'évolution de l'emploi n'est actuellement pas analysée dans ANAIS mais celle-ci est indispensable si on veut prévoir l'évolution des coûts unitaires de production dont une composante importante est constituée par les charges salariales. On se propose donc en premier d'endogénéiser l'emploi par branche dans ANAIS mais pour des raisons statistiques ceci ne pourra être fait (du moins au départ) qu'au niveau plus agrégé de 40 branches ;

TABLEAU 3

SIMULATIONS RETROSPECTIVES ANAIS 1973-1980

Production (en volume) par branche

Erreur absolue moyenne (EAM) en %

MODELE ANAIS : SIMULATION RETROSPECTIVE, ANALYSE DES RESIDUS

ERREUR MOYENNE DE LA PRODUCTION SUR LA PERIODE 1973-1980

		(%)
01	AGRICULTURE	EXOGENE
02	SYLVICULTURE	4.0
03	PECHE	2.9
04	HOUILLE, LIGNITE, AGGLOMERES	EXOGENE
05	COKEFACTION	3.9
06	PETROLE BRUT	EXOGENE
07	GAZ NATUREL	EXOGENE
08	PETROLE RAFFINE	2.9
09	ELECTRICITE DISTRIBUEE	1.0
10	GAZ DISTRIBUE	3.7
11	EAU ET CHAUFFAGE URBAIN	2.0
12	MINERAI DE FER	7.6
13	SIDERUGIE	3.8
14	PREMIERE TRANSFORMATION DE L'ACIER	2.4
15	MINERAIS NON FERREUX	EXOGENE
16	METAUX NON FERREUX	4.5
17	MINERAUX DIVERS	5.9
18	MATERIAUX DE CONTRUCTION	1.5
19	VERRE	4.3
20	CHIMIE MINERALE	4.6
21	CHIMIE ORGANIQUE	5.0
22	PARACHIMIE	2.9
23	PRODUITS PHARMACEUTIQUES	2.0
24	FONDERIES	2.1
25	TRAVAIL DES METAUX	1.7
26	MACHINES AGRICOLES	6.2
27	MACHINES OUTILS	8.1
28	EQUIPEMENT INDUSTRIEL	5.2
29	MATERIEL MTPS	4.4
30	MATERIEL D'ARMEMENT	EXOGENE
31	MACHINES DE BUREAU	7.7
32	MATERIEL ELECTRIQUE	5.4
33	MATERIEL ELECTRONIQUE PROFESSIONNEL	5.1
34	MATERIEL ELECTRONIQUE MENAGER	5.4
35	EQUIPEMENT MENAGER	5.9
36	VEHICULES AUTOMOBILES	3.8
37	MATERIEL FERROVIAIRE ROULANT	7.5
38	CONSTRUCTION NAVALE	EXOGENE
39	CONSTRUCTION AERONAUTIQUE	EXOGENE
40	INSTRUMENTS ET MATERIEL DE PRECISION	8.6
41	VIANDES ET CONSERVES DE VIANDES	1.9
42	LAIT ET PRODUITS LAITIERS	1.9
43	CONSERVES	6.1
44	PAIN ET PATISSERIE	1.0
45	PRODUITS DU TRAVAIL DU GRAIN	1.5

TABLEAU 3 (Suite)

SIMULATIONS RETROSPECTIVES ANAIS 1973-1980

Production (en volume) par branche

Erreur absolue moyenne (EAM) en %

MODELE ANAIS : SIMULATION RETROSPECTIVE, ANALYSE DES RESIDUS

ERREUR MOYENNE DE LA PRODUCTION SUR LA PERIODE 1973-1980

		(%)
46	CORPS GRAS ALIMENTAIRES	7.9
47	SUCRE	13.7
48	AUTRES PRODUITS ALIMENTAIRES	1.3
49	BOISSONS ET ALCOOLS	2.2
50	PRODUITS A BASE DE TABAC	6.6
51	FILS ET FIBRES ARTIFICIELS ET SYNTHETIQUES	8.3
52	FILS ET FILES	2.3
53	BONNETERIE	3.8
54	OUVRAGE EN FILES	3.4
55	CUIRS ET PEAUX	5.2
56	ARTICLES EN CUIR	7.9
57	CHAUSSURES	3.9
58	HABILLEMENT	4.4
59	PRODUITS DU TRAVAIL DU BOIS	3.7
60	MEUBLES	2.4
61	PAPIER, CARTON	4.0
62	PRESSE, IMPRIMERIE, EDITION	3.4
63	PNEUMATIQUES	4.6
64	TRANSFORMATION DES MATIERES PLASTIQUES	5.0
65	INDUSTRIES DIVERSES	5.1
66	BATIMENT ET GENIE CIVIL	1.0
67	RECUPERATION	EXOGENE
68	COMMERCE	0.7
69	COMMERCE ET REPARATION AUTOMOBILE	1.5
70	REPARATIONS DIVERSES	5.5
71	HOTELS, CAFES, RESTAURANTS	2.5
72	TRANSPORTS FERROVIARES	1.3
73	TRANSPORTS ROUTIERS DE MARCHANDISES	3.0
74	AUTRES TRANSPORTS TERRESTRES	2.0
75	NAVIGATION INTERIEURE	4.5
76	TRANSPORTS MARITIMES	2.7
77	TRANSPORTS AERIENS	1.7
78	SERVICES AUXILIAIRES DE TRANSPORT	1.6
79	SERVICES DE TELECOMMUNICATIONS	4.9
80	SERVICES RENDUS AUX ENTREPRISES	1.1
81	LOCATION ET CREDIT-BAIL MOBILIER	6.1
82	LOGEMENT	0.4
83	CREDIT-BAIL IMMOBILIER	4.0
84	ENSEIGNEMENT (MARCHAND)	6.3
85	SANTE (MARCHAND)	1.3
86	AUTRES SERVICES MARCHANDS	1.4
87	ASSURANCES	2.1
88	ORGANISMES FINANCIERS	5.1
89	SERVICES NON MARCHANDS	3.2
90	ENSEMBLE DES PRODUITS	0.5

- les revenus. L'endogénéisation de l'emploi dans ANAIS permettra de déterminer facilement (compte tenu de la dynamique des salaires nominaux par tête) les salaires versés par branche (du moins au niveau de la nomenclature 40). Il conviendra alors d'analyser, au moins en quelques composantes, les autres revenus les ménages pour calculer leur revenu total et leur revenu disponible. Ceci permettrait de tenir compte du bouclage important qui existe par l'intermédiaire des revenus.

Ultérieurement, la détermination des investissements totaux par agent pourrait être faite dans ANAIS. Pour les investissements productifs,elle serait faite au niveau des branches ce qui permettrait de prendre alors en compte l'incidence des "effets d'offre" au niveau des capacités de production disponibles.

Ces améliorations devraient être faites progressivement en 1982-1983 mais d'ores et déjà, malgré ses limites, la version actuelle d'ANAIS qui est opérationnelle depuis juin 1981 peut être utilisée (en liaison avec le modèle MOGLI).

REFERENCES

COURBIS,R., FONTENEAU, A., LE VAN, C. et VOISIN, P. (1978), "Le modèle MOGLI, modèle économétrique plurisectoriel de prévision glissante de l'économie française", Papier GAMA, n°220, juin 1978.

COURBIS,R., FONTENEAU,A., LE VAN,C. et VOISIN,P., (1980), "Le modèle MOGLI", Prévision et Analyse Economique, vol. 1, n°2-3 (Paris, Editions Economica).

HENAFF,P. (1981), "An Input-Output Model of the French Economy", Master thesis, University of Maryland.

LEE,Y.S. et ALMON,C., (1978), "An Input-Output Forecasting Model of the French Economy", Inforum research report, n°20, janvier 1978.

NYHUS,D.E. et ALMON,C. (1980), "Linked Input-Output Models for France, Germany and Belgium", I.I.A.S.A., Laxenburg, juillet 1980 (à paraître en français dans Prévision et Analyse Economique, vol. 2, n°3, juil-sept. 1981).

E.I. INFORUM MODEL FOR BULGARIA - ADJUSTMENT PROBLEMS AND SOME RESULTS

A. Dimitrov

INTRODUCTION

Since the beginning of 1980 The Economic Institute of the Bulgarian Academy of Sciences began to work on the adjustment, experimenting and development of the INFORUM multisectoral macroeconomic model for economic growth. The task of the team was jointly with the International Institute for Applied System Analysis to study the INFORUM model and to find out possibilities for its application in the conditions of our country. The concrete results obtained and the methodology described, the computer software and the recommendations given for the further development and expansion of its scope, are supposed to be put at the disposal of the State Planning Committee and to be useful in making out medium-term perspective foreplans.

A description of a longer-term variant of the INFORUM adjustment was presented to IIASA in January this year. The present report covers the results of these activities of the Economic Institute with completion term - end of 1981 . Preliminary results of the INFORUM model use with Bulgarian data were obtained a year ago. These tests, the first for our country proved useful because they showed the necessity of a large and profound preliminary study of INFORUM and the particularities and differences of the socialist economy. This brought to studies and evaluations of given spheres of the economy and individual sectors and to forecasts preparation . These were essentially the final demand elements, previously examined independently, irrespectively of INFORUM model as a whole.

This year subject to our research were personal consumption, capital investment process, foreign trade sector, changes in the technical coefficients of matrix A and the computer program for each sphere individually and of the model as a whole. The problems which arose in these fields and some preliminary results are the main contents of the present report. The final and complete variant of these research activities and their results are to be finished by the end of 1981 and followed by a discussion in the Scientific Council of the Economic Institute of the Bulgarian Academy of Sciences. Some problems are subject to an wider study and presentation and will be treated in other papers and reports which are under way.

The initial variant of the research is done on a 25 sectors nomenclature. These are the subsectors of the industry and the basic sectors of the country's material production . The main input data used is the input-output balance of People's Republic of Bulgaria for each year of the period I971-I979. Forecasts are being made for I980-I990, as well as for each year of the period. According to the particularities of the different spheres other information as well as data for a longer period of time is made use of.

I. CONSUMPTION

The final demand's element "Consumption" is represented in the Bulgarian Statistics as a sum of "consumption of the population"and"other types of consumption ". Their values have to be forecasted for the period I98I - I990. Both elements of "Consumption" embrace a 25 sectors structure of the national economy.

The forecast of vectors "Consumption of the population" and "Other types of consumption" is done in two directions using two different approaches. This makes a comparative study of the results obtained by the two approaches possible and the choice of the better one / according to set criteria / and thus enriching considerably the retrospective analyses of the studied processes. In a way the methods applied are mutually complimentary one to another, because the first reflects the change of the separate components /sectors/ in time and the second the change of the consumption demand of given commodities and commodities groups, aggregated to the level of sectors. The deaggregation of the sectors in respective commodities and commodities groups and their examination by means of a type of consumption functions affords the analysis of the consumption demand depending on the income and the prices /permanent or current/

and at the same time being defined by the reverse process / aggregation of the respective commodities and commodities groups into sectors/ the consumption of the population by sectors of the material sphere and the industry. Due to the fact that this method reflects only the dependence of the consumption on the income and the prices it is applicable solely to vector"Consumption of the population".

According the first method the examination of the general development tendancies and the respective trend extrapolation is effected in sectors, i.e. the vectors "Consumption of the population" and Other tyoes of consumtion" are analyzed and forecasted for each sector individually. There is a total volume forecast for each vector as well and the results obtained are compared to the sum of the forecasted volumes of the individual sectors. The length of the dynamic rows tested is 9 years /since I97I till I979 /. The analysis of the dynamics and the corresponding extrapolation is done by means of four single development models different in type linear function, second degree non-linear function, linear-semi-logarithmic function and second degree semi-logarithmic function which enables the choice of the one describing best the process under examination for each sector.

The following results were obtained for "Consumption of the population" for I980 - I990. Average annual growth rates for : Electrical energy - about 5%, Machinebuilding and metalworking industry - about 6%, Chemical industry -4,5%, Food and tobacco industry -5,5%, Crop-growing - 2,5% and Stock-breeding -3,0% . The total volume of "Consumption of the population" will develop with about 4% and the summed volume also with about 4%.

In respect to "Other types of consumption" vector the results of the forecast are as follows: Electrical energy - 2,4%, Machine-building and Metalworking industry - 2,4%, Chemical industry - 2%, Crop-growing -2,6% and the same percentage for Stock-breeding . The forecasted total volumes are very close -4,2% for the total volume and 4,7% for the summed one.

Besides the abovementioned method for direct consumption forecast by sectors, one indirect method to forecast "Consumption of the population" was also used. It was based on a system of consumption functions. The essence of this method is in the construction / on the basis of consumption real data and prices indices / of some dependencies of the consumption on the retail prices and the income and the consumption forecast on this ground as a function of the forecasts of the retail prices indices and the income.

For the realization of the second method, starting from the system of consumption functions [1,2] suggested by prof.C.Almon and applied in INFORUM and on the ground of other types of known consumption functions, we have developed and tested a system of hierarchical consumption functions. These are distinguished from Almon's functions by the way of the prices indices aggregation. In Almon's functions the prices indices irrespective of the values elasticities are aggregated only multiplicatively.

One may prove that accepting a more general form of aggregation of the prices indices allows on one side the consumption functions to meet Almon's requirements, at least in the degree satisfying them as C. Almon's functions and in some additional conditions even to a greater degree. On another side our way of aggregation affords a more natural transition between the consumption functions, meeting the well-known neo-classic requirements to the Consumption Choice Theory/ the main one being that the consumption functions are to be engendered by the extremum problems solutions for maximization of given preference functions at incomes' constraints.

At this stage of the research a methodology for the parameters estimation of the consumption functions system was developed at condition based on the multiple use of the least square method /LSM/ ,parameters estimation programs and consumption forecast according to these functions. The necessary real data for the period 1960 -1980 was collected and computed with respect to consumption and the prices for 60 basic goods groups aggregated into four levels. The dynamic rows of the income and the prices till 1990 were forecasted. By means of the above programs a parameters' estimation for the different report subperiods is being done. One has to forecast now the consumption and the consumption functions and the aggregation to the level of sectors.

There are some methodological and information difficulties connected with :the definition of some of the prices indices, the differences in the demand and the consumption in periods of temporary shortage of some goods, the evaluation of the parameters in more significant changes of the nominal retail prices and the incomes, the transition from forecasts by commodities goups to forecasts by sectors / due to the availability of a double price system retail and wholesale prices /etc, which are natural for this type of research and consumption forecast.

It is of interest to realize the more general form of the consumption functions system but it needs a great volume of information from the family budgets in order to study the alterations income influence over the consumption.

2. CAPITAL INVESTMENT

The forecast of the volume of the capital investments generally and by technological structure is effected for the whole national economy by sectors of the material sphere and by industry's sectors. What is not included however is the determination of the expenditure for general overhaul and major expenditures as for geological research, for the formation of basic herds, expenditures for normative, research and other work of experimental nature. The real data by types of capital expenditure do not show a given tendency and their respective equations have a very high variation coefficient.

To define the capital investments volume in perspective different methods and models are used. The data for all indices included in the models are for the period I960-I979(I980) .

The increase of the capital investments volume for the period I960-I980 is not rhythmical in its variations. One can differentiate two sub-periods I960-I969 and I970-I979/I980/ .It is possible to divide the second subperiod to another two subperiods : from I970 till I975 and from I976 till I980.No matter what the alterations are, the average highest alterations rates are detached in the second subperiod, and the lowest - during the first one. The general tendency for a gradual decrease of the relative rates is characteristic.

The original models for the definition of the capital investments volume are the single development models .Here also the afore said four different models are used.For a partial parameters stability test all equations are computed four times : for all " n " members /n = I960,I96I,...,I980 /of the dynamic rows, and also for the first and second subperiods and for n-2 members.The comparative parameters estimations of the equations in all the four cases lead to some conclusions and give an idea about the directions and the rates of their alterations. The total capital investments volume for all the national economy, by sectors and by industry's sectors is defined by means of the equations.The number of the sectors is 22. Simultaneously the volume of the capital investments for all the national economy is obtained also as a sum of the capital investments volumes by its sectors. The comparison of the estimated and the summed value is an index for the choice of the models chosen. An analogical comparison is being done for sector "Industry" too.

Trend equations are used too for the definition of the capital investments volume by their technological structure /machines and equipment and construction and assembly works / for the same sectors and for the whole national economy.

The regression type of models is the following type used to forecast the capital investments volume.

Originally as an independent variable at the definition of the capital investments volume by sectors of the material sphere are the capital investments for the whole national economy. At the same time for the sectors of the industry the independent variable is the capital investment of sector"Industry" . Here also it is possible to compare one and the same quantity computed differently.

Since capital investments represent the means of / productive/ "Accumulation" and amortization then the definition of their volume for the whole national economy in a regression model is effected considering them as independent variables.

The volume of the total gorss product and the national income in a given period depend on the scale of the production. This dependence reflects the indirect influence of the capital investments over the gross product's size and the national income. The multilateral connection of the social production's development with the capital investment's size and structure defines them as a factor of a paramount importance. Through the capital investments the use of the available assets is improved and new ones are created which influences directly the gross product's volume increase and that of the national income's size.

The interaction among the capital investments, the national income and the gross product is important. The national income is the source for capital investments and therefore its size affects theirs. This connection demands a detailed study of their interaction. That is why regression type equations are used for the national economy's sectors where the independent variable is the capital investment and the variable one - the gross national product's growth and the feed back. For all sectors a two year lag was accumulated bearing in mind that the capital investments of the current year /t/ are meant for the completion of the construction of this year, to go on with it in the current year and the beginning of a new construction during the same period. Therefore in case of a capital investments lag τ, in t-th year the construction started in $(t-\tau)$-th year will be done, construction started in $(t-\tau+1)$, $(t-\tau+2)$, ..., $(t-1)$ years will continue and construction which is to be finished in t, $(t+1)$, ..., $(t+\tau-1)$ year and a construction was started in the same year which is to be completed in $(t+\tau)$-th year. The following type of linear dependences were used:

$$\Delta X_{ti} = a_{0i} + a_{1i} + a_{1i} I_{ti} + a_{2i} I_{(t-1)i} + a_{3i} I_{(t-2)i}$$

or

$$I_{ti} = a_{0i} + a_{1i}(X_{(t-1)i} - X_{(t-2)i}) + a_{2i}(X_{(t-2)i} - X_{(t-3)i}$$
$$+ a_{3i}(X_{(t-3)i} - X_{(t-4)i})$$

and

$$I_{ti} = b_{0i} + b_{1i}\Delta X_{ti} + b_{1i}\Delta X_{(t-1)i} + b_2\Delta X_{(t-2)i} + b_{3i}Y_t$$

where ΔX_t is the gross product's growth at the respective i-th sector of the year t ;

y_{ti} -the net output of the i-th sector for t-th year ;

The distribution of the capital investments lagging allocation during three years is modelled in the following equation :

$$\Delta K_{ti} = c_{oi} + c_{Ii}I_{ti} + c_{2i}I_{(t-I)i} + c_{3i}I_{(t-2)i}$$

It expresses the capital investments effect over the growth of the main assets due to them /ΔK_{ti} / .

There is the feed back as well but it will reflect the constant effect of the main assets put into action to the augmentation of the capital investments volume.

Finally in order to fix the capital investments volume one has to proceed from the output ratio and the temporary lag between the total output's growth and the investments allocated to it.

Since capital investments represent a sum of the worn out main assets means for restoration and the expenditures for their expansion, one defines their volume by the following model :

$$I_{ti} + K[Z_i X_{ti} + \sum_{j=1}^{\tau} W_{ji}\Delta X_{(t-j)i}]$$

where k_i is the capital output ratio coefficient at the i-th sector. Z_i - renewals assets norm, W_{ji} - lag coefficients $\sum_{j=1}^{\tau} W_{ji} = I$.

If one assumes that the replacement is proportional to the main assets, and they are in proportion with the smooth quantity $\bar{X}_{jti}$ then the renewal will be in proportion with the smooth quantity output .It is defined by the equation

$$\bar{X}_t = 0.5X_{t-1} + 0.3X_{(t-1)i} + 0.2X_{(t-2)i}$$

where X_{ti} is the output quantity of the investing sectors K for the i-th sector. A two years lag is used. To make the sum of W_{0i}, W_{1i} and W_{2i} equal to one, we assume $W_{2i} = 1 - W_{0i} - W_{1i}$. When replacing this expression for W_{2i} in the equation for I_i a regression type of equation is obtained :

$$I_i = K_i[Z_i\overline{X}_{ti} + \Delta X_{(t-2)i}] + K_iW_{0i}[\Delta X_{ti} - \Delta X_{(t-1)i}]$$

$$+ K_iW_{1i}[\Delta X_{(t-1)i} - X_{(t-2)i}]$$

By means of this regression three parameters K_i , $K_i W_{\cdot i}$, K_iW_{Ii} are estimated. I_i is forecasted by means of the equality $K_iW_{2i} = K_i - K_iW_{0i}$ The negative values for W_{ji} (j=0,I,2) are set to zero.

For our country the output quantity by sectors increases and usualy the first differences are positive but for the cases when this is not so X is defined by the equality $\overline{X}_{1960} = X_{1960}$,

$$\overline{X}_{ti} = \max [X_{ti};\ 0{,}95\,\overline{X}\,(t-i)\,i]$$

$$\Delta X_{it} = \max[(X_{ti} - \overline{X}_{(t-1)i});0] .$$

These dependences ensure non-negative values for the capital investments when the output's volume is decreased and the reconstruction of the machines and the equipment at the expense of the capital investments in case the sector does not reach its previous output level.

The regression equations for the determination of the capital investments volume include factor Time as an independent variable.

For example, the simplified form is as follows:

$$\ln I(t) = \lambda_0 \ln X(t) + (I - \lambda_0 - \lambda_1) \ln X(t-1) + \lambda_1 \ln I(t) \quad ,$$

The analysis of the real and forecasted data for the capital investments volume for the whole economy and for the sectors of the material sphere and those of the industry show a constant upward tendency. For the period I970,I980 the yearly growth rate for sector "Industry" rises from 5,90 to 8,49 while for the whole economy it is from 8,45 to 7,05. The highest yearly growth rate for I975 in respect to I970 have the sectors : construction materials industry, apparel,transport, glass and porcelain etc. The same tendency nearly is observed for the following period under consideration /I975-I980/.

For the whole period of the examination one notices an increase of the relatiove share of sector "Industry" in the capital investments total volume, which is preserved for the sectors :construction and agriculture .

In perspective,increasingly greater becomes the share of industry in the capital investments total volume / it augments from 56,75% for I985 to 57,84% for I990/. The biggest relative share of capital investments show the sectors : machinebuilding, chemical industry, construction materials, electrical and thermal energy and fuel and gas. For the remainign sectors the relative share is nearly the same.Thete is however a decrease of the yearly growth rate for all sectors, for the national economy it changes from 4,75 (for I985) to 4,35 (for I990) and for Industry - from 5,00 (for I985) to 4,70 (for I990) .

3. EXPORTS AND IMPORTS

By means of the abovementioned four single development models one can establish the alteration's trend of the export, import, the output's volume and the apparent consumption one/ the demand for the industry's sub-sectors and for the remaining sectors of the material sphere. As we know the apparent consumption equals the output plus import minus export for each sector individually. These four models characterize the average absolute volumes'growth for the examined period, the change of the absolute growth, the average exponential growth rate and the alteration of the medium growth rate. The four indices define nearly correctly the basic changes for the period. The export a nd import development is determined as a function of the output's volume, respectively the apparent consumption' volume.

These functions are analogical to those of C.Almon and his co-authors [4].

.The difference lies in the fact that there are no expressions for the prices and there is no lag in the export function / it was found out that the analysis of our country's export development did not prove a lag between the export and the production/ The inclusion of price correlations creates additional difficulties and requires a considerable amount of work. This variant was left for the next stage of work. The relations between the export and the import on one side, and the output and the apparent consumption - on the other were expressed by elementary linear models. Our goal was to get a general idea of these variables ratios. The differences by sectors were great and in the dynamics - one observed insignificant changes for the examined period for nearly all sectors.

In most cases the extrapolation on the ground of single development models did not give real results for the export and import for the period till 1990. More realistic estimations were obtained on the base of the export and import functions but then the problem was how to estimate the future meanings for the exogenous variables, finally this is what is aimed at, since their values are internal for the model as a whole. In this case the approach chosen was to determine the values of the separate variants . External information was also made use of - given development of given sectors in some of our norm documents , expected development of some outputs and sectors in the world in accordance with foreign institutes and organizations forecasts. Technically one had to get forecasts for two years : 1985 and 1990. For the years from 1980 till 1990 an interpolation was made by means of the medium geometrical growth rates. The forecasts by sectors were summed and compared to the general forecast made for the export and the import by analogical methods. For each sector and for each year two figures were obtained : lower and upper limit of the export and import volumes. Both values met differences in the premises for the development rates, the type and the contents of the correlations between the dependent and independent variables. By sectors for ewery year one established the relations of the difference between the upper and lower limit and the average quantity.

Most generally the results obtained may be characterized by the following data. The average yearly growth rate for the export during the ten years period till 1990 is 6,5 -7,9% when summed and 4,7 -6,8% in the total forecast sum .For the import the data are 6,0-6,7% and 6,3-8,6% .

According the XII-th Congress of the Bulgarian Communist Party's documents one envisages in 1985 an increase of the foreign trade of PR of Bulgaria 1,4 times. According to our forecast this figure is 1,40-1,48 times and 1,35-1,54 times when summing and in the forecast of the total export and import volume correspondingly. In I990 3/4 of the export will be formed by four sectors : machinebuilding, chemistry, food and tobacco and transport. The biggest sectors by import will be: machinebuilding, chemistry,ferrous metals and fuel and gas covering also a 3/4th of the expected import volume. The expected forecast's limits by years and sectors vary from 2% to 34% in export / from 8 to 27 on the average/ and from 2% to 57% - in import / from 6 to 28% on the average /.The machinebuilding's limits are II-34% in export and 8-I7 % in import. For chemsitry - 5 to IO and 5 to 7% etc. As a rule the limits are widest in the last year of the forecast /I990/ but there are exceptions too for some sectors. Comparisons were made between the export and import volumes - by sectors and in gene ral in view of the balance's changes and level. Bearing in mind the connection of the foreign trade and the country's output all calculations for the export and import are on internal prices at a constant price base.

The forecasted results obtained are discussed only as apreliminary variant which will be used in the general model's solution. The export's and import's volumes forecast, particularly for a country like Bulgaria, must answer a number of additional requirements and to take into consideration many other factors and outside conditions.

4. INPUT-OUTPUT MATRIX

An input-output matrix forecast was done for I980,I985 and I990 on the ground of the report data for the annual input-output tables for the period I97I-I979 with dimensions 25x25.on the following principles :

A hypothesis was assumed for a minimum change of the coefficients with respect to the level of the last years. The reason is that if you don't know sufficiently well the production conditions of every sector and the concrete expected significant technological improvements, the next five-years plans aswell, it is most expedient to envisage a minimum development of the production expenditure for one unit of the output, than considering these expenditures as absolutely constant or a risk to reflect the main changes. We think that the period 1971-1979 is indicative enough in order to be used for a feasible forecast for a near future - till 1985 which renders an account of an already achieved level and as far as a forecast till 1990 - with some additional and visible even at present changes. One may consider that the approach used is necessary in principle as one initial stage of the work .

Practically first of all one forecasts the columns sums /sectors materials - output ratio /. As aggregated quantities these sums are thought to have a greater stability in their development than their separate components - the individual input-output coefficients The dynamic rows for all coefficients and for the above sums are smoothed and extrapolated initially till 1985 by single development models in view of establishing admissible durable trends in the variables' development. These are foremost the linear and non-linear functions. Ordinary exponential, hyperbolic functions and polynomeals fall in the latter category. On another hand one used complicated exponential functions asymptotically limited. The different functions fitness was compared on the ground of past developments and then forecasts were done choosing for every case the most appropriate functions.

The values obtained for the relative expenditure were tested by experts and if necessary minimum corrections of their forecasting significance were made. On the base of the final data for these sums, the forecasted values of their components - the individual technological coefficients were corrected too. The corrections were in proportion with the size of these coefficients and taking into consideration the direction of their development especially for the energy sector.

This is how one got from this forecast the input-output coefficients.

Two variants were made till 1980 and 1985 and for 1990 - one variant.

5. SOFTWARE PACKAGE

For the solution of the INFORUM model IIASA's SLIMFORP software package was used . It was adapted to a II/70 PDP minicomputer The first attempts on ICL failed.The model's solutions were of a considerably altered SLIMFORP variant. Now one nearly finishes adjusting the programs on the IBM 370/135 VS system.
Here too were some difficulties.They arise from the fact that the UNIX system of IIASA is highly organized. Its simplification for a IBM 370/135 VS is actually creating another new SLIMFORP system which is quite a long and workconsuming process. Explicitly the difficulties were :

I , A great software package size - I2 main and 22 sub-programs with about 6I 00 operators /comment operators including /.

2. Initially one had to run every program one by one.This was caused by some differences in the FORTRAN PDP II/70 and IBM 370/I35 VS translators. They are :

a) define file operators

b) stop operators for emergencies

c) some specific requirements of the COMMON blocks

d) impossibility to close the DO cycle by a transfer operator / conditional or unconditional /.

3. Syntactical errors in the programs .

4. Necessity to prepare the control cards for each one of the programs, caused by point 2.

5. Necessity to correct the data in some of the programs according the input/output operators demands .

After all programs were run and final results obtained one made source, object and load libraries for a more efficient use of the package .So the package programs could be corrected again, translated and connected differently in accordance with the package users needs. Procedures for direct execution of different computation parts were created.

Adjusting the program on a IBM 370/I35 VS is only an initial stage; next is the adjustment on a IBM II30 of the Economic Institute in view of a repeated calculation of the model and obtaining different variant solutions. We suppose that the model's users /The State Planning Committee/ will utilize a UNIVAC system. The preliminary calculations for each model's block were done on the minicomputer COMPUCORP 625 -MARK II on basic FORTRAN.

REFERENCES

1. Almon, C. 1979. A System of Consumption Functions and Its Estimation for Belgium. Southern Economic Journal vol.46, pp. 85-106.

2. Nyhus, D. and C. Almon. 1980. Linked Input/Output Models for France, Germany and Belgium. WP-80-122. International Institute for Applied Systems Analysis.

3. Intriligator, M. 1978. Econometric Models, Techniques and Applications. North Holland, Amsterdam.

4. Almon, C., M. Buckler, L. Korwitz, T. Reimbold. 1974. 1985 Interindustry Forecast of the American Economy. Lexington, Mass. 125-127.

OUTLINES OF AN ECONOMETRIC INPUT-OUTPUT MODEL OF THE HUNGARIAN ECONOMY

G. Fink and A. Simon

This paper describes the draft of a model which is to be built as a member of the series of input-output models in the INFORUM international system. (1) The aim of the model is to produce economic forecasts in a very detailed breakdown with special attention to effects of international economic developments on the Hungarian economy. This special treatment would be possible by the ties of the model to the INFORUM system. The project is partly run by the Vienna Institute for Comparative Economic Studies in cooperation with IIASA.

The construction of the model is now in the stage of data collecting. For modeling inter-industry commodity flows, the 1972 input-output table will be used with 79 commodity categories. In the various blocks of the model we shall try to be as close to this breakdown as possible. Where the classification is different, bridge-equations connect the various blocks. The foreign trade model fits into the 79 categories of the input-output table and a bridge is built to transfer to the 119 categories of the INFORUM system.

The model will be developed gradually, adding to the basic model more elaborated blocks one be one.

The development of some of the blocks is already in progress (consumption, investments), others need future research. Let us see one by one, how we inmagine the model blocks at this stage.

Investments and capital accumulation

In our approach on investment we shall start from the assumption that total investments and the distribution of investments between individual branches of the economy follows solely economic criteria derived from past developments and thus could be endogenized

(1) About the INFORUM-project see D. Nyhus – C. Almon: 'The INFORUM International System of Input Output Models and Bilateral Trade Flows', Paper presented at the IIASA Fifth Global Modeling Conference, Laxenburg, 1977.

in the model. We know that this assumption eliminates the possibilitiy of a more active investment policy of central authorities. Thus, at a later stage, we will have to allow for the possibility of intervention of central authorities by introducing exogenously determined investments. Exogenous decisions could affect the level of total investment, and could include decisions on individual projects and investment programs, which create a new production structure in the economy. The framework of INFORUM models gives the opportunity to integrate the effects of exogenous decisions into the model in the form of 'fixes', which may override or adjust the endogenous model projections.

The attempts to endogenize investments in the model could start from two points. In a more narrow approach it could be assumed that expected future output and the capital coefficient are the only determinants of investment decisions. A second approach could try to add to these two influencing factors possible effects of prices or profits through a cost of capital index. Investment policy could enter the model here by including effects of taxes on the user cost of capital.

Given that a cost of capital index could be found and projected, the investment model could follow the lines of the US model: (1)

relative user cost of capital:

$$r_t = \frac{P_e\,(c+d)}{P_v}$$

and desired capital stock:

$$K_t = B_t Q_t r_t^{-\sigma}$$

Where P_e — price of equipment

P_v — value added price (value added per unit of output)

c — cost of capital

d — depreciation rate

K_t — desired capital stock

B_t — trend value (function of time)

Q_t — output

σ = $1/(1-\beta)$; β — elasticity of substition in a CES production function

In addition to the estimation of the above equations the actual model would need further refinement e.g. as to reaction time on changes in output and lead of investment outlays before implementation of new capital equipment.

Consumption (2)

Available data about consumption are the following. The 79 sector input-output table of 1972 has one vector of consumption, which includes both personal consumption and government consumption. Personal consumption statistics give a breakdown of personal consumption in 51 categories for the whole observation period (presently for 1960 – 1978). Time series on total government consumption are available.

---

(1) C. Almon and A. J. Barbera: Investment in Producer Durable Equipment 1976 – 1990, INFORUM Research Report No. 26, October 1978, University of Maryland

(2) This section is a brief account of the work of Gabor Kornai, Institute for Economic and Market Research, Budapest, who is responsible for the consumption model. The estimation of the model is in progress and the results will be published in a separate paper.

A bridge matrix was set up for 1972 which shows the composition of each of 51 personal consumption categories in terms of the input-output sectors. Government consumption data for the base year were approximated sector by sector as the residual remaining after deducting personal consumption from total consumption. These residuals give the 52nd vector of the bridge matrix. Assuming that the shares of I-O categories in the consumption categories and in the government consumption are constant, the bridge matrix helps us to set up the time series of consumption by I-O categories.

The model of personal consumption expenditures assumes, that in Hungary the personal consumption pattern is determined by the choices of the consumers, i.e. scarce supply is not characteristic of the consumer goods market. There are two major exceptions, housing and cars. Housing is permanently and cars are mostly in scarce supply. For these categories a different approach is to be used.

The model generally assumes that consumer demand is determined by relative consumer prices and personal income. The equation system of Clopper Almon was chosen as the form of the demand function. (1) This allows grouping of goods, calculating different cross price elasticities within the groups and across the groups, still preserving the properties required by consumption theory.

As to the two exceptional goods, they were treated as follows:

In the car market there are two different sets of prices. The second-hand prices clear the market and the first-hand prices show the consumer's expenses for a new car. Both prices have to be taken into consideration, because changes in the second -hand prices have substitution effects on consumption, but they have no income effect (they affect only income distribution). Therefore in the demand function an estimated second-hand price was used as price of cars while for calculation of real income the first-hand (official) prices were used.

Housing expenditures were simply excluded from both personal consumption and personal income and added to government consumption. In contrast to the car market, where strong complementarities exist among cars, gasoline, and spare parts, changes in the housing supply affect consumption of other goods only by allowing to spend more or less income on other goods.

Government expenditure could be regarded exogenous as a first step. Later constraints on the supply side could be built in the model. Housing is clearly constrained by housing capacities. Health care, education, administration are similarly limited by both capital and labor constraints. Investments and the hiring of labor in these sectors seem to depend on exogenous government decisions.

(1) C. Almon: A System of Consumption Functions and its Estimation for Belgium, Southern Economic Journal, July, 1979.

The equations of the consumption block are the following:

$$c_k = f\,(pc_k, y) \qquad i = 1,\dots,51$$

$$pce_i = \sum_{k=1}^{51} t_{ik} \cdot c_k \qquad k = 1,\dots,79$$

where c_k – personal consumption by consumption categories in constant prices

y – personal income

pc_k – consumer prices

pce_i – personal consumption by input-output categories

t_{ik} – share matrix, showing shares of pce in c.

Inventory investments and input-output coefficients

Data on inventory investments in a 79 sector detail are very scarce. They are available only for 1968, 1972 and 1976, at current prices. Considering the volatility of inventory investments interpolation and extrapolation of these would hardly give reliable approximations of the missing data. Assuming that the composition of inventory investment is constant over the whole period would not give better results either. (For several sectors inventory investment was negative in 1972.)

The statistical difficulties suggested the idea to determine inventory investments indirectly. In principle the following indentity gives inventory investment:

$$iinv_i = q_i - (\sum_j a_{ij}\, q_j + pce_i + gce_i + ifix_i + exp_i - imp_i) \qquad i = 1,\dots,79$$

where $iinv_i$ – inventory investment

q_i – gross output

$ifix_i$ – fixed investments

exp_i – exports

imp_i – imports

a_{ij} – input-output coefficients.

Actually we have data on the I-O coefficients only for the base year 1972. Even for this year the two sides of the equation differs by a statistical discrepancy. Using the 1972 matrix for every year, we have the following identity:

$$iinv_i + coefc_i = q_i - (\sum_j a_{1972ij}\, q_j + pce_i + gce_i + ifix_i + exp_i - imp_i) \qquad i = 1,\dots,79$$

where $coefc_i$ – residual arising from the fact, that the coefficient matrix of the given year is different from that of 1972, (plus a statistical discrepancy arising from the fact, that the items on the right hand side are from different statistical sources).

Statistics on the right-hand side variables are available, but the left-hand side variables cannot be statistically separated. Thus only the sum of inventory investments and the effects of the shifts in the technology matrix can be modeled.

Let us make the following regression estimation:

$$\frac{\sum_j a_{1972ij}\, q_j + iinv_i + coefc_i}{a_{1972ij}\, q_j} = f(t) + u$$

where f (t) is a logistic function of t

t — time

u — residual of the estimated equation

The numerator of the left-hand side is calculated as the difference of gross output and the final demand items. Fitting a logistic curve to this ratio has many advantages, (1) but other versions with some additional variables (supply pressures or constraints, etc.) could be tried later as well.

Expressing the equation for the sum of inventory investments and the effect of coefficient changes, we have

$$iinv_i + coefc_i = [1 - f(t)] \,.\, \sum_j a_{ij}\, q_j + u_i \,.\, \sum_j a_{ij}\, q_j$$

The fitted values of $iinv_i$ + $coefc_i$ may be assumed to indicate the shifts in the technology matrix and longer-run trends in inventory investments. The form of the function assumes that both of them are proportionate to intermediate use. The residual $\sum_j a_{1972ij}\, q_j \,.\, u_i$ may contain not only random errors, but short-run developments in inventory investment as well. Empirical investigation of these residuals may lead us to some explanation of their development.

Output

Output could be assumed to be basically demand determined according to the following identity:

$$q_i = \sum_j a_{ij}\, q_j + pce_i + gce_i + ifix_i + exp_i - imp_i + iinv_i + coefc_i \qquad i = 1,\ldots,79$$

Thus output is defined independent on the utilization of capacities. Data for capacities could be defined by the Wharton capacity indicator (peak-to peak interpolation of output data).

To keep output reasonably close to capacities in forecasts, a feedback of capacities on output could be built in the model through foreign trade. (Low capacity utilization spurts exports, curbs imports and thus increases output via the output identity.)

Productivity and Employment

As a first step, productivity could be estimated by simple trends or logistic curves:

$$\frac{emp_i(t)}{q_i(t)} = f_i(t) \qquad i = 1,\ldots,71$$

where emp_i (t) — employment

f_i (t) — function of time

(1) See C. Almon — M. Buckler — L. Horwitz — T. Reimbold: 1985: Interindustry Forecasts of the American Economy, Lexington, Mass., 1974, pp. 157 — 165

Data on employment are set up for 71 sectors. More ambitious functions (inverted Cobb-Douglas, etc.) could be tried later, depending on the available research capacities.

Forecasts of employment would be based on the above function.

Labor force is exogenous in the model. It is determined by demographic factors and its forecast could be obtained from population projections.

Forecasts of employment do not necessarily fit to forecasts of the labor force. The model however makes it possible to search for an income policy that assures consistency.

Prices

In the past 20 years the Hungarian price system underwent many changes owing to subsequent price reforms. However, the principles of these reforms were too various and the time of their execution was too unpredictable to estimate price behaviour by econometric methods. Thus prices have to be exogenous for the model. This limits the scope of the model, but this limitation has much less consequences than in a model of a maket economy. Namely the role of the prices is much less important in the Hungarian economy:

1. The firms in Hungary are less profit-motivated, thus their decisions depend much less on prices than in the market economics.

2. Price changes were always accompanied by changes in the subsidy and tax system which would make the net effects of the changes unpredictable.

When forecasting prices, we are not restricted to econometric relationships. We may assume for example, that future prices will behave according to the principles of the 1980 reform. The functions for this price behaviour are known through these principles, but the parameters of these functions have to be set up by assumptions.

The principle for the future price formation is the following: a) prices of the internationally traded or competetitive goods should develop according to the export or the import prices in that sector. b) prices of the non-traded or non-competitive goods should develop according to the capital and labor inputs. Although the principles are clear, in a model there is much room for various assumptions about the share of competitive goods in a sector or the extent of substitutability of goods with export or import goods, etc. Although these assumptions cannot be based on empirical evidence, the model still could give much information about future trends in prices.

Exports and imports

Setting up equations for foreign trade the following requirements would be desirable to be met:

1. The equations should capture the domestic factors producing changes in the sectoral structure of trade.

2. They should indicate effects of domestic demand policy on the balance of trade.

Both of these requirements are met in the model of a market economy by introducing prices into the model. Relative prices of sectors explain long-run developments in the trade structure. The domestic price level (income and monetary policy) and the world market price level together with the exchange rate keeps domestic demand in a long-run equilibrium with output.

In an economy like the Hungarian, it is sure that prices play much less role and it is even doubtful, whether they have any significant influence on the developments of Hungarian foreign trade. It is clear, that the long-run equilibrium between domestic use and output is not a result of a pure market mechanism, but the result of various control measures of the central authorities influencing the decisions of the firms, taken in various incentive systems. It is an empirical question, whether profitability considerations determined significantly the decisions of the policy makers or the firms in the past 10 - 20 years. Regression equations of the model could give an answer to this question. One way to formulate price effects would be to include the ratio of the domestic price index and the export (import) price index into the equation. Another way would be to replace domestic prices by calculative prices of value added. (1) In both cases it seems useful to deflate the price indices by the general domestic and the world market price index, respectively.

These deflated price indexes would ensure the fulfilment of the first requirement. If the data would not show significant relationships in this respect, simple trends could replace the price ratios. To meet the second requirement, it seems that monetary and exchange rate policy are not so important or have not been as widely used in Hungary that variables of this kind would be suitable to indicate balance of payments policy. Instead of price variables it would be probably better to include capacity utilization variables into the equations to show the effect of domestic demand (and demand policy) on the balance of trade.

The export equations include of course foreign demand variables. Data on foreign demand in the detailed breakdown of the model is not easy to obtain. Information could be received probably from the models of the partner countries of the INFORUM-project.

In the import equations domestic demand could be represented by total domestic sales.

---

(1) Using calculative prices instead of actual prices would assume that social costs are still somehow considered in economic decision-making, even if they do not explicitly appear in actual prices. These social costs (calculative prices) of value added could be approximated by the following expression:

$$price_i = \frac{emp_i \,.\, w_i + cap_i \,.\, int + depr_i \,.\, cap_i}{va_i}$$

where emp_i – employment in persons
w_i – wage rate per man-year
cap_i – capital stock
int – uniform interest (profit) rate, constant over time
$depr_i$ – depreciation rate
va_i – value added in constant prices, defined as

$$va_i = q_i - \sum_j a_{ij} q_j$$

The typical export and import equation of the model would be the following:

$$exp_i = f\left(\frac{p_i}{pw_i}, \frac{qcap_i}{q_i}, dw_i\right)$$

$$imp_i = f\left(\frac{p_i}{pw_i}, \frac{qcap_i}{q_i}, dd_i\right)$$

where exp_i – exports in constant prices

imp_i – imports in constant prices

p_i – domestic prices (actual or calculated) deflated by the general domestic price index

pw_i – world market price index deflated by the general world trade price index

$qcap_i$ – capacity output

dw_i – world demand indicator

dd_i – domestic demand indicator, defined as

$$dd_i = \sum_j a_{ij}\, q_j + pce_i + gce_i + ifix_i$$

q_i – actual output

Up to this point the peculiarities of trade with the socialist countries were ignored. During the construction of the model however it may turn out necessary to separate socialist trade and nonsocialist trade. In this case the above equations would apply only to the nonsocialist trade, and barter agreements between the socialist countries would have to be modeled.

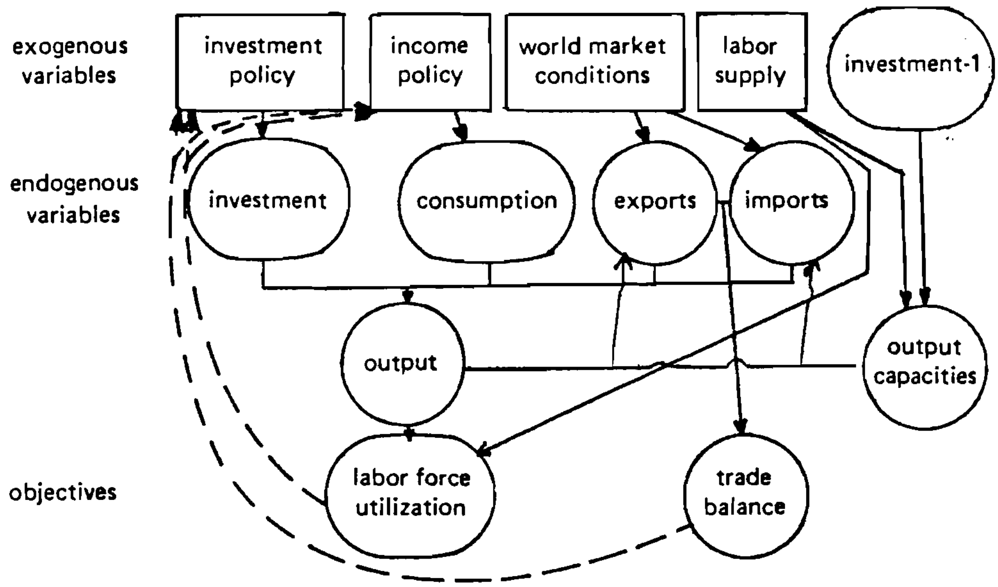
These agreements have some features which are unusual in international trade models: exports and imports may depend on each other within single sectors, the supply of imports in most sectors may have both an upper and a lower limit. As these inter-relationships are not only complicated but changing frequently, a complete model in a 79 sector breakdown seems to be a too ambitious goal for the next future. Probably the major barter trade flows should be treated exogenously in the first versions of the model.

How the model works

With the described model we are able to make forecasts which may be used either as tools of medium-term planning or for the simulation of the planning procedure. It contains all those relationships which are considered when the medium-term plan for the main macroeconomic variables of the economy is set up. One of the advantages of this model over the traditional planning approach is, that it relies upon a consistent set of relationships, based on statistical evidence. Another advantage is that besides the aggregate forecasts it may produce infomration of future development in such detail, which has not been available or producable by the planners up to now. This feature of the model is mostly due to its connections to the international INFORUM system.

The forecasts of the model are inferior to a traditional plan or forecast in some other respects: the statistical evidence of relationships is available only for the past, new relationships or changes in these relationships are not automatically considered by the model. Thus any application of the model has to be coupled with an analysis of new events, possible future unprecedental developments; the effects of which could be integrated into the model in the form of fixes.

Let us see a chart of the aggregate variables of the model. For sake of clearness only the most important relationships of the equation system of the model are shown.



During the forecasting exercises we have to solve the same optimal control problem, that the planners face. Their objective is a reasonable trade balance and labor force utilization. (The objective could be reformulated as consumption and trade balance maximization, etc., but the problem would be essentially the same.) Their tools are income policy and investments policy. In the present actual situation (by the construction of the 1981 - 85 plan) the trade balance had an overwhelming weight in the objective function. Generally however the model shows the usual dilemma of the Keynesian economy between a stagnating economy or a deficit in the trade balance.

The model calculates the values of the objectives attained by the tools. By iterations we can reach those levels of investments, consumption, output etc., which are compatible with the desired objectives.

AN ECONOMETRIC INPUT-OUTPUT MODEL OF THE WEST GERMAN ECONOMY

D.E. Nyhus

This paper describes an econometric input-output model of the Federal Republic of German having 51 sectors, together with a forecast made using the model. Most of the data for the model comes from the Rheinische-Westfälische Institut für Wirtschaftsforschung in Essen. The model presented here is now part of the INFORUM system of dynamic input-output models.

Equations have been estimated to explain 46 categories of consumer expenditure, investment demands (plant and equipment separated) for 34 industries, exports for 51 industries, imports and inventory change for 51 industries. Equations to forecast input-output coefficients have been estimated. Forecasts of prices and incomes have not been made and have now been left as future work.

INTRODUCTION

This is a report on the construction of a dynamic econometric interindustry model of the West German economy. The basic purpose of the model is to make projections for the West German economy, year-by-year for approximately ten years ahead, of industrial output, employment, investment, and other components of final demand within the structure of an everchanging input-output table. Structural change over the past 15 years has been carefully examined in the construction of the model. At present, the model lacks a price and income side; this is a research topic for the future.

This model, which is now part of the INFORUM system of dynamic input-output forecasting models, has been successfully linked with those of the U.S.A., Canada, France and Belgium (see Almon and Nyhus (1981) and INFORUM (1981)).

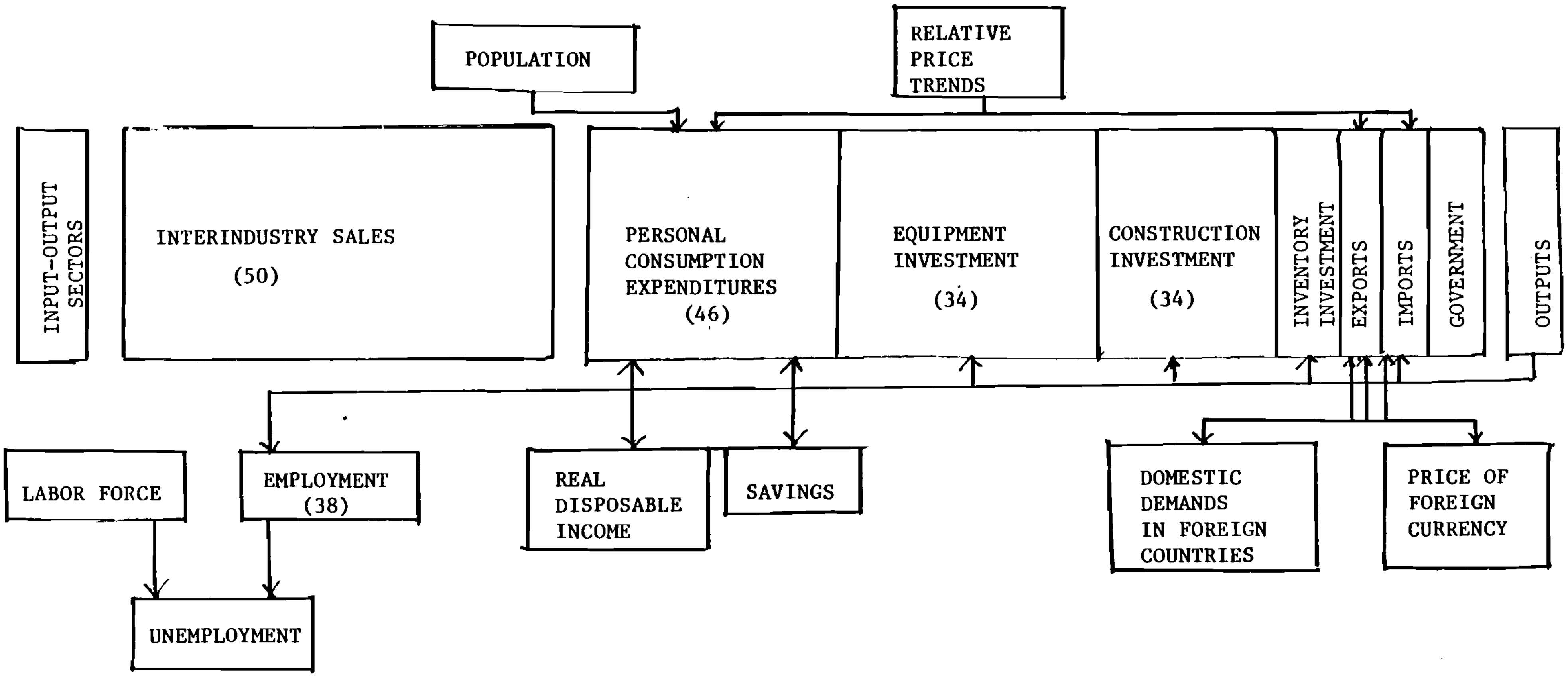
Section I describes the general structure of the model and the results that are available. Section II describes the data and its preparation. Section III shows the model's equations for each of the final demand components, labor productivity and input-output coefficient change. Section IV explains the specific assumptions used in the model and the results of the forecast.

I. MODEL STRUCTURE

The major parts of the model are shown in Figure 1. Also shown are the principal functional connections in the model. The center block is the input-output table where some fifty-one industries are distinguished. This center block shows the sales of each sector to each of the others.

On the right-hand side is the final demand block, which shows the projection of 46 types of consumer expenditure, equipment investment for

FIGURE 1. The Model - major parts and principal functional connections



NOTE: Figures in parentheses show number of columns

34-industries (aggregates of the 51), the construction investment for these same 34 industries plus residential construction. Inventory investment, exports and imports, and government purchases for each of the 51 sectors are also estimated.

To the left of the center block are the sector outputs. The outputs, of course, are the sum of all sales including the negative sales imports; hence production here is domestic output. Productivity and employment for 38 different industries are distinguished here (again aggregates of the 51 sectors).

Estimates of the labor force, per capita disposable income, total government spending, foreign demand for German goods, the price of foreign currency (exchange rate) and relative price trends, are all exogenous in the model at this time. In Secton IV, where a forecast result is shown, there will be more discussion of the exogenous assumptions.

Perhaps the most important element in the structure of the model is not visible in Figure 1, e.g. the numerous ways in which it is possible for the user to intervene in its workings. It is easy to modify or override the output of any behavorial equation in the model: one can add to what the equation produces; multiply by what it produces, or simply replace the equation altogether. Thus, the model is not just a forecasting model, but is designed from the start to make policy analysis easy.

The Model in Motion

The best way to understand the interworkings of the model is to follow it through the forecasting of one year.

We begin with assumptions for the population and real per capita disposable income. The personal consumption expenditure equations use the above assumptions, together with relative price trends, to predict expenditure by consumer category. A bridge matrix will later transfer these demands by consumer category, i.e. butter, into demands for the products, i.e. dairy products, in the input-output table. Next, government demands by product are calculated. A constant structure of government spending is assumed.

Next we use project foreign demands (some derived from other models, e.g. France, U.S.A., Belgium and Canada; some by assumption, e.g. Netherlands, Italy and U.K.) and projected trends in domestic to foreign price ratios by product, in order to calculate the demand for exports by input-output sector. Foreign demands and relative German-to-foreign price trends are used to produce exports by input-output category.

Investment functions generate equipment and construction investment on the basis of past and present changes in output. There are equations relating imports to domestic demand and relative foreign-to-domestic prices. Other equations exist to calculate inventory change by product employing domestic use and stocks of inventory of the products.

The final demand projections are of great interest, particularly since they have a great effect on industrial structure.

When the final demands are ready, we then use the projected direct input-output coefficients -- the inputs of each product required to produce one unit of output. When these are ready, we begin the solution procedure known as the Seidel loop. Here, consumer demand for autos is converted into demand for steel, steel into mining, and mining into gasoline, etc. This continues until all demands for all products, directly or indirectly required to make the final goods, have been accounted for. Note that this process does not call for matric inversion. At no time do we invert a matrix in the solution procedure. It is also to be noted that the whole process needs to be repeated due to the dependence of investment and construction spending on current output. The extent of this dependence is relatively small and therefore we only need to repeat the process twice. A third cycle was performed but found to be unnecessary and is no longer performed

Product outputs are now ready and we are prepared to calculate employment. Labor requirements per unit of output are calculated on the basis of the level of output, its change and trends. We then multiply by output to get employment. Industry employment is then calculated and the result compared with that of the labor force. The difference between the two, unemployment, is then scrutinized. If this figure is unrealistically low, then per capita disposable income, government spending, and the price of foreign currency are examined and revised, in order to produce a larger economy, thus creating more employment.

II. DATA

The Economic Research Institute for Rhineland-Westfalia in Essen, F.R.G., prepared input-output tables in current and 1970 prices for the years 1962-1975 (RWI, 1979). These data were the primary source of most of the calculations. Employment and investment by industry data came from detailed national accounts (Statistisches Bundesamt, 1980). The investment matrix was drawn from a 1972 matrix prepared by the German Institute for Economic Research in Berlin (Baumgart, 1976). Only modifications to the above data base will be discussed below.

Only minor changes to the RWI tables were made:

(1) Most of the government sales were moved from intermediate to final demand sales. Only government production necessary to supply the government's sales to private consumption (presumably government

publications such as maps, statistical yearbooks, etc.) were left in the intermediate column.

(2) Row 51 contains investment subsidies. It has no column or row intermediate entries.

(3) Imports were treated as a final demand column of negative entries. Thus, the row sums are now domestic production.

(4) The RWI row for secondary production (Kuppleproduktion) was allocated back to the primary sector. A new sector, Row 50, contains domestic production of natural gas, which is a produced in Germany as a byproduct of other processes.

The sum of the industry investment did not match the total equipment investment. The difference, approximately one percent, was due to some slight statistical definitional differences and probably, later, to revisions in the national accounts data. The differences, however, were small.

The investment matrix, on the other hand, had many, more substantial, differences. The DIW matrix contained flows for construction while only flows of investment goods were desired. Matches were made between the 57 branches of the DIW and the input-output rows of the RWI table. Matches were also made between the DIW matrix and the detailed national accounts series of investment by purchaser. Starting with the DIW matrix, we produced an investment matrix which showed the national account industries as columns and the RWI sectors as rows. A RAS procedure was used to ensure compatibility. This is an area where further work needs to be done.

III. EQUATION FORMULATION AND ESTIMATION

A. Personal Consumption Expenditures

There are numerous forms of consumption functions in existence. For use in input-output work, one of the most useful forms is that developed by Almon (1979) which has the following major properties:

- homogenity of degree zero in all prices and income
- commodities should be complements for some goods and substitutes for others.
- asymptotic consumption pattern depends, as income increases, on relative prices. The system requires that we list our commodities by group and subgroup, i.e.

Group	Subgroup
Transportation	Public, Private
Food	Protein, non-protein,
	Beverages, Tobacco

The form of the equation for good i in group G and subgroup S is:

$$C_i = [b_{1i} + b_{2i}t + b_{3i}(y/\bar{p}) + b_{4i}A(y/\bar{p})] \left(\frac{p_i}{p_S}\right)^{-\lambda_S} \left(\frac{p_i}{p_G}\right)^{-\lambda_G} \left(\frac{p_i}{\bar{p}}\right)^{-\lambda_0} \tag{1}$$

where

C_i = consumption per capita in constant prices of good i in year t

y = disposable income per capita in current prices in year t

p_i = the price index of good i year t.

$$p_S = \left(\prod_{y \in S} p_j^{s_j}\right)^{\frac{1}{s_S}} , \quad p_G = \left(\prod_{j \in G} p_j^{s_j}\right)^{\frac{1}{s_G}} \tag{2}$$

$$\bar{p} = \prod_{\text{all } j} p_j^{s_j}$$

where s_j is the budget share of commodity j in the base year, and

$$s_S = \sum_{j \varepsilon S} s_j \quad ; \quad s_G = \sum_{j \varepsilon G} s_j \quad ; \quad \sum_{all\ j} s_j = 1 \tag{3}$$

and b's and 's are parameters to be estimated statistically.

Note that b_3 was not estimated but rather, econometrically, we first estimated the income elasticity of each product. Since the b_3's are directly obtainable from the individual elasticity estimates (elasticity - $\frac{\partial C}{\partial y}\ \frac{y}{C} = b_3 \frac{Y}{C}$), we then summed the b_3's to see how close we were to the correct total expenditure in 1975 (the base year). Since our sum was low, all of our individual estimates of b_3 were scaled upwards by 11 percent in order to match the correct total spending rate. Why should we resort to such a method when empirical estimation is so obvious? Two reasons both econometric, are put forward. The first is that income and time are generally very colinear and hence we would have significant problems in pinpointing the exact effects of each. The second is that income development plays a major role in forecasting. However, the level of income encountered in the forecast was never experienced in the past. Hence we could estimate income parameters that fit well historically, but forecast poorly. Our "hand-picked" coefficients could certainly be improved by referring them to cross-section data where much greater ranges of income could be studied. That work must be left to others.

It is a property of these functions that the elasticities with respect to p_i of the demand for all goods not in the same group with i are the same, say η_i. Likewise, the elasticities with respect to p_i of all other goods in the same subgroup with product i are the same, say η_i^s, as are all the elasticities within the same group but not in the subgroup (if any), say η_i^g. Finally, the elasticity of its own price is different.

All items in the Food Group, for example, were substitutes. In the Beverages and Tobacco subgroup, however, the items are complements to each other.

B. Investment and Construction Equations

Investment data for 34 industries, separating plant and equipment, enable us to study the essentials of investment behavior; the capital output ratio and the pattern of the lag between increases in outputs and the increase in capital stock they stimulate.

The investment equation formulates gross investment on the sum of replacement plus expansion investment.

$$V = gk\,\bar{Q}_t + k \sum_{i=0}^{2} w_i\,\Delta Q_{t-1} \qquad (4)$$

where

V = gross investment

g = replacement capital stock ratios

k = capital output ratio

w_i = lag weight

$\bar{Q}_t$ = smoothed output = $.5_t + 3Q_{t-1} + 2Q_{t-2}$

In the first term, $k\bar{Q}_t$ approximates the current capital stock and g is the rate of its replacement. In the second term $k\Delta Q_t$ shows the amount of capital needed to accommodate the output expansion in year t and w_i shows the fraction of that capital bought in the i^{th} year after t.

We constrain the sum of the lag weight, s_i's, to unity by setting $w_2 = 1 - \sum_{i=0}^{1} w_i$. Thus, the actual regression equation estimated is

$$V = k(g\bar{Q}_t + \Delta Q_{t-2}) + kw_0(\Delta Q_t - \Delta Q_{t-2}) + kw_1(\Delta Q_{t-1} - \Delta Q_{t-2}) \qquad (5)$$

Here we must reiterate that we are only trying to get the essentials and that the fits of the regression equations could be improved.

C. Foreign Trade Equations

Merchandise Imports

The equation given below is similar to that used for the U.S.A., Canada, France and Belgium. Only a brief explanation and the empirical results are given here.

The equation form for each commodity (dropping commodity and time subscripts) is:

$$M = (a + bU)P^n \qquad (6)$$

where

M is the volume of imports of the commodity

U is the domestic demand - output + imports - exports

P is the price term

$$P_t = \sum_{i=0}^{5} w_i \, (P_f/P_d)_{t-i} \qquad (7)$$

where

$$P_f = \sum_{m=1}^{9} s_m \; P_{dm} \; E_m = \text{foreign price} \qquad (8)$$

where

Pdm is the domestic price index in country m.

s_m is the share of imports from country m of total imports of the commodity

E_m is an index of the price of m's currency in DM

P_d is the domestic price index of the commodity in the FRG

m = Canada, U.S.A., Japan, Belgium, France, Italy, Netherlands, U.K., and the Rest of the World

where

X is the volume of exports of a commodity

D is an index of foreign demands defined by:

$$D = \sum_{m=1}^{9} v_m I_m \quad , \qquad (12)$$

where

I_m is the industrial production index of country m and

v_m is the share of the total exports going to country m.

m is Canada, U.S.A., Japan, Belgium, France, Italy, Netherlands, U.K., and Rest of the World.

$$P = \sum_{i=0}^{5} w_i \left(\frac{P_d}{P_f}\right)_{t-i} \qquad (13)$$

where

P_d is the domestic price index in the FRG

P_f is an index of competitors prices defined by:

$$P_f = \sum_{m=0}^{10} u_m P_{dm} E_m \quad , \qquad (14)$$

where

P_{dm} is the domestic price index of the commodity in country m E_m is the price of m's currency w's are weights for lagged prices. The weights are derived from Nyhus (1975).

Where estimates of the coefficient b were negative, a time trend equation was estimated. Its form is

$$\ell nM = a + b \text{ time} + cP \qquad (9)$$

In estimating the price parameter n, we started with an a priori value and then maximized the following utility function.

$$U = \bar{R}^2 - .05 \frac{\| n - apri \|}{\| apri \|} \qquad (10)$$

We found that the a priori estimates were sometimes accepted and sometimes rejected. The demand elasticity for all 45 items shows that for 1975 the aggregate price elasticity was -.94 and the aggregate demand elasticity was 1.30.

Merchandise Exports

The form of the equation is identical but the meaning of the variables is different.

$$X = (a + bD)P^n \quad , \qquad (11)$$

u_m is the share of world exports of the commodity by country m.

As with merchandise imports, here we used an a priori estimate of the price elasticity n. The aggregate export price elasticity, using 1975 exports as weights, is estimated to be -.92 and the average demand elasticity to be 1.38.

Nonmerchandise Trade Equations

Transportation and service exports and imports were related very simply as follows:

$$V_{ik} = a + bS_k \tag{15}$$

where

V_{ik} is the flow of exports (k=1) or import (k=2) for a given year

S_k is the merchandise total for exports (k=1) or imports (k=2).

D. Productivity Equations

The equation for labor productivity for each of 38 industries is formulated to show the effects of technological change and growth in the economy. The form is

$$\log (E/Q) = b_0 + b_1 t + b_2 Q \tag{16}$$

where

E = level of employment in an industry in man years

Q = output of that same industry.

The coefficient b_1 represents technological change and b_2 the size of the economy. Because of high collinearity between time and output, the validity of the distinction is sometimes in doubt. An alternative interpretation is that Q is really a proxy for capital accumulation. Hence, b_2 may be interpreted with this assumption as the effect of capital accumulation on productivity.

E. Coefficient Change

Changes in input-output coefficients are crucial to any meaningful study of structural change in an economy. The method chosen here is designed to account for wide-spread, pervasive coefficient change in a rather simple manner.

A logistic curve defined by the differential equation

$$\frac{1}{c}\frac{dc}{dt} = b(a - c) \tag{17}$$

is used. "c" denotes the coefficient, "a" its asymptote and "b" a constant. Thus the rate of change of the coefficient slows as the coefficient approaches its "saturation" or as it nears its minimum use point.

The solution of this differential equation is

$$c_t = a/(1 + Ae^{-bat}) \tag{18}$$

where A is a constant of integration.

To apply ordinary least squares the equation is rearranged as follows:

$$\ln\left(\frac{a}{c_t} - 1\right) = \ln A - bat \text{ if } \frac{a}{c_t} \geq 1 \tag{19}$$

or

$$\ln\left(1 - \frac{a}{c_t}\right) = \ln(-A) - bat \text{ if } \frac{a}{c_t} \leq 1. \tag{20}$$

The first is used for rising coefficients; the second for declining ones.

The application of the above equations to the data presents some problems. Time series on individual coefficients do exist, but because they were not derived from basic data but rather from a form of the RAS method, we feel that the estimates based on the movements of individual cells of the matrix are probably not meaningful. Therefore, we introduce a new C_{it} referring to the entire row i as follows:

$$U_{it} = X_{it} - F_{it} - a_{ij} X_{it} \qquad (21)$$

$$V_{it} = \sum_{j=1}^{49} a_{ij} X_{jt} \qquad (22)$$

$$C_{it} = U_{it}/V_{it} \quad , \quad i=1,...,50. \qquad (23)$$

where

U_{it} = actual intermediate use of commodity i

X_{it} = domestic output of commodity i

V_{it} = indicated use if coefficients have remained constant over the entire period

a_{ij} = the matrix of direct coefficients for 1975

C_{it} = index for the movement of all the coefficients in the i^{th} row.

For each industry we find the predicted coefficient change for the first two years of the forecast, then the change for the last two years. The equation ensures that all absolute rates of change are reduced and that the direction of change is held constant. The third column of numbers shows the fit of the equation as it predicts the movements in C_{it} -- or in the last column -- the fit in prediting U_{it}.

IV. A FORECAST

Any forecast of an economy using an econometric model is the result of joining the equations used to certain assumptions. Assumptions may interact, with each other, as they do here, but the outline of the result is still determined by the interaction of assumptions and equations. Since the equations have clearly been described in Section 3, we will now amplify on the assumptions and then describe the main features of the forecast.

Assumptions

There are three important assumptions concerning personal consumption expenditures for the years 1975-1990. They are the population (known to 1979), disposable income per capita (known to 1978), and the spending rate (also known to 1978). The total population is projected to decline slightly from the current figure of about 61.5 million to 60.0 million in 1990. Disposable per capita income is projected to increase by only 0.4 percent in 1980-81. The period 1981-85 foresees a slightly faster path of 3.1 percent per year; a slowing to 2.5 percent per year is forecast for the 1985-90 period. The West Germans spent an average of 84.5 percent of their incomes in 1975. We project an 84 percent constant rate for the whole 1980-90 period. We must state at this time that the assumptions regarding income are under constant revision and are heavily influenced by the models projection of unemployment. The assumptions regarding labor are discussed later.

The projections of foreign demands, while exogenous to the FRG model, are partially the results of other models in the system. Specifically, the results of the French, Belgian, U.S.A. and Canadian models are used. To these are added assumptions concerning the U.K., Italy, Dutch, Japanese, and the Rest of the World economies. The linked economies (France, etc.) were forecast interactively and with the same non-varying assumptions about the others (U.K, etc.) until insignificant changes appeared in the projected outputs. So, the "assumed" foreign demand projections show a drop of 0.4 percent in 1980-81, a rise of 3.5 percent in 1981-85, and 4.3 percent in 1985-90. The projection of the exchange rate is based upon two factors. The first is that we try to maintain some sense of the balance of trade. The second is that we project the foreign/domestic price ratios from trends estimated historically. Built into these trends is the past revaluation of the DM. We have estimated the effects of the past DM revaluations sector by sector and subtracted them from the trends. Thus, the assumption about the price of foreign currency which we make here must, in some sense, incorporate the past trends in the foreign currency price. The result is an assumed revaluation of the DM of 2.8 percent per year in 1981-85 and 3.3 percent per year in 1985-90. One must keep in mind that these are revaluations against a weighted average of the FRG's trading partners and not just against the dollar.

The final major group of assumptions involves employment. The size of the labor force is projected to remain unchanged during the 1979-90 period following a small drop in 1975-79. Unemployment appears as an assumption. Technically it is an outcome of the subtraction of the total employment from the labor force and, therefore, endogenous to the model. In fact, however, an "unreasonable" unemployment figure leads to revised estimates of disposable income and government expenditure and so here unemployment is called, as in fact, it is, an exogenous assumption. The forecast has unemployment risng until 1983, peaking at 1.13 million and then falling to .9 million in 1985, and falling further to .8 million in 1990.

The last group of assumptions involves coefficient changes in the energy sectors. The across-the-row change equation forecasts were overridden for Electricity, Gas, Coal Mining, and Petroleum refining. The assumed pattern (indexes) is as follows:

Seller	Buyer	1975	1980	1985	1990
electricity	all intermediate	1.0	1.04	1.0	.95
gas	all intermediate	1.0	1.25	1.4	1.4
crude oil	all intermediate	1.0	1.0	1.0	1.0
refining	all intermediate	1.0	.9	.8	.7
refining	electricity	1.0	.9	.6	.3
coal	electricity	1.0	.84	1.2	1.2
gas	electricity	1.0	.8	.7	.4
gas	plastics	1.0	1.0	1.0	1.0

Note: patterns referring to specific buyers have priority over those referring to all buyers.

As one can see, we have imposed a scenario of electricity generation which assumes more emphasis upon coal and nuclear power and less upon oil and gas than would have been the case using the estimated across-the-row function. In addition, natural gas feed stocks sold to make plastics are held fixed since, in this case, gas is not a fuel but a feedstock.

The Forecast Results

Table 1 shows the main macroeconomic results. It should be emphasized that the numbers shown are merely sums of the detailed sector by sector flows and hence are not imposed from above. (The only exception is the item for government spending.) As can be seen, we expect exports to be growing significantly faster than total production, and private consumer expenditures somewhat slower in the coming decade. The table below shows the very marginal shifts in the main GNP:

TABLE 1. The Main Macroeconomic Results.

prognose:	brd 29 may 1981 75-79	79-80	80-81	81-82	82-83	wachstumsraten: 81-83	81-84	summary table 81-85	81-90	85-90
gross nat product	4.33	1.50	-1.00	2.04	2.60	2.32	2.84	3.19	2.99	2.83
output	4.63	1.56	-1.08	2.26	2.77	2.52	3.09	3.49	3.13	2.85
staatsverbrauch	4.31	2.80	0.10	2.00	3.00	2.50	2.67	2.82	3.09	3.30
ausfuhr	5.60	2.67	-0.82	3.33	5.50	4.42	4.80	4.94	5.22	5.45
einfuhr	6.00	0.63	-1.08	2.93	3.53	3.23	4.17	4.92	4.31	3.83
privat verbrauch	3.85	1.63	0.03	2.97	2.87	2.92	2.87	2.82	2.50	2.23
ausruestungen	4.91	2.17	-5.78	-2.75	-3.75	-3.25	0.41	4.00	2.48	1.27
bauten	3.72	-3.82	-0.56	-1.85	-1.99	-1.92	1.01	2.75	2.19	1.75
vorratsveraenderung	0.00	-50.38	-192.55	126.65	47.05	86.85	75.10	63.54	24.08	-7.49
erwerbstaetige	0.14	-0.65	-1.28	-0.45	-0.28	-0.36	-0.08	0.11	0.10	0.08

prognose: brd 29 may 1981 ab. werte: summary table

	1975	1979	1980	1981	1982	1983	1984	1985	1988	1990
gross nat product	1136080.	1350893.	1371267.	1357581.	1385595.	1422039.	1478159.	1542416.	1689504.	1776796.
output	2180708.	2623933.	2665240.	2636699.	2696888.	2772730.	2893106.	3031910.	3328645.	3495789.
staatsverbrauch	331412.	393807.	404989.	405395.	413584.	426179.	439158.	453892.	501127.	535316.
ausfuhr	259275.	324358.	333139.	330432.	341632.	360946.	381568.	402579.	476474.	529573.
einfuhr	249733.	317410.	319416.	315996.	325401.	337077.	358155.	384737.	435282.	465869.
privat verbrauch	572751.	663180.	679145.	679712.	700187.	720551.	740806.	760950.	815222.	850900.
ausruestungen	100640.	122456.	125141.	118112.	114912.	110681.	119589.	138597.	152378.	147700.
bauten	124806.	144846.	139413.	138635.	136099.	133424.	142907.	154741.	167666.	168903.
vorratsveraenderung	-3071.	14656.	8856.	1291.	4582.	7335.	12286.	16395.	11920.	11272.
erwerbstaetige	25479.	25620.	25454.	25129.	25016.	24947.	25067.	25245.	25351.	25346.

	Percent of GNP		
	1975	1980	1990
PCE	50.4	49.5	47.9
GOV	29.2	29.5	30.1
EXP	22.8	24.4	26.2
INVESTMENT (plant & Equipment)	19.8	19.3	17.8

Components predicted by the model.

Some highlights of the sectoral details are shown below:

	Sum of Percent of Gross Outputs		
	1975	1980	1990
Agriculture	3.29	2.92	2.57
Oil refining	1.61	1.54	1.20
Machinery	4.38	4.33	4.34
Textiles	1.55	1.38	1.22
Construction	7.06	6.60	4.20
Retail trade	3.83	3.80	3.79
Banking	1.92	2.05	2.26

	Percent of Total Employment		
	1975	1980	1990
Agriculture	7.15	5.62	3.53
Machinery	4.48	4.57	4.46
Textiles	1.57	1.29	0.85
Trade	12.48	12.22	11.39
Banking	1.92	2.26	3.03
Government	13.78	15.91	20.86

As one can see from these tables, employment is moving strongly toward the service side of the economy, while the output measures show no such pronounced trend. To be sure, for output, we see relative declines in Agriculture and Textiles and an increase in Banking services; on the other hand, both Machinery's and Trade's proportion of the total remain remarkably constant. For employment, Agriculture's and Textile's proportion is halved, while Government's share is predicted to rise by nearly 50 percent.

REFERENCES

Baumgart, E., "Investitionsmatrix fur Die Bundesrepublik Deutschland 1972", (1976) in Deutsches Institut fur Wirtschaftsforschung. Heft 1.

INFORUM, "Spring Meeting" (1981), University of Maryland, May.

Nyhus, D. and Almon, C. (1982) "Linked Input-Output Models for France, the Federal Republic of Germany, and Belgium" in Global International Economic Models ed. B. Hickman, Forthcoming.

RWI, "Input-Output-Tabellen Fuer Die Bundesrepublik Deutchland 1962 BIS 1975", (1979) Rheinisch-Westfaelisches Institut fuer Wirtschaftsforshung, Rapilpe Nr. 8.

Statisches Bundestamt, "Volkswirtschaftliche Gesamtrechnungen, Fachserie 18, Reihe 1" (1981) Wiesbaden.

A DESCRIPTION OF THE BELGIAN INFORUM MODEL

D. Vanwynsberghe

This input-output model was constructed along standard Inforum lines and has been in use since 1976. It considers 52 sectors, for which an input-output table for the year 1970 is available. Final demand is separated into six sectors each of which has its own commodity classification. Econometric equations based on time-series data starting in 1960 are used to derive 59 personal consumption functions, 35 export functions, 35 import functions, and 36 inventory functions.

Investment is divided into two parts, which leads to 72 investment functions for 36 industries aggregated over 52 sectors. The same aggregated industries are used in the employment functions.

The main exogenous variables are:

- Disposable income per capita
- Labor supply
- Rate of savings in national income
- Two categories of government expenditure (disaggregated by sector)
- Aggregated data on foreign demand (for 10 commodities)
- Exchange rate

The model generates a solution in constant prices, and therefore requires only information on relative prices, i.e. indices of the growth in prices treated with the GDP price deflator. At present it assumes that foreign trade shares remain constant, which means that structural changes in the environment of the Belgian economy cannot be taken into account.

The most important part of the model is that dealing with foreign prices. The international market is split into four groups, the first consisting of the U.S.A. and Canada, the second of Japan, the third of the F.R.G., France, the U.K. and the Netherlands, and the fourth of all countries other than those already mentioned.

The study of foreign trade between Belgium and these four 'regions' is based on 1961-1975 import and export data for 119 commodities, which can easily be aggregated to represent 35 sectors. This data is used to construct market share matrices which are assumed to be constant over time, and aggregate prices are calculated for exports and imports.

Domestic prices are derived using equations based on the cost of goods and prices for competitive and complementary imports.

The export prices depend on whether the country of origin is a price-taker or a price-maker. In the first case the export price is a function of the prices charged by competitors and of lagged dependent variables, in the second case it is a function of wholesale prices and lagged dependent variables.

PART II

WORK ON MODELING OF PERSONAL CONSUMPTION EXPENDITURE

A CROSS SECTION AND TIME-SERIES ANALYSIS OF HOUSEHOLD CONSUMPTION

P. Devine

This paper describes a system of equations for forecasting personal consumption expenditures. These equations are unique in that they incorporate all of the following information:

<u>Size Distribution of Income</u> - Because rich and poor households do not consume in a similiar fashion, information on the distribution of income (and not just its average value) is included in the equations.

<u>Age Structure of the Population</u> - The equations treat individuals of different ages differently by assigning to separate age groups relative weights for each consumption item.

<u>Demographic Composition</u> - The structure of the American household is changing in ways that influence consumption patterns: One and two member households are increasing, the educational level is rising, population is migrating from the northern sections of the country, and more households have two wage earners. These demographic changes affect consumption items differently. The combined effects of these demographic changes are included in the consumption equations.

<u>Relative Prices</u> - The system of equations allows for a great deal of flexibility with regard to product substitutability and complementarity.

To measure the magnitude of the demographic and economic influences on consumption required that both time-series and cross-section data be utilized. Because of the wide variety of households in the sample, cross-section data provide a perfect enviroment in which to measure the effects of household composition on consumption paterrns. However, cross-section data are not overly useful when one wants to estimate the price effects that will be used to shape long run forecasts of consumption. Time-series data, on the other hand, can be used for measuring price elasticities but not for measuring the effects of slowly moving demographic variables. Since neither data source alone would have sufficed, both were used, with each contributing their special attributes to the formulation of the final system of equations.

The equations have been estimated for 77 components of US personal consumption expenditures and are used in the Input-Output of the U.S. model developed by the INFORUM project.

This paper is in four sections. The first section describes the cross-section analysis in which the demographic influences on consumption were measured. The second section outlines the transition that was necessary in going from the cross-section results to useful time-series variables. The third section describes the form of the time-series consumption function. Results of the different estimations appear in the fourth section.

CROSS-SECTION

The objective of the cross-section analysis is to measure the effects of both income and demographic influences on household consumption. The analysis allows not only for the effects of income and number of household members, but also for such influences as the age structure of household members, region, labor force participation of spouses, and education.

Apart from the fact that demographic variables have a natural place in the investigation of cross-section consumption data, the reason for studying their effects is to acquire information that can help to explain time-series consumption behavior. Since the demographic make-up of the population contributes to the determination of personal consumption expenditures, it is necessary to account for changes in the demographic profile that occur through time. These changes, however, are many and slow; it is therefore impossible to identify their complex interactions by using only time-series data. Cross-section data, on the other hand, provide a very rich source of information on the demographic influences on consumption. For this reason, if the effects of changing composition of the population are to be accounted for in the time-series analysis of consumption, the magnitude of the demographic effects must be measured by using cross-sectional data.

The theory of the consumer tells us that a consumer's demand for a commodity depends both upon his income and upon prices. In addition, the demand for a commodity might well depend upon demographic characteristics of the consumer. We can write the general demand function for commodity i as

$$C_i = f(Y, P_1, \ldots, P_m, D_1, \ldots, D_L) \qquad (1)$$

where

C_i is the quantity of the i^{th} good demanded,

Y is income,

P_1 through P_m are the prices of all goods, and

D_1 through D_L are demographic variables.

In a cross-sectional analysis, however, all consumers face essentially the same prices. Therefore, different patterns of consumption among consumers are the result of differences in income and demographic characteristics alone. If we consider prices as being constant, we can rewrite equation (1) as

$$C_i = f(Y, D_1, \ldots, D_L) \qquad (2)$$

Therefore, cross-section data provides a framework in which we can study the effect of income and demographic characteristics on consumption without having to account explicitly for the effect of prices.

The task at hand is to specify a functional form for equation (2) which is flexible enough to be used as the consumption function for a long list of goods. (Table 1 contains a complete listing of the 50 separate commodities under investigation.) The function should be able to represent the demand for luxury items, necessities, or even inferior goods. It should also incorparate demographic influences in a straight forward manner.

The form of the cross-section consumption function used, designed with the above requirements as a guide, is as follows:

$$C_i = (a + \sum_{j=1}^{K} b_j Y_j + \sum_{j=1}^{L} d_j D_j) \cdot (\sum_{g=1}^{G} W_g N_g) \qquad (3)$$

$i=1,\ldots,M.$

where

C_i = household consumption of good i

Y_j = the amount of per capita household income within income category j. (An expanded description follows.)

D_j = a zero/one dummy variable used to show inclusion in a demographic group

N_g = the number of household members in age category g

K = the number of income groups

L = the number of demographic categories

G = the number of a age group

a,b,d,w = parameters to be estimated.

Incorporated into equation (3) are three special features. The first is the treatment of the relationship between income and consumption, the second is the method used to measure the effects of demographic variables and, the third feature is the way in which the equation accounts for the age structure of the household. These separate features will be discussed in turn, but first, a few comments about the equation as a whole are in order.

The consumption function can be viewed as explaining household consumption as the product of two components -- consumption per person and the 'size' of the household. The consumption per person is determined by the level of income per person and the demographic characteritics of the household. (This consumption per person is represented by the expression within the first pair of parentheses of equation (3).) The 'size' of the household varies by commodity. This 'size' depends not only on the number of members of the household, but also on the specific age distribution of the members as well. (The 'size' is represented as the summation in the second pair of parenthesis.) The consumption function (3) can be viewed as an expansion of a simple equation which relates per person consumption to per person income, such as

$$\frac{C_i}{N} = f\left(\frac{Y}{N}\right) \qquad (4)$$

where N is the number of members in the household. To this simple form we have added demographic characteristics to $f(\cdot)$ and generalized on the size of the family, n, to make it a function of number of members in separate age categories, say $g(n_1,\ldots,n_g)$. For convience, the equation is then written in product form. That is, equation (3) can be represented as

$$C_i = f\left(\frac{Y}{N}, D\right) \cdot g(N_1,\ldots,N_g)$$

This formulation, therefore, keeps intact the conventional notion of relating per capita expenditures to per capita income.

The component pieces of equation (3) will now be discussed in detail, starting with the per capita portion of the equation (income and demographic characteristics) and then the age distribution of the household.

Consumption As A Function of Income

Consumption has been related to income in a number of fashions. Brown and Deaton, in their fine survey of models of consumer behavior[1], discuss four possible candidates for Engel Curve analysis. They are the double logrithmic, semi-logrithmic, log-reciprocal, and linear functional forms. Each is thought to have special merit in certain circumstances. For goods with income elasticities less than unity, Brown and Deaton recommend the semi-logrithmic form. When the elasticity is close to unity, the linear form is recommended and when income elasticites are greater than unity, the double logrithmic (or constant elasticity) form is considered useful. For goods that approach saturation levels, the log-reciprocal form is suggested.

The suggestions of Brown and Deaton indicate that the functional form used to relate consumption to income should be tailored to match the particular characteristics of each commodity under study. Therefore, no single form among the four functional forms discussed is flexible enough to be used in the investigation of the 50 commodities. For this reason, a more general form for the Engel curve is proposed. We will call this functional form the Piecewise Linear Engel Curve. As the name implies, the curve is made up of linear segments over specified income ranges. The slope of the segments are allowed to differ for different income ranges but with the requirement that the curve be continuous. Figure 1 depicts a Piecewise Linear Engel Curve for the case where five income brackets are considered.

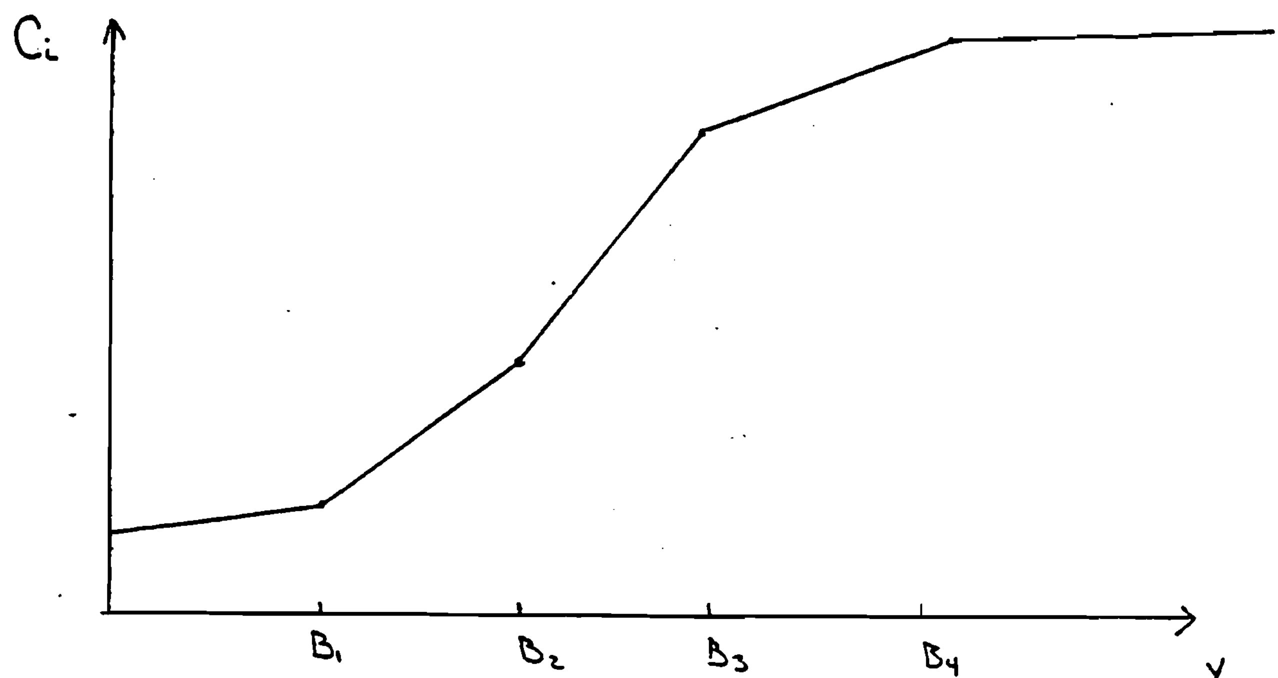


FIGURE 1

The principle advantage of th Piecewise Linear Engel Curve is the flexibility that it possesses. The curve can be likened to a rod that has flexible joints at fixed intervals -- such a rod (curve) can be transformed into a myriad of shapes. Therefore, the Piecewise Linear Engel Curve can approximate quite closely each of the four functional forms listed above. This flexibility allows us to use just this one functional form yet reap the benefits that would accrue from fitting different functional forms to different commodities.

To represent the Piecewise Linear Engel Curve algebraically, we must first define B_j to be the upper bound for the j^{th} income bracket. For example, if we consider the lowest income bracket to be from $0 to $5,000, then B_1 would be $5,000. We can now represent the Piecewise Linear Engel Curve (hereafter PLEC) for good i as

$$C_i = a + \sum_{j=1}^{K} b_j Y_j \qquad (5)$$

WHERE

$$Y_j = \begin{cases} B_j - B_{j-1} & \text{if} \quad B_j \leq Y \\ Y - B_{j-1} & \text{if} \quad B_{j-1} \leq Y < B_j \\ 0 & \text{if} \quad Y < B_{j-1} \end{cases}$$

and

Y = household income

K = the number of income brackets.

B_0 is defined to be zero and B_k as infinity.

For a household with an income of Y, the Y_j variables are defined to be that amount of income the household holds in each income bracket. That is, for brackets below the bracket in which the household's income lies, the amount of income held (the value of Y's) is equal to the full range of income in that bracket. For example, suppose a household has an income of $12,000. Then this household would hold $5,000 in the bracket defined by $5,000 to $10,000. The amount of income held by the household in brackets above the one in which their income lies is zero. Within their income bracket, the household holds that amount by which its income exceeds the lower bound of the bracket. Consider the following example where there are five income brackets. Let

$B_0 = 0$, $B_1 = 5{,}000$, $B_2 = 10{,}000$, $B_3 = 15{,}000$, $B_4 = 20{,}000$,
B_5 = infinity

The following table gives the values of the five Y_j variables for four families with different incomes.

Table 2

FAMILY	Y	Y_1	Y_2	Y_3	Y_4	Y_5
1	3,000	3,000	0	0	0	0
2	12,000	5,000	5,000	2,000	0	0
3	18,000	5,000	5,000	5,000	3,000	0
4	27,000	5,000	5,000	5,000	5,000	7,000

The b_j parameters in equation (5) correspond to the slopes of the PLEC in the different income brackets. To see that this is so, consider the income derivative of equation (5).

$$\frac{dC_i}{dY} = \frac{d}{dY} \quad a + \sum_{j=1}^{K} b_j Y_j$$

$$= \sum_{j=1}^{K} b_j \frac{d}{dY} (Y_j)$$

$$= b_J \qquad \text{where } B_{J-1} \leq Y \leq B_J$$

Since

$$\frac{d}{dY}(Y_j) = \begin{cases} \frac{d}{dY}(B_j - B_{j-1}) = 0 & \text{if} \quad B_j \le Y \\ \frac{d}{dY}(Y - B_{j-1}) = 1 & \text{if} \quad B_{j-1} \le Y < B_j \\ \frac{d}{dY}(0) = 0 & \text{if} \quad Y < B_{j-1} \end{cases}$$

The slopes of these Engel curves tell us what proportion of an additional dollar of income is devoted to the consumption of good i. Let us dub this proportion the Specific Propensity to Consume (or the SPC) on good i. That is

$$SPC_i = \frac{dC_i}{dY} \qquad (6)$$

The term "specific" is used since the propensity to spend on a good will vary from good to good. For example, an additional dollar of income may lead to an increase of expenditure of ten cents on food but only an additional two cents on alcohol. (This concept should not be confused either with that of income elasticity or with that of the total marginal propensity to consume, although it is similar to both.)

The advantage of the PLEC is not just that the SPC varies from good to good but also that the specific propensity to consume a particular good need not be the same for different income groups. This desirable feature is obviously the result of allowing the slope of the Engel curve to vary over income brackets.

Throughout the above discussion of the Piecewise Linear Engel Curve we have repeatedly refered to household income. The precise form of the income variable used is this study, however, is per capita total expenditures for the household. That is, we make the following definition

$$Y = \sum_{i=1}^{M} C_i / N \tag{9}$$

where

C_i = household expenditure on good i

M = the number of commodities

N = the number of members in the household.

Ideally, the level of household consumption would be related to levels of current and past incomes as well as to household wealth. However, it is difficult in the cross-section to account for either past income or for wealth. Brown and Deaton comment that

> "Since on a cross-section of households wealth is in general positively correlated with current income, the calculation of Engel curves without allowance for the separate influence of wealth is likely to be misleading if the relationship is used for prediction through time, since a sudden increase in income will not be matched by a similar increase in wealth.[2]

Since the purpose of this cross-section analysis is indeed to obtain Engel curves that will be used in the examination of time-series consumption patterns, we seek to avoid the problems associated with using current income without accounting for either wealth or past incomes by using total expenditures. The effects of current and past incomes, as well as the effect of wealth, will jointly determine the level of total household expenditures. Suppose, for example, we observe in the cross-section a wealthy household with a history of high income experiencing, for whatever reason, a period of low income. We would expect in this case that the level of total expenditures will have declined to a lesser degree than current income because the household views the period of low income to be transitory.

An additional difficulty with using income as the explanatory variable in the Engel curves is explaining the proportion of income devoted to savings. In the cross-section, there is a strong positive correlation between the level of income and the average propensity to save. The same correlation is not evident in the historical data. Average incomes have increased substantially over time while the savings rate has remained roughly constant and even fallen slightly in recent years. Therefore, using Engel curves fitted with income, as opposed to total expenditure, would result in an overprediction of savings in periods when average incomes are high and an underprediction of savings when average incomes are low.

The Effect of Demographic Variables

The most general method for allowing demographic variables to influence consumption patterns would be to estimate separate consumption equations for each demographic category. This would allow each demographic group to have intercepts and specific propensities to consume out of income that are distinct from those of all the other demographic groups. This approach has the obvious problem of having to estimate a very large number of separate equations. (Consider the case where there are four regions, four family sizes, two educational levels, and working and non-working spouses. To completely categorize the population, we would need 64 separate groups.) Even if the cross-sectional data were sufficiently rich to allow for this method of accounting for demographic effects, severe difficulties would arise when we attempted to incorporate the results into the time-series analysis. With separate equations, we would need to know the past and future size and income distribution of each demographic group.

The procedure adopted for this study allows only the intercept of the Piecewise Linear Engel curves to be different for different demographic groups. This assumes that the effect of demographic variables is to shift the entire curve upwards, or downwards, in a parallel fashion. Therefore, all demographic groups have the same specific propensities to consume out of income. This is accomplished in the usual fashion by creating zero-one dummy variables to indicate inclusion within various demographic categories. With this modification, we can rewrite the consumption function as:

$$C_i = a + \sum_{j=1}^{K} b_j Y_j + \sum_{j=1}^{L} d_j D_j \qquad (10)$$

where

$$D_j = \begin{cases} 1 & \text{if the household is a member of the } j^{th} \text{ demograhic group} \\ 0 & \text{otherwise} \end{cases}$$

L = the number of demographic categories

Consider a specific example of equation (10) where the only demographic variables included are region and the educational status of the household head. Then we could write (10) as:

$$C_i = a + \sum_{j=1}^{K} b_j Y_j + d_1 S + d_2 NC + d_3 W + d_Y E$$

where

S - is 1 if the household lives in the South, 0 otherwise

NC - is 1 if the household lives in the North Central Region, 0 otherwise

W - is 1 if the household lives in the West, 0 otherwise

E - is 1 if the household head is a college graduate, 0 otherwise.

This example illustrates two properties of the proposed technique. First, a separate dummy variable indicating that households reside in the Northeast or a dummy variable indicating that the household head has less than a college level education have not been included. If these variables had been included, there would be perfect multicollinearity between the set of variables indicating region and the set of variables indicating educational attainment (as well as with the constant term). Therefore, the constant term 'a' represents the intercept of a household residing in the Northeast whose head does not have a college degree. Households that live in different regions, or who have a college educated household head, have an intercept that is the sum of the constant term 'a' and the appropriate demographic coefficients.

The second property of the technique used to measure the influences of demograhic characteristics is that no provision is made for interaction between the variables. The effects of the different demograhic variables are assumed to be additive. For example, the effect of having a college education or the consumption of tobacco is exactly the same for all regions. This assumption is made for two reasons. First, if we were to allow for full interaction between the demographic groups, a large number of additional parameters would need to be estimated. Second, and most important, is the fact that the transition from the cross-section to the time-series would be made far more difficult if interaction terms were included. To incorporate the interaction terms into the time-series would require that we know the historical size of more narrowly defined demographic groups. For example, we may want college educated households with working spouses living in the North Central region to have their own special intercept. If this were the case, we would need to find out how many households fit this description over time — a difficult task, at best. (One method of obtaining this data would be to use the marginal totals in the RAS balancing approach, starting with a table obtained from the cross-section data.) If we are content to allow the effects of the different demographic variables to be additive, obtaining the necessary historical data is a much more managable task. (For example, we would only need to know the population of the different regions of the country, the proportion of all household heads who have college degrees, and the proportion of households with two wage earners.) For the above reasons, and because it does not seem to be overly restrictive, we make the assumption that the effects of the different demographic variables are additive.

Age Structure

As mentioned above, age structure can be an important determinant of household expenditures. This is true , to varying degrees, for each of the 50 commodities under investigation. However, if we were to stop the development of the consumption function as it stands in equation (10) and estimate it in per capita form, we would be implicitly assuming that the age structure of the household holds no useful information other than telling us the size of the household.

To utilize fully the information on the age structure of the household, a set of Adult Equivalency Weights are estimated in conjunction with the consumption function. These Adult Equivalency Weights (AEW's) allow us to construct a measure of the size of the household which weights members of different ages differently. This weighted household size varies by commodity. Define the size of the household specific to good i as

$$N_i = \sum_{g=1}^{G} W_{ig} N_g \tag{11}$$

where

N_i = weighted household size for good i

G = number of distinct age groups

N_g = number of household members in the g^{th} age group

W_{ig} = weight of the g^{th} age group in the consumption of good i.

This technique permits us to give high weights to the likely consumers of a good while giving relatively low weights to those less likely to consume the good. Consider the tobacco example and, for contrast, consider medical services as well. Suppose we have three age groups -- children, adults, and the aged. Suppose further that the adult equivalency weights in the consumption of tobacco for the three age groups are 0.2. 1.0, and 0.5 respectively, while the corresponding weights for medical services are 1.5, 1.0, and 2.0. The following table shows the weighted household sizes of five separate households for both tobacco and medical services.

Family	Number of Children	Adults	Aged	Weighted Household Size for Tobacco	Medical Services
1	3	2	0	2.6	6.5
2	0	2	3	3.5	8.0
3	4	1	0	1.8	7.0
4	1	2	2	3.2	7.5
5	0	5	0	5.0	5.0

We can see from Table 3 that while each of the households has five members, the weighted household sizes differ markedly given the hypothetical adult equivalency weights. This is true when comparing all household sizes with regard to a specific good (a range of 1.8 to 5.0 for tobacco and of 5.0 to 8.0 for medical services) and when comparing the two household sizes for each particular household (the last household is the largest relative to tobacco but the smallest relative to medical services.)

This weighting scheme generalizes on the concept of a per capita consumption function in a very natural way by relaxing the assumption that all individuals contribute equally to the size of the household regardless of their age. In addition, the weighting scheme is not required to be the same for different goods as it is under the naive 'per capita' approach. However, since these weights are estimated (through a technique described later), if the naive assumption is correct, or at least sufficient, the estimated parameters (weights) will be roughly equal.

The desirability of this AEW approach can be made even more apparent if we consider the benefits it will produce for use in the coming time-series analysis. For example, suppose we find in the cross-section that the adult equivalency weight for the very young is half the size of the weight for adults with regard to the consumption of tobacco. We could then incorporate this information into the time-series estimation in such a way that the spurt in population associated with the baby boom becomes relevant to the consumption of tobacco only when the "boomers" grow old enough to smoke. (This is accomplished by using the cross-secton AEW's and detailed population information to construct a time-series of weighted populations specific to each good.) This example makes it clear that these age specific weights will be useful, in light of shifts in the age structure of the population, both in the estimation of the time-series equations and when making forecasts of consumption.

The nature of the AEW scheme allows young children to contribute less than an adult to the size of household for some goods (eg. alcohol and tobacco) while contributing more to the size of the household for other goods (eg. furniture). This leads to the question of whether or not children are more or less expensive to maintain than the typical adult. That is, if we average the children's weights over all goods (using consumption shares to construct a weighted average) will this average be more or less than average corresponding to adults? To address this question, other researchers have included in the estimation of Adult Equivalency Weights a general set of weights which they use to construct per capita income. The income weights are defined to be

$$W_{oj} = \sum_{i=1}^{M} s_i W_{ig}$$

where s_i is the share of good i in total consumption. The 'size' of the household, for purposes of constructing per capita income, is then

$$N_o = \sum_{g=1}^{G} W_{og} N_g$$

While this approach has merit, it has not been adopted in this study for a number of reasons. First, the added complication of the income weights makes an already nonlinear equation severly nonlinear making a difficult estimation even more difficult. Second, preliminary estimations indicated that the income weights did not differ significantly across age groups. Finally, and most importantly, the estimation of income weights would have made it nearly impossible to use the Engel curves in the time-series analysis. The difficulty that would have been caused by the income weights is that we would have needed to know not just the distribution of income but the age structure at each income level as well. (This point will be clarified in the next section when the Engel curves are includeded into the time-series analysis.) For the above reasons , when we construct per capita income, all age groups are given identical weights. That is, we assume that

$$W_{o1} = W_{o2} = \dots = W_{og} = 1.0$$

One final point needs to be made about the Adult Equivalency Weights. Consider a simple representation of equation (3) where there are only two income variables, one demographic variable, and three separate age groups

$$C_i = (a + b_1 Y_1 + b_2 Y_2 + d D) \cdot (W_1 N_1 + W_2 N_2 + W_3 N_3) \qquad (12)$$

Consider now the estimation of the parameters of equation (12). It is clear that the parameters are not uniquely identified in the sense that doubling a, b_1, b_2, and d while halving w_1, w_2, and w_3 will leave the value of the product unchanged. This fact makes it impossible to uniquely determine all of the parameters using regression techniques.

The problem of underidentification is avoided by setting the weight for one of the adult age groups equal to 1.0. This normalization allows us to estimate the other weights relative to the weight for adults, hence the name - Adult Equivalency Weights.

Results

The exact form of equation (3) used has five separate income categories, ten demographic variables, and eight age categories. The equation was estimated using data from the Bureau of Labor Statistics Consumer Expenditure Survey. Approximately 8000 households from the 1972 data set were used to estimate the equation. The technique used to estimate the non-linear consumption function was an iterative one which can be shown to converge to the maximun likelihood estimate of the function. The estimation scheme first assumes that all of the AEW's are 1.0 and estimates the other parameters in the equation using OLSQ. Then, with the estimate of the per-person portion of the equation, estimates are made of the AEW's. The procedure is repeated until convergence is reached.

The five income classes were chosen so that each held 20 percent of the individuals in the sample. The demographic variables included:

Region: three variables, one each for the South, North Central and West

Education: one variable, distinguishes families with college educated household heads

Working Spouse: one variable for households where the spouse is employed

Family Size: three variables, one variable each for households with one member, two members, and five or more members

Age of Household Head: two variables, one for households with heads less than 35, and the other for households with heads over 55

Remember then that the intercept represents households with three or four members residing in the Northeast where the spouse does not work and with a household head aged 35-54 without a college degree.

The eight age groupings are as follows:

0 - 5	years	old
6 - 15	"	"
16 - 20	"	"
21 - 30	"	"
31 - 40	"	"
41 - 50	"	"
51 - 65	"	"
66	years and over	

The normalization group is the fifth, 31 to 40 years old. Therefore, our Adult Equivalency Weights might more properly be called "Thirties" Equivalency Weights.

TRANSITION FROM CROSS SECTION TO TIME SERIES

In the cross-section, we were able to quantify to what extent household consumption was affected by demographic composition, age structure, and, through the use of the non-linear Engel curve, the distribution of income. The task now is to combine these cross-section results with information on the make-up of the population as a whole to create variables for use in the coming time-series analysis. First, by using the adult equivalency weights, a time-series of weighted populations is created for each commodity. Then, a time series is calculated of the consumption of each commodity which would be predicted using all of the results of the cross-section analysis combined with historical changes in income distribution and demographic composition. These constructed variables then enter directly into the time-series consumption functions described in the next section. Producing the time-series of weighted populations is the easier of the two tasks and is discussed first.

Weighted Populations

The Census Bureau publishes historical series on population by age. These series can be aggregated to provide separate population totals for each of our eight age groupings. Let N_{gt} represent the number of individuals in age group g in year t. We can then construct our weighted population series for commodity i (P_{it}) as follows:

$$P_{it} = \sum_{g=1}^{8} W_{ig} N_{gt} \qquad (13)$$

This is done for each of the 50 cross-section commodities giving us 50 commodity-specific weighted populations.

The overall benefits of the commodity specific populations are twofold. First, the fact that we can define a more relevant population size for a good should enable us to make better estimates of the price and income effects on the demand for the good. And secondly, when we use the estimated time-series functions to forecast consumption expenditures, we will have explicitly accounted for the changing age structure in the forecast.

Time-Series Predictions from the Cross-Section

From the historical time series on income distribution and demographic shifts, "predictions" of consumption per adult equivalent are made for past years. That is, we use the estimated cross-section parameters in conjunction with information on the level and distribution of income and the other demographic variables to compute a "prediction" of expenditure per adult equivalent. This "prediction" for good i in year t is referred to as C^{*}_{it}. That is,

$$C^*_{it} = a_i + \sum_{j=1}^{5} b_{ij} Y_{jt} + \sum_{j=1}^{10} d_{ij} D_{jt} \qquad (14)$$

where

Yjt = average amount of income received in the kth income bracket in year T

Djt = population proportion falling into the jth demographic category in year T.

ai, bij, dij = the estimated cross-section parameters for good i

Clearly, equation (14) can be viewed as the result of evaluating the cross-section equation for each individual in the population and then averaging the results. (The average value of the 0-1 dummy demographic variable will be the number of individuals in the category divided by the total number of individuals, that is, the population proportion.)

The C^*_{it} variable will be used as an explanatory variable in the time-series consumption function for good i. The benefits that it has in this role are many. Incorporated into C^*_{it} are the changing population proportions of the different demographic categories, together with information on the level and distribution of income but, and most importantly, the inforrmation is interpreted fo product i through the use of the cross-section coefficients. Consider the demographic composition variables. In the cross-section these variables showed that they could be helpful in explaining household expendituues; therefore, they properly belong in the time-series analysis. Imagine, however, the folly of incluidng ten separate, slowly changing population proportions into a time-series estimation. Even if it were possible to circumvent the severe problem of collinearity and get meaningful estimates, the toll taken on the degrees of freedom would be prohibitive. In the construction of the C^*_{it} variable however the ten population proportions are combined in such a way that the most relevant variables to good i are given the most weight. That is, the demographic variables are summed up into one value with the weights being the cross-section paramters (the second summation in Equation (14). For instance, since family size wasn't overly helpful in the cross-section equation for tobacco, the shift to smaller family size will not greatly influence the value of C^*_{it} variable for tobacco. On the other hand, since education was an important variable in the cross-section equation for tobacco, it will have a relatively large parameter and therefore the trend towards higher educational attainment will significantly affect the tobacco C^*_{it} variable.

The income component of the C^*_{it} variable is somewhat difficult to construct, but in its final form it is responsive to shifts in the underlying distribution of income, changes in the overall savings rate and in the level of taxation, and to increases and decreases in the average level of income. Recall that the cross-section income variables, Y_j, were the amount of income received by a household in a specific income bracket, with income defined to be the total household expenditures per person. We now must compute the average value of these five income variables for each year through the use of a distribution of per capita total expenditures.

The distribution of per capita total expenditures is arrived at only after a number of steps which are only outlined here. The process starts with historical distributions of money income in current dollars by family size. These are combined to create, yearly, a distribution of money income per person. The next step is to use a tax function, estimated in the cross-section, to remove taxes from money income and create a distribution of disposable income in current dollars. The assumption is that tax rates are roughly fixed in current dollars so that a function relating tax rates to income estimated using 1972 data can also be used in 1959 or 1979. The total amount of taxes removed from the distribution by the tax function is not automatically guaranteed to be the correct amount, so the next step was to scale the distribution of disposable income so that it had the correct average value. (The required scaling over the period 1955-1979 was never more than two percent in any year and was usually much less. This fact speaks well for the tax function and the assumptions involved in its use.) The distribution of disposable income was then deflated to produce a distribution of disposable income per capita in constant dollars.

The last step required in the progression to the desired goal was to remove savings from the distribution. This was accomplished through the use of a spending function, also estimated in the cross-section, which relates the spending rate at each income level to the income level relative to average income. When this spending function is used to transform the distribution of income to one of total expenditures, the functional form quarantees that the distribution's average value is exactly what it should be for any level of average disposable income and average spending rate. That is, the resulting distribution of expenditures need not be scaled to give the correct average value for expenditures.

Fom the distribution of expenditures, as it emerged from the preceeding step, we now need to calculate the five Y_j variables. First, consider the representation of the distribution of per person total expenditures given in Figure 2. Along the horizontal axis are the N individuals in the population strating with the poorest and continuing on to the richest. The curve gives the value of expenditure for each individual. Note that the area under the curve is precisely total expenitures for the population.

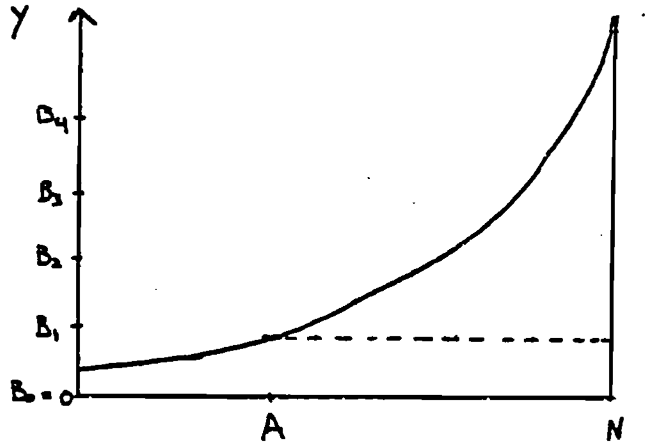


FIGURE 2

To construct the five Y_j variables, we need to find the are in the horizontal bands defined by the B_j boundaries. To see why this is so, it is much easier to think and speak of the total expenditures as "income", and we shall do so. Now, consider Y_1, which is the average amount of "income" received in the B_0 to B_1 range. To get the total "income" in this range, we must sum over all individuals that amount of "income" which each has in this range. For individuals to the left of A, this "income" is the height of the curve. Individuals to the right of A receive the maximum amount, that is B_1-B_0. The total amount of "income" in group 1 can then be divided by population to give Y_1. In a similar fashion, Y_2 through Y_5 can be computed. This procedure is followed for each year of the historical data.

In using this procedure, we make one key decision, namely to define income classes in absolute rather than relative terms. That is, the B_j brackets remain fixed so that the level of "income" that defined the poorest class in 1972 is fixed in real terms and defines the poorest class in all other years. Then, as the the distribution of income drifts upwards, the poorest class becomes smaller and smaller. For instance, suppose that there are 100 individuals in the population, and that in the base peiod the 20^{th} individual is in the poorest category. This person will consume along the first segment of the piecewise-linear Engel curves. If all real incomes increase at the same rate through time, this same individual will slide up into the next income bracket. He will then be assumed to consume along the second segment of the Engel curve. Thus, in this system, it is not relative but absolute real "income" which determines consumption patterns. Note, however, that we are not saying that absolute income determines the share of savings, for our "income" is, in fact, total expenditures.

With the five income variables computed, we can now combine them with the cross-section parameters to construct the expenditure on each item. (This is represented as the first summation in equation (14).)

As mentioned before, the advantage of this procedure is that it is sensitive to shifts in the underlying structure of the distribution of income and not just its average value. If, for example, the distribution shifts such that there are more poor individuals we will see an increase in the C^* variables for necessities and a decrease in the C^* variables for luxuries. This is possible even with no change in average income.

To illustrate this advantage, the income elasticities presented in the time-series section of this paper are measured under four separate assumptions about how a one percent change in average incomes come about. Case I and Case IV are polar cases; in the first case, all of the increase is obtained by giving income to those in the poorest category, and in the fourth case, the entire increase is in the richest category. In Case II, each individual is given the same dollar amount and in Case III, (the most standard) all incomes are increased by one percent. Under all four assumptions, average incomes increase by the same amount but, as the results in the next section will show, the differential effects on individual consumption items can be quite large.

TIME-SERIES

The system of consumer demand equations used in the time-series analysis is a slight extension of the system of Symmetric Consumption Functions developed by Clopper Almon.[3] In both systems, the demand for a good depends upon the prices of all other goods, however, the price effects are simplified by assuming that consumption items can be combined into economically relavent groups. Through this technique, a commodity can be a strong compliment/substitute for other items in its own group while interacting less strongly to the prices of goods in other groups. The difference between the system of equations used in this study and the Symmetric Consumption Functions is the treatment of the group price interactions. In the Symmetric Consumption Functions, all groups are assumed to be weak substitutes for one another. In the new system, groups can be substitutes or compliments and the magnitude of the group price interaction is estimated rather than being specified in advance. Another major difference between the two is that the equations in this study utilize the demographic variables developed in the preceeding sections of this paper.

The system of consumption functions distinguish 77 seperate components of Personal Consumption Expentitures. This data is part of the U.S. National Income and Product Accounts. The equations were estimated over the 1959-79 data period.

The 77 commodities are combined into M seperate groups. (Ten groups to be specific.) In the equation that follows, capital letters represent group indices while small letters refer to individual commodities. For example, good i is an element of group I. Using this notational convention, the system of consumption functions is

$$C_i = (a_i + b_i\ C_i^* + c_i\ \bar{\Delta}\ C_i^* + d_i t) \cdot \prod_{J=1}^{M} \left(\frac{p_i}{P_J} \right)^{S_J\ \lambda\ IJ} \qquad (15)$$

where $i \in I$ and $i = 1,\ldots,77$

and

C_i is expenditure per adult equivalent on good i.

C_i^* is our prediction of C_i using the cross-section results

t is time

p_i is the price of good i

P_J is the average price in group J

S_J is the total share of group J

$a_i, b_i, c_i, d_i, \lambda_{IJ}$ are the parameters to be estimated

The system is made to conform to Slutsky symmetry by requiring that λ_{IJ} equal λ_{JI} for all I and J. (Impossing this constraint on the estimation required that all 77 equations be estimated jointly.) In the exact formulation of the equation for estimation, the (p_i/P_I) term is expanded to allow for subgroups within each group.

The dependant variable in equation (15) is economy wide expenditures on good i divided by the weighted population variable developed in the preceeding section. The C_i^* variable serves as an income variable in the equation, but, as mentioned before, it is much more than that. The ΔC_i^* variable is a cyclical variable since its value is determined primarily by changes in average income. The time trend in the equation is designed to pick up any changes in tastes not explained by the demographic shifts subsumed into C_i^*.

The 77 time-series commodities are associated with the most relavent cross-section commodities. In some cases, the results from one cross-section equation will be used in more than one time-series equation. There are some time-series commodities with no corresponding cross-section results. In these cases, the unweighted population total and simple average income were used in place of the cross-section variables.

ENDNOTES

1. Brown, J.A.C. and A.S. Deaton (1972), "Models of Consumer Behaviour: A Survey," Economic Journal, Vol. 82, pp. 1145-236

2. Brown and Deaton, pp. 1172

3. Almon, C. (1979), "A System of Consumption Functions and Its Estimation for Belgium," Southern Economic Journal, Vol. 46, pp. 85-106

ON THE ESTIMATION OF ENGEL ELASTICITIES WITH APPLICATION TO ITALIAN DATA

L. Grassini

1. After the work of N.S. Iyengar /3/ , who showed under restrictive hypotheses, the relationship between expenditure inequality and Engel elasticities, the concentration curve has been widely used by N.C. Kakwani /4,5,6/ for an "alternative method" to estimate expenditure elasticities from grouped data. In that case elasticities are obtained in terms of the estimated parameters of the concentration curve for total expenditure and for expenditure on each commodity.

Here an application of this "alternative method" for Italy is presented. The data used are obtained from Istat Surveys on Family Budgets, carried out in 1978. The observations are grouped into 19 classes according to the total monthly expenditure of the families; for each class t (where t = 1, 2,..., 19) only the arithmetic means of total expenditure and of the expenditure on each commodity i are available (where i = 1, 2,..., 40).

2. The most important problems that arise in the usual procedure of estimating Engel elasticities from grouped data -we refer in particular to the work of S.J. Prais and H.S. Houthakker /11/ - may be summarized as follows:

1) the choice of the functional form for the Engel curve is based too often on "crucial" values of certain statistical indices, since economic theory or considerations of common sense rarely suggest a specific functional form; further this problem becomes more serious when we deal with data which are highly disaggregated both in terms of consumption units and in terms of commodities;

2) the use of grouped data introduces bias in the estimates when non-linear Engel functions are regressed. For example: the usual procedure for the logarithmic form ($y = a + b \log x$) and for the inverse form ($y = a + b/x$) is to use the arithmetic mean within each expenditure class as proxy for the geometric and harmonic means. This procedure implies a linear Engel function within each class while the "true" Engel curve is supposed - and estimated - as a non-linear one;

3) family budget data require the use of total expenditure as the independent variable of Engel curves. This procedure, which has been widely justified from an economic point of view /11/, has been strongly objected by some Authors who have observed that the application of the least squares (LS) method in family budget

studies introduces biased estimates /9/. It has been pointed out that the total expenditure and its components (the dependent variables of the Engel curves) are endogenous to the consumption unit and are determined simultaneously by its choice's process.

For this "alternative method" of estimating Engel elasticities one needs to specify the form of the concentration curve for which one may find out well defined analitic properties independently from the knowledge of its equation /8, 6/.

3. Let $x>0$ be the total per-capita expenditure with distribution function $F(x)$ and $g_i(x)\geq 0$ be the Engel curve of the expenditure on the i-th commodity, the concentration curve of $g_i(x)$ is defined the relationship between $F_1(g_i(x))$ and $F(x)$ where

$$F_1(g_i(x)) = \frac{1}{E(g_i(x))} \int_0^x g_i(t)\, dF(t)$$

Since

$$\lim_{x\to\infty} F(x) = 1$$

$$\lim_{x\to\infty} F_1(g_i(x)) = 1$$

the curve can be represented in an unit square, whose diagonal - - called egalitarian line - describes the case in which $F(x) = F_1(g_i(x))$ for each x.

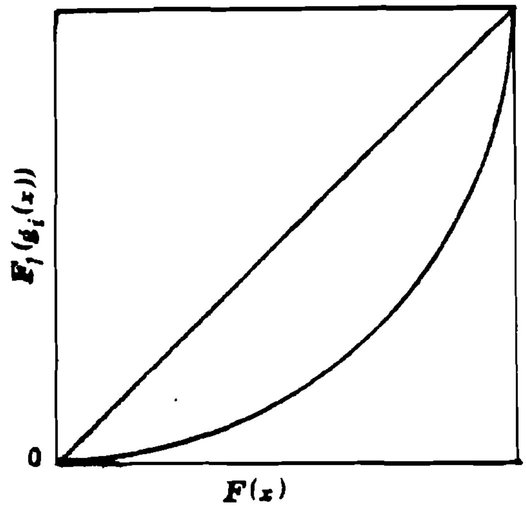


Fig. 1

When $g_i(x) = x$ that relationship is called Lorenz curve; the concentration curve can be then considered as a generalized Lorenz curve, where the Lorenz curve is the concentration curve for total expenditure x. In their works on inequality, N.S. Iyengar /3/ and J.L. Gastwirth /12/ specify the distribution function F(x) and then, eliminating the parameter x from the two equations F(x) and $F_1(g_i(x))$, obtain the function which represents the analitic relationship between F and F_{1i}. N. C. Kakwani and N. Podder /8/ do not use any distribution function of x but a new representation of the concentration curve by passing from the original coordinates $(F;F_{1i})$ to the system $(\pi_i;\eta_i)$, where π_i coincides with the egalitarian line.

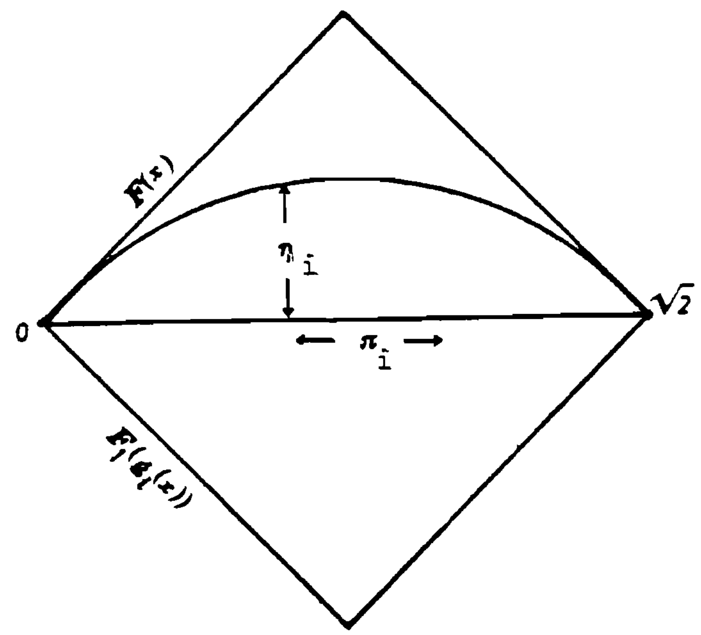


Fig. 2

The transformation matrix which, premoltiplying the vector $(F;F_{1i})$, gives the vector of the new coordinates $(\pi_i;\eta_i)$ is

$$\begin{bmatrix} \cos\gamma & \text{sen}\,\gamma \\ \text{sen}\,\gamma & -\cos\gamma \end{bmatrix} \qquad \text{where} \quad \gamma = 45°$$

Then

$$\pi_i = \frac{F + F_{1i}}{\sqrt{2}} \qquad\qquad \eta_i = \frac{F - F_{1i}}{\sqrt{2}}$$

and the concentration curve of $g_i(x)$ is defined as

$$\eta_i = \Psi_i(\pi_i)$$

where

$$0 \leqslant \pi_i \leqslant \sqrt{2} \qquad\qquad -\sqrt{2}/2 \leqslant \eta_i \leqslant \sqrt{2}/2$$

and the Lorenz curve for total expenditure x is represented by /8/

$$\eta = \Psi(\pi) \qquad \text{with} \qquad 0 < \pi < \sqrt{2} \qquad\qquad 0 \leq \eta < \sqrt{2}/2$$

The expenditure elasticity for the i-th commodity, for each given x, $L_i(x)$, is obtained in terms of $\Psi_i(\pi_i)$ and $\Psi(\pi)$, of their first and second derivatives, as follows /6/:

$$L_i(x) = \frac{\Psi_i''(\pi_i)}{\Psi''(\pi)} \frac{\left[1 + \Psi'(\pi)\right]^2}{\left[1 + \Psi_i'(\pi_i)\right]^2} \quad \frac{1 - \Psi'(\pi)}{1 - \Psi_i'(\pi_i)}$$

4. The method for estimating concentration curves which has been suggested by N. C. Kakwani and N. Podder is analogous to that developed by D. Aigner and A. S. Goldberger in the case of Pareto's law /1/. That method provides asymptotically unbiased and consistent LS estimates and the more efficient generalized least squares (GLS) ones.

Here concentration curves of the per-capita expenditure on 40 commodities and the Lorenz curve for total per-capita expenditure have been estimaded.

Per-capita observations were available, since the average family size within each expenditure class was known. Therefore, at least the effects of family size changes on expenditure have been eliminated.

The change in the total expenditure of the families between two successive classes is, sometimes, proportionally smaller than the change in family size, then we had to put per-capita total expenditure according to increasing values of per-capita total expenditure, modifying the original sequence of the expenditure classes.

Since the total number of family components belonging to each expenditure class were available, in this case the variable $F(x)$ represents the proportion of individuals with total per-capita expenditure less than or equal to x and $F_1(g_i(x))$ the proportion of per-capita expenditure on the i-th commodity of those indidivuals.

The "new" coordinate system for the concentration curve allowed us the fitting of a function which has the analitic properties of the concentration curve /5,6,8,10/ and which is represented, for each expenditure i, by the following equation.

$$\eta_i = \Psi_i(\pi_i) = a_i \pi_i^{\alpha_i}(\sqrt{2} - \pi_i)^{\beta_i}$$

In Tab. 1 we have presented the values of the estimated parameters after the application of the LS method to the logarithmic transformate of the function $\Psi_i(\pi_i)$ and the correspondent determination index R^2, which is really high for each curve. The nature of the stochastic component of the econometric model used would need the application of the GLS method, which provides more efficient estimates /8/. However the gain in precision one can obtain does not seem to justify the resort to a more cumbersome method of estimation /8/. Further, in a recent application to Italian data, the GLS method was suspected to provide downward biased estimates /2/.

The estimated elasticities of Tab. 2 have been calculated at mean per-capita total expenditure.

It is clear that the "alternative method" for estimating Engel elasticities relates the elasticity $L_i(x)$ with the level and the distribution of x and $g_i(x)$ among the consumption units, since each elasticity is calculated in terms of the parameters a, α and β of the Lorenz curve for x and a_i, α_i and β_i of the concentration curve for $g_i(x)$. $L_i(x_0)$ can be defined as the elasticity of the expenditure on the i-th commodity of an individual with total expenditure x_0, given the distribution of x and $g_i(x)$ among the consumption units.

In the usual procedure of estimating Engel elasticities one fits expenditure on i-th commodity, say y_i, on x and the result is an estimation of y_i by e function $g_i(x)$. In the "alternative method" one obtains also estimation of the distribution function of x. The usual procedure refers to a situation in which it is supposed equal distribution within each expenditure class; in that case inequality is implicitely underestimated.

Tab. 1 : ESTIMATED PARAMETERS OF CONCENTRATION CURVES

	COMMODITIES	a	α	β	R^2
1	Bread and cereal	.0337	.9256	.5846	.989
2	Meat	.0952	.8006	.7755	.999
3	Fish	.0604	.6464	.9979	.988
4	Dairy	.0586	.8976	.6282	.994
5	Butter, margarine, oil	.0915	.7628	.5429	.994
6	Fruits and vegetables	.0925	.8303	.8096	.998
7	Potatoes	.0309	.7261	.2979	.977
8	Sugar	.0664	1.6050	.8584	.980
9	Coffes, tea, cocoa	.0580	.7557	.5724	.975
10	Other food	.1855	.8932	.9046	.999
11	Soft drinks	.2362	.9024	.8882	.999
12	Alcoholic drinks	.1030	.7440	.5162	.996
13	Tobacco	.1192	.6718	.9133	.997
14	Clothing and repairs	.3555	.8894	.9389	.999 (°)
15	Shoes " "	.2638	.8360	.9334	.999
16	Housing rents	.0949	1.0053	.4193	.996
17	Fuel and electricity	.2113	.8648	.6585	.997
18	Furniture	.5369	1.0114	.6031	.999 (°)
19	Textiles	.4927	.9369	.8803	.999
20	Household appliances	.5369	.9408	1.0226	.999
21	Glasswork and pottery	.4030	.9027	.7905	.999
22	Nondurables and services	.1474	.7921	.8001	.999
23	Domestic servants	.6977	1.0020	1.0319	.999
24	Medicine	.1169	1.0252	.5642	.976
25	Medical appliances	.4716	.9537	.8153	.999
26	Medical care	.3813	.9127	.6241	.999
27	Hospital care	.5411	.9494	.5223	.992
28	Means of transportation	.9669	1.0065	.1501	.999 (°)
29	Users cost of transportation	.3732	.8603	1.0896	.998
30	Transport service	.2064	.7932	.6018	.999
31	Communication	.3607	.8907	.9741	.999
32	Radio and television sets	.3662	.8995	.7875	.999
33	Newspapers and books	.3073	.8364	.9397	.999
34	Education	.3158	.8612	.9606	.999
35	Entertainment and recreation	.6218	.9761	1.0763	.999
36	Personal toilet articles	.2119	.7960	.8377	.999
37	Hotels, Cafes, Restaurants	.4246	.9073	1.0120	.999
38	Other goods	.3358	.8772	.8164	.999
39	Financial services and Insurances	.4795	.9481	1.1642	.998
40	Other services	.5505	1.0690	.8201	.996
	TOTAL EXPENDITURE	.2050	.8790	.6645	.999 (°)

(°) The adjustment on the last figure gives R^2's equal to 1.0.

Tab. 2 : ESTIMATED ELASTICITIES AT MEAN PER-CAPITA TOTAL EXPENDITURE

	COMMODITIES	ELASTICITY
1	Bread and cereal	.1772
2	Meat	.5132
3	Fish	.3286
4	Dairy	.3078
5	Butter, margarine, oil	.4050
6	Fruits and vegetables	.5168
7	Potatoes	.1031
8	Sugar	.4931
9	Coffee, tea, cocoa	.2692
10	Other food	1.0962
11	Soft drinks	1.3652
12	Alcoholic drinks	.4347
13	Tobacco	.6414
14	Clothing and repairs	2.0361
15	Shoes and repairs	1.5237
16	Housing rents	.4158
17	Fuel and electricity	1.0152
18	Furniture	3.8944
19	Textiles	2.5636
20	Household appliances	3.0904
21	Glasswork and pottery	2.0010
22	Nondurables and services	.7892
23	Domestic servants	3.8456
24	Medicine	.5912
25	Medical appliances	2.4112
26	Medical care	1.6397
27	Hospital care	2.0410
28	Means of transportation	8.8767
29	Users cost of transportation	2.3657
30	Transport services	.9112
31	Communication	2.1197
32	Radio and television sets	1.8436
33	Newspapers and books	1.7629
34	Education	1.8490
35	Entertainment and recreation	3.6258
36	Personal toilet articles	1.1430
37	Hotels, Cafes, Restaurants	2.5211
38	Other goods	1.7475
39	Financial services and Insurances	3.1622
40	Other services	2.7314
	ADDING UP - CRITERION	1.1360

N. C. Kakwani /4/ showed that the more concentrated $g_i(x)$ with respect to x, in the point x_0, the greater is the correspondent elasticity $L_i(x_0)$. The greatest values of the estimated elasticities can be observed for those expenditures which are higher concentrated and show null values for some expenditure class. For example the item 'Means of Transportation" shows in the last class a crude jump in the per-capita expenditure, which passes from 758 to 36362 Lire. The estimated concentration curve is sensible to null values of expenditure, which make that curve "lower" -referring to Fig. 1- with respect to the Lorenz curve for total expenditure /4/ and cause high values of the correspondent elasticity. The value obtained for the adding-up criterion must be considered too high and should show the presence of upward bias in estimating elasticities.

This occurrence must be analyzed also from another point of view: the presence of null values of some expenditures - which appears as the main responsible of the upward estimation - must be referred to the particular nature of the data used. The size of the sample in Family Budgets Surveys is not determined for a particular disaggregation of family expenditures. Further, the aggregation-disaggregation into 40 commodities is made "a posteriori" and it may happen that the sample be underdimensioned for some items. Secondly, some objections are raised against the reliability of observations for the expenditure on durables. In Family Budgets Surveys the note of the expenditures refers to a very short time (a week) and the expenditure on durable goods may escape for its postponability and rareness.

REFERENCES

1 D.J. AIGNER and A.S. GOLDBERGER (1970),"Estimation of Pareto's law from grouped observations", Journal of the American Statistical Association, vol. 65, n. 330.

2 L. GRASSINI (1980), Curve di concentrazione e curve di Engel. Stima econometrica delle elasticità engeliane per alcuni capitoli di spesa delle famiglie italiane, Università di Firenze, Facoltà Economia e Commercio.

3 N.S. IYENGAR (1960), "On a method of computing Engel elasticities from concentration curves", Econometrica, vol. 28, n. 4.

4 N.C. KAKWANI (1977), "Application of the Lorenz curve in economic analysis", Econometrica, vol. 45, n. 3.

5 N.C. KAKWANI (1977), "On the estimation of Engel elasticities from grouped observations with application to Indonesian data", Journal of Econometrics, vol. 6.

6 N.C. KAKWANI (1977), "A new method of estimating Engel elasticities", Journal of Econometrics, vol. 8.

7 N.C. KAKWANI and N. PODDER (1973), "On the estimation of Lorenz curve from grouped observations", International Economic Review, vol. 14, n. 2.

8 N.C. KAKWANI and N. PODDER (1976), "Efficient estimation of the Lorenz curve and associated inequality measures from grouped observations", Econometrica, vol. 14, n. 1.

9 N. LIVIATAN (1961), "Error in variables and Engel curve analysis", Econometrica, vol. 29, n. 29.

10 P. MUSGROVE (1980), "Income distribution and the aggregate consumption function", Journal of Political Economy, vol. 88, n. 31.

11 S.J. PRAIS and H.S. HOUTHAKKER (1955), The analysis of family budgets, Cambridge University Press.

12 J.L. GASTWIRTH (1971), "A general definition of the Lorenz curve", Notes and Comments, The Review of Economics and Statistics, vol. 54.

AN ESTIMATION OF A SYSTEM OF CONSUMPTION FUNCTIONS FOR ITALY. PRELIMINARY RESULTS.

M. Grassini

It is a common practice to derive demand equations from utility maximization. Applied consumption analysis is in fact considered the art of exploiting theoretical restriction in order to reduce, by means of "prior informations", the number of parameters to be estimated.

Nevertheless, if one is looking for operating system of demand equations with given peculiarities, it is possible to consider them at first relying upon the existence of an indirect utility function to retain theoretical restrictions. If "In the present state of the art, the best description and therefore the best forecasts are obtained when using the assumption (or the implications) of utility maximization" /6/, the use of alternative approaches cannot a priori be rejected; being the above criterium -the goodness in fitting and forecasting- applicable to whatever demand systems.

The demand system used has been proposed by C. Almon /2/; it is taylored to impact a set of desirable properties which can be summarized recalling that for forecasting the private consumption expenditures in an I/O model, we should have; adding-up, homogeneity of degree zero in all prices and income, substitution and complementarity between goods, if possible, Slutsky symmetry. Furthermore, price changes should alter the effet of income and non-income determinants, marginal propensity to consume as income rises must be capable of being different for different good and not necessarily independent of the price system, budget shares should depend on prices as income increases, the effet of other variables should be easily included. All these peculiarities may be cheked later on.

The model is revisited in Section 1. The data preparation and the "exogenous" informations are discussed in Section 2. A final section is devoted to the presentation of the results.

The model

The applied demand system is composed by equations of the following form

$$q_i = f(.)\ g(.) \qquad i = 1,2,3,\ldots,n$$

where:

- n = number of commodities
- q_i = consumption per-capita in constant prices of good i
- $f(.)$ = is a function of consumption determinants of good i out of prices; e.g. income, trend and others
- $g(.)$ = is a function homogeneous of degree zero in the prices.

The variables considered in the present application of the model are real income, first difference of real income and a trend. The functional form of f(.) is the following

$$f(.) = b_0 + b_1 \frac{y}{\bar{p}} + b_2\ \Delta\left(\frac{y}{\bar{p}}\right) + b_3 t$$

where

- y = income
- t = trend
- p = price index (income deflator)

The price index is computed as follows

$$\bar{p} = \Pi_j\ p_j^{s_j} \qquad j = 1,2,\ldots,n$$

where s_j's are the budget shares at the base point, t_0, where all prices are set equal to 1.0.

The proposed function g(.) for good i is

$$g(.) = \Pi_j \; p_j^{\;c_{ij}}$$

where the assumed homogeneity implies

$$\sum_j c_{ij} = 0$$

then the demand equation for good i will be

$$(1) \qquad q_i = (b_{0i} + b_{1i} \frac{y}{\bar{p}} + b_{2i} \, \Delta(\frac{y}{\bar{p}}) + b_{3i} \, t) \;\; \Pi_j \; p_j^{\;c_{ij}}$$

To give constant price adding up (at t_0) the conditions

$$\sum_i b_{2i} = \sum_i b_{3i} = 0 \qquad \text{and} \qquad \sum_i b_{1i} = 1$$

are imposed.

Equation (1) is now studied to figure out the implications on parameters to match Slutsky symmetry at a given base point. The "income compensated" derivative of q_i with respect to p_j

$$\left(\frac{\partial q_i}{\partial p_j} \right)_c = \frac{\partial q_i}{\partial p_j} + q_j \; \frac{\partial q_i}{\partial y}$$

evaluated at the base point (which is equivalent to considering $y/\bar{p}$ constant) is

$$\left(\frac{\partial q_i}{\partial p_j} \right)_c = c_{ij} \; \frac{q_i^o}{p_j^o}$$

where $^{\circ}$ denotes values at the base point.

Imposing Slutsky condition implies

$$\left(\frac{\partial q_i}{\partial p_j}\right)_c = \left(\frac{\partial q_j}{\partial p_i}\right)_c$$

that is to say

$$c_{ij} \frac{q_i^{\circ}}{p_j^{\circ}} = c_{ji} \frac{q_j^{\circ}}{p_i^{\circ}}$$

from which

$$\frac{c_{ij}}{p_j^{\circ} q_j^{\circ}} = \frac{c_{ji}}{p_i^{\circ} q_i^{\circ}}$$

or

$$\frac{c_{ij}}{s_j} = \frac{c_{ji}}{s_i}$$

Defining $\lambda_{ij} = c_{ij}/s_j$, we have in $\lambda_{ij} = \lambda_{ji}$ the Slutsky symmetric condition in (1). The demand equation for good i becomes

$$q_i = f(.) \quad \Pi_j \; p_j^{\lambda_{ij} \; s_j}$$

considering for a while $c_{ii} = \lambda_{ii} \; s_i$

Here a reduction on parameters has been obtained but we are still far from a workable model. Infact, using no more than two decades yearly data resorting on Slutsky condition is effective when the commodities are relatively few; that is to say, when goods are grouped in a small number of items.

But the present model is conceived for treating large size demand equations systems, being the objective the forecast of private consumption components in the final demand of an input-output model, preserving as much as possible the required detail. The effective reduction in parameters is then reached by means of the often-used concept of groups and subgroups commodities. The grouping criteria will be based upon the between goods complementarity and substitution we would expect to find.

Now, the hypotheses are that the same λ prevails within the group or subgroup and the same λ will prevail everywhere among the groups which are not closely related by definition.
This means that for a given good i belonging to the group C, $\lambda_{ij} = \lambda_C$ for $j \epsilon C$ and $\lambda_{ij} = \lambda_0$ for $j \epsilon \bar{C}$ where $\bar{C}$ can be divided (and usually it is) in many other groups. In this case the demand equation for good i becomes

$$q_i = f(.) \ \Pi_j \ p_j^{\lambda_C s_j} \ \Pi_k \ p_k^{\lambda_0 s_k}$$

for $j \epsilon C$ and $k \epsilon \bar{C}$.

We can work out $\bar{p}$ using its definition

$$q_i = f(.) \ \Pi_j \ p_j^{(\lambda_C - \lambda_0) s_j} \ \Pi_k \ p_k^{\lambda_0 s_k}$$

where now $k = 1,2,\ldots,n$. Considering $\sum_j \ p_j^{s_j/s_C} = \bar{p}_C$, where $\sum_j s_j = s_C$ for all j in C, and introducing

a) $$(\lambda_C - \lambda_0)\, s_C = \lambda'_C \qquad (2)$$

then what is left on c_{ii} after the "contribution" to set up $\bar{p}$ and $\bar{p}_C$, will be

b) $$c'_{ii} = c_{ii} - (\lambda_C - \lambda_0)\, s_i - \lambda_0\, s_i = c_{ii} - \lambda_C\, s_i$$

The demand equation for good i becomes

$$q_i = f(.)\, p_i^{c'_{ii}}\, \bar{p}_C^{\lambda'_C}\, \bar{p}^{\lambda_0}$$

and taking into account the homogeneity condition

$$c'_{ii} + \lambda'_C + \lambda_0 = 0$$

from which

$$c'_{ii} = -\lambda'_C - \lambda_0 \qquad (3)$$

we are led to the function

$$q_i = f(.) \left(\frac{p_i}{\bar{p}_C}\right)^{-\lambda'_C} \left(\frac{p_i}{\bar{p}}\right)^{-\lambda_0}$$

which gives evidence to the common practice to get homogeneity of degree 0 by means of relative prices.

The present procedure for defining the demand equation can be extended to goods belonging to subgroups and limited to goods not having close relation to any other one. In the first case, we distinguish three λ's. λ_0 and λ_C as before and a specific λ, let us to say

λ_A, where A is a subgroup of C. In this case we still have $\bar{p}$ and $\bar{p}_C$; furthermore, $\bar{p}_A$, the price index of the subgroup A commodities, is computed. Then it is easy to work out the equation

$$q_i = f(.) \left(\frac{p_i}{\bar{p}_A}\right)^{-\lambda'_A} \left(\frac{p_i}{\bar{p}_C}\right)^{-\lambda'_C} \left(\frac{p_i}{\bar{p}}\right)^{-\lambda_O}$$

In the second case, when the good has not any peculiar relation with the others in the consumer's budget, we are led to

$$q_i = f(.) \left(\frac{p_i}{\bar{p}}\right)^{-\lambda_O}$$

Considering income-compensated elasticities, we need to figure out the c_{ij}'s from the estimated λ's. The cross-price elasticity with respect to a good in $\bar{C}$ is

$$c_{ij} = \lambda_O \, s_j$$

which means that the effects on good i of changes in the prices of any goods not in its group are proportional to their budget shares. On the other hand each good affects goods not in its own group by the same amount which is proportional to its budget share.

Coming into the group, let it be C, the cross-price elasticity with respect to good j price is given by

$$c_{ij} = \lambda_C \, s_j$$

Working out λ_C from λ'_C, λ_0 and s_C, we get

$$c_{ij} = (\lambda'_C \frac{1}{s_C} + \lambda_0) s_j$$

Following the above procedure, we can find formula for cross-price elasticity within and between subgroups when they are considered.

The own price elasticity is not so strictly related to the budget share as the cross ones; this is not surprisingly since the assumptions on λ's do not involve the main diagonal of the Slutsky conditions matrix. In the case of no subgroups, from (3) we have, e.g.,

$$c_{ii} = c'_{ii} + \lambda_C s_i = -\lambda_0 - \lambda'_C + \lambda_C s_i =$$

$$= -\lambda'_C (1 - \frac{s_i}{s_C}) - \lambda_0 (1 - s_i)$$

If a good has no special connection with each other or with other groups, given the value of λ_0, all the cross elasticities are proportionally determinated through the budget shares.

2. Estimation for Italy

The new series on national account after the introduction of SEC scheme supplies yearly data on private consumption expenditure (PCE) from 1970 up to now (1980) of 40 goods covering all consumption and services.

PCE data are available in both current and constant prices; from the ratio, prices indexes are computed (implicit deflators).

The family disposable income has been computed from national account figures and the long-run average ratio of consumption to income has been assumed equal to .76; this is an average of results obtained on estimating aggregate consumption function.

Other exogenous informations used are the income elasticities; they were determined from cross-section family-survey data for Italy produced by ISTAT. These data arranged in 40 items, corresponding to the ones of national account, and in 19 expenditure classes represent the information base for estimating Engel's elasticities. Two estimation methods have been used. The first one, described in /1/ assumes that as aggregate income increases by one percent, all families income increases by the same amount and the families recorded in a given expenditure class will move their own consumption pattern toward the one of the families in the next higher class. The second method, described in /5/ and applied in /4/, derives income elasticities from the estimation of concentration curves. The family-survey used is relative to 1978 and is composed by 31764 responses for 157 categories of consumer purchases. Despite the sample size, after the aggregations of the data into the 40 goods and services, corresponding to the national account scheme, for some items the recording is still rare so that many spending classes are empty. For those categories, both methods can exibith clear biases but with different intensity. The comparison of the two methods estimates has given the opportunity for some expost revisions.

A prior information on modelling this demand equations system is the group selection. Here, the chosen grouping is the following.

1 Food

Sub 1	Bread and cereal Fruits and vegetables Potatoes
Sub 2	Meat Fish Dairy
Sub 3	Butter, margarine, oil Sugar Caffes, tea, cocoa Other food

2	Beverages and tobacco	
		Soft drinks
		Alcoholic drinks
		Tobacco
2	Clothing	
		Clothing and repairs
		Shoes and repairs
4	Housing	
		Housing rents
		Fuel and electricity
5	Durables	
		Furniture
		Textiles
		Household appliances
		Glasswork and pottery
		Non durables and services
		Radio and television sets
6	Health	
		Medicine
		Medical appliances
		Medical care
		Hospital care
7	Transportation	
		Means of transportation
		Users cost of transportation
		Transport services
8	Culture	
		Newspapers and books
		Education
		Entertainment and recreation
9	Other goods	
		Personal toilet articles
		Hotels, Cafes, Restaurants

10 Services

Domestic servants
Communication
Other goods
Financial services and Insurances
Other services

The estimation of the parameters is done iteratively. First of all, the income elasticity is used to get a first guess on the income coefficient, b_1. Then, a λ_o is chosen (and used in all groups); λ_o is not altered in the estimation procedure and a scanning can be done looking at the fitting as a choice criterium. Since the assumption on Slutsky symetric condition, the estimation is done simultaneously for each group. First, within each group λ's are set equal to zero and the f(.) is regressed; then, after a Taylor expantion with respect to the parameter of the g(.), a regression is applied to estimate λ's. Coming back to f(.), the iterative procedure is carried on as long as the sum of squared errors is falling faster than a given percentage.

Preliminary Results

The estimation relies upon three exogeneous informations. The first one is the set of income elasticities which are used to compute the income coefficients in f(.)'s. Then, the b_1 values are fixed and the constant-price adding-up turns out to be matched; on forecasting, this property is preserved attaching a "spreader" /2,7/. The second information is the grouping which is not strictly a priori determined. The third information is λ_o which initializes the iterative procedure; it remains constant and implicitely determines price elasticities of the goods in $\bar{C}$.

If the first information, the income elasticities, is strictly exogenous, the others belongs to the art of estimating this kind of demand equations system. Different grouping can be considered and the use of different λ_o's is in order.

Here, four λ values have been checked. For each λ the sum of squared residuals in each group has been computed and the total for all the sectors. The data are shown in the Table 1 where a * marks the lowest entry in each row.

The choice of λ is usually a compromise; the value for which we get the highest frequency of the minimum sum of squared residuals is chosen.

Table 1

λ_o	.1	.3	.5	.7
Food	116*	143	200	285
Beverages	20	17	16*	18
Clothing	40	39*	39	40
Housing	24*	33	73	143
Durables	130*	147	168	192
Health	49	45	42	41*
Transportation	211*	229	254	185
Culture	56*	60	65	72
Other goods	42*	48	58	71
Services	148*	156	165	176
T O T A L	835	917	1,080	1,323

This is the case of λ = .1.

The summary of the estimate is shown in Table 2. The own-prince elasticities are all negative. The food group and subgroups commodities present complementarity as well as housing, health and education.

The preliminary results are slightly gratifying, but a closer analysis of the grouping and a refinement of the data give room to further research

Table 2a.

food

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
1	2	pane e cereali	0.114	0.0	-0.034	-0.007	-.001	0.004	0.8	0.03
2	3	carne	0.471	1.3	-0.032	-0.018	-.025	0.010	0.8	-0.58
3	3	pesce	0.004	0.4	-0.010	-0.002	-.003	0.001	2.2	0.22
4	3	latte, formaggi	0.230	1.0	-0.017	-0.007	-.010	0.004	1.1	0.03
5	4	oli e grassi	0.752	-1.1	-0.016	-0.004	-.009	0.002	2.0	0.19
6	2	frutta	0.171	0.2	-0.035	-0.011	-.002	0.006	2.1	0.35
7	2	patate	0.138	0.6	-0.033	-0.001	-.000	0.000	4.6	0.46
8	4	zucchero	0.200	1.4	-0.009	-0.001	-.002	0.001	1.5	-0.14
9	4	caffe, te, cacao	0.216	0.4	-0.010	-0.001	-.003	0.001	2.2	-0.07
10	4	altri generi alimentari	0.662	-1.1	-0.010	-0.001	-.003	0.001	1.9	-0.15

beverages & tobac

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
11		bevande analcoliche	-0.080	1.8	-0.101	0.000		0.000	3.0	0.13
12		bevande alcoliche	0.759	-2.0	-0.098	0.003		0.003	1.4	-0.14
13		tabacco	0.791	2.8	-0.098	0.003		0.002	3.2	0.42

clothing

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
14		vestiario, comprese riparazi	1.391	-0.5	-0.291	0.834		0.007	1.2	0.21
15		calzature, comprese riparazi	1.008	2.3	-0.925	0.200		0.002	2.3	0.15

housing

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
16		affitti per abitazioni	0.355	1.1	-0.064	-0.073		0.010	0.5	0.41
17		combustibili ed energia elet	0.943	2.0	-0.014	-0.023		0.003	1.6	0.12

Table 2b.

durbles

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	aape	rho
18		mobili	2.985	-0.7	-0.223	0.029		0.002	3.2	0.49
19		biancheria e altri articoli	1.809	-0.4	-0.237	0.015		0.001	3.3	0.45
20		elettrodomestici	1.978	0.9	-0.233	0.019		0.001	3.0	0.22
21		cristallerie, vasellame	1.895	2.2	-0.241	0.011		0.001	2.3	-0.07
23		servizi domestici	1.327	3.3	-0.210	0.042		0.002	5.3	0.33
32		apparecchi radio, tv, ecc	1.155	3.8	-0.206	0.046		0.003	3.5	0.05

health

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	aape	rho
24		medicinali e farmaceutici	0.504	4.8	-0.033	-0.034		0.001	3.2	0.30
25		apparecchi e materiale terap	2.057	2.9	-0.001	-0.002		0.000	5.1	0.42
26		servizi di medici, infermier	1.399	3.2	-0.049	-0.050		0.002	5.2	0.50
27		cure in ospedali e clinche	1.741	-4.3	-0.010	-0.011		0.000	3.4	0.18

transportation

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	aape	rho
28		acquisto di mezzi di traspor	3.423	-0.1	-0.429	0.121		0.002	5.2	0.13
29		spese di esercizio dei mezzi	1.346	1.4	-0.288	0.262		0.005	2.0	0.26
30		acquisto di servizi de trasp	1.062	0.5	-0.473	0.077		0.002	3.2	0.33

culture

sector	subgroup	commodity	income elasticit	time in % of last yr.	own	price elasticities group	subgroup	general	aape	rho
33		libri giornali e perodici	1.702	-0.2	-0.028	-0.029		0.001	5.8	0.60
34		serizi per l7istruzione	1.805	-1.5	-0.007	-0.008		0.000	6.1	0.54
35		spettacoli, servizi ricreati	1.983	-1.5	-0.059	-0.060		0.003	4.2	0.56

other goods

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	aape	rho
36		beni e servizi per l'igiene	1.083	1.4	-0.072	-0.030		0.003	2.5	0.39
38		altri beni no altrove calssi	1.262	-0.1	-0.065	-0.023		0.002	3.8	0.07

Table 2c.

services

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	aape	rho
22		articoli non durevoli e serv	0.928	-3.3	-0.667	0.037		0.001	3.1	0.23
31		communicazioni	1.638	0.6	-0.644	0.060		0.001	5.6	0.33
37		spese relative agli alberghi	1.550	0.6	-0.246	0.458		0.007	2.8	0.42
39		servizi finanziari e assicur	2.127	-0.3	-0.686	0.018		0.000	5.7	0.43
40		altri servizi non altrove cl	0.836	1.3	-0.663	0.041		0.001	2.3	0.26

REFERENCES

1 ALMON, C. (1966), The American Economy to 1975, Harper and Row, New York

2 ALMON, C. (1979), "A system of Consumption Functions and its Estimation for Belgium", Southern Economic Journal

3 BARTEN, A.P. (1977), "The System of Consumer Demand Functions Approach: A Review", Econometrica, vol. 45

4 GRASSINI, L. (1981), "On the Estimation of Engel Ela- sticities to Italian Data", paper presented at the Task Force Meeting on "Input-Output Modelling", IIASA, Laxenburg (Wien)

5 KAKWANI, N.C. (1976), "Efficient Estimation of the Lorenz Curve and Associated Inequality Measures from Grouped Data", Econometrica, vol. 44

6 PHLIPS, L. (1974), Applied Consumption Analysis, North Holland Pub. Co., Amsterdam

7 YOUNG, S.L., ALMON, C. (1978), "An Input-Output Fore- casting Model of the French Economy", Inforum Reasearch Report no. 20, University of Maryland, College Park, Maryland, USA.

THE CONSUMPTION BLOCK OF THE HUNGARIAN INFORUM MODEL

G. Kornai

The Hungarian model of the international INFORUM model system is being prepared in cooperation with the Wiener Institute für Internationale Wirtschaftsvergleiche and the Budapest Institute for Economic and Market Research.[2] In this paper I intend to enlarge upon the questions raised by the elaboration of the personal consumption block of the Hungarian model.

Should the task of elaborating the consumption block mean that a given frame of functions - namely Almon's demand model /Almon /2/ / - be filled with Hungarian data and estimated, I could rest satisfied as the estimated model is completed and practically feasible.

I think, however, that the questions to be answered are far more general, more fundamental. It is well-known that several specific

---

1. I am greatly indebted to Clopper Almon for letting me have the computer programs and to Andras Simon, who lent valuable advice to my work. For the errors that may occur in the paper I am to blame alone.

2. For the outline of the model see: Fink - Simon /1/

factors missing from the traditional demand models account for the changes in personal consumption in the Central-European socialist countries. It is also recognized that there does not exist a special Hungarian consumption model at present, and it is highly questionable that the demand functions elaborated for developed market economies are applicable to Hungary.

In the light of these questions I cannot be content with achievement as an economist. I do not believe that I succeeded in setting up a Hungarian consumption model on firm theoretical bases. Yet I do hope I have been able to contribute to the development of the analysis of Hungarian consumption, which I greatly owe to Clopper Almon's model.

As my knowledge is limited I cannot strive here to develop a complete consumption theory for the Hungarian / or a socialist/ economy. My aims are far more pragmatic. I confine myself to recalling some basic features of the Hungarian consumption recognized by others and to discussing how the unobservance of these factors limit the validity of the model to be presented.

The first part of my paper will cover these questions, to be followed by the presentation of the Almon model estimated on the basis of Hungarian data.

I. Consumer prices and demand[3]

Let me here deal with a single but fundamental question, that of the relationships between consumer prices and demand. Neither do I intend nor have I the means to argue whether the traditional demand functions can be applied to the US or Belgium.
I should merely like to emphasize why they cannot be applied to describe consumption in Hungary /and in the rest of the socialist countries/ satisfactorily.
Let us see the demand function of the total population for a single good. /see Fig. 1./

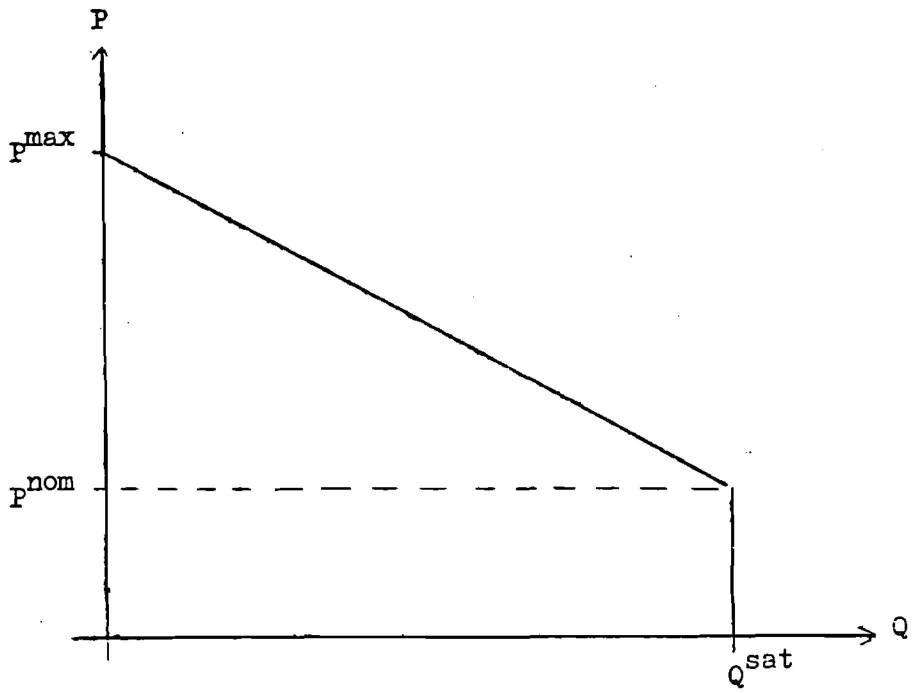


Figure 1

3. The notions to be touched upon below I have adopted - or rather picked from Janos Kornai's book Economics of Shortage /3/.

If the price of the commodity in question is zero, that is the consumer gets it gratis, demand appears on saturation level $/Q^{sat}/$.[4]

As the price of the commodity is raised in a certain interval demand is completely inelastic to the price. In this range prices are called nominal. The upper limit $/P^{nom}/$ to nominal prices appears when at least one consumer gives up the idea of purchasing. Raising the price further, demand will gradually drop until the withdrawal of the very last consumer. Demand above this P^{max} price is again inelastic.

While nominal prices in $/0, P^{nom}/$ interval play no role whatever in regulating consumption, prices in $/P^{nom}, P^{max}/$ interval effectively influence demand.

Consumers in the economic system of socialism have access to a part of goods gratis or at nominal prices.[5] Irrespective of the question whether supply can in fact saturate demand, the consumption of these goods cannot be described by the traditional demand functions.

Our attention should now be focussed on the $/P^{nom}, P^{max}/$ interval of effective prices. Let us consider the supply of the commodity under scrutiny be given. $/Q^{s}/$ at a single but any moment. In the traditional theory of demand functions a given market

---

4. It is easy to see that the demand for any commodity is finite.

5. In Hungary in 1979 some 20% of personal consumption can be considered such.

clearing price $/P^{mc}/$ belongs to a given demand.

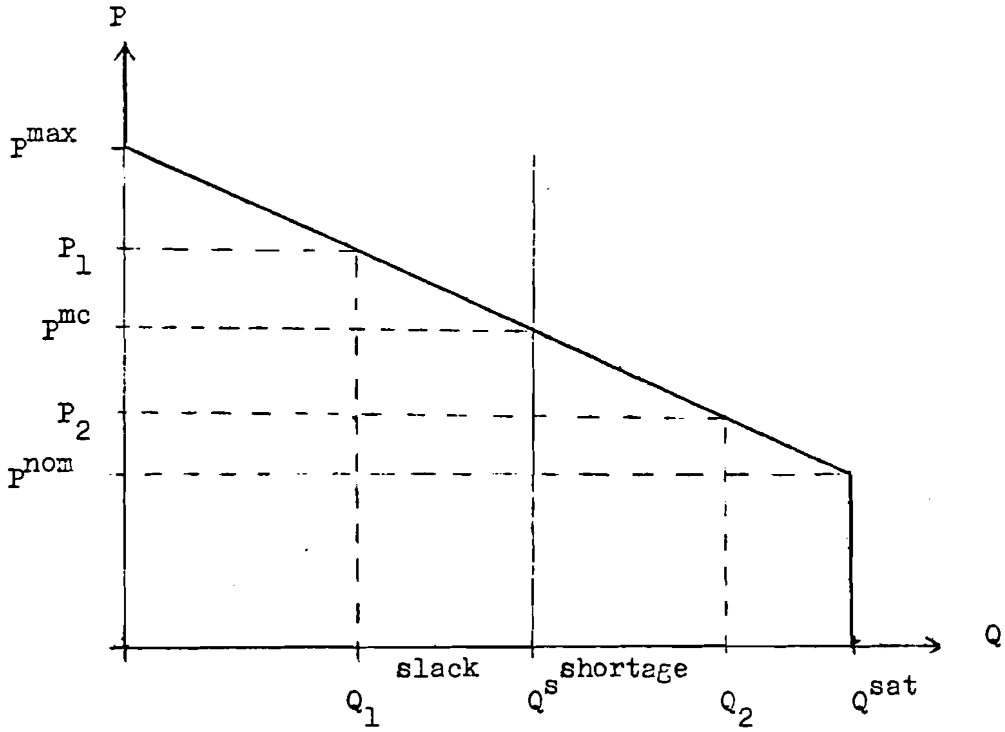


Figure 2

Let us suppose that, say, the price authority permanently diverts the price of a good from the P^{mc} market clearing price. If the price falls within the $P^{mc} \leq P_1 < P^{max}$ interval, the traditional demand models can be applied easily: the price effectively restricts demand $/Q_1/$ and a certain slack of supply develops. The presence of the slack, however, does not influence the attitude of the consumer as anyone who is willing to purchase the commodity can do so easily.

If, on the other hand, the price /P_2/ falls within the $P^{nom} < P_2 < P^{mc}$ interval, it results in a Q_2 demand and in a Q_2--Q^S shortage. Obviously, the price is not able to clear the market alone. Some other form of distribution is necessary to select the customers: rationing commodities and/or queuing for commodities. Neither rationing nor queuing are price-like factors, thus traditional demand models cannot be applied to this price interval either.[6.]

To sum up: traditional demand models fail to describe the consumption of commodities marketed at a price lower than P^{mc}.

II. Preparation of the data base

Personal consumption in the Hungarian and the other socialist economics does not have a theoretically founded econometric model. It is due to the fact that until quite recently there did not exist a specific theory of demand for a "shortage economy". The consumption models estimated so far were nothing but automatic adaptions of some or another test-proven western model and their estimation on Hungarian data. Though in a way my workconstitutes no exception[7.] I did strive throughout the adaptation of the model not to

6. Though an exact guess is hardly possible to give, it is probable that at least half of personal consumption falls into this price interval today.
7. I rather changed the Hungarian data to fit Almon's model than change the model to fit the Hungarian reality.

forget about shortages and other non-price factors in the Hungarian consumption. For this reason I had to give up having an explanation to the whole range of goods consumed. The loss in the range of goods is probably compensated for by a stronger theoretical backing in the remaining relationships of the model.

The Hungarian INFORUM model is built on a 79-sector input-output table. The consumption vector of final consumption comprises the personal and government consumption. As time-series on personal consumption is not broken down by I-O sectors but by consumption categories a bridge-matrix serves to link the categories of goods to the sectors of the I-O table. Naturally, we have to assume that the share of I-O sectors within each consumption category is constant in time.

Time series for 196 groups of commodities where available for the period 196o-1979 covering the whole consumption. I added to the list of groups personal expenditures on housing investments. This group is treated as investment in the statistics, but it is to be modelled as a consumption good.

The consumption data issued by the Central Statistical Office are generally calculated from three sources. A detailed, complete observation of retail trade turnover yield the purchased consumption of a certain commodity. If necessary, self-consumption estimated on the basis of representative household statistical surveys is added, together with the so-called social fringes in kind provided by the government budget and the enterprises.

For the estimation of the Almon model only the price - dependent consumption should be considered for the reasons explained earlier. However, the data base available allowed to single out only a part of the social fringes: free medical, educational, cultural /sports, holidays, etc./ and social /creche, nursery school, etc./ allowances. These items amounted to 10,33 % of total consumption in 1979. We have no definite clue how to recognize commodities marketed at nominal prices making up a further considerable share of social fringes. Therefore I had to fall back upon my own consumer's experience and attitude to decide item by item which price is nominal or effective. I found the following kinds of goods available at nominal prices: rent, depreciation and maintenance costs of state-owned flats, pharmaceutical products, passanger and cargo transport, telecommunication services and purchased creche and nursery school care. I excluded these items amounting to 7,25 % of total consumption from the data base of the model. Yet there remained some /unrecognizable/ social fringes in the data /some consumption at heavily subsidized prices/.

The expenditure items thus excluded from the data base of personal consumption were added to government consumption.

In 1960 self-consumption amounted to nearly one-third of total consumption while in 1979 this share was down to 10 %. Self-consumption was almost exclusively restricted to two major groups of goods: food and alcoholic drinks. Regarding that statistical practice evaluates this consumption at retail prices I do not think it causes any great distortion if it remains both part of income and total consumption.[8.] By excluding, however the goods having nominal prices, I also got rid of some fundamental relationships in the model. It is namely evident that life requires a wide range of commodities and thus none of the elements of the consumption structure can be disregarded arbitrarily. For example, excluding from the model the function explaining the "consumption" of state-owned flats at nominal prices /as explained by a mixture of rationing and queuing distribution rather than by <u>price</u> and <u>income</u> effects/ the validity of several consumption functions of closely related inputs /furniture, household textile, labour saving devices, etc/ becomes questionable. But housing conditions also influence "more remote" elements of the consumption structure as the level of food consumption and or entertainment. As a result of the above "purging" <u>the model contains only commodities at effective prices</u>. Yet - as mentioned earlier - a traditional demand model cannot be directly applied even to this data base. Goods marketed at lower and

---

8. It is also confirmed by the fact that the majority of these goods /e.g. eggs, fruits, vegetables, etc./ can be easily turned into cash at the market places.

higher prices than the /theoretical/ market clearing price should have been separated although we have no information about the market clearing price of these commodities. I had to put up with selecting the commodities on the market of which shortages were apparent according to my experience during the period 1960-79.

In examining the list of consumption categories item by item I did not find any category which was not characterized by a quantitative or qualitative, temporary or chronic shortage in the past 2o years. I also realized that the some 18o aggregate categories quite well "conceal" the individual shortages of tens of thousands of consumer's goods. For example, the shortage of salami is not visible in the category of meat products, and the shortage of stereo turntables in the category of record players. In fact, most of the forced substitution caused by shortages takes place among goods closest to the one sought in vain. The customer buys frankfurter instead of salami if he wishes to eat cold meat, or a mono record player instead of a stereo one if he wishes to listen to records.

To get around the problems generated by forced subsitution is at hand: commodity categories have to be merged until the majority of forced substitution falls into the same category. After the reduction I finally arrived at 38 categories of commodities.

Another part of forced substitution is carried out between goods excluded from and those retained in the model. If, for instance, a patient gets no free medical treatment /excluded commodity/, he is forced to <u>purchase</u> medical care /retained commodity/. If a consumer does not get a flat at a nominal price /excluded commodity/, he is forced to get involved in costly house construction /retained commodity/. The Almon model provides for the handling of such problems by adding any variable to the right-hand side of the equations. In the given case the model can include the changes in free medical care, state housing projects, etc.

Shortage, however, does not only cause forced substitution but <u>forced spending</u>[9] and <u>forced saving</u> as well. The consumer argues: I couldn't get a color TV /and as I don't want a black and white one/ I'll take part in an expensive package tour, or put the money aside for the case I can buy a color TV set next week, month or year. A part of forced saving - and the closely related queuing[lo] - is hidden in the data base of the model broken down by year, as it is seldom necessary to queue or wait for more than a year for a good. The queues stretching from one year to the other or spanning several years have to be taken

---

9. Forced subs cution and forced spending are kin categories, but while in the former case initial demand is more or less met forced spending leaves a void behind it.

lo. In theory, all queuing can be considered forced saving of varying extent, depending on the size of the queue.

into account. Such queues stand for state-owned housing, telephones and new cars. In the data base I retained only the latter. To build it in the model, special indices, e.g. those referring to the intensity of the shortage, had to be used.

The model - due to aggregation through both time and commodity categories - thus conceals such elementary characteristics of Hungarian consumption as a host of queues of varying size, the propensity of consumers to wait, e.t.c. Neglecting these phenomena means the same as assuming that the normal /average, ordinary/ intensity of shortages only slowly changed in the past 2o years, so variations in consumers' demand are basically defined by relative prices and real income.

Almon's model is most suitable for finding a wide range of price-dependent /voluntary/ substitutions and complementarities in consumption. Even this ability of the functions was limited here by the purging and aggregation of the data base.

III. The estimation of the Almon-model[11]

To estimate the model I used the computer program worked out by Clopper Almon. This program does not estimate income coefficients directly, it calculates them on the basis of externally given income elasticities. For the case of Hungary - not

---

11. The reader, I presume, is familiar with Almon's paper /2/.

having any cross section estimation of income elasticities of the 38 commodity categories to be modelled here - I had to take the following /and rather questionable/ course.

I picked first the income elasticity coefficients of a Deaton-Muellbauer model estimated for Hungary by Muszély /4/[12]. About half of the 33 commodity categories selected by Muszély coincide with those of mine. The coefficients of these categories could be applied to my model directly. Ten out of the 21 income elasticities still missing I could compute from a /not very recent/ study of the Central Statistical Office on family income /5/[13], analizing the consumption in 55 commodity sectors. This "computation" was no more, than simply marking the given income elasticities up or down, using the categories coinciding with those of Muszély's as a numeraire. For the remaining 11 commodities no cross section elasticities could be found. I had to pick the starting values of these coefficients from different consumption functions based on time series. /Hulyák: /6/, Hulyák - Losonczy: /7/, Éltető: /8/ / Then I scaled them to fit into the range of the cross section parameters.

Income was calculated to be the sum of consumption from the 38 categories plus the personal savings obtained from the national account statistics. The

---

12. This estimation was based on the family survey data of 8500 households from 1976 to 1978.

13. On the 1969 data of 4000 families Engel curves were estimated.

income elasticities then were converted to linear income coefficients and scaled by the program. The long run average ratio of consumption to income turned out to be .96 . This figure seems to be rather high, especially because our narrowed definition of income and consumption should lower this figure.

The grouping which was selected is shown in the Appendix. Subgroups were selected only within the FOOD group. Then I made a series of rough estimations to find out the proper value of λ . The equations have been estimated with λ_o = .3, .5 and .7. To my surprise, the results of the λ_o = .3 case were already acceptable, since all own-price elasticities were negative, and complementarity appeared only within the MOTORCARS AND OPERATION group, as presumed. Inspite of this sudden success the improvement of the functions - both in terms of economic and statistical plausibility - should follow.

For some sectors - as I mentioned before - special explanatory variables had to be used to capture the "shortage" effects not incorporated into the original model. For reasons of programming convenience, these "special" variables - if used - replaced the first difference of income term in the linear part of the respective equations. The following time series were applied as special explanatory variables:

- A time trend beginning in 1970 is assumed to represent in some cases changes in the quality of supply after the introduction of the so called "new economic management" in 1968, or in other cases just changes in tastes. Where to use this trend variable was decided simply on empirical basis. It was used in the equations of Meat Products, Other Meat and Fish Products, Spirits, Misc. Textiles, Other Services and of Other Products.

- Instalment loans are limited in Hungary and they are given to get rid of surplus supply in a market with unflexible prices. The variable of instalment loans is a semi-price variable playing the role of discounts.

- Supply constraint variables were applied to represent the effects of limited supply on consumption. The time series of non-alcoholic beverages production and imports of personal motorcars were used in the corresponding equations.

- The data on construction of state owned apartments serves as a proxy to represent the substitution between housing at nominal and effective prices.

- The stock of personal motorcars is assumed to determine the demand for spare parts and repair services, as well as car fuel.

Besides these 14 special variables were used successfully, I made some other attempts to improve the plausibility of some equations. E.G. the time series of social fringes for education and healthcare were believed to represent the substitution between free and purchased consumption of these services, but complementarity occured. For the case of cars I tried a different approach. In the cars' market a semi-legal secondhand market has developed, where the reallocation of used cars can take place on the basis of consumers's choices. It is of no question, that there is a market clearing second hand price. That is why a situation can arise when second-hand prices are higher than the price of a new car. Therefore in the demand function an estimated second-hand price was tested instead of the centrally fixed price of new cars. This test was made without satisfactory results.

Applying special variables did alter the range of the estimated within-group elasticities, but did not effect the intergroup relations. This way it can be shown how the supply of non-alcoholic beverages modifies the consumption /and price elasticities/ of tobacco, but it cannot be detected, how many more shoes are sold in lack of soft drinks.

The estimation results are not less satisfactory than those of the model for Belgium /see the Appendix /. Own-price elasticities range from -.23o for Car Fuel to -1.449 for Underwear and Socks. The within-group range of own-price elasticities is reassuring about the appropriateness of the grouping, since the values significantly differ from $-\lambda_0$ in most of the groups. Only the commodities in the groups of STIMULANTS and DURABLES occur to be independent.

The results show that price and income are doing an important part of the explaining. However, the high average absolute percentage errors /AAPE/ of some sectors /e.g. Household Machines, Motorcars and Car Fuel/, as well as the high autocorrelation of residuals /RHO/ in the majority of equations suggests that the improvement of the model would be worth trying.

A possible - and maybe fruitful - way of modelling the Hungarian consumption with Almon's system of functions would be to work out a method suitable for the estimation of the unobservable /theoretical/ "market clearing" prices. These proxy-prices should incorporate the cost of queuing, the "price" of rationing, and the other shortage phenomena. Naturally, the income is to be redefined to express the propensity of consumers to wait, to substitute and to save. Any idea concerning these problems is welcome.

References

/1/ ALMON, C.: A System of Consumption Functions and its Estimation for Belgium. Southerns Economic Journal, July 1979.

/2/ FINK, G. - SIMON A: Outline of an Econometric Input-Output Model of the Hungarian Economy, WIIW, 1981.

/3/ KORNAI, J.: Economics of Shortage. North-Holland, 1981.

/4/ MUSZELY, Gy.: A fogyasztás társadalmi rétegek szerinti differenciáltságának vizsgálata a Deaton-Muelbauer-féle keresleti modellel. /Analysis of consumption by social strata with the Deaton-Muellbauer model./ SZÁMKI, Budapest,1980 october

/5/ ----------- Fogyasztási cikkek és szolgáltatások jövedelemrugalmassága /Income elasticities of consumers' goods and services./ KSH, Budapest, 1972, april

/6/ HULYÁK, K.: A lakosság fogyasztásának vizsgálata dinamikus keresleti függvényekkel. /A dynamic demand function analysis of personal consumption/. Statisztikai Szemle, december 1980.

/7/ HULYÁK, K.- LOSONCZY, I.: Keresleti modellek számszerüsitése idősoros adatok alapján /Estimation of demand models on time series data/. Szigma, 1978, No. 3-4

/8/ ÉLTETŐ, Ö.: A fogyasztói kiadások lineáris modelljének néhány tulajdonsága a hazai tapasztalatok tükrében. /Some properties of the LES model in the view of Hungarian experiences./ Paper presented at the 7th Hungarian Conference on Operation Research, Pécs, 1977.

/9/ SIMON, A.: An Econometric Study on Personal Consumption and Savings. Matekon, Summer 1979/Vol. XV, No.4.

Appendix: Elasticities and Fits for $\lambda_s = .3$

Sector	Subgroup	Commodity	Income Elasticity	Time in % of Last Year	Price Elasticities Own	Group	Subgroup	General	AAPE	RHO
		FOOD								
1	2	PORK	.559	.03	-.434	.023	.091	.014	2.7	.37
2	2	Chicken-Meat	.285	.55	-.484	.010	.041	.006	2.5	.29
3	2	Meat Products	.724	.23	-.477	.012	.048	.007	3.4	.41
4	2	Other Meat & Fish Prod	.428	.61	-.492	.008	.033	.005	3.3	.32
5	3	Milk	.581	-.37	-.396	.013	.019	.008	4.3	.66
6	3	Eggs	.548	.60	-.402	.009	.013	.005	3.1	.59
7	3	Fats & Vegetable Oils	.274	-.95	-.392	.016	.023	.010	1.8	.31
8	3	Cereals	.055	.35	-.384	.022	.031	.013	1.9	.70
9	3	Potatoe	.077	-.94	-.406	.006	.009	.004	2.9	.59
10	3	Sugar	.351	.24	-.405	.007	.010	.004	3.4	.08
12	3	Vegetables & Products	.614	.21	-.395	.014	.020	.009	3.1	.51
13	3	Fruits & Products	.756	.09	-.391	.017	.024	.010	3.8	.33
14	3	Spices & Other Food	.570	2.10	-.403	.008	.012	.005	7.4	.31
		STIMULANTS								
16		Coffee & Tea	1.425	.77	-.350	.025		.005	3.6	.55
17		Mon-Alcoholic Drinks	1.283	-2.28	-.364	.011		.002	6.4	.28
21		Tobacco	.910	.47	-.341	.034		.007	2.6	.43
11		Other Sweets	1.085	.36	-.352	.023		.005	3.7	.49
		ALCOHOLIC BEVERAGES								
18		Wines	1,250	-.30	-.748	.277		.013	4.3	.30
19		Beers	1.338	-.77	-.794	.231		.011	3.0	.39
20		Spirits	1.535	-1.10	-.773	.252		.012	4.3	.36

Appendix: Elasticities and Fits for $\lambda = 3$ Continued

Sector Subgroup	Commodity	Income Elasticity	Time in % of Last Year	Price Elasticities Own	Group	Subgroup	General	AAPE	RHO
			DURAILES						
38	Real Estate Invest	.965	.45	-.337	.064		.017	8.2	.21
27	Furniture	1.403	-.40	-.374	.027		.007	3.3	.32
28	Machines, Household	1.042	2.55	-.382	.019		.005	14.1	.75
34	Cultural Durables	.932	-1.08	-.388	.013		.003	8.6	.26
36	Other Products & Goods	1.031	.35	-.387	.014		.004	5.0	.40
			MOTORCAIS & OPERATION						
31	Motorcars	2.456	-1.99	-.261	-.045		.006	16.9	.47
32	Car Repair & Spareparts	1.491	-2.68	-.232	-.016		.002	8.4	.22
33	Car Fuel	1.491	-1.84	-.230	-.014		.002	14.5	.64
			CLOTHING						
22	Textile, Misc.	1.107	-1.60	-1.328	.346		.009	4.3	.69
23	Upper Wear	.746	.81	-1.086	.588		.016	4.0	.71
24	Underwear & Socks	.998	-1.25	-1.449	.225		.006	4.3	.70
25	Footwear	.680	.73	-1.421	.253		.007	3.9	.49
			MISC. GOODS & SERVICES						
15	Catering /price Gap/	2.291	-1.06	-.704	.031		.004	5.0	.67
26	Energy	.767	.41	-.650	.085		.010	2.8	.35
29	Misc. Household Goods	1.359	-.77	-.591	.144		.017	2.9	.46
30	Health & Phisical Cultu	1.546	-.95	-.684	.051		.006	2.8	.58
35	Other Cultural Goods	1.995	-1.55	-.618	.117		.014	3.7	.55
37	Other Services	1.590	-1.53	-.668	.067		.008	4.5	.64

PART III

PROBLEMS CONNECTED WITH INVESTMENT FUNCTIONS AND ESTIMATION

THE ESTIMATION OF THE INVESTMENT FUNCTIONS FOR THE ITALIAN MODEL

M. Ciaschini

This is a report on the estimation of the investment functions for the construction of a dynamic econometric interindustry forecasting model of the Italian economy [(1)] The model is built for the purpose of medium or long term analysis. The basic idea is to forecast, year-by-year for ten years ahead, industry output, employment, investment, and other final demands within the framework of an input-output table. The model lacks the price side and income side of the economy; they are research projects for the future.

The model is part of the INFORUM (INterindustry FORecasting model University of Maryland) international input output forecasting system. The Italian model will be linked via a central trade model to similar model.to similar models of eigh industrial countries.
The general structure of the model is explained in Section I. The data for the estimation of the investment functions is then described in Section II. Section III explains the investment equations used in the model, the results obtained in estimation and the simulation results with the estimated investment functions.

---

(1) INFORUM and INTIMO (Modello Italiano Interindustriale) are the organizations involved in constructing the model. All the necessary data were provided by INTIMO and the modeling techniques by the INFORUM staff.

I. STRUCTURE AND SOLUTION PROCEDURE

Any model is based on a description of the economy. Macro models rely on the summary descriptions offered by the tables of the na – tional accounts; input-output models rest on the expansion of the- se accounts to distinguish types of products and the users of each product. Figure 1 shows schematically the table used for the Ita – lian model. The output of the economy is divided into 44 branches or products. The sales of anyone of these branches in a particular year are shown across a row of the table in Figure 1 in the column corresponding to the buyer. There are a total of 113 of these buyer columns as follows:

Matrix Name	Number of Buyer Columns	Content
A	44	Intermediate sales: The sale of one pro duct to be used in making another, e.g. steel sold to auto makers.
B	23	The buyers are 23 sectors, aggregates of the 44, for which annual spending on ca pital investment, equipment and plant, is available. The sellers are 15 sectors other rows of B are all zero.
G	4	Four columns of government and non-gover nment spending are distinguished: (1) - Health (2) - Education (3) - Other (4) - Non Government Social Institution.
C	40	Consumption: The "buyers" are categories of consumption from the family-budgets in national accounts.
	1	Exports
	1	Imports
	1	Inventory change
total	114	

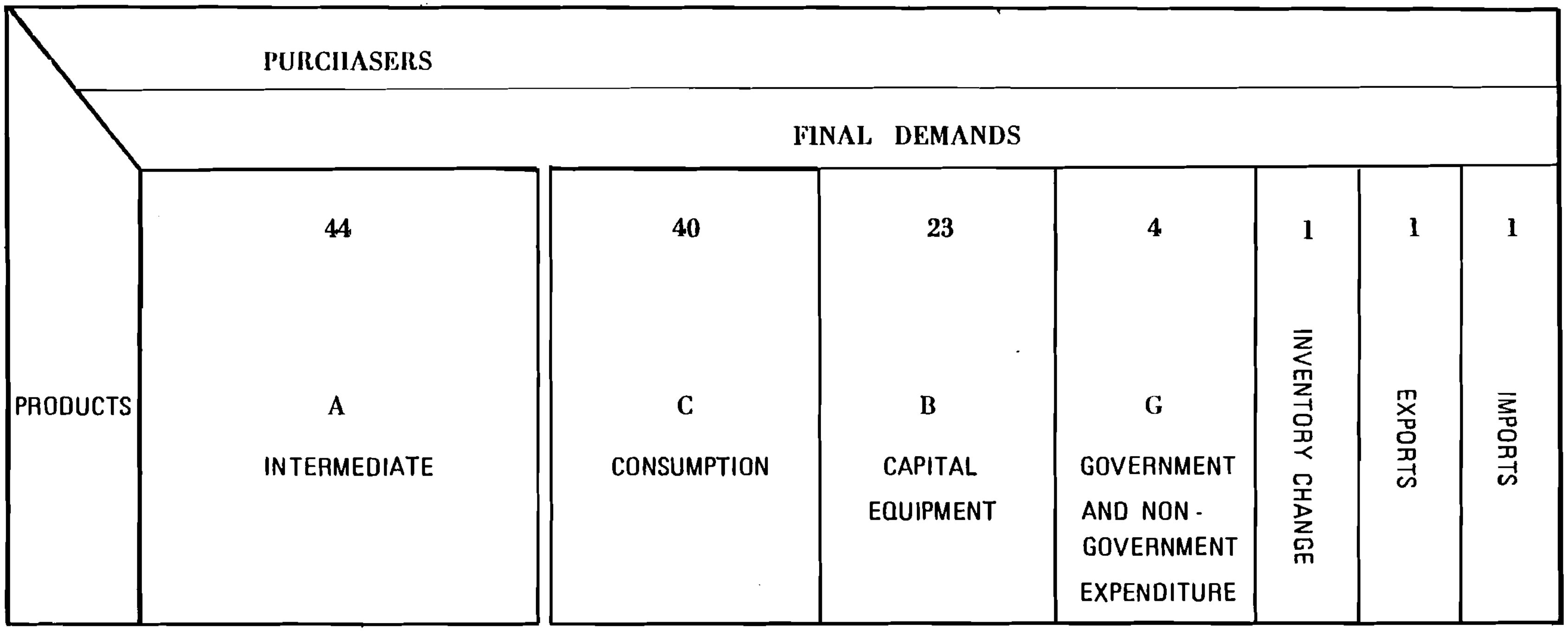


FIGURE 1. What the Italian Model Shows for Each Year.

The elements to the right of the double line in Figure 1 are known as final demands. The sum of all the final demands is the gross domestic product.

The forecasts consist of tables such as this, one for each year out ten years or so into the future. The basic logic begins with the forecast of consumption, exports, imports, and inventory change, product-by-product. Column totals for other final demands are forecast and distributed to industries according to their percentage structure in the base year. Then we take the vector sum of all the final demand columns, and calculate, by solution of linear equations, the outputs which would be necessary to yield these final demands, due account being taken of the requirements for intermediate goods. Thus, a final demand for clothes necessitates production not only of clothes but also of textiles, chemicals, electricity, coal, and so on. From outputs so obtained, employment may then be calculated.

In order to forecast the personal consumption portion of final demand however, we need to know personal disposable income. We start the process by assuming a future course for this variable. Logically, we may then revise it in either of two ways:

1) so that the resultant employment matches a projection of the labor force

2) so that the disposable income resulting from the wages, dividends, interest and rental payments, less the taxes, should be equal to the disposable income assumed in the consumption functions.

We use technique 1. All short-term macro-economic models use technique 2.

Obviously, Number 2 requires a complete system of wage, income, and tax equations to spell out the disposable income implied by the industry outputs and employment.

At present, this complete structure is available only for the U.S. model. Even for this model, however, the second technique is seldom used because it assumes no changes in tax policy. For forecasting ten years ahead, it seems safer to assume that taxes will be revised to give satisfactory employment levels, and that is what techniques 1 assumes. For one or two years ahead, we can use the disposable income forecasts from a macro model.

One further problem remains with the basic logic of solution: some of the final demands depend on the levels of output in the same year, namely investment, imports and inventory change. In the case of imports and inventory change, the dependence is substantial and direct in the sense the imports, say, of a product depend on the domestic demand of that same product. The import and inventory change equations are, therefore, simply included in the simultaneous solution of the equations for the outputs. This solution is accomplished by what is known as the Seidel process. In this process, initial guesses are made of the solution, then all of the demands for good 1 are computed and the guess for good 1's output revised to be equal to its demand. Then, with the revised value of good 1's output, the demand for good 2 is computed and its output revised, and so through the whole list of 44 and then back to good 1 and through the whole list again and again until the solution converges. (Three or four times through is usually sufficient because we can both start with good guesses and also take the sectors in a sequence which puts first the ones close to the consumer - thus clothes come before textiles and automobiles before steel). In this process, it is a simple matter to use the import and inventory equations each time the demand for a good is being calculated to say how much will be supplied by imports and how much required for inventory building.

A different approach must be used to account for the dependency of, say, investment by the apparel industry on the output of apparel, for investment by apparel effects not only apparel output but also output of machinery. Fortunately, this dependency is weak. We deal with it simply by calculating investment on the basis of our first guess of outputs, completing the Seidel solution of the simultaneous equations for outputs, then going back to the investment equation, recomputing investment, and then coming back through the Seidel computation. (Twice through this process is generall enough).

When the outputs for one year have been computed in this way, we can move on to the next, and then on to the next. Thus, we build up the sequence of tables such as one shown schematically in Figure 1.

II. DATA

The main data source is the I/O table for 1975 current prices, the family budgets, the current and constant prices output index from 1959-'80 compiled by ISTAT (Istituto Centrale di Statistica). The I/O table shows the sales of 44 branches to intermediate use by the same 44 to personal consumption, to investment, to government, to exports, and to inventory changes.

Offsetting imports are shown by branch. All the figures in the I/O table are in producer's prices. The family budgets are arranged in 40 consumption items corresponding to the 40 consumption sectors. For the estimation of investment equations we have the ISTAT series of the investment by selling and purchasing sector from 1970 to 1978, the purchasing sectors are 23 according to the N.A classification. The ISTAT input-output tables give only one column for gross capital formation. Because the investment decision is made by the purchasers of the investment goods, we wish to estimate investment by purchases and then to distribute it over the 44 sectors by using a capital flow matrix. The capital flow matrix (B) showing sales from each of the 44 producing sectors to each of the 23 equipment purchasing sector is available for the Italian economy 1975.

III. INVESTMENT FUNCTIONS

The estimation of sectoral investment functions for Italy is not a minor problem. On the one hand the available series are short and their profile rather flat and sparse, on the other hand sectoral investment is influenced in different measure by particular phenomena such as the change of technologies through replacement and the unused capacity. Given such a situation our study concentrated on the essentials of investment behaviour: the capital output ratio and the pattern of the lag between increases in outputs and the increase in capital stock they stimulate, leaving to a further investigation some programmed hypothesis on the sectoral investment functions. Investment data for Italy distinguish 23 investing industries and includes equipment and plant.

Total investment is the sum of replacement plus expansion investment. If replacement is proportional to the stock of capital, and stock is proportional to output, or better, to smoothed output, then replacement is proportional to the smoothed output, thus:

$$\text{Replacement}_t = r\,k\,\bar{Q}_t,$$

where k is the capital output ratio, r is the replacement rate, and $\bar{Q}_t$ is smoothed output. Expansion investment we take to be just the capital-output ratio times a distributed lag on changes in output, thus:

$$\text{Expansion}_t = K \sum_{i=0}^{n} w_i \,\Delta Q_{t-i}$$

where:

$$\sum_{i=0}^{n} w_i = 1.$$

Total investment, V, is therefore:

$$V = r\,k\,\bar{Q}_t + k \sum_{i=0}^{n} w_i \,\Delta Q_{t-i} \qquad (1)$$

or:

$$V = k\,[r\,\bar{Q}_t + \sum_{i=0}^{n} w_i\,\Delta Q_{t-i}] \qquad (2)$$

with $\Sigma\, w_i = 1.$

For smoothed output we took

$$\bar{Q}_t = .5Q_t + .3Q_{t-1} + .2Q_{t-2},$$

where Q_t is output of the investing industry in 1975 prices. To enforce the requirement that the w's sum to 1.0, we set:

$$w_2 = 1 - w_0 - w_1$$

Substituting this equation into (2) gave the actual regression equation for the three years lag (n=2).

$$V = k\,[r\,\bar{Q}_t + \Delta Q_{t-2}] + k\,w_0[\Delta\,Q_t - \Delta\,Q_{t-2}] + k\,w_1[\Delta\,Q_{t-1} - \Delta\,Q_{t-2}] \quad (3)$$

Only three parameters, k, kw_0, and kw_1, are estimated by the regression. For use in the forecasting model, kw_2 is calculated from:

$$kw_2 = k - kw_0 - kw_1$$

We emphasize that the main point of this equation is to obtain a reasonable estimate k for forecasting the investment responses to increases in output. It would be easy to improve the fit to historical data by adding, for example, an intercept or a lagged value of the dependent variable.

We tried the estimation of the three an four years lag (n=2, n=3). Given a capital replacement rate equal to 10 percent we obtained the results shown in table 1 and II. The goodness of the fit in terms of the avera – ge absolute percentage error (aape) in slightly better in the four-years-lag estimation but this is obtained with an higher percentage of w_i's bee ing negative. For some investment equation the fit in terms of AAPE seems to be reasonable: in seven cases AAPE is smaller than 10%. In eight cases AAPE is equal to 10-13%. In four cases equal to 18-20% and in the remaining for equal to 30%.

All capital output ratios appear sensible in terms of the standard error of the coefficient; while at least in one third of the cases the w_i's show a standard too low.

Other estimation experiments were performed with different assumptions on the capital replacement rate for the three and four years lag.

Tabel III show how the fit of each investment function varies, in terms of AAPE, when varying the capital replacement rate. In general we can say that the fit investment functions with aape not greater than 13% in the previous estimation (r=10% becomes better when varying the capital replacement rate in the range from 2,25% to 25%. In the other cases aape varies much slightly even for substantial variations in r. The fitting of some investment equation is shown in tab IV. The simulation of the entire model was performed using the coefficients shown in tab. V for the sectoral investment functions.

An aggregated summary of the results obtained from a preliminary simulation experiments is shown in table IV. Even is not refined the investment functions seem to work reasonably in the simulation. The last kown data for investment in the 78 figure. Next values for the rate of change of investment show a 12.45 per cent for 79 a 9.40 per cent for 80 abd a decrease of 1.88 per cent for 81.

REFERENCE

C. ALMON - D. NYHUS:
The INFORUM International Systems of Input Output Models and Bilateral Flows - UNIVERSITY OF MARYLAND, 1977.

TABLE I : RESULTS OF THE ESTIMATION OF THE SECTORAL INVESTMENT FUNCTIONS WITH REPLACEMENT RATE EQUAL 10 PERCENT

		CAP/OUT	ΔQ	ΔQ(t-i)	SEE	RHO	AAPE
1.	AGRICOLTURA, SILVICOLTURA E PESCA	1.148 (0.038)	0.440 (0.182)	0.529 (0.156)	196.90	0.461	7.63
2.	PRODOTTI ENERGETICI	0.877 (0.025)	0.242 (0.092)	0.242 (0.091)	189.003	0.273	6.16
3.	MINERALI E METALLI FERROSI E NON FERROSI	0.678 (0.143)	0.316 (0.205)	0.084 (0.239)	588.36	0.834	55.65
4.	MINERALI E PRODOTTI DI BASE DI MINERALI NON METALLIFERI	0.611 (0.044)	0.212 (0.243)	0.257 (0.159)	105.96	0.301	18.59
5.	PRODOTTI CHIMICI E FARMACEUTICI	0.783 (0.038)	0.266 (0.076)	0.040 (0.090)	175.90	0.576	13.19
6.	PRODOTTI METALLICI ESCLUSO MACCHINE, TRASPORTO	0.317 (0.028)	0.119 (0.067)	0.082 (0.061)	80.73	0.078	18.76
7.	MACCHINE AGRICOLE	0.309 (0.017)	0.166 (0.041)	-0.028 (0.051)	61.96	-0.348	16.17
8.	MACCHINE PER UFFICIO, STRUMENTI E OTTICA	0.342 (0.046)	0.153 (0.090)	0.106 (0.081)	30.308	0.723	33.21
9.	MATERIALI E FORNITURE ELETTRICHE	0.343 (0.027)	0.1538 (0.065)	0.087 (0.065)	76.662	0.345	21.59
10.	MEZZI DI TRASPORTO	0.625 (0.026)	0.403 (0.056)	0.111 (0.057)	89.956	0.338	10.30
11.	ALIMENTARI, BEVANDE, TABACCO	0.168 (0.006)	0.040 (0.023)	0.030 (0.025)	56.154	0.455	10.66
12.	TESSILI, ABBIGLIAMENTO PELLI, CALZATURE	0.201 (0.018)	0.072 (0.042)	0.051 (0.038)	120.147	0.372	20.92
13.	CARTA, CARTOTECNICI	0.279 (0.011	0.092 (0.021)	0.059 (0.025)	30.179	0.352	10.50
14.	GOMMA, MATERIE PLASTICHE	0.371 (0.058)	0.287 (0.136)	-0.071 (0.175)	99.632	0.733	40.14
15.	LEGNO, MOBILI E ALTRI INDUSTRIALI	0.103 (0.014)	0.054 (0.032)	0.005 (0.033)	65.114	0.555	33.17
16.	COSTRUZIONI	0.141 (0.006)	0.185 (0.032)	-0.119 (0.042)	38.973	0.461	9.72
17.	COMMERCIO	0.586 (0.030)	0.536 (0.153)	-0.035 (0.204)	244.199	0.658	12.40
18.	ALBERGHI E PUBBLICI ESERCIZI	0.410 (0.013)	0.181 (0.060)	-0.075 (0.085)	38.324	0.364	8.01
19.	TRASPORTI	1.464 (0.026)	0.643 (0.087)	0.145 (0.107)	108.448	0.395	4.70
20.	COMUNICAZIONI	3.018 (0.265)	-0.276 (1.694)	1.613 (0.914)	257.721	0.658	23.14
21.	CREDITO E ASSICURAZIONI	0.396 (0.025)	0.062 (0.147)	0.168 (0.136)	91.532	0.754	16.95
22.	SERVIZI VARI	3.177 (0.110)	1.737 (1.019)	1.518 (0.796)	1007.532	0.373	7.58
23.	SERVIZI NON DESTINABILI ALLA VENDITA	1.395 (0.070)	1.249 (1.065)	-0.459 (1.472)	381.611	0.755	11.59

TABLE II : RESULTS OF THE ESTIMATION OF THE SECTORAL INVESTMENT FUNCTIONS WITH REPLACEMENT RATE EQUAL 10 PERCENT

		CAP/OUT	q(t)	Δq(t-1)	Δq(t-3)	SEE	RHO	AAPE
1.	AGRICOLTURA SILVICOLTURA E PESCA	1.150 (0.037)	0.409 (0.176)	0.429 (0.181)	0.154 (0.160)	187.447	0.659	7.60
2.	PRODOTTI ENERGETICI	0.877 (0.025)	0.219 (0.100)	0.297 (0.090)	0.066 (0.125)	186.157	0.378	6.39
3.	MINERALI E METALI FERROSI E NON FERROSI	0.958 (0.153)	-0.133 (0.234)	0.026 (0.183)	1.005 (0.390)	446.613	0.355	32.96
4.	MINERALI E PRODOTTI DI BASE DI MINERALI NON METALLIFERI	0.613 (0.038)	0.338 (0.224)	0.444 (0.177)	-0.323 (0.191)	92.322	-0.182	13.56
5.	PRODOTTI CHIMICI E FARMACEUTICI	0.782 (0.036)	0.312 (0.097)	0.025 (0.089)	0.102 (0.134)	170.491	-0.182	13.56
6.	PRODOTTI METALLICI ESCLUSO MACCHINE TRASPORTO	0.317 (0.025)	0.112 (0.074)	0.076 (0.067)	0.017 (0.074)	80.489	0.127	18.37
7.	MACCHINE AGRICOLE	0.308 (0.015)	0.110 (0.048)	-0.016 (0.044)	0.078 (0.044)	53.469	-0.265	11.77
8.	MACCHINE PER UFFICIO, STRUMENTI E OTTICA	0.345 (0.046	0.137 (0.096)	0.088 (0.090)	0.047 (0.108)	29.994	0.786	32.10
9.	MATERIALE E FORNITURE ELETTRICHE	0.344 (0.25)	0.119 (0.067)	0.066 (0.063)	0.680 (0.068)	71.362	0.524	20.01
10.	MEZZI DI TRASPORTO	0.624 (0.025)	0.451 (0.078)	0.118 (0.055)	-0.066 (0.078)	89.956	0.663	10.56
11.	ALIMENTARI, BEVANDE E TABACCO	0.167 (0.006)	0.056 (0.031)	0.035 (0.025)	-0.027 (0.036)	54.432	0.221	10.63
12.	TESSILI, ABBIGLIAMENTO PELLI, CALZATURE	0.201 (0.018)	0.073 (0.061)	0.051 (0.095)	-0.003 (0.095)	120.138	0.365	20.89
13.	CARTA, CARTOTECNICI	0.279 (0.011)	0.092 (0.024)	0.059 (0.025)	0.000 (0.039)	30.179	0.355	10.49
14.	GOMMA E MATERIE PLASTICHE	0.371 (0.058)	0.281 (0.144)	-0.068 (0.177)	0.017 (0.140)	99.546	0.736	39.36
15.	LEGNO, MOBILI E ALTRI INDUSTRIALI	0.098 (0.013)	0.087 (0.037)	0.013 (0.030)	-0.092 (0.064)	58.769	0.462	28.94
16.	COSTRUZIONI	0.139 (0.005)	0.149 (0.040)	-0.065 (0.057)	0.038 (0.029)	35.799	0.572	9.46
17.	COMMERCIO	0.585 (0.030)	0.559 (0.164)	-0.051 (0.207)	-0.077 (0.206)	242.296	0.634	12.12
18.	ALBERGHI, PUBBLICI ESERCIZI	0.413 (0.003)	0.134 (0.015)	-0.022 (0.021)	0.162 (0.013)	9.334	0.268	2.15
19.	TRASPORTI	1.468 (0.023)	0.517 (0.101)	0.185 (0.094)	0.191 (0.105)	92.719	0.108	4.01
20.	COMUNICAZIONI	3.056 (0.145)	-0.422 (0.929)	0.065 (0.603)	2.391 (0.521)	141.033	-0.005	13.97
21.	CREDITO E ASSICURAZIONI	0.395 (0.025)	0.112 (0.181)	0.174 (0.135)	-0.068 (0.147)	90.454	0.720	18.44
22.	SERVIZI VARI	3.166 (0.102)	1.824 (0.936)	2.048 (0.835)	-0.970 (0.744)	923.996	-0.106	7.53
23.	SERVIZI NON DESTINABILI ALLA VENDITA	1.350 (0.057)	1.802 (0.852)	-0.558 (0.145)	-1.857 (0.765)	296.753	0.270	9.12

TABLE III - AVERAGE ABSOLUTE PERCENTAGE ERROR OF SECTORAL INVESTMEN FUNCTIONS WITH DIFFERENT REPLACEMENT RATES

INVESTING SECTORS	REPLACEMENT - RATES										
	r=.025	r=.05	r=.10	r=.15	r=.20	r=.25	r=.30	r=.35	r=.40	r=.45	r=.50
1	13.18	9.59	7.60	7.13	6.98	6.91	6.86	6.82	7.40	6.77	6.75
2	15.59	10.69	6.39	5.52	5.86	6.08	6.02	6.50	6.13	6.89	6.32
3	38.62	36.00	32.96	31.85	31.39	31.06	30.82	30.63	30.48	30.36	30.26
4	12.95	13.25	13.56	14.00	14.29	14.48	14.62	14.72	14.80	14.87	14.92
5	9.85	10.63	11.69	12.36	12.77	13.04	13.24	13.39	13.51	13.60	13.67
6	19.70	18.98	18.37	18.58	18.72	18.81	18.88	18.93	18.97	19.01	19.03
7	14.51	12.73	11.77	11.26	11.07	11.30	11.47	11.60	11.70	11.78	11.84
8	33.23	32.74	32.10	31.70	31.43	31.23	31.08	30.96	30.87	30.79	30.72
9	20.36	21.21	20.01	29.90	19.82	19.76	19.72	19.69	19.66	19.64	19.62
10	14.20	11.97	10.56	10.93	11.71	12.38	12.85	13.21	13.49	13.71	13.89
11	7.15	4.68	6.64	8.10	8.95	9.51	10.19	10.19	10.41	10.59	10.73
12	22.21	21.58	20.58	20.53	20.30	20.15	20.04	19.95	19.89	19.83	19.78
13	11.62	10.26	10.49	10.90	11.15	11.32	11.44	11.53	11.60	11.66	11.71
14	41.84	40.76	39.36	38.50	37.92	37.50	37.19	36.94	36.74	36.58	36.44
15	37.48	33.25	28.94	26.47	24.88	23.79	22.18	22.37	21.89	21.50	21.18
16	22.10	14.04	9.46	7.70	6.70	6.36	6.13	5.96	5.83	5.73	5.71
17	24.92	18.01	12.12	9.54	8.11	7.20	6.57	6.11	5.76	5.48	5.26
18	14.60	7.97	2.15	3.16	5.29	6.70	7.70	8.44	9.02	9.47	9.85
19	11.15	7.25	4.01	2.83	2.33	2.22	2.49	2.82	3.09	3.31	3.49
20	21.09	17.44	13.97	12.02	10.79	9.96	9.35	8.89	8.53	8.31	8.16
21	20.18	19.30	18.44	18.01	17.76	17.59	17.47	17.38	17.31	17.26	17.21
22	9.99	8.28	7.36	7.04	6.92	6.99	7.04	7.08	7.11	7.13	7.15
23	11.22	10.05	9.12	8.82	8.70	8.62	8.56	8.52	8.49	8.47	8.45

TABLE IV SOME REGRESSION RESULTS WITH DIFFERENT REPLACEMENT RATES

SECTOR 1 AGRICOLTURA, SILVICOTURA E PE BUILDINGS

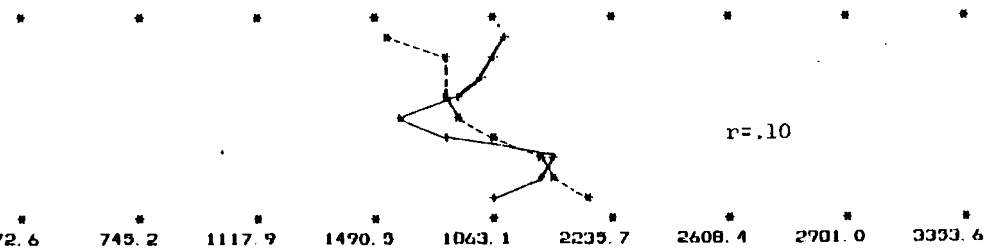
RHO = 0.659 AAPE = 7.60

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	1.150624	0.037151	30.9717
DQ(T)	0.409448	0.176677	2.3175
DQ(T-1)	0.429145	0.181208	2.3682
DQ(T-3)	0.154637	0.160218	0.9652

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	1558.00	1911.02	353.02
71	1726.00	1898.16	172.16
72	1835.00	1833.90	-1.10
73	1735.00	1779.85	44.85
74	1763.00	1601.38	-161.62
75	1870.00	1723.16	-146.84
76	2045.00	2068.16	23.16
77	2069.00	2019.87	-49.13
78	2167.00	1894.63	-272.37

DATE ACTL=* PREDIC=+ MISS=P-A*



SECTOR 4 MINERALI E PROD. DI BASE DE BUILDINGS

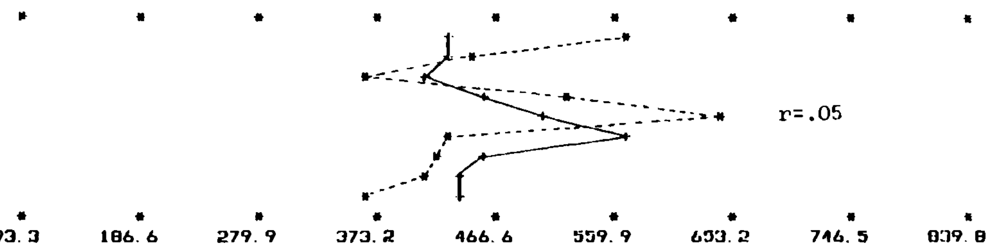
RHO = 0.128 AAPE = 16.12

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.903704	0.058983	15.3214
DQ(T)	0.313450	0.222603	1.4081
DQ(T-1)	0.387548	0.145548	2.6627

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	577.00	433.34	-143.66
71	449.00	435.19	-13.81
72	364.00	417.34	53.34
73	525.00	463.95	-61.05
74	646.00	508.02	-137.98
75	434.00	571.66	137.66
76	426.00	463.91	37.91
77	414.00	440.47	26.47
78	364.00	445.82	81.82

DATE ACTL=* PREDIC=+ MISS=P-A*



SECTOR 5 CHIMICI E FARACEUTICI BUILDINGS

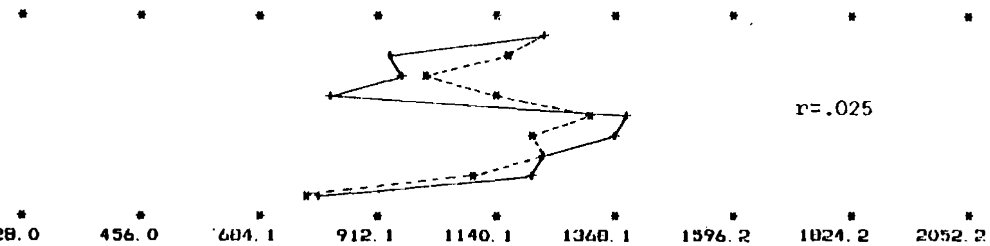
RHO = 0.061 AAPE = 9.57

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	1.439371	0.062441	23.0518
DQ(T)	0.480208	0.072457	6.6275
DQ(T-1)	0.242796	0.080666	3.0099

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	1233.00	1246.96	13.96
71	1180.00	946.04	-233.96
72	1023.00	978.94	-44.06
73	1147.00	837.83	-309.17
74	1324.00	1413.02	89.02
75	1225.00	1370.88	145.80
76	1245.00	1251.62	6.62
77	1103.00	1223.93	120.93
78	781.00	810.64	29.64

DATE ACTL=* PREDIC=+ MISS=P-A*



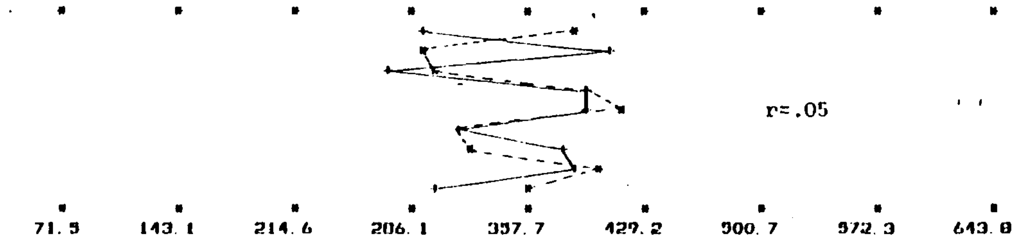
SECTOR 7 MACCHINE AGRCOLE BUILDINGS

RHO =-0.235 AAPE = 12.73

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.441450	0.024460	18.0475
DQ(T)	0.128284	0.054715	2.3446
DQ(T-1)	0.016037	0.051093	0.3139
DQ(T-3)	0.126292	0.050984	2.4771

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	389.00	293.39	-95.61
71	299.00	408.73	109.73
72	303.00	276.33	-26.67
73	398.00	398.08	0.08
74	421.00	398.65	-22.35
75	316.00	320.08	4.08
76	325.00	382.85	57.85
77	404.00	388.70	-15.30
78	364.00	304.76	-59.24
DATE	ACTL=*	PREDIC=+	MISS=P-A*



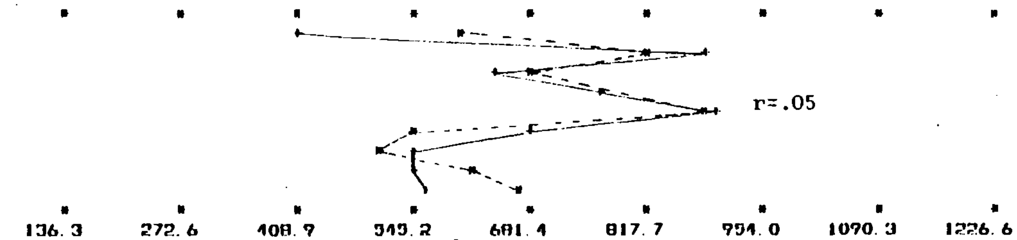
SECTOR 10 MEZZI DI TRASPORTO BUILDINGS

RHO = 0.223 AAPE = 11.93

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.894251	0.041031	21.7947
DQ(T)	0.489650	0.062537	7.8297
DQ(T-1)	0.201002	0.063573	3.1617

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	609.00	421.41	-187.59
71	819.00	897.66	78.66
72	683.00	645.91	-37.09
73	771.00	769.81	-1.19
74	892.00	906.34	14.34
75	549.00	682.82	133.82
76	509.00	555.86	46.86
77	624.00	557.62	-66.38
78	677.00	571.95	-105.05
DATE	ACTL=*	PREDIC=+	MISS=P-A*



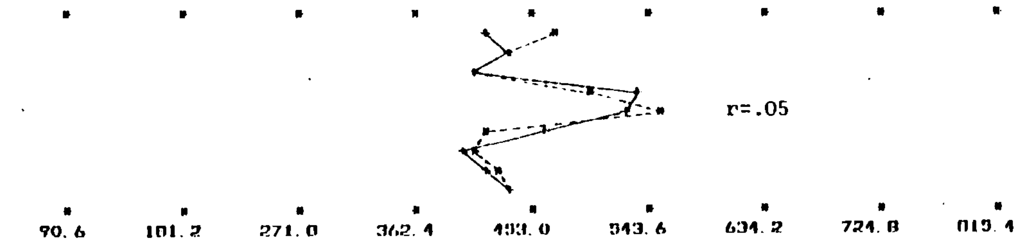
SECTOR 11 ALIMENTARI, BEVANDE E TOBACC BUILDINGS

RHO =-0.143 AAPE = 4.68

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.302514	0.006718	45.0308
DQ(T)	0.065450	0.017250	3.7943
DQ(T-1)	0.067710	0.015737	4.2405
DQ(T-3)	0.032670	0.019978	1.6362

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	476.00	420.02	-55.98
71	440.00	438.99	-1.01
72	409.00	408.10	-0.90
73	499.00	541.14	42.14
74	555.00	530.30	-24.70
75	417.00	465.59	48.59
76	412.00	399.57	-12.43
77	432.00	423.40	-8.60
78	437.00	438.37	1.37
DATE	ACTL=*	PREDIC=+	MISS=P-A*



SECTOR 13 CARTA, CARTOTECNICHE BUILDINGS

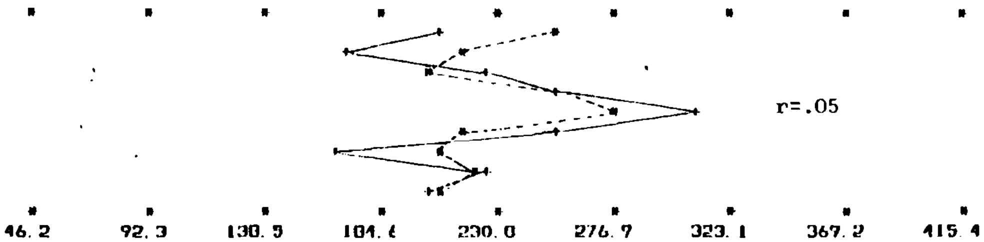
RHO = 0.177 AAPE = 11.47

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.395764	0.018462	21.4471
DQ(T)	0.128457	0.024507	5.2242
DQ(T-1)	0.096307	0.027807	3.4634

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	258.00	210.52	-47.48
71	217.00	170.96	-46.04
72	206.00	227.39	21.39
73	255.00	254.23	-0.77
74	279.00	310.10	31.10
75	220.00	257.66	37.66
76	208.00	166.37	-41.63
77	224.00	230.72	6.72
78	210.00	206.25	-3.75

DATE ACTL=* PREDIC=+ MISS=P-A*



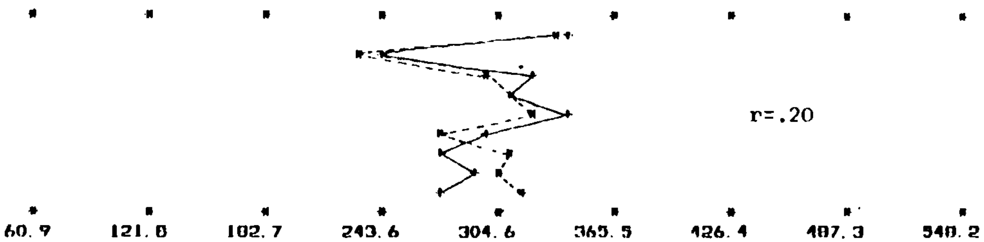
SECTOR 16 CONSTRUZIONI BUILDINGS

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.077476	0.002195	35.2731
DQ(T)	0.129153	0.021059	6.1330
DQ(T-1)	-0.087873	0.028079	-3.1295

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	341.00	345.88	4.88
71	236.00	248.86	12.86
72	302.00	323.61	21.61
73	315.00	316.42	1.42
74	326.00	345.20	19.20
75	278.00	303.25	25.25
76	315.00	274.12	-40.88
77	306.00	295.44	-10.56
78	322.00	277.75	-44.25

DATE ACTL=* PREDIC=+ MISS=P-A*



SECTOR 18 ALBERGHI E PUBBLICI ESERCIZI BUILDINGS

RHO = 0.268 AAPE = 2.15

VARIABLE	REGRES-COEF	STD. ERROR	T-VALUE
CAPITAL/OUTP	0.413970	0.003208	129.0462
DQ(T)	0.134607	0.015341	8.7743
DQ(T-1)	-0.022642	0.021294	-1.0633
DQ(T-3)	0.162601	0.013616	11.9423

GROSS INVEST DEPENDENT VARIABLE

DATE	ACTL=*	PREDIC=+	MISS=P-A*
70	278.00	291.46	13.46
71	395.00	391.82	-3.18
72	371.00	374.67	3.67
73	437.00	440.45	3.45
74	480.00	485.34	5.34
75	372.00	379.65	7.65
76	364.00	347.28	-16.72
77	366.00	358.92	-7.08
78	356.00	347.99	-8.01

DATE ACTL=* PREDIC=+ MISS=P-A*

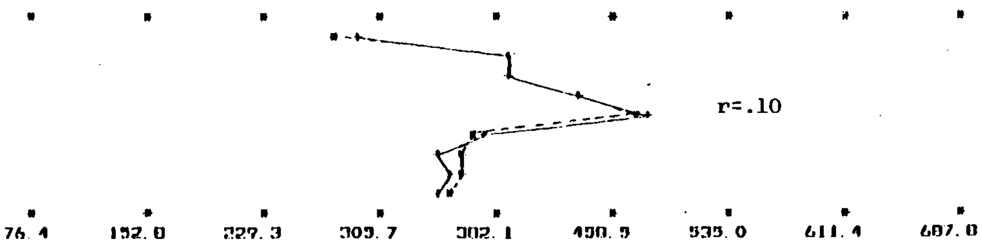


TABLE V : INVESTMENT FUNCTIONS COEFFICIENTS USED IN THE SIMULATION

		rk	w_0	w_1	w_2	w_3
1.	AGRICOLTURA SILVICOLTURA E PESCA	0.11506	0.40945	0.42915	0.15739	0.15464
2.	PRODOTTI ENERGETICI	0.08780	0.21976	0.29767	0.29449	0.6604
3.	MINERALI E METALLI FERROSI E NON FERROSI	0.09689	0.	0.02367	0.5293	0.88227
4.	MINERALI E PRODOTTI DI BASE DI MINERALI NON METALLIFERI	0.06131	0.22173	0.29095	0.10045	0.
5.	PRODOTTI CHIMICI E FRAMACEUTICI	0.07821	0.27624	0.02294	0.48289	0.
6.	PRODOTTI METALLICI ESCLUSO MACCHINE TRASPORTO	0.03174	0.11223	0.07642	0.11123	0.01751
7.	MACCHINE AGRICOLE	0.03086	0.10476	0.	0.12888	0.07497
8.	MACCHINE PER UFFICIO, STRUMENTI E OTTICA	0.03452	0.13775	0.08617	0.07186	0.04738
9.	MATERIALE E FORNITURE ELETTRICHE	0.03446	0.11902	0.06645	0.07862	0.08048
10.	MEZZI DI TRASPORTO	0.06248	0.40835	0.10691	0.10956	0.
11.	ALIMENTARI, BEVANDE E TABACCO	0.01678	0.04877	0.03076	0.08822	0.
12.	TESSILI, ABBIGLIAMENTO PELLI, CALZATURE	0.02018	0.07263	0.05075	0.07844	0.
13.	CARTA, CARTOTECNICI	0.02797	0.09285	0.05996	0.12646	0.00045
14.	GOMMA E MATERIE PLASTICHE	0.03720	0.23781	0.	0.11936	0.01481
15.	LEGNO, MOBILI E ALTRI INDUSTRIALI	0.00980	0.04502	0.00681	0.04622	0.
16.	COSTRUZIONI	0.01398	0.10174	0.	0.01171	0.02631
17.	COMMERCIO	0.05851	0.45769	0.	0.12740	0.
18.	ALBERGHI, PUBBLICI ESERCIZI	0.04140	0.12763	0.	0.13217	0.15417
19.	TRASPORTI	0.14690	0.51760	0.18563	0.57410	0.19162
20.	COMUNICAZIONI	0.30564	0.	0.05714	0.89829	2.10099
21.	CREDITO E ASSICURAZIONI	0.03954	0.09583	0.14868	0.15083	0.
22.	SERVIZI VARI	0.31666	1.39620	1.56772	0.20266	0.
23.	SERVIZI NON DESTINABILI ALLA VENDITA	0.13502	0.64629	0.	0.70389	0.

TABLE VI : AGGREGATED RESULTS OF THE SIMULATION PERCENT CHANGES

	75-76	76-77	77-78	78-79	79-80	80-81	82-85	80-85	85-90	75-90
GNP	3.61	4.64	1.55	6.03	4.63	-0.80	3.43	2.40	4.36	3.62
RISORSE DISPONIBILI	3.75	5.30	1.47	6.44	5.01	-1.09	3.64	2.50	4.47	3.79
AMMINISTRAZIONE	3.64	2.00	0.50	0.05	1.09	0.40	2.61	2.20	3.08	2.24
ISTRUZIONE	3.64	2.00	0.50	0.05	1.09	2.79	1.71	1.94	3.08	2.16
SANITA' SERVIZI SANITARI	3.64	1.67	0.83	0.05	1.09	0.41	2.49	1.58	2.27	1.77
CONSUMI COLL. PRIVATI	3.64	2.00	0.50	0.05	1.09	0.41	1.48	1.34	3.93	2.24
CONSUMI DELLE FAMIGLIE	3.36	2.20	2.97	5.20	4.28	-0.70	2.60	2.09	4.11	3.26
ESPORTAZIONI	11.91	9.88	3.10	5.43	4.13	3.48	5.30	4.80	4.02	5.24
IMPORTAZIONI	13.09	0.12	3.97	7.48	5.51	-1.54	3.04	2.01	4.36	4.13
VARIAZIONI DELLE SCORTE	0.	50.39	-14.39	50.16	8.02	-119.52	17.63	-5.58	6.39	0.
INVESTIMENTI FISSI LORDI	1.32	0.47	-0.21	12.46	9.40	-1.88	3.17	0.83	6.31	3.94
OCCUPAZIONE INTERNA	0.50	0.53	0.71	0.86	0.89	1.20	0.80	0.46	1.81	0.99

CHANGES OF INPUT COEFFICIENTS AND POSSIBILITIES TO FORECAST THEM: A FINNISH CASE

O. Forssell

Characteristics of the Finnish project, sensitivity of input coefficients, preliminary results of observed changes of coefficients and possibilities to forecast coefficients are considered in this paper.

1. Characteristics of the Finnish Input-Output Model System.

Several circumstances affect on compilation of a national model. The most important ones are the goals of the model, characteristics of the project and available statistical data. The Finnish input-output model system (FMS) is built in order to evaluate the alternative strategies for the development of the Finnish economy in the long range. Strategies are examined in connection with (a) the realization of the economic possibilities of social goals, (b) the projections of the perspectives of international trade to the possibilities of the development of the Finnish economy, and (c) the new possibilities and the needs for adjustment created by tecnological change. This means that the model is more a simulation than forecasting model. It has some characteristics of growth model, when it is used mainly for analyzing long range development. It should have links with international trade flows. Technological change should have special considerations in formulating the model.

The model building project is going on in the Department of Economics at the University of Oulu [1]. This means that the project has academic features. We try to get as solid theoretical base as possible for it: simple structures, behavioral equations clear and wellfounded solutions for problems. This is at least the aim, but when construction of the model is in practice, as much art as science, some compromises must always be made. Another acadenic feature is lack of finance and very limited resources. This leads to stepwise development of the model. We start with a slim version and then proceed little by little toward a more developed system. The Academy of Finland has financed us during three years about 4 research years and from other sources we have got 3 research years. In addition we have had about 3 assistant years. Use of computer time is free in the university but programming must be paid for or made by oneselves.

The members of the team pursue their studies at the university simultaneously as they work at the project. It was then purposefull to divide responsibilities in the project so that the members could do their doctoral thesis on the subject they study in the model. Sectors like incomes and their use, investment and growth, foreign trade were responsibility areas. This kind of organisatory division seems to work and members of the team learn by doing[1].

Available statistical data affects on structure of the model, classification, and estimation possibilities. The starting point for industrial classification in the model was the latest available input-output table of the Finnish economy. It was for the year 1970 and it had 65 industries. As an input-output man I prefer the model with as a detailed industrial classification as possible. It is always easier to aggregate than to disaggregate. But we had a lot of trouble and work to be done in order to get time series for gross outputs, final demand and primary input items, prices, employment, and capital stocks.

---

1 The team works at the Department of Economics at the University of Oulu and its members are Osmo Forssell, Jukka Kärrymäki, Ilmo Mäenpää and Rauli Svento.

Time series are not available in national accounts according to the same industrial classification as input-output table has. This was the first reason to reduce number of industries to 38 industries. Another reason was heavy work in gathering, formulating, quessing, linking and deflating data. Making of the data bank for the model took about 2/3 of the time available for the project and the figures available are not final, yet. It was necessary that the real work, let us say productive work, the model building itself had to be started. Classification problems were not finished yet. In Central Statistical Office the revision of SNA was going on and as a result of that work still another classification scheme appeared. Unfortunately it has fewer industries than we planned in the beginning of our work. So the final or better to say updateted industrial classification which we now have comsists of 30 industries.

Revision of SNA had some advantages also from our point of view. Income accounts are better and consists more suitable information for building income and outlay submodel. However input-output table is not a consistent part of national accounts.

2. The 59 most sensitive input coefficients

The following table indicates how the 59 most sensitive input-output coefficients are distributed among industries of the model. Coefficients are arranged by rows and by columns. The row coefficients indicate that the output of the industry is sensitive for intermediate demand of its products (output). When we examine coefficients columnwise their sensitively is studied from production process point of view (inputs). For instance we take the industry n:o 22 Basic metal industries: it has four coefficients on output side and three on input side. They are the following ones:

output of basic metal industries were used by

- basic metal industries	1,21
- manufacture of machinery	6,57
- manufacture of electrical machinery	9,57
- manufacture of metal products	11,09

The 59 most sensitive input-output coefficients

out-put	in-put	The classification of industries
3	3	01. Agriculture, hunting and fishing
3		02. Forestry and logging
2		03. Mining and quarrting
2	2	04. Slaughtering, prepairing and preserving meat
1	2	05. Manufacture og dairy products
1	1	06. Grain mill products, manufacture or bakery products
2	1	07. Other manufacture of food products
2	1	08. Beverage and tobacco industries
2	1	09. Manufacture of textiles
1	2	10. Manufacture of clothings, fur and leather products
3	1	11. Sawing, planing and preserving
1	1	12. Other manufacture of wood
1	4	13. Pulp mills
1	2	14. Manufacture of paper and paperboard
	1	15. Manufacture of pulp, paper and paperboard articles
3	1	16. Printing and publishing
3	1	17. Manufacture of chemicals
1		18. Manufacture of chemical products
3		19. Petroleum refineries, miscellaneous products of petroleum and coal
		20. Manufacture of rubber and plastic products
3	2	21. Pottery, glass and earthenware products
4	3	22. Basic metal industries
4	1	23. Manufacture of metalproducts
	2	24. Manufacture of machinery
2	1	25. Manufacture of electrical machinery and related products
		26. Shipbuilding and repairing
2	1	27. Other manufacture of transport equipment
		28. Other manufacturing industries
4	1	29. Electricity, gas and water
	6	30. Building
	3	31. Other construction
	4	32. Trade
	2	33. Restaurants and hotels
1	3	34. Transport
2		35. Communications
	3	36. Letting and operating of dwellings
1	2	37. Other real estate, financing, incurance and business services
1		38. Private social and personal services
59	59	

basic metal industries used inputs produced by

	d_{rs}-value[2]
- basic metal industries	1,21
- mining	2,19
- manufacture of metal products	9,33

The measure d_{rs} indicates how many percent an input coefficient a_{rs} may change such that the output of any sector does not change more than one percent. The final demand is supposed to be constant. The smaller the value d_{rs} is the more important (sensitive) the coefficient a_{rs}.

There are only three industries which have no coefficient among those 59 most sensitive coefficients. On the output side there are ten industries and on the input side nine industries without their own coefficient among the most sensitive ones. There are 14 diagonal coefficients among the 59.

The most sensitive 59 coefficients are only 4,4 % of all the coefficients (1363 coefficients). Their d-value varies from 1,21 to 11,73. These coefficients are rather evenly distributed among three blocks of industries.

---

2 Ilmo Mäenpää has studied sensitivity of input-output coefficients [3]. He used the following d-measure:

$$d_{rs} = \frac{1}{a_{rs}(\max_i \frac{b_{ir}}{x_i} x_s + 0.01 b_{sr})},$$

where a_{rs} = rs-element of the original input coefficient matrix A;

b_{ir}, b_{sr} = elements of the original Leontief's inverse matrix $B = (I-A)^{-1}$;

x_i, x_s = original gross products of the sectors i and s.

block of industries	output side	input side
agriculture, food, beverage, tobacco and textile industries (1,4–10)	14	13
foresty, wood and paper industries (2,11-16)	12	10
mining, metal industries (3,22-27)	14	8
building and construction (30,31)	-	9
other industries	19	19
TOTAL	59	59

It is then purposeful to study changes of the most sensitive coefficients by set of industries, which are closely related to each others through interindustrial flows. Causes of changes may be expected to be similar or at least related to each others inside the block of industries.

The changes of the most sensitive input coefficients of the Finnish Input-Output Model System has not been examined yet. The earlier studies indicate that changes are rather great[3]. It is then important and necessary to find out possible causes for changes in other to be able to forecast these coefficients in the future.

3 The change in the greatest input-output coefficients of manufacturing industries were observed to increase with the length of the observation period in 1963-65, 1956-59, 1959-63, 1959-65, 1956-63 and 1956-65. The regression equation between the square of median of weighted absolute change in input-ouput coeffecients on rows and time was: $(\text{median})^2 = 0.0074t^{0.8}$. Consequently the median of changes in input-output coefficients was 8.6 % in the first year and later increased less than proportionally. After 10 years it was 21.7. The size and rapidity of changes could not be considered to be different from those observed in other countries. 2

3. Studying impacts of changes of input coefficients on intermediate demand

In studying changes of input coefficients annual input-output tables would be needed. In most countries this kind of full information situation does not exist. Input-output tables are available only for some bench-mark years and spaced years must be covered by data on gross output, public and private expenditures, fixed capital formation, change in stock, imports and exports. Data is usually got from national accounts. Necessary data on final demand is then not available by industries, but must be generated by transformation matrices. These matrices relate industrial final demand to different expenditure categories. Transformation matrices can usually be compiled only for the years, which are covered by input-output tables. All this means that only limited information is available for making observations on changes of input coefficients. Observations are then subject to different sources of errors.

I suppose that the case of Finland is typical. We have input-output tables for 1959, 1965, 1970 and 1978. These tables have different classifications, there are minor continuity between tables and tables are not consistent with national accounts. In order to get estimates on final demand by industries, the following conversions and adjustments had to be done:

Private consumption expenditures are available from national accounts for 30 expenditure categories[4] at marked prices.

---

4 1. Bread and cereals
2. Meat
3. Fish
4. Milk, cheese and eggs
5. Fats and oils
6. Fruit and vegetables
7. Sugar and sweets
8. Coffee, tea cocoa
9. Other food
10. Non-beverages
11. Beverages
12. Tobacco
13. Clothings
14. Footwear
15. Housing
16. Heat and light
17. Furniture
18. Household equipment
19. Other household consumption
20. Householdservices
21. Healt and cleannes
22. Private means of conveyance
23. Expences of private means of conveyance
24. Public transportation
25. Communication
26. Personal things
27. Entertaimet and cultural service
28. Books and magazines
29. Hotels, restaurants and cafes
30. Other services

Classification of indirect taxes is (1) import duties and levies, (2) sales tax, and (3) other commodity tax, net.

Time series are at constant and current prices. These series were converted to the FMS industrial classification at producer's prices by a matrix converter available from 1970 input-output table. Summation across the 30 categories for individual components gave the estimate of private consumption expenditure demanded from an industry.

Public consumption expenditures are available from national accounts for such categories as health services, educational services, etc. It was assumes that every public consumption expenditure item produced by an industry developed as the expenditures on an average. These items are usually only a small part of the output of the industry.

Gross domestic fixed capital formation is available from national accounts for the following four categories:

1. Residential bildings
2. Non-residential buildings
3. Other construction
4. Machinery, transport and other equipment

Market price time series are available at current and constant prices. These series were converted to the FMS industrial classification at producer's prices by a matrix converter estimated from the 1970 input-output table. Summation accross the four categories for individual components gave the estimate of fixed capital formation produced by an industry.

Change in stocks and statistical discrepancy is known from national accounts only as total sum. It was assumed, that the same fraction of output as in 1970, was every year delivered from an industry to this item of final demand.

Exports and imports are presented in foreign trade statisties for goods classified by commodities at f.o.b. and c.i.f. prices. These goods were classified by producing industries (the FMS industries). Time series for export and import items were got directly on basis of these classifications. Price indeces were calculated according to foreign trade statistics information. Constant price series were estimated by means of these indeces. Exported and imported services were evaluated on basis of information got from balance-of-payments statistics.

Time series for final demand of industries were got summing across the five component estimates given above. When these time series were not consistent with final demand of input-output table used in studying changes of coefficients they are adjusted to the level of the input-output table.

When the estimates on final demand by industries are available the impacts of changes of input coefficients on intermediate demand may be examined as follows:

$$\ln \frac{\Sigma_j a_{ij}(o)x_j(t)}{x_i(t)+m_i(t)-y_i(t)} \qquad i = (1,\ldots,n)$$

were $a_{ij}(o)$ is a base year input coefficient including both domestic and imported flows

$x_i(t)$ is the output of industry i at the year t

$m_i(t)$ is imports of commodities characteristic to the industry i at the year t

$y_i(t)$ is the final demand of industry i at the year t

The measure inficates how much in percents the observed intermediate demand deviates from the corresponding demand evaluated by fixed input coefficients. Positive measure means that input coefficients have decreased. Measures are symmetric.

In the Finnish case the calculaitons were made by 1970 coefficients for the year 1971-75 and by 1965 coefficients for 1966-1975. It is planned to continue these calculations up to 1978 and to make similar evaluations by 1959 coefficients for 1960-1978. Some preliminary results of the calculations are presented next. There are some industries, where deviations are rather tolerable. Measures are presented for 1965 and 1970 coefficients.

	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975
Agriculture hunting and fishing	-.019	-.048	-.063	-.094	-.061	-.133	-.177	-.172	-.169	-.136
						-.071	-.122	-.121	-.122	-.090
Forestry and logging	-.107	-.090	-.085	-.129	-.063	-.188	-.197	-.231	-.279	-.234
						-.060	-.130	-.160	-.205	-.152
Pulp mills	-.073	-.026	-.062	-.023	-.053	-.108	-.264	-.308	-.254	.011
						-.064	-.089	-.133	-.087	.182
Printing and publishing	.090	.013	.033	.021	.079	-.192	-.200	-.360	-.154	-.168
						-.039	-.053	-.133	-.042	-.035
Manufacture of chemicals	.001	.026	.069	.143	.246	.313	.410	.437	.453	.394
						.086	.169	.180	.207	.156
Basic metal industries	-.038	-.053	-.109	-.086	-.046	-.358	-.410	-.393	-.328	-.506
						-.152	-.145	-.132	-.101	-.208
Manufacture of metal products	-.025	-.054	-.171	-.332	-.047	-.008	-.049	-.017	-.013	-.173
						-.036	-.052	-.012	-.027	-.150
Electricity gas and water	-.070	-.038	-.064	-.069	-.060	-.040	-.034	.001	.021	.070
						.059	.053	.094	.129	.192
Trade	.004	-.009	-.108	-.240	-.159	-.105	-.077	.028	.056	.076
						.074	.099	.197	.226	.243
Transport	-.007	-.051	-.072	-.073	-.092	-.113	-.124	-.099	.054	.055
						-.026	-.047	-.031	.115	.116

These industries have delivered major part of their output for intermedeiate demand.

The following examples are industries which have great changes of input coefficients. Major part of their output is used as final demand.

	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975
Manufacture of dairy	.061	.119	.170	.255	-.657	-.310	-.218	-.031	-.765	-.024
products						.088	.118	.178	-.067	.188
Sawing, planing	-.317	-.247	-.357	-.122	-.247	-.217	-.424	-.412	-.303	-.734
and preserving						-.016	-.225	-.199	-.100	-.517
Manufacture of rubber and	.171	.158	.169	.315	.438	.399	.293	.365	.333	.288
plastic p.						-.178	-.372	-.255	-.329	-.400
Manufacture of	-.232	-.155	-.372	-1.762	-.106	.105	-.267	.007	.144	.263
machinery						.187	-.188	.094	.217	.309
Shipbuilding	.056	.522	-.103	-.097	.326	.494	.547	.385	.580	1.325
and repairing						.045	.094	-.225	.049	1.097
Building	-.146	.008	-.205	-.117	-.069	-.094	-.113	-.112	.007	.030
						-.026	-.037	-.037	0.50	.062

The following reasons have generated the extent of the discrepancy between fixed coefficient intermediate demand and the corresponding estimates got by different approaches:

1. Imperfections in the underlying statistics, for instance lack of stock statistics
2. Unstability of convertes with fixed distributions
3. Problems related to deflation to constant prices
4. Changes of input coefficients

These preliminary results with the Finnish data point out that it is too early to make conclusions on changes of input coefficients. The third first mentioned reasons have obviously so great impacts on results. Used data must be further elaborated.

4. Possibilities to forecast coefficients

Possibilities to forecast changes of input coefficient depend on the lenght of forecasting period. In short term forecasts and in updating input-output tables more mechanical approaches may be used than in long term forecasts. Observed trends of individual sensitive cofficients as well as forecasting errors of the whole intermediate demand of an industry may then be utilized.

When changes of input coefficints are analysed in the longrange evaluation concerns first of all technological development. It is closely related to return of invested capital and changes of relative prices of inputs. Substitution among inputs are then relevant phenomenon to study at. Evaluation of substitution among inputs from technological development point of view may proceed as fallows:

1. Analysis of trends in technological development and specification of the input coefficients directed by these trends.
2. Evaluation of time-path of substitution process.
3. Forecasting of future changes of input coefficents according to time-path of substitution process.

This approach demands much detailed work. In our case it can be applied at most to the most sensitive input coefficients. When technological development is international phenomenon some international project (for instance at IIASA) studying technological trends were useful perhaps for several countries.

Evaluation of changes of input coefficients in the longrange is a difficult problem. Many factors have effects on cahnges and many purpose oriented measures may steer these changes. Acconrding to our aims it is then purposed to study which were the most desirable and reasonable technical development patterns for the Finnish economy. Consideration of different alternatives gives more range with in which to move for the study. The model system should be build such a way that choices of technologies according to return of capital or according to some other goals may be analysed by the model.

References

1 Osmo Forssell, Experiences in Compilation of the Finnish Inpu-Output Model System, A paper presented in a workshop on the Inforum - IIASA international system of input-output models, October 1980.

2 Osmo Forssell, Explaning changes in input-output coefficients for Finland, an article in Input-Output Techniques, eds. A Brody and A.P. Carter, 1972, North Holland Publishing Company.

3 Ilmo Mäenpää, The sensitivity of the FMS input-output model to the changes of input-output coefficients, an appendix to Osmo Forssell, Problems using input-output methodology in industrial planning. A paper presented in a Finnish-Soviet seminar, 1979 at Tampere.

INTERCOUNTRY PATTERNS OF STRUCTURAL CHANGE IN ECONOMIES

A. Smyshlyaev

ABSTRACT

The purpose of a comparative study of the economic structure between developed countries is to discover, numerically, the similarities in economic growth, i.e., to identify the main factors influencing the great difference in their pattern of structural changes. This paper considers a set of interrelated indicators of the economic development and of the production structure for six developed countries (USA, United Kingdom, France, Federal Republic of Germany, Canada, Japan) during the period of 1960-1977 years. Attention is given to the intercountry data, this is necessary in such a long-range prediction (10 years ahead). Some questions of the usage of the econometric techniques on pooled data (time-series for different countries) are also raised.

1. Problem

Frequent use is made of most Input-Output modeling elements in applied economic forecasting when the interaction between industries (branches of an economy) is represented by demands for intermediate products $X(i,j)_t = a(i,j)_t * X(j)_t$. The total demands (values of the industries' outputs $X(i)_t$) are generated by exogenously given final demands; this also applies to the exogenously given so-called technical coefficients $a(i,j)$. Summing all the values of $X(i,j)$ and the final demand components we find that the value of $X(i)$ needs to produce:

$$X(i) = \sum_j X(i,j) + Final\ Demand(i)$$

If we look at any global, multiregional or national multisectoral forecasting model (static or dynamic) it will definitely contain the exogenously given $a(i,j)_T$ and for models of a very different level of aggregation the same expression "technical coefficients" is used. The interindustry interaction is treated by constant or changing over time coefficients $a(i,j)$. Therefore in order to find only N values of industry gross outputs $X(i)$ one needs to have a precise prediction of $N*N$ parameters $a(i,j)$.

Our interest relates to the long-term economic forecasting, i.e., to the prediction of an economic structure for 5-10 years. It can be said that this time interval is quite uncertain and that the problems of a 5 year forecast are rather different from those for a 10 year forecast, but it is not a case, assuming that we have a multisectoral model, and start with a forecast for the year 1985 in 1981. Thus the starting point is 1981 and the time horizon is only 5 years but the last input-output table $a(i,j)$ can be available only after 2-3 years of the survey being carried out. If the table of $a(i,j)$ for 1978 is available in 1980, and if we have also reestimated all the model parameters according to this new data within the year 1981, then the interval will be 7 years. If the last table is available for 1976 only, and a new one for 1979 or 1980 has not been published, then the lag will increase to 9-10 years. In both cases care should be taken that the gap between the data available year and the year of forecast to be developed is approximately 7-10 years. Such great lags can be definitely be found in models applied in the past few years.

It should be also noted that such lags do not exist for other parts of multisectoral models. Elements of final demands and labour inputs are generated by statistical offices without any gap, or with only a one year lag between the starting year and the data becomes available year. For example, the structure of the personal consumption expenditure can also be given by a system of equations developed for different income groups on old surveys but nevertheless the corresponding components of consumption used in a model run are given for each year and the equations can be adjusted to these data prior to the starting year. Therefore one has *a priori* knowledge about N elements of consumption which can be used to adjust a model that will generate the same N inputs in the *Input – Output* model. For the elements of $X(i,j)$ we find only $2*N$ values of the corresponding sums (by rows and by columns)

$$\sum_{i=1}^{N} X(i,j) = X(j) - \textit{Value added}(j)$$

and

$$\sum_{j=1}^{N} X(i,j) = X(i) - \textit{Final demand}(i)$$

to produce $N*N$ values of $a(i,j)$. Some artificial tables for the later years can exist but their development is also based on some models, for example the *RAS* technique is frequently used. Thus the $a(i,j)$ matrix is an unique part of a model in that it cannot be adjusted to the current statistic data so well as other components of a model. Note that in most studies using an input-output model for forecasting the explanation of the $a(i,j)$ projection technique is very short and vague.

In long-term forecasting we find rather simple equations to connect directly most parameters of the interindustry structure with the final demand growth. This enables the use of some simple approaches in order to forecast $X(i)$ over time and in our opinion an intercountry set of data should be applied in long-term economic forecasting.

2. Model

It is assumed that the entire set of interrelated structural characteristics is moving in time to the common "intercountry" pattern of growth. A generalization of historical patterns of economic growth across nations is usual in macroeconomic studies. The aim is to determine some stages in economic growth that influence global changes of an economic structure for every nation. In other words the approach is the quantifying of general trends of economic structure development in several economies.

The analytical purposes should be divided: the first is to understand whether there are common "intercountry" patterns of economic growth and the second is the estimation of parameters to be applied in the long-term forecasting of a particular country's development. The application of such estimates cannot be done on the statistical data for every single country. Therefore, the results of modeling can be used in different ways and estimates of the parameters are not the only goal of an application of models. Some conclusions can be also drawn on the impact of short- and long-term changes in explanatory variables. The strong implicit assumption is that the evolutionary development of an economy, represented for example by *GDP* (*GDP* is Gross Domestic Product) rates of growth and its level per capita, determines the main structural changes in every economy. The *GDP* per capita level for different countries can be interpreted as the factor which reflects an impact of some other macroeconomic variables on the movement of economic structure. However for every economy there are some changes in time that do not depend on the rates of economic growth.

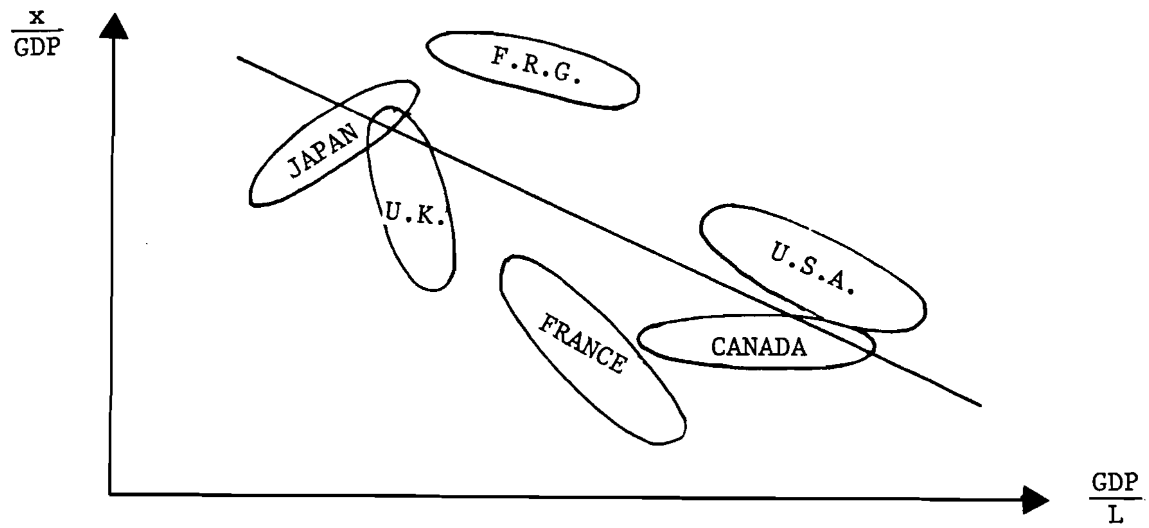
Some geographic indicators such as climate, availability of resources, availability of trade partners should be also included in the list of explanatory variables and the comparison of long-term trends of the structure of economies across countries can orient us in searching out those factors.

The average or the common tendency of economic development is named "intercountry pattern" and the differences between the trend of a particular country and its "intercountry pattern" are of main interest. Theoretically the question of the similarity of economies cannot been proved without quantitative analysis. It means that if we have a model to explain why countries' patterns are different then we can estimate the impact of these differences on the structure of macro variables based on the development of each nation. Applications of the intercountry analysis show a similarity in the main tendencies across developed countries. It is possible to distinguish some essential factors determining most gaps of the economic structures.

The most significant trends in economic structure related to the increase of *GDP* per capita are frequently cited in intercountry studies. Many papers investigate $\frac{Energy\ Consumption}{GDP}$ ratio across nations in order to estimate the effect of an energy price change on the rates of energy consumption. The comparison of product structure is, in itself, of interest, but its greatest value is as a starting point for investigations of the causes or consequences of international differences and of those socio-economic structural features of economies usually associated with the level of wealth or income per capita. Many macro and micro-economic studies are also concerned with the causes of industrial structure differences, and also with the consequences of consumption patterns and resource availability on industrial structure. These include international comparisons of energy consumption to determine whether the *USA* is particularly wasteful in its use of energy. Because a valid comparison and modeling requires price comparability such macroeconomic studies also require information on relative prices.

Some obvious trends in economic structure can be easily predicted; the slowdown in the share of agriculture in the labor force for example. We should stress that less obvious trends at least at the aggregated level of economic structure such as $\frac{X(i)_{s,t}}{GDP_{s,t}}$ can be also predicted for all N industries $i = 1,\dots N$ only if certain characteristics of the future, such as *GDP* rates of growth, share of *investment* and the share of *equipment* in *investment*, are given. (Note that s is an index for the country ($s = 1,\dots,6$), t is a time variable and $\frac{GDP}{L}$ is GDP per capita). Various indicators of a country economic "history" determine existent gaps between its own trend and the intercountry pattern, and Graph 1 shows the standard situation in applying regression techniques to the pooled data for industry output per *GDP*.

Graph 1. Clusters of data and "intercountry pattern" produced by the regression $\frac{X(i)_{s,t}}{GDP_{s,t}} = \alpha + \beta * \frac{GDP_{s,t}}{L_{s,t}}$



The aggregated econometric model consists of about 20 equations that reflect changes in gross output and energy inputs. In such study the industries should be divided in two groups. The first one includes those which development (and relative level) is mostly associated with the growth and composition of final demand. Obviously it is rather difficult to obtain good results for them unless some socio-economic and resource variables are included in analysis. The second group consists of industries whose growth is related mainly to other industries (through interindustry interaction process).

Thus the model of intercountry interindustry interaction is as follows:

$$\frac{X(i)_{s,t}}{GDP_{s,t}} = F\left[\frac{GDP_{s,t}}{L}, Z_{s,t}\right]$$

for the first set of industries, where $Z_{s,t}$ are predetermined variables in $I-O$ modeling, and

$$\frac{X(i)_{s,t}}{GDP_{s,t}} = F\left[\frac{X(j)_{s,t}}{GDP_{s,t}}, \ldots, \frac{X(k)_{s,t}}{GDP_{s,t}}\right]$$

for the second set of industries where $j,\ldots,k$ are those industries mostly using products of i-th industry.

3. Usage of Econometric Techniques

There are many problems related to the econometric analysis of such data and therefore developed techniques should be used. It is easy to convert all the data in a "homogeneous" set by applying a "Dummy" model, i.e. if we add a set of binary dummies that imply different levels of the dependent variable for each of the countries to each equation. This means that, between countries, we equalize all the mean values of the dependent variable by the binary dummies and therefore the causes for the very different levels of the industry development between countries are removed from the study. The formal reason for the rejection of such modeling is that most of differences of the "pooled" dependent variable are explained by the "dummies" which themselves cannot be explained and R^2 is an inappropriate estimate of goodness of fit for dummy models based on pooled samples. For the dummy variable model the greatest part of the dependent variable variance

$$Var\ X(i)_{s,t} = \sum_{s,t}[\ X(i)_{s,t} - \bar{X}(i)\ ]^2$$

is *explained* by the dummy variables, but not by the real variables, because the average value of the dependent variable ($\bar{X}(i)$) used in the calculation of R^2 is assumed to be the same for all the data, and the difference between this value and the corresponding average values for countries is mostly reduced by the dummies.

The main reason for using rather simple equations for the second set of industries is that only a few of the significant interindustry interactions in economic growth between industries exist. If the relation between industries is strong across economies then few parameters can be found instead of forecasting all the values needed in $I-O$ model for each industry output. The complicated interaction which exists between rates of economic growth measured by *GDP* per capita and rates of *investment* and changes in the composition of industry output should also be investigated.

Note that in intercountry analysis it is necessary to distinguish between the impact of the ***level*** of economic growth (or of the level of wealth of nation) and the impact of the ***rate*** of economic growth on the economic structure by using more complicated models.

Thus in order to find sensible relations between indicators of socio-economic growth and economic structure the approach is to combine the $I-O$ methodology with econometric techniques. For example, if we divide energy consumption between the most energy intensive industries with the corresponding values of output $(X(1),\ldots,X(M))$, we then obtain a consistent forecast of the $\frac{Energy\ Consumption}{GDP}$ according to obvious changes in industrial structure:

$$\frac{Energy\ Consumption_{s,t}}{GDP_{s,t}} = \sum_{i=1}^{M} \beta_i * \frac{X(i)_{s,t}}{GDP_{s,t}}$$

Moreover, if some of the technical coefficients $a(i,j)$ for all countries are known we can implement them into the model as *a priori* given restrictions on values of β_j. Therefore, the general simplified model to be estimated in a linear form for each industry looks as follows:

$$\frac{X(i)_{s,t}}{GDP_{s,t}} = \alpha_0 + \sum_j \alpha_j * \frac{X(j)_{s,t}}{GDP_{s,t}} + \sum_\alpha \beta_\alpha * \frac{Y(\alpha)_{s,t}}{GDP_{s,t}} + \gamma * \frac{GDP_{s,t}}{L_{s,t}} + \delta * Time + \varepsilon(i)_{s,t} \quad (1)$$

where $\varepsilon(i)_{s,t}$ is an error term, and *Time* is time trend. Dividing all the values by *GDP* we eliminate possible heterogeneity of errors, because the scale of economies is quite different. We assume that the variance of the residuals should be roughly equal in percentage to the size of economy (represented by *GDP* value). Note that it looks very similar to the balance equation:

$$X(i)_{s,t} = \sum_{j=1}^{N} \alpha_{i,j} * X(j)_{s,t} + \sum_{\alpha} \gamma_{i,\alpha} * Y(\alpha)_{s,t} \qquad (2)$$

where $Y(\alpha)_{s,t}$ are final demand components given exogenously in forecasting and therefore they should be considered as predetermined variables in the model rather that $X(j)_{s,t}$ which are endogenous variables. All the parameters in (1) can vary in time and space to reflect some essential features mentioned above. At the first stage of the study they are supposed to be constant, therefore we can use standard regression techniques.

The first set of industries' outputs can be estimated by the ordinary least squares. For the second set we must take into account errors in explanatory variables if some of them are industry' outputs expressed as regressions in the first set. The simplest way to improve the quality of parameter's estimates is to use a reduced form for the second set, i.e., to consider all $X(i)_{s,t}$ as functions of predetermined variables only (right-hand variables of regressions in the first set). It means that we propose a recursive system of econometric equations whose parameters can be estimated by the ordinary and the two-stage least squares techniques.

4. Data

As it is stated above, we carried out the efforts at the aggregated level of industry output. These selected industries cover about 85 percent of the total industry output (mining and manufacturing) for every developed country. The first problem arising in a comparative study is the availability of homogeneous data and their capability for applying econometric techniques. International comparisons of changes in economic structure over time depend mainly on the quality of data for a selected set of countries. This study confines comparisons to the last 20 years. Investigation of long-term trends and patterns of economic structure development needs a reliable set of variables for all countries to be made on similar assumptions (same methodology). Therefore, valid modeling requires a price comparability that cannot be obtained by use of exchange rates to convert different currencies to one common currency. It stems from the fact that only small parts of goods are exchanged (in trade) between countries and such calculated ratio as that, selected for foreign trade goods cannot be applied to an economy as a whole, especially for any certain aggregated industry output. It is also obvious that the weights to be applied in the calculation of converting coefficients to composite industry (as far as our study of aggregated industry is concerned) can be derived by no one strict method. The use of two countries' price weights would yield two different industry output comparisons. Fisher's ideal index is computed to reestimate all the outputs into the *$USA*.

This study deals mainly with values of gross output for selected aggregated industries, therefore we began with an aggregation of activities (products) from 3 digit *ISIC* to 13 industry classification according to the system based on the *USSR* statistic methodology (the concept of the material product accounting) used in the model of interindustry interactions [1] - [3]. The values of gross outputs for 3 digit *ISIC* are available in national currency for all countries. We developed the time-series of the outputs in constant 1970 prices for all countries and then reestimated them in 1970 *$USA*. To do this we needed to apply purchasing power parities for all products or for aggregated industries. These values have been developed in the Institute of World Economy and International Affairs of the *USSR* Academy of Sciences.

With the exception of the Gross Industry Outputs values, the *Energyconsumption* values by type of fuels are used in this study. These values are measured in physical terms and have been taken from "*WorldEnergySupplies*". The value of the output of *construction* as a branch of an economy is also used.

In the first stages, we operated with only one indicator of socio-economic development: Final Domestic Product (*FDP*). Its value is was obtained from the corresponding (*GDP*) components measured in constant *$USA*. The difference between *FDP* and *GDP* is attributed to the fact that in an examination of the interindustry interaction we had to test the connection between exchange of materials and services and growth of industries. Accounting of economic indicators in the *USSR* is based on the concept of two kinds of activities in economy: productive and nonproductive spheres. It considers such services as finance, housing, health care, and education, as nonproductive activities. The reason for this is proved by the system of distribution of value added in an economy. Those services whose output is a result of redistribution of material sphere profits are excluded from *GDP* to obtain *FDP* value. Nevertheless, intermediate deliveries to nonproductive services are included in *FDP* as a part of final demand; they are elements of government consumption (social consumption), and these differences do not significantly change those ranges of countries measured by *GDP* or *FDP* per capita (see for example the difference between *National Income* and *Net Material Product* estimated by two techniques for the most developed countries [4,p.123]).

To obtain the comparable data for *FDP* we used the values of the Purchasing Power Parities (*PPP*) developed by Kravis and et. [5] for all components of *GDP* for most countries.

Thus, all *FDP* values are measured in constant *$USA* for the year 1970 by usage *PPP* (one exception is the *Canada* data). Because the *USA* and *Canada* economies are strongly interconnected and the majority of products are on trade between countries, this was a reason enough to use an exchange rate equal to 1.01 in the reestimation of all *Canadian* data to the *\$USA*.

5. An Example of the Econometric Model (Preliminary Results)

To clarify the technique used we concentrate on one industry, *Ferrous Metals*. It is clear that there is no strict relationship between *FDP* per capita and *ferrous metals* industry gross output per *FDP* across 6 countries, but there is an obvious decreasing trend (in time or in respect to an increase in *FDP* per capita) for the six countries' sample.

If we estimate simple regressions with dummy variables

$$\frac{X(1)_{s,t}}{FDP_{s,t}} = \sum_{s=1}^{6} Dummy_s + \beta_1 {*} FDP^* + \varepsilon_{s,t} \tag{3}$$

$$\frac{X(1)_{s,t}}{FDP_{s,t}} = \sum_{s=1}^{6} Dummy_s + \beta_2 {*} Time + \varepsilon_{s,t} \tag{4}$$

the goodness of fit are found to be quite appropriate: $R^2 = 0.9181$ and 0.9316, $V = 13.48\%$ and 12.31%, respectively; the parameter estimates of β_1 and β_2 are -0.0037 and -0.0007 and both are insignificant. (Here and below, R^2 is a coefficient of determination, V is an average error in percentage to the average of a dependent variable, values in parentheses (below parameter estimates) are standard errors of those estimates). Note that in both equations only the dummies' estimates are significant that is clarified by the simplest form in which only the dummy for each country is an explanatory "variable":

$$\frac{X(1)}{FDP} = \sum_{s=1}^{6} Dummy_s + \varepsilon_{s,t} \tag{5}$$

The dummies are "explaining" the variance of $\frac{X(1)_{s,t}}{FDP_{s,t}}$ with $R^2 = 0.9147$ and $V = 13.69\%$ and all the estimates are significant at 95% level. In respect to the *U.K.'* level the dummies have a positive effect on the values of $\frac{X(1)}{FDP}$ for *FRG* and *Japan* and a negative impact on those for *Canada*, *France* and *USA*.

The very opposite approach considers the relation between the values

$\frac{X(1)_{s,t}}{FDP_{s,t}}$ and *FDP* per capita assuming that the slope coefficients are the same in space and time. It is clear that the results of estimation are very poor:

$$\frac{X(1)}{FDP} = \underset{(0.0082)}{0.0999} - \underset{(0.0034)}{0.0170} * \frac{FDP}{L} \qquad (6)$$

with R^2 = 0.1907 and V = 41.34%.

$$\frac{X(1)}{FDP} = \underset{(0.0213)}{0.0645} + \underset{(0.0191)}{0.0167} * \frac{FDP}{L} - \underset{(0.0041)}{0.0073} * \left[\frac{FDP}{L}\right]^2 \qquad (7)$$

with R^2 = 0.2148 and V = 40.92%.

$$\frac{X(1)}{FDP} = \underset{(0.0283)}{0.1569} - \underset{(0.0072)}{0.0305} * \frac{FDP}{L} - \underset{(0.0253)}{0.0532} * \left[1 / \frac{FDP}{L}\right] \qquad (8)$$

with R^2 = 0.2234 and V = 40.69%.

Thus the list of explanatory variables for the *Ferrous Metals* industry have to consist of the outputs for at least two metal consuming industries: *Machine Building* and *Basic Building Materials Producing* industry. The last one can be expressed by *Construction* industry output itself.

Assuming that the error term $\varepsilon_{s,t}$ has zero mean value and a constant (equal across the countries) variance we can estimate the parameters of a model by ordinary least squares. This means that we believe in roughly equal errors for all countries included in the pooled data. This is proved partly by the fact that the dependent variable is measured as the ratio $\frac{\textit{Industry output}}{FDP}$.

Note that in future we shall use the following abbreviations for the variables:

industry output per *FDP* is written as $x(i)$; i.e., $x(i) = \frac{X(i)_{s,t}}{FDP_{s,t}}$;

fdp is used for *FDP* per capita value;

Inv denotes the share of *investment* in *FDP* (in percentage);

Equi denotes the share of *equipment* in *investment* (also in percentage);

and the following indices are used:

i = 1 is an index for the *Ferrous Metals* industry,

i = 7 – for *Machine Building* industry,

i =10 – for *Basic Building Materials Producing* industry,

i = 14 – for *Construction*;

$\bar{x}$ denotes the average (for each country) values of corresponding variables;

x' denotes the deviation on time of x around its mean value, i.e.,

$x' = x_t - \bar{x}$.

An important variable should be added into the model: the ratio between levels of the *basic building materials producing* industry output and *construction* output $\frac{x(10)}{x(14)}$ reflecting the structure of raw material inputs in construction.

Although there are many explanatory variables, not all of them are seriously correlated and we can choose those which are approximately orthogonal. Obviously there is no need to represent in the model the level of metal consuming industries output $\overline{x(7)}$, $\overline{x(10)}$, $\overline{x(14)}$ because they are closely connected with the predetermined variables $\overline{fdp}$, $\overline{Inv}$, $\overline{Equi}$.

With the full set of the variables mentioned above, the model looks as follows:

$$x(1) = \beta_0 + \beta_1{*}x'(7) + \beta_2{*}x'(14) + \beta_3{*}fdp' + \beta_4{*}Inv' + \beta_5{*}Equi' \qquad (9)$$
$$+ \beta_6{*}\frac{x(10)}{x(14)} + \beta_7{*}Time + \beta_8{*}\overline{fdp} + \beta_9{*}\overline{Inv} + \beta_{10}{*}\overline{Equi} + \varepsilon_{s,t}$$

By this approach one does not need any "unexplained" dummies for each country because intercountry differences in the dependent variable are neglected by the whole set of real explanatory variables.

Assuming that the level of *FDP* per capita effects the development level of the *Ferrous Metals* industry (measured by its mean values for each country), and that the growth rates of the two metal consuming industries effect the growth over time of the *Ferrous Metals* industry, we have estimated the alternative versions of the model and the results of the estimation of the model with the above mentioned full set of variables are given in Table 1. This means that the output of the *Ferrous Metals* industry is closely connected with the growth of the *Machine Building* and the *Basic Building Materials Producing* industries. As far as the levels of the both industries are in close relation (and is considered so in the model) to the *FDP* per capita and $\overline{inv}$, $\overline{equi}$ the terms $\bar{x}(7)$ and $\bar{x}(14)$ can be replaced by the last terms. The parameters estimates of (9) are shown in Table 1.

Because not all estimates are significant, we found that simplified versions of the model are preferred when some variables are omitted, for example fdp'. It gives approximately the same R^2 at 0.96 and V at 9.5%.

The similar equations can be estimated for this industry in logarithms:

$$\begin{aligned} log[\, x(1)\,] = \beta_0 + \beta_1{*}\log[\, x''(7)\,] + \beta_2{*}log[\, x''(14)\,] + \beta_3{*}log[\, fdp''\,] \\ {*}log[\, fdp''\,] + \beta_4{*}log[\, Inv''\,] + \beta_5{*}log[\, Equi''\,] + \beta_6{*}\log[\, \frac{x(10)}{x(14)}\,] \\ + \beta_7{*}Time + \beta_8{*}\overline{fdp} + \beta_9{*}\overline{Inv} + \beta_{10}{*}\overline{Equi} + \varepsilon_{s,t} \end{aligned} \qquad (10)$$

where x'' denotes the ratio $\frac{x(i)}{\bar{x}(i)}$) and high estimates of elasticities are found (see also Table 2). Thus 1% average annual growth in $x(7)$ and $x(14)$ leads to 1.2% and 0.7% growth in $x(1)$ respectively. That seems reasonable in view of the great impact of technological progress, i.e., -2.75% (see β_7 in Table 2).

This "intercountry" approach is also fruitful for finding other important factors that determine existed large differences in patterns of economic growth across developed countries. In future, it is useful when testing the results in

Table 1. Parameters' estimates for the general linear model (9).

$\hat{\beta}_i$ when insignificant variables consequently are omitted					
$\hat{\beta}_0$	-0.1536 (0.0118)	-0.1840 (0.0181)	-0.1538 (0.0118)	-0.1577 (0.0118)	-0.1137 (0.0078)
$\hat{\beta}_1$	0.1375 (0.0315)	0.1582 (0.0322)	0.1373 (0.0313)	0.1080 (0.0284)	0.2076 (0.0204)
$\hat{\beta}_2$	0.3169 (0.1870)	0.4026 (0.1012)	0.3695 (0.1019)	0.1706 (0.0336)	0.2295 (0.0341)
$\hat{\beta}_3$	0.0383 (0.0099)	0.0308 (0.0103)	0.0387 (0.0098)	0.0444 (0.0095)	
$\hat{\beta}_4$	-0.0014 (0.0013)	-0.0019 (0.0008)	-0.0017 (0.0008)		
$\hat{\beta}_5$	-0.0002 (0.0006)				
$\hat{\beta}_6$	0.3664 (0.0135)	0.3656 (0.0132)	0.3662 (0.0135)	0.3675 (0.0137)	0.3462 (0.0141)
$\hat{\beta}_7$	-0.0023 (0.0003)	-0.0021 (0.0004)	-0.0023 (0.0003)	-0.0027 (0.0003)	-0.0015 (0.0002)
$\hat{\beta}_8$	0.0080 (0.0014)	0.0110 (0.0019)	0.0080 (0.0014)	0.0082 (0.0014)	0.0067 (0.0015)
$\hat{\beta}_9$	0.0033 (0.0001)	0.0036 (0.0002)	0.0033 (0.0001)	0.0033 (0.0001)	0.0032 (0.0001)
$\hat{\beta}_{10}$	0.0001 (0.0006)	0.0005 (0.0002)			
R^2	0.9657	0.9653	0.9637	0.9621	0.9539
$V(in\%)$	8.85	8.90	9.07	9.21	10.11

applications obtained by the usage of complicated models, if between trends an increasing gap is given for different economies. Obviously high or low rates of *FDP* per capita are associated with significant long-term shifts in the economic structure and these shifts are more similar in developed countries than one would expect. This is not surprising because the interindustry interactions are strong enough in every economy with respect to restricted amounts of resources to be allocated between them and because the impact of different

Table 2. The most important parameters' estimates for the log-linear model (10) obtained for the reduced samples with one particular country dropped.

Estimate of β_i	Country name that is dropped from the sample						
	No	FRG	Canada	Japan	France	USA	UK
$\hat{\beta}_1$	1.1938	1.3374	1.2854	0.6976	1.0858	1.2453	1.2677
	(0.1495)	(0.1500)	(0.1629)	(0.1806)	(0.1511)	(0.1169)	(0.1755)
$\hat{\beta}_2$	0.6893	0.8542	0.6420	0.5072	0.8121	0.4970	0.9141
	(0.1633)	(0.1735)	(0.1638)	(0.1434)	(0.1919)	(0.1468)	(0.1914)
$\hat{\beta}_6$	1.1432	1.2738	1.1171	1.2642	1.0020	1.0058	1.4946
	(0.0491)	(0.0573)	(0.0487)	(0.0941)	(0.0780)	(0.0403)	(0.0996)
$\hat{\beta}_7$	-0.0284	-0.0298	-0.0268	-0.0276	-0.0203	-0.0274	-0.0259
	(0.0106)	(0.0034)	(0.0034)	(0.0027)	(0.0038)	(0.0026)	(0.0043)
R^2	0.9343	0.9227	0.9342	0.9606	0.9174	0.9691	0.9425
$V(in\%)$	4.41	4.07	4.47	3.43	4.41	3.33	4.30

industries in economic growth measured by *FDP* is not the same.

Some preliminary results show that expansion of intercountry analysis to interindustry interaction modeling provides a consistent basis for long-term forecasting when all industry output values are generated by a system of simultaneous equations estimated on the pooled data. However changes in time for every economy do not depend only on the rates of economic growth, so we need to assume that the level of *FDP* per capita is not the only main factor.

The results of estimation do not change significantly when data from one country is dropped from the sample (see Tables 2 and 3). However we notice difference between fitted and real values are large for two countries (Canada and USA), therefore it is better to use the data for the model when those countries are considered as one region or dropping both of them. This is also justified by the following test. Assuming that all parameters' estimates differ slightly for each specific country, we can reestimate them singly by the following:

To find $\gamma_{s,0}$ and $\gamma_{s,1}$ for the model

Table 3. Parameters' estimates for the linear model (9) obtained for the reduced samples (the results correspond to the 5-th column of Table 1).

	Country name that is dropped from the sample						
Estimate of β_i	No	*FRG*	*Canada*	*Japan*	*France*	*USA*	*UK*
$\hat{\beta}_0$	-0.1577 (0.0118)	-0.1490 (0.0141)	-0.1655 (0.0118)	-0.1441 (0.0158)	-0.1794 (0.0164)	-0.1390 (0.0116)	-0.1491 (0.0206)
$\hat{\beta}_1$	0.1080 (0.0284)	0.1332 (0.0295)	0.0862 (0.0297)	0.0908 (0.0396)	0.0675 (0.0329)	0.1484 (0.0280)	0.1028 (0.0310)
$\hat{\beta}_2$	0.1706 (0.0336)	0.1875 (0.0394)	0.1575 (0.0321)	0.1577 (0.0410)	0.2077 (0.0383)	0.1617 (0.0322)	0.1664 (0.0440)
$\hat{\beta}_3$	0.0444 (0.0095)	0.0367 (0.0095)	0.0525 (0.0098)	0.0277 (0.0152)	0.0568 (0.0112)	0.0311 (0.0094)	0.0480 (0.0112)
$\hat{\beta}_6$	0.3675 (0.0137)	0.3643 (0.0153)	0.3615 (0.0133)	0.3714 (0.0147)	0.3939 (0.0228)	0.3424 (0.0140)	0.3947 (0.0300)
$\hat{\beta}_7$	-0.0027 (0.0003)	-0.0025 (0.0003)	-0.0030 (0.0003)	-0.0022 (0.0004)	-0.0028 (0.0003)	-0.0024 (0.0003)	-0.0028 (0.0004)
$\hat{\beta}_8$	0.0082 (0.0014)	0.0078 (0.0020)	0.0104 (0.0014)	0.0084 (0.0015)	0.0093 (0.0016)	0.0047 (0.0015)	0.0053 (0.0039)
$\hat{\beta}_9$	0.0033 (0.0001)	0.0033 (0.0001)	0.0033 (0.0001)	0.0033 (0.0001)	0.0034 (0.0001)	0.0034 (0.0001)	0.0031
R^2	0.9621	0.9484	0.9652	0.9642	0.9539	0.9727	0.9646
$V(in\%)$	9.21	9.78	7.99	9.91	8.56	8.20	9.88

$$x(1) = \gamma_{s,0} + \gamma_{s,1}*[\ \hat{\beta}_1*x'(7) + \hat{\beta}_2*x'(14) + \hat{\beta}_3*fdp' + \hat{\beta}_4*Inv' + \hat{\beta}_5*Equi' \qquad (11)$$
$$+ \hat{\beta}_6*\frac{x(10)}{x(14)} + \hat{\beta}_7*Time + \hat{\beta}_8*\overline{fdp} + \hat{\beta}_9*\overline{Inv} + \hat{\beta}_{10}*\overline{Equi}\ ] + \varepsilon_{s,t}$$

where $\hat{\beta}_i$ is the i-the parameter estimate found for general sample. The following set of γ_1 estimates is usually obtained:

Country					
FRG	*Canada*	*Japan*	*France*	*USA*	*UK*
0.940	0.278	0.970	0.815	0.543	0.936

One can find that the smallest slope soefficients are for *USA* and Canada and for every model the greatest errors are attributed to these two countries therefore the following results were obtained on a sample including only *FRG*, *Japan*, *France*, *UK*:

$$x(1) = -0.1538 + 0.1178*x'(7) + 0.1554*x'(14) + 0.0442*fdp' + 0.3533*\frac{x(10)}{x(14)}$$
$$(0.0144) \quad (0.0351) \quad (0.0342) \quad (0.0118) \quad (0.0156)$$

$$- 0.0029*Time + 0.0078*\overline{fdp} + 0.0034*\overline{Inv}$$
$$(0.0004) \quad (0.0024) \quad (0.0001)$$

with R^2 at 0.9688 and V at 7.69%, and $\beta_1 = 1.4531$, $\beta_2 = 0.3218$, $\beta_6 = 0.9794$, $\beta_7 = -0.0287$ for the log-linear model with R^2 at 0.9690 and V at 3.44%, in comparison with the results given in Table 2.

During 1960-1977 the residuals in the regression are of different signs for every country and their absolute level has decreased over the last years. It is very useful to incorporate this equation into a forecasting model. The validity of the equation is proved by the stability of slope coefficients when the size of the sample has been changed (see tables 2 and 3). The level of economic growth and share of investment in *FDP*, together with share of equipment in investment, provide a significant goodness of fit. Other for pooled sample equations were estimated to check the nonlinear effects in the intercountry pattern of structural changes.

By this approach one can predict the future structure of an developed economy with *a priori* knowledge on the *GDP rates* of growth and about the *investment* policy. Also, it is necessary to take into account that the quality of the future structure prediction will be good enough because the estimates of the recursive model parameters are quite stable.

References:

[1]. Y.V.Yaremenko. Strusturnye izmenenia v socialisticheskoj ekonomike (Structural changes in a socialist economy, in Russian), *Moscow*, *Mysl*, 1981, *pp.* 160–211.

[2]. Y.V.Yaremenko, E.B.Ershov, and A.S.Smyshlayev. A Model of Interindustry Interactions in the *USSR*, IIASA, 1980, CP-80-40.

[3]. A.S.Smyshlayev. Model of Interindustry Interactions as a System of Simultaneous Equations, IIASA, 1982, WP-82-28.

[4]. Narodnoe Khoziajstvo SSSR v 1975 godu (Statistical Yearbook: USSR Economy in 1975, in Russian), *Moscow*, *Statistica*, 1976.

[5]. Kravis, Irving B., Heston, Alan W., and Summers, Robert. Real *GDP* per capita for more than one hundred countries, *Economic Journal*, 1978, *June*, *pp*. 215–242.

PART IV

GENERAL TOPICS

INTERNATIONAL TRADE LINKAGES IN INTER-REGIONAL I/O ECONOMIC MODELING: THE MODEL FOR THE TUSCANY CASE STUDY

A. Cavalieri

1. INTRODUCTION

Even if the national economic variables of regions within a nation and the linkages between them are the core of the analysis in most interregional and multiregional models, in some regions, the international trade linkages have a specific and relevant local role.

The impacts of the changes in the international division of labor over a nation are not uniform with respect to the single regions. In fact, the national import and export figures are pure statistical entities, because the different export and import specialization and foreign market orientations at regional level.

In addition, some regions have a specific role in acting as trade intermediaries towards foreign countries by distributing import flows and by collecting export flows via interregional trade.

In spite of these considerations, international trade flows are very seldom regarded as affected by regional determinants.

This paper deals with the treatment of the international linkages and relationships between foreign flows and interregional ones in the context of a biregional I/O model which will be built for the Tuscany region, within the Tuscany Case Study Project.*

The specific interest in analyzing the international trade from a bottom-up point of view, derives from the fact that the regional export, in Tuscany, has shown a high elasticity of its weight on national export according to the international demand cycles (A. Cavalieri, 1980).

The regional high specialization in some typical "mature" products is the main explanatory factor of such strong fluctations. The reaction to the international variables affecting the export of the kind of commodity is fairly different in Tuscany and in the rest of Italy. A top-down appraoch based on fixed share of national export flows does not produce, in this case, significant results at the regional level.

In addition, the model is aimed to take into consideration the interrelations between interregional and international trade which is normally neglected, even if they plan an important role in a small, open regional economy.

In the first part of the paper, different approaches are surveyed with reference to some existing models including regional I/O matrices even if they were not designed at a biregional level. In fact, we have faced the nature of such models by reformulating them in order to carry on some possible approaches in treating international and interregional trade in an I/O biregional model.

The choice of the models as well as their reformulation into a biregional context, is a very personal one.

Our selection was made in order to introduce the second part of the paper which aims at describing the approach we have followed in designing the model for the Tuscany region with respect to the international trade components.

---

*This project is being carried out by the Regional Development Group of IIASA in cooperation with IRPET (Regional Institute for Economic Planning of Tuscany) and IASI (Institute for Systems Analysis and Informatics).

2. SOME EXAMPLES ON DIFFERENT APPROACHES IN THE TREATMENT OF INTERNATIONAL AND INTERREGIONAL TRADE

As we stated above, in this paragraph, some examples derived from existing models will be shown. It is important to note that we will not deal with the procedures used in estimating the trade coefficients or the gross flows; attention will be given only to the approach used in including the international and interregional trade into an I/O model.

Because the models analyzed were designed in order to focus attention on different aims, the original structures are quite different. The examples we have chosen are nevertheless the closest to the model specified for the Tuscany region, even if this one is a biregional one whereas most surveyed models are multiregional ones.

An extensive comparative study of multiregional economic models is currently performed by the Regional Development Task at IIASA, and the Free University, Amsterdam (see Nijkamp and Rietveld, 1980). Within this study, a preliminary comparison on interregional and international linkages in multiregional economic models was carried out by F. Snickars (1981).

Our limited survey is strictly oriented to the specification of the biregional I/O model for the Tuscany region; for this reason, a common list of relations is used in order to make easier the comparison between the different approaches and the scheme designed for the Tuscany model.

In the rest of the paper, therefore, the following notations are used:

Variables:

X_T, X_R = gross production vector in Tuscany and the rest of Italy;

F_T, F_R = internal final demand vector in Tuscany and the rest of Italy;

M_T^R, M_R^T = interregional import vector in Tuscany and the rest of Italy;

E_T^W, E_R^W = foreign export vector in Tuscany and the rest of Italy;

M_T^W, M_R^W = international import vector in Tuscany and the rest of Italy;

E_T^R, E_R^T = interregional export vector in Tuscany and the rest of Italy;

E_N^W, E_{NT}^W = vector and scalar of national export;

NET, NER = net interregional trade in Tuscany and the rest of Italy.

Parameters:

A_T, A_R = I/O matrix of technological coefficient;

μ_T, μ_R = diagonal matrix of foreign import shares of demand components, Tuscany and the rest of Italy;

$\hat{\mu}_T$, $\hat{\mu}_R$ = diagonal matrix of foreign import shares of gross production, Tuscany and the rest of Italy;

μ'_T, μ'_R = diagonal matrix of foreign import shares of intermediate consumption, Tuscany and the rest of Italy;

μ''_T, μ''_R = diagonal matrix of foreign import shares of internal final demand, Tuscany and the rest of Italy;

m_T, m_R = diagonal matrix of interregional import shares of demand components, Tuscany and the rest of Italy;

$\hat{m}_T$, $\hat{m}_R$ = diagonal matrix of interregional import shares of gross production, Tuscany and the rest of Italy;

m'_T, m'_R = diagonal matrix of interregional import shares of intermediate consumption;

m''_T, m''_R = diagonal matrix of interregional import shares of internal final demand, Tuscany and the rest of Italy;

ε_T, ε_R = vector of foreign export share of national export (scalar), Tuscany and the rest of Italy;

ε'_T, ε'_R = diagonal matrix of foreign export share of national export (vector), Tuscany and the rest of Italy.

2.1 The Scheme of the REGAL Model

This multiregional optimization model for the allocation of private and public investment, production, employment and population over economic sectors and regions, (see F. Snickars and A. Granholm, 1981) is not aimed to treat explicitly interregional and international trade.

Nevertheless, the linkages between the regional economic variables and the external trade flows are treated in a way which is normally followed in many other interregional models based on the top-down approach.

If we reformulate the REGAL model in a biregional one, and we specify only the external flows, we get the following structure:

$$X_T + M_T^W + M_T^R = (A_T X_T + F_T) + E_T^W + E_T^R \quad ,$$

$$X_R + M_R^W + M_R^T = (A_R X_R + F_R) + E_R^W + E_R^T \quad ,$$

where

$$M_T^W = \hat{\mu}_T X_T \quad ; \quad E_T^W = \varepsilon_T \, E_N^{\bar{W}} \quad ; \quad E_T^R = M_R^T = \hat{m}_R X_R \quad ,$$

$$M_R^W = \hat{\mu}_R X_R \quad ; \quad E_R^W = \varepsilon_R \, E_N^{\bar{W}} \quad ; \quad E_R^T = M_T^R = \hat{m}_T X_T \quad .$$

In terms of the multiplier, we have:

$$X_T = [(I + \hat{\mu}_T + \hat{m}_T) - A_T]^{-1} \cdot (F_T + \varepsilon_T \, E_N^{\bar{W}} + \hat{m}_R X_R) \quad ,$$

$$X_R = [(I + \hat{\mu}_R + \hat{m}_R) - A_R]^{-1} \cdot (F_R + \varepsilon_R \, E_N^{\bar{W}} + \hat{m}_T X_T) \quad .$$

Both the imports from the other region and from the rest of the world are determined endogenously as a fixed share of the regional gross production; in the current literature that it means they are supposed to be of a complementary type. The import flows do not affect the matrix A and the final demand components according to the scheme: final demand → I/O matrix → gross production → import.

No specification is given on the import share of the I/O coefficients and of the final demand components as can be shown in Figure 1.

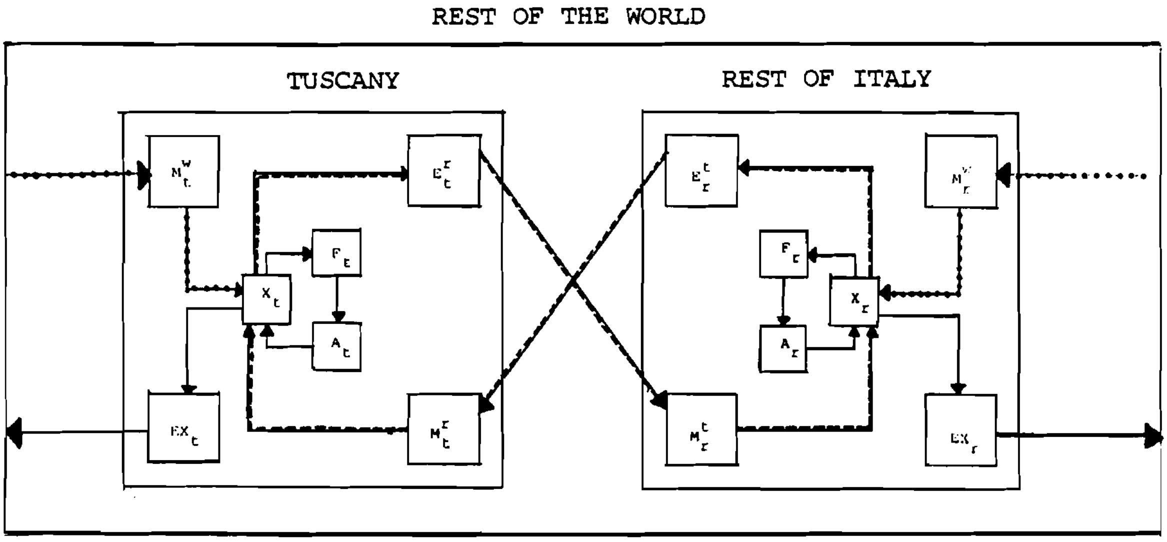


Figure 1. External trade scheme in the REGAL model.

Looking at Figure 1 it is also clear that the interregional trade does not include foreign production, i.e. it is supposed that the regional import meets only complementary needs <u>inside</u> the importer region itself.

As far as the regional export flows to the rest of the world are concerned, they are estimated as a fixed share of the national vector, which is exogenously derived from a national model where constraints by BOP (Balance of Payment) are supposed to exist.

This model is designed in order to focus attention on the interregional impact of some planning policies. All the variables that are not subjected to planning policy (as the international flows are) are treated in a simple way which becomes even simpler when the model is reformulated in a biregional form.

2.2 The Scheme of the VERDI Model

This biregional input-output model (P. Costa, L. Malfi, D. Martellato, 1980) was designed in order to analyze the interdependencies between a single region and the rest of the country.

It is an open and static model where the components of the final demand are exogenously determined.

The input-output model is closed only with regard to inter-regional trade, according to the Chenery-Moses approach (Chenery and Cao-Pinna, 1953), applied to Italy more than twenty years ago.

The general structure of the VERDI model is the following:

$$X_T = [(A_T X_T + \overline{F}_T) - M_T^W - M_T^R] + \overline{E_T^W} + E_T^R \quad ,$$

$$X_R = [(A_R X_R + \overline{F_R}) - M_R^W - M_R^T] + \overline{E_R^W} + E_R^T \quad ,$$

where

$$M_T^W = \mu_T \; (A_T X_T + \overline{F_T}) \quad ; \quad E_T^R = M_R^T = m_R \; (A_R X_R + \overline{F_R}) \quad ,$$

$$M_R^W = \mu_R \; (A_R X_R + \overline{F_R}) \quad ; \quad E_R^T = M_T^R = m_T \; (A_T X_T + \overline{F_T}) \quad ,$$

by substituting, we have:

$$X_T = [I - (1-m_T-\mu_T)A_T]^{-1} \cdot [(1-m_T-\mu_T)F_T + m_R(A_R X_R+F_R) + \overline{E}_T^{\overline{W}}] \quad ,$$

$$X_R = [I - (1-m_R-\mu_R)A_R]^{-1} \cdot [(1-m_R-\mu_R)F_R + m_T(A_T X_T+F_T) + \overline{E}_T^{\overline{W}}] \quad .$$

The imports from the other region as well as from the rest of the world are specified as a fixed share of the demand components, both intermediate and final ones, excluding export flows as shown in Figure 2.

The internal demand is split into domestic and external components; by this way, imports are supposed to be competitive and the multiplier effect is estimated according to the following scheme: final demand (reduced by import propensity) → I/O matrix (where the coefficients are reduced by the same import propensity) → gross production.

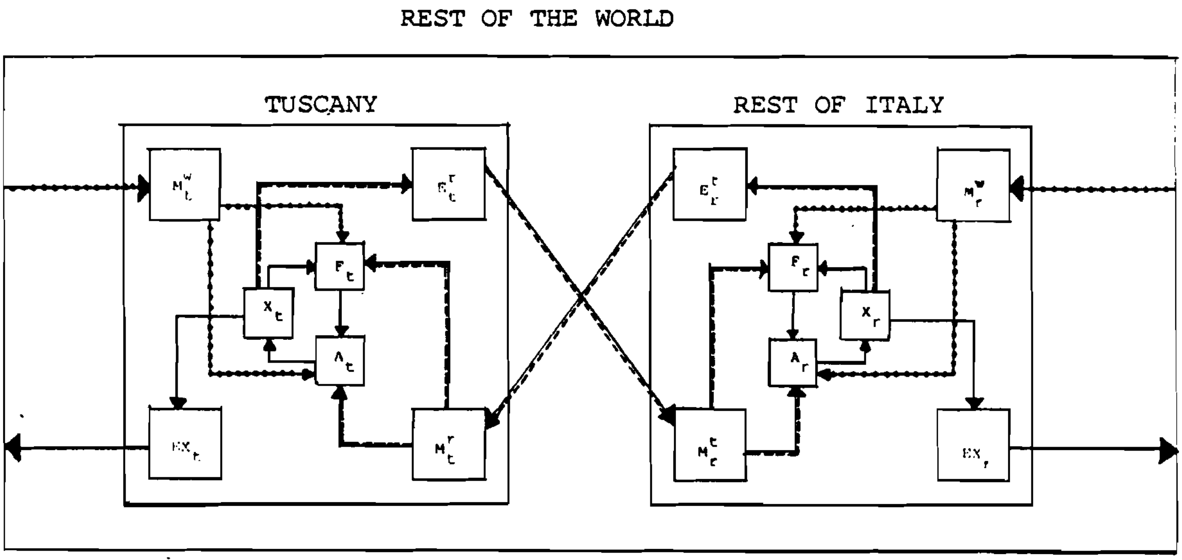


Figure 2. External trade scheme in the VERDI model.

As in the former model, also here, the foreign production goods are not included in the biregional trade which is concerned only with national production consumed inside the country.

The model is designed in order to evaluate the impact effects on the economy of a region from changes in the national economic variables, with particular reference to interdependencies due to the trade patterns between the regions and the rest of the country.

2.3 The Scheme of the MORSE Model

The MORSE model, (L. Lundqvist, 1981), is a dynamic multiregional I/O model aimed to integrate economic, employment and energy planning in a regional perspective.

Theoretically, the model combines traits from input-output analysis, interregional programming and equilibrium growth theory.

As the model is concerned mainly with an objective function representing goals in economic, employment and energy planning, the external trade flows are not the core of the analysis. Nevertheless, some specifications in the treatment of the linkages to the rest of the world within a top-down approach, could be interesting even in a biregional I/O model more oriented to a better specification of the international trade.

By reformulating the MORSE model's structure into a bi-regional model, we get the following structure:

$$X_T + M_T^W = [(A_T X_T + F_T) - M_T^R] + E_T^W + E_T^R \quad ,$$

$$X_R + M_R^W = [(A_R X_R + F_R) - M_R^T] + E_R^W + E_R^T \quad ,$$

where

$$M_T^W = \hat{\mu}_T X_T \quad ; \quad E_T^R = M_R^T = m_R' A_R X_R + m_R'' F_R \quad ; \quad E_T^W = \varepsilon_T' \, E_{TN}^W \quad ,$$

$$M_R^W = \hat{\mu}_R X_R \quad ; \quad E_R^T = M_T^R = m_T' A_T X_T + m_T'' F_T \quad ; \quad E_R^W = \varepsilon_R' \, E_{TN}^W \quad .$$

In terms of the multipliers, we have:

$$X_T = [(I+\hat{\mu}_T) - (1-m_T')A_T]^{-1} \cdot [F_T(1-m_T'') + m_R' A_R X_R + m_R'' F_R + \varepsilon_T E_{TN}^W \quad ,$$

$$X_R = [(I+\hat{\mu}_R) - (1-m_R')A_R]^{-1} \cdot [F_T(1-m_R'') + m_T' A_T X_T + m_T'' F_T + \varepsilon_R' E_{TN}^W \quad .$$

As far as the scalar of national export is concerned, E_{TN}^W is constrained at the national level by BOP goals:

$$E_{TN}^W = \overline{BOP} - M_N^W \quad ,$$

$$E_{TN}^W = \overline{BOP} - (\hat{\mu}_T X_T + \hat{\mu}_R X_R) \quad .$$

A specific feature of this model is the different treatment of import between interregional and international flows (Figure 3). Whereas the first ones are considered as competitive and therefore are linked to demand components, the second ones are supposed to be complementary and are linked to regional gross production.

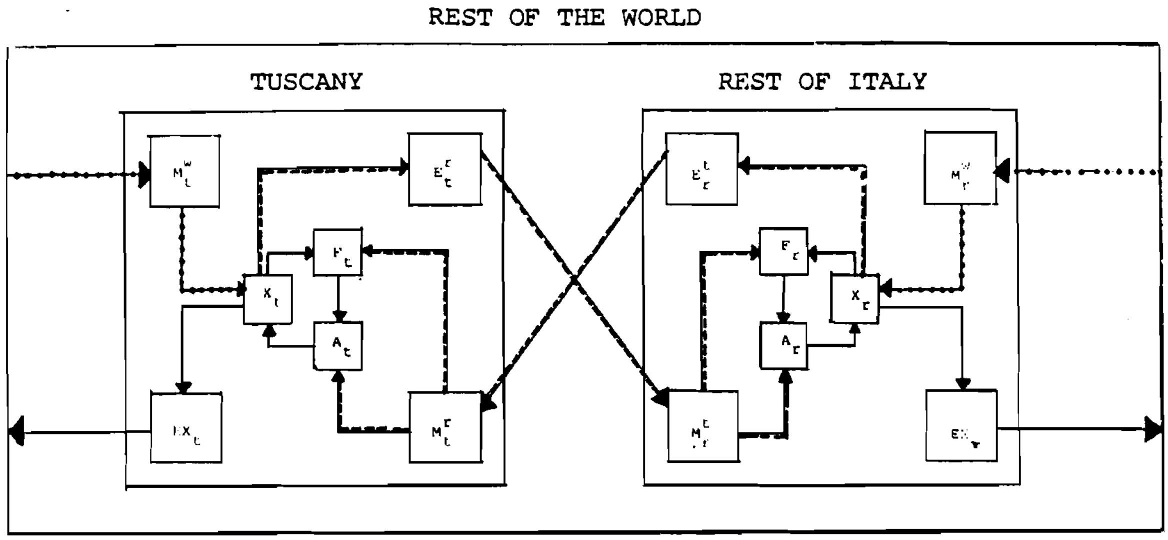


Figure 3. External trade scheme in the MORSE model.

In addition, by permitting different rates of self-sufficiency for intermediary goods and for final demand, the Chenery-Moses framework is somewhat extended.

As in the other models surveyed until now, the interregional trade is concerned only with domestic production demanded inside the country.

As far as the regional export is concerned, a matrix share is used to breakdown at the regional and sectoral level the total national export which is derived from a national econometric model.

In order to avoid the strong assumption on the fixed coefficients of that matrix the parameters ε'_T, ε'_R are subjected to change over time, according to the trends in the sectoral mixing of the national export.

The same procedure is adopted for parameters $\hat{\mu}_T$, $\hat{\mu}_R$ which link regional import to gross production in the region.

In matrix terms, the vector ε'_i (in region $_T$ and region $_R$) as well as the diagonal matrix $\hat{\mu}$ (in region $_T$ and region $_R$), is split into two components in the following way:

$$\begin{bmatrix} \varepsilon'_1 \\ \vdots \\ \varepsilon'_i \\ \vdots \\ \varepsilon'_n \end{bmatrix} \begin{bmatrix} \varepsilon^*_1 & & & & \emptyset \\ & \ddots & & & \\ & & \varepsilon^*_i & & \\ & & & \ddots & \\ \emptyset & & & & \varepsilon^*_n \end{bmatrix} \begin{bmatrix} \varepsilon^t_1 \\ \vdots \\ \varepsilon^t_i \\ \vdots \\ \varepsilon^t_n \end{bmatrix}$$

where

$$E^t_i = \frac{E_{Ni}}{E^W_{TN}} \text{ at time } t \quad ,$$

and

$$\begin{bmatrix} \hat{\mu}_1 & & & & \emptyset \\ & \ddots & & & \\ & & \hat{\mu}_1 & & \\ & & & \ddots & \\ \emptyset & & & & \hat{\mu}_n \end{bmatrix} = \begin{bmatrix} \mu^*_1 & & & & \emptyset \\ & \ddots & & & \\ & & \mu^*_i & & \\ & & & \ddots & \\ \emptyset & & & & \mu^*_n \end{bmatrix} \cdot \begin{bmatrix} \mu^t_1 & & & & \emptyset \\ & \ddots & & & \\ & & \mu^t_i & & \\ & & & \ddots & \\ \emptyset & & & & \mu^t_n \end{bmatrix}$$

where

$$\mu^t_i = \frac{M^W_{Ni}}{M^W_{TN}} \text{ at time } t \quad .$$

While both the diagonal matrix ε^* and μ^* are taken as fixed over time, the diagonal matrix μ^t and the vector ε^t are subjected to change over time, according to historical trends or short-term forecasts.

In this way, the consistency between the sectoral mixing at national and regional levels is achieved for each time t.

The model contains a typical national policy variable in order to constrain the foreign flows: the balance of payment (BOP). Because of the high dependence of the export from exogenous international variables, by determining BOP, the model (closed with respect to the other final demand components) indicates the level of the gross production which is consistent with the export flows.

2.4 The Scheme of the REGIS Model

The REGIS model (R. Courbis and G. Cornilleau, 1978) is a reduced form of the national REGINA model. Both models are of the multiregional, interderdependent kind, build around a core constituted by a production model of input-output type.

The sectoral disaggregation, as well as the territorial one, is highly detailed; the model takes into account the interregional mobility of many economic and social factors.

Even if the REGIS is one of the most comprehensive and sophisticated multiregional models, the interregional trade is not specified in terms of gross flows. On the other hand, the REGIS model is a good example of a detailed and theoretically justified treatment of foreign trade at a regional level.

By reducing the REGIS model to a biregional one, we lose the main specific characteristics of this model; in addition, by focussing our attention on the external trade flows at a regional level, we analyze a part of the model which is not specified as the multiregional trade is concerned.

By bearing in mind these strong limitations, the biregional structure of the REGIS model is a very simple one:

$$X_T = (A_T X_T + F_T) - M_T^W + E_T^W + NET \quad ,$$

$$X_R = (A_R X_R + F_R) - M_R^W + E_R^W + NER \quad ,$$

where

$$NET = -NER \quad .$$

The multiplier of the model is the traditional Leontieff inverse matrix:

$$X_T = [I - A_T]^{-1} \cdot (F_T - M_T^W + E_T^W + NET) \quad ,$$

$$X_R = [I - A_R]^{-1} \cdot (F_R - M_R^W + E_R^W - NET) \quad .$$

In this structure no direct impact of the interregional trade flows is supposed to operate in the production and demand formation (see Figure 4), where as many other factors (not taken into consideration here) have many indirect effects via interregional mobility or interdependencies.

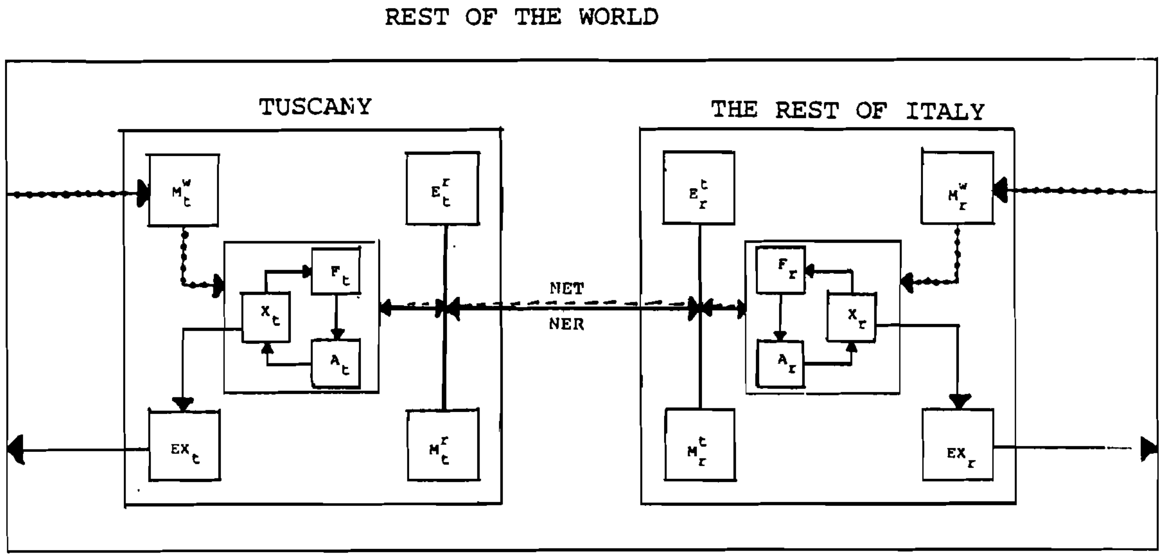


Figure 4. External trade scheme in the REGIS model.

The REGIS model employs import and export functions econometrically specified at the regional level; there is a different treatment of import with respect to competitive or complementory products. Here the difference between competitiveness and complementarity is more theoretically based than in most models. Competitiveness is assumed to be determined only by national variables. In this case, the regional import is estimated as a fixed share of the national one:

$$M_T^W = K_T\, M_N^W \;, \qquad M_R^W = K_R\, M_N^W \;.$$

For complementary goods, instead, it is assumed that import flows are linked to the growth of regional demand and to the production prices related to import prices, via an econometric function at the regional level:

$$M_T^W = \alpha_T\, DT_T^{bT} \cdot (P/PM)^{CT} \;,$$

$$M_R^W = \alpha_R \; DT_R^{b_R} \cdot (P/PM)^{CR} \quad ,$$

where

DT = vector of regional total demand (index numbers);
P = vector of production price at the national level;
PM = vector of import at the national level;
b,c = elasticity parameters (demand, relative price).

Foreign export is also estimated by econometric functions with regional specifications based on different elasticity parameters, where the independent variables are the export prices at the national level (PX):

$$E_T^W = \gamma_T + \beta_T PX^{K_T} \quad ,$$

$$E_R^W = \gamma_R + \beta_R PX^{K_R} \quad .$$

In addition to the equations there are national sums corresponding to regional export and import, both endogenous. Thus, foreign trade is altogether regionally specified in this model, which is a very unusual property in the current modeling practice where the top-down approach is normally used.

3. A BIREGIONAL I/O MODEL DESIGNED FOR THE TUSCANY REGION

This model is designed in order to have a core to which are linked some submodels oriented to analyzing specific aspects of the regional realty and the interdependencies with the rest of the nation and the rest of the world.

The core of the model is constituted by the I/O matrix for the Tuscany region which is at present available for the years 1975 and 1977 (built by an indirect procedure) and for the year 1978 (built by a direct survey procedure).

The linkages between Tuscany and the rest of Italy are based on the trade coefficients matrix which links the Tuscany I/O matrix and the rest of Italy, and on an interregional population model which will be developed into a demo-economic interregional model.

The linkages between Tuscany and the rest of the world via international trade are provided both by a regional model of export (bottom-up approach) and by a breakdown from the INFORUM project (top-down approach) on which IRPET is working in order to link Italy to the INFORUM model (C. Almon and D. Nyhus, 1977).

Particular emphasis is given in the model to the treatment of public components of the final demand in order to achieve a detailed specification of the linkages between the public authority expenditures (especially at the local level) and the productive sectors.

The general scheme of the models system that is developed within the Tuscany case study is shown in Figure 5.

This paper deals only with the parts of the model, as well as their linkages, that are concerned with external trade. These parts are shown in the scheme by doubled-lined squares; they are concerned with regional export and tourism model (box A), international export and import linkages - INFORUM (box B), regional linkages demand for consumption (only tourist consumption) and exports (box C-1), and interregional trade linkages (box C-2).

3.1 General Structure of the Model with Reference to Interregional, International Trade Flows and Their Linkages

The interregional trade interdependencies are treated in this model, to some extent, as in the VERDI model; in addition, a linkage between interregional flows and international ones is modeled in order to take into account also the re-exports from the importer region, according to the following two schemes:

> foreign production → foreign regional import → interregional export

> foreign regional export ← interregional import ← regional production

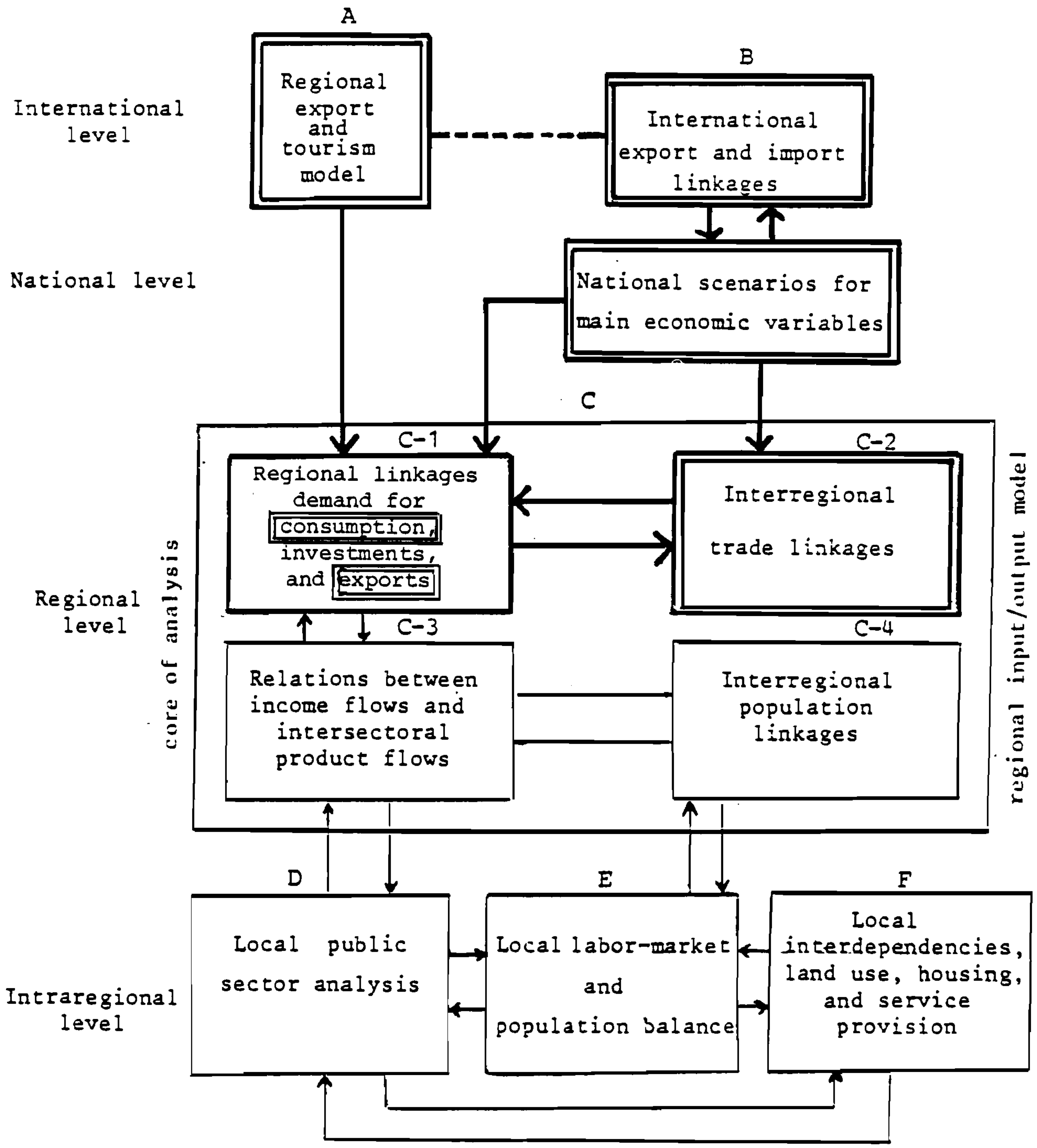


Figure 5. Tuscany case study: general model scheme.

Foreign regional import is not supposed to be re-exported (via interregional trade) to foreign markets.

According to these assumptions, the general structure of the model can be designed in the following terms:

$$X_T = [(A_T X_T + F_T) - M_T^W - M_T^R] + E_T^W + E_T^R \quad ,$$

$$X_R = [(A_R X_R + F_R) - M_R^W - M_R^T] + E_R^W + E_R^T \quad ,$$

where

$$M_T^W = \mu_T \ (A_T X_T + F_T) + \mu_T^{**} \ m_R \ (A_R X_R + F_R) \quad ,$$

$$M_R^W = \mu_R \ (A_R X_R + F_R) + \mu_R^{**} \ m_T \ (A_T X_T + F_T) \quad ,$$

and

$$M_T^R = m_T^* \ [(A_T X_T + F_T) + m_T^{**} \ E_T^W)] \quad ;$$

$$M_R^T = m_R \ [(A_R X_R + F_R) + m_R^{**} \ E_R^W)] \quad ,$$

by taking into account that

$$E_T^R = M_R^T \quad ; \qquad E_R^T = M_T^R \quad ,$$

we have

$$X_T = (1-\mu_T^*-m_T^*)(A_T X_T+F_T) + (1-\mu_T^{**}) \ m_R^* \ (A_R X_R+F_R) + m_R^{**} E_R^W + (1-m_T^{**}) \ E_T^W \quad ,$$

$$X_R = (1-\mu_R^*-m_R^*)(A_R X_R+F_R) + (1-\mu_R^{**}) \ m_T^* \ (A_T X_T+F_T) + m_T^{**} E_T^W + (1-m_R^{**}) \ E_R^W \quad .$$

The final structure of the model in terms of input-output inverse matrix is given by the following equations:

$$X_T = [I - (1-\mu_T^*-m_T^*)A_T]^{-1} \cdot [(1-\mu_T^*-m_T^*)\ F_T$$
$$+ (1-\mu_T^{**})\ m_R^*\ (A_R X_R+F_R)$$
$$+ m_R^{**}\ E_R^W + (1-m_T^{**})\ E_T^W] \quad ,$$

$$X_R = [I - (1-\mu_R^*-m_R^*)A_R]^{-1} \cdot [(1-\mu_R^*-m_R^*)\ F_R$$
$$+ (1-\mu_R^{**})\ m_T^*\ (A_T X_T+F_T)$$
$$+ m_T^{**}\ E_T^W + (1-m_R^{**})\ E_R^W] \quad .$$

The interregional and international trade flows are connected as can be illustrated by Figure 6.

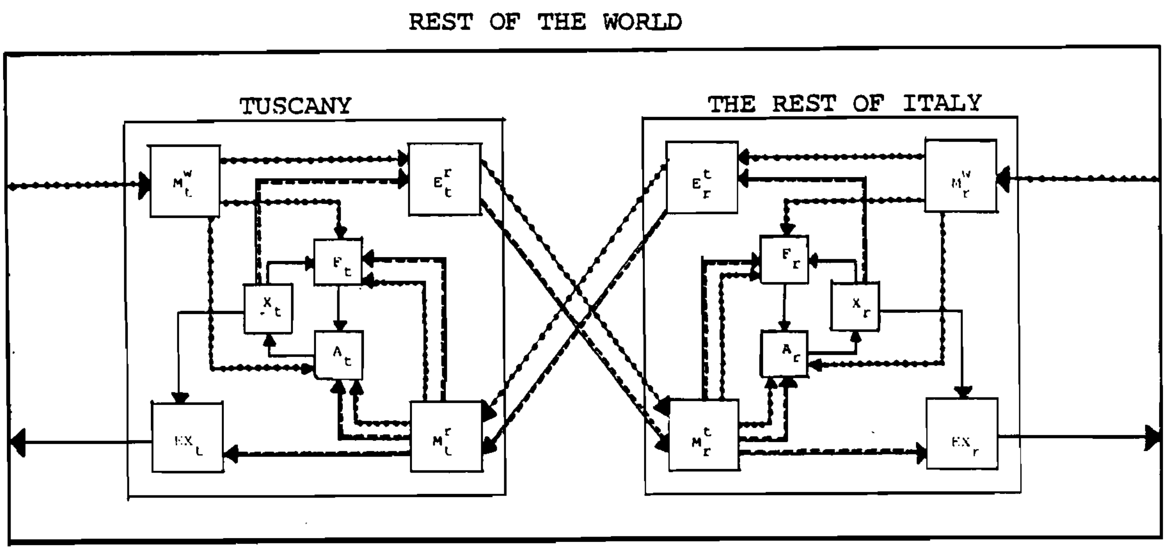


Figure 6. External trade scheme in the Tuscany model.

As it is shown in this figure, the interregional trade is concerned also with imported foreign production and with exported domestic production.

The split of the national import into two parts (used inside the importer region and traded outside the imput region) is clear when we aggregate the two regions.

By defining $A_T X_T + F_T = D_T$ and $A_R X_R + F_R = D_R$ we obtain

$$
\begin{aligned}
(X_T+X_R) &= (D_T+D_R) - (\mu_T D_T+\mu_R D_R) - (\mu_R m_T D_T+\mu_T m_R D_R) \\
X_N &= D_N - M_N^{W*} - M_N^{W**} \\
&\qquad + (E_T^W+E_R^W) \;, \\
&\qquad + E_N^W \;,
\end{aligned}
$$

where

M_N^{W*} = national import used <u>inside</u> the importer regions (Tuscany and the rest of Italy);

M_N^{W**} = national import trade <u>outside</u> the importer regions (re-exported import).

3.2 The Treatment of Import Flows and Interregional Trade

As far as the import flows are concerned, the main distinction which is usually made is between competitive and complementary imports. Whereas in the economic theory such a distinction is clear (complementary when no domestic substitute is available in the region, competitive when it is), in reality, as well as in modeling work, it is not so. Firstly, by working on a sectoral level (branches of the input-output matrix), each sector includes both competitive and complementary products. Secondly, even if goods are produced in the region, the level of the regional production could not be high enough to meet the needs of the internal demand.

In the regional modeling work it is often assumed that all the import flows are either competitive, or complementary; in addition, by applied fixed coefficients also in the treatment of competitive imports, the distinction with the complementary import is smooth and it has no real economic basis. Taking into account these problems, the imports are often considered either as competitive or complementary according to a very crude assumption. If the import does not affect the production formation, the production is determined by the following equation.

$$X = (I - A)^{-1} (D + E) \quad ,$$

whereas the imports are determined by a second equation:

$$M = m_X \qquad \text{or} \qquad M = m(D + E).$$

In this case, the import flows are supposed to be complementary.

In an opposite way, the imports are treated as competitive when they are included in the determination of the production via the following equation:

$$X = (I - mA)^{-1} (D + E) \quad ,$$

if they are competitive with respect to the intermediate products demand, and via the following equation:

$$X = (I - A)^{-1} (D + E - M) \quad ,$$

if they are competitive with respect to the final demand components.

In this last case, the imports are modeled in a similar way to that used in the Keynesian models within an open economy. Taking into account these crude definitions, in the biregional I/O model for the Tuscany region, the imports are considered competitive both for interregional and international flows.

In fact, this approach implies only the assumption that a proportional linkage is supposed to link import and internal demand components via fixed coefficients, i.e. no substitution effect between internal and foreign production is taken into consideration.

As far as the interregional trade coefficients are concerned, the traditional Chenery-Moses procedure is modified in order to include in the interregional trade also the foreign imports and exports. The trade coefficients equations are reformulated in the following way if $m_T^{*} = m_T^{**} = m_T$; and $m_R^{*} = m_R^{**} = m_R$.

$$X_T + M_T^W = (1-m_T)(D_T+E_T^W) + m_R(D_R+E_R^W) \quad ,$$

$$X_R + M_R^W = (1-m_R)(D_R+E_R^W) + m_T(D_T+E_T^W) \quad ,$$

where $(1-m_T)$, as well as $(1-m_R)$ are the self-sufficiency rate with regard to interregional trade.

M_T^W and M_R^W are linearly dependent both from the regional demand and from the interregional export (i.e. import of the other regions) demanded for internal utilization through the parameters μ_T^*, μ_R^* and μ_T^{**}, μ_R^{**}. If $\mu_T^* = \mu_T^{**} = \mu_T$ and $\mu_R^* = \mu_R^{**} = \mu_R$, we get:

$$\mu_T = \frac{M_T^W}{D_T + m_R D_R} \quad \text{and} \quad \mu_R = \frac{M_R^W}{D_R + m_T D_T} \quad ,$$

As in the MORSE model, the parameters μ_T and μ_R are subjected to change over time according to the expected changes in the sectoral mixing of the import at the national level derived via the INFORUM model.

Future steps in the modeling work will take into account the possibility of treating import flows (or some of them) as real competitive ones. A possible approach could be the one used in the REGIS model, where an economic function links the import flows to some independent variables such as regional demand, price levels, etc., via elasticity parameters. Even in this case the INFORUM model could provide consistent scenarios at the national level.

3.3 The Treatment of Export Flows and Tourism

Because of the important economic role of the linkages to the foreign markets, the model for the Tuscany region devotes great attention to a regional determination of the export flows, both of commodities and services (tourists).

A bottom-up approach is used for those commodities which have a high weight in the export mixing of the region and, at the same time, are supposed to have specific behavioral functions as far as the regional level is concerned.

On the other hand, for commodities whose export functions are not supposed to have regional specifications, we assume that a top-down approach is the most appropriate one.

Whereas in the first case (bottom-up approach), econometric functions at the regional level are used, in the second case (top-down approach), the regional vectors are derived from national forecasts, achieved via INFORUM model to which Italy will be linked in the near future.

In both cases transition matrices are used in order to convert commodities into I/O sectoral branches (31 in Tuscany I/O matrix).

In addition to a highly detailed commodity disaggregation each commodity export function is separately estimated for some groups of foreign commodities, between Tuscany and the rest of Italy.

Figure 7 describes the submodel for the export flows of commodities as it is designed in the Tuscany biregional I/O model.

The following notations are used:

$E^{W}_{t,g,c}$, $E^{W}_{r,g,c}$ = export of Tuscany and the rest of Italy by commodity g (UICC classification*) and country c;

$E^{W}_{n,g,c}$ = export of Italy by commodity g (SITC classification**) and country c;

$T^{UICC}_{i,g}$, $T^{SITC}_{i,g}$ = transition matrices converting commodities in SITC and UICC classification into I/O sectoral branches;

* UICC (Unione Italiana Camere di Commercio) classification is adopted in Italy for export and import data at the provincial level, derived from foreign payment flows.

** SITC (Standard International Trade Classification) classification is adopted by the United Nations and by OECD for the international trade.

$E^W_{t,i(o,m)}$, $E^W_{r,i(o,m)}$ = vectors of regional export by sectors (o-m), derived from the bottom-up approach;

$E^W_{t,i(m,n)}$, $E^W_{r,i(m,n)}$ = vectors of regional export by sectors (m-n), derived from the top-down approach;

$$E^W_{n,g};\ E^W_{t,g};\ E^W_{r,g} = \sum_c E^W_{n,g,c};\ \sum_c E^W_{t,g,c};\ \sum_c E^W_{r,g,c} \quad ,$$

$$E^W_{n,i(o,m)};\ E^W_{n,i(m,n)} = E^W_{t,i(o,m)} + E^W_{r,i(o,m)} \quad ;$$

$$E^W_{t,i(m,n)} + E^W_{r,i(m,n)} \quad .$$

The econometric regional export functions are estimated by techniques applied to independent variables at the international and national level.

Both time series and cross-section data are used, the latter by grouping the single countries into market areas. All the variables are expressed in constant prices.

The regional specification is achieved by estimating different elasticity parameters between Tuscany and the rest of Italy. In addition, by getting each set of parameters for each country and commodity combination, the country share composition, by itself, implies different estimations of the export trends between Tuscany and the rest of Italy, even if the elasticity was the same.

Regional exports are treated by an additive demand and price relationship:

$$E^W_{T,g,c} = K_{T,g,c} + a_{T,g,c} \left(D_c \cdot \frac{D_c}{D_N}\right)^{b_{T,g,c}} \cdot RP_{g,c}^{\ c_{T,g,c}} \quad ,$$

$$E^W_{R,g,c} = K_{R,g,c} + a_{R,g,c} \left(D_c \cdot \frac{D_c}{D_N}\right)^{b_{R,g,c}} \cdot RP_{g,c}^{\ c_{R,g,c}} \quad .$$

These functions assume that only a part of the exports are price and/or demand dependent due to the constants $K_{T,g,c}$ and $K_{R,g,c}$.

The variable $RP_{g,c}$ is the index of the relative export prices as they are taken into account by the foreign customers. $RP_{g,c}$ is equal to the index of the national export price, divided by the index of price in country C; finally, the result is multiplied by the index of exchange between Italy and country C.

The second independent variable is a composite one. The index of the domestic demand in country$_c$ (D_c) is, in fact, corrected by the ratio between the same index and that of domestic demand in Italy.

Here it is assumed that a partial substitution effect between foreign market and the national one can influence the business cycle of regional export.

The role of the foreign demand is reduced or amplified according to the differentials in the growth rate between the foreign demand itself and the national one.

By adopting this composite variable, a linkage, even if a very simple one, is established between the regional foreign export and the interregional trade which is dependent, via trade coefficients, by the domestic demand in both regions.

As far as the part of the regional export vector derived from the national forecast (via the INFORUM model) is concerned, the following algorithm is used:

$$E_T^W = \varepsilon_T \, E_N^W \quad ; \qquad E_R^W = \varepsilon_R E_N^W \quad ,$$

for sectors i from m to n (see Figure 7).

Due to the role of external tourism in Tuscany, the tourist flows are treated as an additional commodity. The procedure is similar to that applied for the export flows, i.e. an econometric function in estimated for each country of origin (including the rest of Italy and Tuscany).

The dependent variable is the number of the tourist-days (T^C), whereas the independent variables, are the matrix of the demand in country $_c(D_c)$ and the national price index, deflated

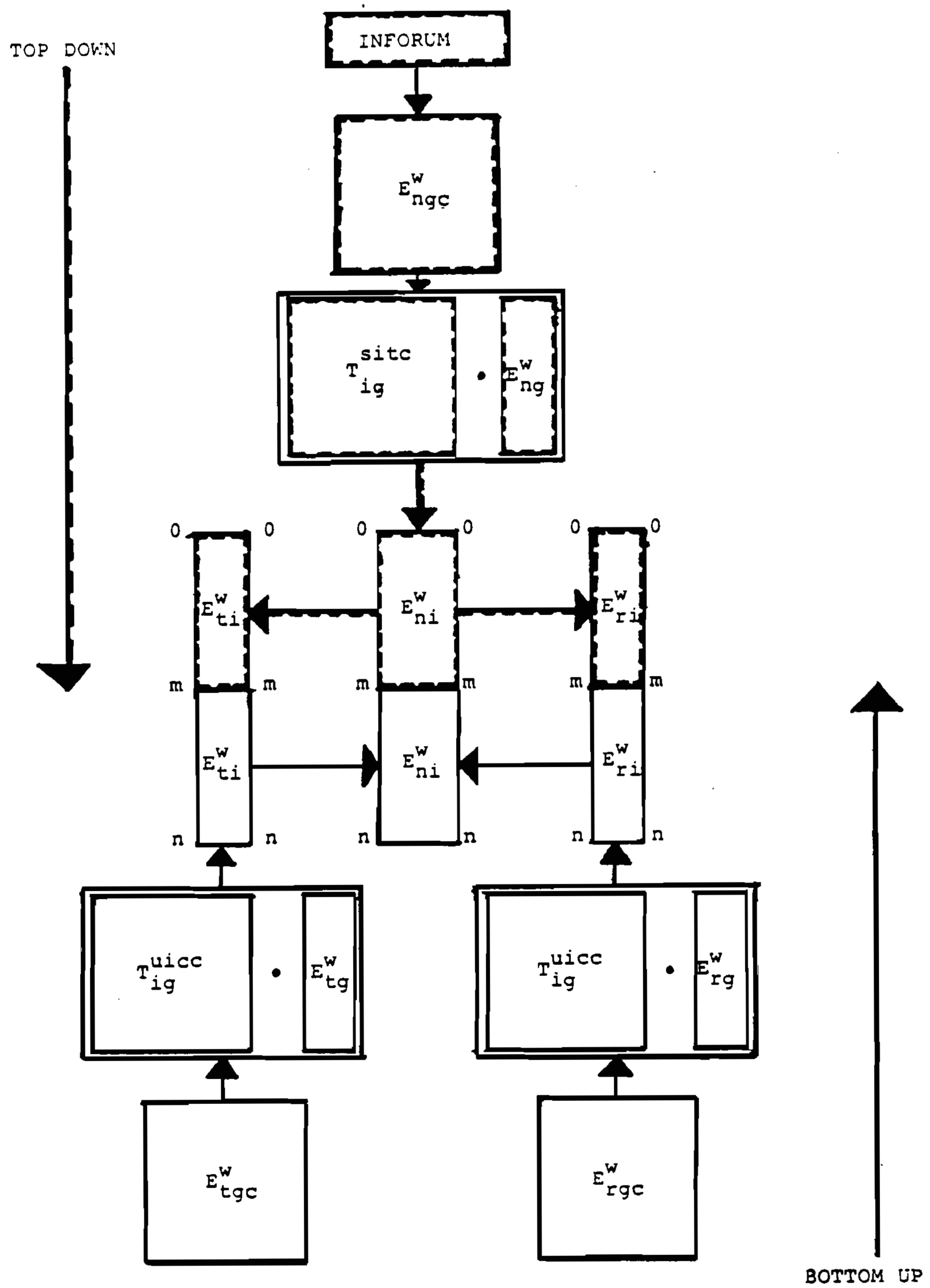


Figure 7. Export flows scheme.

by the price index and exchange index in country $_c(RP_c)$; this last variable is assumed to be equal to 1 for the Italian tourists.

The function used has the following form:

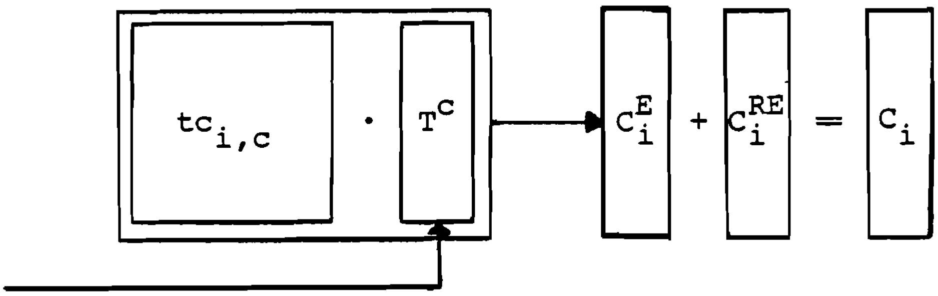
$$T_T^C = K_{T,c} + a_{T,c} \cdot D_c^{b_{T,c}} \cdot RP_c^{c_{T,c}} \quad ;$$

$$T_T^R = K_{T,R} + a_{T,R} \cdot D_N^{b_{T,R}} \quad ;$$

$$T_R^C = K_{R,c} + a_{R,c} \cdot D_c^{b_{R,c}} \cdot RP_c^{c_{R,c}} \quad ;$$

$$T_R^T = K_{R,T} + a_{R,T} \cdot D_N^{b_{R,T}} \quad ;$$

The estimations of the tourist-days are converted into the tourist-consumption through a transition matrix of the sectoral tourist comsumption by sector i and country$_c$ $(t_{ci,c})$ according to the following scheme:



,

where

C_i^E = consumption of non-residents;

C_i^{RE} = consumption of residents;

C_i = total private consumption in the region.

In the biregional model, the total private consumption, as a component of the final demand, assumes the following form in Tuscany and in the rest of Italy.

$$C^T = C_T^W + C_T^R + C_T^T \quad ,$$

$$C_R = C_R^W + C_R^T + C_R^R \quad .$$

It should be noted that a regional specification of the tourism functions, as well as those for export, is made possible by the availability of regional data over a long period of time.

4. CLOSING REMARKS (to be written)

The international trade is a weak point in many multiregional economic models, especially as far as the treatment of export is concerned.

The most common treatment of the export flows is to regionalize results of a national economic model. This methodology is appropriate for commodities where export functions are not supposed to have regional specifications. On the other hand, for those commodities supposed to have specific behavioral functions at the regional level, a bottom-up approach is preferred.

The model for the Tuscany region devotes great attention to a regional determination of the export flows both of commodities and services (foreign tourism), as well as the interdependencies between international and interregional trade. Even if the methodology adopted in modeling such linkages is a very simple one, the external trade relationships at the regional level is, to some extent, more comprehensive.

REFERENCES

Cavalieri, A. 1980. Development paths and placement within the international and intranational context: the Tuscany case. Paper presented at the Italian Regional Sciences Conference, November, Rome.

Nijkamp, P. and P. Rietveld. 1980. Towards a Comparative Study of Multiregional Models. WP-80-172. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Snickars, F. 1981. Interregional and International Linkages in Multiregional Economic Models. Laxenburg, Austria: International Institute for Applied Systems Analysis (forthcoming).

Snickars, F. and A. Granholm. 1981. A Multiregional Planning and Forecasting Model with Special Regard to the Public Sector. Laxenburg, Austria: International Institute for Applied Systems Analysis (forthcoming).

Costa, P., L. Malfi, and D. Martellato. 1980. Interregional Input-Output Analysis. Some Experiments with the Regional Model VERDI. Paper presented at the Italian Regional Sciences Conference, November, Rome.

Chenery, H.B. and V. Cao-Pinna. 1953. The Structure and Growth of the Italian Economy. U.S. Mutual Secutury Agency, Rome.

Lundqvist, L. 1980. A dynamic, multiregional input-output model for analyzing regional development, employment, and energy use. Paper presented at the 20th European Meeting of the Regional Science Association, August 26-29, Munich.

Courbis, R. and G. Cornilleau. 1978. The REGIS model: a national-regional REGINA model. Paper presented at the 18th European Congress Meeting of the Regional Science Association, Fribourg, Switzerland, August 29th to 1st September.

Almon, C. and D. Nyhus. 1977. The INFORUM international system of input-output models and bilateral trade flows. INFORUM Research Report 21.

ON THE CONCEPT OF GENERALIZED SETS OF ACCESSIBILITY AND THEIR CONSTRUCTION FOR LINEAR CONTROLLED SYSTEMS

A.V. Lotov

(Presented by Academician A. A. Dorodnitsyn, July 6, 1979)
(Submitted August 20, 1979)
Dokl. Akad. Nauk SSSR 250, 1081–1083 (February 1980)

PACS numbers: 02.30.Wd

1. In the investigation of controlled systems with the aid of mathematical models when there are several operation indicators for the system under study, use is made of the following basic approaches, which enable us to analyze the properties of the system and to give the person making the decision (PMD) the ability to choose the most suitable input signal to the system: 1) Estimation of the values of the indicators when there are a finite number of versions of the signal to the system (the simulation approach)[1]; 2) mathematical contraction (combination) of the indicators into a single indicator and subsequent optimization of it,[2] or solution of a series of optimization problems with gradual refinement of the nonformalized contraction of the indicators that is available in the representation of the PMD (vector optimization)[3]; 3) construction of the set of effective (nonimprovable) values of the indicators or the signals (the Pareto set in the space of indicators or controls).[3] In this article we present a new approach to the analysis of controlled systems with several indicators, an approach based on constructing generalized sets of accessibility.

2. Any mathematical model can be regarded as a set of constraints singling out admissible values of the variables among all possible values. If the number of variables is finite, then the model can be represented in the form

$$x \in G_x \subset X, \tag{1}$$

where x is the vector of variables, G_x is the set of admissible values, and the space X of variables of the model can be either finite-dimensional or some function space if, say, a part of the constraints consists of differential equations. Since the system under study is controlled, more than one point $x \in X$ satisfies the equations of the model (1). Suppose that the operation of the system is evaluated by a finite-dimensional vector of indicators f which is connected with the variables of the system by a mapping F,

$$f = Fx, \quad f \in E^r. \tag{2}$$

The proposed approach is based on the construction of the set G_f of all accessible values of the indicators f from the point of view of the model (1).

<u>Definition.</u> The set G_f defined by the relation

$$G_f = \{f \in E^r : f = Fx, \ x \in G_x\}, \tag{3}$$

will be called the generalized set of accessibility (the set of potential possibilities) of the model (1) with respect to the indicators (2).

This definition gives the set G_f in implicit form. By the approach on the basis of generalized sets of accessibility we mean the construction of the sets G_f in explicit

0038-5689/80/02 0082-03 $01.40

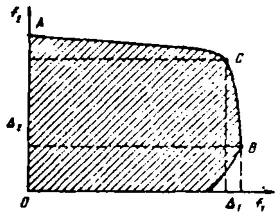


FIG. 1

form and their investigation. The proposed method makes it possible to investigate all possible values of the indicators (unlike in the simulation approach) and does not require us to contract the indicators in this or that form, as is done in vector optimization. Part of the boundary of G_f is the Pareto set in the space of indicators, and for convex sets G_x and linear indicators (2) the set G_f is convex, i.e., it can be (at least approximately) represented as the intersection of a finite number of half-spaces, while even in this case the Pareto set is not convex (see Fig. 1, where a typical set G_f is pictured; here the Pareto set is the curve ACB). For sufficiently aggregated linear models and indicators the set G_f can be constructed numerically with the help of the the methods presented here. Up to the present time sets of the type G_f have been used for explaining vector-optimization methods, but have not been used directly in the investigations because of the absence of effective methods for constructing them.

3. If X is the n-dimensional Euclidean space E^n and the set G_x is polyhedral, then the set G_f can be constructed as follows. The relations (1) and (2) determine a certain set Y in the direct sum of the spaces E^n and E^r:

$$Y = \{\{x, f\}: x \in G_x \subset E^n, f \in E^r, f = Fx\}. \quad (4)$$

The set G_f is the orthogonal projection of the set Y into the space E^r. Methods for constructing the projections of polyhedral sets specified in the form of a system of linear inequalities were considered in Ref. 4. Certain modifications of these methods requiring less operational computer memory and complemented by devices for discarding those inequalities in the description of the polyhedral sets whose removal does not change (or changes only slightly) the set described make it possible to construct the sets G_f for many practically interesting systems in the form

$$\{G_f = f: Df \leqslant d\}, \quad (5)$$

where D is a matrix and d is a vector. If the model (1) is nonlinear, but the set G_x is convex, then G_f can be approximated by a polyhedral set.

4. Suppose now that (1) is a linear dynamic model with linear constraints on the controls and phase coordinates, i.e., the model is described by:
the equations

$$y_{i+1} = A_i y_i + B_i u_i + a_i, \quad i = 0, \ldots, N-1, \quad (6)$$

where $y_i \in E^{m_y}$ is the vector of the phase coordinates, $u_i \in E^{m_u}$ is the control vector, and A_i, B_i, a_i are specified matrices and vectors;
the system of constraints

$$C_i^1 y_i + C_i^2 u_i \leqslant c_i, \quad i = 0, \ldots, N-1, \quad (7)$$

where C_i^1, C_i^2, c_i are specified matrices and vectors;
the initial condition

$$y_0 \in \Gamma_0 \subset E^{m_y}, \quad (8)$$

where Γ_0 is a specified polyhedral set.

This dynamic model can be reduced to the static model considered in the preceding section if we introduce the vector $x = (y_0, y_1, \ldots, y_N, u_0, \ldots, u_{N-1})$, which has dimension $(N+1)m_y + Nm_u$, after which we construct the generalized set of accessibility G_f by the method described above. However, it is possible to approach the construction of the set G_f on the basis of the usual set of accessibility Γ_N, i.e., the set of all points of the space of phase coordinates E^{m_y} into which the system (6), (7) can be transformed from Γ_0 after N steps. If we take the vector f to be the vector of the phase coordinates y_N, then the generalized set of accessibility G_f coincides with the set Γ_N (this is the origin of the term "generalized set of accessibility"). The sequence of sets of accessibility Γ_0, $\Gamma_1, \ldots, \Gamma_N$ can be constructed in the form (5) also by using algorithms for orthogonal projection of polyhedral sets,[5] after which, for arbitrary indicators depending on the final state of the system, the set G_f can easily be obtained from the set Γ_N and the relation (2), where the image depends only on those coordinates of the vector x that represent y_N. But if some linear indicator has "integral" form, then it is known[6] that this case can be reduced to the previous introduction of an additional phase coordinate.

5. Suppose that the model (1) under study is a system of controlled differential equations on the segment of time [0, T], but differs from the model (6)-(8) in that the dynamics of the phase variables is described by a differential equation instead of a multi-step equation. Then under certain natural conditions it turns out[7] that the set of accessibility $\Gamma(T)$ of this system can be approximated by the set of accessibility of a multi-step system of the type (6)-(8), and, thus, the problem of constructing the generalized set of accessibility G_f for such a system reduces to the problem considered in Sec. 4.

6. The set G_f constructed can be represented to the investigator or the PMD in the form of two-dimensional projections on the coordinate axes of the space E^r of the indicators, as well as in the form of all possible two-dimensional sections. The Computing Center of the Academy of Sciences of the USSR has now implemented the first version of a package of applied programs designed for constructing and analyzing ordinary and generalized sets of accessibility both in an automatic mode and in a dialogue mode with representation of the projections and sections on the screen of a terminal unit and on an alphanumeric printer.

To illustrate the approach described we consider some results of an investigation of one region in a model of the world economy. The economy of the region was described with allowance for the extraction and cleansing of pollutants, on the basis of a modification of the world-economy[8] model presented in Ref. 9 and plausible information. The model had the form considered in Sec. 3. The following vector of indicators was chosen: f_1 is a scalar characteristic of the consumption, f_2 is a characteristic of the cleanliness of the environment, f_3 is the balance of foreign trade. The section of the set Gf for $f_3 = 0$ is depicted in Fig. 1. A typical feature is the presence of a turning point C on the Pareto curves ACB such that on AC the growth of consumption f_1 leads to insignificant pollution of the environment, but after the level C, a relatively small increase Δ_1 in the consumption leads to a sharp increase Δ_2 in the pollution. This example demonstrates in an explicit graphic form the ability of the proposed approach to represent the limiting possibilities of the systems under study.

[1]Yu. P. Ivanilov and A. V. Lotov, Mathematical Models in Economics [In Russian], Nauka, Moscow (1979).

[2]Yu. B. Germeier, Introduction to the Theory of Operations Research [In Russian], Nauka, Moscow (1971).

[3]Problems in Analysis and Decision-Making Procedure [Collection of Russian translations], Mir, Moscow (1976).

[4]S. N. Chernikov, Linear Inequalities [In Russian], Nauka, Moscow (1968).

[5]A. V. Lotov, Zh. Vychisl. Mat. Mat. Fiz. 15, 67 (1975) [USSR Comput. Math. Math. Phys. 15, 63 (1975)].

[6]N. N. Moiseev, Elements of the Theory of Optimal Systems [In Russian], Nauka, Moscow (1975).

[7]A. V. Lotov, Zh. Vychisl. Mat. Mat. Fiz. 19, 44 (1979) [USSR Comput. Math. Math. Phys. 19, No. 1 (1979)].

[8]W. Leontief et al., The Future of the World Economy, New York (1977).

[9]A. G. Granberg and A. G. Rubinshtein, Ekon. Mat. Metody 15, 307 (1979).

Translated by H. McFaden

ON THE STRUCTURAL CHANGE OF THE RELATIONSHIP BETWEEN CONSUMPTION AND PRIMARY RESOURCES IN THE GDR ECONOMY

U. Ludwig, M. Kraft, J. Behr

The article outlines the structural change of the input of labour, fixed capital and primary energy caused by the growth of private und public consumption in the 70ies. The standard input-output model compiled with empirical data is used to find out the interdependence of the various inputs and the growth of consumption according to individual needs. The article presents some results obtained by comparisons of actual input-output tables from 1972 and 1977.

1. Introduction

2. Analyses of the development of structures of consumption as well as of the employment of resources

2.1. Development of the consumption according to complexes of needs

2.2. Development of the structure of labour and fixed capital input for the satisfaction of consumptive needs

2.3. Development of the energy consumption according to complexes of needs

3. Conclusion

1. Introduction

The national economy of the GDR year by year supplies the population with a growing volume of commodities that serve the increasing satisfaction of private and public needs. This is a basic feature of the unity between socialist economic and social policy in the GDR.

The division of labour, specialization and cooperation in the course of the history of social production together with its industrialization have resulted in a general stage of development where between the extraction of raw materials and the manufacturing of final products which are the direct object for the satisfaction of private and public needs, there are continually emerging more mediating links. Thus only part of all produced goods and services is included in the final consumption whereas their majority is used for productive purposes. The link between the production of goods and the satisfaction of needs becomes more and more complicate. Investigating this relationship in the highly developed modern national economies requires the application of certain mathematical techniques, by means of which the interdependence between the extreme moments of each reproduction process of the national economy, i.e. between the stage of production and the stage of consumption, can be represented in quantitative terms. National input-output tables are well suited for this purpose, since they reveal the relationship between social production and final demand not only in terms of volume but also in structural terms.

With the aid of national input-output tables can be calculated, whether the goals of final demand set up in long range programs of social and economic development can be met materially by the national economy. This isn't a task faced by the planning authorities only but also a challenge to fundamental economic research. Discarding the numerous mediating links concerning the production of commodities for the final use, their supply boils down to the proportionate allocation of the entire social labour to sectors, branches and stages of the reproduction process of the national economy as well as to their efficient utilization. Input-output models are suitable to investigate this problem too, allowing to deduce conclusions originating at the given level of technological and organizational development of the national economy from the volume and the structure of final demand and resulting in the allocation of entire social labour in the form of primary resources. Simultanously, by this a step can be done to reduce the analysis of the development of market relations necessitated in the framework of private consumption to its underlieing essence, i.e. to the reallocation of the entire social labour according to the shifts of needs. Subject of this paper is the presentation and analysis of the relationship between the consumption and the input of labour, fixed capital as well as of primary energy resources, calculated on the base of two actual input-output tables of the GDR economy from the 70ies. The paper presents results of similar research efforts continued from former years.[1]

2. Analyses of the development of structures of consumption as well as of the employment of resources

2.1. Development of the consumption according to complexes of needs

Depending on the detailed character of an investigation of the mentioned interrelation and on its horizon in time, it is appropriate to distinguish different levels of abstraction . In investigations for the short-range planning of the national economy, a relatively great degree of specification is necessary. Thus also the first calculation as a starting point for the annual plan of the GDR is performed by means of a static input-output model of about 600 items.[2] For the long-range planning, on the contrary, investigations with a lesser number of stronger aggregated positions provide more indications. To this end, in the GDR, input-output models of a medium aggregation level with about 30 items are suggesting themselves.

Something similar applies to the final use.
A number going into the thousands of different services and goods are consumed. Their combination into homogeneous entities which are called complexes of needs is advocated by the fact that according to our experience, there are certain dependencies between the satisfaction of various needs (accomodation and furniture, motor cars and fuel ...). Besides, it has been proved by consumption research that major aggregates of needs

are more correlated with such economic variables as the national income. Additionally, the formation of complexes of needs which are relatively independent of each other, provides the possibility to define additive utility functions which correspond well with the starting point of the linear equation of the static input-output model.[3]

In the concrete formation of such complexes of needs, it is necessary to take into account a series of other conditions. Thus, for instance, the horizon in time of the investigation, the degree of homogeneity of the aggregated goods and services, the uniqueness of the attribution of goods and services to needs, the organizational structure of the economy, the manner of the acquisition and of the processing of statistical data, as well as other factors are playing a major role in this respect.

Each ordering of the goods and services according to complexes of needs is therefore up to a certain degree a compromise solution between the aim of the analysis and the available statistics Here, we are proceeding from the following complexes of needs:[4]

1- nutrition
2- clothing
3- accommodation
4- health care
5- education/culture/recreation
6- communication/information
7- transport
8- general public needs

These complexes of needs are easy to survey and they develop in a relatively independent manner. From them, it is possible to better investigate and guide by purposeful measures certain continual changes of the structures of needs and of consumption habits, than this would be possible, if individual products are measured in isolation from each other. The definition of the conceptions for development and the substantiation of priorities or ranking scales in the satisfaction of needs for longer periods of time are facilitated in this manner.

Proceeding from the basic equation of the standard static input-output table

$$Y = (I-A)X \qquad (1)$$

at first, the individual components of the consumption vector are attributed to the 8 complexes of needs. For this end, on the basis of various statistical documents, it is estimated, which goods or services must be attributed to which degree to the individual complexes of needs.[5]

We use the following designations:

A - inter-industry coefficient matrix
X - column vector of the gross domestic output
Y - column vector of the final demand
C^D - vector of consumption expressed as a diagonal matrix
N - matrix of the distribution of the goods and services of the economic sectors for the consumptive final demand according to the complexes of needs
$N^{(k)}$ - column vector of those goods and services from matrix N which constitute the complex of needs k
Z - bridge matrix transforming the nomenclature of goods and services by sectors into the nomenclature by complexes of needs.

The distribution of the individual goods and services of the consumption according to the complexes of needs is calculated then from the equation:

$$N = C^D Z \qquad (2)$$

In the investigation, we here proceed from time-independent, constant elements of the matrix Z.

Table 1 conveys an impression of the dynamism of growth and of the structural development of those consumer goods and services which were provided in 1972 and 1977 for the individual complexes of needs of consumption. These changes are in accordance with regularities in the upward development of needs. Thus with regard to the dynamism of growth, it is possible to state a more than average development in the provision of such consumer goods and services which serve to satisfy the complexes of needs of transport, general public needs, health care, as well as of education/culture/recreation. There has been an average development in the provision of consumer goods and services for the complexes of needs of accommodation and communication/information. However, in this respect, it is necessary to bear in mind that the indications about accommodation do not include the final product for residential construction and for the modernization of housing which belongs to the investments. There has been a below than average rise in the provision of consumer goods and services for the complexes of needs of nutrition and clothing.

In this manner, in the seventies, in the GDR a regularity has been strengthened which could be observed within former periods of time, according to which in the entire structure of the final consumption the relative share of those needs which can be called primary, elementary or basic needs in a certain sense, in the entire needs, according to their limit of saturation remains rather constant or is even decreased. In the other complexes of needs which can be called in the same limited sense as secondary, raised or luxury needs and which are more derived from the social development of Man and of the social institutions, an opposed tendency is imposing itself. This structural shift between primary and secondary needs can be attributed above all to a relative decline in the increase of those goods and services which can be included into the complex of needs of nutrition.

Table 1

Structure and growth of the final demand for consumption in the GDR from 1972 to 1977 according to complexes of need

Complex of needs	Structure of consumption in %		Average annual growth rate in %
	1972	1977	
Nutrition	46,8	42,7	3,3
Clothing	12,1	11,6	4,4
Accomodation[1]	16,6	16,8	5,5
Σ Primary needs	75,5	71,1	4,0
Health care[1]	4,7	5,5	8,5
Education[1]/culture/ recreation	3,2	3,3	6,2
Communication/ information	4,0	3,9	4,9
Transport	8,1	10,8	11,4
General public needs	4,5	5,4	9,2
Σ Secondary needs	24,5	28,9	8,8
Consumption total	100,0	100,0	5,2

[1]without the non-productive investments in residential construction, the health service and the education system

Source: Calculations of the Central Institute for Economic Sciences

2.2. Development of the structure of labour and fixed capital input for the satisfaction of consumptive needs

The subdivision of the final demand according to complexes of needs serves the aim of making accessible to scientific analysis the extent and proportionality of the distribution of entire social labour according to the individual complexes of needs and the changes occurring in this process. Within the framework of the standard input-output model, such a possibility exists above all on the level of calculating the total input of labour force and of fixed capital.

For all complexes of needs of consumption there applies:

$$X^{(k)} = (I-A)^{-1}N^{(k)} \quad (3)$$

$$L^{(k)} = L^{D}X^{(k)} \quad (4)$$

$$G^{(k)} = G^{D}X^{(k)} \quad (5)$$

Here, the following designations are used:

L^{D} - vector of labour coefficients per unit of gross domestic output by sectors, expressed as a diagonal matrix,

G^{D} - vector of capital coefficients per unit of gross domestic product by sectors, expressed as a diagonal matrix,

$L^{(k)}$ - vector of total labour input of the sectors for the complex of needs k,

$G^{(k)}$ - vector of total fixed capital input of the sectors for the complex of needs k,

$X^{(k)}$ - vector of total gross domestic output of the sectors for the complex of needs k.

After these calculations, each vector $N^{(k)}$, $X^{(k)}$, $L^{(k)}$ and $G^{(k)}$ is further summarized by the addition of its components. From an investigation of the linking of labour force and of fixed capital for consumption it is shown that the efficiency of social production in the seventies was increased above all via relative savings of live labour. The growth of consumption from 1972 to 1977 by almost 30 per cent was obtained without any substantial change of the number of labour force. Hence, the increase in the final product for consumption was entirely based upon the rise in the productivity of labour in the time under review.

A comparison of the linking of the labour force to the individual complexes of needs reveals considerable differences. If one discards problems connected with the acquisition, evaluation and aggregation of data, then they result from the various conditions in which the individual products are manufactured. In 1972, in the national economy of the GDR, 48 persons of the labour force were engaged in providing to the population goods and services worth one million marks for nutrition. The level of labour productivity expressed in this number has increased by 1977 in the corresponding production and processing lines. In 1977, about 43 working people were needed for this purpose. While the total input of labour for 1 million marks worth of goods for nutrition was in 1972 below the then average level of labour productivity for the production of consumer goods in general, this input rose in 1977 above this average, however. This is connected with the fact that the productivity of labour for the satisfaction of the other complexes of needs has increased substantially faster.

The relative low increase in the productivity of labour along the production and processing lines for the satisfaction of the need of nutrition furthermore entailed the fact that the level of labour productivity which was above the average in 1972 in the satisfaction of the primary needs has fallen below the average in 1977. The satisfaction of the secondary needs took place in 1977 on a higher level of productivity than was the case for the primary needs.

We had a similar differentiated development in the field of fixed capital. Whereas the satisfaction of the primary needs required year by year 1,7 per cent more fixed capital per 1 million marks worth of goods and services for consumption, in the case of secondary needs a relative release of capital can be noted.

The input of fixed capital for consumption increased from 1972 to 1977 in absolute and relativ terms. The correspondent decrease of the output capital ratio based above all on its decline in the framework of satisfying the need of nutrition. This tendency couldn't be made up for nor by the growing output capital ratio in the framework of satisfying the most expanding complex of needs of transport nor by the growing or remaining constantly output capital ratio in the framework of satisfying the other complexes of needs.

Table 2 gives a summarizing survey of the effects due to the differentiated growth both on the side of labour productivity and of the output capital ratio, and on the side of consumption, effects which affected the structure of the input of fixed capital and of labour. From the different dynamism of structural changes in goods and services for consumption on the one hand and in the total input of labour and of fixed capital which is necessary for this purpose on the other, within the same period of time there has taken place a reallocation of the entire social labour over the individual complexes of needs. This was connected with a certain change in the employment of resources. If for the satisfaction of the primary needs in 1972 still about 75 per cent of the labour forces and 71 per cent of the fixed capital stocks were engaged in the field of production for consumption, then in 1977, it was still 73 per cent of the labour force and 69 per cent of the fixed capital. In 1977 in comparison with 1972, a higher share of the entire social labour could be allocated to those complexes of needs which are not elementary vital needs any longer.

Table 2

Structure of the total labour and fixed capital input for consumption in the GDR according to complexes of needs in %

Complex of needs	Labour		fixed capital	
	1972	1977	1972	1977
Nutrition	44,8	47,2	42,6	44,2
Clothing	12,8	10,4	9,7	8,5
Accomodation	17,3	15,2	18,5	16,4
Σ Primary needs	74,9	72,8	70,8	69,1
Health care	4,1	4,3	5,1	5,6
Education/culture/recreation	3,6	3,1	3,0	3,2
Communication/information	4,4	4,5	4,3	3,8
Transport	8,3	10,3	11,4	12,5
General public needs	4,7	5,0	5,4	5,8
Σ Secondary needs	25,1	27,2	29,2	30,9
Consumption total	100,0	100,0	100,0	100,0

Source: Calculations of the Central Institute for Economic Sciences

2.3. Development of the energy consumption according to complexes of needs

The allocation of the entire social labour is oriented not only according to the extent and structure of the needs to be satisfied, as well as according to the state of technological and organizational development of production, but also depends on the type and amount of the available raw materials and energy carriers. The scarcity in some fossile energy carriers led to structural changes also in the GDR's economy in the second half of the seventies. In order to make this development amenable to a scientific analysis, the interrelation between consumption according to complexes of needs and to the primary energy consumption is investigated in greater detail.

The analysis of the consumption of primary energy by means of an input-output table of a medium level of aggregation is made more complicate owing to the fact, that the consumption of specific energy carriers is not expressed as the utilization of any primary resource but is contained in the intermediate consumption within the first quadrant of the table. Hence, according to the basic equations of the input-output table the supply of primary energy carriers represented as the output of individual sectors of the economy lastly appears to be limited only by the availability of labour and fixed capital. Due to this different acquisition of data the total input of primary energy carriers for consumption cannot be calculated similarily to the total input of labour and fixed capital.

By a disaggregation of those branches in which primary energy carriers are supplied, it is possible, however, to determine the total input of primary energy carriers in terms of gross output for the satisfaction of the individual complexes of needs. This characteristic contains double countings, but nevertheless, the special structure of the input coefficients of the branches supplying primary energy allows to use that total input coefficien of primary energy carriers in terms of gross output as a good approximation for the total input according to the value of the primary energy.

The following designations are used:

$\tilde{A}$ - augmented inter-industry coefficient matrix which is formed by separating the branches of primary energy from matrix A,

W - matrix of aggregation coefficients which summarizes the columns of $(I-\tilde{A})^{-1}$ into nomenclature of A,

H - matrix by which all coefficients which are not related to primary energy are eliminated from an initial matrix,

E_S - matrix of the total input coefficients of all the sectors supplying primary energy

E_N - matrix of the total input of primary energy according to complexes of needs.

Then the following equations apply:

$$E_S = H\,(I-\tilde{A})^{-1}W \qquad (6)$$

$$E_N = E_S N \qquad (7)$$

The sum of the rows of E_N gives the total input of primary energy in terms of gross output according to complexes of needs.

The investigation revealed that in 1977, as expected, the utilization of primary energy carriers varies according to complexes of needs.

In general, it was revealed that the satisfaction of the secondary needs as compared with the satisfaction of the primary needs required the 1,7-fold amount of the total input of primary energy. The structural change of consumption towards the secondary needs therefore aggravates the tense situation of the primary energy balances.

3. Conclusion

The main result of the investigation is the trade-off revealed in the use of different primary resources according to the satisfaction of primary and secondary needs. While the allocation and utilization of the entire social labour in the form of employees and of fixed capital to the satisfaction of primary and secondary needs was more or less in line with the structure of the consumer goods and services in 1977, it was clearly distinct from it with regard to the utilization of primary energy. Favourable results are displayed above all by the complex of needs of nutrition which claims in comparison with the great entire share in the consumption of 42,7 per cent only about 30 per cent of the utilization of primary energy.

Hence, concluding any reallocation of production in favour of the satisfaction of primary needs, however, cannot be justified. Rather these results point out, where new energy-saving technologies must be developed and used most urgently from the viewpoint of the satisfaction of consumptive needs.

N o t e s

[1] See J. Behr, M. Kraft, U. Ludwig, The Utilization of Input-Output Analysis forPlanning Basic Structures Affecting the Satisfaction of Needs in the German Democratic Republic, Seventh International Conference on input-output techniques, Innsbruck 1979.

[2] See Die Erprobung der Natural-Wert-Verflechtungsbilanz im Planungsprozeß, "Wirtschaftswissenschaft", 3/1978, p. 283-295.

[3] See H.S. Houthakker, Additive Preferences, "Econometrica", 1960, p. 244.

[4] See also H. Wold/L. Jureen, Demand Analysis - A Study in Econometrics, New York 1953, p. 20, 226.

[5] In contrast with the system of national accounts (SNA) in this paper the attribution of commodities to complexes of needs is built upon the flow of the so-called material products and services (MPS) going to the housholds and to the government. Hardly there arises any difficulty of understanding, in principle, if the satisfaction of private needs is considered such as nutrition, clothing, accommodation, transport and communication. But this may happen, if such needs as health care, education, culture and general public needs are considered which are satisfied to a high degree collectively. In these cases the income of the employees (doctors and medical staff, teachers etc.) is not taken into account. Thus the complex of general public needs mustn't mixed up with the traditionally used term of government expenditure.