# ESTIMATION OF THE CONSUMER DEMAND SYSTEM IN POSTWAR JAPAN

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April 1982
CP-82-14

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# PREFACE

This paper is intended to present an analytical model of a consumer demand system and to give an example of its application to empirical data in postwar Japan. The model is called Powell's system, a type of linear expenditure system.

The linear expenditure system has been used by IIASA as a method of carrying out the analysis and prediction of the demand side in various national models concerned with the Food and Agriculture Program (FAP). It is hoped that this paper will be of some help in assessing the efficiency and usefulness of the linear expenditure system in the process of advancing the task at IIASA.

ACKNOWLEDGEMENTS

The author is very grateful to Mr. E. Arakawa, Secretary General of the Japan Committee for IIASA, and the parties concerned, who nominated him as a participant to the Status Report Conference of the Food and Agriculture Program in Laxenburg, February 16-18, 1981. It provided the author with the opportunity to present the original version of this paper in the Conference. He is also grateful to Professor K.S. Parikh, Dr. K. Frohberg and Professor Y. Maruyama, who made possible the distribution of this paper as an IIASA Collaborative Paper. Mr. Y. Fukagawa was very helpful in conducting the computation required for this study. Ms. Marianne Spak, Bonnie Riley and Stefanie Hoffmann were also very helpful and efficient in preparing the paper.

## FOREWORD

Understanding the nature and dimensions of the world food problem and the policies available to alleviate it has been the focal point of the IIASA Food and Agriculture Program since it began in 1977.

National food systems are highly interdependent, and yet the major policy options exist at the national level. Therefore, to explore these options, it is necessary both to develop policy models for national economies and to link them together by trade and capital transfers. For greater realism the models in this scheme are being kept descriptive, rather than normative. In the end it is proposed to link models to twenty countries, which together account for nearly 80 per cent of important agricultural attributes such as area, production, population, exports, imports and so on.

The linear expenditure system used by the Food and Agriculture Program for analysis and description of the demand side within the national models is examined in both a static and a dynamic version by Kozo Sasaki for the case of postwar Japan. This is a further step towards the development of a detailed agricultural model for Japan.

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CONTENTS

## 1. INTRODUCTION

The objectives of this study are to estimate the demand system for subgroups of commodities and to clarify the changes in consumer demand and their characteristics, using the time series of family budget data in postwar Japan.

As is commonly known, the condition of consumption improved remarkably from the deficient state to the present high level, as the Japanese economy rapidly recovered from the war-devastated state and reached the high standard of living of today. In the meantime, consumer demand has apparently undergone a marked change. Data used in the analysis are those of *All Households in Cities with Population of 50,000 or More*. The period covered is 27 years, from 1951 to 1977, and excludes the immediate postwar years. The analytical method adopted is the linear expenditure system developed by A.A. Powell (1966, 1968), which corresponds to a particular utility function and is effective in analyzing a number of commodities under the assumption of directly additive preferences. It is a variant of J.R.N. Stone's classical linear expenditure system (Stone 1954), which was simplified for estimation purposes.

First, a linear approximation by the static version of the linear expenditure system is made for appropriate segments of the entire period, since there have been remarkable changes in consumption patterns over the past three decades. Estimated parameters of the demand model yield estimates of income and price elasticities of demand, income flexibility or Frisch's money flexibility, subsistence consumption levels, etc. At the same time, changing patterns of those demand and utility parameters are examined. Second, taste variables are introduced into the demand system in order to make it a dynamic one. It is of some interest to see to what extent the estimated parameters are stable over time and across alternative specifications of demand model.

Nonlinearity of the demand system to be estimated necessarily arouses our interest in the convergence process of estimates. Various statistical tests are undertaken for the results obtained.

## 2. METHOD

### 2.1. Static Model of the Linear Expenditure System

Powell's system, which is directly applied to this analysis, enables us to derive a number of commodity expenditure functions from empirical data on such few variables as expenditures and prices. It is designed to reduce considerably the number of parameters to be estimated and to avoid the burdensome problem of multi-colinearity.

A brief description of Powell's system may be necessary for this discussion. Generally, a simple static model of the linear expenditure system is written as a set of linear expenditure functions in prices and income with fixed coefficients. In the case of marked changes in tastes, such a static model may be easily transformed into a dynamic one by introducing an additional term which allows for shifts of expenditure functions. With the three theoretical restrictions of additivity, homogeneity, and symmetry incorporated into the static model, the linear expenditure system of the Stone type is obtained.

The distinction of Powell's system is that the assumption of directly additive preferences is locally imposed upon this demand system at the sample means of all variables. The underlying assumption is characterized by the symmetry restriction. Apart from both additivity and homogeneity, symmetry is given as the condition that, at sample means, the substitution effect between any pair of different commodities is proportional to the two relevant income derivatives.[1]

Thus the static model is transformed into the following expression:

$$v_{it} = p_{it}\overline{x}_i + \lambda z_{it} + b_i u_t + \varepsilon_{it}, \qquad (i = 1,2,\ldots,N) \qquad (t = 1,2,\ldots,T) \tag{1}$$

where

$$z_{it} = b_i \textstyle\sum_j b_j (p_{jt}/\overline{p}_j - p_{it}/\overline{p}_i), \qquad (i, j = 1, 2,\ldots,N) \tag{2}$$

$$u_t = m_t - \textstyle\sum_j p_{jt}\overline{x}_j \tag{3}$$

The $v_{it}$, $p_{it}$, and $x_{it}$ indicate per capita expenditure on the $i^{th}$ commodity, the price of the $i^{th}$ commodity, and the quantity consumed per capita of the $i^{th}$ commodity during time t, respectively. The $m_t$ denotes per capita income or total expenditure during time t. The $\overline{p}_i$, $\overline{x}_i$ ($\overline{x}_i \equiv \overline{v}_i/\overline{p}_i$) and $\overline{v}_i$ represent the sample means of $p_i$, $x_i$, and $v_i$. The $\lambda$ is Houthakker's income flexibility;[2] that is, a proportionality factor which appears in the cross substitution of effects under the additive preference postulate. It is related to the marginal utility of income $\omega$ in the following manner:

$$\lambda = -\omega/(\partial\omega/\partial m) \tag{4}$$

Moreover, $b_i$ and $\varepsilon_{it}$ are the marginal budget share of the $i^{th}$ commodity and the residual respectively. Both $\lambda$ and $b_i$ are behavioral parameters to be estimated, so that equation (1) proves to be nonlinear in unknown parameters. The estimating equation is written as

$$y_{it} = \lambda z_{it} + b_i u_t + \varepsilon_{it} \tag{5}$$

where

$$y_{it} \equiv v_{it} - p_{it}\overline{x}_i$$

The $y_{it}$ is the difference between actual expenditure and the presumed expenditure for the purchase of sample mean quantity of the $i^{th}$ commodity during time t. The $z_{it}$ is the variable associated with substitution effects.[3] The $u_t$ is the difference between the actual total expenditure and the total presumed expenditure for the purchase of sample mean quantities of all commodities during time t. According to this analytical model, changes in the quantity of each commodity consumed are represented by its variations around the sample mean, while changes in income during each time t are represented by changes in the remaining income after deduction of the total expenditure for all sample mean quantities from the current total expenditure. The average values of $y_i$, $z_i$, and u are all set equal to zero.

Income and price elasticities of demand, evaluated at sample means, are derived from equation (1) as

$$\overline{E}_i = b_i/\overline{w}_i \tag{6}$$

and

$$\overline{e}_{ij} = \begin{cases} \varphi\overline{E}_i - \overline{w}_i\overline{E}_i(1 + \varphi\overline{E}_i), & (i = j), \\ -\overline{w}_j\overline{E}_i(1 + \varphi\overline{E}_j), & (i \neq j) \end{cases} \tag{7}$$

$\overline{E}_i$ is the income elasticity of the $i^{th}$ commodity calculated at sample means, $\overline{e}_{ij}$ is the price elasticity of the $i^{th}$ commodity calculated at sample means with respect to the $j^{th}$ price, $\overline{w}_i$ is the sample mean average budget share or budget proportion of the $i^{th}$ commodity, and $\varphi$ is Theil's income flexibility, that is, the reciprocal of the income elasticity of the marginal utility of income.

The corresponding utility function is of the Stone-Geary type:

$$U = \sum_i b_i \log(x_i - \beta_i),\ b_i > 0,\ \sum_i b_i = 1,\ (x_i - \beta_i) > 0 \tag{8}$$

where

$$\beta_i = \overline{x}_i - b_i\lambda / \overline{p}_i$$

## 2.2. Dynamic Model: Introduction of the Taste Variable

A dynamic factor should be taken into account in constructing a demand system which covers a long period of time. As previously stated, the static model can be readily converted into a dynamic one by introducing a taste variable into the demand system. Thereby, equation (1) is transformed into

$$v_{it} = p_{it}\overline{x}_i + \lambda z_{it} + b_i u_t + c_i s_t + \varepsilon_{it} \tag{9}$$

where $s_t$ stands for the level of taste variable during time t, and $c_i$ is its coefficient. The $s_t$ is common to all of the N expenditure functions. Similarly, equation (2) is modified as

$$y_{it} = \lambda z_{it} + b_i u_t + c_i s_t + \varepsilon_{it} \tag{10}$$

Although time trend is often used as a proxy for the taste variable, it does not seem to serve as such an explanatory variable in this model, because of its high correlation with the income variable $u_t$. Let us consider two alternative specifications for the proxy; that is, an annual increase in income and an annual rate of increase in income. They are written as follows:

$$s_t = m_t - m_{t-1} \tag{11}$$

and

$$s_t = (m_t - m_{t-1}) \ /\ m_{t-1}, \text{ respectively} \tag{12}$$

All values of the $s_t$ for each specification can be adjusted in such a way that they sum to zero, and the average is also equal to zero. In this case, additivity is globally satisfied, but both homogeneity and symmetry are fulfilled only at sample means. Furthermore, $\beta_i$ is rewritten as

$$\beta_{it} = \overline{x}_i - (b_i\lambda / \overline{p}_i) + (c_i s_t / p_{it}) \tag{13}$$

## 3. ESTIMATION PROCEDURE

The estimating equation (5) in the static model is compactly expressed by Zellner's block diagonal specification:

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = \begin{bmatrix} z_1 & u & 0 & \cdots & 0 \\ z_2 & 0 & u & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & 0 \\ z_N & 0 & \cdots & 0 & u \end{bmatrix} \begin{bmatrix} \lambda \\ b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_N \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_N \end{bmatrix} \tag{14}$$

where $y_i$, $z_i$, u, and $\varepsilon_i$ are (T×1) vectors. Equation (14) is also written as

$$y = X\gamma + \varepsilon \tag{14'}$$

where

$$y = (y_1', \ldots, y_N')'$$
$$\varepsilon = (\varepsilon_1', \ldots, \varepsilon_N')'$$
$$\gamma = (\lambda, b_1, \ldots, b_N)'$$

and X indicates the $(NT\times(N+1))$ matrix on the right hand side of equation (14). For simplicity of estimation, systems least squares method is used.[4] Least squares estimator of $\gamma$ is obtained by minimizing the residual sum of squares over all equations and all observations:[5]

$$\hat{\gamma} = (X'X)^{-1}X'y \tag{15}$$

This result is partly described as

$$\hat{\lambda} = \textstyle\sum_i N_i / \sum_i D_i \tag{16}$$

where

$$N_i = \begin{vmatrix} \sum_t z_{it}y_{it} & \sum_t z_{it}u_t \\ \sum_t u_t y_{it} & \sum_t (u_t)^2 \end{vmatrix} \tag{17}$$

$$D_i = \begin{vmatrix} \sum_t (z_{it})^2 & \sum_t z_{it}u_t \\ \sum_t u_t z_{it} & \sum_t (u_t)^2 \end{vmatrix} \tag{18}$$

The equation for estimating $b_i$ results in

$$y_{it}' = b_i u_t + \varepsilon_{it}, \quad (i = 1,2,\ldots,N) \quad (t = 1,2,\ldots,T). \tag{19}$$

where

$$y_{it}' \equiv y_{it} - \hat{\lambda} z_{it}$$

The estimates of $b_i$'s are obtained by applying ordinary least squares to each equation in (19) separately. Since $z_{it}$ is a function in unknown parameters as shown in equation (2), equation (5) or (14) is nonlinear in unknown parameters, and its estimation requires an iterative procedure.

Under the simple assumption of the error structure[6], behavioral parameters $\lambda$ and $b_i$ are estimated by an iteration of linear regressions. If $z_{it}$ is regarded as an exogenous variable for the present, unbiased estimates are obtained for $\lambda$ and $b_i$, and their standard errors can be computed.[7] Thus, it is possible to test the significance of estimated parameters.

Prior to the iterative estimation of Powell's system, starting values of the marginal budget shares $b_i$'s should be sought to treat $z_{it}$ as an exogenous variable. For this purpose, it is convenient to get the estimates of $b_i$'s from Leser's system[8] (Leser 1960 and 1961), which is along the lines of Powell's. Examination of the convergence of estimated parameter $\hat{\lambda}$ is sufficient to show the convergence of the whole system. It is decided that convergence has been reached when the relative deviation of $\hat{\lambda}$ between two consecutive iterations has dropped below 0.01 percent.[9]

On the other hand, the estimating equation (10) in the dynamic model is also transformed, and hence equations (17) – (19) have to be modified[10] in estimating a set of $\lambda$, $b_i$, and $c_i$.

## 4. DATA AND ESTIMATION

The above models are fitted to empirical data on household expenditures and prices to derive the consumer demand system on a per capita basis in the postwar period. The data sources are *Annual Report on the Family Income and Expenditure Survey* and *Annual Report on the Consumer Price Index*.

As regards the classification of commodities, total expenditure is first decomposed into 24 subgroups of commodities as listed in Table 1 with 11 subgroups of food commodities and 13 subgroups of nonfood commodities. Some of the 24 subgroups are further aggregated into fewer groups in specified periods where required. It is of our great concern to analyze as many commodity groups as possible under given assumptions.

As for the classification of food commodities, the following would be noteworthy: the subgroup of other cereals contains barley, wheat flour, bread, rice-cakes, etc.; meat includes beef, pork, chicken, ham, and sausages; milk and eggs subgroup also includes powdered milk, butter and cheese; processed food involves dried food (beans, mushrooms, laver, etc.), cooked and canned food, and condiments (sugar, fat and oil, etc.); fruits comprise oranges, apples, strawberries, grapes, etc.; and beverages is composed of alcoholic ("sake," beer, whiskey, wine) and nonalcoholic (tea, coffee, fruit juice, lactic drinks, etc.) beverages. In regard to the non-food commodities, subgroups of public transportation, communication and private transportation; education and stationery; and of recreation, reading, and other miscellaneous items are respectively aggregated at the start into a single group.

The major notations and data used in the analysis are as follows:[11]

$p_i$ = price of the $i^{th}$ subgroup, deflated by the General Consumer Price Index ($i = 1, 2, \ldots, 24$; 1970 prices = 1)

$x_i$ = quantity yearly consumed per capita of the $i^{th}$ subgroup (expenditure in constant 1970 prices)

m = yearly income per capita, deflated by the General Consumer Price Index (total expenditure in 1970 yen)

Price index is taken as an individual price for each subgroup of commodities. The base year is 1970, the prices of which are all set equal to unity.

At the first step of estimation, Leser's system[12] was fitted to the same data to obtain the starting values of $b_i$ estimates. This system also has a common parameter to all equations, which is viewed as the average elasticity of substitution. Its value was confined to the range between 0 and 1 in the static model, as considered in Leser (1960, 1961). However, this restriction was relaxed in the dynamic model because, in a few cases, estimates of the common parameter centered about unity, and their empirical meaning seemed reasonable.

Starting with the estimation of the static model, an iterative procedure was undertaken by least squares to find the estimates of demand parameters for various segments of the whole period. Then, such a static approach ensured the linearity of expenditure functions over the specific subperiods at the particular levels of commodity breakdown. The estimates of static parameters converged so fast that many of the iterative estimations ended within ten rounds.

The dynamic model was fitted to longer time series of a similar data set, using a 21-commodity breakdown. The iteration took more rounds, but the speed of convergence was such that iteration terminated within 19 rounds in all cases undertaken.

Table 1. Estimates of Demand Parameters $\hat{b}_i$, $\hat{\lambda}$ by Subperiod (Static Model)

| Subperiod | 1951–1960 | | | | | 1961–1970 | | | | | 1963–1977 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Marginal budget Share | | Correlation coefficient | | Serial corr. | Marginal budget share | | Correlation coefficient | | Serial corr. | Marginal budget Share | | Correlation coefficient | | Serial corr. |
| i | $\hat{b}_i$ | \|t\| ratio | $r_{y',u}$ | R | coeff. | $\hat{b}_i$ | \|t\| ratio | $r_{y',u}$ | R | coeff. | $\hat{b}_i$ | \|t\| ratio | $r_{y',u}$ | R | coeff. |
| 1 Rice | .0415 | 5.891 | .901 | .952 | .274 | -.0740 | 16.595 | -.986 | .943 | .019 | -.0518 | 14.475 | -.970 | .957 | .562** |
| 2 Other cereals | -.0403 | 10.058 | -.963 | .953 | .310 | .0064 | 4.192 | .829 | .834 | .499 | .0034 | 3.399 | .686 | .957 | .073 |
| 3 Fish | .0109 | 3.953 | .813 | .837 | .185 | .0047* | 2.092 | .595* | .986 | -.191 | .0036* | 1.727 | .432* | .984 | -.183 |
| 4 Meat | .0357 | 16.740 | .986 | .986 | .316 | .0461 | 18.641 | .989 | .990 | .613 | .0526 | 2.283 | .988 | .983 | .282 |
| 5 Milk + eggs | .0447 | 22.506 | .992 | .991 | .125 | .0327 | 7.197 | .931 | .841 | .592 | | | | | |
| 6 Vegetables | -.0028* | .935 | -.314* | .756 | -.133 | .0100 | 4.199 | .829 | .970 | -.125 | .0142 | 15.636 | .974 | .991 | -.293 |
| 7 Processed food | .0521 | 13.530 | .979 | .965 | .349 | .0181 | 10.211 | .964 | .944 | .310 | .0169 | 15.274 | .973 | .987 | .034 |
| 8 Cakes | .0336 | 10.646 | .966 | .920 | -.003 | .0167 | 17.715 | .987 | .983 | -.097 | .0115 | 15 .305 | .973 | .978 | .480 |
| 9 Fruits | | | | | | .0301 | 12.385 | .975 | .969 | .341 | .0214 | 8.708 | .729 | .850 | .467 |
| 10 Beverages | .0323 | 16.824 | .986 | .981 | .405 | .0467 | 51.334 | .998 | .998 | -.529 | .0376 | 21.049 | .986 | .975 | .570** |
| 11 F.a.f.h.[a] | .0479 | 15.532 | .984 | .983 | .425 | .0451 | 20.343 | .990 | .991 | .585 | .0495 | 44.984 | .997 | .997 | .195 |
| 12 Rent | .0229 | 5.135 | .876 | .958 | .527 | .0352 | 8.636 | .950 | .968 | .599 | .0422 | 30.986 | .993 | .993 | -.042 |
| 13 Repairs[b] | .0257 | 8.707 | .951 | .949 | .105 | .0158 | 7.620 | .937 | .945 | .078 | .0127 | 5.961 | .856 | .947 | .476 |
| 14 Water charges | .0049 | 8.322 | .947 | .966 | .210 | .0069 | 17.510 | .987 | .985 | .337 | | | | | |
| 15 Furniture | .0822 | 8.318 | .947 | .948 | .559 | .0767 | 8.279 | .946 | .925 | .710** | .0692 | 9.294 | .932 | .888 | .799** |
| 16 Fuel + light | .0345 | 15.828 | .984 | .990 | -.041 | .0436 | 19.379 | .990 | .937 | .397 | .0476 | 19.073 | .983 | .955 | .444 |
| 17 Clothes | .1364 | 11.660 | .972 | .935 | -.157 | .0769 | 16.999 | .986 | .979 | .570 | .0802 | 23.088 | .988 | .988 | .207 |
| 18 Personal effects | | | | | | .0174 | 7.826 | .940 | .914 | .231 | .0091 | 7.449 | .900 | .746 | .276 |
| 19 Medical care | .0325 | 18.431 | .988 | .979 | .361 | .0393 | 20.137 | .990 | .987 | -.159 | .0381 | 46.541 | .997 | .995 | -.281 |
| 20 Toilet care | .0257 | 10.173 | .963 | .972 | .581 | .0234 | 18.168 | .988 | .986 | .362 | .0167 | 13.187 | .965 | .966 | .309 |
| 21 Transportation[c] | .0237 | 21.311 | .991 | .993 | .222 | .1090 | 27.203 | .995 | .994 | -.688** | .1180 | 20.100 | .984 | .982 | .188 |
| 22 Education | .0365 | 8.034 | .943 | .962 | .594 | .0125* | 1.463 | .459* | .775 | .699** | -.0061 | 2.404 | -.555 | .859 | .389 |
| 23 Tobacco | .3195 | 25.638 | .994 | .994 | .264 | .0080 | 9.575 | .959 | .939 | .403 | .4135 | 33.346 | .994 | .994 | .771** |
| 24 Recreation[d] | | | | | | .3528 | 82.152 | .999 | .999 | .354 | | | | | |
| $\hat{\lambda}$ | 47.027 | 2.388 | ($\phi$ = .407) | | | 82.652 | 6.062 | ($\phi$ = −.410) | | | 110.626 | 3.357 | ($\phi$ = −.450) | | |

[a]F.a.f.h. indicates food away from home.
[b]Repairs include maintenance.
[c]Transportation also contains communication.
[d]Recreation includes miscellaneous.
*insignificant at 5 percent ($\hat{b}_i$)
**significant at 5 percent (serial correlation coefficient)
$\phi = -\hat{\lambda}/\overline{m}$

## 5. EMPIRICAL RESULTS OF THE STATIC MODEL

### 5.1. Estimates of Demand Parameters

Demand parameters estimated by the static model for three sample periods, which are relatively good from a statistical viewpoint, are summarized in Table 1. As demand relations have not been stable since the mid-1960s, sample periods partly overlap.

It seems relatively difficult to estimate demand relations in later subperiods owing to a change in consumers' behavior. Consumers are considered to have lately become quite moderate in purchasing, facing simultaneously a steep rise in prices and considerable slowdown of economic growth. Per capita deflated income (or total expenditure) increased by 80 percent in 1951-60, 58 percent in 1961-70, and only 35 percent in 1968-77.

In the first subperiod (1951-60), other cereals belonged to an inferior good, while vegetables did not show an increase in consumption. Although inferior goods are ruled out from an additive utility function, a few of them do exist at the subgroup level of commodity classification. After parameters $b_i$ and $\lambda$ are estimated, a system of expenditure functions (1) is determined as well as demand elasticities (6) and (7). As a measure of goodness of fit, the multiple correlation coefficient was indirectly computed for each expenditure function,[13] in addition to the simple correlation coefficient in equation (19). The multiple correlation coefficients obtained in this way are generally high. There is no first order serial correlation in the error term. The fitted model shows a high fit in the total test, as most of the measures of fit[14] indicate an accuracy of 80 percent or more.

In the second subperiod (1961-70), rice changed to an inferior good, while other cereals and vegetables turned to normal goods. Expenditures except for rice increased steadily. The multiple correlation coefficients are high as a whole, and the measures of fit are mostly at the level of 90 percent in the total test.

In the third subperiod (1968-77), during which the national economy grew substantially less than in previous subperiods and prices went on rising sharply, consumer demand was restrained to a considerable degree. The income coefficient for education is negative, as is that of rice. As for rice, both deflated expenditure and the expenditure in 1970 prices declined. In the case of education, the expenditure in 1970 prices showed a downward tendency due to the steep rise in its relative price in recent years, although the deflated expenditure increased. In this respect, it may not be appropriate to call it an inferior good indiscriminately. Serial correlation is not serious, but the Durbin-Watson test appears more severe.

### 5.2. Demand Elasticities

Price and income elasticities computed from estimated parameters and observed data are given by subperiod in Tables 2-4.

In the first subperiod (Table 2) income elasticity is particularly large for furniture, food away from home, milk and eggs, repairs, recreation, etc.; and their own price elasticities are also relatively high. The own price elasticity for furniture exceeds unity in absolute value. For this subgroup, the estimate of subsistence parameter $\beta_i$ shows a negative sign. An inferior good has necessarily positive own price elasticity and is a net complement for all normal goods.

In the second subperiod (Table 3) income elasticity is quite large for transportation, furniture, medical care, beverages, and food away from home. It

Table 2 Demand Elasticities Estimated for Twenty-one Subgroups at the Sample Means of all Variables in 1951–1960 [$\overline{e}_{ij}$, $\overline{E}_i$] and Sample Mean Average Budget Shares [$\overline{w}_j$]

| i \ j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8-9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17-18 | 19 | 20 | 21 | 22 | 23-24 | $\overline{E}_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Rice | -.170 | -.017 | -.014 | -.004 | -.002 | -.012 | -.018 | -.010 | -.004 | -.000 | -.004 | -.001 | -.001 | .001 | -.012 | -.022 | -.003 | -.008 | -.003 | -.005 | -.020 | .328 |
| 2 Other cereals | .129 | .540 | .050 | .013 | .008 | .044 | .066 | .035 | 015 | .001 | .014 | .004 | .003 | -.005 | .044 | .080 | .010 | .027 | .011 | .019 | .071 | -1.180 |
| 3 Fish | -.025 | -.012 | -.105 | -.003 | -.002 | -.009 | -.013 | -.007 | -.003 | -.000 | -.003 | -.001 | -.001 | .001 | -.009 | -.016 | -.002 | -.005 | -.002 | -.004 | -.014 | .233 |
| 4 Meat | -.154 | -.071 | -.059 | -.580 | -.010 | -.052 | -.079 | -.042 | -.017 | -.001 | -.016 | -.005 | -.003 | .006 | -.053 | -.095 | -.012 | -.032 | -.013 | -.023 | -.085 | 1.408 |
| 5 Milk + eggs | -.194 | -.090 | -.075 | -.019 | -.736 | -.066 | -.100 | -.053 | -.022 | -.001 | -.021 | -.006 | -.004 | .008 | -.067 | -.120 | -.015 | -.041 | -.016 | -.029 | -.107 | 1.776 |
| 6 Vegetables | .008 | .004 | .003 | -.001 | .001 | .034 | .004 | .002 | .001 | .000 | .001 | .000 | .000 | -.000 | .003 | .005 | -.001 | .002 | .001 | .001 | .005 | -.077 |
| 7 Processed food | -.074 | -.034 | -.028 | -.007 | -.005 | -.025 | -.312 | -.020 | -.008 | -.001 | -.008 | -.002 | -.002 | .003 | -.025 | -.046 | -.006 | -.015 | -.006 | -.011 | -.041 | .673 |
| 8-9 Cakes + fruits | -.084 | -.039 | -.032 | -.008 | -.005 | -.029 | -.043 | -.337 | -.010 | -.001 | -.009 | -.002 | -.002 | .003 | -.029 | -.052 | -.007 | -.018 | -.007 | -.013 | -.047 | .770 |
| 10 Beverages | -.139 | -.064 | -.053 | -.014 | -.009 | -.047 | -.071 | -.038 | -.531 | -.001 | -.015 | -.004 | -.003 | .005 | -.048 | -.086 | -.011 | -.029 | -.011 | -.021 | -.077 | 1.266 |
| 11 F.a.f.h. | -.258 | -.119 | -.099 | -.025 | -.016 | -.087 | -.133 | -.071 | -.029 | -.962 | -.028 | -.008 | -.006 | .010 | -.089 | -.160 | -.020 | -.054 | -.021 | -.039 | -.143 | 2.358 |
| 12 Rent | -.119 | -.055 | -.046 | -.012 | -.008 | -.040 | -.061 | -.033 | -.013 | -.001 | -.457 | -.004 | -.003 | .005 | -.041 | -.074 | -.009 | -.025 | -.010 | -.018 | -.066 | 1.090 |
| 13 Repairs | -.201 | -.093 | -.078 | -.020 | -.013 | -.068 | -.103 | -.055 | -.023 | -.001 | -.021 | -.756 | -.005 | .008 | -.069 | -.125 | -.016 | -.042 | -.017 | -.030 | -.111 | 1.840 |
| 14 Water charges | -.120 | -.055 | -.046 | -.012 | -.008 | -.041 | -.062 | -.033 | -.014 | -.001 | -.013 | -.004 | -.448 | .005 | -.041 | -.074 | -.009 | -.025 | -.010 | -.018 | -.066 | 1.094 |
| 15 Furniture | -.308 | -.142 | -.119 | -.030 | -.020 | -.104 | -.158 | -.084 | -.035 | -.002 | -.033 | -.010 | -.007 | -1.134 | -.106 | -.190 | -.024 | -.065 | -.025 | -.046 | -.170 | 2.812 |
| 16 Fuel + light | -.073 | -.034 | -.028 | -.007 | -.005 | -.025 | -.038 | -.020 | -.008 | -.001 | -.008 | -.002 | -.002 | .003 | -.297 | -.045 | -.006 | -.015 | -.006 | -.011 | -.040 | .667 |
| 17-18 Clothing + personal effects | -.121 | -.056 | -.047 | -.012 | -.008 | -.041 | -.062 | -.033 | -.014 | -.001 | -.013 | -.004 | -.003 | .005 | -.042 | -.526 | -.009 | -.025 | -.010 | -.018 | -.067 | 1.106 |
| 19 Medical care | -.163 | -.075 | -.063 | -.016 | -.010 | -.055 | -.084 | -.045 | -.018 | -.001 | -.017 | -.005 | -.004 | .006 | -.056 | -.101 | -.620 | -.034 | -.013 | -.025 | -.090 | 1.491 |
| 20 Toilet care | -.084 | -.039 | -.032 | -.008 | -.005 | -.028 | -.043 | -.023 | -.009 | -.001 | -.009 | -.003 | -.002 | .003 | -.029 | -.052 | -.007 | -.331 | -.007 | -.013 | -.047 | .768 |
| 21 Transportation | -.139 | -.064 | -.053 | -.014 | -.009 | -.047 | -.071 | -.038 | -.016 | -.001 | -.015 | -.004 | -.003 | .005 | -.048 | -.086 | -.011 | -.029 | -.528 | -.021 | -.077 | 1.267 |
| 22 Education | -.128 | -.059 | -.049 | -.013 | -.008 | -.043 | -.066 | -.035 | -.014 | -.001 | -.014 | -.004 | -.003 | .005 | -.044 | -.079 | -.010 | -.027 | -.011 | -.494 | -.071 | 1.166 |
| 23-24 Tobacco + recreation | -.183 | -.085 | -.071 | -.018 | -.012 | -.062 | -.094 | -.050 | -.021 | -.001 | -.020 | -.006 | -.004 | .007 | -.063 | -.113 | -.014 | -.038 | -.015 | -.028 | -.784 | 1.675 |
| $\overline{w}_j$ | .126 | .034 | .466 | .025 | .025 | .036 | .077 | .044 | .026 | .020 | .021 | .014 | .004 | .029 | .052 | .123 | .022 | .033 | .019 | .031 | .191 | |

$\overline{e}_{ij}$ = elasticity of subgroup i with respect to the j-th price calculated at sample means
$\overline{E}_i$ = income elasticity of subgroup i calculated at sample means
$\overline{w}_j$ = budget share of subgroup j calculated at sample means

Table 3. Demand Elasticities Estimated for Twenty-four Subgroups at the Sample Means of all Variables in 1961–1970 [$\overline{e}_{ij}$, $\overline{E}_i$] and Sample Mean Average Budget Shares [$\overline{w}_j$]

| i \ j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | $\overline{E}_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Rice | .617 | .017 | .043 | .020 | .021 | .032 | .061 | .018 | .010 | .014 | .014 | .019 | .013 | .002 | .020 | .031 | .062 | .032 | .011 | .025 | -.009 | .035 | .007 | .116 | -1.231 |
| 2 Other cereals | -.035 | -.102 | -.013 | -.006 | -.007 | -.010 | -.019 | -.005 | -.003 | -.004 | -.004 | -.006 | -.004 | -.001 | -.008 | -.010 | -.019 | -.010 | -.003 | -.008 | .003 | -.011 | -.002 | -.036 | .382 |
| 3 Fish | -.011 | -.002 | -.056 | -.002 | -.002 | -.003 | -.006 | -.002 | -.001 | -.001 | -.001 | -.002 | -.001 | .000 | -.002 | -.003 | -.006 | -.003 | -.001 | -.003 | .001 | -.004 | -.001 | -.012 | .126 |
| 4 Meat | -.118 | -.018 | -.046 | -.557 | -.022 | -.034 | -.065 | -.019 | -.011 | -.015 | -.015 | -.020 | -.014 | -.003 | -.021 | -.033 | -.065 | -.034 | -.011 | -.026 | .010 | -.038 | -.007 | -.122 | 1.305 |
| 5 Milk + eggs | -.097 | -.015 | -.038 | -.018 | -.458 | -.028 | -.053 | -.015 | -.009 | -.012 | -.012 | -.017 | -.011 | -.002 | -.017 | -.027 | -.054 | -.028 | -.009 | -.022 | .008 | -.031 | -.006 | -.101 | 1.072 |
| 6 Vegetables | -.030 | -.005 | -.011 | -.005 | -.006 | -.143 | -.016 | -.005 | -.003 | -.004 | -.004 | -.005 | -.003 | -.001 | -.005 | -.008 | -.016 | -.009 | -.003 | -.007 | .003 | -.009 | -.002 | -.031 | .327 |
| 7 Processed food | -.029 | -.004 | -.011 | -.005 | -.005 | -.008 | -.145 | -.005 | -.003 | -.004 | -.004 | -.005 | -.003 | -.001 | -.005 | -.008 | -.016 | -.008 | -.003 | -.006 | .002 | -.009 | -.002 | -.030 | .316 |
| 8 Cakes | -.071 | -.011 | -.028 | -.013 | -.013 | -.021 | -.039 | -.335 | -.007 | -.009 | -.009 | -.012 | -.008 | -.002 | -.013 | -.020 | -.040 | -.021 | -.007 | -.016 | .006 | -.023 | -.004 | -.074 | .790 |
| 9 Fruits | -.132 | -.021 | -.051 | -.024 | -.025 | -.039 | -.073 | -.021 | -.611 | -.016 | -.017 | -.023 | -.015 | -.003 | -.023 | -.037 | -.073 | -.038 | -.013 | -.029 | .011 | -.042 | -.008 | -.137 | 1.460 |
| 10 Beverages | -.139 | -.022 | -.054 | -.025 | -.026 | -.041 | -.077 | -.022 | -.013 | -.648 | -.017 | -.024 | -.016 | -.003 | -.025 | -.039 | -.077 | -.040 | -.013 | -.031 | .012 | -.044 | -.009 | -.144 | 1.537 |
| 11 F.a.f.h. | -.137 | -.021 | -.053 | -.025 | -.026 | -.040 | -.075 | -.022 | -.013 | -.017 | -.638 | -.024 | -.016 | -.003 | -.024 | -.038 | -.076 | -.040 | -.013 | -.030 | .012 | -.044 | -.009 | -.142 | 1.513 |
| 12 Rent | -.106 | -.016 | -.041 | -.019 | -.020 | -.031 | -.058 | -.017 | -.010 | -.013 | -.013 | -.498 | -.012 | -.002 | -.019 | -.030 | -.059 | -.031 | -.010 | -.023 | .009 | -.034 | -.007 | -.110 | 1.168 |
| 13 Repairs | -.084 | -.013 | -.033 | -.015 | -.016 | -.025 | -.046 | -.013 | -.008 | -.010 | -.011 | -.015 | -.393 | -.002 | -.015 | -.024 | -.047 | -.024 | -.008 | -.019 | .007 | -.027 | -.005 | -.088 | .933 |
| 14 Water charges | -.129 | -.020 | -.050 | -.024 | -.024 | -.038 | -.071 | -.020 | -.012 | -.016 | -.016 | -.022 | -.015 | -.590 | -.023 | -.036 | -.072 | -.038 | -.013 | -.029 | .011 | -.041 | -.008 | -.134 | 1.430 |
| 15 Furniture | -.146 | -.023 | -.057 | -.027 | -.026 | -.043 | -.061 | -.023 | -.013 | -.016 | -.018 | -.025 | -.017 | -.003 | -.689 | -.041 | -.081 | -.042 | -.014 | -.032 | .012 | -.047 | -.009 | -.152 | 1.618 |
| 16 Fuel + light | -.091 | -.014 | -.035 | -.017 | -.017 | -.027 | -.050 | -.014 | -.008 | -.011 | -.011 | -.016 | -.011 | -.002 | -.016 | -.439 | -.051 | -.026 | -.009 | -.020 | .006 | -.029 | -.006 | -.095 | 1.009 |
| 17 Clothes | -.085 | -.013 | -.033 | -.015 | -.016 | -.025 | -.047 | -.013 | -.008 | -.011 | -.011 | -.015 | -.010 | -.002 | -.015 | -.024 | -.434 | -.025 | -.008 | -.019 | .007 | -.027 | -.005 | -.088 | .942 |
| 18 Personal effects | -.047 | -.007 | -.018 | -.009 | -.009 | -.014 | -.026 | -.007 | -.004 | -.006 | -.006 | -.008 | -.005 | -.001 | -.008 | -.013 | -.026 | -.227 | -.005 | -.010 | .004 | -.015 | -.003 | -.049 | .521 |
| 19 Medical care | -.143 | -.022 | -.056 | -.026 | -.027 | -.042 | -.079 | -.023 | -.013 | -.018 | -.018 | -.025 | -.016 | -.003 | -.025 | -.040 | -.079 | -.041 | -.662 | -.032 | .012 | -.046 | -.009 | -.148 | 1.579 |
| 20 Toilet care | -.071 | -.011 | -.028 | -.013 | -.013 | -.021 | -.039 | -.011 | -.007 | -.009 | -.009 | -.012 | -.008 | -.002 | -.013 | -.020 | -.040 | -.021 | -.007 | -.340 | .006 | -.023 | -.004 | -.074 | .789 |
| 21 Transportation | -.266 | -.041 | -.103 | -.048 | -.050 | -.078 | -.146 | -.042 | -.024 | -.033 | -.033 | -.046 | -.031 | -.008 | -.047 | -.075 | -.147 | -.077 | -.026 | -.059 | -1.184 | -.085 | -.017 | -.276 | 2.941 |
| 22 Education | -.033 | -.005 | -.013 | -.006 | -.006 | -.010 | -.018 | -.005 | -.003 | -.004 | -.004 | -.006 | -.004 | -.001 | -.006 | -.009 | -.018 | -.010 | -.003 | -.007 | .003 | -.162 | -.002 | -.035 | .368 |
| 23 Tobacco | -.081 | -.013 | -.032 | -.015 | -.015 | -.024 | -.045 | -.013 | -.007 | -.010 | -.010 | -.014 | -.009 | -.002 | -.014 | -.023 | -.045 | -.024 | -.008 | -.018 | .007 | -.026 | -.373 | -.084 | .897 |
| 24 Recreation | -.134 | -.021 | -.052 | -.024 | -.025 | -.039 | -.074 | -.021 | -.012 | -.017 | -.017 | -.023 | -.015 | -.003 | -.024 | -.037 | -.074 | -.039 | -.013 | -.030 | .011 | -.043 | -.008 | -.745 | 1.479 |
| $\overline{w}_j$ | .060 | .017 | .037 | .035 | .030 | .030 | .057 | .021 | .021 | .030 | .030 | .030 | .017 | .005 | .047 | .043 | .082 | .033 | .025 | .030 | .037 | .034 | .009 | .239 | |

$\overline{e}_{ij}$ = elasticity of subgroup i with respect to the j-th price calculated at sample means

$\overline{E}_i$ = income elasticity of subgroup i calculated at sample means

$\overline{w}_j$ = budget share of subgroup j calculated at sample means

Table 4. Demand Elasticities Estimated for Twenty-one Subgroups at the Sample Means of all Variables in 1963–1977 [$\bar{e}_{ij}$, $\bar{E}_i$] and Sample Mean Average Budget Shares [$\bar{w}_j$]

| i \ j | 1 | 2 | 3 | 4-5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13-14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23-24 | $\bar{E}_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 Rice | .833 | .018 | .044 | .048 | .027 | .053 | .018 | .013 | .016 | .013 | .015 | .020 | .020 | .023 | .054 | .032 | .011 | .025 | -.004 | .041 | .106 | -1.226 |
| 2 Other cereals | -.014 | -.101 | -.008 | -.008 | -.005 | -.009 | -.003 | -.002 | -.003 | -.002 | -.003 | -.004 | -.004 | -.004 | -.010 | -.006 | -.002 | -.004 | .001 | -.007 | -.019 | .218 |
| 3 Fish | -.006 | -.001 | -.046 | -.004 | -.002 | -.004 | -.001 | -.001 | -.001 | -.001 | -.001 | -.002 | -.002 | -.002 | -.004 | -.002 | -.001 | -.002 | .000 | -.003 | -.008 | .095 |
| 4-5 Meat, milk,etc. | -.055 | -.012 | -.030 | -.409 | -.019 | -.038 | -.012 | -.009 | -.011 | -.009 | -.011 | -.014 | -.014 | -.016 | -.037 | -.022 | -.007 | -.017 | .002 | -.028 | -.073 | .836 |
| 6 Vegetables | -.032 | -.007 | -.018 | -.019 | -.233 | -.021 | -.007 | -.005 | -.006 | -.005 | -.006 | -.008 | -.008 | -.010 | -.022 | -.013 | -.004 | -.010 | .001 | -.016 | -.043 | .492 |
| 7 Processed food | -.022 | -.005 | -.012 | -.013 | -.007 | -.163 | -.005 | -.003 | -.004 | -.004 | -.004 | -.005 | -.005 | -.006 | -.015 | -.009 | -.003 | -.007 | .001 | -.011 | -.029 | .330 |
| 8 Cakes | -.038 | -.008 | -.021 | -.023 | -.013 | -.025 | -.272 | -.006 | -.008 | -.006 | -.007 | -.010 | -.009 | -.011 | -.026 | -.015 | -.005 | -.012 | .002 | -.019 | -.051 | .584 |
| 9 Fruits | -.070 | -.015 | -.038 | -.042 | -.024 | -.046 | -.018 | -.491 | -.014 | -.012 | -.013 | -.017 | -.017 | -.020 | -.047 | -.027 | -.009 | -.021 | .003 | -.035 | -.093 | 1.066 |
| 10 Beverages | -.062 | -.018 | -.045 | -.049 | -.028 | -.055 | -.018 | -.013 | -.581 | -.014 | -.016 | -.020 | -.020 | -.023 | -.055 | -.032 | -.010 | -.025 | .004 | -.042 | -.109 | 1.253 |
| 11 F.a.f.h. | -.078 | -.021 | -.054 | -.058 | -.033 | -.065 | -.022 | -.016 | -.019 | -.686 | -.019 | -.024 | -.024 | -.028 | -.065 | -.038 | -.013 | -.030 | .004 | -.049 | -.129 | 1.488 |
| 12 Rent | -.087 | -.019 | -.048 | -.052 | -.030 | -.058 | -.019 | -.014 | -.017 | -.015 | -.618 | -.022 | -.022 | -.025 | -.059 | -.034 | -.011 | -.027 | .004 | -.044 | -.116 | 1.334 |
| 13 Repairs + water | -.036 | -.008 | -.021 | -.023 | -.013 | -.025 | -.008 | -.006 | -.008 | -.006 | -.007 | -.268 | -.009 | -.011 | -.025 | -.015 | -.005 | -.012 | .002 | -.019 | -.450 | .575 |
| 15 Furniture | -.096 | -.021 | -.053 | -.057 | -.033 | -.064 | -.021 | -.015 | -.019 | -.016 | -.018 | -.024 | -.682 | -.027 | -.064 | -.038 | -.013 | -.029 | .004 | -.049 | -.127 | 1.461 |
| 16 Fuel + light | -.078 | -.017 | -.043 | -.047 | -.027 | -.052 | -.017 | -.012 | -.016 | -.013 | -.015 | -.019 | -.019 | -.556 | -.052 | -.031 | -.010 | -.024 | .003 | -.039 | -.103 | 1.186 |
| 17 Clothing | -.086 | -.014 | -.036 | -.039 | -.023 | -.044 | -.015 | -.010 | -.013 | -.011 | -.013 | -.016 | -.016 | -.019 | -.495 | -.026 | -.009 | -.020 | .003 | -.033 | -.087 | 1.002 |
| 18 Personal effects | -.020 | -.004 | -.011 | -.012 | -.007 | -.013 | -.004 | -.003 | -.004 | -.003 | -.004 | -.005 | -.005 | -.006 | -.013 | -.145 | -.003 | -.006 | .001 | -.010 | -.026 | .305 |
| 19 Medical care | -.097 | -.021 | -.053 | -.058 | -.033 | -.064 | -.022 | -.015 | -.019 | -.016 | -.019 | -.024 | -.024 | -.028 | -.065 | -.038 | -.678 | -.030 | .004 | -.049 | -.128 | 1.478 |
| 20 Toilet care | -.040 | -.009 | -.022 | -.024 | -.014 | -.026 | -.009 | -.006 | -.008 | -.067 | -.008 | -.010 | -.010 | -.011 | -.027 | -.016 | -.005 | -.285 | .002 | -.020 | -.053 | .605 |
| 21 Transportation | -.154 | -.034 | -.065 | -.092 | -.053 | -.102 | -.034 | -.025 | -.031 | -.028 | -.030 | -.038 | -.038 | -.044 | -.103 | -.061 | -.020 | -.047 | -1.050 | -.078 | -.204 | 2.347 |
| 22 Education | .013 | .003 | .007 | .008 | .004 | .009 | .003 | .002 | .003 | .002 | .003 | .003 | .003 | .004 | .009 | .005 | .002 | .004 | -.001 | .097 | .017 | -.200 |
| 23-24 Tobacco + recreation | -.099 | -.022 | -.055 | -.059 | -.034 | -.066 | -.022 | -.016 | -.019 | -.017 | -.019 | -.025 | -.025 | -.028 | -.067 | -.039 | -.013 | -.030 | .004 | -.050 | -.813 | 1.514 |
| $\bar{w}_j$ | .042 | .016 | .038 | .063 | .029 | .051 | .020 | .020 | .030 | .033 | .032 | .022 | .047 | .040 | .080 | .030 | .026 | .028 | .050 | .030 | .273 | |

$\bar{e}_{ij}$ = *elasticity of subgroup i with respect to the j-th price calculated at sample* means
$\bar{E}_i$ = income elasticity of subgroup i calculated at sample means
$\bar{w}_j$ = budget share of subgroup j calculated at sample means

also increased for cakes, fruits, rent, water charges, fuel and light, etc. Demand for transportation is highly responsive to a change in its price.

In the third subperiod (Table 4) transportation, recreation, food away from home, medical care, and furniture are rather high in income elasticity, while income elasticities of rent, fuel and light, clothes, etc., increased in comparison with the second subperiod.

It is apparent that the demand for subgroups of food commodities has become less elastic with respect to both income and own prices over time. It is also notable that the housing demand as a whole has been substantially elastic during the entire period. More conspicuous is the fact that transportation has the largest income and own price elasticities, reflecting a strong demand for private cars in recent times.

### 5.3. Money Flexibilities

From all the estimated linear expenditure systems, some good results were chosen and their estimates of money flexibility $\check{\omega}^*$ were tabulated in Table 5. These estimates are liable to depend on the sample period, the level of commodity aggregation, and so on. However, they range from -2.0 to -2.5 without wide variations. Until comparatively recently, they tended to decline in absolute value. The corresponding $\hat{\lambda}$'s and $\hat{\varphi}$'s were all estimated as statistically significant values.

Table 5. Estimated Money Flexibility by the Sample Period and by the Commodity Classification (Static Model)

| Subperiod | $\check{\omega}^*$ | Number of subgroups | Subgroups further aggregated |
|---|---|---|---|
| 1951–60 | -2.455 | 21 | Cakes and fruits, clothes and personal effects, tobacco and recreation |
| 1951–60 | -2.533 | 22 | Cakes and fruits, clothes and personal effects |
| 1959–73 | -2.401 | 23 | Meat, milk, etc. |
| 1959–73 | -2.284 | 24 | |
| 1960–72 | -2.547 | 24 | |
| 1961–70 | -2.438 | 24 | |
| 1961–73 | -2.295 | 23 | Meat, milk, etc. |
| 1961–73 | -2.240 | 24 | |
| 1962–77 | -1.957 | 21 | Meat, milk, etc, repairs and water, tobacco and recreation |
| 1963–77 | -2.221 | 21 | Meat, milk, etc, repairs and water, tobacco and recreation |

$\check{\omega}^* = 1/\hat{\varphi} = -\overline{m}/\hat{\lambda}$

Lastly, sample mean estimates of subsistence parameter $\beta_i$ were calculated, but they are not mentioned here. The concept of subsistence consumption levels are not applicable to inferior goods in an additive utility function. It is discussed again with the economic implications of the dynamic model.

## 6. EMPIRICAL RESULTS OF THE DYNAMIC MODEL

### 6.1. Estimates of Demand Parameters

In estimation, equation (10) was used with alternative specifications of the proxy for changing tastes, as shown in equations (11) and (12). It was fitted to longer time series of per capita expenditure and price data. Several favorable results were obtained from various data sets, which are somewhat different in terms of sample period, proxy for taste variable, and commodity aggregation. One of the good results can be seen in Table 6. It reveals recent trends in consumption patterns to some extent. The commodity classification is the same as in the third subperiod (1963-77) in Table 1.

Table 6. Estimates of Demand Parameters $\hat{b}_i$, $\hat{c}_i$, $\hat{\lambda}_i$ in 1958–1977 (Dynamic Model)

| Coefficient | Marginal budget share | | Coefficient of $s_t$ variable | | Correlation coefficient | | Serial correlation |
|---|---|---|---|---|---|---|---|
| i | $\hat{b}_i$ | \|t\| ratio | $\hat{c}_i$ | \|t\| ratio | $R_{y'.us}$ | R | coefficient |
| 1 Rice | -.0540 | 22.003 | -.0205* | .750 | .983 | .975 | .575 |
| 2 Other cereals | .0032 | 4.322 | -.0052* | .640 | .752 | .957 | .322 |
| 3 Fish | -.0009 | .657 | .0159* | 1.110 | .309* | .990 | .008 |
| 4-5 Meat, milk, etc. | .0672 | 20.618 | .0278* | .766 | .981 | .975 | .804** |
| 6 Vegetables | .0079 | 5.946 | .0069* | .464 | .822 | .976 | .508 |
| 7 Processed food | .0205 | 21.288 | .0116* | 1.081 | .982 | .987 | .493 |
| 8 Cakes | .0151 | 19.858 | .0085* | 1.001 | .979 | .976 | .515 |
| 9 Fruits | .0228 | 15.718 | .0197* | 1.218 | .967 | .958 | .578 |
| 10 Beverages | .0411 | 29.288 | .0249* | 1.596 | .990 | .983 | .522 |
| 11 F.a.f.h. | .0442 | 38.650 | -.0089* | .695 | .994 | .995 | .666** |
| 12 Rent | .0371 | 26.152 | -.0046* | .294 | .988 | .990 | .376 |
| 13-14 Repairs + water | .0170 | 13.912 | .0050* | .366 | .959 | .976 | .414 |
| 15 Furniture | .0698 | 16.391 | .0786* | 1.658 | .970 | .947 | .514 |
| 16 Fuel + light | .0469 | 32.171 | -.0137* | .844 | .992 | .976 | .393 |
| 17 Clothes | .0863 | 30.472 | .0615* | 1.951 | .991 | .988 | .449 |
| 18 Personal effects | .0166 | 11.618 | .0218* | 1.371 | .942 | .946 | .702** |
| 19 Medical care | .0381 | 78.074 | .0106* | 1.957 | .999 | .998 | .023 |
| 20 Toilet care | .0205 | 25.204 | .0255 | 2.817 | .987 | .986 | .654** |
| 21 Transportation | .1063 | 23.728 | -.0472* | .947 | .985 | .984 | .332 |
| 22 Education | .0089 | 3.279 | -.0120* | .396 | .630 | .886 | .832** |
| 23-24 Tobacco + rec. | .3853 | 47.164 | -.2061 | 2.267 | .996 | .996 | .568 |
| $\hat{\lambda}$ | 96.471 | 3.277 | ($\hat{\varphi}$ = -.4354) | | | | |

Taste variable $s_t = m_t - m_{t-1}$
*insignificant at 5 percent ($\hat{b}_i$, $\hat{c}_i$, $R_{y'.us}$)
**significant at 5 percent (serial correlation coefficient)

Estimated marginal budget shares are all significant except for fish. All subgroups other than rice and fish are defined to be normal goods. Significance of the coefficients of the taste variable turns out to be low on the whole. It would imply that changes in the quantities demanded of many subgroups are substantially explained by income and price changes within the framework of economic theory. It is noteworthy, however, that the introduction of taste variable into the expenditure functions had a noticeable effect in stabilizing other relevant parameters in the regressions. The multiple correlation coefficients indirectly computed are very high; on the other hand, the serial correlation in the residuals is not a serious problem in this case. Measures of fit in the total test are mostly at the level of 90 percent. These two facts indicate a high predictive power of the model. Only a couple of values of this measure are rather low, i.e., for transportation in the early years of the period under consideration.

Table 7. Demand Elasticities Estimated for Twenty-one Subgroups at the Sample Means of all Variables in 1958–1977 [$\overline{e}_{ij}$, $\overline{E}_i$] and Sample Mean Average Budget Shares [$\overline{w}_j$]

| i \ j | 1 | 2 | 3 | 4-5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13-14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23-24 | $\overline{E}_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 rice | .663 | .016 | .040 | .034 | .027 | .047 | .014 | .010 | .012 | .013 | .015 | .015 | .017 | .022 | .045 | .025 | .009 | .020 | -.001 | .028 | .100 | -1.044 |
| 2 other cereals | -.014 | -.067 | -.007 | -.006 | -.005 | -.009 | -.003 | -.002 | -.002 | -.002 | -.003 | -.003 | -.003 | -.001 | -.008 | -.005 | -.002 | -.004 | .000 | -.005 | -.018 | .192 |
| 3 fish | .002 | .000 | .011 | .001 | .001 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .001 | .000 | .000 | -.000 | .001 | .002 | -.022 |
| 4-5 meat, milk, etc. | -.061 | -.016 | -.042 | -.506 | -.028 | -.049 | -.015 | -.011 | -.013 | -.014 | -.016 | -.016 | -.018 | -.023 | -.047 | -.026 | -.009 | -.021 | .001 | -.029 | -.103 | 1.060 |
| 6 vegetables | -.020 | -.004 | -.010 | -.009 | -.124 | -.012 | -.004 | -.003 | -.003 | -.003 | -.004 | -.004 | -.005 | -.006 | -.012 | -.006 | -.002 | -.005 | .000 | -.007 | -.026 | .270 |
| 7 processed food | -.028 | -.006 | -.014 | -.012 | -.010 | -.181 | -.005 | -.004 | -.004 | -.005 | -.006 | -.006 | -.006 | -.006 | -.016 | -.009 | -.003 | -.007 | .000 | -.010 | -.036 | .377 |
| 8 cakes | -.056 | -.011 | -.029 | -.025 | -.019 | -.034 | -.336 | -.007 | -.009 | -.010 | -.011 | -.011 | -.012 | -.016 | -.032 | -.018 | -.006 | -.014 | .000 | -.020 | -.072 | .748 |
| 9 fruits | -.067 | -.018 | -.045 | -.038 | -.030 | -.052 | -.016 | -.516 | -.014 | -.015 | -.017 | -.017 | -.019 | -.025 | -.050 | -.027 | .010 | -.022 | .001 | -.031 | -.111 | 1.158 |
| 10 beverages | -.104 | -.021 | -.053 | -.046 | -.036 | -.063 | -.019 | -.014 | -.620 | -.018 | -.020 | -.020 | -.023 | -.030 | -.060 | -.033 | -.012 | -.027 | .001 | -.037 | -.133 | 1.387 |
| 11 f.a.f.h. | -.103 | -.021 | -.053 | -.045 | -.036 | -.062 | -.019 | -.013 | -.016 | -.617 | -.020 | -.020 | -.023 | -.029 | -.060 | -.033 | -.012 | -.027 | .001 | -.037 | -.132 | 1.376 |
| 12 rent | -.091 | -.016 | -.046 | -.040 | -.031 | -.055 | -.016 | -.012 | -.014 | -.016 | -.542 | -.018 | -.020 | -.026 | -.052 | -.029 | -.010 | -.023 | .012 | -.032 | -.115 | 1.205 |
| 13-14 repairs + water | -.058 | -.012 | -.030 | -.026 | -.020 | -.035 | -.011 | -.008 | -.009 | -.010 | -.011 | -.349 | -.013 | -.017 | -.034 | -.018 | -.007 | -.015 | .001 | -.021 | -.074 | .776 |
| 15 furniture | -.112 | -.023 | -.057 | -.049 | -.038 | -.067 | -.020 | -.015 | -.017 | -.019 | -.022 | -.022 | -.672 | -.032 | -.065 | -.035 | -.013 | -.029 | .001 | -.040 | -.142 | 1.486 |
| 16 fuel + light | -.084 | -.017 | -.043 | -.037 | -.029 | -.051 | -.015 | -.011 | -.013 | -.014 | -.016 | -.016 | -.019 | -.513 | -.049 | -.027 | -.010 | -.022 | .001 | -.030 | -.107 | 1.123 |
| 17 clothes | -.080 | -.016 | -.041 | -.035 | -.028 | -.048 | -.015 | -.010 | -.012 | -.014 | -.016 | -.016 | -.018 | -.023 | -.510 | -.025 | -.009 | -.021 | .001 | -.029 | -.102 | 1.065 |
| 18 personal effects | -.040 | -.008 | -.021 | -.018 | -.014 | -.024 | -.007 | -.005 | -.006 | -.007 | -.008 | -.008 | -.009 | -.011 | -.023 | -.247 | -.005 | -.010 | .001 | -.014 | -.051 | .538 |
| 19 medical care | -.114 | -.023 | -.058 | -.050 | -.039 | -.068 | -.021 | -.015 | -.018 | -.019 | -.022 | -.022 | -.025 | -.032 | -.066 | -.036 | -.671 | -.029 | .005 | -.041 | -.145 | 1.511 |
| 20 toilet care | -.055 | -.011 | -.028 | -.024 | -.019 | -.033 | -.010 | -.007 | -.009 | -.009 | -.011 | -.011 | -.012 | -.016 | -.032 | -.017 | -.006 | -.331 | .001 | -.020 | -.070 | .728 |
| 21 transportation | -.177 | -.036 | -.090 | -.077 | -.061 | -.106 | -.032 | -.023 | -.028 | -.030 | -.034 | -.034 | -.039 | -.050 | -.102 | -.056 | -.020 | -.045 | -1.020 | -.063 | -.225 | 2.347 |
| 22 education | -.022 | -.004 | -.011 | -.010 | -.008 | -.013 | -.004 | -.003 | -.003 | -.004 | -.004 | -.004 | -.005 | -.006 | -.013 | -.007 | -.003 | -.006 | .000 | -.134 | -.028 | .290 |
| 23-24 tobacco + rec. | -.110 | -.022 | -.056 | -.048 | -.038 | -.066 | -.020 | -.014 | -.017 | -.019 | -.021 | -.021 | -.024 | -.031 | -.064 | -.035 | -.013 | -.028 | .015 | -.039 | -.779 | 1.463 |
| $\overline{w}_j$ | .052 | .017 | .038 | .062 | .029 | 0.54 | .020 | .020 | .030 | .032 | .031 | .022 | .047 | .042 | .081 | .031 | .025 | .028 | .045 | .031 | .263 | |

$\overline{e}_{ij}$ = elasticity of subgroup i with respect to the j-th price calculated at sample means

$\overline{E}_i$ = income elasticity of subgroup i calculated at sample means

$\overline{w}_j$ = budget share of subgroup j calculated at sample means

### 6.2. Demand Elasticities

Elasticities of demand with respect to deflated income and prices are given in Table 7, evaluated at the sample means in the past 20 years. At first sight, Table 7 closely resembles Table 4 in the static model. There are only slight differences in income elasticities between the two tables. Education is now apparently a normal good. As regards the food category, beverages, food away from home, fruits, and meat are elastic with respect to income. In the nonfood category, transportation, medical care, furniture and recreation have very high income elasticities.

Own price elasticities in Table 7 are similar to those in Table 4. This implies that the money flexibility estimated by the dynamic model in 1958-77 is close to that of the static model in 1963-77. Estimated money flexibilities vary rather widely in the dynamic model, depending mainly on the length of sample period in this analysis. Nevertheless, most of those estimates fell in the interval between -1 and -4.

### 6.3. Cost of Living Index and Subsistence Cost

There are three exceptional subgroups in estimating the cost of living index and the subsistence cost. They are rice, fish, and transportation. The first two subgroups have negative marginal budget shares, and the last one has a negative subsistence parameter. In disregard of their peculiarities, an attempt is made to estimate the cost of living index and the subsistence cost. In fact, these three subgroups possess only small shares of the total budget. The calculation of the cost of living index follows the formula (see Hoa 1969a, 1969b, and Theil 1980).

$$C_{ot} = (1+\hat{\varphi})\left(\sum_i p_{it}\beta_{it} / \sum_i p_{io}\beta_{it}\right) - \varphi\prod_i (p_{it}/p_{io})^{\hat{b}_i} \tag{20}$$

where $p_{it}$ and $p_{io}$ denote the $i^{th}$ price in the comparison and base periods respectively. The $\hat{\beta}_{it}$ can be obtained by equation (13) after the estimates $\hat{b}_i$, $\hat{c}_i$, and $\hat{\lambda}$ have been determined.

If the values of the cost of living index were all equal to 100, the General Consumer Price Index and the 'true' cost of living index would be the same. Though the values of the index in Table 8 are very close to 100, many of them do not attain this level. It would follow from the fact that the General Consumer Price Index in the Laspeyres form tends to have an upward bias as a deflator.

Table 8. Estimates of Cost of Living Index and Subsistence Cost by Year

| Year | Cost of living index | Subsistence cost | Year | Cost of living index | Subsistence cost |
|---|---|---|---|---|---|
| 1958 | 100.0 | 122,120 | 1968 | 98.8 | 124,148 |
| 1959 | 100.4 | 122,251 | 1969 | 99.1 | 125,288 |
| 1960 | 100.3 | 122,080 | 1970 | 99.1 | 125,832 |
| 1961 | 100.0 | 122,159 | 1971 | 99.5 | 126,261 |
| 1962 | 99.5 | 122,187 | 1972 | 99.5 | 126,586 |
| 1963 | 99.1 | 122,520 | 1973 | 99.3 | 128,037 |
| 1964 | 99.2 | 122,826 | 1974 | 99.3 | 128,031 |
| 1965 | 98.3 | 123,014 | 1975 | 99.5 | 129,726 |
| 1966 | 98.6 | 122,689 | 1976 | 100.0 | 131,402 |
| 1967 | 98.7 | 123,437 | 1977 | 100.2 | 131,455 |

Cost of living index in 1958 = 100.0
Subsistence cost = $\Sigma_i p_{it}\hat{\beta}_{it}$

Estimated subsistence cost, as shown in Table 8, changes quite slowly over time. It results from the weak influence of the taste variable.

## 7. CONCLUDING REMARKS

It was intended in this paper to systematically analyze the consumer demand at subgroup levels on the basis of family budget data in 1951-77. All the commodities were classified into 21 to 24 subgroups in estimating the linear expenditure system. Powell's system was applied to the annual data in various segments of the whole period, estimating both static and dynamic parameters of the expenditure system.

The static model yielded well-defined demand relations and their characteristics in various subperiods, particularly in the three subperiods 1951-60, 1961-70, and 1963-77. Such a static approximation was attempted to preserve the linearity of expenditure functions and to take account of the possible changes in preferences during the whole period. Evidently from the empirical results, price and income elasticities of demand have changed over time, and the values of money flexibility show a little variation in dependence on sample period, commodity classification and so on.

In the dynamic model, many of the estimated parameters for the taste variable were not statistically significant, but some important demand and utility parameters were obtained. Estimates of money flexibility were fairly changeable according to the income level, specification of the taste variable and so on. They were more or less different from those of the static model. Price elasticities in the dynamic model are also at variance with the static results. The striking features of the results are that the measures of fit of the model were very high in interpolation test, and that the estimated parameters were rather stable as a whole.

Consumer demand estimation in more recent years will be discussed on another occasion.

NOTES

1. Let the cross substitution term in the Slutsky equation be $K_{ij}$. Then the symmetry condition is

   $K_{ij} = \lambda(\partial x_i / \partial m)\,(\partial x_j / \partial m),\ (i \neq j),\ (\lambda\text{: constant})$

2. The $\lambda$ is related to Theil's income flexibility $\varphi$ and to Frisch's money flexibility $\check{\omega}$ as follows:

   $(\lambda / m) = -\varphi = -(1 / \check{\omega}),\ (m\text{: income})$

   Frisch's money flexibility $\check{\omega}$ is equivalent to the income elasticity of the marginal utility of income. Since the supernumerary ratio is defined as (see Goldberger 1970):

   $-\varphi = (m - \sum_i p_i \beta_i) / m$

   $\lambda$ is interpreted as the supernumerary income in the linear expenditure system.

3. Denote the substitution term by $K_{ij}$. Then $z_{it}$ is of the form:

   $z_{it} = (p_{it} / \lambda) \sum_j K_{ij} (p_{jt} / \overline{p}_j)$

4. The maximum likelihood method entails a greater burden of computation as compared with the least squares method. As regards the convergence of demand parameters in nonlinear regressions, the maximum likelihood method appears to involve some difficulty. Lluch and Powell (1975) and Lluch and Williams (1975) reported the results that maximum likelihood estimates did not converge in some cases, but that convergence was achieved in those cases by the least squares method in the estimation of the linear expenditure system and of the extended linear expenditure system, respectively.

5. Assume that X and y are the matrix and vector whose elements consist of sample data on exogenous variables. Furthermore, if we assume in regard to the error structure that there is no serial correlation either within or across equations, and that there is no contemporaneous correlation across equations but a common error variance for all equations, maximum likelihood method reduces to least squares method (see Goldgberger and Gamaletsos 1970, Lluch and Williams 1975). The error structure in this case is of the form

$$E(\varepsilon_{it}) = 0$$

$$E(\varepsilon_{it}\varepsilon_{jt'}) = \begin{cases} \sigma^2 \ (i = j \text{ and } t = t'), \\ 0 \text{ otherwise} \end{cases}$$

However, this error specification is practically implausible, as was pointed out by Goldberger and Gamaletsos (1970).

6. The simple assumption is that there is no serial correlation either within or across equations and that there is no contemporaneous correlation across equations but a constant error variance for each equation. The error specification in this case is of the form

$$E(\varepsilon_{it}) = 0$$

$$E(\varepsilon_{it}\varepsilon_{jt'}) = \begin{cases} \sigma_i^2 \ (i = j \text{ and } t = t'), \\ 0 \text{ otherwise} \end{cases}$$

7. The variances of the estimators $\hat{b}_i$ and $\lambda$ under least squares postulates are mentioned below:

$$\sigma^2_{\hat{b}_i} = \sigma_i^2 / \sum_t u_t^2 \ , \ (i = 1,2,\ldots,N)$$
$$\sigma^2_{\hat{\lambda}} = (\sum_i D_i \cdot \sigma_i^2) \cdot \sum_t u_t^2 / (\sum_i D_i)^2$$

$\sigma_i^2$ indicates the error variance in the estimating equation for the $i^{th}$ commodity, and its unbiased estimator ordinarily takes the expression

$$\hat{\sigma}_i^2 = \sum e_{it}^2 / (T-2)$$

with $e_{it}$ being the residual and $(T-2)$ the degree of freedom.

8. For the theoretical features of Leser's system, see Sasaki and Saegusa 1974.

9. The criterion of convergence is written as below, denoting the estimate $\hat{\lambda}$ in round r by $\hat{\lambda}_r (r = 1,2,\ldots)$:

$$\left| (\hat{\lambda}_{r-1}) - \hat{\lambda}_r) / \hat{\lambda}_r \right| < 10^{-4}$$

10.

$$N_i = \begin{vmatrix} \sum_t z_{it} y_{it} & \sum_t z_{it} u_t & \sum_t z_{it} s_t \\ \sum_t u_t y_{it} & \sum_t (u_t)^2 & \sum_t u_t s_t \\ \sum_t s_t y_{it} & \sum_t s_t u_t & \sum_t (s_t)^2 \end{vmatrix}$$

$$D_i = \begin{vmatrix} \sum_t (z_{it})^2 & \sum_t z_{it} u_t & \sum_t z_{it} s_t \\ \sum_t u_t z_{it} & \sum_t (u_t)^2 & \sum_t u_t s_t \\ \sum_t s_t z_{it} & \sum_t s_t u_t & \sum_t (s_t)^2 \end{vmatrix}$$

$$y_{it}' = b_i u_t + c_i s_t + \varepsilon_{it}$$

The variances of estimators $\hat{b}_i$, $\hat{c}_i$ and $\hat{\lambda}$ are

$$\sigma^2_{\hat{b}_i} = \sigma_i^2 \cdot r_{11}$$
$$\sigma^2_{\hat{c}_i} = \sigma_i^2 \cdot r_{22}$$

$$\sigma_\lambda^2 = \sum_i \sigma_i^2 D_i \{(\sum_t u_t^2)(\sum_t s_t)^2 - (\sum_t u_t s_t)^2\} / (\sum_i D_i)^2$$

$\sigma_i^2$ is the error variance of the $i^{th}$ equation, and its unbiased estimator is

$\hat{\sigma}_i^2 = \sum e_i^2 / (T - 3)$, ($e_i$: residual)

$r_{ii}$ (i = 1,2) indicates a diagonal element in the inverse matrix:

$$\begin{bmatrix} \sum_t u_t^2 & \sum_t u_t s_t \\ \sum_t s_t u_t & \sum_t s_t^2 \end{bmatrix}^{-1} \equiv \begin{bmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \end{bmatrix}$$

In this paper, the sample size is not reduced by taking differences in annual income for the specification of the taste variable.

11. For details on data, see Sasaki (1981).
12. The static model of Leser's system is expressed as

$$v_i = p_i\overline{x}_i + \overline{\alpha}(\overline{w}_i \sum_j p_j\overline{x}_j - p_i\overline{x}_i) + b_i(m - \sum_j p_j\overline{x}_j)$$

It does not require an iterative estimation. The taste variable $s_t$ is added to the above equation to extend it to a dynamic model in this analysis.

13. The multiple correlation coefficient R was computed as the simple correlation coefficient between actual and estimated expenditures for each subgroup.
14. The measure of fit in the total test is the ratio of calculated expenditure $\hat{v}_{it}$ to actual expenditure $v_{it}$. This is equivalent to taking the ratio of calculated quantity consumed per capita $\hat{x}_{it}$ to its actual value $x_{it}$.

Measure of fit = $(\hat{v}_{it} / v_{it}) = (\hat{x}_{it} / x_{it})$

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