PROSPECT THEORY AND INSURANCE BEHAVIOR

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EXECUTIVE NOTE ON INSURANCE BEHAVIOR AND PROSPECT THEORY

This paper provides analysts with an alternative way of thinking about insurance purchase behavior that may be more natural than the traditional approach which assumes that people purchase specific contingent claims. In our formulation individuals are assumed to insure themselves against specific amounts of coverage which will protect them from a series of different states of nature where losses fall within a specific range. Insurance firms tend to market coverage in this manner: they offer $x worth of protection or $y deductible against a class of different events and set their premiums accordingly.

Here we investigate the implications on consumer choice of a convex utility or value function. The same conceptual framework can be utilized to characterize behavior if the utility or value function is concave in the loss domain.

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I. INTRODUCTION

Prospect theory was developed by Kahneman and Tversky (1979) as an alternative model of choice under risk to help correct the deficiencies of expected utility theory. On the basis of a series of laboratory experiments, the authors postulated that:

a) Probabilities of uncertain events are evaluated according to a probability weighting function π,

b) Gains and losses due to such events are evaluated according to a value function (v) which is concave for gains and convex for losses. This note demonstrates that if this theory is applied to insurance behavior, then the optimal decision for any given risk involves one of two extremes: either purchase full coverage or no insurance at all.

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II. A TWO STATE MODEL

To illustrate the above properties consider the simplest model where the consumer faces a single loss, x, which is assumed to occur with probability p. He assigns a decision weight $\pi(p)$ to this event and $\pi(1-p)$ to the event where no loss occurs. To protect himself against a negative outcome, the consumer can purchase an amount of insurance I at a rate (r) per dollar coverage. The objective is to find the optimal amount of insurance coverage, I^*, which maximizes the overall value V of the prospect given by:

$$V(I) = \pi(p)\left[v(-x + (1-r)I)\right] + \left[1 - \pi(p)\right]v(-rI) \qquad (1)$$

subject to $0 \leq I \leq x$ where $v'(y), v''(y) > 0$ for $y \leq 0$ and $v(0) = 0$. Since $V(I)$ is a convex function for all permissible values of I,[1] the optimal solution cannot be in the (open) interval $(0,x)$ but must be at a boundary. In contrast, standard utility theory assumes a concave utility function for which there may be an optimal interior solution. These two situations are shown in Figure 1.

The two possible options, $I^* = 0$ or x, yield the following values:

$$V(0) = \pi(p)v(-x) \qquad (2)$$

$$V(x) = v(-rx) \qquad (3)$$

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[1] Specifically,

$$V''(I) = \pi(p)(1-r)^2 v''(-x + (1-r)I) + \left[1-\pi(p)\right]r^2 v''(-rI) > 0$$

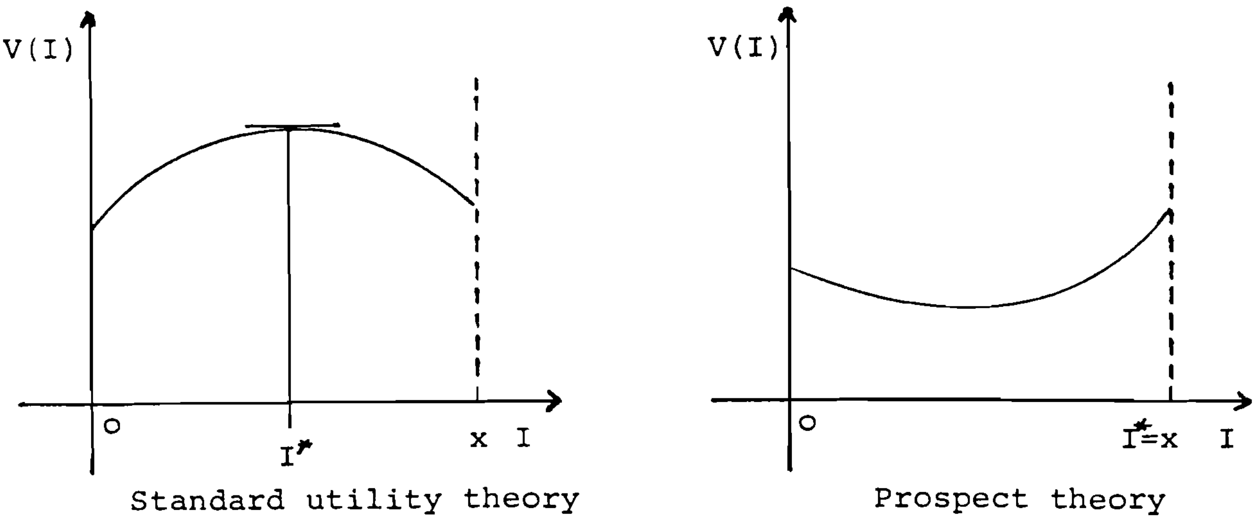

Figure 1.

Equating (2) and (3) we can find the critical insurance rate r^* for given values of $\pi(p)$ and x, where a consumer is indifferent between purchasing 0 or x units of coverage.

To illustrate this result consider the case where $x = 1$ and $v(y) = e^y - 1$ for all $y \leq 0$. In this case $r^* = -ln\left[\pi(p)(e^{-1} - 1) + 1\right]$. If $p = .001$ and the expectation principle holds so that $(\pi(p) = p)$ then $r^* = .0006$. Suppose an individual overweights low probability events, as suggested by Kahneman and Tversky, so that $\pi(.001) = .01$. In this case $r^* = .006$.

III. AN N-STATE MODEL

In reality, the decision problems related to insurance behavior are much more complex than the simple model above. As suggested by Kahneman and Tversky (1979) their model may be extended to several states of nature, each causing a different type of loss. Furthermore, their results suggest that individuals tend to recognize only a relatively small number of such states, which are derived from an even more complex reality by some editing mechanism.

Let n be the number of different events that are considered by an individual and x_i be the loss associated with event i assumed to occur with probability p_i. We order the states according to the losses so that $x_i > x_{i-1}$, $i = 1 \cdots n$ with $x_0 = 0$. The probability of experiencing a loss of at least x_i dollars is thus given by

$$P_i = \sum_{j=i}^{n} p_j \tag{4}$$

with an associated decision weight $\pi(P_i)$. The relevant rates for covering losses can also be specified analogously. Let r_i denote the rate associated with insuring against losses caused by event i.

Consider losses of a given size defined by the interval (l,u). To protect against all such losses one has to buy insurance against all events i with $x_i \geq l$. The rate for one dollar's coverage for such protection is given by

$$R_{(l,u)} = \sum_{i \epsilon S} r_i \tag{5}$$

where $S = \{i \,|\, x_i \geq l\}$ is the set of all events that may cause such a loss.

Insurance obtained at rate $R_{(l,u)}$ will only cover that part of the loss that actually falls within this given interval. It is comparable to a deductible of size l and a maximum coverage of u. Many insurance companies actually offer such packages that cover losses in an interval regardless of its cause.

To simplify the following exposition, we assume that the boundaries l and u of these packages correspond to possible losses x_{i-1} and x_i. The rate for protection against risks within the interval (x_{i-1}, x_i) then becomes

$$R_i = \sum_{j=i}^{n} r_j \tag{6}$$

To illustrate, suppose $x_1 = 1000$, $x_2 = 5000$, and $x_3 = 10{,}000$. There would then be rates R_1 for purchasing insurance to cover losses between 0 and up to 1000 from any event, R_2 to cover losses greater than 1000 and up to 5000, and R_3 to cover losses in the interval between 5000 and up to 10,000. If an individual purchased insurance to cover losses up to 3000, then $I_1 = 1000$ and $I_2 = 2000$;[2] if he purchased coverage for any losses greater than 1000, then $I_1 = 0$, and $I_2 = 4000$, and $I_3 = 5000$.

The value function given by (1) is now modified to be:

$$V(I_1, \ldots, I_n) = \sum_{i=1}^{n} \pi(P_i) v\left[-(x_i - x_{i-1}) + (1-R_i)I_i\right] + \left[1-\pi(P_i)\right] v(-R_i I_i) \tag{7}$$

subject to $0 \leq I_i \leq x_i - x_{i-1}$.[3]

[2] Such behavior, however, is inconsistent with prospect theory as will be shown below.
[3] A more detailed model would include a budget constraint that relates insurance expenditures to the individual's income. For this simple exposition, we assume that the income is sufficiently large so this constraint will not influence insurance decisions.

Due to the convexity of the value function there is a critical value R_i^*, which determines what ranges of loss for which a consumer will want to to purchase insurance coverage I_i, $i = 1 \cdots n$. Depending on the actual rate schedule an individual will want to cover all, some or none, of the risks facing him. For example, full coverage against all risks will be purchased only if $R_i \le R_i^*$, $i = 1, \cdots, n$; full coverage with a deductible will be chosen if $R_i > R_i^*$, $i = 1, \cdots, j$ and $R_i \le R_i^*$, $i = j+1, \cdots, n$ and no coverage will be purchased if $R_i > R_i^*$, $i = 1, \cdots, n$. An implication of the convexity of the value function is that an individual will never choose partial coverage within any given interval. Either $I_i = 0$ or $I_i = x_i - x_{i-1}$.

IV. IMPLICATIONS FOR EMPIRICAL RESEARCH

The implication of prospect theory for insurance behavior is that individuals do not make choices on the basis of asking how much coverage should I buy against a certain risk. Rather they estimate the possible losses they can suffer from each of a number of different events and then determine whether or not they what to protect themselves fully or not at all within each possible range of losses. This process differs from the one implied by the traditional concave utility function for losses utilized in the economic analysis of insurance decision. Prospect theory suggests that individuals utilize simplified heuristics in their choice processes. Suppose people focus on only one or two possible events (e.g., an automobile accident causing x dollars damage; a minor or severe flood causing x_1 and x_2 dollars losses), then one would expect to find individuals having a limited menu of alternative insurance options and choosing between them on the basis of wholistic comparisons rather than making decisions at the margin as implied by standard utility theory. The validity of each of these models for describing insurance purchase behavior against specific types of losses still awaits empirical verification.

REFERENCES

Kahneman, D., and Tversky, A. (1979). "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, 47:263-291.