A CONCEPT OF MODELING A HEALTH MANPOWER EDUCATIONAL SYSTEM

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

This paper focuses on problems of health manpower education modeling, analyzing the formation of an important source of health care resource supply. The structure of the overall system has been discussed and presented, and illustrative computations for medical academies in Poland have been performed. Finally, possible directions for model development have been outlined.

Related publications in the Health Care Systems Task are listed at the end of this report.

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ABSTRACT

The paper presents some mathematical concepts of modeling a health manpower educational system. The importance of manpower resources, i.e., doctors, nurses, and other supporting staff, in the health services delivery process is widely recognized. Therefore, the research on resource supply models analyzing health manpower education was undertaken.

First, the general structure of the health manpower educational system (HMES) was presented. Next the adapted methodology of modeling was described, followed by more detailed presentations of:

-- secondary medical school subsystems

-- medical academy subsystems

-- postgraduate courses

Numerical examples from Poland of the application of proposed simulation techniques to medical academies were given. In addition, the forecasts of the number of medical doctors with Ph.D. degrees were presented.

Then the utilization of resources in the education process was briefly described. The paper focused its attention on models for simulation purposes, but an optimization approach to the modeling of an educational system was also presented, proceeding naturally from simulation models.

CONTENTS

A CONCEPT OF MODELING A HEALTH MANPOWER EDUCATIONAL SYSTEM

1. INTRODUCTION

The health care research at IIASA is aimed at the development of a family of interrelated computerized submodels of national health care systems, the goal being to prepare an assistant tool for planning and decision-making bodies on national and regional levels. The HCS Model and some of its main interactions with external systems are presented in Figure 1.

The submodels so far considered deal with population, disease prevalence, resource need, resource allocation, and resource supply. To build the resource supply model it is necessary to develop:

-- the health manpower education model
-- the facility construction (investment process) model

In this way some important interactions of the Health Care System (HCS) with other subsystems or spheres of the economy, i.e., education, construction, industry, etc., can be modeled (see Figure 1). Such a necessity has been underlined several times in IIASA papers, e.g., Kiselev (1975) and Shigan (1977). In fact, it is quite a natural way of thinking if one wants not only to model the separated health care system but also to consider it as an important part of the model of socioeconomic development.

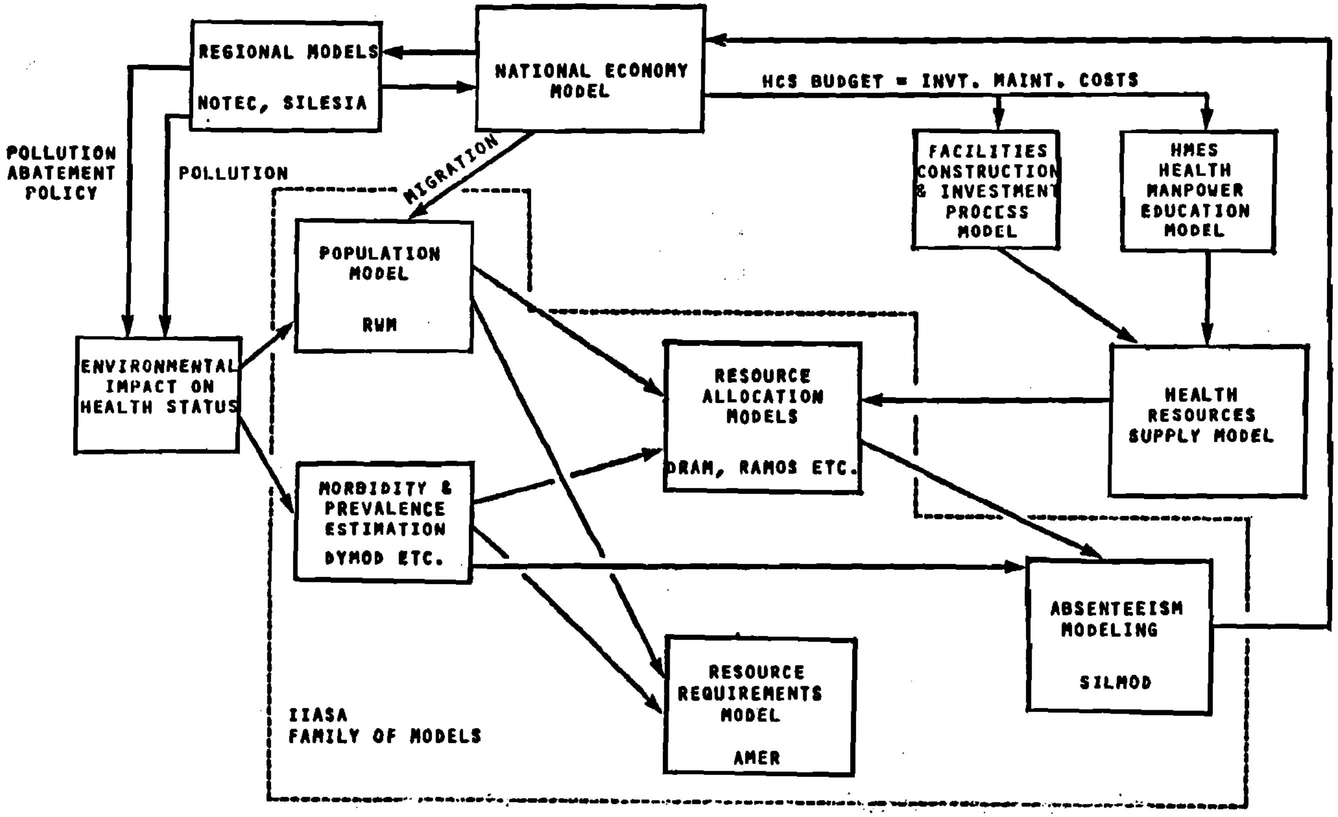

Figure 1. Health care system model and its main interconnections. Specification of IIASA's submodels: RWM, Rogers-Willekens model for demographic analysis (RM-76-58); DRAM, disaggregated resource allocation model (RR-78-8); RAMOS, resource allocation model over space (WP-80-125); DYMOD, dynamic approach to the estimation of morbidity (WP-80-71); AMER, aggregate model for estimating resource requirements (RM-78-21); SILMOD, sick leave model (WP-78-28). See also Shigan et al. (1979).

Recently, at the Systems Research Institute, Polish Academy of Sciences, the research on the modeling of socioeconomic development for planning and management purposes is being carried on. Within this research activity the sector models for health care systems (Bojańczyk 1978, 1979) and education systems (Rokicki 1978, 1979a, 1979b, 1980) are being created.

In the present paper some concepts of health care and education models are merged to produce the outline of health manpower education system (HMES) modeling. In the beginning the general structure of the system is presented, followed by the more detailed descriptions of three different levels of education. The paper is ended by the resource utilization (in the educational system) analysis and optimization approach recommendations.

2. THE GENERAL STRUCTURE OF THE HEALTH MANPOWER EDUCATIONAL SYSTEM

The place and role of the health manpower educational system in Poland is somewhat peculiar and probably unique. The educational system of physicians, dentists, and all other medical personnel (all specialties) is relegated to the Ministry of Health and Social Welfare (not to the Ministry of Education which is the case in most countries).* This fact results in the rather obvious necessity of including the educational model to the HCS Model.

But in other countries as well, the arguments for analyzing the HMES goals and activities and considering the system outputs in a planning context are too numerous to mention.

Let us limit our attention just to the four most important:

-- the medical staff is one of the crucial resources in

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*In analyzing health manpower problems, it is of course important to consider as well other specialties rather than just the strictly medical ones. In administrative staff there are, for example, economists and management science faculty graduates and the technical staff which usually comes from technical universities. Thus, one should remember these facts when formulating complex and comprehensive models.

health care services delivery

-- the education of highly qualified staff (like senior doctors or consultants) requires many years of resource consuming studies (medical academies and then post-graduate courses)

-- the educated and trained staff utilizes its knowledge and experience for a long time (30-40 years) with rather limited possibilities of dramatic changes of specializations (e.g., switching from psychiatry to ophthalmology).

Summarizing, the decisions on enrollment policy (the structure and numbers of entrants to the educational system) made now have their consequences in a relatively distant future. Since the HCS is linked with external subsystems too, the introduction of a possibly complex model of educational systems seems to be unavoidable.

In Figure 2 the general structure of the Polish HMES is presented. It is a kind of hierarchical system with three levels of education (and many specialties within each of the levels):

-- secondary medical schools (denoted by index l=s)

-- medical academies (s=a)

-- postgraduate training courses systems (l=pg).

HMES being itself an independent subsystem of the Ministry of Health Care and Welfare is nevertheless interconnected with the general educational system mostly through the input flows of primary and general secondary school graduates. HMES educates medical doctors (in medical academies and postgraduate courses) and nursing and paramedical staff (in secondary medical schools).

3. TOWARDS BUILDING THE SIMULATION MODEL OF HMES

3.1 Introduction

As it can be noted from Figures 1 and 2, HMES itself seems to be a rather complicated system with non-trivial dynamics and

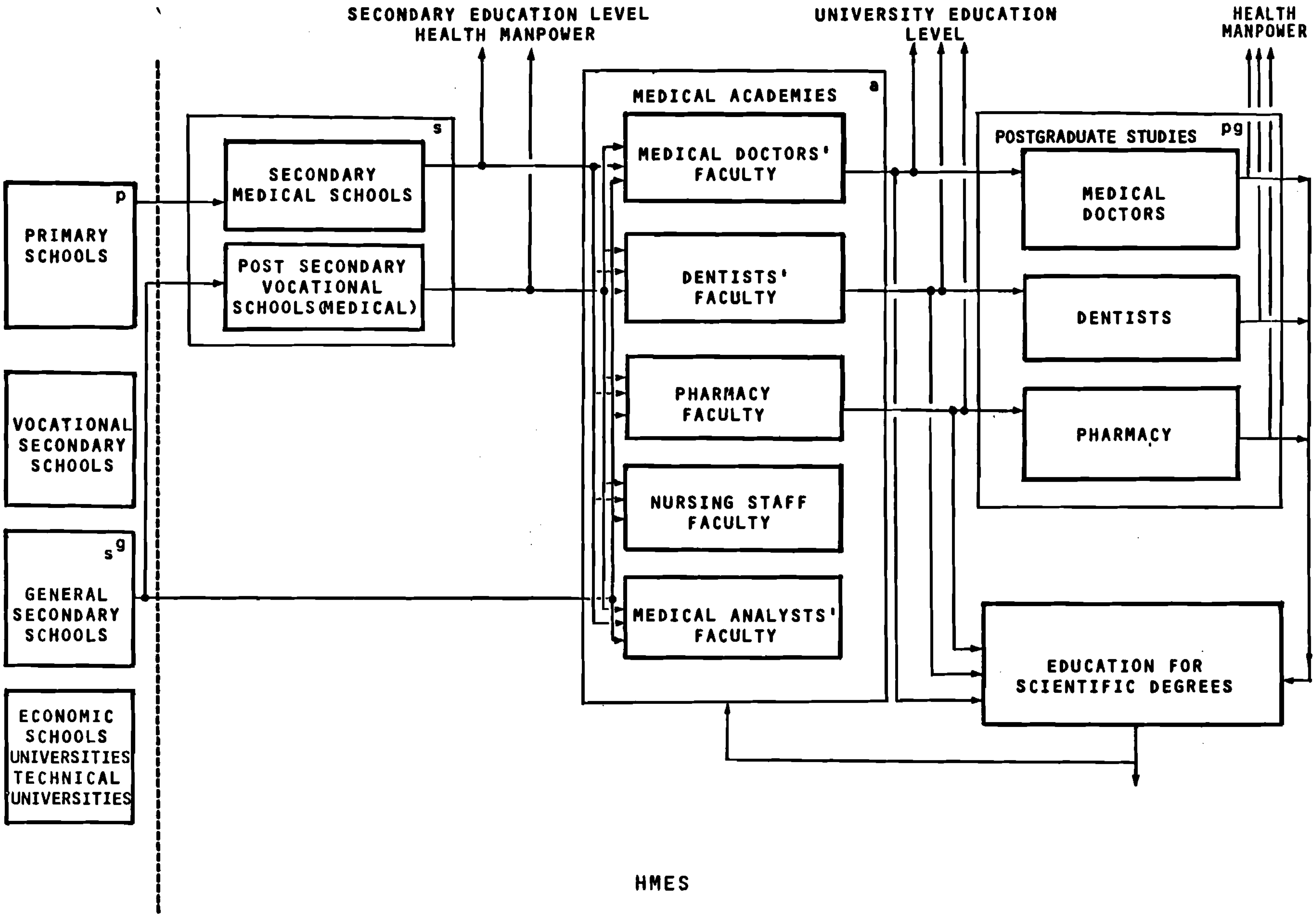

Figure 2 The general structure of health manpower educational systems.

numerous interconnections with external systems.* From the analysis carried out in Rokicki (1980) it follows that different modules of educational systems (different types of schools for different levels of education) can be described using the same methodology.

Therefore in this section two models of the basic typical subsystem of HMES will be presented and analyzed. They are derived to describe the dynamics of physical flows (flows of pupils, students, drop-outs, and graduates) in the system.

The basic subsystem - for i-th type of training - is presented in Figure 3.** The hierarchical model consists of such subsystems and its structure depends on the real structure of the considered educational system.

Only the postgraduate studies system will be modeled in a slightly different way.

The simulation models proposed could be used in many ways, two of them being the most important from the planning and decision-making bodies' viewpoint:

S1 - given the enrollment strategy to calculate number of graduates (within different levels and types of schools)

S2 - given the demand for health manpower (in some chosen aggregated or more specified categories) to determine the enrollment levels and structure and confront them with available resources.

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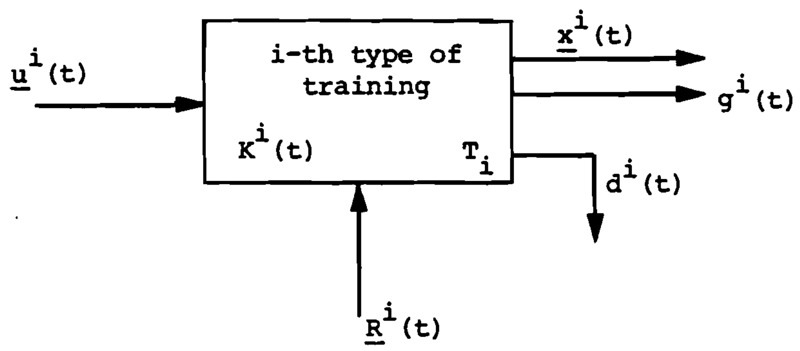
*For different approaches to educational systems modeling see e.g.: the econometric model in Boardman et al. (1978) and the Markov model in Stewman (1978).

**The i-th type of training is considered within given level of education ℓ e.g., Dentist's Faculty within medical academies.



Remarks:

- index ℓ omitted for clarity
- $\underline{R}^i(t) = [R_1^i(t),\ldots,R_j^i(t),\ldots,R_J^i(t)]$
- $j \in \{1,\ldots,j,\ldots,J\}$
- other notations are introduced and explained in text further on

Figure 3. The basic subsystem of HMES for i-th type of training.

3.2 Mathematical Models of Educational Process

Model A

The basic assumptions of this simulation model are as follows:

(i) the numbers of people trained, graduates and drop-outs, depend linearly on the number of entrants (with obvious time delays introduced to the equations)

(ii) the maximal (or potential) numbers of the people trained (pupils or students) in schools depend on available resources (maintenance costs and investments)

(iii) the standards or normes of resource utilization in the educational process are introduced.

In this section only (i) is important while the assumptions (ii) and (iii) will be necessary to formulate resource utilization rules in section 4.

Let us introduce the notation:

T_i - the duration of i-th type of training

$\underline{x}^i(t) = [x_1^i(t), x_2^i(t), x_j^i(t), \ldots, x_{T_i}^i(t)]^T$ - numbers of pupils (or students) beginning the consecutive classes (or university years) in i-th type of training in year t

$g^i(t)$ - number of graduates in i-th type of training in year t

$\underline{u}^i(t)$ - number of entrants to the schools in i-th type of training in year t.

Then the pupils (students) flows could be described in a following way:

$$\underline{x}^i(t + 1) = \underline{K}^i(t) \cdot \underline{x}^i(t) + \underline{u}^i(t + 1) \qquad (1)$$

where matrix $K^i(t)$ has the form

$$\underline{K}^i(t) = \begin{bmatrix} r_1^i(t) & & & & \\ p_1^i(t) & r_2^i(t) & & & \underline{0} \\ & p_2^i(t) & r_3^i(t) & & \\ \underline{0} & & \ddots & \ddots & \\ & & & p_{T_{i-1}}^i(t) & r_{T_i}^i(t) \end{bmatrix} \tag{2}$$

and coefficients $p_j^i(t)$, $r_j^i(t)$ satisfy the balance equation

$$p_j^i(t) + r_j^i(t) + w_j^i(t) = 1 \qquad \forall j \in [1, T_i] \tag{3}$$

$p_j^i(t)$ ratio of number of pupils of j-th class in year t who become the pupils of j+1-th class one year later to the total number of pupils of j-th class in year t

$r_j^i(t)$ ratio of number of pupils of j-th class in year t who repeat this class in next year to the total number of pupils of j-th class in year t*

$w_j^i(t)$ ratio of drop-outs in j-th class in year t to the total number of pupils of j-th class in year t

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*In Model A multiple repeating of classes are allowed for. Naturally $r_j^i(t)$ coefficients should be adjusted according to the existing administrative regulations - e.g., in Poland one is not allowed to repeat the first year of university degree studies in medical academies. However, in this case, it does not introduce a significant bias.

$$\underline{u}^i(t) = [u_1^i(t), 0, 0, \ldots, 0]^{T^*} \tag{4}$$

The numbers of graduates and drop-outs are given by equations:

$$g_t^i(t) = p_{T_i}^i(t-1) \cdot x_{T_i}^i(t-1) \tag{5}$$

$$d^i(t) = \underline{w}^i(t-1)^T \cdot \underline{x}^i(t-1) \tag{6}$$

where

$$\underline{w}^i(t)^T = [w_1^i(t), w_2^i(t), \ldots, w_j^i(t), \ldots, w_{T_i}^i(t)] \tag{7}$$

In model A the knowledge of parameters $\underline{p}^i(t)$, $\underline{r}^i(t)$, and $\underline{w}^i(t)$ is required to describe the dynamics of the subsystem. For the past they can be identified from past flows of people trained. For the future (for the expected behavior simulation purposes) some prognosis has to be performed (possibly incorporating important administrative regulations and limitations as expressing the policy options and strategies for the educational system development).

The presented model can be applied in both modes of use S1 and S2. Unfortunately, it requires significantly more data than model B, which will be described later. Illustrative numerical examples presented in Section 3.4 (and Appendix A) concern only the model B formulation. However, the research on model A, which has a more "physical" structure, is carried on in close collaboration with Department of Education (of the Ministry of Health Care and Welfare, Warsaw) in order to apply this methodology in a real planning context.

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*The entrants to the first class (admissions to the universities) only are considered. For the regional models, including migration, some other non-zero elements of $\underline{u}^i(t)$ should be introduced.

Model B

This model can be treated as a simplified version of the previous one.

The basic assumption (i) of model A holds for this model with some modifications:

- only the numbers of graduates that depend linearly on the combination of number of entrants are considered*
- in mathematical formulation the possibilities of repeating are limited

The assumptions (ii) and (iii) will be discussed in section 4.

If the notations for T_i, $g_i(t)$, and $u^i(t)$ from model A remain unchanged then the following equation

$$g^i(t) = a_1^i u^i(t - T_i) + a_2^i u^i(t - T_i - 1) \qquad (8)$$

will be proposed for the graduate's formation description, where

$$a_1^i(t) + a_2^i(t) < 1 \qquad (9)$$

It can be interpreted as follows - two components contribute to the graduate's formation:

- the first one represents straight forwardly the number of entrants to the educational system T_i years earlier (and the majority - see the range of coefficients a_1^i in the analysis of section 3.4 - of pupils or students graduate after a normal sequence of steps)
- the second can be considered as a rough measure of the repeating cycles (only one repeating chance during the educational process is allowed for).

*Nevertheless some proposition for the estimation of number of people trained, being the prerequisite for the resource utilization analaysis, will be proposed in section 4.

3.3 Remarks on Secondary Medical Schools Subsystem

It can be seen from Figure 2 that there exist two types of medical schools on secondary education levels:

SSA - vocational secondary schools (usually 4-5 years in the educational cycle)

SSB - post-secondary vocational schools (usually 1-2.5 years in the educational cycle)

In the group SSA there are two types of schools for:

- nursing staff (5 years, 9500 graduates in 1979 in Poland)
- baby care personnel (4 years, 348 graduates in 1979)

The entrants are recruited from the primary school graduates. Statistical data for this group of secondary medical schools were very few and none of the models proposed could be tested. This lack of consistent data will be overcome in the future because vocational secondary schools of group SSA account for two-thirds of the total number of nursing staff graduates.

As far as the group SSB is concerned there were about 20 specialities represented in 1980 - mostly nursing staff, medical technical laboratories staff, social workers, dentist technique assistant staff, etc. The entrants were recruited from the graduates of general secondary schools. The schools of the SSB group (total number of graduates in 1980 = 12444) graduate on average (with an exception for nursing staff being rather numerous: 5580 graduates in 1980) 450 persons per speciality - see Appendix A. Only few data points (2-4) on the number of entrants and graduates were available for SSB schools. Therefore only apparently oversimplified type B model (but practically not introducing significant bias) could be proposed, i.e., of the form

$$g^i(t) = a^i \cdot u^i(t - T_i) \qquad (10)$$

where a^i can be understood as an estimate of educational process efficiency. In Table A of the Appendix A some results of a^i parameters estimation were presented.

3.4 Simple Model of Medical Academy Subsystems

Students of medical academies are recruited from the vocational secondary schools (both groups SSA and SSB as denoted in section 3.3) and the general secondary schools. In Figure 2 the structure of medical academies subsystem was presented. The following faculties are represented for Poland:

- Medical Doctors' Faculty (T_1 = 6 years, in 1980: 3951 entrants, 3376 graduates)
- Dentists' Faculty (T_2 = 5 years, in 1980: 682 entrants, 740 graduates)
- Pharmacy Faculty (T_3 = 5 years, in 1980: 659 entrants, 856 graduates)
- Nursing Staff Faculty (T_4 = 4 years, in 1980: 333 entrants, 276 graduates)*
- Medical Analysts' Faculty (T_5 = 5 years, new speciality, first entrants in last year, no graduates).

The first three faculties are well-established, broad specialities traditionally taught in the medical academies.** Therefore the statistical data are fairly easy to be obtained and pretty long time series are available. It is a very important and nice feature from the point of view of the parameter identification (estimation) process.

As far as the next two faculties are concerned the data are either non-existing, as in the case of the Medical Analysts' Faculty,or too few,as in the case of the Nursing Staff Faculty (four-year long time series against four years for studies).

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*Poland is one of the only country in the world where the studies are organized for the nursing staff at university level. The graduates usually occupy some high posts in hospital administration - senior nurses, etc.

**The graduates of the Medical Doctors' Faculty having passed the general curriculum (including the most important medical specialities) enter an extensively developed system of post-graudate courses which provides fairly comprehensive professional orientation (see section 3.5).

Therefore the authors decided to estimate the type B (of section 3.2) model parameters only for:

- Medical Doctors' Faculty
- Dentists' Faculty
- Pharmacy Faculty

Some measures of goodness-of-fit of the proposed model's parameters and the forecasts of the number of graduates will be presented.*

Two measures of goodness-of-fit were calculated:

- correlation coefficient R_i

$$R_i = \sqrt{1 - \frac{\sum_{t=1}^{N_i} [g_S^i(t) - g_M^i(t)]^2}{\sum_{t=1}^{N_i} [g_s^i(t) - \bar{g}_2^i]^2}} \qquad (11)$$

where

$g_S^i(t)$ - data point (number of graduates from i-th faculty in year t)

$g_M^i(t)$ - from the model [number of graduates from i-th faculty in year t calculated from equation (8)]

N^i - number of data points for estimation procedure

$\bar{g}_s^i = \left[\sum_{t=1}^{N_i} g_s^i(t)\right] / N^i$ - the average number of graduates in time interval N^i [estimation period $(t_0^i, t_0^i + N^i - 1)$]

and

*The data for estimation are given in Appendix B.

- average error δ_i

$$\delta_i = \sqrt{\frac{\sum_{t=1}^{N^i} [g_s^i(t) - g_M^i(t)]^2}{N^i}} \cdot \frac{1}{\bar{g}_s^i} \qquad (12)$$

<u>Medical Doctors' Faculty</u>

$g_M^1(t) = a_1^1\, u^1(t - 6) + a_2^1\, u^1(t - 7),\ t \in [1965, 1986]$

$a_1^1 = 0.619\ ,\ a_2^1 = 0.349\ ,\ t_0^1 = 1965\ ,\ N^1 = 16\ ,\ R_1 = 0.943\ ,$

$\delta_1 = 6.0\%$

In Figure 4 statistical data and model values for number of graduates together with the forecast [based on the model equation (8) and the data for number of entrants from Table B in Appendix B] are presented.

<u>Dentists' Faculty</u>

$g_M^2(t) = a_1^2\, u^2(t - 5) + a_2^2\, u^2(t - 6),\ t \in [1963, 1985]$

$a_1^2 = 0.689\ ,\ a_2^2 = 0.034\ ,\ t_0^2 = 1963\ ,\ N^2 = 18\ ,\ R_2 = 0.7452\ ,$

$\delta_2 = 8.9\%$

In Figure 5 as in Figure 4, the corresponding graphs for Dentists' Faculty graduates were given.

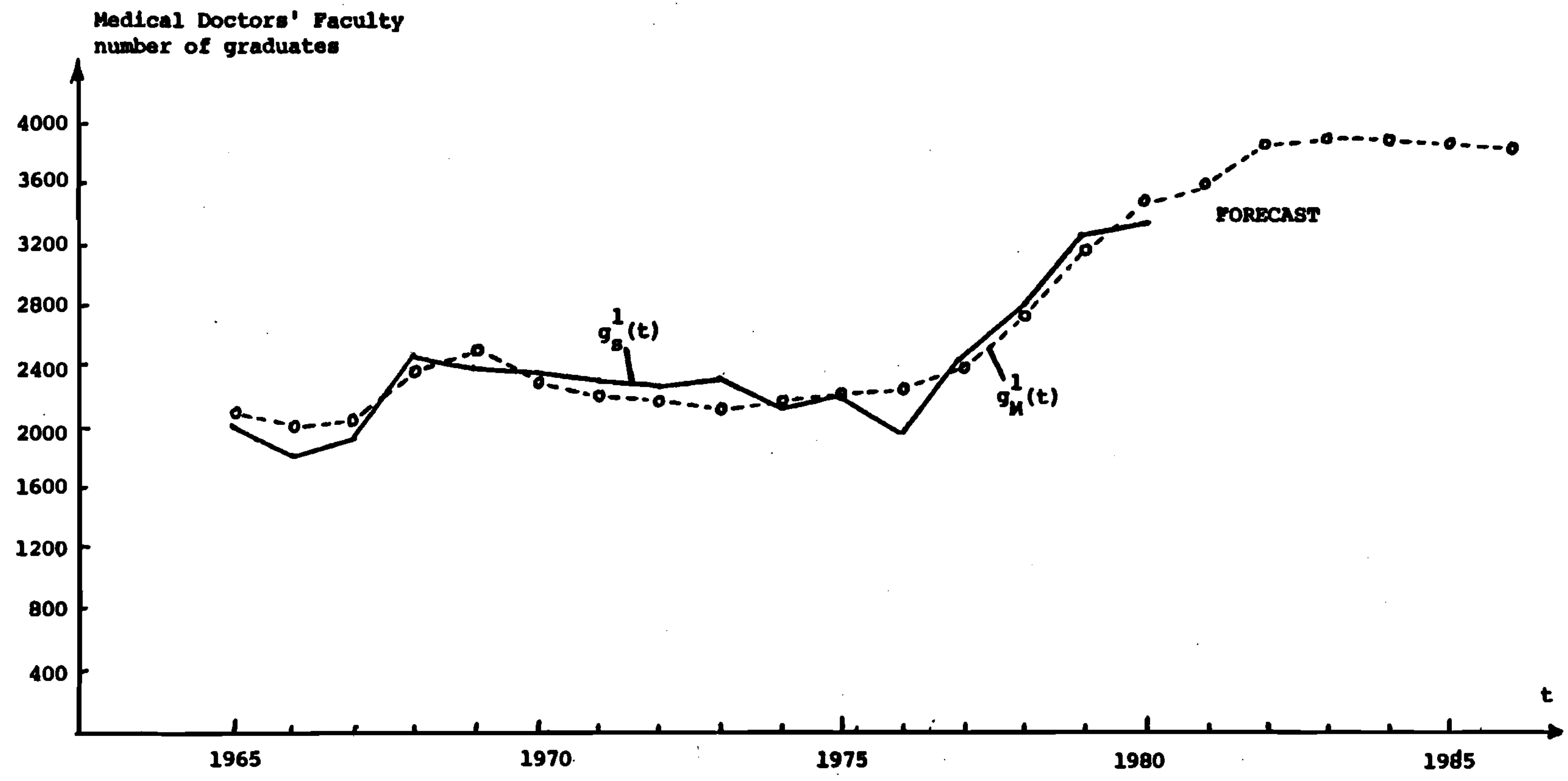

Figure 4. Number of graduates from Medical Doctors' Faculty. Actual and from the model.

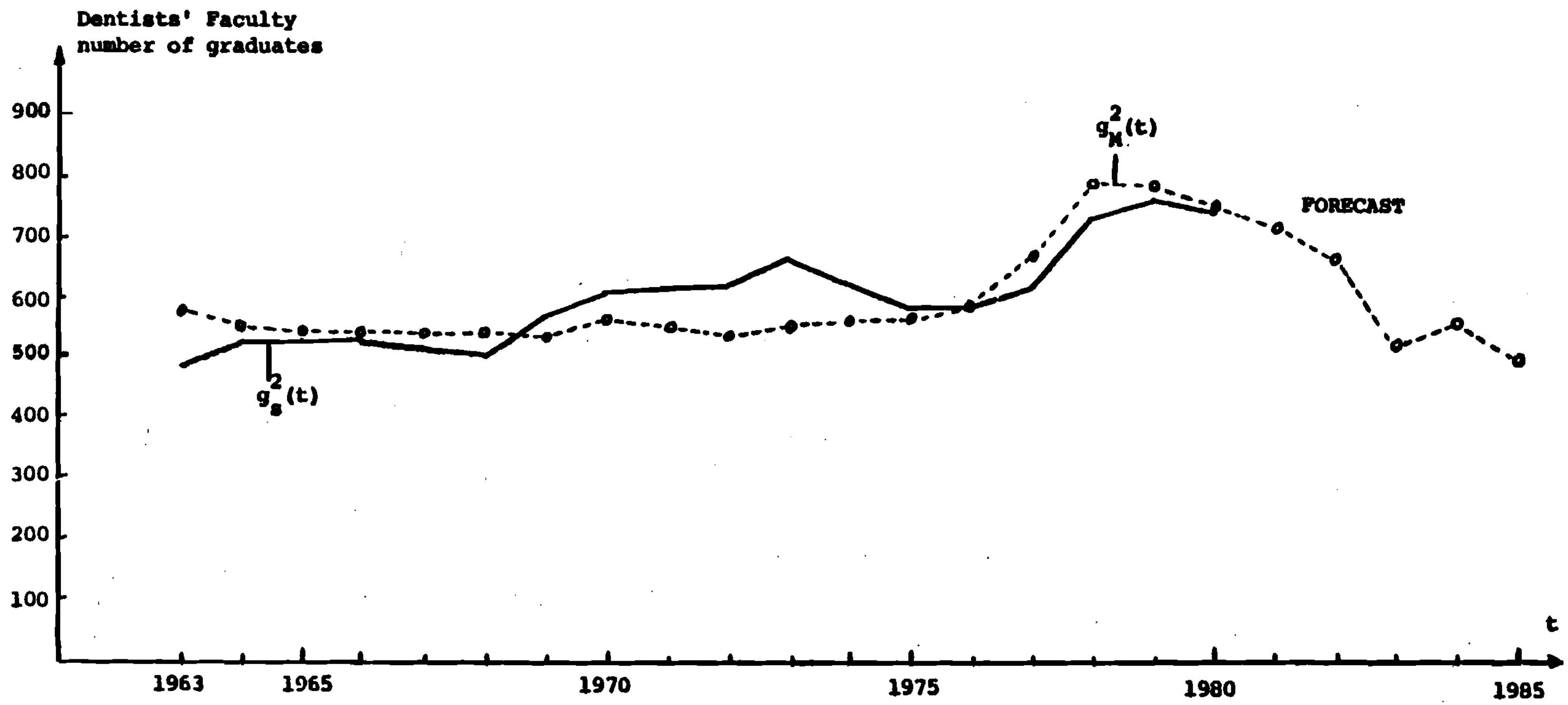

Figure 5. Number of graduates from Dentists' Faculty. Actual and from the model.

The number of graduates is slightly decreasing, which is in close reference to the broadly accepted opinion that the present ratio of number of dentists per population is not far from the "ideal standard" defined by the health care planners. Therefore the enrollment policy with respect to this faculty in the last few years was to lower the limits of entrants.

Pharmacy Faculty

$$g_M^3(t) = a_1^3\, u^3(t-5) + a_2^3\, u^3(t-6),\ t \in [1965 - 1985]$$

$$a_1^3 = 0.479\ ,\ a_2^3 = 0.255\ ,\ t_0^3 = 1965\ ,\ N^3 = 16\ ,\ R_3 = 0.748\ ,$$

$$\delta_3 = 9.6\%$$

The results of estimation and the forecast for Pharmacy Faculty graduates are shown in Figure 6. The behavioral pattern of the number of graduate changes is very similar to that observed in Figure 5. It again reflects the enrollment policy of the Ministry.

A summarization of the three type B models presented above may be used for forecasting purposes (in both —S1 and S2— modes of use described in section 3.2) provided the efficiency of the educational process expressed in terms of a_1^i and a_2^i coefficeints does not change much.

The move through investigations leading to the type A model are recommended and in fact they are in progress.

3.5 Conceptual Framework for Postgraduate Studies Modeling

In this section a methodology for postgraduate studies modeling will be presented. This approach was adapted from Kulikowski, Mierzejewski and Rokicki (1975) where the model of obtaining scientific degrees had been proposed and analyzed. The model presented was elaborated to make forecasts of teaching staff at the universities.

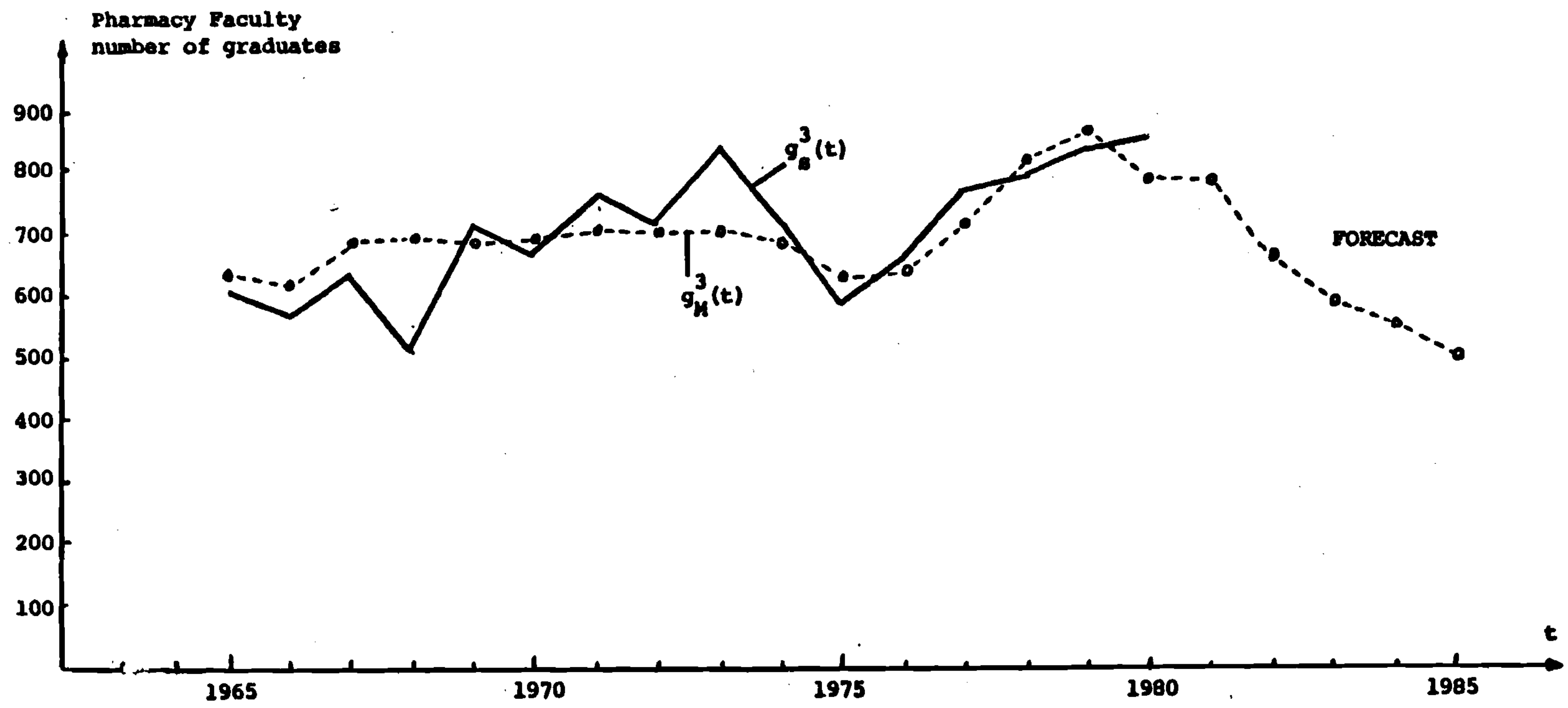

Figure 6. Number of graduates from Pharmacy Faculty. Actual and from the model.

At present there exists in Poland a well-established system of postgraduate studies for medical doctors, dentists, and pharmacists (see Figure 2). The graduates of medical academies can apply for so-called specialization degrees I and II.*

The general requirements are as follows:

- postgraduate studies cannot be undertaken earlier than 12 months after graduation from the medical academy
- a certain kind of professional experience is recommended for that period
- the process of "specialization" lasts from 18 months to 48 months** depending on the speciality (30 specialities in degree I and 55 specialities in degree II)
- the specializing graduates of medical academies usually get scholarships during this time

The specialists of degrees I and II are better paid and occupy more important and responsible posts. Thanks to fairly high requirements and comprehensive programs of the training, they form a highly qualified group of medical personnel.

Apart from the system of postgraduate studies some graduates of medical academies during their professional activities in pitals or ambulatory care units (and of course those teaching at the academies) obtain scientific degrees such as a Ph.D. in medical sciences.***

*It can be observed that the share of specialists (both degrees I and II) in the total number of medical doctors, for example, increases: 54% in 1960, 63% in 1970, and 72% in 1977.

**These are minimal times according to existing administrative regulations - but usually it takes longer to accomplish in practice and to pass all the exams required.

***In the Polish educational system a traditional hierarchy of university degrees exists:

- equivalent of Ph.D.
- so-called Habilitation Doctorate
- Professor Degree (not post but degree of title)
- Full Professor Degree

Each of them has its own requirements like writing and defending in public a special thesis for Ph.D., writing and presenting at the Scientific Council of the Institute or University another special thesis for Habilitation Doctorate, supervising several Ph.D. theses and making extensive research and widely publishing for Professor Degrees, etc. In this paper we focus our attention only on the Ph.D. group. The entire system was modeled in Kulikowski, Mierejewski, and Rokicki (1975).

Both processes of professional perfectioning:

- for specialization degrees I and II
- for scientific degrees

could be described mathematically using the same approach.

The idea of the model will be presented below (the attention in formulation being paid to "specialization" model). In Figure 7, the illustration for a chosen speciality i is given.

The number of specialists (medical doctors) of degree I in speciality i in year t can be derived from the following equation

$$S_i^I(t) = \int_{-\infty}^{t} k_i^I(t, \tau) \cdot g(\tau) d\tau \qquad (13)$$

or from its discrete equivalent

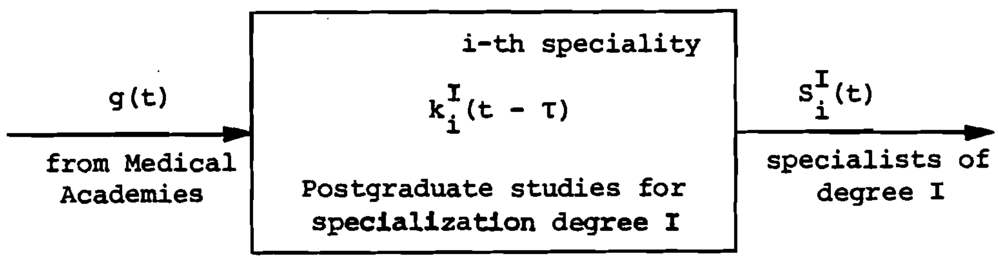

Figure 7. Postgraduate studies model for i-th speciality. Specialization of degree I.

$$S_i^I(t) = \sum_{\tau=-\infty}^{t} k_i^I(t, \tau) \cdot g(\tau)$$

$$= k_i^I(t, t) \cdot g(t) + k_i^I(t, t-1) \cdot g(t-1) \qquad (14)$$

$$+ k_i^I(t, t-2) \cdot g(t-2) + \ldots + k_i^I(t, t-k) \cdot g(t-k) + \ldots$$

where

$g(t)$ - number of graduates of medical academies in year t .

$S_i^I(t)$ - number of new specialists of degree I in speciality i in year t (i.e., the specialists who obtained this degree during this very year)

$k_i^I(t, \tau)$ - core function of integral Volterra-type operator.

The following interpretation of the components of $S_i^I(t)$ (discrete version) could be given:

$$k_i^I(t, t-j) \cdot g(t-j) = K_i^I(t, t-j) \qquad (15)$$

is the contribution of the graduates of medical academies in year (t-j) to the total number of new specialists $S_i^I(t)$.

It is assumed that the process of specialization is stationary, i.e., this contribution depends only on $\xi = t - \tau$ but not on t and τ independently. In Figure 8 the typical shape of $k_i^I(\xi)$ function is shown - there are two curves: - T - represents the theoretical function and A - is an approximation useful for computational purposes. T_{iMIN}^{I} * - denotes a certain minimal time required for specialization to be completed (given in regulations elaborated by the Ministry of Health Care and Welfare).

*It is an integer number.

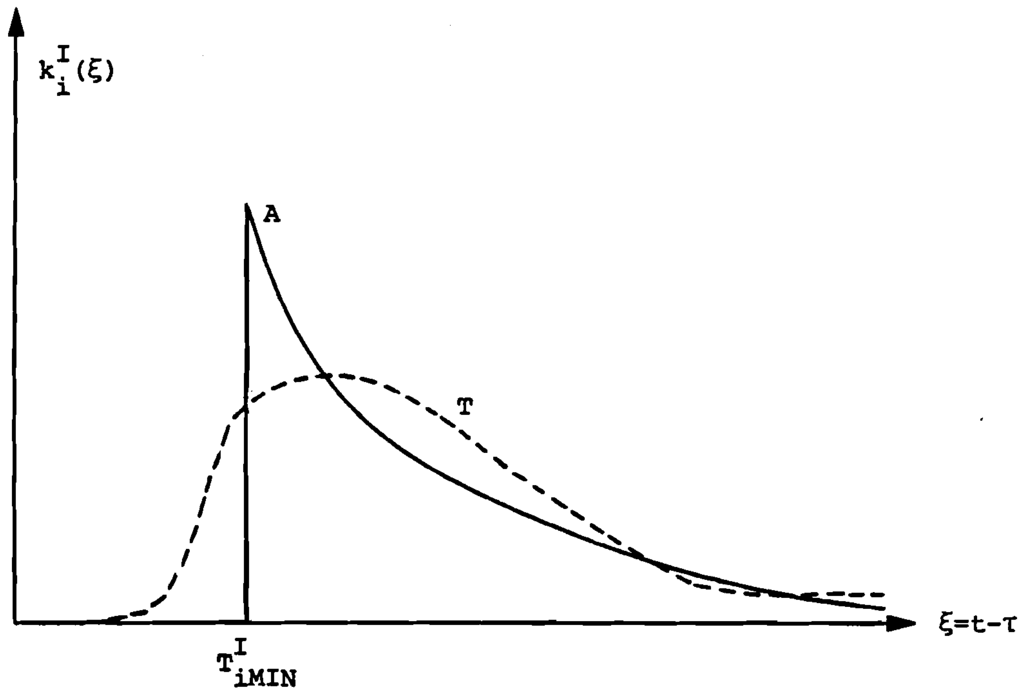

Figure 8. The typical shape of k_i^I function.

The approximating function $k_i^I(\xi)$ is of the form

$$k_i^I(\xi) = \begin{cases} 0, \text{ for } \xi \le T_{iMIN}^I - 1 \\ C_i^I \quad e^{-\alpha_i^I(\xi - T_{iMIN}^I)} & \text{, otherwise} \end{cases} \tag{16}$$

the parameters C_i^I, α_i^I, T_{iMIN}^I should be taken from experts or derived from the following formula:

$$(\hat{C}_i^I, \hat{\alpha}_i^I, \hat{T}_{iMIN}^I) = \arg\min \left[\sum_{t=t_0+1}^{t_f} (g_{iS}^I(t) - g_{iM}^I(t))^2 \right] \tag{17}$$

where

t_0, t_f - initial and final time of observations, respectively

$(\hat{c}_i^I, \hat{\alpha}_i^I, \hat{T}_{iMIN}^I)$ - optimal values of parameters, i.e., minimizing the deviation of model ($g_{iM}^I(t)$ values) from statistical data ($g_{iS}^I(t)$).

Naturally if any of the parameters is given or somehow limited (like T_{iMIN}^I - see footnote on page 20) then the minimization procedure should be carried out taking into account this constraint.

Now the fundamental equation describing the dynamic input output relationship between the numbers of graduates and specialist can be rewritten as follows:

$$S_i^I(t) = S_i^I(t_0) \cdot e^{-\alpha_i^I(t-t_0)} + C_i^I \sum_{\tau=t_0-T_{iMIN}^I+1}^{\tau=t-T_{iMIN}^I} e^{-\alpha_i^I(t-\tau-T_{iMIN}^I)} \cdot g_i^I(\tau) \tag{18}$$

or in a different form

$$S_i^I(t) = S_i^I(t-1) \cdot e^{-\alpha_i^I} + C_i^I \cdot g_i^I(t - T_{iMIN}^I) \qquad (19)$$

which seems to be very handy for computations (if the parameters are known).*

Proceeding from the formulation presented for the specialists of degree I, it is very easy to derive a similar description for the specialists of degree II (see Figure 9)

$$S_i^{II}(t) = S_i^{II}(t_0) \cdot e^{-\beta_i^{II}(t-t_0)} + C_i^{II} \sum_{\tau=t_0+1-T_{iMIN}^{II}}^{\tau=t-T_{iMIN}^{II}} e^{-\beta_i^{II}(t-\tau-T_{iMIN}^{II})} \cdot S_i^I(\tau) \qquad (20)$$

where

C_i^{II}, β_i^{II}, T_{iMIN}^{II} are the parameters describing the dynamics of specialists of the degree II formation process.

Summarizing, the model of postgraduate medical studies (not only for graduates of Medical Doctors Faculty because the methodology can be immediately extended in a rather natural way to Dentists' Faculty and Pharmacy Faculty where similar post-graduate studies exist - see Figure 2) to be identified and used in forecasting mode requires:

---

*Note that $S_i^I(t)$ is understood as the number of medical doctors who obtained degree I of specialization in year t - briefly speaking it is the number of "new" specialist of degree I. The interpretation of $S_i^{II}(t)$ is identical.

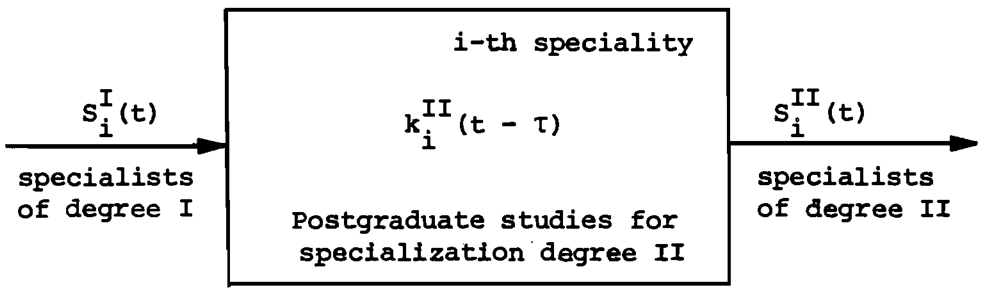

Figure 9. Postgraduate studies model for i-th speciality. Specialization of degree II.

- statistical data on the numbers of "new" specialists of degrees I and II and on the number of graduates of corresponding faculties of medical academies
- statistical material for 10-15 years time series
- experts' evaluation of some of the model parameters, e.g., T^{I}_{iMIN}, T^{II}_{iMIN}
- taking into account all existing administrative limitations and regulations having the impact on dynamic features of the process.

We close the present section with a numerical example of the above presented methodology application - this time for the Ph.D. degrees' case (see Figure 10).

The fundamental equation - in the earlier introduced recurrence form - is as follows:

$$D(t) = e^{-\alpha_D} \cdot D(t-1) + D_C \cdot g(t-T_D) \qquad (21)$$

where α_D, C_D, T_D parameters are to be estimated.

From the experts' opinion it followed that T_D equals 3 years thus only α_D and C_D were to be estimated from statistical data. The period 1964-1979 was considered as a data base and the calculations have been performed using the formula:

$$(a_D, C_D) = \operatorname{argmin} \left\{ \sum_{t=1969}^{1979} [D_S(t) - a_D D_S(t-1) - C_D g_S(t-T_D)]^2 \right\} \qquad (22)$$

where

- $a_D = e^{-\alpha_D}$
- $D_S(t)$, $g_S(t)$ - statistical data on numbers of: medical doctors with Ph.D. degree and, graduates from medical academies, respectively.

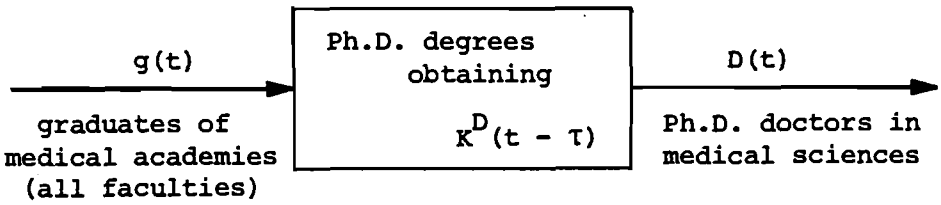

Figure 10. Presentation of the model for Ph.D. degrees.

The very simple optimality conditions:

$$a_D \sum_t D_s(t-1)^2 + C_D \sum_t g_s(t-T_D) \cdot D_s(t-1) = \sum_t D_s(t) \cdot D_s(t-1) \quad (23)$$

$$a_D \sum_t D_s(t-1) \cdot g_s(t-T_D) + C_D \sum_t g_s(t-T_D)^2 = \sum_t D_s(t) \cdot g_s(t-T_D) \quad (24)$$

allow one to find a_D and C_D.

For the data as in Table C in Appendix C the following values were obtained:

$$a_D = 0.1434 \qquad C_D = 0.1385$$

In Figure 11 the data points and the model values (together with prognosis of medical doctors with a Ph.D. degree resulting from the adopted modeling approach) were presented.

4. RESOURCES UTILIZED IN THE EDUCATIONAL PROCESS

In previous sections only the dynamics of pupils (or students or even graduates) flows in the health manpower educational system were considered thus leaving apart such important problems as:

- demand for resources: teaching staff and facilities
- maintenance and investment costs estimation

As it was pointed out in Rokicki (1979a) the attempts to estimate cost-effect relationships* were rather discouraging. The ratio of promotions in the Polish educational system in general

*Cost was understood as cost of teaching and effect calculated as e.g., ratio of number of graduates to the number of pupils (students). The rather obvious and important components of effects - quality of knowledge, being very subjective and difficult to be measured has not been considered up to now.

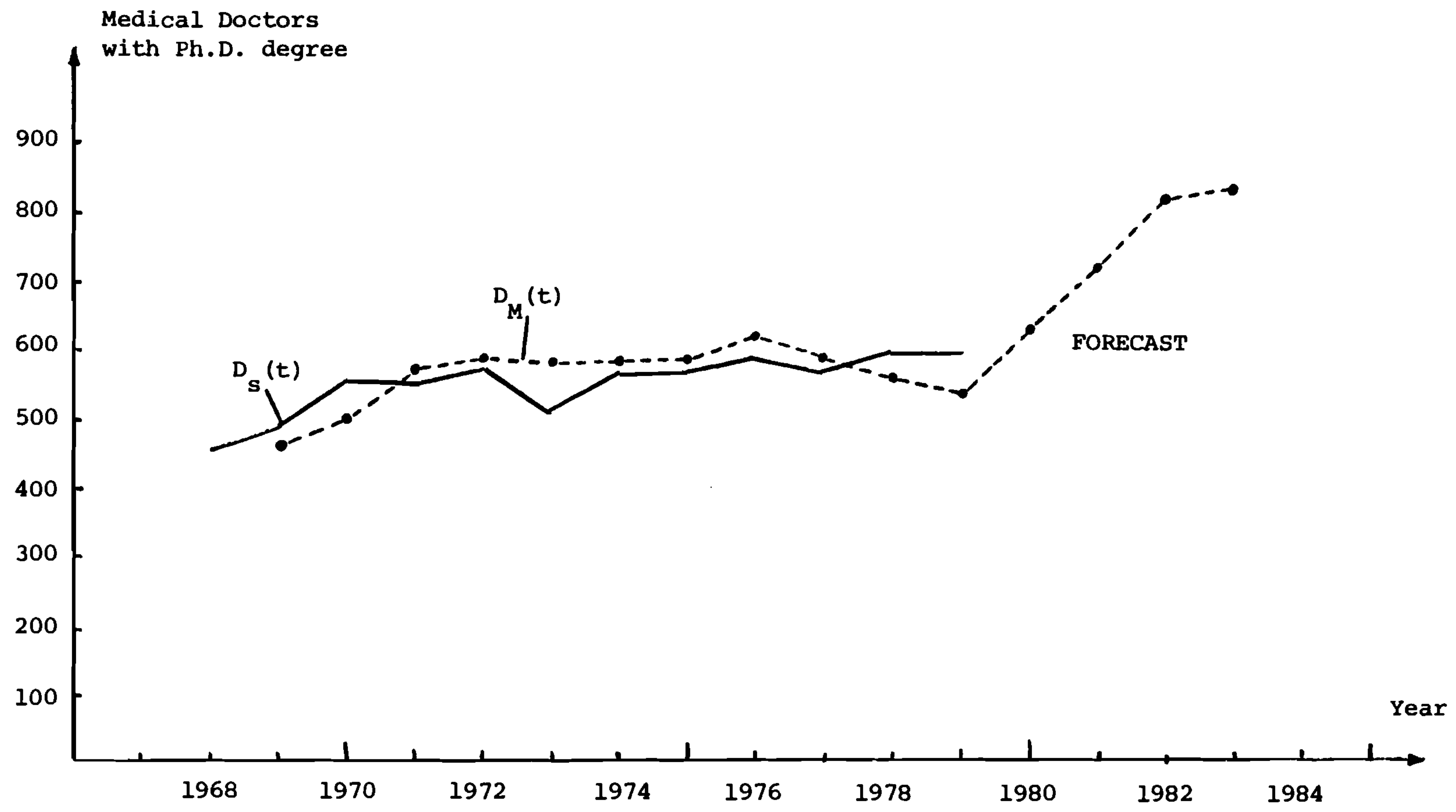

Figure 11. Number of Medical Doctors with a Ph.D. degree. Actual and from the model.

is rather high and therefore its dependence on resources appears to be weak [see Rokicki (1979a)].

Nevertheless, to guarantee the desired standard of educational process and thus the knowledge of graduates, some constraints on the resource norms (or standards) are introduced to the models.

If the certain type of training (education) is considered - and index i from Figure 3, for example, is omitted - then the following constant is proposed:

$$\mu_j(t) \leq \frac{R_j(t)}{P(t)} \quad , \quad j = 1,2 \tag{25}$$

where

- $R_1(t)$ - number of teachers (or another teaching staff required for the considered type of education)*
- $R_2(t)$ - capital used in this education subsystem facilities**
- $P(t)$ - total number of pupils (students) learning (or studying) in these facilities
- $\mu_j(t)$ - corresponding resource norm or standard.

---

*The number and structure of teaching staff could be modeled using the approach presented in section 3.5 because nowhere else but at the medical academies (and for other than medical professions at the universities) the future candidates for teaching staff are educated. For more details see Rokicki (1980).

**Several investment-capital models can be adapated in order to describe the dynamics of facility construction, e.g., discrete nonlinear equivalent of Volterra operator

$$R_2(t) = \sum_{\tau=-\infty}^{t} k_I(t-\tau) \cdot I(\tau)^{\beta}$$

where: - $I(t)$ - investments, - $k_I(t-\tau)$ certain function similar to those introduced in section 3.5 for postgraduate studies modeling and -β parameter to be estimated.

It is assumed that coefficients $\mu_j(t)$ should be increasing in time.

In model A from section 3.2 number P(t) could be calculated as follows:

$$P(t) = \sum_{j=1}^{T} x_j(t) \tag{26}$$

where T is the time of training (education). In model B it is somewhat more difficult (and less essential) while the number of pupils has to be estimated indirectly from the number of graduates. The following scheme is proposed:

$$g(t) = a_1 \cdot u(t-T) + a_2 \cdot u(t-T-1) \tag{27}$$

$$d(t) = \delta \cdot P(t) \tag{28}$$

and if one assumes

$$\delta \cong 1 - (a_1 + a_2) \tag{29}$$

then the number of pupils P(t) can be obtained from the recurrent equation -

$$P(t) = P(t-1) \cdot (1-\delta) + u(t) - g(t-1) \tag{30}$$

provided the initial value $P(t_0)$ is known.

The both types of resources engaged in the educational process:

- teaching staff (manpower)
- the facilities (the capital)

require the expenditures to cover the corresponding maintenance costs:

- salaries of teaching and supporting staff
- other current expenditures (administration, estate management, etc.)

In the present section only general concepts of resource utilization and resource supply models for a health manpower educational system were presented and hence they constitute important elements of HMES. A more detailed analysis is to be carried out in the near future.

5. OPTIMIZATION APPROACH TO THE MODELING OF EDUCATIONAL SYSTEMS

Section 3 of the present paper was devoted to the presentation of the possible simulation model formulations for the purposes of a health manpower education system. The more detailed analysis of that problem was carried out in section 3.2 where models A and B had been proposed.

Now the model A will be slightly reformulated to allow its use in an optimization mode. Only a very simple model will be presented the emphasis being put rather on the methodological aspects. The development directions, depending strongly on statistical data availability, are quite natural.

Let us consider a two-level hierarchical structure system (reduced from three-level system in Figure 2) for one aggregated speciality as shown in Figure 12. If the levels of education are denoted - as in section 2, Figure 2 - by:

- $\ell = s$ - secondary vocational medical schools
- $\ell = a$ - medical academies

then the following equations describe the optimization model:

- state equations

$$\underline{x}^{\ell}(t+1) = \underline{K}^{\ell}(t) \cdot \underline{x}^{\ell}(t) + \underline{u}^{\ell}(t+1) \qquad (31)$$

$$\ell \in L = \{s, a\}$$

- output equations

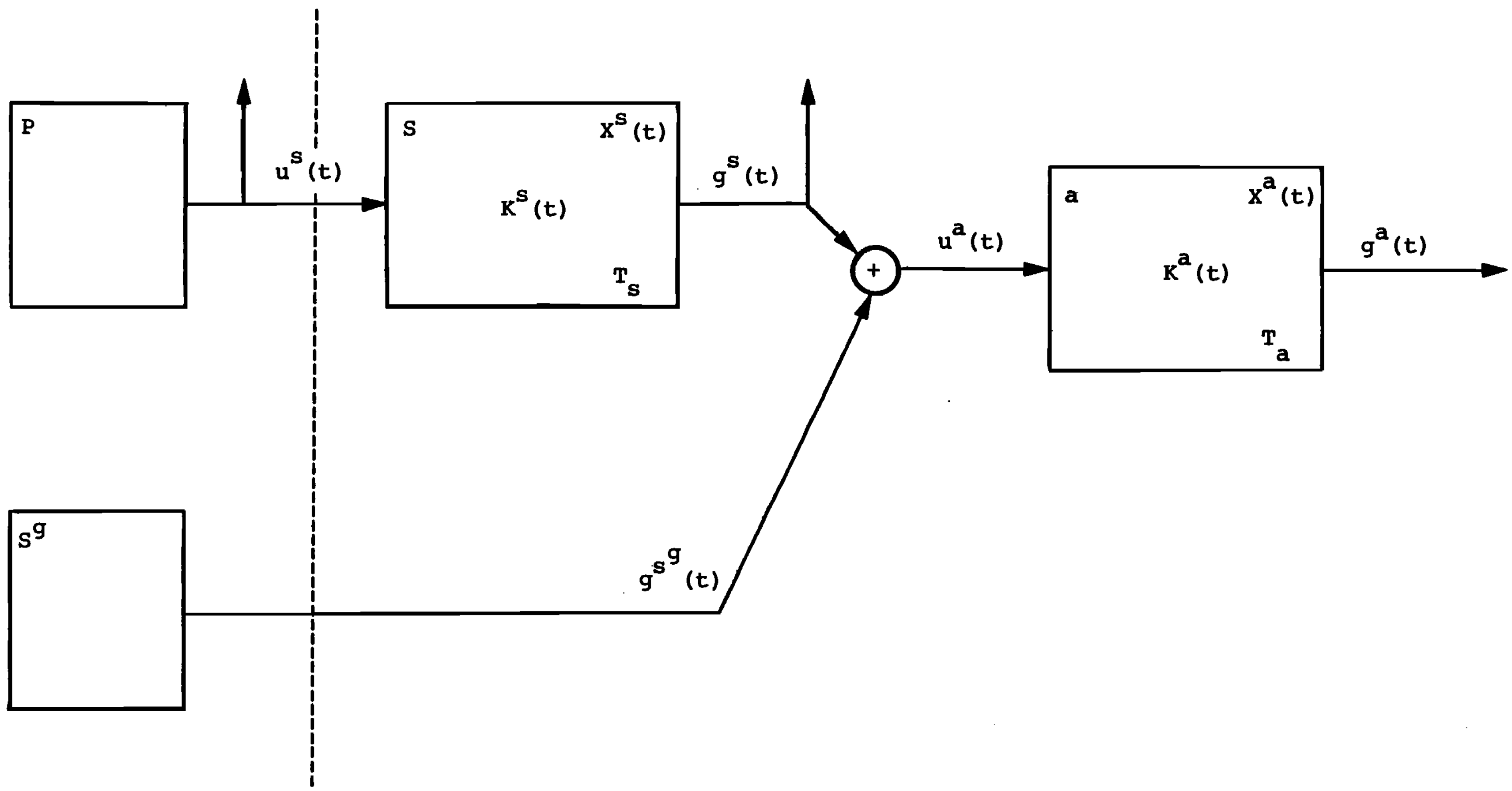

Figure 12. Flows of pupils (students) and graduates in a two-level hierarchical structure of reduced HMES for one aggregated speciality.

$$g^{\ell}(t) = p_{T_\ell}(t-1) \cdot x_{T_\ell}(t-1) \tag{32}$$

$$d^{\ell}(t) = \underline{w}^{\ell}(t-1)^T \cdot \underline{x}^{\ell}(t-1) \qquad \ell \in L \tag{33}$$

- state variable constraints (resulting from resource scarcity - see section 4)

$$X^{\ell}(t) \cdot \mu^{\ell j}(t) \le R^{\ell j}(t) \tag{34}$$

where

$$X^{\ell}(t) = \sum_{k=1}^{T_\ell} x^k(t) \tag{35}$$

- constraints on number of entrants

$$u^s(t) \le U^s(t, p) \tag{36}$$

$$u^a(t) \le U^a(t, p, s, s^g) \tag{37}$$

where the notation is identical with that introduced in section 3.2 with obvious modifications.

$U^s(t, p)$ denotes the maximal possible number of entrants to secondary vocational medical schools which depends on the efficiency of the educational process in primary schools (index p) and its dynamics.

Similarly - $U^a(t, p, s, s^g)$ - is a certain limit or constraint on the number of entrants to the medical academies. It depends directly on the outputs from secondary vocational medical schools (index s) and general secondary schools (index s^g) and of course indirectly on the outputs from primary schools.

$U^s(t, p)$ and $U^a(t, p, s, s^g)$ result generally speaking from the educational policy in the external (from the point of view of HMES) part of the educational system and they can be treated as exogenously given parameters for the optimization problem considered later. However, in $U^a(t, p, s, s^g)$ the impacts of external and internal educational systems have to be differentiated because the flows from s to a are to a substantial extent under the control of the Ministry of Health Care and Welfare.

If the demand for HMES graduates could be determined e.g., applying the IIASA family of computerized models for morbidity estimation [Klementiev (1977) and Kitsul (1980)] and resource allocation [Hughes and Wierzbicki (1980)] then the HMES model would be recommended for the optimization mode i.e.: to minimize the discrepancy between demand for health manpower and the supply of graduates.

In a formal way it can be written as follows:

$$\min \sum_{\ell \varepsilon L} \sum_{t=t_0}^{t_f} w^{\ell}(t)\,[g^{\ell}(t) - \tilde{g}^{\ell}(t)]^2 \tag{38}$$

Subject to constraints (31) - (37) where

$w^{\ell}(t)$ = weighting factors

$\tilde{g}^{\ell}(t)$ = demand for health manpower

$[t_0, t_f]$ = planning time interval

The minimization has to be performed with respect to input flows to s and a educational levels, i.e., the numbers and structure of entrants are to be determined.

Disaggregation to more specialities and the consideration of more levels of education (e.g. postgraduate studies) make the structure of the model more diversified and new dimensions in the space of decision variables have to be introduced.

The above presented optimization model can be treated as a natural extension of the simulation model preserving its mathematical structure and formulation by adding the performance criterion and some constraints on decision variables.

Therefore the simulation model has to be developed along the lines proposed in section 3.2 with some refinements and extensions requiring the linkage with other submodels of the health care system, for example morbidity estimation procedures are needed [see also Propoi (1978)].

CONCLUSION

In the paper the general methodology of the health manpower educational system modeling was presented. The "physical" flows (flows of pupils, students, and other persons entering the educational system) were described in orderto simulate the behavior of the system. The conceptual framework for describing resource utilization in the educational process was added to complete the idea of a simulation model. Finally, the optimization approach to the modeling of educational systems was presented to stimulate further research on health manpower within the health care system.

Only few parts of the HMES model were identified, but it is hoped that further collaboration with the Department of Education of the Ministry of Health Care and Welfare will enable the authors to prepare the model along the lines proposed in near future.

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APPENDIX A: PRESENTATION OF SSB GROUP FOR THE SECONDARY MEDICAL SCHOOL SUBSYSTEMS OF POLAND

	Type of school	Duration of training in years	Number of graduates in 1980	Estimated parameter A in Model B*
1	nursing staff	2	3534	0.785
2	nurse assistants	1	306	0.934
3	nurses for psychiatry	2.5	251	0.687
4	midwives	2.5	1489	0.788
5	medical analysts	2	1406	0.734
6	pharmaceutical technique	2	893	0.794
7	physiotherapy	2	1039	0.850
8	electrocardiology	2	799	0.790
9	social workers	2	700	0.889
10	social workers	2	567	0.793
11	dentist hygienists	1	320	0.793
12	school hygienists	1	244	0.771
13	hygienists	2	223	0.790
14	child care nurses	1	220	0.871
15	therapeutists	2	34	0.953
16	dietitians	2	188	no information
17	dietitians	1	96	0.810
18	biomechanics	2	42	0.934
19	medical electronics	2	67	0.678
20	orthopedists	1	26	0.963

*simplified version $g^i(t) = a^i \cdot u^i(t - T_i)$

APPENDIX B: SOME STATISTICAL DATA ON THE MEDICAL ACADEMY SUBSYSTEMS OF POLAND

Year	Medical Doctors' Faculty			Dentists' Faculty			Pharmacy Faculty		
t	$g_S^1(t)$	$g_M^1(t)$	$u^1(t-6)$	$g_S^2(t)$	$g_M^2(t)$	$u^2(t-5)$	$g_S^3(t)$	$g_M^3(t)$	$u^3(t-5)$
1986		3832	3951						
1985		3849	3968		497	682		501	659
1984		3873	3988		560	777		554	727
1983		3907	4020		519	708		590	806
1982		3823	4062		656	903		662	801
1981		3604	3747		709	977		784	1092
1980	3376	3476	3677	740	751	1036	856	791	1022
1979	3272	3175	3434	757	788	1088	834	865	1181
1978	2797	2750	3005	733	796	1109	798	820	1176
1977	2463	2425	2547	609	661	920	772	712	1008
1976	1955	2293	2430	583	578	801	653	638	899
1975	2207	2189	2257	587	558	772	585	625	814
1974	2115	2148	2267	624	557	771	706	686	921
1973	2321	2090	2131	657	550	761	834	700	960
1972	2290	2153	2206	618	537	741	718	699	944
1971	2339	2189	2255	617	550	760	763	705	967
1970	2372	2313	2270	604	556	771	670	689	947
1969	2396	2522	2600	566	530	732	716	679	922
1968	2489	2373	2614	499	543	751	520	693	933
1967	1935	2064	2161	508	545	753	630	679	967
1966	1810	2016	2079	532	544	752	560	620	845
1965	2007	2061	2088	528	543	750	600	629	843
1964				524	553	764			
1963				489	574	786			

APPENDIX C: CHOSEN STATISTICAL DATA ON MEDICAL DOCTORS WITH PH.D. DEGREE IN POLAND

Year	Number of graduates of medical academies (all faculties)	Number of medical doctors with Ph.D. degree	Number of medical doctors with Ph.D. degree - Model
t	$g_S(t)$	$D_S(t)$	$D_M(t)$
1964	4247		
1965	3135		
1966	2902		
1967	3073		
1968	3508	455	
1969	3678	486	467
1970	3646	558	495
1971	3719	551	566
1972	3626	576	588
1973	3861	513	587
1974	3604	569	589
1975	3439	569	584
1976	3253	594	616
1977	3928	570	584
1978	4493	600	558
1979	5146	599	537
1980	5248		630
1981			713
1982			815
1983			844

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