TAXES AND RESOURCE PRICES: A NORTH–SOUTH GAME

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FOREWORD

This is one of three papers derived from research on North–South resource trade performed in the System and Decision Sciences Area during the summer of 1982. The aim of this research was, first, to construct a model of North–South trade, and then to use it as a framework for further work in gaming, negotiations and interactive decision making.

In this paper, two models already developed by the author are modified and combined to produce a simple model of North–South trade. This model is then reformulated in such a way that it can be used as the basis of an experimental game for the development of resource pricing policies.

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ABSTRACT

The purpose of this note is to construct and analyze a model of North–South trade in resources, which could then be used as the basis of an experimental game for the formulation of resource pricing policies. The South trades oil for industrial goods with the North. Each region has one policy parameter: the North, a tax on oil, and the South, the price of oil. Each region has two economic objectives: the North, higher output and higher real wages, and the South, higher revenues from exports and higher output levels. The policy instruments are used to define strategies: the payoffs of these strategies are given by the general equilibrium solutions of the model.

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Graciela Chichilnisky*

1. INTRODUCTION

The economic model of North–South trade presented here is a modification, and synthesis, of two models already described by Chichilnisky (1983a,b). The model is given in the form most suitable for experiments on North–South trade policies, following an approach developed by Fortuna (1983) and Wierzbicki (1983).

We consider a two-region model, in which one region (the South) exports oil to the other (the North) in exchange for industrial goods. Each region has two outputs, basic goods and industrial goods. Each good is produced using three inputs: capital, labor and oil.

This model differs from that presented in Chichilnisky (1983a) in that here we have a full macroeconomic description of both regions, rather than of

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one region only. This model also differs from the two-region model given in Chichilnisky (1983b) in that (i) the policy variable for the South is here the price of oil, rather than the level of oil exports, and (ii) we also consider a policy variable for the North, namely a tax on oil.

Finally, we present the two-region model in a so-called "reduced form", i.e., as one equation that can be used to solve the whole model; this is the most suitable form for experimental work on North–South trade policies.

2. THE MODEL FOR THE NORTH: TAX POLICIES

The economy of the North is described by six behavioral equations, and seven equilibrium conditions. Considering first the behavioral equations, we have

$$B^S = \min (L^B / a_1 , \vartheta^B / b_1 , K^B / c_1) \quad , \tag{1}$$

where B is the basic good. The superscript S indicates supply, L^B is labor employed in producing B, ϑ^B and K^B are respectively the oil and capital employed in producing B, and a_1, b_1, c_1 are positive real numbers. This is a typical production function with fixed factor proportions.

Similarly, for the industrial good I, the production function is

$$I^S = \min (L^I / a_2 , \vartheta^I / b_2 , K^I / c_2) \quad , \tag{2}$$

while for oil ϑ we have

$$\vartheta^S = 0 \quad , \tag{3}$$

i.e., the North produces no oil (we could equally well put $\vartheta^S = C$, where C is an arbitrary constant). Labor supply is positively linked to real wages:

$$L^S = \frac{\alpha w}{p_B}, \quad \alpha > 0 \quad , \tag{4}$$

where L^S represents labor supply, w wages, and p_B the price of B. Similarly,

$$K^S = \beta r \ , \quad \beta > 0 \quad . \tag{5}$$

where K^S represents capital supply and r the rate of return on capital. We postulate the following demand behavior at equilibrium:

$$p_B B^D = wL \quad , \tag{6}$$

where the superscript D denotes demand, i.e., wage income is spent on B, so that B is a "wage good".

The equilibrium conditions are:

$$K^S = K^D \quad , \tag{7}$$

i.e., $K^S = c_1 B^S + c_2 I^S$;

$$L^S = L^D \quad , \tag{8}$$

i.e., $L^S = a_1 B^S + a_2 I^S$;

$$B^D = B^S \quad , \tag{9}$$

i.e., B is not traded internationally;

$$X_I^S = I^S - I^D \quad , \tag{10}$$

where X_I^S denotes exports of I;

$$X_\vartheta^D = \vartheta^D = \vartheta^D - \vartheta^S \quad , \tag{11}$$

where X_ϑ^D denotes oil imports;

$$b_1 B^S + b_2 I^S = \vartheta^D \quad , \tag{12}$$

and

$$p_I X_I^S = p_\vartheta X_\vartheta^D \quad , \tag{13}$$

which is a balance of payments condition, where p_I and p_ϑ denote the price of industrial goods and of oil, respectively.

The national income identity (or Walras' law) is always satisfied at equilibrium:

$$p_B B^D + p_I I^D = wL + rp_I K$$

which (at equilibrium) is equal to

$$p_B B^D + p_I(I^S - X_I^S) = wL + rp_I K \quad .$$

The model of the northern economy therefore consists of 13 equations in 15 variables: I^S, I^D, X_I^S, ϑ^S, ϑ^D, X_ϑ^D, B^S, B^D, r, p_B, p_I, p_ϑ, w, L, and K. It follows that for a given price of oil, p_ϑ, the model is "closed" in relative prices. Since the price of oil is given, there is thus no room for policy in the model as formulated above.

We now introduce a new parameter, a tax on oil, into the model of the North. The ***tax rate*** is denoted by the variable t. The above 13 equations must now be modified to take this into account.

First note that the production functions (1) and (2) are equivalent to the following price equations (assuming competitive behavior), ***before tax***:

$$p_B = a_1 w + b_1 p_\vartheta + c_1 r p_I \tag{14}$$

$$p_I = a_2 w + b_2 p_\vartheta + c_2 r p_I \quad . \tag{15}$$

From now on we shall use price equations (14) and (15) instead of production functions (1) and (2). ***After tax***, these price equations become:

$$p_B = a_1 w + b_1 p_\vartheta (1 + t) + c_1 r p_I \quad , \tag{14'}$$

$$p_I = a_2 w + b_2 p_\vartheta (1 + t) + c_2 r p_I \quad . \tag{15'}$$

The total tax revenue is thus $t\vartheta^D$. If a fraction λ, $0 < \lambda < 1$, of this revenue is allocated to wage income, and the complement $(1 - \lambda)$ to capital income, equation (6) becomes:

$$p_B B^D = wL + \lambda t \vartheta^D \tag{6'}$$

and Walras' law now becomes (at equilibrium):

$$p_B B^D + p_I I^D = wL + r p_I K + t \vartheta^D \tag{16}$$

so that at equilibrium we have

$$p_I I^D = r p_I K + (1 - \lambda) t \vartheta^D \tag{17}$$

The tax on oil having been introduced, the northern economy is now described by a system of 13 equations in 16 variables, where the original equations (14), (15) and (6) are replaced by their equivalents under tax, (14′), (15′) and (6′). Since the tax rate represents a new variable, the equilibrium positions of this model now ***depend on two parameters*** rather than one working in relative prices. We take these parameters to be the price of oil p_ϑ, which is the policy variable of the South, and the tax rate t, which is the policy variable of the North. In the next section we present a model of the southern economy, and show that given the tax rate t and the price of oil p_ϑ, there exists a (locally) unique equilibrium for the North–South trade model. This means that when the North chooses its policy parameter t and the South its policy parameter p_ϑ, the equilibria of both regions, and of the world economy, are determined. In particular, the output of goods in both regions, the North–South terms of trade p_ϑ / p_I, the volume of industrial goods exported by the North, and the real wages and rates of profits in both regions are all determined by the choice of t and p_ϑ. Therefore, our North–South model can actually be interpreted as a ***game form*** G, i.e., a map from strategies to outcomes:

North–South strategies		Outcomes
(t, p_ϑ)	———— G ⟶	North–South trade equilibrium

The *game form* G is a map that assigns an equilibrium to each pair of strategies t and p_{ϑ}. If in addition we define the objectives of the two regions, i.e., the set of variables that each region wishes to maximize, we would then have obtained a well-defined *game* from our model.

The model of the southern economy is defined in the next section, we then give the "reduced form" of the North–South model, and the game G is described in more detail in the final section.

3. THE MODEL FOR THE SOUTH: OIL PRICING POLICIES

The economy of the South is described by five behavioral equations and seven equilibrium conditions. The behavioral equations are similar to those of the North (except that the parameters a_1, a_2, b_1, b_2, c_1, c_2, α and β may be different):

$$B^S = \min(L^B / a_1 \, , \vartheta^B / b_1 \, , K^B / c_1) \tag{18}$$

$$I^S = \min(L^I / a_2 \, , \vartheta^I / b_2 \, , K^I / c_2) \tag{19}$$

$$L^S = \frac{\alpha w}{p_B} , \quad \alpha > 0 \tag{20}$$

$$K^S = \beta r \, , \quad \beta > 0 \tag{21}$$

$$p_B B^D = wL \quad , \tag{22}$$

where the output of oil is *not* constrained to zero; there is only a total endowment condition $\vartheta^S \leq \bar{\vartheta}$, which in some cases will not be binding. The output of oil is therefore "passive", adjusting to demand. We assume that extraction of oil is cost-free, i.e., it uses no factors of production.

The equilibrium conditions are, as in the North:

$$K^S = K^D = c_1 B^S + c_2 I^S \tag{23}$$

$$L^S = L^D = a_1 B^S + a_2 I^S \tag{24}$$

$$B^D = B^S \tag{25}$$

$$X_I^D = I^D - I^S \quad , \tag{26}$$

where X_I^D denotes imports of industrial goods;

$$X_\vartheta^S = \vartheta^S - \vartheta^D \quad , \tag{27}$$

where X_ϑ^S denotes exports of oil;

$$\vartheta^D = b_2 I^S + b_1 B^S \quad , \tag{28}$$

and

$$p_I X_I^D = p_\vartheta X_\vartheta^S \quad , \tag{29}$$

which is the balance of payments condition.

The national income identity is always satisfied at equilibrium:

$$p_B B^D + p_I I^D = wL^S + rp_I K^S + p_\vartheta \vartheta^S \quad . \tag{30}$$

Equivalently, (30) can be written

$$p_B B^D + p_I(I^S + X_I^D) = wL^S + rp_I K^S + p_\vartheta(\vartheta^D + X_\vartheta^S) \quad , \tag{31}$$

where ϑ^D denotes the amount of oil employed in the South. Equations (30) and (31) are identical at equilibrium, because of the balance of payments condition. The economy of the South is therefore represented by a system of 12 equations ((18) to (29)) in 15 variables, I^S, I^D, X_I^D, ϑ^S, ϑ^D, X_ϑ^S, B^S, B^D, r, p_B, p_I, p_ϑ, w, L and K. The equilibria of this model once again form a two-parameter family, assuming relative prices. One of these parameters is taken to be the price of oil p_ϑ. In the next section we show that, when the equilibrium conditions for international trade are taken into consideration, the equilibrium of the North—South model is fully determined up to two parameters, the *price of oil* p_ϑ and the *tax rate* t in the North.

4. THE NORTH–SOUTH MODEL

Here we consider simultaneously the 13 equations for the North [(3)–(5), (6′), (7)–(13), (14′) and (15′)], and the corresponding 12 equations for the South [price equations (14) and (15), and (20)–(29)]. (Recall that the South does not have taxes and that $\vartheta^S \neq 0$ in the South.) We also require a set of world equilibrium conditions, which are given below.

The international market-clearing conditions for an equilibrium are (i) that oil exports must equal oil imports:

$$X_\vartheta^S(\mathrm{S}) = X_\vartheta^D(\mathrm{N}) \tag{32}$$

(where the S and N in parentheses indicate the South and the North, respectively); and (ii) that exports of industrial goods must equal imports of industrial goods:

$$X_I^S(\mathrm{N}) = X_I^D(\mathrm{S}) \quad . \tag{33}$$

The following two equations state that the prices of internationally traded goods must be equal at equilibrium:

$$p_\vartheta(\mathrm{S}) = p_\vartheta(\mathrm{N}) \tag{34}$$

$$p_I(\mathrm{S}) = p_I(\mathrm{N}) \quad . \tag{35}$$

We may now determine the number of equations in the North–South model. First note that the balance of payments conditions of the two regions (13) and (29) are equivalent, from (32) to (35). We therefore have 12 equations for the North and 12 for the South, plus the four international trade conditions (32) to (35), making 28 equations altogether.

There are 16 variables in the North, including the tax on oil, and 15 in the South. We thus have 31 variables and 28 independent equations. The North–South equilibria can therefore be determined in relative prices if we make two parameters exogenous; we choose these to be the two policy variables, the oil tax rate t for the North and the price of oil p_ϑ for the South.

The next section shows this explicitly, producing one equation which gives a "reduced form" solution to the model.

5. SOLVING THE NORTH–SOUTH MODEL: A REDUCED FORM

In order to solve the model in relative prices, we choose B in the North as the 'numeraire', i.e., $p_B(\mathrm{N}) = 1$. From now on, all prices are given relative to the price of B in the North.

The production functions (1) and (2) yield equations for the demand for factors K, L and ϑ. At each level of output, when factors are used efficiently, we have

$$L^D = B^S a_1 + I^S a_2 \tag{36}$$

$$K^D = B^S c_1 + I^S c_2 \tag{37}$$

$$\vartheta^D = B^S b_1 + I^S b_2 \quad . \tag{38}$$

Equations (36) and (37) imply

$$B^S = (c_2 L - a_2 K)/D \tag{39}$$

$$I^S = (a_1 K - c_1 L)/D \quad , \tag{40}$$

where D is $a_1 c_2 - a_2 c_1$.

The (taxed) price equations for the North (14′) and (15′) can be regarded as a system of two equations in two variables (taking p_ϑ as a constant), yielding

$$w = \frac{[1-b_1(1+t)p_\vartheta]c_2-[p_I-b_2(1+t)p_\vartheta]c_1}{D} \tag{41}$$

$$r = \frac{a_1[p_I-b_2(1+t)p_\vartheta]-a_2[1-b_1(1+t)p_\vartheta]}{D} \tag{42}$$

Then, substituting L and K from (4) and (5), and w and r from (41) and (42) into (39) we obtain an equation relating the supply of basic goods B to the price of industrial goods p_I and the price of oil p_ϑ, i.e.,

$$\begin{aligned} B^S &= \frac{(c_2\alpha w-a_2\beta r)}{D} \\ &= \frac{\alpha c_2}{D^2}[c_2 + (1+t)p_\vartheta N - c_1 p_I] + \frac{\beta a_2}{D^2}\left[\frac{p_\vartheta(1+t)M}{p_I} + \frac{a_2}{p_I} - a_1\right] \quad , \end{aligned} \tag{43}$$

where $M = a_1b_2 - a_2b_1$ and $N = c_1b_2 - b_1c_2$.

Similarly, we obtain

$$I^S = \frac{\beta a_1}{D^2}\left[a_1 - \frac{p_\vartheta(1+t)M}{p_I} - \frac{a_2}{p_I}\right] + \frac{\alpha c_1}{D^2}[p_I c_1 - c_2 - Np_\vartheta(1+t)] \quad . \tag{44}$$

Consider now the demand relation

$$B^D = wL + \lambda t\vartheta^D \quad . \tag{45}$$

Since $B^S = B^D$, we obtain, using (41), (42), (44) and (12)

$$\begin{aligned} &\frac{\alpha c_2}{D^2}[c_2 + (1+t)p_\vartheta N - c_1 p_I] + \frac{\beta a_2}{D^2}\left[\frac{(1+t)p_\vartheta M}{p_I} + \frac{a_2}{p_I} - a_1\right] \\ &= \alpha\{[1 - b_1(1+t)p_\vartheta]c_2 - [p_I - b_2(1+t)p_\vartheta]c_1\}^2 + \lambda t(b_1B^S + b_2I^S) \quad , \end{aligned} \tag{46}$$

i.e.,

$$-p_I^3(1+\lambda t b_1)$$

$$-p_I^2\left[\frac{\alpha c_2 c_1}{D^2}+2[(1+t)p_\vartheta N+c_2]+\frac{\alpha c_1^2}{D^2}\right]$$

$$+p_I\left[\frac{\alpha c_2}{D^2}[c_2+(1+t)p_\vartheta N]-\frac{\beta a_2 a_1}{D^2}-\frac{\beta a_1^2 b_2}{D}\right.$$

$$\left.+\frac{\alpha c_1}{D^2}[c_2+Mp_\vartheta(1+t)]-(1+\lambda t b_1)[(1+t)p_\vartheta N+c_2]^2\right]$$

$$+[p_\vartheta M(1+t)+a_2][\frac{\beta a_2}{D^2}+b_2]=0 \quad . \tag{47}$$

Equation (47) above is the ***reduced form*** of the North—South model, linking the price of oil p_ϑ and the tax rate t with the price of industrial goods at equilibrium, p_I^*. It is now a simple matter to check that when p_ϑ and t are known, and therefore an equilibrium value of p_I^* can be obtained from (47), the North—South model is "closed", i.e., the equilibrium values of all other variables can be computed. We start with the North.

Given p_ϑ and t, we obtain the equilibrium price p_I^* from (47). From (41) and (42) we obtain the equilibrium wage level w^* and profit rate r^*. We can then obtain the equilibrium capital supply K^{S^*} and labor supply L^{S^*} from (4) and (5). Equations (39) and (40) yield B^{S^*} and I^{S^*}, and thus we obtain $\vartheta^{D^*}=X_\vartheta^{D^*}(\mathrm{N})$. Since $p_\vartheta X_\vartheta^{D^*}(\mathrm{N})=p_I^* X_I^{S^*}(\mathrm{N})$, it follows that $X_I^{S^*}$ can be found, so that $I^{D^*}=I^{S^*}-X_I^{S^*}(\mathrm{N})$ can also be found. The model for the North is thus closed.

We show next that the model for the South is also closed. Consider the two price equations for the South:

$$p_I = a_2 w + b_2 p_\vartheta + c_2 r p_I \qquad (48)$$

or equivalently,

$$w = \frac{1}{a_2}[(1 - c_2 r)p_I - b_2 p_\vartheta] \quad ,$$

and

$$p_B = a_1 w + b_1 p_\vartheta + c_1 r p_I \quad .$$

These equations lead to

$$\frac{w}{p_B} = -\frac{(1-c_2 r)p_I - b_2 p_\vartheta}{(a_1 - Dr)p_I - Mp_\vartheta} \quad . \qquad (49)$$

Note that p_ϑ is an exogenously given policy variable, and p_I is given by (47) as the equilibrium solution corresponding to p_ϑ, $p_I^* = p_I^*(p_\vartheta)$. Therefore (49) yields a relation connecting w/p_B and r. There is an implicit assumption here that the price of industrial goods is determined both by the price of oil (the policy parameter of the South) and by the market activity of the North.

Next, consider the equilibrium condition $B^D = B^S$. From $B^D = wL/p_B$ and $B^S = (c_2 L - a_2 K)/D$, since $L = \alpha w/p_B$ and $K = \beta r$, we obtain

$$\frac{w}{p_B} = \frac{c_2}{2D} + \sqrt{\left(\frac{c_2}{2D}\right)^2 - \frac{\beta a r}{\alpha D}} \quad , \qquad (50)$$

which is another relation between w/p_B and r. Equating (49) and (50), we obtain the equilibrium values of real wages $(w/p_B)^*$ and the rate of profit r^* in the South. From $(w/p_B)^*$ and r^* we obtain L^{S*} and K^{S*}, and hence B^{S*} and I^{S*}, using (39) and (40). Finally, since $X_I^{D*}(S) = X_I^{S*}(N)$ at equilibrium, we obtain the equilibrium demand for I in the South, $I^{D*}(S)$, which equals $I^{S*}(S) + X_I^{D*}(S)$. This completes our calculation of the equilibrium for the South.

6. THE NORTH—SOUTH MODEL AS A GAME

We have previously mentioned that the model can be used to construct a *game form*, which is the generalization of a payoff matrix,

$$G : (t, p_{\vartheta}) \rightarrow \text{Equilibrium outcomes: } p_{\vartheta}{}^{*}, p_I{}^{*}, r(\mathrm{N})^{*}, r(\mathrm{S})^{*}, X_I^{S*}, \ldots$$

In order to fully define a game, we now only have to specify the objectives which the two players (the North and the South) pursue when choosing their strategies.

We assume that the North wishes to optimize two variables, ***the total value of net output***

$$Y(\mathrm{N}) = p_B B^S + p_I I^S - p_{\vartheta} \vartheta^D$$

and ***the real wage***

$$\frac{w}{p_B}(\mathrm{N}) \quad .$$

The South wishes to optimize two variables, ***total net output***

$$Y(\mathrm{S})$$

and ***the level of industrial goods imported in exchange for oil***

$$X_I^D(\mathrm{S}) \quad .$$

In a traditional game-theoretical approach, we would define preferences over the outcomes, and subsequently adopt an equilibrium concept to determine the solutions of this game. The approach pioneered by Fortuna (1983) and Wierzbicki (1983) is different, since it is not necessary to have well-defined preferences over all possible outcomes in order to find a (Pareto) solution.

It would also be of interest to resolve a number of questions related to market behavior, such as:

- What is the range of oil prices for which factor prices (profits r and wages w) are positive in both regions?
- How do *real* wages w/p_B in the South depend on the price of oil p_ϑ?

A numerical solution of the model would provide approximate answers to these questions.

7. CHANGES IN TECHNOLOGIES

We shall close with a discussion of the possible impact of tax policies on the technologies used in the North: the model of the North may be altered to include the impact of the oil tax on these technologies under a number of different assumptions, such as "learning by doing", or more direct government intervention.

For instance, consider the case where $\lambda = 0$, so that all the proceeds from the oil tax are transferred to capital income (increasing the demand for industrial goods) *under the condition* that oil/output coefficients $1/b_1$ and $1/b_2$ be increased (i.e., investment should be in technologies which are less oil-intensive). In this case we may postulate* that the inverses of the oil/output coefficients b_1, b_2 *decrease* as the tax revenue allocated to the improvement of technology in each sector increases, i.e.,

$$\tilde{b}_1 = \frac{b_1}{1+\alpha_1 t \vartheta^D} \tag{51}$$

$$\tilde{b}_2 = \frac{b_2}{1+\alpha_2 t \vartheta^D} \quad . \tag{52}$$

*This formulation was suggested by Wierzbicki.

Equations (51) and (52) now become part of the model of the North. Since there are two more variables ($\tilde{b}_1$ and $\tilde{b}_2$) and two more equations, the solutions have the same characteristics as before.

However, it should be pointed out that some of the computations made in the previous sections no longer apply, and in particular that the reduced-form equation must be modified to include $\tilde{b}_1$ and $\tilde{b}_2$ as variables depending on t and ϑ^D.

This completes the formulation of the North–South trade game. The next step is to implement this game experimentally, in an attempt to discover resource pricing policies that are consistent with the goals, and the strategic behavior, of both North and South.

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