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PRICE ADJUSTMENTS AND MULTIREGIONAL RIGIDITIES IN THE ANALYSIS OF WORLD TRADE*

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FOREWORD

This paper reviews a set of approaches to modeling interregional and international trade flows. It also suggests a framework which is capable of combining price formation mechanisms with factors which cause resistance to change and barriers to trade. In certain respects the framework adheres to the Takayama-Judge tradition, but differs by using a probabilistic framework which generates equilibrium solutions which are not perfect competition equilibria.

The paper was presented at the 8th Pacific Congress, Regional Science Association, Tokyo, 17-19 August 1983; and at IIASA's Forest Sector Project Workshop in Sopron, 29 August-3 September, 1983. One of the motivations for the paper has been to provide a background to the world trade analysis in IIASA's Forest Sector Project.

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1. INTRODUCTION

A number of different conceptual approaches can be adopted in the development of economic models to study trade flow patterns between regions or nations. Since the early work of Ohlin (1933), which stressed the unification of the theories of trade and location, a variety of techniques have emerged. Prominent amongst the *trade share* or *pooling* school, typified by Project LINK and Leontief's World Model, is the assumption that imports may be treated as competitive, implying the adoption of the classical principle of comparative advantage based on Ricardo-Heckscher-Ohlin assumptions.

Although such global modeling exercises have a certain intellectual appeal, in reality most suffer from a number of serious deficiencies. Firstly, consistency in these models is usually obtained by applying a variety of ad hoc assumptions and adjustments. This clearly obscures the analysis of various equilibrium or disequilibrium properties of their solution. Secondly, it is impossible to capture structural changes in the pattern of trade because of the aggregate trade matrices utilized. Thirdly, such models fail to recognize the effects of specific bilateral trade agreements or the fact that trade in some commodities may be strongly influenced by link-specific factors in addition to the supply and demand-related considerations. Finally, the previous drawback suggests a more general criticism, namely that the abovementioned methods tend to ignore completely the theoretical foundations of regional science, in so much as they do not consider distance and location effects explicitly.

With regard to methods of trade analysis developed within the regional science community we may identify two quite different approaches, both of which emphasize the importance of location, distance and other link-related frictions. One alternative is offered in the work by Takayama and Judge (1971) which combines various spatial frictions and trade flow barriers with a neo-classical price equilibrium, and which refers to the early analysis by Samuelson (1952). The second approach has heavily concentrated on the gravitational assumption which provides a means to identify forces of friction and attraction without explicit reference to relative prices. Besides attempts to incorporate transportation costs, a glaring omission from such models has been considerations of price and cost differences between exporters and importers. In an international context, such an omission is unacceptable.

In this paper, we attempt to develop a framework which is capable of combining supply and demand-related factors in each of the trading regions (countries) with link-related factors (in particular trade inertia); these two categories of factors are connected by price-sensitive mechanisms which affect supply and demand behavior and determine trade flows under the influence of trade inertia and other barriers to trade. In certain respects this ambition brings the approach rather close to the Takayama-Judge analysis, but differs by using a probabilistic framework which generates solutions which are not perfect competition equilibria. The resulting formulations extend and enrich earlier work by both authors (see Batten, 1982a, c; Johansson and Persson, 1982; Batten, Johansson and Kallio, 1983). In this paper we show that it is possible to modify certain information-theoretical techniques based on the notion of trade inertia to allow for price adjustments, supply-demand equilibrium, and price dispersion.

2. INTERNATIONAL TRADE THEORY AND MULTIREGIONAL ANALYSIS

2.1 Comparative Advantage and the Friction of Distance

An important element in neo-classical trade theory is the analysis of the relation between the trading nations' factor endowments and the international commodity flows. This is a static analysis of efficient allocation. The general conclusion is that the sequence autarky--free trade--factor mobility implies increasing international welfare. In this perspective, freight costs, tariffs, import quotas and other barriers represent distortions of efficient allocation (Stern, 1973).

The paradigm of comparative advantage provides a long-term image of how different countries tend to specialize their production and associated export and import flows. It also characterizes the corresponding price structures (including adjustments due to transportation costs). In the tradition of Hecksher-Ohlin-Lerner-Samuelson, allocation adaptions in accordance with prevailing comparative advantage patterns are assumed to bring the world economy towards a situation of equalized factor prices (Ohlin, 1933; Samuelson, 1971a,b).

Two features of this analysis are crucial. First, comparative advantages are not static properties but are largely created by capital formation, education, research and development and the transfer of technology. Second, the adaption of activity allocations and trade patterns is a slow process. In particular, the interrelations between exporting and importing countries are influenced not only by geographical distance but also by "distance frictions" such as established contracts between countries, information channels, etc.

In the tradition of multiregional analysis the influence of different distance frictions has always been treated as a fundamental factor which strongly affects the interaction between regions. This is reflected in the attention paid to concepts like transportation costs, accessibility, and the diffusion of information.

The long-term character of the comparative advantage approach is contrasted by the short-term character of standard multi-regional models. The latter frequently apply constant input/output coefficients combined with fixed interaction schemes as regards interregional trade.

2.2 A Review of Linkage Systems

The two traditions introduced in section 2.1 are not directly applicable to the analysis we shall outline. An important feature of our approach is that we distinguish between allocation mechanisms operating within regions and between regions. The first category comprises supply and demand-related factors, the second link-related factors. Associated with the first category are the gradual processes of changes in production capacity and rigidities of production structures. The inertia of established trade channels and other trade preferences, tariffs, import quotas and other trade barriers such as transportation costs constitute link-related factors.

At the international level, a number of systems have been devised to connect separate economic models for an exhaustive set of world regions in order to obtain consistent projections of world trade. Notable among these are the Project LINK System pioneered by Klein and Hickman (see Klein, 1976 and Waelbroeck, 1976), the United Nations World Input-Output Model developed under the direction of Leontief (see Leontief et al, 1977), the OECD's global linkage model (OECD, 1979), and a ten-region model developed by Thorbecke and Field (1974). We shall restrict our immediate attention to Project LINK and the World Model, since these two are largely representative of the rest.

Project LINK attempts to link structural econometric models of 13 developed market economies, 7 socialist economies and 4 developing regions through their trade accounts. Import functions are used to explain the imports of each commodity group into each region. A trade share matrix is used to estimate the exports from each trading partner, and this matrix takes the following form:

$$\beta^{rs} = x^{rs} / \sum_r x^{rs} \qquad (2.1)$$

where x^{rs} denotes the exports from region r to region s in constant value terms. Using this export share assumption, four alternative mechanisms have been tried to complete the full table of trade flows:

(i) constant β^{rs} in value terms;
(ii) constant β^{rs} in volume terms;
(iii) a linear expenditure system;
(iv) a constant elasticity of substitution method.

One drawback of these alternative trade share assumptions is that consistency must be obtained by ad hoc adjustments, and therefore any equilibrium or disequilibrium properties of the solution are obscured. A similar criticism applies to the complimentary import share approach, namely

$$\alpha^{rs} = x^{rs}/\sum_{s} x^{rs} \qquad (2.2)$$

which has been used more frequently in trade share analysis (Nagy, 1983).

There seems to be no particular evidence favoring the use of either import or export shares as a basis for projecting structural change in trade patterns. In reality, the investigations of the United Nations Economic Commission for Europe (ECE;1971) have revealed large inconsistencies arising from the application of α^{rs} or β^{rs} share assumptions and the resulting trend extrapolations.

Project LINK aims at a comprehensive analysis of all commodity flows. Since it does not provide any specific method for disaggregation and aggregation of commodity groups, it is incapable of reflecting the underlying processes of structural change which are largely microscopic.

A similar criticism can be levelled at the Leontief World Model, which is a special type of interregional input-output model of the world economy. It links fifteen regional economies, each of which is represented by an input-output matrix describing the structure of production and consumption. Projections over a thirty year time horizon (1970-2000) are made. The linkage between these regional models in terms of trade flows is accomplished using a "trade pool" approach. According to this assumption:

(i) each region's imports of a particular commodity are assumed to be a fixed proportion or share of corresponding regional production;

(ii) the level of the trade pool is the sum of every region's imports; and

(iii) each region's exports of a particular commodity are assumed to be a fixed share of aggregate world exports of that commodity (Duchin, 1980).

A further weakness of the World Model is that the particular share assumptions adopted commit the model to a specific bilateral pattern of trade flows which is unlikely to ever coincide with the actual flow pattern (Costa, 1981). If the World Model is used for forecasting purposes, its linkage assumptions yield a *biased* interregional distribution of sectoral outputs, engendering cumulative errors in the whole set of variables forecasted.

The use of various trade share assumptions in Project LINK and the World Model typify a general weakness of most of the current global trade models which attempt to link together a set of regional models. By placing too much emphasis on the supply and demand-related factors (the "push" and "pull" factors), at the expense of the important link-related factors or trade intensity effects, they are unable to capture the essence of structural change. This oversight can partly be rectified by drawing upon recent advances in the field of regional science, in which link-related factors associated with distance and other interregional frictions have been explored.

2.3 Link-Related Factors and Gravitation-Type Models

Link-related factors have been examined within the framework of gravitational models which emphasize the importance of distance, location and other types of friction and attraction. Here the distribution of trade flows is influenced by bilateral characteristics such as trade preferences, trade barriers, and transportation costs (Batten, Johansson and Kallio, 1983).

Gravity-type models can account for most of the link-related factors which are not directly attributable to changes in relative prices. Elementary examples include the models of Tinbergen (1962), Pöyhönen (1963), Pulliainen (1963), Leontief and Strout (1963), Linnemann (1966) and Theil (1967). For a comprehensive review of the use of gravitational models in international trade flow analysis, see Nagy (1979). The most conspicuous feature of the various parameters estimated with the aid of these models is that trade flows are generally more strongly influenced by the supply-related (pull) factors of the exporting region than the demand-related (pull) factors of the importing region.

In the sequel, we shall limit the review of gravitational models to the work of Leontief-Strout and Theil, since their gravity formulations contain useful introductory insights for the approach outlined in subsequent sections. Theil's original model can be written as

$$x_i^{rs} = [X_i^r \; D_i^s / X_i] \; Q_i^{rs} \tag{2.3}$$

where x_i^{rs} is the flow of commodity i from region r to region s, X_i^r is the total production of i in region r, D_i^s is the total consumption (realized demand) in region s, X_i is the total global production of commodity i, and Q_i^{rs} is an interregional friction parameter which can be estimated from historical data, namely

$$Q_i^{rs} = \bar{x}_i^{rs} \bar{X}_i / \bar{X}_i^r \bar{D}_i^s \tag{2.4}$$

where the bar indicates that values of the variables are observed values.

Theil's model has been written in the form of (2.3) because this is exactly the form postulated by Leontief and Strout, except that the latter assumed that independent estimates of the regional totals, X_i^r and D_i^s, were unavailable.

Both models are simple variants of the classic Newtonian gravity model, but with the distance or cost function being replaced by an empirically determined matrix of friction parameters, $\{Q_i^{rs}\}$. Those parameters are reflecting link-related influences. A similar gravity assumption was also adopted by Polenske (1980) in her treatment of multiregional trade.

There are some obvious deficiencies in Theil's gravity model approach, including the fact that the estimates resulting from the use of (2.3) are unlikely to satisfy the pertinent set of consistency constraints, namely

$$\sum_r \sum_s x_i^{rs} = X_i \tag{2.5}$$

$$\sum_s x_i^{rs} = X_i^r \tag{2.6}$$

$$\sum_r x_i^{rs} = D_i^s \tag{2.7}$$

without some adjustments. To overcome this problem, Theil replaced his normalized estimate of x_i^{rs} by $\hat{x}_i^{rs}$, which he obtained by minimizing the following function:

$$I = \sum_r \sum_s x_i^{rs} \log (x_i^{rs}/\hat{x}_i^{rs}) \tag{2.8}$$

subject to equations (2.6) and (2.7) as constraints. At the time, he could only do this approximately; efficient algorithms for this type of problem were first formulated during the 1980s.

The gravity type of approach described in this subsection does not include any price and cost differences between exporters and importers. An attempt to incorporate transportation costs explicitly is provided in Wilson (1970). He built on the original Leontief-Strout framework of supply-demand pooling, in which all goods produced in each region r are notionally delivered to a single supply pool, and all goods used in r are extracted from a single demand pool. He utilizes an entropy formulation of the following type

$$\text{Min} \sum_i \sum_r \sum_s x_i^{rs} \log x_i^{rs} \qquad (2.9)$$

subject to

$$\sum_s x_i^s = \sum_j a_{ij}^r \Sigma x_j^{rs} + y_i^r \qquad (2.10)$$

$$\Sigma \Sigma c_i^{rs} x_i^{rs} = c_i \qquad (2.11)$$

where a_{ij}^r is the input-output coefficient in region r, y_i^r is the final demand in region r, and c_i^{rs} is the unit transportation cost of delivering commodity i from region r to region s.

Wilson also examines three other models which retain (2.9) and (2.11). By adding (2.6), he obtains a supply-constrained model, and by adding (2.7) he obtains a demand-constrained formulation. In addition, a doubly-constrained model is examined. Modeling exercises of this kind have recently been extended to interregional input-output systems (see Snickars, 1979; Batten, 1982a, 1982b). One main objective of such methods has been to estimate fixed coefficients, a_{ij}^{rs}, or similar schemes describing delivery relations between each pair (r,s). Constant coefficients of this type presuppose short- to medium-term time horizons for regions which together form a strongly integrated production system, so that imports may be regarded as complementary rather than competitive.

2.4 Spatial Price Equilibrium Models and Trade

Takayama and Judge (1971) have formulated a framework for analyzing and determining multiregional allocation of supply, demand and trade flows. In one version Marshallian supply and demand functions are specified for a single commodity in each region. With this formulation, the concepts of producer and consumer surplus are used to derive a perfect competition equilibrium such that quantities and prices are determined endogenously.

In its non-constrained form the model is based on two functions: (i) the demand conditions determining the demand price $p_i^r = p_i^r(D_i^r)$, where D_i^r denotes the demanded quantity and p_i^r the demand price in region r, and (ii) the supply conditions determining the supply price $\tilde{p}_i^r = \tilde{p}_i^r(X_i^r)$, where X_i^r is quantity supplied and $\tilde{p}_i^r$ the corresponding supply price in region r. Given these functions Takayama and Judge can define a "quasi-welfare function" for region r

$$W_i^r(D_i^r, X_i^r) = \int_0^{D_i^r} p_i^r(\eta)d\eta - \int_0^{X_i^r} \tilde{p}_i^r(\xi)d\xi \tag{2.12}$$

The competitive equilibrium solution is obtained by maximizing the "net social payoff" (Samuelson, 1952) $\Sigma_r W_i^r(D_i^r, X_i^r) - \sum_{rs}\Sigma c_i^{rs} x_i^{rs}$, subject to (i) $D_i^r \leq \Sigma_s x_i^{rs}$ and (ii) $X_i^r \geq \Sigma_s x_i^{rs}$ for all r. Letting ρ_i^r and $\tilde{\rho}_i^r$ be the shadow prices associated with the first and second market constraint, the following price conditions are satisfied by a solution

$$\tilde{p}_i^r \leq \tilde{\rho}_i^r \; ; \; p_i^r \leq \rho_i^r \tag{2.13}$$

$$\rho_i^s \leq \tilde{\rho}_i^r + c_i^{rs} \tag{2.14}$$

where $\rho_i^s < \tilde{\rho}_i^r + c_i^{rs}$ implies that $x_i^{rs}=0$, and where $X_i^r, D_i^r>0$, implies $\tilde{p}_i^r = \tilde{\rho}_i^r$ and $p_i^r = \rho_i^r$.

To this formulation the authors can add constraints and relations that reflect tariffs, subsidies, quotas and add valorem tariffs.

In the subsequent sections we are presenting a model framework within which one may generate market solutions that have several features in common with the Takayama-Judge "spatial price equilibria". At the same time our framework allows for a smooth way of varying the degree of competitiveness and price sensitivity of individual trade flows. The outlined approach combines inertia elements of the type reflected in gravitational analysis with price-dependent demand and supply behavior. In this way our approach partly corresponds to recent suggestions made by Hua (1980) about how to incorporate price related demand and supply conditions into gravity type models. One may argue that our model formulation constitutes a complement to both the gravity-like models and the Takayama-Judge type of models. Compared with the latter, our price formation mechanisms rather reflect the type of smooth price responsiveness that is often introduced in models of oligopolistic competition (or monopolistic competition) with product differentiation and the like.

3. A SEQUENTIAL LINKAGE SYSTEM

3.1 Production Theory, Supply Adjustments, and Time-Scales

In this section, some elements from production theory are introduced. The aim is to indicate the differences between short- and longer-term supply adjustments, and to emphasize that comparative advantage is created by research and development, diffusion of knowledge and investments in a dynamic process.

The production capacity of a sector is fixed in the short run, although the production level may vary. A sector consists of a set of production units or plants with varying production costs. The units may be ordered in a sequence k = 1,2,... such that $v_i^{rk} > v_i^{r,\,k+1} > \ldots.$, where v_i^{rk} denotes the cost level of unit k. These cost differentials stem from differences in production techniques which largely reflect the vintage of the technology embodied in the fixed capital design of each unit. We may then form a cost function, $v_i^r(\ )$ describing the structure of sector i in region r

$$v_i^r(X_i^r) = v_i^{rk*} \text{ for } X_i^r = \sum_{k \geq k*} \bar{X}_i^{rk} \tag{3.1}$$

where $\bar{X}_i^{rk}$ is the production capacity of unit k. For each single unit, the prevailing price, p_i^r, together with the cost level, v_i^{rk}, determines the surplus or loss associated with operating that unit. Assuming that the likelihood of reducing production increases with the difference $(v_i^{rk} - p_i^r)$, we may form a short run supply function, $X_i^r(p_i^r)$, with the following properties

$$x_i^r(p_i^r) < \bar{x}_i^r = \sum_k \bar{x}_i^{rk}$$
$$x_i^r(p_i^r) = \sum_k \phi(p_i^r - v_i^{rk})\bar{x}_i^{rk} \qquad (3.2)$$

where $\phi(\ )$ describes the share of $\bar{x}_i^{rk}$ which is utilized given p_i^r. The most frequent assumption is that $\phi(p_i^r - v_i^{rk}) = 1$ if $p_i^r \geq v_i^{rk} + \xi$ for some $\xi \geq 0$.

The cost structure is illustrated by a step function in Figure 1. As the figure demonstrates, (3.1)-(3.2) imply that in the short run marginal cost increases as the production level approaches the temporary capacity limit $\bar{x}_i^r$. During intervals which are long enough to allow for new investment, new capacities, Δx_i^r, with a reduced cost may be introduced. Hence, the cost effect of increased production changes as the time interval varies. These arguments can be extended to situations in which production costs are derived from technical characteristics of the different production units together with the price structure of production inputs (see, for example, Johansson and Strömqvist, 1981; Johansson, 1983). Estimates of the time invariance of industrial cost structures are presented in Johansson and Marksjö (1983).

The type of cost structure depicted in Figure 1 may be delineated over the set of production sectors and interacting regions. It is on the basis of such a structure, that comparative advantages must be evaluated. Comparative advantage will change over time in accordance with the investment profiles of the interacting regions.

The model we have sketched also sheds light on the relevant time scale for our linkage mechanism. If we analyse only one commodity class in isolation, we may choose time intervals for our trade model which correspond to the time period required to create new capacity. In such a case, the supply function in (3.2) will refer to a single time period, and it will change between time periods in response to investments in new capacities and shut down of old. If we extend the time period beyond this, the supply function must be changed so as to incorporate capacity change behavior, which, for example, may be modeled as a response to price changes and cost opportunities.

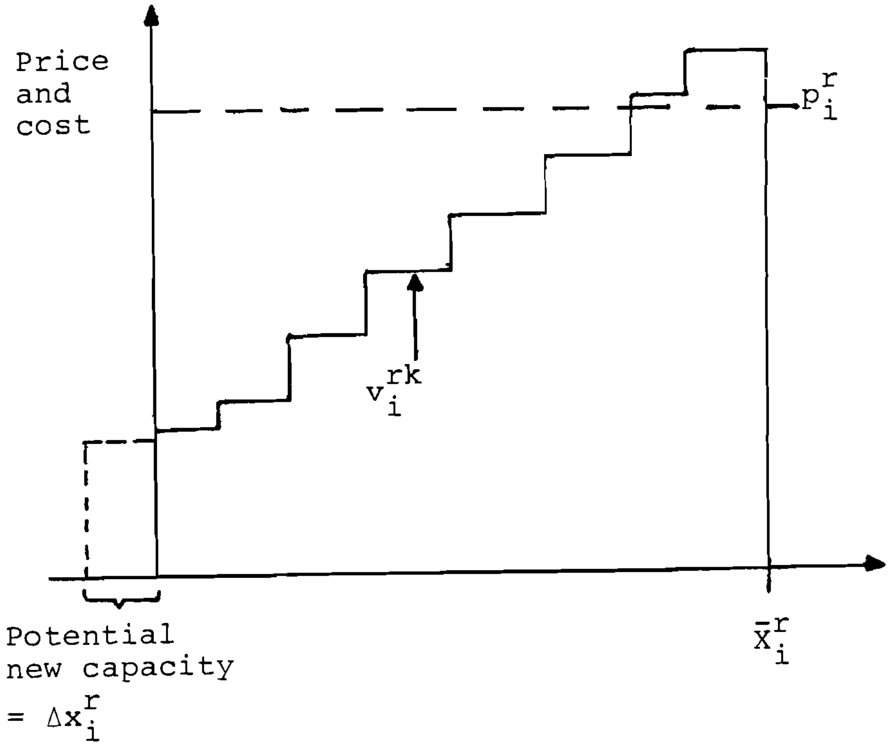

Figure 1. Cost structure in a region.

In the sequel, we are primarily focusing on the single-commodity (product class) case, assuming that each time period is selected in such a way that $\Delta x_i^r/\bar{X}_i^r$ is small.

3.2 Commodity Flows, Prices and Equilibria

With several regions (countries or groups of countries) trading in the same commodity, we may conceive the possibility of some price dispersion within each region. Hence, the price level, p_i^r, in region r may reflect an average price during a given period. For such a period, we express supply, X_i^r, and demand, D_i^r, as [1)]

$$X_i^r = X_i^r(p_i^r) \leq \bar{X}_i^r$$

$$D_i^r = D_i^r(p_i^r) \tag{3.3}$$

where $\bar{X}_i^r$ represents the maximum capacity that can be reached during the period. In the general case, we assume that $\delta X_i^r/\delta p_i^r \geq 0$ and $\delta D_i^r/\delta p_i^r < 0$.

A global price equilibrium, such that excess supply and demand are equal to zero, can be defined in the following way:

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1) These functions could be furnished by a regional component model.

$$\sum_s x_i^{rs} = X_i^r(p_i^r) \text{ and } \sum_s x_i^{sr} = D_i^r(p_i^r), \text{ for all } r \qquad (3.4)$$

where the supply and demand functions are defined for the case with the prices of all other commodities given. The extension to the multi-commodity case is self-evident.

We may observe that (3.4) implies that total supply equals total demand. At this equilibrium point, export and import flows from and to region r take the form

$$X_i^r(p_i^r) - x_i^{rr} = \text{export from region } r$$
$$D_i^r(p_i^r) - x_i^{rr} = \text{import to region } r \qquad (3.5)$$

Whenever the feasible adjustments of market prices are constrained, we may obtain market outcomes in a region which are "one-sided" in the following way:

$$\sum_s x_i^{rs} = X_i^r(p_i^r) \text{ and } \sum_s x_i^{sr} < D_i^r(p_i^r)$$

or

$$\sum_s x_i^{rs} < X_i^r(p_i^r) \text{ and } \sum_s x_i^{sr} = D_i^r(p_i^r)$$

where for example the first situation violates the competitive market constraint which is $\Sigma_s x_i^{sr} \geq D_i^r(p_i^r)$. The type of price constrained trade solutions indicated above could reflect situations in which specific trade agreements exist while at the same time regional prices are not adjusted towards market clearing trade prices.

3.3 Trade Inertia Effects

Amongst the link-related factors which affect the structural change possibilities at the global level, the more important ones include trade preferences and barriers, transportation costs, and freight capacities.

In Batten, Johansson and Kallio (1983), ample evidence is provided to suggest that a considerable degree of inertia is embodied in the trade patterns between various nations.[3)] This inertia manifests itself in patterns of trade which alter quite slowly in terms of variations to earlier bilateral trade intensities or trade shares. Analyses of trade flows must therefore be able to reflect sequential effects of the establishment and maintenance of certain rigidities in the trade adjustment process, such as particular channels of communication, specific contracts between particular buyers and sellers in distinct regions, and bilateral trade agreements. These forces of attraction or alliance may be loosely grouped under the heading of *trade preferences*. For certain trade links and geographical zones, such preferences are extremely binding at the level of aggregate trade flows (see, e.g., Andersson & Persson, 1982).

We can conveniently introduce these preferential potentials into our linkage system by defining probability spaces with respect to elementary contracts which reflect previous trading preferences.[4)] For a commodity class i, we can define an a priori probability distribution Q_i as a function of a sequence of previously realized trade flows such that

$$Q_i = \{q_i^{rs}\}, \sum_r \sum_s q_i^{rs} = 1 \text{ , all } q_i^{rs} \geq 0 \tag{3.6}$$

where q_i^{rs} defines the probability that a specific bilateral contract for the exchange of product i will be established between regions r and s. Assuming that each elementary contract or agreement has an identical probability over the complete set of regional combinations, we can determine the probability, $P(x_i^{rs})$, of a specific flow distribution, $\{x_i^{rs}\}$, using the following combinatorial calculus:

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3) These observations refer to different types of forest products; similar rigidities are likely to prevail for a large spectrum of commodities. See also Anderstig (1983).

4) For a more detailed background description of the probabilistic approach described in this section, see Snickars and Weibull (1977); Webber (1979); Batten (1982a,b).

$$P(x_i^{rs}) = N^{rs} \prod_{r,s} [q_i^{rs}]^{x_i^{rs}} / \prod x_i^{rs}\,! \qquad (3.7)$$

where N^{rs} is a parameter which depends on the number of elementary contracts and the number of regional combinations. Omitting constant terms, we can simplify (3.10) using Stirling's approximation, thereby transforming $P(x_i^{rs})$ into the following function:

$$J(x;q) = \sum_r \sum_s x_i^{rs} \log (x_i^{rs}/q_i^{rs}) \qquad (3.8)$$

The a priori distribution $\{q_i^{rs}\}$ is designed to reflect the preferential influence or bias of earlier contracts or trade agreements. If q_i^{rs} is equal for all pairs r, s formula (3.8) collapses to (2.9).

Let $x_i^{rs}(t-\tau)$ denote flows realized τ periods before t, and let $z_i^{rs}(t)$ denote the normalized trade distribution in any period t, namely

$$z_i^{rs}(t) = x_i^{rs}(t)/\sum_r \sum_s x_i^{rs}(t) \qquad (3.9)$$

We shall assume that the a priori pattern $q_i^{rs}(t)$ is formed by a sequence of historical contracts and established market channels. Moreover, as time develops within the model exercise, the pattern is assumed to be renewed for each time period. Formally, we have that

$$q_i^{rs}(t) = F_i^r(z_i^{rs}(t-1)\ z_i^{rs}(t-2), \ldots) \qquad (3.10)$$

where F_i^r shows the cumulative effect of the sequence of past patterns, with a diminishing contribution from patterns which are more distant in time. For example, F_i^r may be represented by a weighting operator of the following type

$$q_i^{rs}(t) = \sum_{\tau=1}^{T} w_i(\tau) z_i^{rs}(t-\tau) \qquad (3.11)$$

where $\Sigma_\tau w_i(\tau) = 1$ and all $w_i(\tau) \geq 0$. To summarize, the above assumption implies that actual trade flows reflect the establishment and maintenance of specific communication channels and information flows between buyers and sellers in different regions. The larger the value of z_i^{rs}, the greater the probability of a continuation of the communication and future exchange; the higher the weighting $w_i(1)$, the greater the influence of current patterns.

The a priori distribution, Q_i, can also be utilized to implement other link-related rigidities or frictions. For example, Theil's gravity model which was defined in (2.3) contains an interregional friction parameter Q_i^{rs}, which was estimated using historical data. This theoretical assumption is nothing more than a simple disguise for another a priori distribution.[5)] Similar comments would apply to the methods of trade share analysis and trade intensity effects (see Nagy, 1983). All these approaches correspond to a particular theoretical assumption concerning trade adjustment possibilities, and all can be easily implemented using the functional representation given in (3.8).

Moreover, in a prescriptive situation, q_i^{rs} may refer to a preferred future pattern of trade (see Batten, 1983). For problems of this type, the outlined approach can also be of assistance.

Given any a priori distribution, Q_i, we can determine the most likely trade pattern (highest $P(x_i^{rs})$ value) by minimizing $J(x;q)$ in (3.8) subject to the relevant set of constraints. The basic set of constraints consists of the equilibrium conditions expressed in (3.4), or some set of disequilibrium conditions. The calculated trade pattern then represents the most likely outcome given the imposed constraints, and the outcome is consistent with assuming a stochastic search behavior by (optimizing) agents. In section 4 we use such an outcome as a step in an iteration process where prices, supply and demand are gradually adjusted to provide new constraints for (3.8).

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5) Snickars and Weibull (1977) have already demonstrated that the classical gravity model and its variants are less powerful as a tool for describing adjustments in metropolitan trip patterns than models based on historical trip patterns. A similar situation seems likely with respect to world trade patterns.

3.4 Other Link-Related Factors

Although Q_i can reflect earlier trade preferences between different regions, any historical distribution will also reflect certain impedance factors and frictional effects which may be conveniently grouped under the broad heading of *trade barriers*. These are of two types: (i) natural barriers such as distance, transportation network, and climate; (ii) artificial barriers such as tariffs, import duties, quotas, embargoes and bans.

We can formally represent non-price restrictions as additional constraints on the feasible area for (3.8). For example, an import quota may be prescribed in the form

$$\sum_{r \neq s} x_i^{rs} \leq M_i^s \text{ or } x_i^{rs} \leq M_i^{rs} \text{ for each } r, \tag{3.12}$$

where M_i^s (or M_i^{rs}) defines the maximum level of imports of product i into region s. An export ban with respect to a pair of countries yields the following constraint

$$x_i^{rs} = 0 \tag{3.13}$$

where r is the region imposing the ban. If such a ban is of a long-term nature, then $q_i^{rs} = 0$.

Binding constraints between different regions may also emanate from the transportation system. Once again, we can distinguish between price-related (e.g., freight costs) and non-price-related (e.g., freight capacities) factors. With regard to freight capacities, congestion may occur within certain regions themselves or on specific links (rail links, shipping routes, etc.) between different regions. Link capacity constraints may take the form $\Sigma_i k_i^{rs} x_i^{rs} \leq K^{rs}$, where k_i^{rs} represents the capacity requirement as regards transportation of product i on the link (r,s). There may be more than one subset of such constraints needed to reflect various modes of transportation, or each subset may correspond to the particular handling and freight characteristics of different product classes.

The main impedance factor associated with the transportation system relates to the economic cost of distance, which is part of the price formation process analyzed in the next section.

4. PRICE FORMATION AND FLOW ADJUSTMENTS

4.1 Dimension of Price Space and Dispersion of Prices

The optimization problem in (3.8) involves RxR flows for each product, where R is the number of regions. If each flow, x_i^{rs}, is associated with a price, p_i^{rs}, we cannot determine each price using simply the 2R equilibrium conditions given in (3.4).

Before we tackle these problems, we shall introduce a producers' price variable which may correspond to the production unit with the highest cost level specified in (3.1). Introducing an average domestic transportation and handling cost level, c_i^{rr}, in region r, we require that

$$p_i^r \geq v_i^r + c_i^{rr} \qquad (4.1)$$

where p_i^r denotes the regional (market) price which enters the domestic supply and demand functions so that $X_i^r = X_i^r(p_i^r)$, $D_i^r = D_i^r(p_i^r)$. Formula (4.1) may be interpreted as a principle of non-negative profits as regards intraregional deliveries.

Observing that $(p_i^s - c_i^{rs})$[6] represents the net price in region r when selling commodity i in region s we may formulate the following two aggregate versions of the principle of non-negative profits

$$\sum_s (p_i^s - c_i^{rs}) x_i^{rs} \geq v_i^r \sum_s x_i^{rs} \qquad (4.2)$$

$$\sum_r (p_i^s - c_i^{rs}) x_i^{rs} \geq \sum_r v_i^r x_i^{rs} \qquad (4.3)$$

---

6) Artificial price barriers such as tarrifs, which are additive and not proportional to v_i^r or p_i^s, can be included in c_i^{rs}. Proportional price barriers such as import duties have been omitted in order to simplify notation.

where (4.3) implies that $p_i^s D_i^s \geq \Sigma_r (v_i^r + c_i^{rs}) x_i^{rs}$, given that $D_i^s = \Sigma_r x_i^{rs}$.

The two price conditions in (4.2) and (4.3) cannot be compared directly with the conditions in the Takayama-Judge model (TJM) as expressed by (2.13) and (2.14). One reason for this is that TJM adheres to the Marshallian framework, which means that for each quantity supplied and demanded one determines a supply price and a demand price (as described in Figure 2a for a single region). The approach we are outlining is Walrasian in nature, which in our case means that there is one regional (market) price and this determines a quantity supplied and a quantity demanded (see Figure 2b). Hence, the prices in (4.2) and (4.3) cannot be interpreted as supply and demand prices.

A second observation we can make is that the price responsiveness we are formulating in the next subsection is of the smooth kind which is often postulated for oligopolistic competition with product differentiation (see, e.g., Friedman, J., 1977). This is in line with the trade inertia which is modelled as probabilistic supplier-customer ties. The result of this is that the TJM-condition

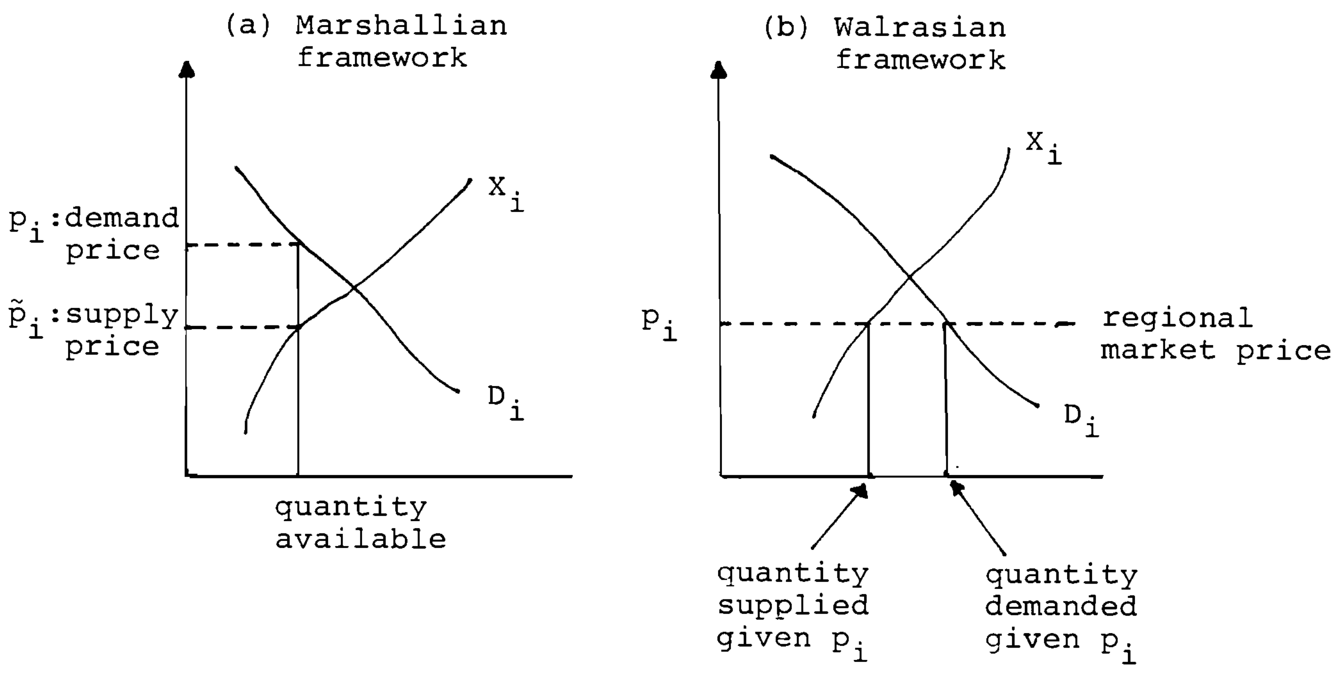

Figure 2. Illustration of (a) Marshallian, and (b) Walrasian frameworks.

$$\rho_i^s < \tilde{\rho}_i^r + c_i^{rs} \Rightarrow x_i^{rs} = 0$$

does not apply to our setting, where ρ_i^s is demand and $\tilde{\rho}_i^r$ supply prices for the flow (r,s). Because of inertia we may, for an individual link (r,s) obtain a small flow which does not satisfy the principle of non-negative profits. That is, we may have in a solution $p_i^s - c_i^{rs} \leq v_i^r$ and $x_i^{rs} \geq 0$. The non-negative profit condition applies to the aggregate level as shown in (4.2) and (4.3). When we sum over flows they are associated with prices that on the average cover costs. Although we are avoiding to stipulate any definite relation between market prices, p_i^r, and "marginal" cost levels or producer prices, v_i^r, it is obviously possible to reflect oligopolistic or monopolistic elements in price formation by explicitly having $p_i^s = v_i^s + c_i^{ss} + \xi_i^s$ or

$$\sum_s p_i^s x_i^{rs} \geq \sum [v_i^r + c_i^{rs} + \xi_i^s(x_i^{rs})] x_i^{rs}$$

or

$$\sum_r p_i^s x_i^{rs} \geq \sum_r [v_i^r + c_i^{rs} + \xi_i^s(x_i^{rs})] x_i^{rs}$$

which could reflect a relation between the size of the flow, x_i^{rs} or x_i^{rs}/D_i^s, and the degree of oligopolistic price mark-up.

Remark 1: One should observe that if we let the inequalities in (4.2) and (4.3) hold for every individual flow, then the two conditions will be equivalent. However, we only require the constraints to hold on averages, since in general the probabilistic assumption introduced in (3.8) will prevent feasible solutions if we do not allow for price dispersion as regards individual flows.

The price dispersion that may obtain in our solutions are due to the trade inertia and will imply that $(p_i^s - c_i^{rs})$ may differ from $(p_i^k - c_i^{rk})$ as defined in (4.2) and that $(p_i^s - c_i^{rs})$ may differ from $(p_i^s - c_i^{ks})$ as defined in (4.3).

Remark 2: In the subsequent model examples we shall interpret v_i^r as the minimum price level that suppliers in region r will accept. Only one explicit relation between v_i^r and p_i^r is introduced: we assume that we can treat v_i^r as a function of p_i^r such that there is a distinct derivative $1 \geq \partial v_i^r / \partial p_i^r \geq 0$.

This assumption means that the cost-related minimum price level, v_i^r, changes parallel with the market price in the region p_i^r.

4.2 Price Constraints with Regard to Buyers

Expression (4.3) describes a price constraint formulated from the perspective of deliveries going into each single region. Hence price patterns are constrained to generate non-negative profits averaged from the perspective of buyers. With this constraint we may formulate the following model of the world trade:

$$\begin{aligned} &\text{Min } J(x;q) \\ &\text{subject to} \\ &\sum_r x_i^{rs} = D_i^s(p_i^s) \text{ , all s (from 3.4)} \\ &\sum_r (v_i^r + c_i^{rs}) x_i^{rs} \leq p_i^s D_i^s(p_i^s) \text{ , all s (from 4.3)} \end{aligned} \qquad (4.4)$$

We can ensure that the constraints in (4.4) will always be satisfied, since $\Sigma_r x_i^{sr}$ is not restrained, and therefore the solution $(v_i^s + c_i^{ss}) x_i^{ss} \leq p_i^s D_i^s(p_i^s)$ together with $x_i^{ss} = D_i^s(p_i^s)$ is always possible.

Remark 3: As shown in section A.2 of the appendix, the model in (4.4) is essentially equivalent to the following formulation Max $\Sigma_s p_i^s D_i^s(p_i^s) - \Sigma_s \Sigma_r (v_i^r + c_i^{rs}) x_i^{rs}$, subject to $D_i^s(p_i^s) = \Sigma_r x_i^{rs}$ for all s, and $J^* \geq J(x,q)$, where J* is the prespecified value of the probability distribution.

In Remark 3 we have reformulated the objective function to reflect sales value minus cost of production and transportation. In appendix A.2 we extend the reformulation in such a way that the optimization problem resembles the Takayama-Judge model.

The solution to the convex optimization problem in (4.4) may be obtained explicitly by differentiating the associated Lagrange function which yields

$$x_i^{rs} = q_i^{rs}\exp\{-\gamma_i^s(v_i^r + c_i^{rs}) - 1 - \beta_i^s\} \tag{4.5a}$$

where γ_i^s, and β_i^s are Lagrange multipliers. By summing over s, the solution in (4.5) can be checked with regard to supply constraints. If $\Sigma_s x_i^{rs} \neq x_i^r(p_i^r)$, we do not have an equilibrium and prices must be adjusted. After a suitable price adjustment, (4.4) can be solved again at the new prices.

In order to examine the price adjustment process we first sum (4.5a) over r which yields

$$\sum_r q_i^{rs}\exp\{-\gamma_i^s(v_i^r + c_i^{rs}) - 1 - \beta_i^s\} = D_i^s(p_i^s)$$

and we may rewrite (4.5a) as follows [7]

$$x_i^{rs} = \frac{q_i^{rs}\exp\{-\gamma_i^s(v_i^r + c_i^{rs})\}}{\sum_k q_i^{ks}\exp\{-\gamma_i^s(v_i^k + c_i^{ks})\}} D_i^s(p_i^s) \tag{4.5b}$$

A guide to the adjustment of the price structure is obtained from the excess supply function

$$\begin{aligned} z_i^r(p_i) &= X_i^r(p_i^r) - \sum_s [A_i^{rs}/\sum_k A_i^{ks}] D_i^s(p_i^s) \\ A_i^{ks} &= q_i^{ks}\exp\{-\gamma_i^s(v_i^k + c_i^{ks})\} \end{aligned} \tag{4.6}$$

where $p_i = (p_i^1,\ldots,p_i^R)$, and $z_i^r(p_i) = 0$ signifies equilibrium for commodity i with regard to region r.

---

7) This "logit-type" expression may be interpreted as the outcome of search behavior for forming "elementary contracts".

The Jacobian of $\{z_i^r(p)\}$, $J(z_i(p_i))$, is of special interest for our investigation. According to Smale (1976, 1981) the process

$$J(z_i(p_i))p_i = \lambda(p_i)z_i(p_i) \tag{4.7}$$

always converges for the real valued function $\lambda(p_i)$ where sign $\lambda(p_i) = [-1]^{n-1}$ sign det $J(z_i(p_i))$, if the process is designed to keep prices away from the boundary of the (compact) price space (see also Hahn, 1981)[8].

In order to evaluate the Jacobian we differentiate $z_i^r(p_i)$ with respect to p_i^r and all p_i^s. By setting $\partial\gamma_i^s/\partial p_i^k = 0$ for all s and k we obtain

$$\frac{\delta z_i^r}{\delta p_i^r} = \frac{\delta X_i^r}{\delta p_i^r} - \frac{A_i^{rr}}{A_i^r}\frac{\delta D_i^r}{\delta p_i^r} + \sum_s \frac{D_i^s(p_i^s)\gamma_i^s A_i^{rs}}{A_i^s}\{1-A_i^{rs}/A_i^s\} > 0 \tag{4.8}$$

$$\sum_{s\neq r}\frac{\delta z_i^r}{\delta p_i^s} = -\sum_{s\neq r}\frac{A_i^{rs}}{A_i^s}\left\{\frac{\delta D_i^s}{\delta p_i^s} + \frac{D_i^s(p_i^s)\gamma_i^s A_i^{ss}}{A_i^s}\right\} \tag{4.9}$$

where $A_i^s = \Sigma_k A_i^{ks}$, and where $\delta A_i^{rs}/\delta p_i^r$ has been assumed to equal $\delta A_i^{rs}/\delta v_i^r$ for all r and s. The effect of omitting the terms $\partial\gamma_i^s/\partial p_i^k$ is examined in the Appendix.

Let there be an international numeraire or standard in relation to which all prices are measured. This means that the excess supply functions are defined for prices evaluated against this numeraire. Consider then a price adjustment process $\dot{p}_i^r = f_i^r(z_i^r(p_i)$ for $p_i = (p_i^1, p_i^2, \ldots)$, and let f_i^r satisfy for all r

8) In the form of an algorithm, this may be referred to as the "Global Newton" method.

$$f_i^r(0) = 0$$

$$f_i^r(\lambda) = -f_i^r(-\lambda) \text{ for } \lambda \text{ real} \tag{4.10}$$

$$f_i^{r'} > 0 \text{ for all positive price vectors}$$

$$z_i^r(p_i) f_i^r(z_i^r(p)) > 0 \text{ for } z_i^r(p_i) \neq 0$$

Proposition 1: Let the Jacobian of $\{z_i^r(p_i)\}$ have diagonal dominance at all prices such that $\partial z_i^r / \partial p_i^r > \Sigma_{s \neq r} \partial z_i^r / \partial p_i^s$ for all r, and let the price space be bounded so that $p_i > 0$. Then the equilibrium p_i^* such that $\{z_i^r(p_i^*)\} = 0$ is globally stable for the process in (4.10).

Proof: We can select a Lyapounov function such that

$$V(p_i) = \max\{\max[-f_i^r(z_i^r(p_i))], \max[f_i^s(z_i^s(p_i))]\} \tag{4.11}$$

$V(p_i)$ is a real valued, continuous function which is positive definite for $\{z_i^r(p_i)\} \neq 0$ by construction. It has a unique minimum at $\{z_i^r(p_i^*)\} = 0$. For $V(p_i) = \max[-f_i^r(z_i^r(p_i))] > 0$ we have that $\dot{p}_i^r > \dot{p}_i^s$ for all s. Hence,

$$\dot{V} = \frac{-\partial f_i^r}{\partial z_i^r} \frac{\partial z_i^r}{\partial p_i^r} \dot{p}_i^r - \sum_{s \neq r} \frac{\partial f_i^r}{\partial z_i^r} \frac{\partial z_i^r}{\partial p_i^s} \dot{p}_i^s < 0 \tag{4.12}$$

Analogously we can ensure that $\dot{V} < 0$ also when $V(p_i) = \max[f_i^s(z_i^s(p_i))] > 0$.

The model which is obtained by combining (4.4) and (4.10) will have a bounded price space if (i) $D_i^s(p_i^s) \to \infty$ as $p_i^s \to 0$, and (ii) because $X_i^s(p_i^s) < \bar{X}_i^s$ for all p_i^s due to capacity limitations during the chosen time period.

Next we refer to the Appendix which demonstrates that the effects of omitting the terms obtained by differentiating γ_i^s with respect to p_i^k (all s and k) are likely to be negligible as regards the diagonal dominance property. Knowing this we inspect (4.8) and (4.9). From this we conclude that the

Jacobian is likely to have diagonal dominance if (i) price elasticities of demand do not vary too much between regions, and (ii) each region has a domestically oriented delivery pattern in the sense that A_i^{rr}/A_i^r is much larger than A_i^{rs}/A_i^s, where $A_i^s = \Sigma_k A_i^{ks}$. This means that region r's share of the domestic market is larger than its share of other markets. More precisely, (4.8) is larger than (4.9) if

$$\frac{-A_i^{rr}}{A_i^r}\frac{\partial D_i^r}{\partial p_i^r} + \sum_{s \neq r} \frac{A_i^{rs}}{A_i^s}\frac{\partial D_i^s}{\partial p_i^s} > 0, \text{ and}$$

$$\sum_s \frac{A_i^{rs}\, D_i^s(p_i^s)\gamma_i^s}{(A_i^s)^2}\,[A_i^s + A_i^{ss} - A_i^{rs}] - \frac{D_i^r(p_i^r)\gamma_i^r(A_i^{rr})^2}{(A_i^r)^2} > 0 \tag{4.13}$$

4.3 Price Constraints with Regard to Sellers

In this section we are changing the perspective of section 4.2 by placing the constraints on the price dispersion with the sellers. Hence, the optimization problem in (4.4) is transformed into the following one

$$\begin{aligned} &\text{Min } J(x,q) \\ &\text{subject to} \\ &\sum_s x_i^{rs} = X_i^r(p_i^r) \quad \text{all } r \quad (\text{from } 3.4) \\ &\sum_s (p_i^s - c_i^{rs})x_i^{rs} \geq v_i^r X_i^r(p_i^r) \quad \text{all } r \ (\text{from } 4.2) \end{aligned} \tag{4.14}$$

Solving this system we obtain in analogy with (4.5b)

$$x_i^{rs} = \frac{q_i^{rs}\exp\{\lambda_i^r(p_i^s - c_i^{rs})\}}{\sum_k q_i^{rk}\exp\{\lambda_i^r(p_i^k - c_i^{rk})\}} X_i^r(p_i^r) \tag{4.15}$$

From this we obtain the following excess demand function

$$\tilde{z}_i^s(p_i) = D_i^s(p_i^s) - \sum_r (A_i^{rs}/\sum_k A_i^{rk}) X_i^r(p_i^r) \tag{4.16}$$

where $A_i^{rk} = q_i^{rk} \exp\{\lambda_i^r(p_i^k - c_i^{rk})\}$.

We shall treat (4.16) in the same way as we treated (4.6). First, we observe that $\lambda_i^r(p_i)$ is a function of all prices p_i through the second system of constraints in (4.14). Referring to the Appendix we omit this price effect when we differentiate (4.16) with respect to prices, since the property of diagonal dominance as regards the Jacobian is not likely to be affected by contributions from $\{\partial\lambda_i^r/\partial p_i^s\}$ - effects. Then we obtain

$$\frac{\partial \tilde{z}_i^s}{\partial p_i^s} = \frac{\partial D_i^s}{\partial p_i^s} - \frac{\partial X_i^s}{\partial p_i^s}\frac{A_i^{ss}}{A_i^s} - \sum_r \frac{\lambda_i^r X_i^r(p_i^r)}{A_i^r}\left[1 - \frac{A_i^{rs}}{A_i^r}\right] \tag{4.17}$$

$$\sum_{r\neq s} \frac{\partial \tilde{z}_i^s}{\partial p_i^r} = -\sum_{r\neq s}\left[\frac{A_i^{rs}}{A_i^r}\frac{\partial X_i^r}{\partial p_i^r} - \frac{A_i^{rs} X_i^r(p_i^r)\lambda_i^r A_i^{rr}}{(A_i^r)^2}\right] \tag{4.18}$$

The diagonal term, given in (4.17), is negative for all prices. Formula (4.18) expresses the sum of off-diagonal terms. With similar arguments as those used in the preceding section, we may conjecture that the Jacobian $\{\partial\tilde{z}_i^s/\partial p_i^r\}$ has a dominant diagonal. Hence, similar stability properties apply also to the model in (4.14).

4.4 Model Solution and Numerical Implementation

The price-sensitive models discussed in sections 4.1 through 4.3 certainly complicate the traditional gravity and entropy formulations which have been implemented for the analysis of trade flows. Trade inertia models of the type specified in (3.8) are quite straightforward to solve using a number of existing computer algorithms for numerical problems of this type (see

Eriksson, 1981). Once we introduce constraints on price dispersion, such as (4.2) or (4.3), explicit solutions can always be obtained by differentiating the associated Lagrange function.

We can also be confident concerning the development of a suitable numerical algorithm which can cater for price dispersion and price adjustments. This algorithm will generate a path of price-quantity iterations converging to an equilibrium solution. Such an iterative price-quantity adjustment process must be designed to keep prices away from the boundary of the (compact) price space.

5. CONCLUDING REMARKS

The model formulations developed in this paper represent an initial attempt to establish a framework for world trade analysis which extends beyond the familiar trade share assumptions and which may complement the "spatial price equilibrium" analysis of Takayama and Judge. Two sets of fundamental forces have been integrated, namely (i) the inertia or resistance to change generated by link-specific factors such as trade barriers and exchange agreements, and (ii) the propensity to change generated by price-sensitive adjustments of the type normally stressed in market equilibrium models of world trade. Although the introduction of price formation and adjustment mechanisms complicates the traditional gravity or entropy models, the associated model enrichment is both feasible and rewarding. In this exploratory paper, our attention has been restricted to aspects such as price adjustment mechanisms generating market equilibria which can accommodate price dispersion within a probabilistic framework.

The framework we have sketched implies that at least two particular dynamic processes affect each trading region's comparative advantage: (i) the gradual change of production technique, and (ii) the evolution of bilateral or multilateral trade preferences. Each of these developments can be viewed as a process of investment. On the supply side, we have emphasized rigidities and patterns of change which can be explained in the framework of vintage theory. The latter focuses on rigidities caused by the creation of fixed capital and other durables at fixed locations. Existing production units have, at every point in time, a given capacity (capital equipment) which embodies the technology of its date of construction. These limitations can only be changed by means of investments which create new operating units. Hence, the dynamics takes the form of a process in which new plants are constructed in a time sequence, taking advantage of the most recent technological developments to achieve greater efficiency than their predecessors. In this sense, regional advantages are created over time and are dependent on investment decisions in the region.

Trade preferences may be viewed in a similar perspective. By investing in the establishment of new trade channels and trade partnerships, a rigid network of preferences and barriers are built. If a single region's network is strengthened by links with other regions where demand is growing, this will improve the region's comparative advantage. These dynamic aspects emphasize the need to further investigate the framework proposed in sections 2 and 3.

The price adjustment mechanisms in the model have to be analyzed more profoundly, and the exporter- and importer-constrained prices used have to be compared in more detail. Moreover, other market arrangements could be considered such as intertia-influenced negotiations (Batten, 1983).

APPENDIX

A.1. *Differentiation of γ_i^s with Respect to Prices*

Consider the optimization model in (4.4). The associated Lagrange function is

$$L = J(x,q) + \sum_s \beta_i^s(p_i^s)(\sum_r x_i^{rs} - D_i^s(p_i^s) + \sum_s \gamma_i^s(p_i)\sum_r [(v_i^r + c_i^{rs})x_i^{rs} -$$

$$- p_i^s D_i^s(p_i^s)]$$

Letting v_i^r be strictly related to p_i^r so that $v_i^r = v_i^r(p_i^r)$ or $p_i^r = p_i^r(v_i^r)$, $\gamma_i^s = \gamma_i^s(p_i)$ will be a differentiable function of all prices p_i^1, p_i^2,... . Letting h_i^r denote the Right Hand Side of (4.8), we can write

$$\frac{\partial z_i^r}{\partial p_i^r} = h_i^r + \sum_s \frac{\partial \gamma_i^s}{\partial p_i^r} \frac{D_i^s(p_i^s)A_i^{rs}}{(A_i^s)^2} [A_i^s(v_i^r + c_i^{rs}) - \sum_k (v_i^k + c_i^{ks})A_i^{ks}] \qquad (a.1)$$

Since $\sum_i A_i^{ks} = A_i^s$, we know that for modest variations in the price terms $(v_i^k + c_i^{ks})$ the difference within brackets in (a.1) will be close to zero. When this is the case (4.8) will be a good approximation of (a.1).

Next, let Σh_i^{rs} represent the Right Hand Side of (4.9). Then we may write

$$\sum_s \frac{\partial z_i^r}{\partial p_i^s} = \Sigma h_i^{rs} +$$

$$+\sum_s \sum_\ell \frac{\partial \gamma_i^\ell}{\partial p_i^s} \frac{A_i^{r\ell} D_i^\ell(p_i^\ell)[A_i^\ell(v_i^r + c_i^{r\ell}) - \sum_k A_i^{k\ell}(v_i^k + c_i^{k\ell})]}{(A_i^\ell)^2} \quad \text{(a.2)}$$

Also in this case we observe that $A_i^\ell = \Sigma_k A_i^{k\ell}$. Hence, if the price terms $(v_i^\ell + c_i^{k\ell})$ vary in such a way that the second part of the formula is close to zero, formula (4.) will be a good approximation of expression (a.2).

From (4.4) we may conclude that $\partial\gamma_i^\ell / \partial p_i^s > 0$ for $s \neq \ell$. Moreover, if the elasticity of demand is greater than unity in absolute terms, we also have $\partial\gamma_i^s / \partial p_i^s > 0$. If the elasticity is less than unity, the derivative is negative.

A similar result is obtained for the model version in section 4.3.

A.2. *Comparison with Other Model Formulations*

First we shall demonstrate that the model in Remark 3 generates solutions which have the same structure as those obtained by using the model in (4.4). The Lagrange function associated with the model in Remark 2 is

$$L = \sum_s p_i^s D_i^s(p_i^s) - \sum_{sr}\Sigma(v_i^r + c_i^{rs})x_i^{rs} + \Sigma\beta_i^s[D_i^s(p_i^s) - \sum_r x_i^{rs}]$$
$$+ \xi(J^* - J(x,q)) \quad \text{(a.3)}$$

which yields $x_i^{rs} = q_i^{rs}\exp\{-(1+\beta_i^s+v_i^r+c_i^{rs})/\xi\}$, from which we obtain

$$x_i^{rs} = \frac{q_i^{rs}\exp\{-(v_i^r+c_i^{rs})/\xi\}}{\Sigma q_i^{rs}\exp\{-(v_i^r+c_i^{rs})/\xi\}} D_i^s(p_i^s) \qquad (a.4)$$

Obviously, (a.4) has the same structure as (4.5b) with the only difference that γ_i^s has been replaced by $1/\xi$. The pleasant result of this is that the problems discussed in Appendix A.1 are resolved, since there is not direct dependence between price alterations and changes in $1/\xi$.

Our second task is to illustrate how a Takayama-Judge framework as sketched in (2.12) can be combined with a probabilistic treatment of inertia in the adjustment of trade patterns. The following Lagrange function illustrates such a hybrid model:

$$\begin{aligned} L = & \sum_s \int_0^{p_i^s} (D_i^s(\eta) - X_i^s(\eta))d\eta - \Sigma\Sigma c_i^{rs} x_i^{rs} \\ & + \sum_s \tau_i^s [\sum_r x_i^{rs} - D_i^s(p_i^s)] + \Sigma\rho_i^r [X_i^r(p_i^r) - \sum_s x_i^{rs}] \qquad (a.5) \\ & + \xi[J^* - J(x,q)] \end{aligned}$$

where the first summation in (a.5) represents the "net social payoff" term. A solution must satisfy $x_i^{rs} = q_i^{rs}\exp\{\tau_i^s-\rho_i^r-c_i^{rs}-1\}$ which gives us the following two alternative formulations

$$x_i^{rs} = \frac{q_i^{rs}\exp\{-\rho_i^r - c_i^{rs}\}}{\sum_k q_i^{rs}\exp\{-\rho_i^k - c_i^{ks}\}} D_i^s(p_i^s)$$

$$x_i^{rs} = \frac{q_i^{rs}\exp\{\tau_i^s - c_i^{rs}\}}{\sum_s q_i^{rs}\exp\{\tau_i^s - c_i^{rs}\}} X_i^r(p_i^r)$$

We may also observe that $X_i^r(p_i^r)$ may be represented by (or constrained by) a step-function of the kind introduced in (3.2).

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