WORKING PAPER

A DYNAMIC INTERACTIVE DECISION ANALYSIS AND SUPPORT SYSTEM (DIDASS)

USER'S GUIDE (MAY 1983)

Manfred Grauer

June 1983 WP-83-60



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PREFACE

The Interactive Decision Analysis group at IIASA has recently developed an interactive decision support system called DIDASS (dynamic interactive decision analysis and support system). The major advantage of this system over most other computerized approaches to decision problems is that it is interactive, that is, it involves the decision maker in the decision process. It is an attempt to combine the analytical power of the "hard" computer model with the qualitative assessments of the decision maker.

DIDASS is an interactive multicriteria programming package based on the reference (aspiration) approach to multicriteria analysis, and is capable of dealing with both linear and nonlinear problems. It has been written in FORTRAN 77, avoiding the use of any operating-system-dependent statements or commands, which means that it can be transferred to almost any computer without difficulty. This guide has been prepared for users both at IIASA and at the many collaborating institutions where DIDASS is now running, and is based on the version of DIDASS available on tape from IIASA.

If you have any comments or suggestions concerning the system or this guide, we would be glad to hear from you - DIDASS is intended to be useful, useable, and used!

ANDRZEJ WIERZBIEKI

Chairman

Systems and Decision Sciences

DIDASS is an interactive multicriteria programming package designed for decision support. It is able to deal with both linear and nonlinear multicriteria programming problems, and is based on the reference point approach to multicriteria analysis.

The basic idea of the reference point method is to rank multidimensional decision alternatives q, defined as points in the R^p ($p \ge 2$), relative to a reference point \overline{q} which reflects the preferences of the user.

The ranking of the decision alternatives is based on a partial ordering of the R^p :

$$q^{1} \leq q^{2}; \quad q_{i}^{1} \leq q_{i}^{2}; \quad i = 1, 2, \dots, p; \quad q^{1}; q^{2} \in \mathbb{R}^{p}$$
 (1)

The decision problem is to determine an n-vector x of decision variables satisfying all given constraints while taking into account the p-vector of objectives. We will assume that each component of q should be as large as possible.

A reference point or reference objective is a suggestion \bar{q} supplied by the user which reflects in some sense the "desired level" of the objective. An achievement scalarizing function $s(q-\bar{q})$ defined over the set of objective vectors q is then associated with each reference point \bar{q} [1]. If we regard the function $s(q-\bar{q})$ as the "distance" between the points q and \bar{q} , then, intuitively, the problem of minimizing this distance may be interpreted as the problem of finding from within the Pareto set the point \hat{q} "nearest" to the reference point \bar{q} . (However, the function s is not necessarily related to the usual notion of distance.) With this interpretation in mind, reference point optimization may be viewed as a way of guiding a sequence $\{\hat{q}^k\}$ of Pareto points generated through an interactive procedure and should result in a set of attainable efficient points $\{\hat{q}^k\}$ of interest to the user. If the sequence $\{\hat{q}^k\}$ converges, the limit may be

seen as the solution to the decision problem.

2. PROBLEM FORMULATION

Let us assume that the decision problem can be clarified by analyzing a general constrained multicriteria problem in the following standard form:

$$\max_{\mathbf{x}_{nl},\mathbf{x}_{l}} \left| \begin{array}{c} f_{1}(x_{nl}) + c_{1}^{T}x_{nl} + d_{1}^{T}x_{l} = q_{1} \\ f_{2}(x_{nl}) + c_{2}^{T}x_{nl} + d_{2}^{T}x_{l} = q_{2} \\ & \ddots & \ddots \\ f_{p}(x_{nl}) + c_{p}^{T}x_{nl} + d_{p}^{T}x_{l} = q_{p} \end{array} \right|$$

$$(2)$$

subject to:

$$g(x_{nl}) + A_1 x_l \le b_1 \tag{3}$$

$$A_2 x_{nl} + A_3 x_l \le b_2 \tag{4}$$

$$l \leq \begin{bmatrix} x_{nl} \\ x_l \end{bmatrix} \leq u \tag{5}$$

where $g(x_{nl}) = \left[g_1(x_{nl}), g_2(x_{nl}), \dots, g_m(x_{nl})\right]^T$ is a vector of nonlinear constraints and $f_1(x_{nl}), f_2(x_{nl}), \dots, f_p(x_{nl})$ in (2) represents the nonlinear parts of the performance criteria. The decision variables are divided into two subsets: a vector of "nonlinear" variables (x_{nl}) and a vector of "linear" variables (x_l). It is clear that when vectors f and g are nonexistent, formulation (2)-(5) is identical with the standard multicriteria linear programming problem. An overview of the various ways in which the reference point approach can be used in the linear case is given in [2], while the nonlinear case is described in [3].

The current computer implementation of the decision analysis and support system DIDASS is based on a two-stage model of the decision-making process. In the first stage - the exploratory stage - the user is informed about the range of his alternatives, thus giving him an overview of the problem. In the second stage - the search stage - the user works with the system in an interactive way to analyze the efficient alternatives { \hat{q}^k } generated by DIDASS in response to his reference objectives { \bar{q}^{k} }. The initial information for the exploratory stage is provided by calculating the extreme points for each of the objectives in (2) separately. A matrix D_{S} which yields information on the range of numerical values of each objective is then constructed. We shall call this the *decision support matrix*.

$$D_{S} = \begin{bmatrix} q_{1}^{*} q_{2}^{1} \cdots q_{p}^{1} \\ q_{1}^{2} q_{2}^{*} \cdots q_{p}^{2} \\ q_{1}^{2} q_{2}^{*} \cdots q_{p}^{2} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ q_{1}^{i} q_{2}^{i} \cdots q_{p}^{i} \\ \vdots & \vdots & \vdots \\ q_{1}^{i} q_{2}^{i} \cdots q_{p}^{i} \end{bmatrix}$$
(6)

Row *i* corresponds to the solution vector x_i which maximizes objective q_i . The vector with elements $q_i^i = q_i^*$, i.e., the diagonal of D_S , represents the *utopia (ideal) point*. This point is not normally attainable (if it were, it would be the solution of the proposed decision problem), but it is presented to the user as an upper guideline to the sequence { \bar{q}^k } of reference objectives. Let us consider , column *i* of the matrix D_S . The maximum value in the column is q_i^* . Let q_i^n be the minimum value, where

$$\min_{1 \le j \le p} \left\{ q_i^j \right\} = q_i^n$$

We shall call this the *nadir* value. The vector with elements $q_1^n, q_2^n, \ldots, q_p^n$ represents the *nadir point*, and may be seen as a lower guideline to the values of the user's objectives.

In the linear case we use the following scalarizing function s(w), where minimization results in a linear programming formulation:

$$s(w) = -\min\left\{\rho \min_{i} w_{i} ; \sum_{i=1}^{p} w_{i}\right\} - \varepsilon w$$
(7)

Here $w_i \equiv (q_i - \tilde{q}_i) / \gamma_i$, ρ is an arbitrary coefficient which is greater than or equal to p, γ_i is a scaling factor, and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)$ is a nonnegative vector of parameters.

In the nonlinear version of the package the following achievement scalarizing function is used:

$$s(w) = -\frac{1}{\rho} \ln \left[\frac{1}{p} \sum_{i=1}^{p} (w_i)^{\rho} \right]$$
(8)

where $w_i = \gamma_i \left[(\tilde{q}_i - q_i) / (\tilde{q}_i - \bar{q}_i) \right]$, \tilde{q}_i is an upper limit to the sequence of reference points, $\rho \ge 2$ is again an arbitrary coefficient greater than or equal to p, and γ_i acts here as a weighting factor. This achievement scalarizing function meets the following requirements:

- It yields scaling factors which make additional scaling of objectives unnecessary.
- It is a smoothly differentiable function that approximates the nonsmooth function $s = \max_{i} w_{i}$.
- It is strongly order-preserving and weakly order-approximating.

The resulting single-criterion programming problems are solved using the solution package MINOS [4].

3. INFORMATION FOR IMPLEMENTATION

The current version of the DIDASS package has been designed specifically to be portable, and it has therefore been written completely in FORTRAN 77, avoiding the use of any operating-system-dependent statements or commands.

The DIDASS source code is normally supplied by IIASA on tape (9-track,

unlabeled, ebcdic, upper case, 800 bpi, block size 800 characters, record length 80 characters) under the following names:

list_of_files

mtibm: Tape	e format: upper	case ebcdic	
mtibm Reco	ord length: 80.	Block length:	800. Density: 800. bpi.
File:1	Records:9	Blocks:1	Filename:constr_l.f
File:2	Records:5	Blocks:1	Filename:constr_nl.f
File:3	Records: 1407	Blocks:141	Filename:didass.f
File:4	Records:27	Blocks:3	Filename:list_of_files
File:5	Records:45	Blocks:5	Filename:model.l
File:6	Records:100	Blocks:10	Filename:model.nl
File:7	Records: 6765	Blocks:677	Filename:nonlp.f
File:8	Records:9	Blocks:1	Filename:object_1.f
File:9	Records:103	Blocks:11	Filename:object_nl.f
File:10	Records:4	Blocks:1	Filename:rfp.l
File:11	Records:4	Blocks:1	Filename:rfp.nl
File:12	Records:22	Blocks:3	Filename:specs.l
File:13	Records:46	Blocks:5	Filename:specs.nl

To prepare DIDASS to solve linear problems only the user must read all files (including data and FORTRAN files) from the tape, compile, link and load the following FORTRAN files:

constr_l.f, object_l.f, didass.f, nonlp.f

To prepare DIDASS to solve mixed linear and nonlinear problems, or nonlinear problems only, the user must read all the files (including data and FORTRAN files, excluding the files constr_l.f and object_l.f), compile, link and load the following FORTRAN files:

constr_nl.f, object_nl.f, didass.f, nonlp.f.

To support the implementation of the system, data for a linear and a nonlinear example are also given on the tape.

4. SOLVING A LINEAR PROBLEM

The solution of a linear problem is demonstrated by example I (hypoth.1):

$$\max \begin{cases} 1.5x_1 + 2x_2 - x_3 + 3x_4 + x_5 + x_7 = obj1 \\ 1.2x_1 + x_2 + x_3 + x_4 + 2.75x_5 + x_6 = obj2 \\ 2.5x_1 + x_3 + 2x_4 + 1.7x_5 - x_6 - x_7 = obj3 \end{cases}$$

subject to :

٢

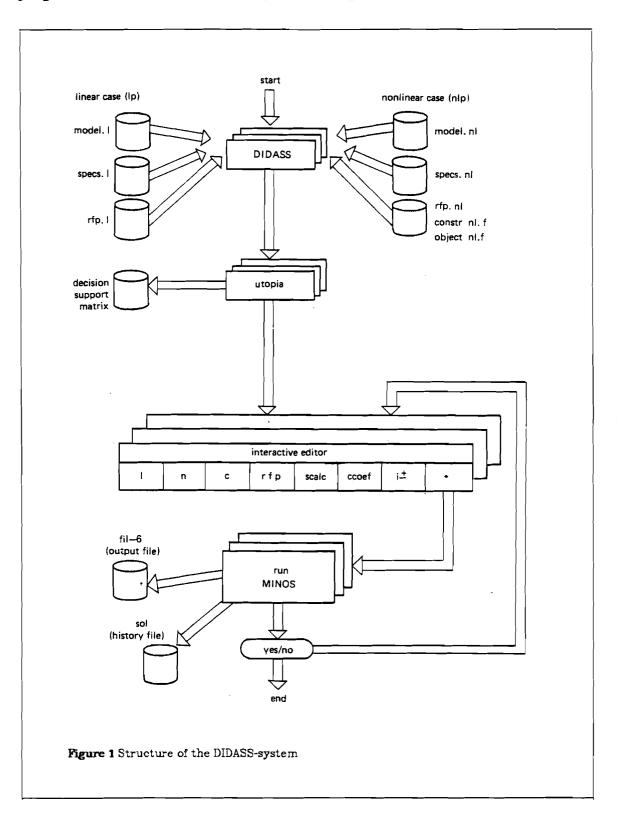
The MPS input file should be prepared in a certain way - the user must list objectives of type e (equalities) as the first entries in the row definition section in the same order as they appear in the reference point file (see later). The input file must contain a section on bounds, even if this section is empty. The MPS file ("model.1") and the specification file ("specs.1") for the above example are given in Appendix 1.

The reference point file ("rfp.1") has the format (2X,2A4,2X,3F12.5). The first two characters are blanks, the next eight characters contain the name of the objective, and there are then two more blanks. The first in F12.5 field gives the value of the reference point. The next field contains the max-min indicator, which is + for maximization, and - for minimization; the corresponding digits can be used as scaling factors. The last contains the values of the control coefficients, $\rho \ge p$ on the first line, and ε on the second line. The last line of the file must contain four dots (....) as characters 5-8 in the A4 field. For example I the reference point file would take the following form:

obj1	3 5.0	+1.0	3.00
obj2	15. 0	+1.0	0.100e-06
obj3	25.0	+1.0	
••••			

The files "constr_l.f" and "object_l.f" contain dummy subroutines which are needed if the user intends to solve linear problems only. The steps taken in the

- 8 -



program for the linear case are explained in Figure 1.

The interactive procedure begins with the program asking which type of problem is to be considered (lp in this case). The program then enters the exploratory stage and informs the user of the range of values possible for each alternative. This leads into the search stage, which is split into two parts: editing and problem solving. In the editing mode the following commands may be used:

- l list the names of the objectives and the components of the reference points
- n neutral solution zero is the reference point

i+- positive infinite reference point $(+10^5)$

- i- negative infinite reference point (-10^5)
- c copy solution from previous session as reference point

The following commands are also available:

rfp - change to reference point definition status scale - change to scaling factor modification status ccoef - change to control coefficient definition status

In order to define a new value of ρ (or ε), it is sufficient to type ccoef and then rho (or eps); the program moves to the corresponding definition status, the new value of the parameter may be typed in and the program returns to waiting status.

The reference point components may be redefined in rfp status. To do this, it is necessary to type two lines - one containing the name of the objective, the other the new value of the reference point component. Redefinition of scaling factors is carried out in scaling factor modification status. Here again it is necessary to type two lines - one containing the name of the objective, the other the new value of the scaling factor. The only way to leave editing status is to type an asterisk (*).

In the next phase of the interactive process the program asks the user for the names of the RHS and BOUNDS sections. (In our example these are "rhs" and "bnd".) Using this option the user can change the set of constraints between sessions, thus modifying the problem. The system then generates a new MPS file and the single-criterion LP problem (7) is solved. Finally, the necessary information from the LP output file is extracted and presented to the user in the same form as the original problem.

The user then repeats the search stage until he obtains satisfactory results.

5. SOLVING A NONLINEAR PROBLEM

The solution of a nonlinear problem is demonstrated by example II (theo):

min	$ \left\{ \begin{array}{l} (x_1-3)^2 + (x_2-2)^2 + (x_6-6)^2 + (x_7-4)^2 &= obj1\\ 0.5(x_3-4)^2 + (x_8-6)^2 + (x_9-11)^2 &= obj2\\ (x_4-1)^2 &+ (x_5-8)^2 + (x_{11}-4)^2 + (x_{12}-1)^2 + (x_{10}-8)^2 &= obj3 \end{array} \right\} $
-----	---

subject to :

$$2x_{1}+0.5x_{2}-x_{6}+x_{11} = 5$$

$$x_{1}+2x_{2}-x_{7}+x_{11} = 0$$

$$x_{3}+0.5x_{6}-x_{8}-x_{12} = 0$$

$$0.5x_{3}+x_{6}-x_{9}+x_{12} = 0$$

$$x_{4}+0.5x_{5}+0.5x_{8}-x_{10} = 0$$

$$2x_{4}+x_{5}-x_{8}-x_{11} = 0$$

$$3x_{4}-x_{5}+x_{8}-x_{12} = 0$$

$$2x_{1}+x_{2}+2x_{11} \le 8$$

$$2x_{6}+3x_{12} \le 12$$

$$5x_{4}+3x_{5} \le 15$$

$$3x_{4}+2x_{5}+3x_{8} \le 12$$

$$3x_{1}+2x_{2} \le 13$$

and

$x_i \ge 0, i=1,2,...,12$

 $x_1 \leq 2$, $x_2 \leq 6$, $x_3 \leq 3$, $x_4 \leq 2$, $x_5 \leq 4$, $x_6 \leq 4$, $x_8 \leq 3$, $x_{11} \leq 3$, $x_{12} \leq 2$

The corresponding MPS file ("model.nl"), including the linear part, is given together with the specification file ("specs.nl") in Appendix 2.

The subroutine constrn ("constr_nl.f") (see Appendix 2) may be used to compute the nonlinear constraint functions g(x) (here f(m)) and the corresponding elements of the Jacobian matrix $\partial g / \partial x_j$ (here g(m,n)). As example II does not contain nonlinear constraints the subroutine is "empty" in this case.

In subroutine objectf ("object_nl.f") the user has to insert the nonlinear objective functions $f_i(x_{nl})$ under the name obj(i) as indicated in Appendix 2. The gradients are calculated automatically.

The reference point file ("rfp.nl") has the format (2X,2A4,2X,3F12.5). The first two characters in each line contain blanks, the next eight characters the name of the objective, and there are then two more blanks. The first F12.5 field contains the value of the reference point, the second a variable that can be used as a weighting coefficient and the third a control variable. In example II the value of the control coefficient is $\rho=24$. The last line must contain four dots (....) as characters 5-8 in the 2A4 field.

rfp1	25.0	1.	2 4.0
rfp2	50.0	1.	1.0
rfp3	45.	1.	1.0
• • • •			

To start the interactive procedure in the nonlinear case, having prepared the files, the user must first initialize DIDASS. The program steps correspond to those already outlined for the linear case (see Section 4 and Figure 1). Appendix 3 contains examples of the interactive use of DIDASS in the linear and nonlinear cases.

ADDITIONAL REMARKS

The (May 83) implementation of DIDASS described in this guide is still being tested and improved. We would be glad to receive any suggestions or comments you might have concerning the system.

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APPENDIX 1

The MPS file ("model.1") for example I is:

e obj2 e obj3 l const1 l const2 l const3 l const4 columns	
x1 obj1 1.5 obj3 2.	5
x1 obj2 1.2	Ŭ
x1 const1 1.0 const2 -2.	0
x1 const3 -1.0	
x2 obj1 2.0 obj2 1.	0
x2 const1 2.0 const2 -1.	0
x2 const4 1.0	
x3 obj1 -1.0 obj2 1.	
x3 obj3 1.0 const1 1.	
x3 const3 1.0 const4 2.	
x4 obj1 3.0 obj2 1.	
x4 obj3 2.0 const1 1.	
x4 const2 1.0 const4 -1. x5 obj1 1.0 obj2 2.	
x5 obj1 1.0 obj2 2. x5 obj3 1.7	75
x5 const1 2.0 const2 2.	n
$\mathbf{x5}$ const3 2.0 const4 1.	
x6 obj2 1.0 obj3 -1.	
x6 const1 1.0 const4 -1.	
x7 obj1 1.0 obj3 -1.	
x7 const1 2.0 const2 1.	0
x7 const3 -2.0 const4 -1.	0
rhs	
rhs const1 12.25	
rhs const2 13.75	
rhs const3 14.0	
rhs const4 16.5	
bounds lobnd x1 0.0	
lobnd x1 0.0 lobnd x2 0.0	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{array}{cccc} 10 \text{ bid} & x0 & 0.0 \\ 10 \text{ bid} & x4 & 0.0 \end{array}$	
$\begin{array}{cccc} 10 & \text{bid} & \text{xf} & 0.0 \\ 10 & \text{bid} & \text{x5} & 0.0 \end{array}$	
lo bnd x6 D.0	
lo bnd x^7 0.0	
endata	

The specification file ("specs.l") for example I is:

<pre>begin hypoth.1 minimize objective rhs bounds rows columns elements aijtol mps file crash option iterations</pre>	mccobj rhs bnd 1400 1500 11000 0.000001 9 1 6000
-	-
log frequency	50
factorize frequen	cy 100
partial price	1
solution	yes
feasibility tol	1.0e-5
problem number endrun	0

•

APPENDIX 2

theo name rows e gl1 g12 е е g13 е g14 е gl5 g16 е е g17 ugl1 1 ug12 l ug13 1 1 ugl4 ug15 1 colums 2.0 g12 1.0 **x**1 gl1 2.0 3.0 **x**1 ugl1 ugl5 0.5 2.0 x2 gl1 g12 1.0 2.0 х2 ugl1 ugl5 xЗ g13 1.0 g14 0.5 **x**4 g15 1.0 g16 2.0 **x**4 g17 3.0 ug13 5.0 ugl4 3.0 **x**4 0.5 **x**5 g15 g16 1.0 -1.0 xб g17 х5 ug13 3.0 ugl4 2.0 -1.0 **x**6 gl1 g13 0.5 **x**6 gl4 1.0 ug12 2.0 -1.0 **x**7 g12 g13 -1.0 0.5 **x**8 g15 x8 g16 -1.0 g17 1.0 **x**8 ugl4 3.0 -1.0 х9 gl4 -1.0 **x**10 **g**15 gl1 **x**11 1.0 g12 1.0 **x**11 g16 -1.0 ugl1 2.0 x12 g13 -1.0 gl4 1.0 x12 g17 -1.0 3.0 ugl2 rhs 5.0 rhs gl1 ug l 1 8.0 rhs 12.0 rhs ugl2 ug13 15.0 rhs 12.0 rhs ugl4 rhs 13.0 ug15 bounds up bnd **x**1 2.0 x2 6.0 up bnd xЗ 3.0 up bnd up bnd **x**4 2.0

The MPS file ("model.nl"), including the linear part, for example II is:

up bnd up bnd up bnd up bnd up bnd fx initial fx initial fx initial fx initial fx initial fx initial fx initial fx initial	x5 x6 x8 x11 x12 x1 x2 x3 x4 x5 x6 x7	4.0 4.0 3.0 3.0 2.0 1.9 1.6 1.7 0.4 1.2 0.6 6.1
fx initial	x6	0.6
fx initial	x7	6.1
fx initial	x8	1.0
fx initial	x9	2.45
fx initial	x10	1.5
fx initial	x11	1.0
fx initial endata	x12	1.0

The specification file ("specs.nl") is:

begin theo minimize nonlinear constrat nonlinear jacobiar nonlinear objectiv	n vars 12
objective = object problem no.	: 1
bounds	bnd
rhs	rhs
rows	20
columns	20
elements	100
aijtol	0.000001
mps file	9
crash option	1
iterations	1000
solution	yes
feasibility tol	1.0e-5
lower bound	0.
completion	full
jacobian	dense
lagrangian	yes
major iterations	10
minor iterations	20
penalty parameter	0.1

```
1.0e-6
dj tolerance
row tolerance
                   1.0e-6
                     0.01
radius of conver
superbasics
                        12
                        12
hessian dimension
linesearch toler
                      0.1
print level (jflxi)
                     101
derivative level
                        2
                     1.0e-06
difference interval
call function routines when optimal
```

end

The subroutine constrn for example II is:

```
subroutine constrn( mode,m,n,njac.x,f,g,nstate,nprob )
implicit real*8(a-h,o-z)
real*8 x(n),f(m),g(m,n)
return
end
```

The subroutine objectf for example II is:

```
subroutine objectf( mode, n, x, f, g, nstate, nprob)
    implicit real*8(a-h,o-z)
   real*8 x(n),g(n)
   logical ityp
   character*4objnam, ipoint
   common/tables/nc, objnam(2, 100), sig(100), ityp(100)
   common/err/ierr
   common/rfp/rfp(100)
   common/gamma/gam(100), obj(100), dif(100)
   common/gamma1/gam1(100), rfp1(100), sig1(100)
   common/gammau/obju(10000), objmin(100), objmax(100), w(100)
   data ipoint/'....'/
   if (nstate .ne. 1) go to 741
С
      first entry
С
С
   nc=0
      repeat
С
23041
         continue
     nc=nc+1
     read(11,290)objnam(1,nc),rfp(nc),gam(nc),sig(nc)
     write(6,290)objnam(1,nc), rfp(nc), gam(nc), sig(nc)
      ityp(nc) = .false.
     if(.not.(objnam(1,nc).eq.ipoint)) goto 23044
```

```
goto 280
23044
         continue
23042
         goto 23041
280
     nc=nc-1
290
      format(2x, a4, 6x, 3f12.5)
      continue
741
С
      normal entry
с
c
      Insert the criteria functions as
С
с
      FORTRAN-statements.
c
      This is the example II ("theo") with quadratic
С
      criteria functions and linear constraints.
С
С
      obj(1)=((x(1)-3)**2+(x(2)-2)**2+(x(6)-6)**2+(x(7)-4)**2)
      obj(2)=(0.5*(x(3)-4)**2+(x(8)-6)**2+(x(9)-11)**2)
      ob_{j}(3)=((x(4)-1)**2+(x(5)-8)**2+(x(11)-4)**2
     *+(x(12)-1)**2+(x(10)-8)**2)
      if (nprob .ne. 1) go to 720
С
      A quadratic scalarizing function is used to
С
      calculate the decision support matrix.
с
c
      f=0.0
      do 751 k=1,nc
         f=(gam(k)*(rfp(k)-obj(k))/sig(k))*
           (gam(k)*(rfp(k)-obj(k))/sig(k))+f
  751 continue
      go to 714
  720 continue
      if (nprob.ne. 2)go to 714
      if (nstate .ne. 1)go to 103
С
      The decision support matrix is stored for
с
      future sessions.
С
С
295
      format(f12.5)
      do 100 i=1,nc
      read (7, 295) objmin(i)
100
      continue
103
      continue
С
      The automatically scaled achievement variables
С
      are calculated.
с
c
      if (nstate .ne. 1) goto 8013
      do 101 i=1,nc
      if (rfp(i) .le. objmin(i)) goto 8011
      obju(i) = .5*objmin(i)
  101 continue
      go to 8013
 8011 continue
      do 8012 i=1,nc
```

```
obju(i)=.5*rfp(i)
8012 continue
С
      The logarithmic scalarizing function
С
      is used.
С
с
8013 continue
     rho=sig(1)
      f=.0
      s=.0
      do 102 i=1,nc
     w(i)=((obju(i)-obj(i))/(obju(i)-rfp(i)))*gan(i)
      s=s+w(i)**rho
  102 continue
      s=s/nc
      f=+(dlog(s))/rho
  714 continue
      if (nstate .ne. 2) return
С
      Final entry
с
С
      do 802 k=1,nc
      write (10,801) obj(k)
  802 continue
  801 format (2x, f12.5)
  800 continue
      return
      end
```

APPENDIX 3

didass Enter the problem type linear(enter lp) or nonlinear(enter nlp) lp Each line of the matrix gives the results of the selfish optimizations. The diagonal represents the utopia point. obj(1) obj(2) obj(3) extreme obj(1) 36.75000 12.25000 24.50000 extreme obj(2) 6.12500 16.84300 10.41200 extreme obj(3) 18.37500 14.70000 30.62500 You can now: list the names of the obj. and components of ref.pts.(enter 1), ask for neutral solution(enter n), ask for plus infinite reference point(enter i+), ask for minus infinite reference point(enter i-), copy solution from last session as ref.p.(enter c), change the values of scal.coef.(enter scalc), change the values of control coef.(enter ccoef), change the values of ref. point components (enter rfp). If you wish to make no more changes, type * to exit from editing status 1 obj.name refpt.value scal.coef. contr.coef. objl 20.0 4.00 3.00 obj2 30.0 1.000 0.100e-06 obj3 28.0 1.000 enter name of rhs set rhs 3 objectives eps 0.100e-06 3.00 rho enter name of bounds set bnd eps 0.100e-06 rho 3.00 objective utopia reference efficient dual scale names point point point obj(1) obj(2) 4.00 36.8 20.0 8.55 Ο. 16.8 30.0 16.4 2.72 1.000 obj(3) 0.288 1.000 28.0 14.4 30.6 Do you want to run the program once more with edited input data(enter yes) or terminate the session (enter no)? no

didass Enter the problem type linear(enter lp) or nonlinear(enter nlp) nlp Each line of the matrix gives the results of the selfish optimizations. The diagonal represents the utopia point. obj(1) obj(2) obj(3) extreme obj(1) 24.01900 89.69800 92.13700 extreme obj(2) 38.56200 38.31200 108.99000 extreme obj(3) 42.50000 128.00301 48.86200 You can now: list the names of the obj. and components of ref.pts.(enter 1), ask for neutral solution(enter n), ask for plus infinite reference point(enter i+), ask for minus infinite reference point(enter i-), copy solution from last session as ref.p.(enter c), change the values of scal.coef.(enter scalc), change the values of control coef. (enter ccoef), change the values of ref. point components (enter rfp). If you wish to make no more changes, type * to exit from editing status 1 obj.name refpt.value scal.coef. contr.coef. 25.0 1.000 24.0 objl 50.0 1.000 1.000 obj2 1.000 1.000 obj3 45.0 enter name of rhs set rhs enter name of bounds set bnd objective reference efficient utopia nadir point point point point names _____ obj(1) 24.0 25.0 31.6 42.5 obj(2) 38.3 50.0 63.7 128.

obj(3) 48.9 45.0 59.5 109. Do you want to run the program once more with edited input data(enter yes) or terminate the session (enter no)? no