

NOT FOR QUOTATION
WITHOUT PERMISSION
OF THE AUTHOR

CRITICAL ESSAY ON THE METHODOLOGY
OF MULTIOBJECTIVE ANALYSIS

Andrzej P. Wierzbicki

January 1983
WP-83-14

Working Papers are interim reports on work of the International Institute for Applied Systems Analysis and have received only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute or of its National Member Organizations.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

This paper presents a discussion of methodological issues in multiobjective analysis, encompassing various approaches to multiobjective optimization and decision making. The main thesis is that while there are already many methods for multiobjective analysis, this field would gain from further methodological reflection.

CRITICAL ESSAY ON THE METHODOLOGY
OF MULTIOBJECTIVE ANALYSIS

Andrzej P. Wierzbicki

1. INTRODUCTION: WHAT IS MULTIOBJECTIVE ANALYSIS?

The various methods for multiobjective optimization and decision making that have been developed since the work of Pareto (1896) have recently been summarized in several books [Keeney and Raiffa (1976), Spronk (1981), Cohon (1978)]; one of the most incisive summaries [Rietveld (1980)] relates to regional planning. In general terms, these methods deal with the situation where one or more persons must generate and choose between various alternatives that cannot be evaluated on the basis of a scalar performance measure (a 'single-objective') alone. Instead, the evaluation must involve a number of performance characteristics ('multiple objectives') which are often not commensurable.

Such situations often arise when technological, economic, social or political decisions are made, and are usually resolved

either by intuition, or by the collective processes of choice that have grown up throughout human history. Thus, there is nothing new in multiobjective decision making--people have been doing it for thousands of years. However, this term has recently taken on a new and much more specific meaning with the application of mathematical methods to the problem. These methods are generally designed to clarify the decision making situation and to generate useful alternatives; sometimes they involve considerable use of computers and computerized models. However, in none of these methods can a single practical decision be made without the involvement and approval of people--and the author hopes that this will never happen, except in the most routine of situations. To call this group of methods 'multiobjective decision making' without further qualification is therefore semantically misleading; we should perhaps rather refer to it as *multiobjective analysis*.

Some researchers concentrating on the mathematical part of the multiobjective analysis prefer to speak of multiobjective optimization. However, this would limit the field of study to a particular area of mathematics, while the motivation and importance of multiobjective analysis come not from mathematics but rather from applied problems. Thus, for methodological clarity, we should consider multiobjective analysis as a part of the multidisciplinary applied science called *applied systems analysis*.

Some readers might object to the definition of 'applied systems analysis' as a 'multidisciplinary applied science'. For example, Rietveld (1980) defines *systems theory* more traditionally as a new science concerned with the functioning of systems in general, and the word *system* itself has a very old meaning as a description of a set of elements and the relationships between them. However, this definition is too broad: on this basis Ptolemy, Copernicus and Bohr were systems analysts, since the first two investigated the solar system, while the third studied the atomic system. The new factor in contemporary systems analysis is the realization that certain methodological principles and mathematical tools can be applied to systems in a multidisciplinary fashion.

Contemporary systems analysis also lays great importance on the applied or empirical aspects of research. *Mathematical systems theory* is a new and still developing branch of applied mathematics which includes the theory of dynamical systems, optimization theory, some aspects of economic equilibrium theory, game theory and multiobjective decision theory. Though the initial practical motivation (for example, mechanics, electronics, economics) underlying any part of mathematical systems theory is responsible for the basic concepts, the theory still remains a branch of applied mathematics, where the fundamental questions are those of *syntactical correctness and completeness of mathematical language*; questions of *semantic importance* are considered valid only in the sense of motivation. This interpretation of mathematics as a language in which empirical statements can be formulated and transformed, but never validated (in the empirical sense) is quite clear in the modern philosophy of science, from the work of Russell (1927) on mathematical logic, through the development of logical empiricism, to the work of Popper (1959). Thus, it is the applied nature of systems analysis that holds the real meaning, for all the beauty of the mathematical language that we can use to describe it.

An *empirical scientific statement* is one that purports to explain some observations made in the real world and *admits an empirical falsification test* [see Popper (1959)]. Such statements may not have any immediate uses, at least none that can be easily perceived. By contrast, the *applied sciences* concentrate on producing empirical statements of perceived direct usefulness, though these might be limited in their precision and validity.

Some researchers distinguish between science and technology on the understanding that science is interested in the universal questions of general validity, while technology considers questions of an approximate, 'good enough', 'mostly', 'can do' character [Rose (1982)]. On this basis, systems analysis is a multidisciplinary methodology for technological thought. However, this understanding of technology is peculiar to the English language; more modern usage and most other languages prefer the broader term of applied sciences.

When using this phrase, however, we must avoid narrow interpretations in terms of utilitarian science. This can be illustrated by the following anecdote about three people who, not knowing anything about electricity, observed that amber sometimes attracts pieces of paper. One of them, a utilitarian scientist, concluded that this amusing fact could have no possible uses. Another one, a technologist, started to produce toys based on this observation. Finally, the third individual, a good scientist, decided to study the phenomenon, with the result that he discovered electricity and all its potential applications.

To summarize these initial remarks, we can state that multi-objective analysis is part of a multidisciplinary applied science called systems analysis, and is concerned with situations in which complex decisions involving many objectives must be made. Its purpose is to clarify the problem by constructing prototypes of decision situations, using certain fundamental concepts based on empirical observations. After the prototype situations and related concepts have been chosen, they are described in mathematical language, and mathematical tools can then be used to suggest how these situations should be handled. While the development of mathematical methods for multiobjective analysis is an important element of this scientific discipline, it is even more important that any statement in the multiobjective analysis should be validated by repeated empirical falsification tests. The generally accepted methodological principle behind the semantic validity of scientific hypotheses is that an empirical scientific hypothesis cannot ever be proven, but may be accepted if it passes various falsification tests. This distinguishes an empirical statement from a mathematical one whose syntactic correctness is subject to rigorous proofs. Since we consider multiobjective analysis to be an empirical scientific discipline, we must choose mathematical tools and language that, while syntactically correct, yield statements that are both empirically testable and semantically valid.

The critical analysis put forward in this paper attempts to show that the above principles, while generally accepted in the methodology of sciences, have been observed only to a limited

extent in the development of multiobjective analysis. Further development of this relatively young discipline will require much stronger adherence to these methodological principles.

2. PROTOTYPES OF DECISION SITUATIONS

2.1 Basic Prototype: Centralized Decisions

Most of the work in multiobjective analysis is based on the prototype decision situation illustrated in Figure 1(a). This involves a 'decision maker' (a single person who has the authority and experience to take the actual decision); an 'analyst' or team of analysts responsible for the analysis of the decision situation; and a 'substantive model of the problem' that is supposed to represent all the pertinent knowledge that the analyst(s) can muster. It should be emphasized that the term 'model' is used here in a very broad sense. It is not necessarily a computerized mathematical model; it may just be a collection of relevant knowledge, data and hypotheses. But this is still a model, not reality, and this fact should be stressed very strongly when examining the methodological implications of the basic prototype. The model is based on the analyst's perception of the decision problem, and this perception may be wrong, or inconsistent with that of the decision maker. Thus, the model should be validated before use. However, before this the model must first be built.

The methodology of model building is itself a separate subject in systems analysis, with its own extensive literature [see, for example, Wierzbicki (1977) and Lewandowski and Wierzbicki (1982)]. Here we shall list only a few general principles.

1. The ultimate purpose of the model should be the most important consideration in model building; the model should also be the simplest possible that serves the purpose. One of the most important tasks of model building is to identify the relevant information, hypotheses, etc.

2. Models should be built in an iterative fashion, at each iteration developing and executing falsification tests examining internal consistency, consistency with other information,

consistency with available empirical data, and consistency with new data gathered specifically for falsification purposes.

3. Models should be built interactively, involving not only analysts but also decision makers, so that the decision maker's perceptions of the problem, the relevant data, and the model validity can be taken into account.

Unfortunately, these principles are not observed in many system-analytic studies, with multiobjective analysis being one of the worst offenders. A possible reason for this is that multiobjective analysis is often influenced by economic traditions, and it is known that the methodological principles of empirical science are sometimes not followed in economic studies [see, for example, a recent critical essay by Leontief (1982)]. However, important as the subject is, this is no place for a detailed discussion of model building. We must assume that the substantive model of the problem has already been built and validated, and concentrate on the second stage: the use of the model to clarify the decision situation.

Before we do this, however, it should be noted that the prototype situation shown in Figure 1(a) is usually oversimplified. Much more common is the situation shown in Figure 1(b), where there is an additional link, a senior analyst responsible for explaining the situation to the decision maker. In other cases individual experts may be involved in evaluating the alternatives proposed by the analysts, as in Figure 1(c), or a group of decision makers may be responsible for the final decision (Figure 1d). The elements of these nontrivial variants of the first prototype can also be combined in other ways. In addition, the 'decision maker' from Figure 1(a) could actually be a 'senior analyst' or 'expert' or 'politician'. However, the main feature of this prototype is that decision-making is actually centralized, even if several parties have to agree upon the decision.

Now, it is the duty of the team of analysts not only to clarify the substantive aspects of the decision situation, but also to formulate proposals taking into account the institutional aspects of this situation, i.e., the characteristics of

the political process that will lead to the actual decision. This principle is not generally followed in contemporary multi-objective analysis, where attention is concentrated primarily on the prototype situation from Figure 1(a). However, there are some notable exceptions.

One of the most common aspects of political processes is that neither the decision makers nor even the experts have much time to study the very detailed reports prepared by the analysts. Even if this is not the case (discussed later), the decision-making process is usually split into two phases. The first phase is usually performed by the team of analysts with some possible interaction from the decision maker, and involves the *generation* of a small number of *alternatives*. The second phase is the responsibility of the decision makers (possibly with the help of experts and senior analysts) and concerns the *choice between alternatives*. Both phases have characteristic features.

Clearly, the stronger the interaction with the decision makers in the first phase, the easier is the second phase. However, in many situations the substantive model is not sufficiently formalized to allow easy interaction. A team of analysts can sometimes have no option but to generate (more or less intuitively) a number of alternatives that seem professionally sound, and submit them to the decision makers.

On the other hand, if the substantive model can be formulated in mathematical terms and computerized, and if the decision makers or experts or even the senior analyst can work interactively with the model to generate alternatives, the chances that the alternatives will be satisfactory are greatly improved. In such a case, it is important to computerize not only the substantive model, but also an interactive decision support system to help the user work with the substantive model (see Figure 1e). It is important to have a clear understanding of the role of interactive decision support systems in this situation. Firstly, they simulate the work of the team of analysts in Figure 1(b), generating alternatives in response to the requirements of the senior analyst. A model user, although supported by the system, must either have some general analytical knowledge about the

problem, or work with an analyst who helps him to interact with the model. Thus, Figure 1(e) represents a situation functionally similar to that illustrated in the lower part of Figure 1(b) but to none of the other cases considered previously. Secondly, the interactive decision support system enables the user to learn about possible alternatives, and assists him in choosing a set for the next stage of the decision process. This second phase, choice between alternatives, can very rarely be suppressed by making the decision via interaction with the model. With these qualifications, however, interactive decision support systems are much more effective than analysts trying to prepare alternatives for the decision maker without his participation.

Thus, decision makers should be involved in the generation of alternatives; conversely, analysts should be involved in the decision making process. Although the choice between alternatives usually has some political character, this does not make it irrational; the analyst should try to understand the rationality of this phase and help to structure it. We should perhaps stress that we do not limit "rationality" to its traditional economic meaning; political processes have their own (mostly procedural) rationality, which arises from experience in making political and social decisions. The best example of procedural rationality is given by the procedures of evidence in courts of law and, more generally, by the rationality of law: this is built on long experience with methods of handling controversial evidence and social disputes. An analyst who understands the rationality of the underlying processes is in a better position to represent the substantive aspects of the problem.

Although there are several methods of multiobjective analysis that can help the analyst to clarify differences of opinion between experts [Keeney and Raiffa (1976)], or even to obtain consensus between decision makers [Rietveld (1980)], most of these methods are based on classical notions of economic rationality. A study of procedural rationality and its possible applications in multiobjective decision making would be an important complement to existing methods for multiobjective analysis.

2.2 The Role of Uncertainty: Normative Core and Procedural Belt in Policy Analysis

Before considering more prototype decision situations, we should perhaps discuss the role of uncertainty in decision making and its impact on planning and policy analysis.

The word 'uncertainty' has two meanings, one mathematical and the other empirical. Mathematically, uncertainty is usually understood in a probabilistic sense: it is represented by some a priori probability distribution which can be modified when additional information becomes available. The basic drawback of this representation is that a probabilistic model actually requires much more information than a nonprobabilistic one, because our assumptions about the probability distribution and its parameters must be validated experimentally. This drawback could be overcome by formulating subjective probability models; however, the question of empirical falsification then becomes even more critical. Other techniques that overcome this problem include simple interval characterization with subsequent interval analysis, and their extension by fuzzy set theory.

Empirically, uncertainty is a much broader concept. When building a model, the analyst might consciously neglect several factors that he considers to be either irrelevant or not sufficiently understood to be modelled. These neglected or unpredictable factors cannot necessarily be represented by a probabilistic model or even by interval characterization.

Before we can examine the effects of uncertainty on planning and policy analysis, we first have to consider what these terms actually mean. Most definitions of planning are in basic agreement [see, for example, Dror (1963)]: "planning is the process of preparing a set of decisions for action in the future, directed at achieving goals by preferable means". However, there is greater disagreement on the definition of policy. Ranney (1968) states that "policy is a course of action conceived as deliberately adopted, after a review of possible alternatives, and pursued or intended to be pursued", but many other definitions stress either

the political process of policy formulation or the implementation aspects. Nonetheless, there is a great similarity in the definitions of planning and policy. As a basis for discussion, therefore, we shall assume that planning is the process of policy formulation or policy specification (in the case when a higher-level policy is accepted as a basis for more detailed planning), while the concept of policy includes both formulation and implementation aspects.

To obtain a comprehensive definition of a policy, we will distinguish between two types of uncertainty: predictable uncertainty and unpredictable uncertainty. The first can be included in a model by probabilistic means, supported by empirical data, while the second should be understood in a pragmatic and semantic (rather than syntactic) sense: due to lack of empirical data, or because of model simplifications, we accept that there are aspects of the problem that cannot be predicted in the basic model that we intend to use for policy analysis. Having made this distinction, we can now define the various elements that comprise a policy (see Figure 2a).

The first of these elements is the substantive *content of policy*--selected knowledge about real situation (economic, ecological, technological, regional) addressed by the policy; the second is the political *process*--the institutional and sociopolitical aspects of policy formation and implementation. Both of these elements are included in the analysis only to a limited degree: both involve neglected, unpredictable or unknown factors as well as known or predictable factors. For this reason, the concept of policy also contains two other elements: a *normative core* and a *procedural belt*. The normative core includes everything that is known and predictable about the policy content and political process; the procedural belt describes *implementation procedures* for handling the neglected and unpredictable aspects.

While the concepts of policy content and political process are well-known in policy analysis, the concepts of the normative

core and procedural belt are new¹⁾ and require further discussion. There are many reasons for introducing these ideas: for example, the discussion on the merits of various planning approaches (blueprint versus process planning, etc.) is clearly related to the lack of any distinction between what we call the normative core and the procedural belt of a policy. Rational comprehensive planning is clearly concerned with the normative core aspects of a policy: set a goal and decide in general how to achieve it, assuming that the world will behave as predicted. However, if anything can go wrong, it will: some aspects are always neglected or unpredictable and must be dealt with by providing specific implementation procedures as well as general normative directions, and by authorizing a 'man on the spot' to deal with developing situations as he finds appropriate.

There are many areas of human activity in which much time is spent considering *what could go wrong* and in devising procedural responses, i.e., emphasis is on the procedural belt. For example, one of the lessons of the Three Mile Island nuclear reactor accident was that the operating procedures were not rich enough; another was that the personnel were not trained in various emergency actions. Consider the case of the shop owner who says "it is our policy not to accept cheques": it is clear that the common language interpretation of 'policy' includes the procedural belt and even concentrates on it. In economics, many widely disputed issues, such as the relative advantages of market and planned economies, are really related more to the robustness of the procedural belt than to the efficiency of the normative core. (However, because this distinction had not been made, and because there were no mathematical tools for investigating the procedural belt, it was tried unsuccessfully to settle these issues by investigating the normative core;) In control sciences, procedural belt issues correspond

¹⁾ These concepts were formulated by the author during discussions with Nino Majone at the International Institute for Applied Systems Analysis in early 1982, and are analogous to the concepts of a normative core and protective belt in scientific programs introduced by Lakatos (1978).

to the problems of stabilizing feedback systems, and these have been investigated quite widely. However, in only a few cases [e.g., Wierzbicki (1977)] is a mode of analysis adopted that could encompass both the normative core and the procedural belt.

Now, how can we investigate something that is unpredictable? In the same way that we train pilots: by imagining the most dangerous--if improbable--situations that can develop, and exposing the pilots to them on a flight simulator. In terms of building models for decision analysis, this approach would mean constructing *two models* (see Figure 2b): a *basic model* and an *extended model*. The first represents the known and predictable, while the second contains possible answers to the question: which of the aspects of reality neglected in the basic model could have the most negative impact on the implementation of the policy? It should be stressed that the extended model is not a better representation of reality, it is simply a different representation of reality, a falsification hypothesis constructed to check the robustness of the conclusions derived from the basic model. When checking this robustness, we would really like to know which implementation procedure to choose; there are usually many implementation procedures that are consistent with the course of action suggested in the basic model, but these procedures might give quite different results when applied to an extended model.

This framework immediately suggests several research questions. First, how should implementation procedures be generated? Second, how should the consistency of an implementation procedure with respect to the basic model (normative core) be characterized? Third, how should the robustness of an implementation procedure be defined operationally? The most natural definition would be the losses that result from the fact that the policy was devised using the basic model rather than the extended one. However, this might not be feasible, since it would involve deriving the normative policy for each extended model, and then comparing the results of applying two policies to the extended model (one policy should be derived from the basic model, with some implementation procedure, and the other derived from the extended model). If such simulation experiments are to be performed on

several extended models, the time necessary for robustness analysis might be excessive. This results in further questions: How could we make such a definition operational? How should we organize robustness analysis? Are there any mathematical methods that would enable us to compare the robustness of various implementation procedures without requiring many solutions for the extended model and the calculation of its normative policy cores?

It turns out that all these questions have an answer, at least for single-objective decision problems [Wierzbicki (1977), Snower and Wierzbicki (1982)]; whether these results can be extended to the multiobjective case remains uncertain.

For some models, particularly those of a probabilistic nature, the distinction between the normative and the procedural aspects of a policy can be less sharp. For example, if we have a stochastic process model we can derive the optimal feedback policy, which suggests that there is a unique best method of implementing the policy. We could go even further: assume a stochastic process model with some parameters that are not known a priori, and derive an adaptive optimal feedback policy, i.e., a procedure that both responds to perturbations and can learn by accumulating information [see Walters (1981) for an empirical application of this mathematical idea]. Surely this would be equivalent to a joint solution of the normative and procedural aspects of a policy, and, in this case, is the distinction really necessary?

In both of the above cases, we really assume predictability: the world will behave largely as we expect, although there may be some nasty stochastic effects and we cannot predict its behavior fully. There is no place here for really unpredictable events, no room for anything to go wrong. Thus, although these cases include some procedural features, they really lie in the normative core: the unique optimal feedback policy might turn out to be wrong if there was some unpredictable parameter change of a type not assumed in the basic model. This is a known paradox in control theory: the optimal stochastic feedback policy suggests proportional controller forms, although a great deal of experimental evidence shows that if we are to achieve robustness we must partly neglect optimality and adopt, for example,

proportional-integral controller forms. This implies a multi-objective approach: if there is a unique implementation procedure that is consistent with the normative core of a policy, we might accept a decrease in the normative efficiency of another procedure if it guarantees a substantial increase in robustness in unpredictable cases. Finally, we should stress that efficiency and robustness might not be the only objectives; another could be adaptability, the ability to learn from experience. Thus, we might try to design policies in a way that takes all three objectives into account.

After this discussion of the procedural belt and normative core, it would perhaps be useful to formulate an extended definition of policy. Policy is a course of action, assumed to include a basic normative direction and procedural implementation rules, which has been deliberately adopted after review of possible alternatives and assessment of predictable and unpredictable aspects of both substantive content and political process. This definition, together with the framework discussed above, still leaves many questions for research; however, it seems to be a constructive point of entry to many important problems. For example, the issue of 'process planning' can clearly be investigated in what we call the procedural belt of policy.

2.3 Second Prototype: Decisions of Independent Actors

Decisions are often made by independent actors (or 'players') who, bearing in mind the fact that the behavior of other actors might influence the final outcome, must choose whether to act independently or to agree on joint action with others. Typical examples are two nations negotiating trade agreements, or two regional authorities, one dealing with ecological protection, the other with industrial development.

This situation is typically represented by the prototype in Figure 3(a). However, although this prototype has been studied in some depth (see later sections), it is not a good representation of a typical decision situation since it assumes that decisions are prepared, evaluated and implemented directly

by the principal actors or decision makers. Much more realistic prototypes are illustrated in Figures 3(b) and 3(c). Here the decision analysis is performed by teams of analysts, possibly with senior analysts serving as links between the teams and the principal actors.

The situations in Figures 3(a), 3(b) and 3(c) may be greatly complicated by antagonism between the actors. Actors and analysts who have common goals or share a cultural background (whether it be political, disciplinary or whatever) can agree relatively quickly on some common model of the problem. They would share their substantive knowledge of the problem, although they may withhold, for strategic reasons, information about the political aspects or about their real goals. This strategic aspect of information is really the most important difference between the centralized situation, in which all information is assumed to be shared, and situations involving independent actors, in which any information given to other actors might change the outcome of the decision process.

In highly antagonistic situations it is possible that the teams of analysts cannot agree on a joint model of the substantive aspects of the problem, or do not want to exchange substantive information because even this might be too revealing. If a joint decision analysis is necessary in a situation where the actors come from completely different cultural backgrounds (not necessarily from different countries; I have observed that even economists from different political backgrounds understand each other better than, say, an economist and a lawyer from the same university), then a neutral mediator (see Figure 3d) has to be employed, even to assist in joint model building. Such mediations might result in a model that incorporates the models of all interested parties; however, the various parties may or may not agree to the mediator transferring information about their models to the other parties. (Clearly, a mediator could theoretically be corrupted by some party; but if his prestige and other benefits depend on the negotiations being successful, he has a strong incentive to remain neutral--if his bias were detected the negotiations might be broken off).

During joint decision analysis or actual negotiation, the role of a neutral mediator would be even more important. Empirical experience in negotiations [see, for example, Fisher and Ury (1981)] shows that, although the interested parties do not like to disclose their real interests to each other, a mediator often finds that their interests are not as antagonistic as they suspect, and that attractive compromises are possible. This empirical evidence contradicts the usual perceptions of antagonists, who tend to believe the worst of their opponents and view negotiations as a zero-sum game in which they should take hard positions and have a definite, single objective mind.

However, if life were really like this even the simplest negotiations over prices would almost always be unsuccessful. For, if both seller and buyer had the single objectives, say, of charging no less and paying no more than the market value, they could agree only on the current market price, without profit for either of them; there would be no reason for the general observation that both the buyer and the seller conclude the bargaining with a feeling of satisfaction. To explain this effect, it is necessary to assume that both sides are working with more than one objective. The buyer might want a present for his wife, he might have taken a fancy to the object in question, or he might be a collector who needs the object to complete his collection. The seller might not have had any customers that day, might have liquidity problems, or might want to renew his stock. Thus, there is not a single price, but a range of prices at which both sides would conclude the bargaining--the ritual of bargaining directs the price to this range by gradually disclosing the strength of interests on either side.

It should be pointed out that our analytical understanding of the multiobjective, multiparty decision situation is as yet rather poor (see later sections), and has begun to improve only recently [Raiffa (1982)]. Much work has yet to be done if we are to describe such situations analytically.

2.4 Third Prototype: Hierarchical Decisions

Although it has long been recognized that decisions are made within hierarchical structures, the prototype decision situations in which the hierarchy of decisions are investigated have until now been influenced more by the syntactic possibilities of the language of mathematics than by their semantic relevance. Two prototypes have received particular attention. The first assumes fully coordinated interests and single objectives, and is such that an upper-level decision maker can influence and modify the (single) objectives of various lower-level decision makers (Figure 4a), thus maximizing his own objective. The second prototype assumes shared information, noncoordinated interests and single objectives, and is such that an upper-level decision maker cannot influence the lower-level decision makers but is fully informed of their interests (single objectives); he can plan his moves to maximize his objective assuming that the lower-level decision makers make certain responses (see Figure 4b). The first prototype began with the Dantzig-Wolfe decomposition principle [see Dantzig and Wolfe (1960) and Findersen et al. (1980)], the second with the concept of Stackelberg equilibrium in game theory [see Stackelberg (1938) and Germeer (1976)]; both have since been the subject of very considerable theoretical interest with only limited success in applications.

Although there has been some attempt at hierarchical multi-objective analysis [see Seo and Sakawa (1980)] only limited attention has been paid to the analysis of useful prototype situations. If we assume full coordination as in the first hierarchical prototype (the hierarchical optimization prototype), we must also describe the means by which the upper-level decision maker influences the choices and preferences of the lower-level decision makers. It is questionable whether we could adopt the assumptions of the second hierarchical prototype (the hierarchical game prototype) without modifications, since the assumption that the higher-level decision maker has full information on the preferences of the lower-level, institutionally independent decision makers is not usually justified by empirical evidence. Much more research based on empirical falsification tests must

be done before we can formulate prototypes for hierarchical decision situations that are both realistic and mathematically tractable.

3. MATHEMATICAL CONCEPTS IN MULTIOBJECTIVE ANALYSIS

Even a short discussion of the mathematical foundations of multiobjective analysis would require a book rather than a short paper. Thus, we will not even attempt to explain these foundations here, just simply state that they are syntactically quite well developed. Instead, we will discuss the semantic usefulness of some of the basic concepts underlying these mathematical methods, and explain the possible syntactic difficulties of making these concepts more meaningful.

We start with the concept of Pareto optimality. A Pareto-optimal decision is one in which no objective or outcome of interest can be improved without worsening other outcomes of interest. Observe that this definition depends critically on the completeness of the list of outcomes of interest (objectives): if the list is incomplete, the 'best' decision may not be Pareto-optimal for the incomplete list, because we could worsen all the objectives on the list in order to improve an unlisted objective. This observation has two interpretations: one, tautological, is that any decision could be considered Pareto-optimal if we choose the objectives carefully enough; the second, empirical, is that we could, under certain additional assumptions, identify the unstated objectives of decision makers who prefer seemingly Pareto-inferior decisions. This empirical interpretation makes the concept of Pareto optimality richer and more useful, although more theoretical research is needed on the conditions under which unstated outcomes of interest can be identified.

The second basic concept is that of expressing preferences by utility functions. This concept, while very important syntactically [Debreu (1959)], has rather limited semantic usefulness: many empirical tests in mathematical psychology which have tried to identify the utility functions of human decision-makers have had very limited success [Tversky(1972)]. Some defenders of this concept try to use a tautological argument

similar to that concerning Pareto-optimality: people do behave as if they were maximizing a utility function, only this function may depend on more variables than we first thought. Since there is an infinite number of functions of various variables that could have a maximum at any chosen decision, this defence cannot be falsified, except in a very concrete situation when we try to identify the additional variables, postulate a limited class of utility functions, and run a specific falsification test; however, most of the known attempts to do this have given rather indeterminate results. We should rather try to accept the fact that utility functions are purely mathematical constructs, very useful whenever we can substantiate their use in a concrete case, but always demanding a careful empirical justification. Many related concepts in mathematical multiobjective analysis, such as the ideas of weighting coefficients and trade-off coefficients, are subject to the same qualification: while mathematically elegant and possibly useful for an analyst, they do not mean anything in applications until checked empirically. This point has been the subject of long and heated discussions: analysts who use certain mathematical concepts extensively are apt to believe that these concepts have some independent existence in the real world.

However, there have also been notable successes in developing alternative 'basic' concepts. The concept of 'satisficing' decision making [see Simon (1958)] assumes that people set up *aspiration levels* for various outcomes of interests, modify them as they accumulate more information, and then make decisions that satisfy or come close to these aspiration levels. Although substantiated by much empirical evidence, this concept generated only limited mathematical interest, and thus had only a limited impact on mathematical decision theory and mathematical psychology. However, many of the methods of multiobjective analysis, such as the displaced ideal point approach [Zeleny (1974)] and goal programming [Charnes and Cooper (1977)] have more or less consciously adopted this concept. A generalized approach that combines the satisficing and aspiration level concepts with mathematical optimization has been proposed by Wierzbicki (1980).

This approach concentrates on the construction of modified utility functions called *achievement functions* that express the utility or disutility of reaching or not reaching given aspiration levels. These aspiration levels are either formed by experience, or established by an accepted authority (say, when a wife gives a shopping list to her husband, or when a boss in a team-like organization proposes goals for his staff). This type of modified utility function is much more likely to be validated empirically than the classical, context-free utility function, since the specification of aspiration levels involves analysis of the variables of interest and of the problem; the achievement function is only used to measure deviations from the agreed aspiration point. However, this concept has not yet been tested empirically; it has been used more to define the success of an interactive decision support system in responding to the wishes of a user (see Figure 1e). This technique is often referred to as the 'reference point method' [see Grauer et al. (1982) and Grauer and Lewandowski (1982)].

3.1 Mathematical Tools for the Multiobjective Analysis of Centralized Decisions

There are a large number of mathematical tools based on multiobjective optimization that can be used for generating alternatives in the first stage of multiobjective analysis. The most advanced tools available are for the situation in which the underlying substantive model can be represented as linear program with many objectives, simply because there are many reliable codes for linear programming. [See Evans and Steuer (1973), Ecker and Kuada (1978), Yu and Zeleny (1975), Gal (1977, 1979), Gal and Leberling (1977, 1981) and Iserman (1974) for various approaches to multiobjective linear programming.] However, many of these methods can also be extended to nonlinear models or discrete optimization models provided a good nonlinear or discrete programming code is available [see, for example, Grauer et al. (1982)]. The main issues in using multiobjective optimization techniques to generate alternatives relate more to other aspects of the problem: the number of objectives and treatment of dynamic models, the way in which alternatives

are selected for presentation to decision makers, the way in which interaction is organized in the interactive decision support systems, and the way in which possible unlisted objectives are treated.

Non-interactive methods of generating alternatives cannot handle very many objectives, since the alternatives should in some sense cover the Pareto set which, in an n -dimensional objective space, is typically an $(n - 1)$ -dimensional manifold. Thus, the number of alternatives that in some sense represents the Pareto set grows exponentially, say as a^{n-1} , with the number of objectives. If we use dynamic models and concentrate on trajectories as outcomes or objectives, the number of objectives increases considerably (since each point on a trajectory is technically equivalent to an objective).

This is not the case in interactive methods, particularly those based on aspiration levels [see Zeleny (1976), Dyer (1972), Ignizio (1976), and Grauer et al. (1982)]. The reason for this is that in each interactive iteration, the user is presented with only a small number of alternatives which corresponds to the current aspiration levels. The number of objectives is then limited by the processing capabilities of the human mind-- established experimentally in psychology as between five and nine. Since the human mind processes 'by gestalt', these objectives may be numbers or trajectories (each theoretically of an unlimited number of points), so that dynamic models do not present any particular problems in the interactive mode. This observation [Wierzbicki (1980)] has resulted in the application of the achievement function method to many dynamic problems [Grauer et al. (1982), Grauer and Lewandowski (1982)].

The organization of interaction involves a number of issues: What information should the user contribute in an interactive system? What sort of questions should the system ask the user, and what sort of questions will the user ask the system? How far should the user be allowed to modify the computerized model? What should be done about unlisted objectives? Methods based on the utility concept assume that the user's utility function should be identified as far as possible by asking him questions

about pairwise comparisons of alternatives, trade-off coefficients, etc. [see Wallenius (1975) and Haimes et al. (1975)]. Methods based on aspiration levels assume that overt questions related to preferences and utility are not legitimate (while these concepts might be used technically we have no right to assume that users think in these terms), and ask instead how aspiration levels should be modified [see Zeleny (1976), Dyer (1972), Ignizio (1976), and Grauer et al. (1982)]. The issue of unlisted objectives can be approached by generating a Pareto-inferior alternative with each Pareto-optimal alternative (for example, by random modification of constraints, particularly those expressing resource availability). The user is then asked to state whether some aspects of the inferior alternative are attractive to him, and to try to express those aspects in terms of model variables. If an additional objective is already represented in the model, say, by a resource constraint, the user should have an easy way of including it in the list of objectives, for example, by reclassifying constraints as objectives.

The second stage of the decision process, the actual choice of alternatives, has also been studied intensively but only for quite specific cases. One problem that has received considerable theoretical and experimental attention is that of deriving the opinion of a group of experts using multiattribute utility methods. [Keeney and Raiffa (1976)]. While these methods are quite useful in analyzing the opinions of several experts, they are of only limited use in promoting actual agreement between the experts. Raiffa (1982) has published some new ideas on this subject only recently. Other methods for group decision making and for setting up choices between discrete alternatives have also been studied by Rietfeld (1980); however, most of those disregard the political aspects of decision making. These issues require further empirical, theoretical and mathematical study.

3.2 Mathematical Tools for the Analysis of Decisions of Independent Actors

The theory of games is an area of mathematics that has expanded very rapidly over the past thirty years. However, game theory has very rarely been applied to practical problems. The

same cannot be said of operational gaming, in which independent actors in conflict situations are supposed to make decisions and a computerized simulation model informs them of the overall outcome of their individual decisions. Some comparisons of game-theoretical and operational gaming approaches to empirical decision situations show that even very experienced actors seldom arrive at the solutions predicted by game theory [see Young et al. (1981)]. One possible reason for this phenomenon is that empirical conflict situations are seldom characterized by independent actors with single objectives. Some work has been done on multiobjective game theory [see, for example, Germeier (1976)], but this field is not as advanced as the multiobjective analysis of centralized decision making.

One possible development²⁾ in this area could be an interactive negotiation and mediation support system that attempts to model the prototype situations represented in Figures 3(b) and 3(d) by the structure illustrated in Figure 5. Two independent actors, users of the system, are assumed to indicate their general wishes in terms of aspiration levels for various variables of interest (these might be different for each actor). The aspiration levels are then used as parameters in achievement functions in two models of negotiating staffs. A noncooperative status quo is established in the first phase of interaction. This involves the computation of the Nash equilibrium [Nash (1950)] defined by the achievement functions of staffs for each specification of aspiration levels; this is then reported to the users, who should modify their aspiration levels until a status quo accepted by both sides is reached. Even in this first phase there is a need for a mediating procedure to try to lead the users to a status quo situation that they will both accept.

2)
Research on this possibility has recently been initiated at the International Institute for Applied Systems Analysis in Laxenburg, Austria. The scientists involved are the author of this paper, who suggested the prototype decision support system, Zenon Fortuna, who has developed the first computerized elements of this system, and Pradeep Dubey, who is working on related game theoretical-questions.

The second phase of interaction is concerned with finding a cooperative Kalai-Smorodinsky solution, starting from the non-cooperative status quo solution [see Kalai and Smorodinsky (1975)]. If each player had only a single objective there would be no room for negotiation once the mediator model had proposed a Kalai-Smorodinsky solution. However, in a multiobjective situation, the proposed Kalai-Smorodinsky solution can be improved for each user provided that he indicates which objectives can be allowed to deteriorate so that other objectives may be improved.

The interactive negotiation and mediation support system currently being developed at IIASA can be considered to be a product of a new approach to operational gaming, in which achievement functions and game equilibria are used to produce more realistic models of conflicts in decision making. Much algorithmic development and game-theoretical work still remains to be done. However, the prototype of the system shows that it is possible to combine methodological reflection on the practical requirements of decision-making situations with developments in game theory and multiobjective optimization. The author hopes that similar advances can also be achieved in hierarchical optimization and game theory, thus making them more useful for applied multiobjective analysis.

REFERENCES

- Aubin, J.P., 1979, Mathematical methods of game and economic theory (North-Holland, Amsterdam).
- Charnes, A. and W.W. Cooper, 1977, Goal programming and multiple objective optimization, European Journal of Operational Research 1, 39-59.
- Cohon, J.L., 1978, Multiobjective programming and planning (Academic Press, New York).
- Dantzig, G. and P. Wolfe, 1960, Decomposition principle for linear programs, Operations Research 8, 101-111.
- Debreu, G., 1959, Theory of value (Wiley, New York).
- Dror, Y., 1963, The planning process, a facet design, International Review of Administrative Science 29, 46-58.
- Dyer, J.S., 1972, Interactive goal programming, Management Science 19, 62-70.

- Ecker, J.G. and I.A. Kuada, 1978, Finding all efficient extreme points for multiple objective linear programs, *Mathematical Programming* 14, 249-261.
- Evans, J.P. and R.E. Steuer, 1973, A revised simplex method for linear multiple objective programs, *Mathematical Programming* 5, 54-73.
- Findeisen, W., F.N. Bailey, M. Brdys, K. Kalinowski, P. Tatjewski and A. Wozniak, 1980, Control and coordination in hierarchical systems (John Wiley, Chichester).
- Fisher, R. and W. Ury, 1981, Getting to YES: negotiating agreement without giving in (Houghton Mifflin, Cambridge, Mass.).
- Gal, T., 1977, A general method for determining the set of all efficient solutions to a linear vector maximum problem *European Journal of Operational Research* 1, 307-322.
- Gal, T., 1979, Parametric programming (McGraw-Hill, New York).
- Gal, T. and H. Leberling, 1977, Redundant objective functions in linear vector maximum problems and their determination, *European Journal of Operational Research* 1, 176-184.
- Gal, T. and H. Leberling, 1981, Relaxation analysis in linear vector valued maximization, *European Journal of Operational Research* 8, 274-282.
- Germeer, Yu.B., 1976, Games with non-opposing interests (in Russian) (Nauka, Moscow).
- Grauer, M., A. Lewandowski and L. Schrattenholzer, 1982, Use of the reference level approach for the generation of efficient energy supply strategies, WP-82-019 (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Grauer, M. and A. Lewandowski, 1982, The reference point optimization approach - methods of efficient implementation, WP-82-026 (International Institute for Applied Systems Analysis, Laxenburg, Austria).

- Haimes, Y.Y., W.A. Hall and H.T. Freedman, 1975, Multiobjective optimization in water resource systems (Elsevier, Amsterdam).
- Harrison, W., 1964, Policy, in: J. Gould and W.L. Kolb, eds., A dictionary of social sciences (Free Press of Glencoe, New York).
- Ignizio, J.P., 1976, Goal programming and extensions (Heath, Lexington).
- Iserman, H., 1974, Linear vectoroptimierung (Dissertation, University of Regensburg).
- Kalai, E. and M. Smorodinsky, 1975, Other solutions to Nash bargaining problem, *Econometrica* 43, 513-518.
- Keeney, R.L. and H. Raiffa, 1976, Decisions with multiple objectives; preferences and value trade-offs (Wiley, New York).
- Lakatos, I., 1978, Philosophical Papers, Vol. 1: The methodology of scientific research programs (Cambridge University Press, Cambridge).
- Leontief V., 1982, Academic economics, *Science* 21, 104-105.
- Lewandowski, A. and A.P. Wierzbicki, 1982, Modeling methodology: issues of model validity and credibility. A Collaborative Proceedings (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Nash, J.F., 1950, The bargaining problem, *Econometrica* 18, 155-162.
- Pareto, V., 1896, Course d'economie politique (Rouge, Lausanne).
- Popper, K.R., 1959, Logic of scientific discovery (Hutchinson, London).
- Raiffa, H., 1982, The art and science of negotiations (Harvard University Press, Cambridge, Mass.).
- Ranney, A., 1968, Political science and public policy (Markham P.C., Chicago).

- Rietveld, P., 1980, Multiple objective decision methods and regional planning (North-Holland, Amsterdam).
- Rose, M.R., 1982, Systems analysis as technological thought, in: A. Lewandowski and A. Wierzbicki, eds., Modeling methodology: issues of model validity and credibility (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Russell, B., 1927, An outline of philosophy (Allen and Unwin, London).
- Seo, F. and M. Sakawa, 1980, A methodology for environmental systems management: dynamic application of the nested Lagrangian multiplier method, RR-80-12 (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Simon, H., 1958, Administrative behavior (MacMillan, New York).
- Snower, D. and A.P. Wierzbicki, 1982, Simplistic modeling, imperfect information, and the robustness of policy rules, WP-82-24 (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Spronk, J., 1981, Interactive multiple goal programming; application to financial planning (Martinus Nijhoff, Boston).
- Stackelberg, H., 1938, Probleme der unvollkommenes Konkurrenz (Weltw Archivraum, Jena).
- Tvershy, A., 1972, Elimination by aspects; a theory of choice, Psychological Review 79, 281-299.
- Wallenius, J., 1975, Interactive multiple criteria decision methods (Helsinki School of Economics, Helsinki).
- Walters, C.J., 1981, Optimum escapements in the face of alternative recruitment hypotheses, Canadian Journal of Fisheries and Aquatic Sciences 38, 678-689.

- Wierzbicki, A.P., 1977, Models and sensitivity of control systems (in Polish) (WNT, Warsaw).
- Wierzbicki, A.P., 1980, A mathematical basis for satisficing decision making, WP-80-30 (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Wierzbicki, A.P., 1980, Multiobjective trajectory optimization and model semiregularization, WP-80-181 (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Young, P., N. Okada and T. Hashimoto, 1981, Sharing costs fairly: a practical introduction to cooperative game theory, Executive Report 5 (International Institute for Applied Systems Analysis, Laxenburg, Austria).
- Yu, P.L. and M. Zeleny, 1975, Set of all nondominated solutions in linear cases and a multicriteria simplex method, Journal of Mathematical Analysis and Applications 49, 430-468.
- Zeleny, M., 1974, Linear multi-objective programming (Springer, Berlin).
- Zeleny, M., 1976, The theory of displaced ideal, in: M. Zeleny ed., Multiple criteria decision making Kyoto (Springer, Berlin).

LEGENDS TO FIGURES

- Figure 1(a). The prototype decision situation usually considered in multiobjective analysis.
- Figure 1(b). A variant of the basic prototype (with senior analyst).
- Figure 1(c). A variant of the basic prototype (with several experts).
- Figure 1(d). A variant of the basic prototype (with a group of decision makers).
- Figure 1(e). The basic prototype for use of interactive models.
- Figure 2(a). Normative core and procedural belt aspects of the concept of policy.
- Figure 2(b). Framework for investigating the procedural belt.
- Figure 3(a). Second prototype: basic variant with independent actors.
- Figure 3(b). Second prototype: variant with teams of analysts providing decision support.
- Figure 3(c). Second prototype: variant with experts and teams of analysts providing decision support.
- Figure 3(d). Second prototype: variant with decision support and mediation.
- Figure 4(a). Hierarchical prototype with fully coordinated interests.
- Figure 4(b). Hierarchical prototype with shared information and non-coordinated interests.
- Figure 5. A prototype of an interactive negotiation and mediation support system.

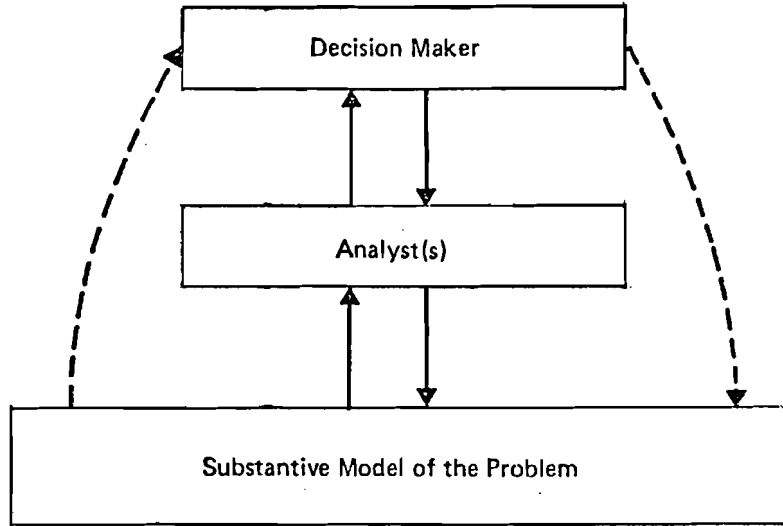


Figure 1(a). The prototype decision situation usually considered in multiobjective analysis.

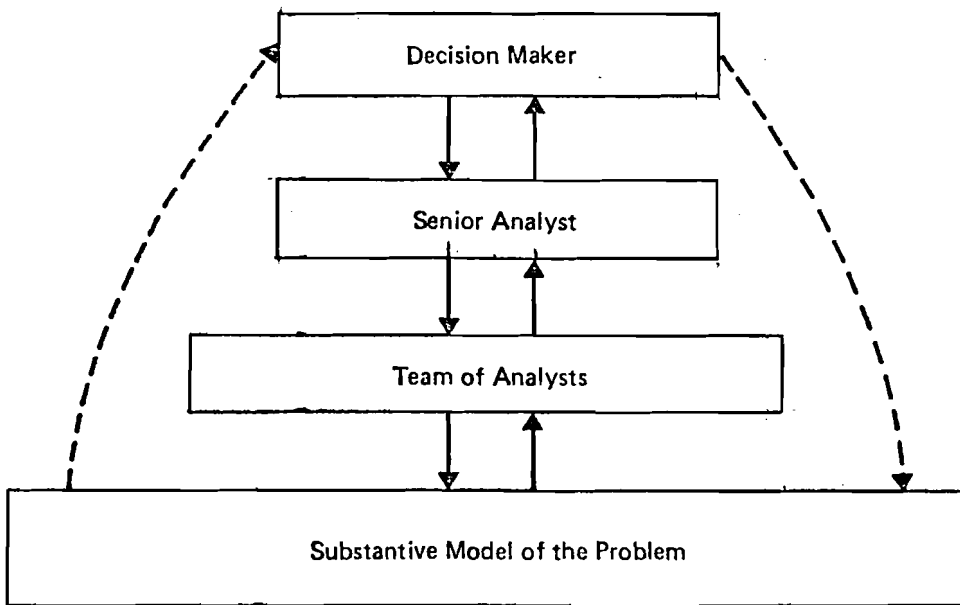


Figure 1(b). A variant of the basic prototype (with senior analyst).

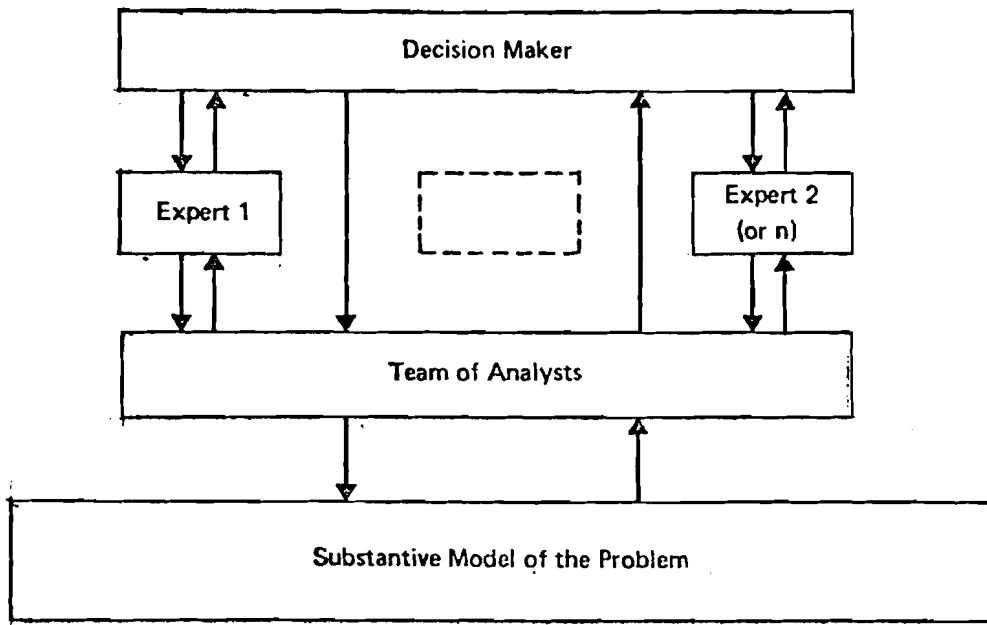


Figure 1(c). A variant of the basic prototype (with several experts).

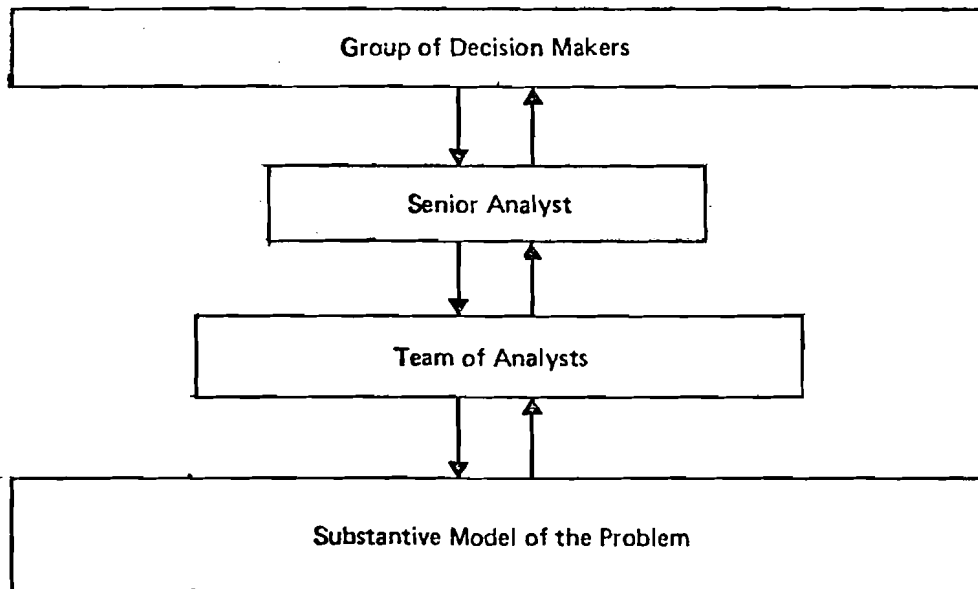


Figure 1(d). A variant of the basic prototype (with a group of decision makers).

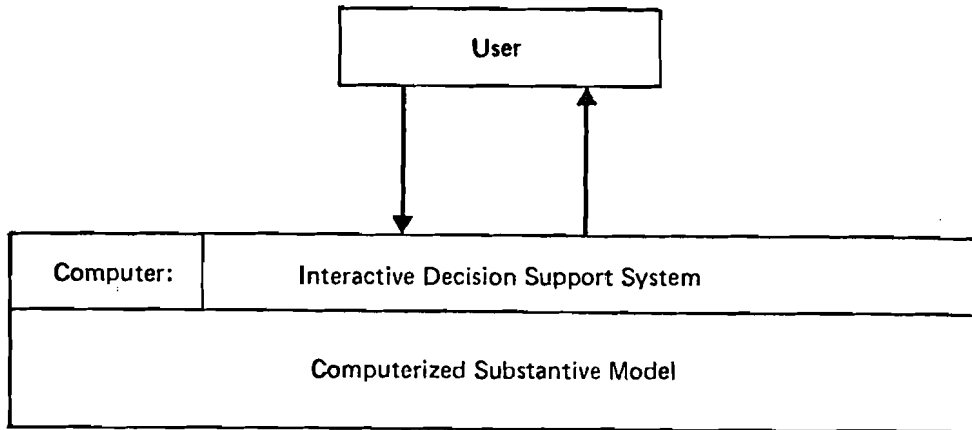


Figure 1(e). The basic prototype for use of interactive models.

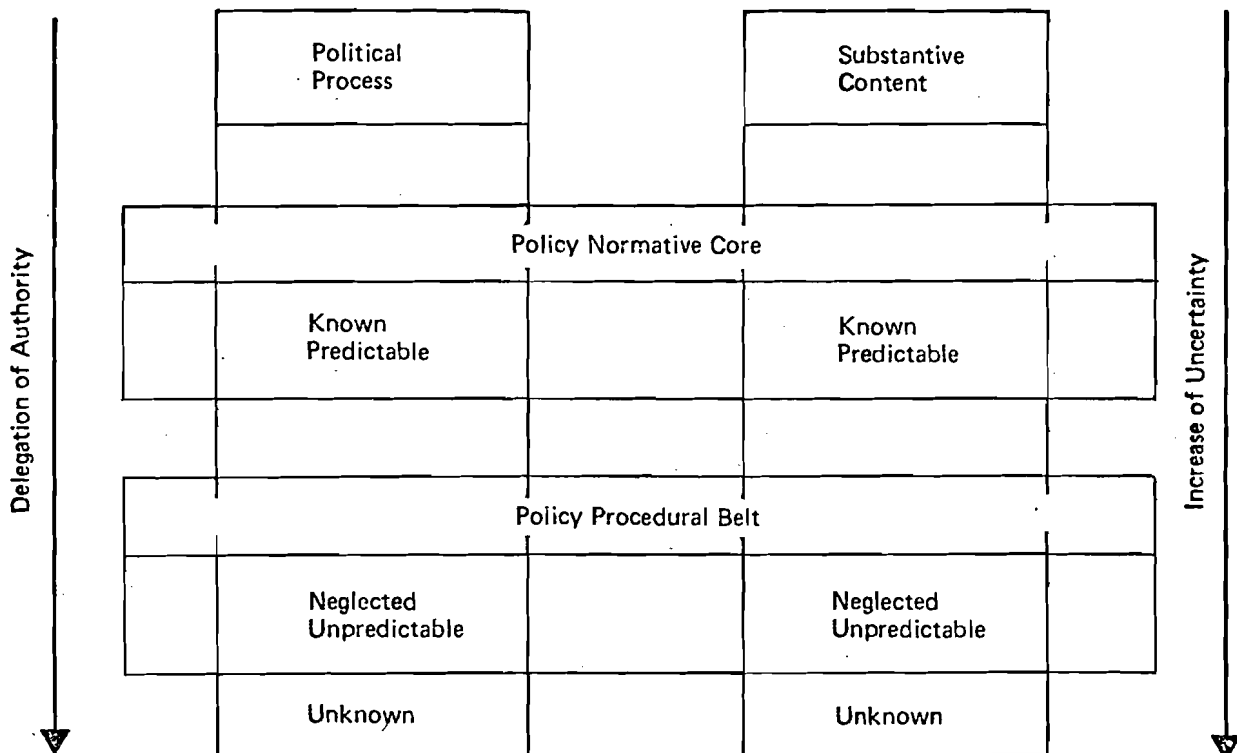


Figure 2(a). Normative core and procedural belt aspects of the concept of policy.

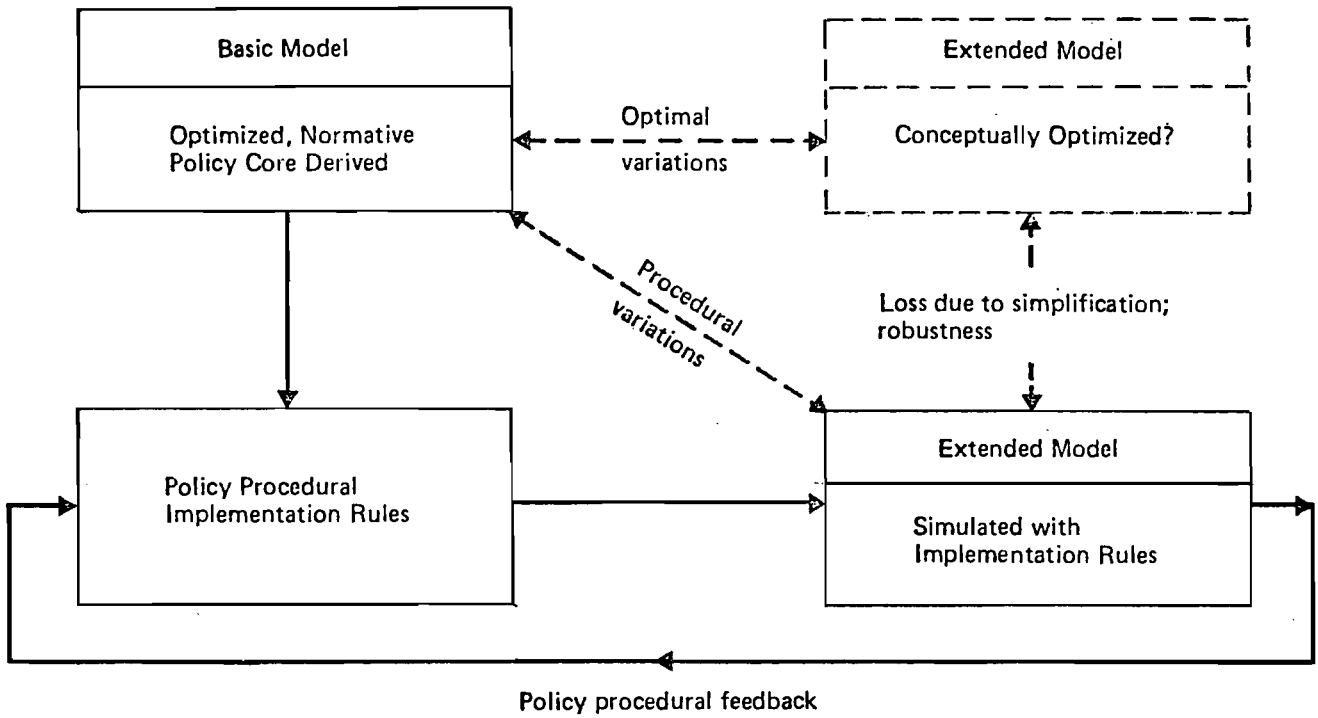


Figure 2(b). Framework for investigating the procedural belt.

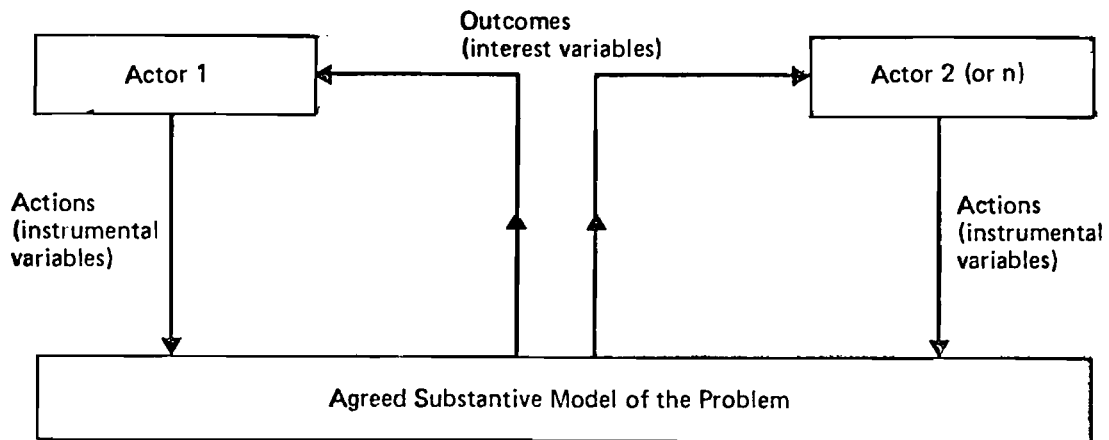


Figure 3(a). Second prototype: basic variant with independent actors.

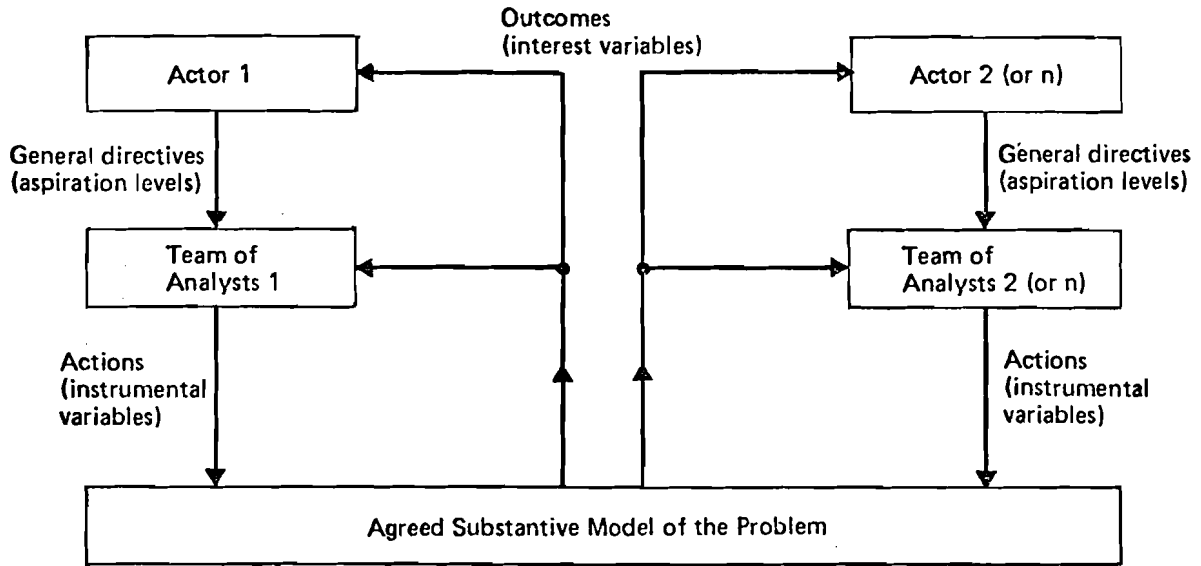


Figure 3(b). Second prototype: variant with teams of analysts providing decision support.

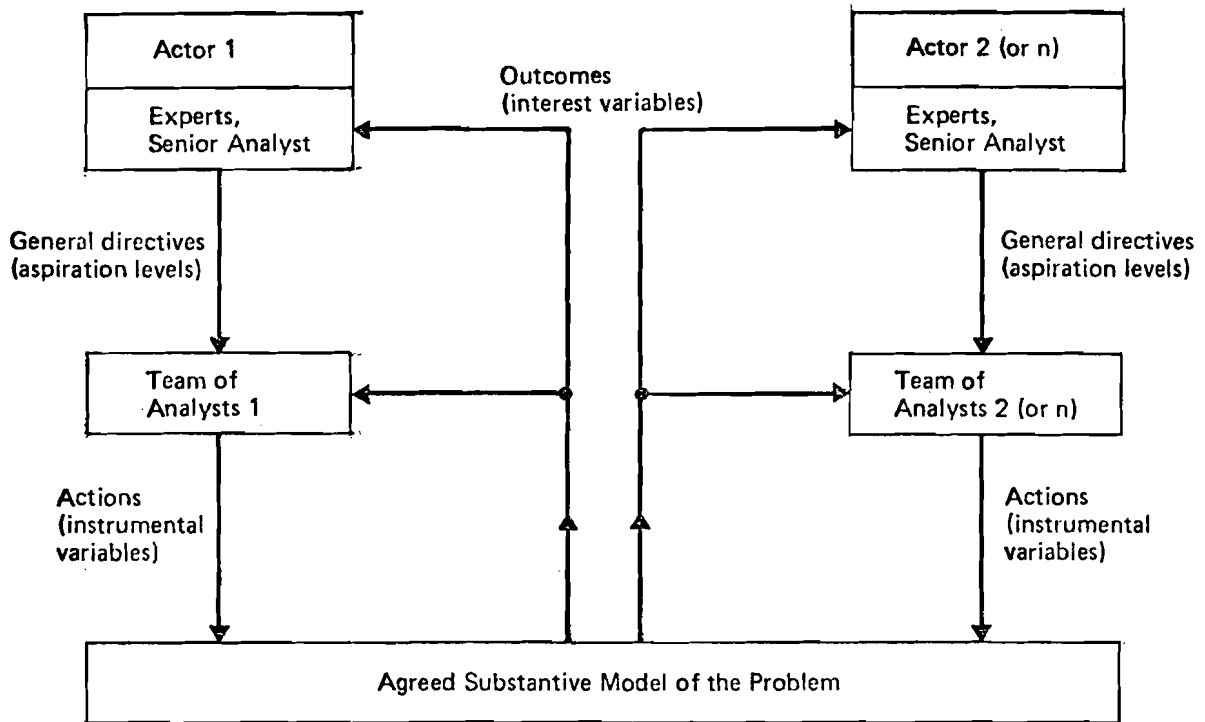


Figure 3(c). Second prototype: variant with experts and teams of analysts providing decision support.

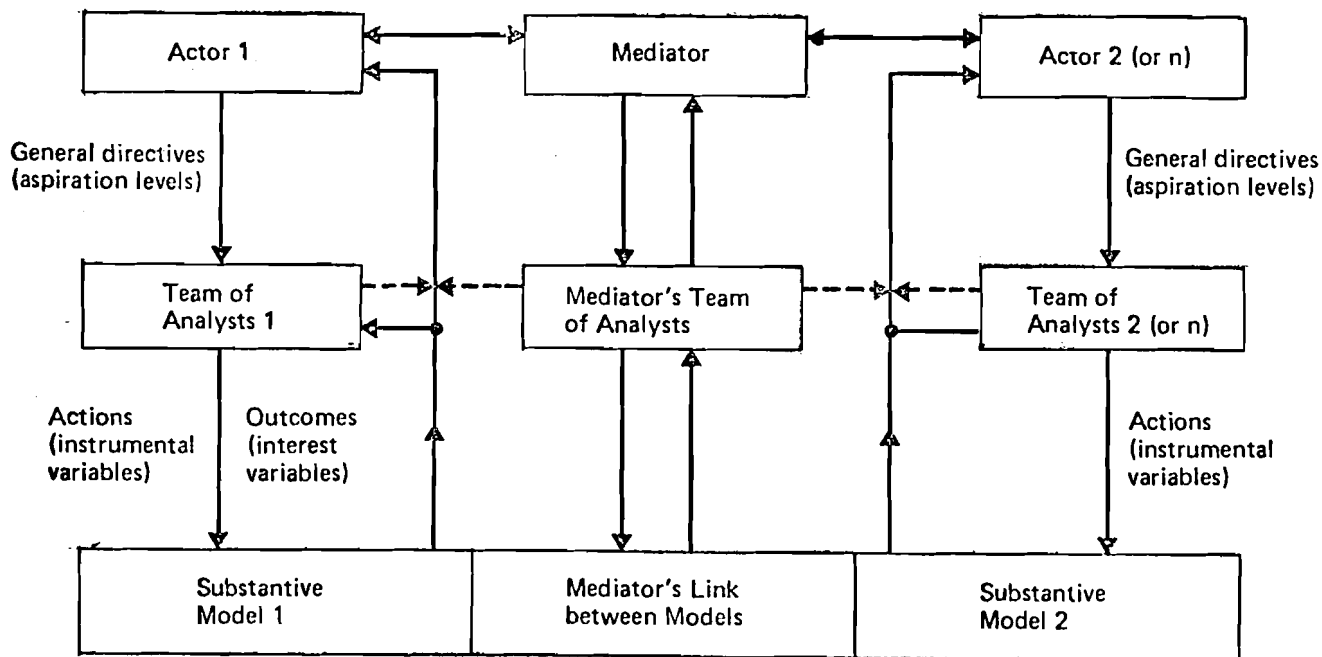


Figure 3 (d). Second prototype: variant with decision support and mediation.

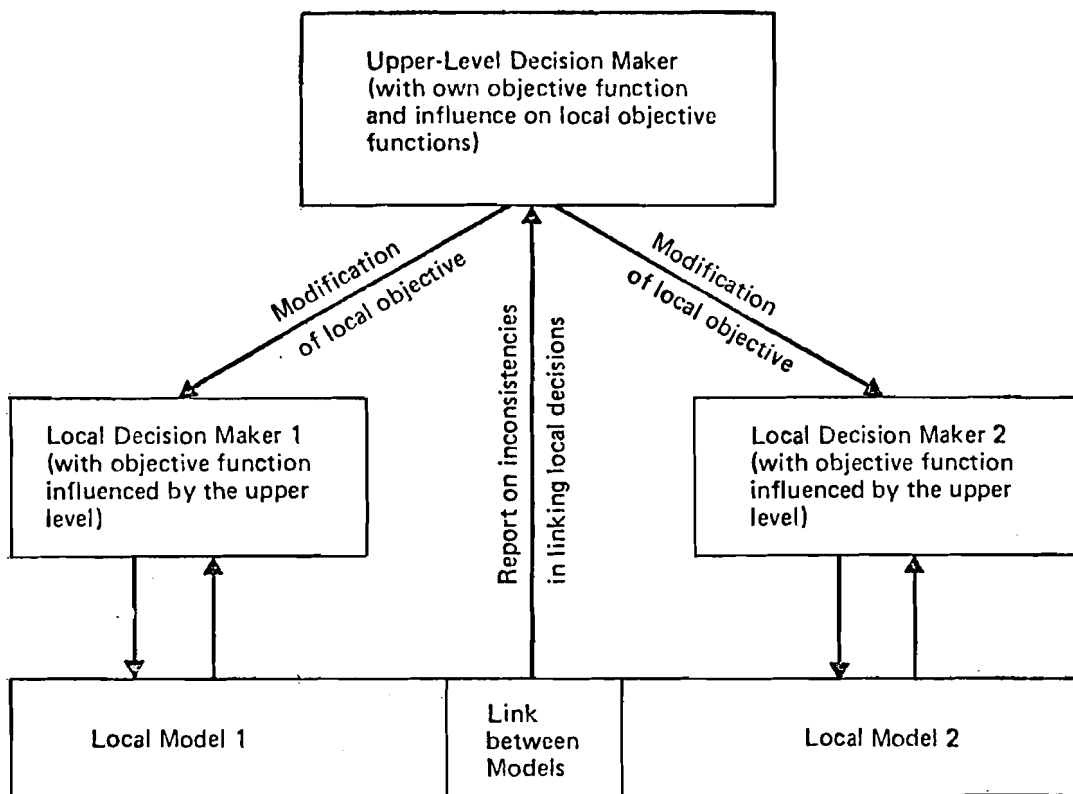


Figure 4 (a). Hierarchical prototype with fully coordinated interests.

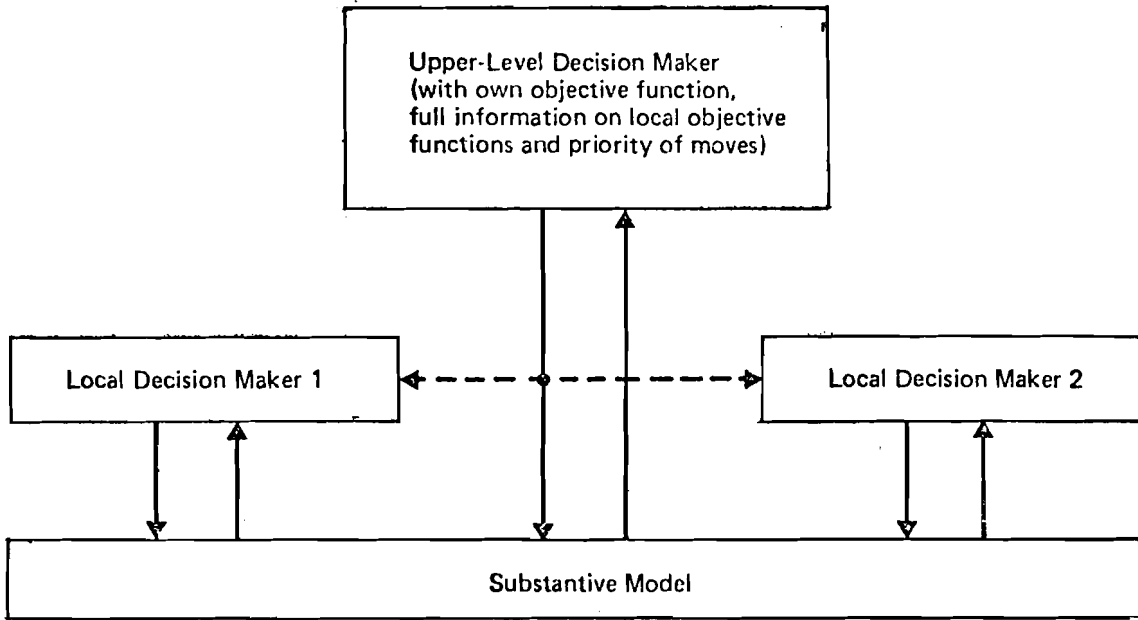


Figure 4(b). Hierarchical prototype with shared information and non-coordinated interests.

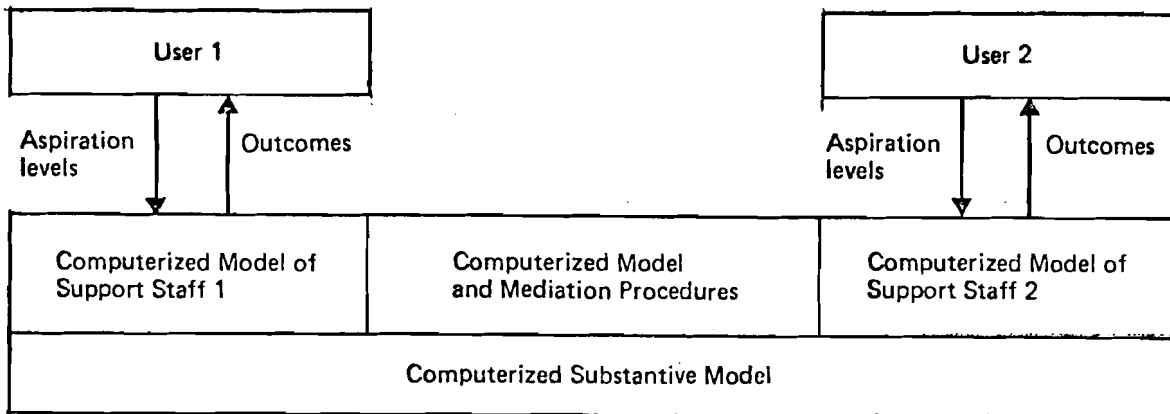


Figure 5. A prototype of an interactive negotiation and mediation support system.