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A BANK ASSET AND LIABILITY MANAGEMENT MODEL

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PREFACE

The area of asset management is rich in potential applications of stochastic programming techniques. This article develops a multiperiod stochastic programming model for bank asset and liability management, it shows that the results are far superior to those of a deterministic version of such a model. The algorithm used to solve the stochastic problem is part of the soft ware packages for stochastic optimization problems under development by the Adaptation and Optimization Task at IIASA.

Roger Wets

ABSTRACT

The uncertainty of a bank's cash flows, cost of funds and return on investments due to inherent factors and variable economic conditions has emphasized the need for greater efficiency in the management of asset and liabilities. A primary goal is to determine an optimal tradeoff between risk, return, and liquidity. In this paper we develop a multiperiod stochastic linear programming model (ALM) that includes the essential institutional, legal, financial, and bank related policy considerations, along with their uncertain aspects, yet is computationally tractable for realistic sized problems. A version of the model was developed for the Vancouver City Savings Credit Union for a five year planning period. The results indicate that ALM is theoretically and operationally superior to a corresponding deterministic linear prgramming model and the effort required for the implementation of ALM and the computational costs are comparable to those of the deterministic model. Moreover, the qualitative and quantitative characteristics of the solutions are sensitive to the stochastic elements of the model such as the asymmetry of the cash flow distributions. ALM was also compared with the stochastic decision tree (SDT) model developed by Bradley and Crane. ALM is more computationally tractable on realistic sized problems than SDT and simulation results indicate that ALM generates superior policies.

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1. INTRODUCTION

The inherent uncertainty of a bank's cash flows, cost of funds, and return on investments has emphasized the need for greater efficiency in the management of its assets and liabilities. This has led to a number of studies concerned with how one should structure a bank's assets and liabilities so that there are optimal tradeoffs among risk, return, and liquidity. These studies focus on the determination of the use of funds for deterministic and stochastic economic scenarios. Factors that must be considered in these decisions include: balancing of anticipated sources and uses of funds to meet liquidity and capital adequacy constraints while concurrently maximizing profitability [Chambers and Charnes (1961), Cohen and Hammer (1967)], allocating funds among assets based on risk and liquidity classification, maturity and rate of return [Bradley and Crane (1972, 1973, 1976)], and adjusting a bank's financial structure in terms of liquidity, capital adequacy and leverage [Chambers and Charnes (1961), Cohen and Hammer (1967)].

Current research has stressed two approaches. The first approach, based on Markowitz's (1959) theory of portfolio selection, assumes that returns are normally distributed and bank managers utilize risk-averse utility functions. The value of an asset then depends not only on the expectation and variance of its return but also on the covariance of its return with the returns of all other existing and potential investments. The second approach assumes that a bank seeks to maximize its future stream of profits (or expected profits) subject to portfolio mix constraints.

The most general example of the use of the first approach is Pyle (1971), where a static model is developed in which the financial intermediary (bank) can select the asset and liability levels to be maintained throughout the period. Pyle's analysis demonstrates the need for financial intermediaries. He only considers the risk of the portfolio and not other possible uncertainties. Trading activity, matching assets and liabilities, transactions costs, etc., are omitted from the model. It is possible to develop dynamic models using constructs along these lines, see, e.g., Kallberg and Ziemba (1981). However, given the severe computational difficulties due to the level of complexity of algorithms for these problems, it is not at present possible to develop useful operational models for large organizations such as banks.

Since our interest is in operational models we concentrate on the second approach which has theoretical and empirical support. Myers (1968) attempted to determine which criteria is most suitable for the asset and liability management problem by showing that: a necessary condition for the existence of security market equilibrium is risk independence; security market equilibrium implies risk independence of securities; and risk independence of investment opportunities implies that the maximization of the expected net present value is the appropriate objective criterion.

Thus, if, as is widely believed, a state of equilibrium exists for the securities which are held by financial institutions, and securities purchased do not have synergetic effect (implying the risk independence of securities) then the appropriate objective functions for a financial institution is the maximization of the expected net present value (ENPV). In a major empirical study Hester and Pierce (1975) used cross-sectional data to analyze the validity of a number of portfolio selection models in bank fund management. They concluded that banks can be well managed using models as a decision aid and that the best objective functions are either ENPV or the maximization of a two variable function where ENPV is dominant.

Asset and liability management models using an ENPV criteria fall in two broad categories: deterministic and stochastic. The deterministic models use linear programming, assume particular realizations for all random events, and are computationally tractable for large problems. These models have been accepted as a useful normative tool by the banking industry [Cohen and Hammer (1967)]. Stochastic models on the other hand have achieved very modest success. This is due to the inherent computational difficulties, the over-simplifications needed to achieve computational tractability, and the practitioners' unfamiliarity with their potential. The stochastic models included the use of the following techniques: chance-constrained programming; dynamic programming; sequential decision theoretic approach; and linear programming under uncertainty.

Essentially all of the deterministic models and many of the stochastic models follow the approach of Chambers and Charnes' (1961) linear programming model. They maximize net discounted returns subject to budget and liquidity constraints using the FRB's capital adequacy formulas, see Section 3 below. The literature contains several examples of successful applications of this model [Cohen and Hammer (1967), Komar (1971), and Lifson and Blackman (1973)]. However criticism continues to be leveled largely because of the omission of uncertainty in the model [Bradley and Crane (1976), Cohen and Thore (1970), and Eppen and Fama (1968)]. Probability distributions can be obtained for different economic scenarios and a linear programming formulation can be applied to each scenario to determine optimal solutions. However, this will not generate an optimal solution to the total problem but rather act as a deterministic simulation to observe portfolio behavior under various economic conditions. One must use care in defining such models as it may happen that no scenario leads to an optimal solution, see Birge (1982).

Charnes and Kirby (1965), Charnes and Littlechild (1968), Charnes and Thore (1966), and others developed chance-constrained models in which future deposits and loan repayments were expressed as joint normally distributed random variables and the capital adequacy formula was replaced by chance-constraints on meeting withdrawal claims. These approaches lead to a computationally feasible scheme for realistic situations, see e.g., Charnes, Gallegos and Yao (1982). However, the chance-constrained procedure does not have the facility to handle a differential penalty for either varying magnitudes of constraint violations or different types of constraints. Moreover, in multi-period models there are conceptual difficulties, as yet unresolved in the literature dealing with the treatment of infeasibility in periods 2,...,n, see, e.g., Eisner, Kaplan, and Soden (1971).

The second approach is dynamic programming. Eppen and Fama (1968, 1969, 1971) modelled two and three asset problems, and their work was extended by Daellenbach and Archer (1969) to include one liability. For a survey of this literature see Ziemba and Vickson (1975). The virtues of these models are that they are dynamic and take into account the inherent uncertainty of the problem. However, given the small number of financial instruments that can be analyzed simultaneously, they are of limited use in practice. See Daellenbach (1974) for estimates of possible gain using these models. For a recent survey of related applications in banking see Cohen, Maier and Van Der Weide (1981).

The third alternative, proposed by Wolf (1969) is a sequential decision theoretic approach which employs sequential decision analysis to find an optimal solution through the use of implicit enumeration. The difficulty with this technique is that it does not find an explicit optimal solution to problems with a time horizon beyond one period, because it is necessary to enumerate all possible portfolio strategies for periods preceding the present decision point in order to guarantee optimality. In an effort to explain away this drawback, Wolf makes the dubious assertion that the solution to a one period model would be equivalent to a solution provided by solving an n period model. This among other things ignores the problem of synchronizing the maturities of assets and liabilities. Bradley and Crane (1972, 1973, 1976) have developed a stochastic decision tree model that has many of the desirable features essential to an operational bank portfilio model. Their model is conceptually similar to Wolf's model; to overcome computational difficulties they reformulated the asset and liability problem and developed a general linear programming decomposition algorithm that minimizes the computational difficulties. This model is discussed in Section 5.

The fourth approach is stochastic linear programming with simple recourse (SLPSR) which is also called linear programming under uncertainty (LPUU). This technique explicitly characterizes each realization of the random variables by a constraint and leads to large problems in realistic situations. This handicapped modellers greatly; in fact Cohen and Thore (1970) viewed their model more as a tool for sensitivity analysis (in the aggregate) rather than a normative decision tool. The computational intractability and the perceptions of the formulation precluded consideration of problems other than those which were limited both in terms of time periods (Cohen and Thore used one and Crane (1971) use two) and in the number of variables and realizations. Booth (1972) applied this formulation by limiting the number of possible realizations and the number of variables considered in order to incorporate two time periods. Although relatively efficient solution algorithms existed for solving SLPSR's [Wets (1966)], these models were solved by using "extensive representation".

With the possible exception of the Bradley-Crane model none of the above mentioned models gives an adequate treatment of the essential features necessary for an adequate operational bank asset and liability management model that is computationally tractable. An ideal operational model should contain the following features:

1. multi-periodicity that incorporates: changing yield spreads across time, transaction costs associated with selling assets prior to maturity, and the synchronization of cash flows across time by matching maturity of assets with expected cash outflows;
2. simultaneous considerations of assets and liabilities to satisfy basic accounting principles and match the liquidity of assets and liabilities;
3. transaction costs that incorporate brokerage fees, and other expenses incurred in buying and selling securities;
4. uncertainty of cash flows that incorporates the uncertainty inherent in the depositers' withdrawal claims and deposits (The model must ensure that the structure of the asset portfolio is such that the capacity to meet these claims is maintained by the bank);
5. the incorporation of uncertain interest rates into the decision-making process to avoid lending and borrowing decisions which may ultimately be detrimental to the financial well-being of the bank; and
6. legal and policy constraints appropriate to the bank's operating environment.

In this paper we develop an SLPSR model that essentially captures these features of asset and liability management while maintaining computational feasibility. Some background concerning SLPSR models and the solution algorithm used appear in Section 2. The model ALM is described and formulated in Section 3. In Section 4 we apply ALM to the operations of the Vancouver City Savings Credit Union. Section 5 provides a comparison of ALM and Bradley and Crane's Model. Final remarks and conclusions appear in Section 6.

2. STOCHASTIC LINEAR PROGRAMS WITH SIMPLE RECOURSE

The basic (SLPSR) model is

$$\min_{x \geq 0} Z(x) \equiv c'x + E_{\xi}\left[\min_{y^+,y^- \geq 0} (q^{+'}y^+ + q^{-'}y^-)\right] \tag{1}$$

$$\text{s.t.} \quad Ax = b$$

$$Tx + Iy^+ - Iy^- = \xi$$

where $c,x \in R^n$, y^+, y^-, q^+, $q^- \in R^{m_2}$, A is $m_1 x n$, T is m_2 x n, I is a m_2-dimensional identity matrix and ξ is a m_2-dimensional random variable distributed independently of x on the probability space $(\Xi,\mathcal{F},F)$. The SLPSR model is the two stage process: choose a decision vector x, observe the random vector ξ then take the corrective action (y^+,y^-). The model is said to have simple recourse because the second stage minimization is fictitious since (y^+,y^-) are effectively unique functions of (x,ξ).

Beale (1955) and Dantzig (1955) independently proposed the SLPSR model as a special case of the general linear recourse model where Iy^+-Iy^- is replaced by Wy for a general matrix W. Detailed presentations of the theory of this model appear in Kall (1976), Parikh (1968), and Ziemba (1974). Assuming Ax = b, $x \geq 0$ has a solution x^0 and $q^+ + q^- \geq 0$, (1) has an optimal solution and is a separable convex program. If ξ is absolutely continuous then Z is differentiable and (1) may be solved using modifications of standard feasible direction algorithms, see, e.g., Wets (1966) and Ziemba (1974). If ξ has a finite distribution then Z is piecewise linear and (1) is equivalent to a large linear program. Wets (1974) noted that the deterministic equivalent linear program can be written in the form

$$\min c'x - \sum_{i=1}^{m_2} \sum_{\ell=1}^{k_i+1} (q_i^- - F_{i\ell} q_i) y_{i\ell} + \sum_{i=1}^{m_2} q_i^+ \bar{\xi}_i$$

$$\text{s.t.} \quad Ax = b, \; x \geq 0$$

$$\sum_{j=1}^{n} t_{ij}x_j - \sum_{\ell=1}^{k_i} y_{i\ell} = \alpha_i$$

$$y_{i1} \leq d_{i1}, \; 0 \leq y_{i\ell} \leq d_{i\ell}, \; 0 \leq y_{i,k_i+1}$$

where $i = 1,\ldots,m_2$, $\ell = 2,\ldots,k_i$, $d_{i1} = \xi_{i1}$, $d_{i\ell} = \xi_{i\ell} - \xi_{i,\ell-1}$, $q_i = q_i^+ + q_i^-$, $\xi_{i1}<\ldots<\xi_{ik_i}$ are the possible values of each ξ_i, the ith component of ξ, with probabilities $f_{i1},\ldots,f_{ik_i}$ and $F_{is} = \Pr(\xi_i < \xi_{is}) = \sum_{\ell=1}^{s-1} f_{i1}$.

It is possible to develop an algorithm using generalized upper bounding constructs that will solve (2) in a number of pivots that is of the same order of magnitude as the number of pivots required to solve the mean linear programming approximation problem, i.e., (1) with ξ replacing $\bar{\xi}$. The linear program (2) has the same number of working basis elements, (m_1+m_2) as the mean problem. Wets (1974, 1983a) has developed an algorithm that has been coded by Collins (1975), Kallberg and Kusy (1976). The code was written to solve problems with up to 70 stochastic constraints, 220 total constraints and 8 realizations per random element. The code can be expanded to solve much larger problems. The development of more sophisticated codes to handle larger problems is currently being undertaken at IIASA. See Wets (1983b) for extension of his algorithm to the convex case.

The formulation (1) is essentially static while the asset and liability management problem is dynamic. We utilize the model (2) and its efficient computational scheme while at the same time retaining as many of the dynamic aspects of the model as possible. To do this we utilize the approximation described below. The general n-stage SLPSR problem is

$$\min_{\substack{x^1 \geq 0 \\ Ax = b^1}} c^{1'} x^1 + E_{\xi^1} \left\{ \min_{y^{1+}, y^{1-} \geq 0} [q^{1+'} y^{1+} + q^{1-'} y^{1-} + \ldots + \right.$$

$$\left. + \min_{\substack{x^n \geq 0 \\ A^n x^n = b^n}} [c^{n'} x^n + E_{\xi^n | \xi^{n-1}, \ldots, \xi^1} \left\{ \min_{y^{n+}, y^{n-} \geq 0} [q^{n+'} y^{n+} + q^{n-'} y^{n-}] \right\}] \ldots] \right\}$$

$$\text{s.t.} \sum_{j=1}^{i} T_{ij} x^i + I y^{i+} - I y^{i-} = \xi^i, \; i=1,\ldots,n. \tag{3}$$

The approximation procedure aggregrates $x^2,\ldots,x^n$ in with x^1 and $\xi^2,\ldots,\xi^n$ with ξ^1. Thus in (1) one chooses $x \equiv (x^1,\ldots,x^n)'$ in stage one, observes $\xi \equiv (\xi^1,\ldots,\xi^n)'$ at the end of stage one and these together determine $(y^+,y^-) \equiv [(y^{1+},y^{1-}),\ldots,(y^{n+},y^{n-})]$ in stage two. This approach yields a feasible procedure for the true dynamic model (3) that is computationally feasible for large problems and incorporates partial dynamic aspects since penalty costs for periods $2,\ldots,n$ are considered in the choice of $x^1,\ldots,x^n$. The decision maker is primarily interested in the immediate revision of the bank's assets and liabilities. The ALM model incorporates immediate revision by setting times 0 and 1 an arbitrarily small time period apart. Point 0 refers to the bank's initial position and point 1 refers to the bank's position immediately after running the model. In practice the model is rolled over continuously. Also to partially overcome the drawbacks of a static solution technique the decision variables are defined so that a security can be purchased in one time period and sold in one or more subsequent periods.

The recourse aspect of the model gives it a dynamic flavour. The model is two-stage: initially the decision variables are chosen, next the stochastic variables are observed and this determines the recourse variables (in order to recover feasibility) and their corresponding penalties. The penalty is a function of both the constraint violated and the magnitude of violation. The recourse cost has the effect of restraining "aggressive" choices of decision variables if the costs involved with regaining feasibility outweigh the benefits. Thus, the rolling over of the ALM model, defining the variables to give them flexibility and the recourse aspect of SLPSR, are the dynamic features of the ALM model.

3. FORMULATION OF THE ALM MODEL

The asset and liability management (ALM) model is an intertemporal decision-making optimization tool to determine a bank's portfolio of assets and liabilities given deterministic rates of returns and cost (interest rates), and random cash flows (deposits). Although the asset and liability management problem is a continuous decision problem as portfolios are constantly being revised over time, the computations and analysis involved with a continuous time process are infeasible for a normative tool. Therefore, the ALM model is developed as a multi-period decision problem in which portfolios are determined at discrete points in time (e.g., the end of each accounting period).

The ALM model has the following features:

1. Objective function:

   maximize the net present value of bank profits minus the expected penalty costs for infeasibility.

2. Constraints:

   a. legal, being a function of the bank's jurisdiction,

   b. budget: initial conditions and the sources and uses of funds,

   c. liquidity and leverage, to satisfy deposit withdrawals on demand, (the FRB's capital adequacy formula is the basis of these constraints),

d. policy and termination: constraints unique to the bank and conditions to ensure the bank's continuing existence after the termination of the model, and

e. deposit flows.

Constraints (a) and (b) are deterministic, (c) consists of both deterministic and stochastic constraints, (d) can consist of either deterministic or stochastic constraints, and (e) contains only stochastic constraints.

Chambers and Charnes (1961) and Cohen and Hammer (1967) have justified the use of linear functions to model a bank's asset and liability management problem. Thus from the point of view of linearity, the appropriateness of SLPSR is established. The recourse aspect is justified with the following argument. In the banking business, constraint violations do not imply that the intermediary is put into receivership. Rather the bank is allowed to restructure its portfolio of assets to regain feasibility at some cost (penalties). With the inherent uncertainties the asset and liability management problem is well modeled as a stochastic linear program with simple recourse.

3.1 Notation for the ALM Model

x_{ij}^{k} - amount of asset k purchased in period i sold in period j; $k=1,\ldots,K$; $i=0,\ldots,n-1$; $j=i+1,\ldots,n$

x_{00}^{k} - initial holdings of security k

$x_{i\infty}^{k}$ - amount of security k purchased in period i to be held beyond the horizon of the model

y_{i}^{d} - new deposits of type d in period i; $d=1,\ldots,D$

y_{o}^{d} - initial holdings of deposit type d

b_i - funds borrowed in period i

y_{js}^{+} - shortage in period j in stochastic constraint s

y^-_{js} - surplus in period j in stochasic constraint s

p^+_{js} - proportional penalty cost associated with y^+_{js}

p^-_{js} - proportional penalty cost associated with y^-_{js}

β^k_{ij} - parameter for shrinkage, under normal economic conditions, in period j of asset type k purchased in period i

α^k_{ij} - parameter for shrinkage, under severe economic conditions, in period j of asset type k purchased in period i

t^k_i - proportional transaction cost on asset k, which is either purchased or sold in period i

r^k_i - return on asset k purchased in period i

T_j - tax rate on capital gains (losses) in period j

τ_j - marginal tax rate on income in period j

z^k_{ij} - proportional capital gain (loss) on security k purchased in period i and sold in period j

γ_d - the anticipated fraction of deposits of type d withdrawn under adverse economic conditions

c^d_i - rate paid on deposits of type d

P_i - discount rate from period i to period 0

K_o - set of possible current assets as specified by the British Columbia Credit Union Act

K_1 - set of primary and secondary assets as defined in the capital adequacy formula (caf)

K_2 - set of minimum risk assets as defined in the caf

K_3 - set of intermediate risk assets as defined in the caf

q_i - penalty rate for the potential withdrawal of funds, in period i, which are not covered by assets in $K_1 \cup \ldots \cup K_3$

P_i - liquidity reserves for the potential withdrawal of funds, in period i, not covered by assets in $K_1 \cup \ldots \cup K_3$

k_{m_i} - m_i-th mortgage

ξ_{js} - discrete random variable in period j for stochastic constraint type s, $s \in S$ where S is the set of stochastic contraints.

3.2 The ALM Model

$$\max_{x,y,b} \sum_{k=1}^{K} \Bigg[\sum_{j=2}^{n} x_{0j}^k \Big\{ \sum_{\ell=2}^{j} r_o^k (1-\tau_\ell) p_\ell + z_{oj}^k (1-T_j) p_j \Big\} + x_{01}^k z_{01} (1-T_j) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} x_{ij}^k \Big\{ \sum_{\ell=i+1}^{j} r_i^k (1+\tau_\ell) p_\ell + z_{ij}^k (1-T_j) p_j \Big\} + x_{0\infty}^k \sum_{\ell=2} r_0^k (1-T_\ell) p_\ell \Bigg] + \sum_{j=\tau_1}^{n} \sum_{\ell=i+1}^{n} x_{i\infty}^k r_i^k (1-\tau_\ell) p_\ell$$

discounted returns and capital gains (net of taxes) on assets

$$- \sum_{d=1}^{i=1} \Bigg[\sum_{j=1} y_0^d (1-\gamma_d/2)(1-\gamma_d)^{j-1} c_j^d p_j + \sum_{j=1}^{n} 1/2 y_j^d c_j^d p_j + \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} y_i^d (1-\gamma_d/2)(1-\gamma_d)^{j-i} c_j^d p_j \Bigg]$$

net discounted cost of deposits (demand and time)

$$- b_0 c_0^b p_1 - \sum_{j=1}^{n} b_j c_j^b p_{j+1}$$

cost of direct borrowing from other banks and a central bank

$$- E_\xi \min_{y^+, y^-} \sum_{j=1}^{n} \sum_{s \in S} p_{js}^+ y_{js}^+ + p_{js}^- y_{js}^-$$

expected penalty costs for constraint violations

Subject to:

(a) Legal constraints

$$\sum_{k \in K_0} \sum_{i=0}^{1} \Big\{ \sum_{\ell=2}^{n} x_{i\ell}^k + x_{i\infty}^k \Big\} - .1 \Bigg[\sum_{d=1}^{D} y_0^d + (1-\gamma_d) y_0^d /2 + y_1^d/2 + b_0 + b_1 \Bigg] > 0 \qquad j = 1$$

$$\sum_{k\varepsilon K_0} \sum_{i=0}^{j} \left\{ \sum_{\ell=j+1}^{n} x_{i\ell}^{k} + x_{i\infty}^{k} \right\} - .1 \left[\sum_{d=1}^{D} \left\{ \sum_{i=0}^{j-1} y_i^d + (1-\gamma_d) y_0^d /2 \ (1 - \gamma_d)^{j-i-1} \right.\right.$$

$$\left.\left. + y_1^d/2 \right\} + b_j \right] \geq 0, \qquad j = 2,\ldots,n$$

(b) Budget constraints

i. Initial holdings

$$\sum_{j=1}^{n} x_{0j}^{k} + x_{0\infty}^{k} = x_{00}^{k}, \qquad k = 1,\ldots K$$

$$y_0^d = y_{00}^d, \qquad d = 1,\ldots,D$$

ii. Sources and uses

$$\sum_{k=1}^{K} \left[\sum_{\ell=2}^{n} \left\{ x_{1\ell}^{k} + x_{1\infty}^{k} \right\} \left\{ 1 + t_1^k \right\} - x_{01}^{k} \left\{ 1 + z_{01}^{k} (1-T_1) \right.\right.$$

$$\left.\left. - t_1^k (1 + z_{01}^k) \right\} \right] + \sum_{d=1}^{D} \left[\gamma_d y^d_{0/2} - y_1^d/2 \right] - b_1 = 0, \qquad j = 1$$

$$\sum_{k=1}^{K} \left\{ \sum_{\ell=j+1}^{n} x_{j\ell}^{k} + x_{j\infty}^{k} \right\} \left\{ 1 + t_j^k \right\} - \sum_{i=0}^{j-1} \left[\left\{ \sum_{\ell=j}^{n} x_{i\ell}^{k} + x_{i\infty}^{k} \right\} \right.$$

$$\left. \cdot \left\{ r_i^k (1 - \gamma_j) \right\} + x_{ij}^{k} \left\{ 1 + z_{ij}^{k} (1-T_j) - t_j^k (1 + z_{ij}^k) \right\} \right]$$

$$- \sum_{d=1}^{D} \left[\sum_{i=0}^{j-2} y_i^d (1 - \gamma_d)^{j-i-2} \left\{ (-\gamma_d)(1 - \gamma_d/2) \right\} \right.$$

$$\left. + (1 - \gamma_d) y_{j-1}^d/2 + y_j^d/2 - (1 - \gamma_d/2) c_{j-1}^d - y_{j-1}^d c_{j-1}^d \ /2 \right]$$

$$+ b_{j-1} (1 + c_{j-1}^b) - b_j = 0, \qquad j = 2,\ldots,n$$

(c) Liquidity constraints

$$\text{i. } - \sum_{k\varepsilon K} \sum_{i=0}^{j} \left[\sum_{\ell=j+1}^{n} x_{i\ell}^{k} \alpha_{ij}^{k} + x_{i\infty}^{k} \alpha_{ij}^{k} \right] + b_j$$

$$+ \sum_{d=1}^{D} \gamma_d \left[\sum_{i=0}^{j-1} y_i^d (1 - \gamma_d)^{j-i-1} \ (1 - \gamma_d/2) + y_j^d/2 \right] \leq P_{1j}/q_{ij}$$

$$\text{ii.} \; - \sum_{k\varepsilon K_1 \cup K_2} \sum_{i=0}^{j} \left[\sum_{\ell=j+1}^{n} x_{i\ell}^{k} \alpha_{ij}^{k} + x_{i\infty}^{k} \alpha_{ij}^{k} \right] + b_j$$

$$+ \sum_{d=1}^{D} \gamma_d \left[\sum_{i=0}^{j-1} y_i^d (1 - \gamma_d)^{j-i-1} (1 - \gamma_d/2) + y_j^d/2 \right] \leq P_{2j}/q_{ij}$$

$$\text{iii.} \; - \sum_{k\varepsilon K_1 \cup K_2 \cup K_3} \sum_{i=0}^{j} \left[\sum_{\ell=j+1}^{n} x_{i\ell}^{k} \alpha_{ij}^{k} + x_{i\infty}^{k} \alpha_{ij}^{k} \right] + b_j$$

$$+ \sum_{d=1}^{D} \gamma_d \left[\sum_{i=0}^{j-1} y_i^d (1 - \gamma_d)^{j-i-1} (1 - \gamma_d/2) + y_j^d/2 \right] \leq P_{3j}/q_{ij}$$

$$\text{iv.} \; - \sum_{k=1}^{K} \sum_{i=0}^{j} \left[\sum_{\ell=j+1}^{n} (1 - \beta_{ij}^{k}) x_{i\ell}^{k} + (1 - \beta_{ij}^{k}) x_{i\infty}^{k} \right]$$

$$+ y_{js}^{+} - y_{js}^{-} \geq P_{1j} + P_{2j} + P_{3j} + b_j$$

$$+ \sum_{d=1}^{D} \left[\sum_{i=0}^{j-1} y_i^d (1 - \gamma_d)^{j-i-1} (1 - \gamma_d/2) + y_j^d/2 \right] + \xi_{js}$$

$$j = 1, \ldots, n, \; s\varepsilon S$$

(d) Policy constraints

$$- .1 \sum_{i=0}^{j} \left[\sum_{\ell=j+1}^{n} x_{i\ell}^{k\,m1} + x_{i\infty}^{k\,m1} \right] + \sum_{i=0}^{j} \left[\sum_{\ell=j+1}^{n} x_{i\ell}^{k\,m2} + x_{i\infty}^{k\,m2} \right]$$

$$+ y_{js}^{+} - y_{js}^{-} \leq \xi_{js} \qquad j = 1, \ldots, n, \; s\varepsilon S$$

(e) Deposit flows

$$y_j^d + \sum_{i=0}^{j-1} y_i^d (1 - \gamma_d)^{j-i} + y_{js}^{+} - y_{js}^{-} = \xi_{js}$$

$$j = 1, \ldots, n; \; d = 1, \ldots D, \; s\varepsilon S$$

(f) Nonnegativity

$$x_{ij}^{k},\; b_i,\; y_i^d,\; y_{js}^{+}, y_{js}^{-} \geq 0 \text{ for all } i,j,k,d$$

There are no discount factors incorporated into the constraints since each constraint refers to conditions in only one period. The ALM model treats the first two types of constraints, legal and budget, as deterministic. The legal constraint states that the current assets cannot be less than 10% of the total liabilities less reserves, surplus and equity [as defined by the British Columbia Credit Union Act (British Columbia Government, 1973)]. The legal constraints are, of course, peculiar to the locale of the institution being studied. The budget constraints include the initial conditions and the accounting identity--uses and sources of funds are equal.

The liquidity constraints follow from the Federal Reserve Board's capital adequacy formula (caf). The requirement that the market value of a bank's assets is adequate to meet depositor's withdrawal claims during adverse economic conditions is the principal constraint in the caf. To develop this constraint, liquidity reserves (for adverse economic conditions) are first defined. The first three liquidity constraints are

$$P_i \geq q_i W - \sum_{k \in K_1 \cup \ldots \cup K_i} \alpha_k, \qquad i = 1,2,3.$$

The principal constraint of the caf is

$$\sum_{i=1}^{K} (1-\beta_i)x_i \geq \sum_{i=1}^{3} P_i + \left\{\begin{array}{l}\text{total right hand}\\ \text{side of balance-surplus-equity}\\ \text{sheet}\end{array}\right. .$$

Thus the market value of the bank's assets should be not less than the liquidity reserves for disintermediation under severe economic conditions plus liabilities. This constraint is the final liquidity constraint in ALM. Although this constraint is not stochastic, a bank portfolio manager may violate it because the caf set forth by the FRB is a suggested guideline for sound bank management rather than a strict regulation. The penalty for a violations of this constraint is $\sum_{i=1}^{3} q_i$ (as prescribed by the FRB). This **elastic** treatment of FRB's regulation allows the constraint to be violated when the benefits of violation exceed the costs. In this manner, the criticism, levelled at modellers using FRB's conservative constraints, can be resolved in a systematic manner. See Section 4.1.3 for more discussion concerning these constraints.

The fourth set of constraints is also **elastic.** These constraints are introduced to capture the internal operational policy of the institution modelled. In reality minor constraint violations of bank policies are usually tolerable while more severe violations are increasingly less tolerable. The introduction of a piece-wise linear convex penalty function (via additional constraints) can capture the dependency between the penalty costs and the extent of the policy violations. This is accomplished by the addition of supplementary constraints to reflect the increased seriousness of the magnitude of constraint violations.

The final set of constraints, deposit flows, is stochastic. Since deposit flows are continually turned over and bear various rates of interest the model has to reflect the gross (and not net) flows during an accounting period. This property of the problem was incorporated in the model by having a proportional outflow [statistically calculated by the FRB and corroborated for use in British Columbia in Credit Union Reserve Board (1973)] of old funds during each period.

The three types of liability expressions in the ALM formulation are now developed. The deposit flow constraints represent the total amount of new deposits in the jth period. The total amount of new deposits of type d generated in period j is

$$y_j^d = BS_j^d - \sum_{i=0}^{j-i} y_i^d(1 - \gamma_d)^{j-i}$$

or

$$y_j^d + \sum_{i=0}^{j-i} y_i^d(1 - \gamma_d)^{j-i} = BS_j^d ,$$

where y_j^d is the total amount of new type d deposits in period j, γ_d is the annual rate of withdrawal of type d deposits, and BS_j^d is the discrete random variable representing balance sheet figure of type d deposits at the end of the jth period.

The second type of liability expression represents the total amount of deposits outstanding during a period. Since the model is discrete, an approximation to the continuous flow is made by assuming that half of a period's net flows arrive at the beginning of the period and the other half arrive at the beginning of the next period. During the first period, the funds available are

$$\left[y_0^d+(1-\gamma_d)y_0^d \right] /2 + y_1^d/2$$

and for period j

$$\sum_{i=0}^{j-1} \left[(1-\gamma)^{j-i-1} y_i^d + (1-\gamma)^{j-1} y_i^d \right] /2 + y_j^d/2$$

or

$$\sum_{i=0}^{j-1} y_i^d(1 - \gamma_d/2)(1-\gamma_d)^{j-i-1} + y_j^d/2.$$

This expression is used in the objective function, and the legal and liquidity constraints. The third liability expression is the incremental increase (decrease) of deposits from one period to the next. This incremental difference is used in the sources and uses constraint. For period j the incremental difference is

$$\left[\sum_{i=0}^{j-1} y_i^d(1 - \gamma_d/2)(1-\gamma_d)^{j-i-1} + y_j^d/2 \right]$$

$$-\left[\sum_{i=0}^{j-2} y_i^d(1 - \gamma_d/2)(1-\gamma_d)^{j-i-2} + y_{j-1}^d/2 \right]$$

$$= \sum_{i=0}^{j-2} y_i^d(1 - \gamma_d)^{j-i-2}[- \gamma_d(1 - \gamma_d/2)] + (1-\gamma_d)y_{j-1}^d/2 + y_j^d/2.$$

3.3 Data Required to Implement the ALM Model

To implement the ALM model requires the following data:

1. the identification of the assets in which the bank can potentially invest (or at least a representative group of assets);
2. point estimates of the returns on these assets;
3. point estimates of capital gains (losses) as a function of the time the bank holds the assets;
4. identification of the liabilities which the bank can potentially sell;
5. point estimates of the costs of these liabilities;
6. the rate at which deposits are withdrawn;
7. an estimated weighted cost of funds to determine the discount rate;
8. pertinent legal constraints;
9. parameters used in the development of the liquidity constraints;
10. policy constraints used by the bank;
11. estimates of the marginal distributions of the stochastic resources; and
12. unit penalties incurred for shortage or surplus in the stochastic constraints.

Remarks:

a. Since the ALM model has a separable objective only the marginal distributions of the components of the resource vector are needed to find the optimal solution.

b. The shortage (y^+) and surplus (y^-) variables have specific meanings in the ALM formulation. Consider a realization $\xi_{js}^{d'}$ of the random deposit ξ_{js}^{d}. If

$$y_j^d + \sum_{i=0}^{j-i} y_i^d(1 - \gamma_d)^{j-i} < \xi_{js}^{d'}$$

then $y^+ > 0$ and $y^- = 0$, assuming $p^+ + p^- > 0$; y^+ would be interpreted as the amount of funds that could have been used for investment purposes in the ALM. Since the cost of deposits is usually lower than the returns on assets, the bank would want to utilize all available funds. A penalty $p^+ > 0$ for the opportunity cost can be determined by assuming that the funds not used can be invested in earning assets. The y^+ dollars would be available at some rate c and could then be invested in some asset at a rate r. The penalty, p^+, would be equal to (r-c) discounted to point 0 plus the net discounted returns on $y^+(r-c)$ to the horizon of the model (that is, the profits that could have been generated).

On the other hand, if

$$y_j^d + \sum_{i=0}^{j-i} y_i^d(1 - \gamma_d)^{j-i} > \xi_{js}^{d'}$$

then $y^- > 0$ and $y^+ = 0$, and a surplus occurs. In this case, the bank would have to divest itself of some earning assets. The cost, p^-, of this action is (r-c) discounted to point 0 plus the net discounted returns on $y^-(r-c)$ to the horizon of the model (that is, the profits that would have been generated with unavailable funds).

Thus both p^+ and p^- are positive and profit is lowered if either too little or too much is invested. The key issue of what r and c should be used to determine the penalties is now addressed where a case study using the ALM formulation is presented.

4. APPLICATION OF ALM TO THE VANCOUVER CITY SAVINGS CREDIT UNION

This section is concerned with an application of the ALM model to the asset and liabilitiy portfolio problem of Vancouver City Savings Credit Union (VCS).

There is also a discussion of procedural aspects of implementing the model to this and related institutions. This study was prompted by the VCS's continual liquidity problem and focuses on the five year planning period 1970-1974.

During this period the firm's assets grew at a compound rate of 57%/year from $26 million to $160 million and there was an aggressive policy of investing in high yielding assets, predominantly mortgages. In 1974, VCS realized that the combination of their aggressive investment policy and changing market conditions was creating serious liquidity problems. Investors were trading low yield term deposits for higher yield deposits. Meanwhile the outstanding mortgage loans were still earning returns on the basis of the lower fixed rates. It was at this moment that this study was initiated.

4.1 Model Details

We now describe the input necessary to implement the ALM model at VCS. The discussion here is on general concepts concerning methods of data collection, choice of decision variables, constraints and objective function. The actual data, a 92 x 257 input matrix, appear in Kusy (1978).

The first stage variables are assets (x_{ij}^k) and liabilities (y_i^d and b_i). There are eleven asset types:

1. cash;
2. British Columbia Credit Union shares;
3-6. federal government bonds maturing in $i = 1,\ldots,4$ years;
7. federal government bonds maturing in five to ten years;
8. provincial government bonds maturing in more than ten years;
9-10. first and second mortgages with a three year term, and
11. personal loans.

Six types of liabilities are considered:

1. demand deposits;

2. share capital of VCS;

3. borrowing from banks; and

4-6. term deposits maturing in i = 1,3,5 years.

These asset and liability types generate 132 and 36 variables, respectively, including initial positions. For example a four year federal government bond purchased at the beginning of the third time period generates decision variables x_{34}^{6}, x_{35}^{6}, and $x_{3\infty}^{6}$, where x_{34}^{6} and x_{35}^{6} are the amounts of the initial investment to be sold in periods four and five, respectively, and $x_{3\infty}^{6}$ is the amount to be held at the horizon. The choice of assets and liabilities was based on VCS's historical portfolios (1968-1975) so that comparison between actual portfolios and ALM generated portfolios could be easily made. Although cash flows are continuous over time the model assumes that all transactions occur at the beginning of periods. Cash flows during any period are modeled by assuming that half the flow occurs at the beginning of the present period and the other half at the beginning of the next period. The model has the following constraints.

4.1.1 Legal Constraints

The source for the legal constraints is the Credit Union Act of British Columbia [British Columbia Government (1973)], which places three operational restrictions on the composition of the portfolio of assets and liabilities. The first constraint is that credit unions maintain at least 10% of the total assets (denoted by the set I) in high liquid assets (denoted by the set I_L):

$$\sum_{i \varepsilon I_L} x_{it} \geq .1 \sum_{i \varepsilon I} x_{it}$$

The second requirement is that credit unions maintain at least 1% of their total debt in cash and term deposits:

$$x_{1t} + x_{2t} \geq .01 \sum_{i \varepsilon D} y_{it}.$$

The final constraint restricts the credit union's borrowing from opportunities denoted by the set B, to one half of the total liabilities:

$$\sum_{b \varepsilon B} y_{bt} \leq .5 \sum_{i \varepsilon D} y_{it}.$$

Since the planning horizon has five periods, the legal requirements account for fifteen constraints.

4.1.2 Budget Constraints

There are twenty-two budget constraints, seventeen establish the initial positions of the eleven asset and six liability types and five require the sources and uses of funds to be equal in each period.

4.1.3 Liquidity Constraints

The liquidity constraints ensure that the firm has sufficient capital reserves to meet severe withdrawal claims under adverse economic conditions. The constraints follow from the Federal Reserve Board's capital adequacy formula [Crosse and Hempel (1973)]. The application of the FRB's caf to British Columbia's credit unions is justified in a study published by the Credit Union Reserve Board (1973).

The first three constraints establish capital reserves based upon the structure of the portfolio of assets and liabilities:

$$P_i \geq q_i(W - \sum_{k \varepsilon K_1 U \ldots U K_i} \alpha_k) \qquad i=1,2,3.$$

where P_i is the required reserve necessary to meet the excess withdrawal claims, q_i measures the reserves required for potential withdrawal claims that exceed the realizable portion of the assets contained in $K_1 U \ldots U K_i$, α_k is a parameter that measures the realizable portion of the value of asset k if the asset is to be liquidated quickly under adverse economic conditions, $W = \sum_{i=1}^{m} \gamma_i y_i$ is the dollar value of the expected withdrawal claims under adverse conditions, where γ_i measures the contraction of liability y_i under adverse economic conditions. The γ 's used were 0.47 for demand deposits, 0.36 for term deposits and 1.0 for borrowing; see Credit Union Reserve Board (1973) for justification.

The assets are classified as per the FRB's caf as follows:

1. "Primary and Secondary Reserves: (K_1) which includes cash, treasury bills, and government bonds of less than five years maturity;
2. "Minimum Risk Assets" (K_2) which include government bonds with more than five years maturity, and municipal bonds; and
3. "Intermediate Assets" which includes mortgage and personal loans.

Finally, the principal constraint in the caf is

$$\sum_{i=1}^{K} (1 - \beta_i)\, x_i \geq \sum_{i=1}^{3} P_i + \begin{cases} \text{total right-equity-surplus} \\ \text{hand side} \\ \text{of balance} \\ \text{sheet} \end{cases}$$

where β_i is a parameter to measure the shrinkage of asset i, when the asset is to be liquidated quickly. The actual numbers used for α_k, q_i, and β_i are those prescribed by the FRB [Cross and Hempel (1973)]. Since the purpose here is not to develop an operational model for VCS, but rather to demonstrate the applicability of the ALM model, the parameter values used provide an adequate proxy. In the development of an operational model it would be necessary to estimate the parameters. Since these constraints have to hold for all five periods, there are twenty liquidity constraints.

4.1.4 Policy Constraints

Two types of policy constraints are included:

1. personal loans should not exceed 20% of the first mortgage loans in any period t, i.e., $x_{tL} \leq 0.2\ x_{tm}$; and
2. second mortgages should not exceed 12.5% of first mortgages, i.e., $x_{tp} \leq 0.125\ x_{tm}$.

The rationale is that returns on first mortgages are less risky than second mortgages or personal loans and some of the latter are desirable (even though they may have lower returns) to respond to management's preference for a less risky overall portfolio. These constraints may be violated without legal implications and are modelled by treating the constraints as stochastic using $(p^+,p^-) = (0,1)$. There are ten such constraints over the five periods.

4.1.5 Deposit Flows

The variable y_j^d represents the new deposits of type d = 1,...,5 generated in period j = 1,...,5 and ξ_{jd} is a discrete random variable representing the balance sheet of deposit type d at the end of period j. The deposit flow constraints are

$$y_j^d + \sum_{i=0}^{j-i} y_i^d(1 - \gamma_d)^{j-i} + y_{jd}^+ - y_{jd}^- = \xi_{jd}$$

where the γ's (1.0 for demand deposits and 0.36 for term deposits) are included to reflect the gross flow of deposit funds. The distribution of ξ_{jd} was estimated using the balance sheet figures of VCS for 1970-1974; see Kusy (1978) for specific estimates.

The penalties for shortages associated with these constraints are:

1. for demand deposits and share capital, p^+ is the total discounted return on a one year term deposit minus the discounted cost of the funds calculated to the horizon of the model;

2. for term deposits maturing in one or three years, p^+ is the total discounted return on a five year term deposit minus the discounted cost of the funds calculated to the horizon of the model; and
3. for term deposits maturing in five years, p^+ is the total discounted return on a ten year provincial government bond minus the discounted cost of the funds calculated to the horizon of the model.

The penalties p^-, for surpluses associated with the deposit flow constraints are the total discounted returns on first mortgages minus the discounted costs of funds calculated to the horizon of the model. The penalty approach attempts to model a conservative management strategy with surplus funds when realized sources exceed uses and when there are shortages.

4.1.6 Objective Function

The objective is to maximize the expected total discounted revenues minus expected total discounted costs including penalty costs. The source for data on the returns on the federal and provincial government bonds is the Central Mortgage and Housing Corporation (1975). The source for the returns on BCCU shares, demand deposits and share capital is Vancouver City and Savings Credit Union (1968-1975).

The discount rate used was the time value of money. The risk free rate (the average yield on three month treasury bills) was [Central Mortgage and Housing Corporation (1975)].:

	1970	1971	1972	1973	1974
Average yearly yield	.0599	.0356	.0356	.0547	.0782
Multiperiod discount factor $\pi(1/(1 + AYY_i))$	.9435	.9110	.8797	.8341	.7736

The returns on the assets are [Central Mortgage and Housing Corporation (1975), Vancouver City Savings Credit Union (1968-1975)]:

Type of Asset	Returns on Asset in Year 1969	1970	1971	1972	1973	1974
1 year federal government bond (Fgb)	.0725	.0620	.0450	.0510	.0610	.0800
2 year Fgb	.0749	.0657	.0490	.0550	.0654	.0803
3 year Fgb	.0758	.0684	.0525	.0590	.0680	.0807
4 year Fgb	.0767	.0710	.0555	.0626	.0698	.0810
5 year Fgb	.0776	.0758	.0615	.0674	.0717	.0827
10 year provincial government bond	.0840	.0904	.0803	.0813	.0836	.0991
first mortgage	.0938	.1040	.0943	.0921	.0959	.1124
second mortgage	.1050	.1220	.1108	.1083	.1123	.1321
personal loans	.1040	.1170	.1075	.1050	.1075	.1275
B.C.C.U. shares	.0600	.0600.	.0600.	.0600	.0700	.0700

For purchase of a five year federal government bond in 1970, the decision variables X^7_{12}, X^7_{13}, X^7_{14}, X^7_{15}, and X^7_1i would be generated. The returns are the interest earned each year discounted to the beginning of the planning horizon, namely:

Decision Variable x_{ij}	Return r^7_{ij}
x^7_{12}	(.0758)(.9435) = .0720
x^7_{13}	(.0758)(.9435 + .9110) = .1410
x^7_{14}	(.0758)(.9435 + .9110 + .8797) = .2070
x^7_{15}	(.0758)(.9435 + .9110 + .8797 + .8341) = .2700
$x^7_{1\infty}$	(.0758)(.9435 + .9110 + .8797 + .8341 + .7736) = .3290

The costs of the liabilities are [Vancouver City Savings Credit Union (1968-1975)]:

	Cost of Liability in Year					
Type of Liability	1969	1970	1971	1972	1973	1974
1 year term deposit	.0712	.0780	.0720	.0680	.0780	.0990
3 year term deposit	.0750	.0820	.0760	.0690	.0820	.0980
5 year term deposit	.0785	.0850	.0800	.0800	.0850	.0975
demand deposit	.0400	.0460	.0410	.0420	.0560	.0770
share capital	.0500	.0500	.0500	.0550	.0575	.0800

The cost of a five year term deposit (y^3_1) sold during 1970 is:

Year	Cost
1970	$(.5)(.0850)(.9435) = .0401$
1971	$(.82)(.0850)(.9110) = .0635$
1972	$(.82)(.64)(.0850)(.8797) = .0392$
1973	$(.82)(.64)^2 (.0850)(.8341) = .0238$
1974	$(.82)(.64)^3 (.0850)(.7736) = .0141$
	Total Cost .1807

4.2 Results of the VCS Application

The purposes of this application are to demonstrate applicability of the ALM model and to test the sensitivity of the solution generated. To accomplish these goals the basic model was run along with several variants that used modified penalty costs and probability distributions as well as a deterministic model where all random variables were replaced by their means.

The basic model has symmetric three point probability distributions (0.2, 0.6, 0.2) for the deposit flow constraints and degenerate probability distributions for the liquidity and policy constraints. The penalties for all stochastic constraints are asymmetric. The optimal value of the basic model is $2,520,316.01 ($8,288,941.53 in expected profits minus $6,282,885 in expected penalties). As shown by Madansky (1960) the mean model provides a lower bound on the optimal value of a stochastic linear program; here the bound is 10.6% below the optimal value of the basic model. The structure of the two portfolios is similar in the initial period however the investment patterns differ in later periods; in particular the basic model invests less heavily in less liquid assets (namely mortgages). See Kusy (1978) for specifics.

The mean model was initially infeasible since the initial portfolio held by VCS violated the liquidity constraints (a situation known to management). To secure feasibility variables were added to the liquidity constraints. The objective coefficients of these variables were the same as the penalties associated with violating the stochastic liquidity constraints in the basic model. As a further insight into the operations of VCS, the penalties could be set arbitrarily high so that the model would violate the liquidity constraints only to attain feasibility. The amount by which the constraints are violated will be the amount of liquid reserves that the firm needs to meet the FRB's liquidity requirements.

Variants of the basic model were run in order to ascertain the effects of different probability distributions, penalty costs and parameter values. The initial change was the alteration of the first legal constraint from the requirement that current assets be at least 10% of the liabilities to at least 1% of the liabilities. This increases the optimal value to $2,906,773.53 ($8,657,619.24 in expected profits minus $5,750,845.71 in expected penalties). For the initial two periods, the investment pattern deviated substantially from that of the basic model in that more of the incremental funds were allocated to longer term assets. After the first two periods there did not seem to be any generalizable behaviour in the investment patterns of the two models.

The basic model was then further altered to include a change in the probability distributions (0.05, 0.50, 0.45) of the cash flows. The optimal value increases sharply to $3,256,500.65 ($8,872,911.53 in expected profits minus $5,661,410.80 in expected penalties). The expected net profit rises compared with the basic model and the model with the parameter change while the expected penalty costs decrease in both cases because:

1. all the violations of stochastic constraints are now feasible only with a probability 0.05 instead of 0.2 (decreased penalties); and
2. constraints which were not previously violated because of excessive penalties are now violated by 15% resulting in more profits.

Although it is not possible to make definitive generalizations from these runs of the ALM model some conclusions may be inferred. First, the asymmetry of the probability distributions may have a substantial effect on the optimal solutions and values. Of particular importance is the sensitivity of the estimate of the probability distribution around the value on the left hand side of the stochastic constraints. Second, the solutions are sensitive to changing penalty costs. Third, the various stochastic models have substantially different solutions than the mean model. This indicates that reliance on the deterministic models as normative tools can lead to erroneous solutions. Fourth, the implementation of this model is no more difficult than the implementation of a similar deterministic model. Finally, the computations necessary to solve the ALM formulation are in the same order as the computations necessary for the mean or related deterministic models. [Similar conclusions on a SLPSR model of short term financial planning were found by Kallberg, White, and Ziemba (1982).] All the runs were made on the University of British Columbia's IBM 370/168. The ALM model is 92 x 257 with 40 stochastic constraints. Using the SLPSR code [Kallberg and Kusy (1976)] the solution of the ALM model took 37 seconds of CPU time. To solve an equivalent sized deterministic problem took 30 seconds using the SLPSR code and 17 seconds on the standard L.P. code UBC LIP. Experience in solving SLPSR models and related deterministic problems indicates that the CPU times are in a ratio of about 1.5-2 to 1. Detailed output appears in Kusy (1978).

5. COMPARISON OF THE ALM AND SDT APPROACHES

The asset and liability management problem is a continuous decision problem in which actions (e.g., portfolio revisions) are made continuously on the basis of new information (e.g., differing forecasts of future interest rates, etc.). The ideal way to model the ALM problem would be via a continuous time adaptive dynamic program. At present such a formulation is computationally intractable for the types of problems considered here. ALM and Bradley and Crane's stochastic decision tree model (SDT), which is described in this section,constitute two types of operational models where time and probability distributions have been discretized. In this section we simulate a large number of economic scenarios to compare these two models.

5.1 The Bradley-Crane Stochastic Decision Tree Model

The Bradley-Crane (1972, 1973, 1976) model depends upon the development of economic scenarios that are intended to include the set of all possible outcomes. The scenarios may be viewed as a tree diagram where each element (economic conditions) in each path has a set of cash flows and interest rates. The problem is formulated as a linear program whose objective is the maximization of expected terminal wealth of the firm. There are four types of constraints:

1. cash flow, which does not allow the firm to purchase more assets than it has funds available;
2. inventory balancing, which ensures that the firm cannot sell and/or hold more of an asset at the end of a period than it held at the beginning;
3. capital loss, which does not allow the net realized capital losses in a period to exceed some pre-specified upper bound; and
4. class composition, which limits the holding of a particular asset.

The basic formulation is

$$\max \sum_{e_N \varepsilon E_N} p(e_N) \sum_{k=1}^{K} \left\{ \sum_{m=0}^{N-1} \left[y_m^k(e_m) + v_{m,N}^k(e_N) \right] h_{mN}^k(e_N) + \left[y_N^k(e_N) + v_{NN}^k(e_N) \right] b_N^k(e_N) \right\}$$

s.t.

1) $$\sum_{k=1}^{K} b_n^k(e_n) - \sum_{k=1}^{K} \left[\sum_{m=0}^{n-2} y_m^k(e_m) h_{m,n-1}^k(e_{n-1}) + y_{n-1}^k(e_{n-1}) b_{n-1}^k(e_{n-1}) \right]$$

$$- \sum_{k=1}^{K} \sum_{m=0}^{n-1} \left[1 + g_{m,n}^k(e_n) \right] s_{m,n}^k(e_n) = f_n(e_n) \qquad \text{(Cash flows)}$$

2) $$- h_{m,n-1}^k(e_n-1) + s_{m,n}^k(e_n) + h_{m,n}^k(e_n) = 0, \; m = 0,\ldots,n-2$$

$$- b_{n-1}^k(e_{n-1}) + s_{n-1,n}^k(e_n) + h_{n-1,n}^k(e_n) = 0$$

$$h_{0,0}^k(e_0) = h_0^k \qquad \text{(Inventory Balance)}$$

3) $$- \sum_{k=1}^{K} \sum_{m=0}^{n-1} g_{m,n}^k(e_n) s_{m,n}^k(e_n) \leq L_n(e_n) \qquad \text{(Capital Losses)}$$

4) $$\sum_{k \varepsilon K^i} \left[b_n^k(e_n) + \sum_{m=0}^{n-1} h_{m,n}^k(e_n) \right] (\lesseqgtr) \; c_n^i(e_n), \qquad i = 1,\ldots I$$

(Category Limits)

$$b_{m,n}^k(e_n) \geq 0, \; s_{m,n}^k(e_n) \geq 0, \; h_{m,n}^k(e_n) \geq 0, \; m = 1,\ldots,n-1$$

(Nonnegativity)

where $e_n \varepsilon E_n$; $n = 1,\ldots,N$; $k = 1,\ldots,K$; e_n is an economic scenario from period 1 to n having probability $p(e_n)$; E_n is the set of possible economic scenarios from period 1 to n; K_i is the number of assets of type i, the total number of assets is K; N is the number of time periods; $y^k_m(e_m)$ is the income yield per dollar of purchase price in period m of asset k, conditional on e_m; $v^k_{m,N}(e_N)$ is the expected terminal value per dollar of purchase price in period m of asset k held at the horizon (period N), conditional on e_N; $b^k_n(e_n)$ is the dollar amount f asset k purchased in period n, conditional on e_n; $h^k_{m,n}(e_n)$ is the dollar amount of asset k purchased in period m and held in period n, conditional on e_n; $s^k_{m,n}(e_n)$ is the dollar amount of asset k purchased in period m and sold in period n, conditional on e_n; $g^k_{m,n}(e_n)$ is the capital gain (loss) per dollar of purchase price in period m of asset k sold in period n; $f_n(e_n)$ is the incremental increase (decrease) of funds available for period n; $L_n(e_n)$ is the dollar amount of maximum allowable net realized capital losses in period n; and $C^i_n(e_n)$ is the upper (lower) bound in dollars on the amount of funds invested in asset type i in period n.

The SDT formulation is a true dynamic model. The first decision (immediate revision, $h^k_{01}(e_1)$, $b^k(e_1)$, $s^k_{01}(e_1)$] has as its feasible set the intersection of all possible realizations. That is the current solution must be feasible for the set E_N. This decision is conditional on the realization of economic events in the first period. Similarly in each succeeding period to the end of the planning horizon the decisions generated are all conditional on the states of nature that have occurred up to the current decision point.

The model has a number of attractive features including its dynamic nature and associated clever solution using decomposition. However the formulation has features that detract from its practicability. The capital loss and category limit constraints have as upper (or lower) bounds amounts (resources) generated arbitrarily by portfolio managers rather than through a systematic procedure. For example, no consideration is given to the portfolio mix in the development of bounds, except in the sense that upper (or lower) bounds are placed on asset categories. At some point in time, this may imply that the bank has invested a disproportionate amount of its available funds in long-term bonds when compared to the amount of short-term liabilities held. Also the formulation does not utilize either the FRB's recommended capital adequacy formula or any other statistically generated systematic procedure in the development of bounds for the constraints. Since the capital loss and category limit constraints actually determine the composition of the solution, the arbitrary nature of the choice may bias the solution.

One feature of the SDT model is that first period feasibility is assured for every possible scenario. As is well known, see e.g., Madansky (1962), such fat formulations shrink the feasible set and give substantial importance to scenarios with low probabilities of occurence. For example, consider the two period problem where an investor:

1. has \$100 in period 1 to invest in asset x_{11} with return $r_{11} = .1$ maturing after one period or x_{12} with return $r_{12} = .2$ per period maturing after two periods;
2. receives in period 2 either an additional \$50 to invest with probability .9 or he loses \$50 with probability .1;
3. has in period 2, the opportunity to invest in a one period asset x_{21} with return $r_{21} = .1$ or can sell off his holdings in x_{12} at a 20% discount; and
4. stipulates that his realized capital losses cannot exceed 10% of the outstanding funds in any period.

The linear programming optimal solution is $b_1^1(\ell_1) = 11.11$, $b_1^2(\ell_1) = 88.89$, $b_2^1(\ell_{21}) = 80.00$, $h_2^1(\ell_{22}) = 0$, $h_{12}^2(\ell_{21}) = 88.89$, $h_1^2(\ell_{12}) = 63.89$, $s_{12}^1(\ell_{21}) = 11.11$, $s_{12}^1(\ell_{22}) = 11.11$, $s_{12}^2(\ell_{21}) = 0$, $s_{12}^2(\ell_{22}) = 25.00$, with optimal value $42.87, where "b" means buy, "h" hold, and "s" sell and the ℓ's denote the possible scenario events. The bound on realized capital losses is binding. If a maximal loss of 15% were allowed it would be optimal to purchase $100 of asset x_{12} and sell $37,50 of x_{12} at the end of period 1, if the $50 is lost. This modification yields an optimal value of $44.11. Thus consideration of this low probability event significantly alters the optimal solution. By contrast in the simple recourse ALM formulation the right hand sides are not binding; recourse at a penalty cost is allowed to compensate for decision infeasibility. The recourse formulation has more first period decision flexibility than the decision tree formulation.

To gain computational tractability the SDT model only considers bonds. In general, if D is the number of possible realizations per period, n is the number of time periods, I is the number of asset claims and K is the number of assets then the number of variables is $(3 + 5D + 7D^2 + \ldots + (2n + 1)D^{n-1})$ and the number of constraints is equal to the sum of the cash flow constraints $(1 + D + D^2 + \ldots + D^{n-1})$, the capital loss constraints $(1 + D + D^2 + \ldots + D^{n-1})$, the category limit constraints $(I)(1 + D + D^2 + \ldots + D^{n-1})$, the inventory balance constraints $(K)(I + 2D + 3D^2 + \ldots + nD^{n-1})$, and the initial conditions K. The effect of differing numbers of assets, possible realizatiions per period, and number of periods on problem size is shown in:

Table 1. Size of Bradley-Crane Model

	Number of: Assets	Asset Classes	Time Periods	Possible Realizations Per Period	Variables	Constraints
1)	8	1	3	3	656	319
2)	30	5	3	3	2,460	1,141
3)	30	5	3	5	6,120	2,827
4)	30	5	5	5	246,120	116,827

Bradley and Crane (1976) solved model (1) in 68 seconds on an IBM 360/65. The subsequent models add more realism and are much larger. In the case of (4) the basis of the master problem if one uses decompostion is 5467 and there are about 850,000 non-zero elements in the problem. The computational and data handling difficulties of (2) and (3) are less striking but, nevertheless, they remain formidable. Bradley and Crane (1976: 112) are well aware of these computational difficulties:

> "Unfortunately, taking uncertainty explicitly into account will make an asset and liability management model for the entire bank computationally intractable, unless it is an extremely aggregrated model. The complexities of the general dynamic balance sheet management problem are such that the number of constraints and variables needed to accurately model the environment would be very large."

In view of this our aim has been to develop a computationally tractable model that still has some dynamic and other desirable features and represents a practical approach to bank asset and liability management. We now compare the ALM and SDT models via simulation.

5.2 The Economic Scenarios

To maintain computational feasibility for the SDT model only three assets and one liability were considered over three periods. The assets are a one period treasury bill, a term deposit maturing beyond the horizon of the model and a long-term mortgage. The liability is a demand deposit. The returns and costs of these financial instruments were generated from 26 consecutive observations using data from the Central Mortgage and Housing Corporation (1975). To obtain a reasonable correlation of interest rates, the returns and costs were made a function of the prime rate using the following distribution of the prime rate (R)

Pr(R = r)	6/26	3/26	1/26	2/26	1/26	2/26	4/26	2/26	2/26	2/26	1/26
r	.06	.065	.0675	.075	.0775	.08	.085	.09	.095	.11	.115

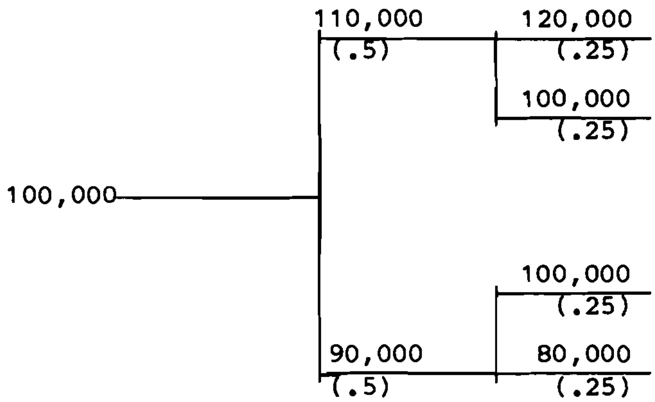
The following distributions were then derived for the difference between the prime rate and the rate of return of each of the four financial instruments, where the random variables M,D,T, and L are defined to be the difference between the prime rate, and the mortgage rate, term deposit rate, treasury bill rate and the liability rate, respectively.

m	$P(M \leq m)$	d	$P(D \leq d)$	t	$P(T \leq t)$	ℓ	$P(L \leq \ell)$
.0037	0.0	-.0104	0.0	-.0388	0.0	-.0275	0.0
.0088	0.2	-.0072	0.2	-.0306	0.2	-.025	0.2
.0198	0.42	+.0008	0.44	-.0253	0.5	-.0225	0.31
.0235	0.62	+.004	0.5	-.0225	0.77	-0.2	0.92
.0297	0.81	+.0118	0.78	-.0174	0.81	-.0175	1.00
.0338	1.00	+.0195	1.00	-.0051	1.00		

At time zero the investor has $100,000 in demand deposits equally invested in the three assets. The demand deposits are assumed to increase (decrease) from one period to the next uniformly on [-20,000, 20,000]. If the demand deposits decrease so that assets have to be liquidated, then the FRB's parameters for quick liquidation are used. The discounts for treasury bills, term depostis, and mortgages are 0.5%, 4%, and 6%, respectively. The constraints on the investor are of the BC type and include 1) cash flows, 2) capital losses, 3) class composition, and 4) terminal conditions. The capital loss constraints assume that the investor does not want to realize net losses of more than 3% of the outstanding demand deposits in periods 1 and 2, and 4% in period 3. The class composition constraints limit the investor from having more than $50,000in total investments in any asset in periods 1 and 2, and $60,000 in period 3. The terminal constraints include a discount on the assets in the current portfolio so that all funds are not simply invested in the highest yielding assets and held to the horizon of the model. These discounts are one-half of the normal discounts. The objective of the model is to maximize the net expected returns.

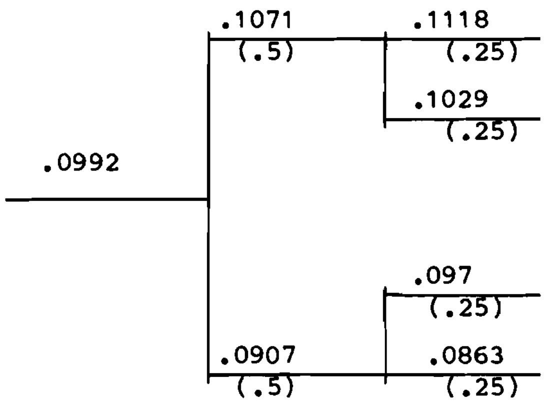
5.3 Formulations of the Stochastic Dynamic Programming Model

To formulate the SDP an economic scenario over the three period horizon must be established. This includes a representative distribution for the cash flows and the rate of return for the various financial instruments. Since for computational tractability SDP requires crude approximations of probability distributions the number of possible realizations of the random variables during each time period was limited to two. With initial demand deposits of $100,000 and an incremental difference in the interval [-20,000, 20,000] a natural two point distribution is $90,000 and $110,000 with equal probabilities. Using this distribution the mean of the underlying distribution is maintained although the variance is smaller (1.0×10^5 versus 1.33×10^5). The distribution was constructed similarly at the third decision point. The cash flows have the distribution

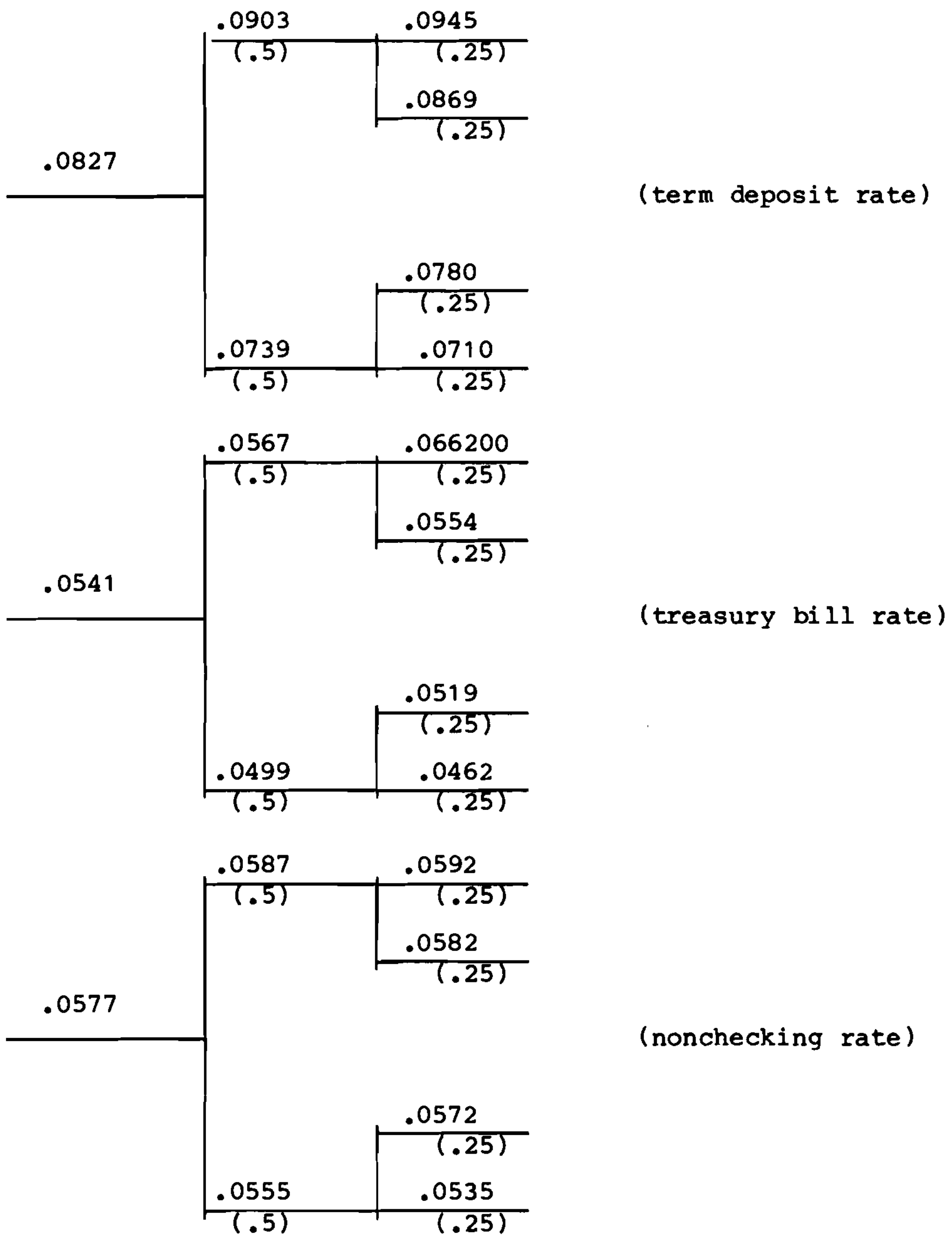


Using the same approach, the first period rate of return for a particular financial instrument (assume mortgage rate) is the median prime rate ($\bar{R}$) plus the median of the difference between the prime rate and the rate of return of the mortgages ($\bar{M}$). The two point estimate in the second period is $\bar{R}$ plus m, where $P(M \leq m) = 0.25$. The four rates of return in the third period are: $\bar{R}+m$ where $P(M \leq m)$ is 0.875, 0.625, 0.375, and 0.125, respectively.

The disribution of the rates of return used were:



(mortgage rate)



For the simulation, 70% of the nonchequing rate was used as the demand deposit rate since the nonchequing rate dominates the treasury bill rate. This would have precluded investment in treasury bills a priori. This ad hoc derivation of the demand deposit rate does not impinge on the usefulness of the simulation because the objective is to demonstrate that one solution technique may be operationally superior.

Treasury bills mature after one period, hence eighteen variables completely define all potential investment opportunities. Since the term deposits and mortgages mature beyond the horizon of the model, 42 variables are required to describe all investment opportunities in each of these categories. The variables necessary to define the demand deposits include, the initial position, the demand deposit flows in period one, two demand deposit flows in period two, and the four demand deposit flows in period three. In all, 110 variables define the investment opportunities in the problem.

There are four types of constraints. Constraints 1 to 7 are the cash flow requirements for each period under each economic scenario; namely uses of funds equal sources of funds. Constraints 8 to 14 require realized capital losses to be less than 3% of the outstanding demand deposits in period one and two, and 4% in period three. Constraints 15 to 35 limit the funds invested in each asset as prescribed in the problem. Constraints 36 to 89 (inventory balancing) consist of the initial holdings of each of the four financial instruments and record the transactions in each economic scenario.

The demand deposit flow constraint for period 1 places an upper bound on the funds potentially available for investment. The capital loss and the composition constraints add another 28 slack variables to the formulation. The total size of the SDT formulation is 89 constraints with 139 variables.

The objective is to maximize the expected value of the net returns from the portfolio over the horizon of the model. Thus the coefficient of each variable is the product of the net return and its probability of occurence.

5.4 Formulations of Asset and Liability Management Model

The ALM uses the same information as the SDT model although it has fewer constraints because of its different treatment of uncertainty. The investment opportunities for treasury bills, term deposits, mortgages, and demand deposits are defined by six, eleven, eleven and four variables, respectively. There are 25 constraints, of which five are stochastic, consisting of:

1. three constraints to balance the initial holding of an asset with the future buying and selling of the asset;
2. three constraints that equate the cash flows for the three periods;
3. three constraints for each of the three assets for composition requirements;
4. four constraints to describe the initial position of the three assets and one liability;
5. three capital loss constraints of which the first period's is deterministic as (1) to (4) above, and the others being stochastic; and
6. three stochastic constraints which describe the flow of demand deposits.

Adding nine slack variables for the class composition constraints and one for the deterministic capital loss constraint, the SLPR formulation has 25 constraints and 42 variables not counting recourse variables.

The right hand sides of the stochastic demand deposit constraints are representative points from the uniform distribution used in the SDT model. However, because of the ability of the Wets algorithm to handle many realizations without creating computational difficulties, the number of points chosen is larger than in the SDT model. The penalty for violations of any of these constraints is the net return to the horizon of the model, generated by a portfolio consisting of 50% mortgages and 50% term deposits, since their portfolio is considered, a priori, to be potentially the highest yielding portfolio. This penalty is

$$.5[(1 + \bar{r}_m)^{4-n} - 1] + .5[(1 + \bar{r}_t)^{4-n} - 1] - [(1 + \bar{r}_d)^{4-n} - 1]$$

where n = 1,2,3 is the period; $\bar{r}_m$ is the median return on mortgages; $\bar{r}_t$ is the median return on term deposits; and $\bar{r}_d$ is the median cost of demand deposits.

The right hand sides of the stochastic capital loss constraints are the representative points used in the SDT formulation. A penalty of 4.1% is used for violations of these constraints.

The objective is to maximize the net returns minus the expected penalties for constraint violations. The coefficient of each variable is the net return for the first stage variables and the penalty for the second stage variables.

5.5 Results of the Simulation

In normative financial planning models, the objective is generally to determine which portfolio changes should be effected immediately. The multi-periodicity of financial models compensates for the shifting economic scenarios across time. However, the purpose of the model is to determine the changes to be implemented immediately. Hence the simulation is intended to determine which model produces the best first period solution. In reality, decisions may be made at any point in a period; however, using a discrete time model, one aggregrates so as to consider all decisions to be made at the start of each period--facing random rates of return. Again, the incremental cash flows are aggregated so that one-half is available at the beginning of the current period. In both formulations the same initial security holdings are given and the cash flows for the next period are random.

The process starts with an initial portfolio. Both the ALM and SDT models determine an optimal solution for the first period. A random cash flow is then generated. If the amount of funds spend during the first period exceeds the random cash flow, then an amount equal to the excess spending is divested from the present portfolio of 45% of mortgages, 45% term deposits and 10% treasury bills. If the random cash flows exceed spending during the first period, then the incremental amount is invested in treasury bills. After this reconcillia-tion, revenues are the sum of the known returns of the assets held since the start of the period and the random returns of the assets bought at the start of the period. The costs are the sum of the random cost of demand deposits and the discount for selling securities prior to maturity. The reconciled portfolio serves as the new initial portfolio which is then used to generate the new solutions for both models. This cycle is repeated eight times. This whole process is repeated fifty times for a total of four hundred scenarios. See Kusy (1978) for the simulation flowchart, computer program and full details of the results.

The simulation results for the ALM and SDT formulations are used to test two hypotheses. The first hypothesis

$$H_1: \mu_d = \mu^1_{SDP} - \mu^1_{ALM} \geq 0$$

is used to test whether or not the initial period profit for ALM is superior to that for SDT.

This hypothesis is tested by examining the paired differences of the profits for the initial run of the 50 cycles for both models. The specific information used is:

1. the mean of the paired difference ($251.37 in favour of ALM); and
2. the standard deviation of the paired differences ($150.43).

The correlation between the ALM and SDT profits is 0.958. Given the large sample, the significance of the paired differences is tested using the t statistic

$$\frac{-251.37}{1.50.43/\sqrt{50}} = -11.81.$$

The test statistic is significant at the 0.001 level hence the null hypothesis is rejected. Thus, ALM yields a statistically significant better initial solution than SDT.

The second hypothesis

$$H_2: \mu_d = \mu_{SDP} - \mu_{ALM} \geq 0$$

is used to test whether or not the mean profit for ALM is superior to that for SDT.

This hypothesis is tested by examining the paired differences of the mean profits of the eight runs of fifty cycles for both models. The specific information used is:

1. the mean of the paired differences ($297.26 in favour of ALM); and
2. the standard deviation of the paired differences ($308.74).

The correlation between the ALM and SDT mean profits is 0.785.

The t statistic is

$$\frac{-297.26}{308.74\sqrt{50}} = -6.81.$$

Since this is significant at the .001 level, the null hypothesis is rejected. Thus the ALM formulation yields a statistically significant better solution than the SDT formulation.

To test the stability of these summary statistics, a second simulation using ALM was run. The results of this simulation are analyzed similarly: 1) a test of the initial solution of the fifty cycles, and 2) a test of the mean profits for the 8 runs of the fifty cycles. The information necessary to test the first hypothesis is: 1) the mean profits for the first and second ALM runs ($4645.85 and $4672.23, respectively), and 2) the standard deviations for the two runs ($421.11 and $482.15, respectively). The hypothesis that both samples have the same mean

$$H_3: \mu_{ALM_1} = \mu_{ALM_2}$$

is tested first.

The standard deviation used for the test statistic is the root of the pooled variance. The test statistic is

$$\frac{4672.23 - 4645.85}{90.53} = 0.291.$$

The test statistic for the final hypothesis is established similarly and is

$$\frac{4783.13 - 4720.15}{86.84} = 0.730.$$

Since the test statistics for H_3 and H_4 are not significant at the .10 level there is no reason that the mean is not stable.

A CDC 6400 at the University of British Columbia was used to perform the computations. The total CPU time to perform the 400 iterations for ALM was 0.240 hours and for SDP 6.385 hours. This explains why only a limited number of financial instruments, time periods and realizations were used in the simulations, and highlights the gap in tractability between the ALM and SDT techniques. Full details of the codes, etc., used to perform the simulations appear in Kusy (1978).

6. FINAL REMARKS

The literature on bank asset and liability management has been based on two approaches: the mean variance portfolio selection model and using an objective of maximizing expected net returns. Deterministic and stochastic models based on these constraints are reviewed in Section 1. In an attempt to determine which approach is most suitable for asset and liability management problems, Myers (1968) showed that existence of security market equilibrium implies that net present value is the appropriate objective function. The most comprehensive model of this type is the Bradley-Crane stochastic decision tree model (1972, 1973, 1976). They attempted to overcome the crucial obstacle to asset and liability management of incorporating uncertainty while maintaining computational tractability. Their model is a useful one and many important insights appear in Bradley and Crane (1976). However, their model does not really maintain computational tractability for realistic sized bank asset and liability management problems. It also possesses some undesirable features, notably arbitrary constraints on capital losses, an absence of portfolio mix constraints and an immediate revision that must satisfy all possible forecasted economic constraints.

The ALM model is an attempt to remedy some of these deficiencies and as shown in Sections 3 and 4 is an implementable model of bank asset and liability management. The results of the application to the Vancouver City Savings Credit Union indicate that the ALM model is superior to related deterministic models and the simulations in Section 5 indicates that ALM generates better first period decisions than does the Bradley-Crane model. The CPU time to solve ALM is 1.5-2 times that of a related deterministic model and much less than that required for the Bradley-Crane model (0.24 hours versus 6.39 hours for simulation in Section 5). Hence it is a feasible option for implementation in large banks. To apply ALM one must determine essentially the same information as with a deterministic model: 1) deposit flow estimates, 2) estimates of the term structure of interest rates, 3) estimates of withdrawal rates of deposits under various economic conditions, 4) legal constraints governing the behavior of the financial institution, 5) policy constraints, 6) the Federal Reserve Board's recommended reserves for maintaining a liquid position, and 7) the initial position of the firm plus discrete probability distributions for the random elements and the penalty costs. The model is quite capable of handling very useful policy constraints. The major drawback of ALM is that it is not a true dynamic model. Simulations such as those in Section 5 provide confidence in the approach taken in ALM. Some bounds on the error associated with aggregration schemes such as that used in this paper in the context of general recourse models appear in Birge (1983). The general problem of the accuracy of various approximations in multiperiod stochastic programs is currently being studied by the second author in collaboration with J. Birge, M.A.H. Dempster, R.C. Grinold and R. Wets. The results of this work hopefully will provide general guidance regarding solution technique tradeoffs in dynamic stochastic modeling.

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