

STRUCTURAL CHANGE IN URBAN SYSTEMS

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Contributions to the Metropolitan Study:5

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FOREWORD

Contributions to the Metropolitan Study:5

The Project "Nested Dynamics of Metropolitan Processes and Policies" was initiated by the Regional & Urban Development Group in 1982, and the work on this collaborative study started in 1983. The series of contributions to the study is a means of conveying information between the collaborators in the network of the project.

This paper by Nijkamp and Schubert outlines a conceptual background for understanding the dynamics of metropolitan regions and urban systems in general. An essential part of the paper reviews existing theoretical explanations of urban change processes with special attention being paid to long term cycles and waves as well as discontinuities and qualitative changes in the evolution of metropolitan regions.

The paper introduces a distinction between constrained and structural dynamics. With reference to this distinction, the authors describe and classify various forms of urban oscillations and changes in urban structures. Relations between technological development and infrastructural change are also discussed.

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1. INTRODUCTION

In the last decade, many cities have exhibited profound changes in the number of residents, the number of workplaces, the sectoral structure of the urban economy, the demographic structure, the spatial lay-out, and so forth. In fact, the post-war urban development can be characterized as urban dynamics.

Urban dynamics can be observed in almost all countries of the world. In some cases, cities display a smooth transition pattern, while in others sudden jumps take place. The Silicon-valley development pattern, the urban systems development in New England, and the rise of metropolitan areas in many developing countries reflect a transition that may be denoted by *structural dynamics*. The latter concept means that a system is not only showing a shift in the (endogenous and exogenous) variables characterizing the state of the system, but also in the parameters and relationships defining its structure. Thus, two concepts of dynamics may be distinguished here, viz. *constrained dynamics* and *structural dynamics*. Constrained dynamics refers to a system where the element of time plays an intrinsically important role in the evolution of state and/or control variables without however affecting the structure of the system itself (in

terms of formal linkages between systems elements, of structure parameters, or of the impact pattern of time itself). Clearly, constrained dynamics may affect the stability of equilibria in a comparative static or evolutionary context.

Structural dynamics, on the other hand, refers to a situation where the systems configuration (in terms of linkage patterns, parameter values or time dimensions) exhibits an incremental or integral change. Such--often qualitative--structural changes may not only affect the stability of equilibrium points in the system at hand, but also lead to a new topology of systems trajectories (cf. Dendrinos, 1981, Nijkamp, 1982a, and Wilson, 1981). Structural dynamics may lead to unstable behavior of the system at hand. Such unstable behavior may emerge, if the parameters of a (usually non-linear) dynamic system reach a critical limit, beyond which the system displays a different set of structural relationships. It is clear that, in general, stability analysis is of major importance for analyzing complex dynamic systems, especially because sometimes marginal changes in parameter values may cause drastic perturbations and structural shifts in urban systems.

Structural dynamics in urban systems may be caused by two (mutually non-exclusive) factors (see also section 3):

- *external* factors outside the urban system that lead to a change in the structural parameters of the urban system at hand;
- *internal* factors within the urban system that affect the structure of the urban mechanism.

In the literature on urban dynamics, these factors are not always clearly distinguished, as will be shown on the basis of a concise review in section 2. Next, in sections 3 and 4, more specific attention will be devoted to the role of innovation in long-term development processes, with a special view of urban systems. The remaining section will be devoted to the design of a non-linear dynamic model that may explain or describe urban fluctuations. Throughout the paper, the term *fluctuations* will

be used to indicate the long-run trajectory of a dynamic system. Fluctuations may be regarded as oscillations in a broad sense including *inter alia* discontinuous jumps, smooth periodic cycles, stable random variations, and even chaotic fluctuations. Thus, fluctuation is an umbrella term encircling various concepts of evolutionary patterns. This term is different from *cycles* (which are usually thought of as periodic and stable) and *waves* (which display regular patterns with regular time intervals in economic dynamics; see also section 3).

2. THEORIES ON URBAN DYNAMICS

In the past decade, several theories have been designed that aim at explaining the background of structural urban dynamics in the Western world. In the present section, six major contributions in this field will briefly be described. A more extensive review can be found in Nijkamp et al. (1983).

I. A. Pred

Pred's analysis (1977) describes growth patterns of (mainly industrial) cities as cumulative and circular feedback processes. Industrial growth and population growth in cities have a mutually reinforcing impact on each other. Economic base multiplier and agglomeration economies induce a process of urban economic development, which is in turn favored by technological progress. Therefore, adoption and diffusion of innovation is of crucial importance for urban growth. Pred has illustrated his theory on the basis of industrial evolution and urban growth patterns in Western Europe. Especially his multiple-nuclei approach including spatial interaction patterns due to innovation diffusion and communication infrastructure may be regarded as a meaningful vehicle for explaining integrated spatial-urban growth processes.

His analysis has also some limitations: it is mainly a *growth* theory that fails to explain urban decline; it mainly pays attention to product innovations and neglects process and intellectual innovations; and it neglects the role of urban regional-national policies in urban dynamics, as well as the interactions between demographic and economic changes.

II. B. Thomas

Thomas (1972) has made an attempt at identifying urban development waves, based on a Schumpeterian view. He paid especial attention to the impacts of migration on cities and was able to show the existence of a wave-like urban growth pattern of U.S. cities before World War II. These migration patterns had also a direct impact on the construction, building and housing sector, so that a link could be found between urban evolution and economic growth patterns. After World War II such urban waves could no longer be identified, due to restrictions on immigration to the U.S.

Thomas' analysis has also shortcomings: it fails to explain post-war de-urbanisation processes; it neglects spatial impacts exerted by the urban system as a whole; and it is not based on an integrated theory of entrepreneurial and residential behavior in urban systems.

III. J. Jacobs

Jacobs (1977) explains urban cyclical processes from the diversity of functions (for instance, living, working, shopping, recreation) in a city. She introduces the notion of optimal urban diversity which is characterized by the following four elements: (1) a variety of functions, (2) a variable age structure of buildings, (3) an accessibility of urban facilities, and (4) an adequate concentration of urban population. The diversity of all these functions guarantees an optimal use of urban facilities. Lack of diversity may lead to a downward spiral movement of cities. On the other hand, in case of too many attractive functions of a city, a self-destruction of diversity in a free-market urban system may take place leading to congestion, land-use competition and environmental decay.

Jacobs' analysis also has various limitations: it does not pay satisfactory attention to external impacts on city life; it does not explain why non-market oriented (planned) cities are suffering from the same problems; and it is not based on a clear comprehensive economic view of entrepreneurial and household behavior in cities.

IV. R.D. Norton

Norton (1979) observes especially city life cycles in the U.S. He demonstrated that especially older cities are suffering from stagnation and decline due to their compact lay-out, the urban segregation and the inadequate tax base (due to the flight to suburbs by wealthier people). Newer cities are more spacious, less segregated and have a more satisfactory tax base. In addition, older cities are based on the industrialisation that took place in the last century. Due to the transition to the tertiary and quaternary sector, these older cities could not satisfactorily compete with newer cities and suburbs that were less rigid as to their economic structure and that were more innovation-oriented. These innovative forces have favored the rise of modern cities and caused the decline of older cities.

The following remarks can be made regarding Norton's analysis: it does not precisely explain the motives of innovation; it does not pay satisfactory attention to specific bottleneck factors in previously established cities; and it neglects the role of exogenous circumstances for urban life cycles (e.g. the role of urban governments).

V. L. van den Berg et al.

Van den Berg et al (1981) have made several investigations into urban development patterns, mainly based on migration and employment. Urban agglomeration are subdivided into two areas, viz. a core (center) and a ring (fringe). Urban development stages can then be characterized by the evolution of both the core and the ring, pending on the growth (or decline) rates of the core and the ring. The following phases in the urban development patterns are then distinguished: urbanisation, suburbanisation, de-urbanisation and re-urbanisation.

This analysis also has some limitations: it is more a descriptive analysis than a unifying economic theory for structural urban dynamics; the role of innovation in urban development is not satisfactorily included; and spatial interaction patterns (e.g., the evolution of an urban system as a whole) have not received sufficient attention.

VI. P.M. Allen et al.

Allen and associates (1981) have developed a set of models of urban settlement and structures as dynamic self-organizing systems. These models were mainly theoretical in nature and served to understand analytically urban systems as dynamic, non-linear entities, based on the application of principles of self-organizing systems to cities and regions. In doing so, several models have been developed with a special emphasis on the role of transportation in the processes of spatial and economic self-structuring. In addition, also links to decision-making, behavioral spatial patterns and hierarchical interactions were taken into account. By simulating urban development patterns, the economic resurgence and the dynamic evolution (including cyclical processes) of cities could be imitated. In the simulation model, especially the interactions between the employment patterns, the residential choice processes, the development of the tertiary sector, and the impacts of transportation sector were taken into account. The model itself was based on a simple non-linear dynamic relationship including attractiveness and bottleneck factors.

This model has clearly some limitations: it does not pay attention to the spatial and economic repercussions of innovative entrepreneurial behavior; it does not contain many clear policy controls; and it neglects the role of the housing market (and other social infrastructure categories) in the dynamic evolution of an urban system.

The foregoing theories demonstrate quite clearly the role of *technological progress* and *innovation* in urban life cycles (especially the contributions made by Pred, Thomas and Norton). The existence of such *cycles* has especially been studied by Thomas, Jacobs, Van den Berg et al., and Allen et al. The importance of *bottleneck* factors is also emphasized by several authors (especially Thomas, Jacobs, Van den Berg et al., and Allen et al.).

Despite relevant partial contributions, a unifying theory for urban evolution patterns is still lacking. It has been suggested by several authors that technological progress may be

an important factor behind urban development patterns, though hardly any attempt has been made to include innovation as an *endogenous* impulse in urban growth patterns. In order to shed more light on the intriguing role of innovation in spatial development patterns, the next two sections will be devoted to a discussion of long wave theories and innovations, and to their relevance for urban development cycles. According to the distinction made in section 1, both *external* and *internal* determinants for urban dynamics will successively be dealt with in section 3 and 4.

3. EXTERNAL FACTORS FOR URBAN FLUCTUATIONS

As mentioned in section 1, a distinction can be made between constrained and structural change. In an urban context, constrained dynamics may lead to a change of the urban structure, while structural dynamics may lead to a different configuration of urban dynamics (behavior of parameters or of relational structures, e.g.). This distinction may be important, as a transition from an upswing to a downswing of the urban economy is not necessarily due to a structural change in dynamics, though it may affect the urban structure.

Urban systems (and spatial systems in general) have never been in a static state, but have always been marked by a state of flux. This dynamics may to a certain extent be ascribed to drastic changes in the environment *outside* the urban system leading to profound changes in the urban system itself. For instance, the rise of oil prices in the seventies has had a great impact on urban transportation systems and urban residential patterns (see also Beaumont and Keys, 1982).

Especially in recent years, a revival of interest in structural economic changes has emerged, not only in a macro-economic sense of innovation patterns, but also in a geographical sense of a reorientation of cities and regions. Before dealing with external factors for urban dynamics, a brief review of long waves theories will be given.

For many decades already, economic fluctuations, long wave patterns and spatial dynamics have always drawn a great deal of attention by economic historians (cf. Adelman, 1965, and Schumpeter, 1939), but the emergence of the current economic recession and its inherent future uncertainty has stimulated a new interest in structural dynamics of economic systems (including *inter alia* such issues as industrial perturbations, (un)balanced growth, (un)stable equilibrium analysis, international and geographical equity, and multi-actor conflicts; see also Olson, 1982).

In this respect, Kondratieff's theory on long cycles has led to new reflections and scientific debates (see, for instance, Clark et al., 1981, Freeman et al., Kleinknecht, 1981, Mandel, 1980 and Mensch, 1979). Kondratieff's original theory distinguished five stages in a long-run cyclical pattern of a free enterprise economy: take-off, rapid growth, maturation, saturation and decline. The real existence of such long-term fluctuations is hard to demonstrate due to lack of historical data; in general, only price data have been used to test the long-wave hypothesis, although fortunately in recent years new efforts have been made to provide a more substantial empirical foundation for the long-wave hypothesis by means of industrial innovation data.

It is still an unresolved research question whether a pattern of long-run economic fluctuations is an *endogenous* phenomenon inherent in a certain socio-economic or political system. Endogeneity of a long wave pattern would require a theory explaining each new stage of a cycle from economic and technological developments during previous ones. A related problem is evidently the length of the cycle itself. Although Kondratieff cycles for a national economy are assumed to last for 40 to 50 years, several other cycles with a shorter time horizon may exist (Kuznets and Juglar cycles, e.g.). Short-run economic fluctuations (such as normal business cycles) are less interesting in this regard, as they do not deal with long-run changes in the structure of the economy.

There are various theoretical explanations--though not always rooted in empirical evidence--that aim at supporting the long wave hypothesis. Some of them regard long-term economic cycles at a national level as exogenous phenomena, but most of them aim at providing an endogenous explanation rooted in the development of the socio-economic system itself. Speaking about *urban* fluctuations, it may also be important to make a distinction between *exogenous* and *endogenous* urban cyclical patterns. Exogenous urban patterns are caused by external developments (e.g., at the (international level), which do not possess a specific urban component (uniform tax changes, e.g.), but are transferred to the urban territory through a top-down diffusion process. Endogenous urban growth patterns are a result of structural dynamics in the urban economy itself. By including the national and urban dimensions in one figure, one obtains Figure 1 describing the causes of fluctuations in (inter)national economies and in urban economies. A situation of an exogenous (inter)national development that is endogenous for a city is regarded as unfeasible.

Clearly, this figure is based on a top-down configuration from an (inter)national system toward cities. This may also lead to a situation where endogenous urban development and exogenous (inter)national developments are compatible, but in this context no analytical contributions to long wave pattern have been made thus far.

		U R B A N E C O N O M Y	
		exogenous causes of fluctuations	endogenous causes of fluctuations
NATIONAL AND INTER- NATIONAL ECONOMY	exogenous causes of fluctuations	I	not relevant
	endogenous causes of fluctuations	II	III

FIGURE 1. Causes of fluctuations in (inter)national-urban systems.

The following theories explaining the emergence of long waves in an economy may be distinguished (see Nijkamp, 1983):

1. Monetary theories. These theories take for granted the validity of the naive quantity theory by assuming an inverse relationship between price level and gold stock (see, for instance, Dupriez, 1947). Consequently, changes in gold stocks (caused *inter alia* by new exploitations of gold mines) might lead to economic fluctuations.

This theory belongs to category I, as it provides only an exogenous explanation for economic changes at both a national and an urban level. It is not particularly interesting for our purposes.

2. Resource theories. These theories argue that--from a global viewpoint--long-term international cyclical patterns may emerge due to variations in the supply of food stuff and raw materials. Such fluctuations are of course also reflected in price patterns (cf. Rostow, 1978). These theories provide an important exogenous explanation for changes at a national and urban level, but do not take into account the internal adjustment mechanism of urban systems. Clearly, they fall into category I of Figure 1.

3. Profit theories. In a competitive economy, profit rates are related to an acceleration and deceleration of capital accumulation, leading to fluctuating profit rates. In a downswing of a cycle, profit rates tend to decline until a depression is reached. However, once such a critical level has been reached, a counter-movement leading to a reverse growth pattern, may start. Such a countermovement may be induced by a higher technological efficiency in capital composition, by capital saving innovations or by a wage decline (cf. Mandel, 1980). The latter theory attempts to give an endogenous explanation, at least at the national level. It is not specifically an endogenous urban theory. Hence it belongs to class II.

4. Bottleneck theories. These theories are mainly related to the primary-secondary sector. Due to inertia in the primary production sector, a continuing rise in the industry will be hampered due to lack of intermediate products from the primary sector. This

may lead to overproduction and to lower profit rates in the primary sector. Then it is relatively more profitable to invest in the industrial sector, and so forth (cf. Delbeke, 1981). Bottleneck theories are providing relevant endogenous explanations for a growing economy, in which the service sector does not play a major role. They are not particularly interesting for an urban analysis and belong to category II.

5. Investment theories. The demand for productive capital demonstrates often a fluctuating pattern: a rapid expansion during a period of economic growth will increase the costs of capital, so that next less capital goods will be produced, followed by a price decline. This cyclical pattern of investment behavior may be explained from several reasons (cf. Clark, 1980 and Graham and Serge, 1980):

- the existence of indivisibilities in capital stocks may lead to shocks in the rate of use of existing capital;
- the stimuli provided by final demand to introduce more capital goods are marked by threshold effects hampering a smooth investment behavior (investment behavior is often a zero-one choice);
- the long gestation period of productive capital implies that when new investments come into operation, an entirely different economic situation may have emerged, so that unstable and/or cyclical growth patterns may be induced.

These investment theories are essentially based on over- and underinvestments due to inertia and rigidity in economic behavior. They provide an endogenous explanation for cyclical growth patterns and may also be relevant in urban systems. These theories may thus belong to classes II and III.

6. Systems dynamic theories. These theories assume that dynamic multiplier and accelerator mechanisms cause fluctuations throughout the economy. Smooth systems behavior are disrupted by discontinuous capital stock adjustments. Usually there is too much capital expansion in an upswing stage of the economy (especially

when the prospects are favorable) and too much contraction in a downswing stage (when prospects are less favorable). This lack of a fine tuning of positive and negative feedback effects may lead to a fluctuating development pattern of a system (cf. Forrester, 1977, and Jacobs, 1977).

The system dynamics theories are essentially a general case of the abovementioned investment theories. They may also belong to classes II or III.

7. Innovation theories. Innovation plays a major role in the efficiency of dynamic economic systems. Innovation is here regarded as a process of research, development, application and exploitation of a technology. Lack of innovation (or of diffusion of acceptance in innovation) may lead to cyclical growth patterns. For instance, economic recovery from a downswing stage will require much emphasis on innovation efforts during previous stages. Usually innovations are not spread uniformly over all sectors of the economy, but usually only over a limited number of key sectors. Consequently, innovation has a strong sectoral (and hence locational) dimension (cf. Kleinknecht, 1981, Mensch, 1979), and Pred, 1972). Innovation may have both a macro-economic component and an urban economic component. Thus these theories may belong to classes II and III, with a particular relevance for class III.

All abovementioned theories provide exogenous (1 and 2) or endogenous (3 - 7) explanations for cyclical growth patterns in an economy. Theories 1 - 4 have no specific urban dimension that links the urban development to its internal growth mechanism. This implies that an urban variant of theories 1 - 4 would require a formal model linking of the urban economy to its national (external) determinants. First, however, the internal (endogenous) mechanism of urban fluctuations will be discussed in section 4.

4. INTERNAL FACTORS FOR URBAN FLUCTUATIONS

It has already been indicated in the brief survey of section 2 that an urban economy may display also endogenous fluctuations caused by a variety of factors: social, demographic, political,

economic, and so forth. In the present section, particular attention will be paid to long-term cyclical patterns associated with economic and technological developments (see also Pred, 1972, and Thomas, 1981).

Urban economic and technological developments are particularly related to innovations, either *basic* innovations (leading to new products, new forms of even new industrial sectors) or *process* innovations (leading to new industrial processes in existing sectors). Especially basic innovations are assumed to take place periodically and cluster-wise, leading to economic fluctuations. In regard to this, it is usually assumed that after a period of growth a period of saturation may take place, leading to a recession. Thus, such growth processes can be described by means of a logistic (s-shaped) curve characterized by the following phases: introduction, growth, maturity, saturation and eventually decline.

Apart from innovations *per se*, also the filtering and diffusion processes through which new inventions evolve have to be mentioned. For instance, new innovations may emerge in city centers, while in the long-run the effects of implementing these innovations may be observed elsewhere (the heartland-hinterland paradigm).

Especially during a phase of saturation and decline, basic innovations and radical technological changes may be effective vehicles for again reaching a growing economy. This so-called 'depression-trigger' hypothesis has been advocated among others by Mensch (1979). However, Clark et al. (1981) and Freeman et al. (1982) have questioned the 'depression-trigger' hypothesis, because in their view radical investments may be too risky in a phase of an economic 'downswing'. Clearly, an economic recovery will only be possible, if the products emerging from basic technological innovations can be sold on the market, the so-called 'demand-pull' hypothesis (cf. Mowery and Rosenberg, 1979, and Norton, 1979).

The 'depression-trigger' hypothesis is extremely relevant for the urban economy, as it states that a stimulus to new economic

growth can only be given, if the necessary basic innovations in the productive sector--either *private* or *public*--are taking place. Private basic innovations would require the production of new commodities and/or the location of new firms within the urban territory. Public basic changes would require the implementation of new urban infrastructure investments. In this respect, the notion of infrastructure indicates all public overhead capital that is necessary for the take-off or growth of private activities. Examples of infrastructure categories are: streets, highways, medical, socio-cultural and educational facilities, housing, recreational and "quality of life" capital, and so forth.

The "demand-pull" hypothesis assumes that a sufficiently large market has to be created for the new products. This may be either the urban market itself or the outside market. Clearly, the "demand-pull" hypothesis is a contemporary variant of a Keynesian view of the urban economy. In this regard, the notion of "economic base" phenomena is especially relevant. Clearly, infrastructure capital also has a direct (Keynesian) demand effect. By combining both hypotheses, it is clear that an urban recovery from an economic down-swing will only take place if:

- the urban system provides a satisfactory supply of R & D capital;
- the urban system stimulates the implementation of directly productive (mainly private) and social overhead (mainly public) capital;
- a sufficient (potential) market for new products can be created (either within or outside the urban system).

Thus, the combination of R & D capital, productive capital, public overhead capital and new markets is a necessary condition to create radical technological changes (cf. Schmookler, 1966). Such changes are essentially the propulsive factors behind the process of structural urban economic developments.

The presence of a satisfactory urban infrastructure is thus a necessary condition for making a city a breeding place for new activities (cf. Rosenberg, 1976). This requires, in general, favorable educational facilities, communication possibilities, market entrance, good environmental conditions and agglomeration

favoring innovative activities. This may also explain why monopoly situations and industrial concentrations (including patent systems) often have greater technological and innovative opportunities. Although the data on innovations are in general poor (cf. Terlecky, 1980), there is a certain empirical evidence that only a limited number of industrial sectors account for the majority of innovations (electronics, petrochemicals and aircraft, for example), although in various cases small firms may also be a source of major innovations (micro-processors, for example) (see also Rothwell, 1979, and Thomas, 1981). This also implies that sectoral specialisation and urban fluctuations may go hand in hand.

Especially in recent years, several geographers have claimed that several urban growth patterns exhibit a clean break with the past (see among others, Berry and Dahmann, 1977; Vining and Kontuly, 1977; and Vining and Strauss, 1977), though this reversal of past trends has been questioned by others (see Gordon, 1982). Clearly, various countries have to a certain extent demonstrated a pattern of spatial and urban fluctuations in the post-war period. It appears that external economies and diseconomies have successively had a deep impact on urban systems in the Western world. Several theories have emphasized the close linkage between economic and urban developments (see Nijkamp, 1982b) such as: economic-base/multiplier models, (inter)regional input-output models, gravity and income potential models, growth pole models, center-periphery models, and unbalanced growth models and development potential models.

Two important questions emerge from the previous remarks, viz:

- is the urban economy autonomous, so that it may generate its own endogenous urban cycle?
- is there a minimum city size favoring urban innovations?

The first question needs a return to the above-mentioned theories on long-term cycles. The arguments given in the present section suggest that indeed an internal and endogenous urban fluctuation may exist, based on investment theories, systems dynamics theories or innovation theories. As indicated before,

these theories may be relevant at both the national and urban level as explanatory devices for long-term wave patterns (see category III in Figure 1). This leads to the following figurative representation:

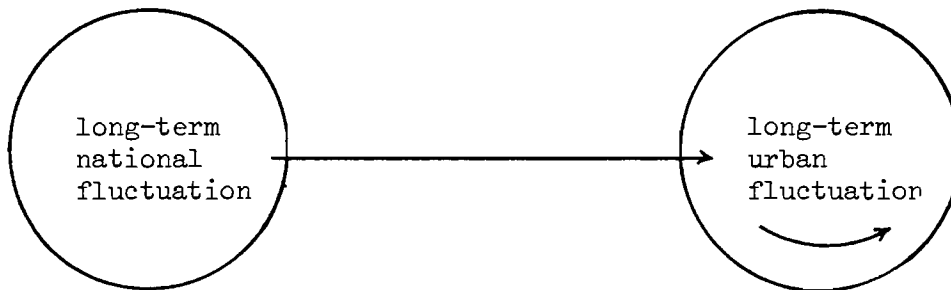


Figure 2. National and urban fluctuations.

Thus urban fluctuations may be explained from national long-term fluctuations (either exogenous or endogenous) or urban long-term fluctuations (exogenous or endogenous). In all such cases, dynamic evolutionary urban models may be used as meaningful operational tools for describing and analyzing urban innovation and diffusion processes (cf. Nelson and Winter, 1977).

The second question relates innovation to large-scale operations, leading to geographical concentration and specialisation. It is often claimed that city size favors innovative ability (cf. Alonso, 1971; Bluestone and Harrison, 1982; Carlino, 1977; Dunn, 1982; Jacobs, 1977; Kawashima, 1981; Pred, 1966; Richardson, 1973; and Thompson, 1977). It should be added, however, that the innovative potential in the U.S. which was traditionally concentrated in large urban agglomerations, is showing a declining trend, especially in the largest urban concentrations (see Malecki, 1979; Norton, 1979; and Sveikauskas, 1979).

A final remark is in order now. Innovative potential as a source of urban dynamics may be suffering from agglomeration diseconomies (so-called urban *bottleneck* factors), but in many cases it also needs a minimum R & D capital and infrastructure endowment (so-called urban *threshold* factors). Within (and also due to) these two limits, urban fluctuations may emerge and lead to unstable urban growth patterns.

5. TOWARD AN INTEGRATED MODEL FOR URBAN FLUCTUATIONS

The growth pattern of an urban system may demonstrate fluctuations, unbalanced growth processes and perturbations. In the present section, a more formal approach to urban long-term fluctuations will be presented, based on the previous sections. At first, an attempt will be made at presenting the main driving forces of an urban system by means of a simplified arrow diagram (see Figure 3). The assumption is made here that R & D capital can be separated from productive capital and infrastructure (social overhead) capital and other production factors, so that it has its own specific impact on the urban production efficiency. R & D capital is assumed to incorporate information and communication technology as well. Various production factors may thus exert an impact on urban dynamics, as reflected in the impact model of Figure 3. In the present paper, diffusion processes of innovations will not be dealt with, so that in this context the urban economy is regarded as a point economy.

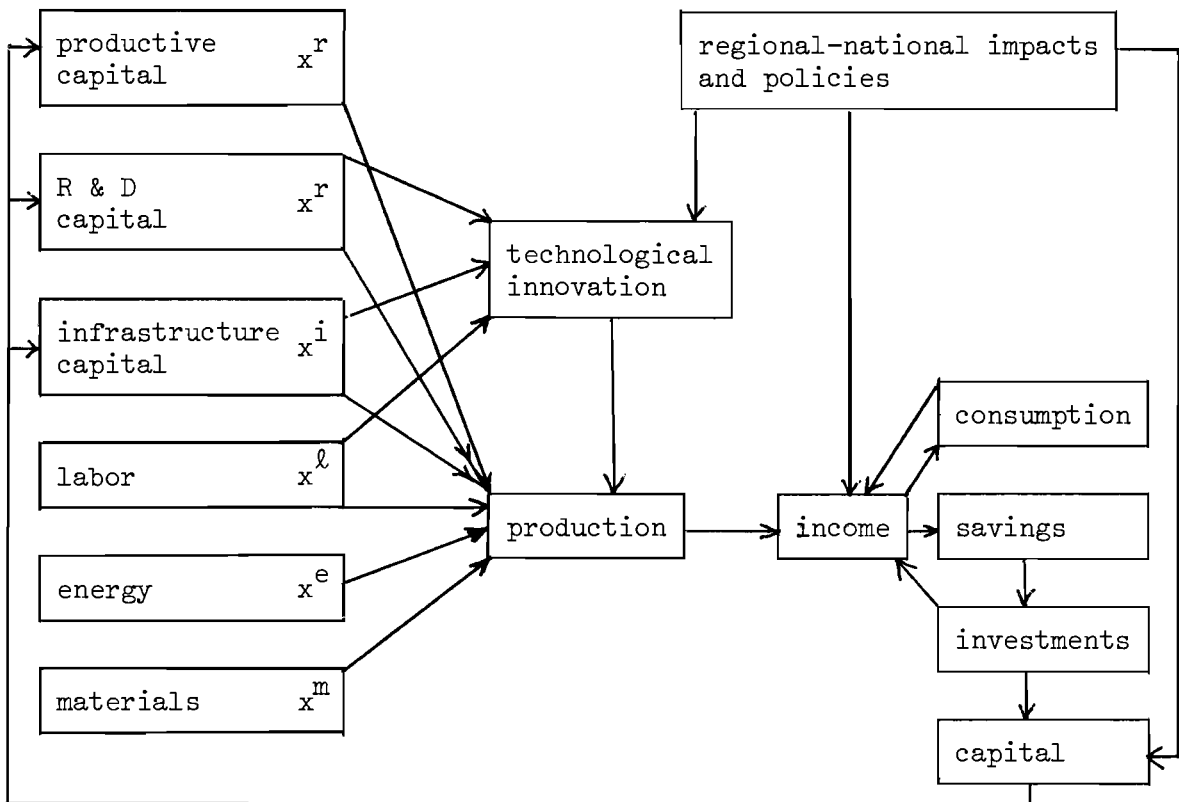


Figure 3. A dynamic urban system.

It is evident that in case of *qualitative* changes in a non-linear dynamic system several shocks and perturbations may emerge (see also Allen and Sanglier, 1979; Batten, 1981; Casetti, 1981; Dendrinis, 1981; Isard and Liosattos, 1979; and Wilson, 1981b). A first simple mathematical representation of the driving forces of such a system can be found in Nijkamp (1983). This simplified model was based on a so-called *quasi-production function* (including productive capital, infrastructure and R & D capital as arguments). The dynamics of the system was described by motion equations for productive investments, infrastructure investments and R & D investments. Several constraints were also added, for instance, due to maximum congestion effects and maximum consumption rates. Equilibrium solutions of the model were obtained by using optimal control theory.

In the present paper, the issue of *non-linear dynamics* will be further taken up. Specific attention will be given to a *specific kind of Volterra-Lotka equation* for describing a complex dynamic urban economy.

The boundaries of this urban system are assumed to be known, and diffusion processes to other areas are assumed away.

Suppose now a (closed) urban economy characterized by a "generalized" production function including productive capital (x^k), labor (x^l), energy (x^e), materials (x^m), public infrastructure (x^i) and R & D activities (x^r) as arguments. The first four components (x^k, x^l, x^e and x^m) are often found in modern KLEM production functions dealing with substitution effects between capital, labor, energy and materials (see for instance Lesuis et al., 1980). The fifth component indicates the necessary *public overhead capital* needed as a complement to private productive capital, along the lines suggested by Hirschman (1958) in order to achieve a balanced growth strategy. The inclusion of this infrastructure component (in a broad sense) had led to the notion of the above-mentioned *quasi-production function* in recent literature (see for instance Biehl, 1980; and Nijkamp, 1982b). Finally, the sixth component is reflecting the *innovation* effects due to R & D investments (including information technology) in the urban agglomeration. Hence, the following generalized production function may be assumed:

$$y = f(x^k, x^l, x^e, x^m, x^i, x^r) \quad , \quad (1)$$

where y is the volume of urban production. The parameters of the urban production technology depend on the general state of the technology (at a national-regional level) and on the specific agglomeration factors (at the urban level). If a normal Cobb-Douglas specification is assumed, one may write (1) as the following static generalized production function:

$$y = \alpha(x^k)^\beta (x^l)^\gamma (x^e)^\delta (x^m)^\varepsilon (x^i)^\xi (x^r)^\eta \quad , \quad (2)$$

where the parameters β, \dots, η reflect the production elasticities concerned. The production elasticities are assumed to be positive on the range (y^{\min}, y^{\max}) . Below a certain *minimum threshold level* y^{\min} , the urban size may be too small for agglomeration advantages, so that then a marginal increase in one of the production factors may have a zero impact on the urban production volume. This situation indicates that a city needs a minimum endowment with production factors before reaching a self-sustained growth. Furthermore, beyond a certain *maximum capacity level* of urban size, bottlenecks (congestions, for example)--due to a high concentration of capital--may cause a *negative* marginal product of some of the production factors (e.g., productive capital, R & D). If the static model (2) is used in a dynamic context, then within the relevant range (y^{\min}, y^{\max}) the shifts in the urban production volume in a certain period of time may be approximated by means of the following discrete time version of (2):

$$\Delta y_t = (\beta k_t + \gamma l_t + \delta e_t + \varepsilon m_t + \xi i_t + \eta r_t) y_{t-1} \quad , \quad (3)$$

with

$$\Delta y_t = y_t - y_{t-1} \quad (4)$$

and

$$h_t = \frac{x_t^h - x_{t-1}^h}{x_{t-1}^h} \quad , \quad h = k, l, e, m, i, r \quad (5)$$

Thus the arguments of (5) are written as relative changes of the original variables. This discrete approximation of a model with continuous time is valid here within the range for which the structure of the system is stable.

Within the range (y^{\min}, y^{\max}) , the urban system will exhibit a non-cyclical growth. This self-sustained growth path may be drawing to a close due to two causes:

- external: scarcity of production factors or lack of demand
- internal: emergence of congestion effects leading to negative marginal products.

External factors will only imply that the system will move toward an upper limit set by the constraint concerned. Internal factors may lead to perturbations and qualitative changes in systemic behavior. Suppose for instance, a congestion effect caused by too high a concentration of capital in an urban agglomeration. Then each additional increase in productive capital will have a negative impact on the urban production level. In other words, beyond the capacity limit y^{\max} an auxiliary relationship reflecting a negative marginal capital product may be assumed:

$$\beta_t = \hat{\beta} (y^{\max} - \kappa y_{t-1}) / y^{\max} \quad (6)$$

This implies that the production elasticity has become a time-dependent variable. Analogous relationships indicating a negative marginal product may be assumed for all remaining production factors. Substitution of all these relationships into (3) leads to the following adjusted dynamic urban production function:

$$\Delta y_t = (\hat{\beta} k_t + \hat{\gamma} \lambda_t + \hat{\delta} e_t + \hat{\epsilon} m_t + \hat{\xi} i_t + \hat{\eta} r_t) (y^{\max} - \kappa y_{t-1}) y_{t-1} / y^{\max} \quad (7)$$

This is seemingly a fairly simple non-stochastic dynamic relationship, but it can be shown that this equation is able to exhibit unstable and even erratic behavior leading to a-periodic fluctuations.

The standard format of (7) may be written as:

$$\Delta y_t = v_t (y_t^{\max} - \kappa y_{t-1}) y_{t-1} / y_t^{\max} \quad , \quad (8)$$

with:

$$v_t = (\hat{\beta} k_t + \hat{\gamma} \ell_t + \hat{\delta} e_t + \hat{\epsilon} m_t + \hat{\xi} i_t + \hat{\eta} r_t) \quad (9)$$

Equation (8) is essentially a part of a Volterra-Lotka type model which has in recent years often been used for modeling predator-prey relationships in population biology (see also Goh and Jennings, 1977; Jeffries, 1979; Pimm, 1982; and Wilson, 1981a). This model in difference equation form has been dealt with among others by May (1974), Li and Yorke (1975) and Yorke and Yorke (1975). Applications in a geographical setting can be found in Brouwer and Nijkamp (1983) and Dendrinis (1983) among others. In the present context, the dynamic trajectory of the urban economy can be studied more precisely by rewriting (8) as:

$$\Delta y_t = v_t (1 - \kappa y_{t-1} / y_t^{\max}) y_{t-1} \quad (10)$$

Equation (10) is a standard equation from population dynamics. It should be noted that logistic evolutionary patterns may also be approximated by a (slightly more flexible) Ricker curve (see May, 1974). In that case, the exponential specification precludes the generation of negative values for the y variables in simulation experiments, a situation that may emerge in relation to equation (10). Model (10) has some very unusual properties. On the basis of numerical experiments, it has been demonstrated by May (1974) that this model may exhibit a remarkable spectrum of dynamical behavior, such as stable equilibrium points, stable cyclic oscillations, stable cycles, and chaotic regimes with a-periodic but bounded fluctuations. Two major elements determine the stability properties of (8), viz. the initial values of y_t and the growth rate for the urban system (which is depending on v_t). Simulation experiments indicated that especially the growth rate has a major impact on the emergence of cyclic or a-periodic fluctuations.

May has also demonstrated that a stable equilibrium may emerge if $0 \leq v_t \leq 2$; otherwise stable cyclic and unstable fluctuations may be generated. Li and Yorke (1975) have later developed a set of sufficient conditions for the emergence of chaotic behavior for general continuous difference equations. Clearly, in a discrete model the potential chaotic behavior depends on the absolute value of v_t , which in turn depends on the metric of measuring the relevant time units.

The general problem of discrete versus continuous model specification is very intriguing. Though time is essentially a continuum, for practical reasons (data availability, observations, sampling) a discretization is usually necessary. Clearly, in a space-time context this may lead to specification errors in a way analogous to the scale and aggregation problem in geography. Thus the formulation of appropriate discrete-time analogues for continuous processes is far from easy (see also Sonis, 1983).

Consequently, the conclusion may be drawn that--due to the presence of a capacity limit y^{\max} --a city may exhibit a wide variety of dynamical or even cyclical growth patterns. A long wave pattern of an urban economy is compatible with the above-mentioned urban production technology, but this is only a specific case. A wide variety of other dynamic (and sometimes unstable) trajectories may arise as well. This heterogeneity in urban development patterns is also reflected in current trends of cities all over the world. The shape of urban fluctuation curves is determined by the initial city size and by the growth rate of the urban production system. This growth rate is a weighted average of the individual growth rates of the urban production factors.

In contrast with many biological growth functions, however, the growth rate v_t is not a constant, but a time-dependent variable. Consequently, it may be used as a control variable so as to generate a more stable urban growth path. In this respect, relationship (8) may be used in the context of an optimal control approach. It should be noted that equation (8) is essentially a signomial specification, for which in the framework of geometric programming analysis appropriate solution

algorithms have been developed (see among others Duffin and Peterson, 1973; and Nijkamp, 1972).

Apart from a programming approach, one may also introduce an auxiliary relationship for R & D investments, as one may assume that technological progress may be one of the tools to attack urban capacity constraints (the so-called "depression-trigger hypothesis"). This might imply that the efforts made in the R & D sector have to increase as a city is surpassing its critical upper limit. Thus R & D investments can be used to improve the locational profile of a city, for both entrepreneurs (e.g., by improving accessibility) and residents (e.g., by improving urban quality of life). Then the following auxiliary relationship may be assumed:

$$r_t = \lambda (y_{t-1} - \pi Y^{\max}) / Y^{\max} \quad (11)$$

Substitutions of (11) into (10) yields the following result:

$$\Delta y_t = \{v_t^* + \hat{\eta} \lambda (y_{t-1} - \pi Y^{\max}) / Y^{\max}\} (Y^{\max} - \kappa y_{t-1}) y_{t-1} / Y^{\max} \quad (12)$$

where:

$$v_t^* = \hat{\beta} k_t + \hat{\gamma} \ell_t + \hat{\delta} e_t + \hat{\epsilon} m_t + \hat{\xi} i_t \quad (13)$$

Relationship (11) may also be related to a vintage view of urban capital. If after some time periods the existing capital becomes less efficient (including a decline in urban development), R & D capital may be used to compensate for this decline. This implies that--after the implementation of a new technology--an upswing may take place based on a more efficient capital stock. It is of course a major problem to start R & D activities in the right time period so as to achieve a balanced growth path. Due to lack of insight and monopoly tendencies (innovations may be monopolized through patent systems), a fine tuning is not likely to take place. This may of course lead to various fluctuations (see also Figure 4).

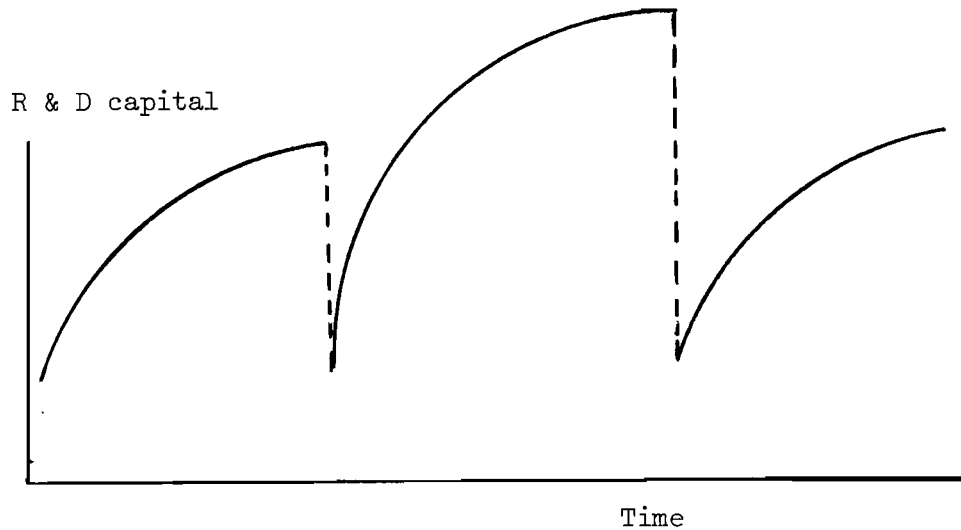


Figure 4. Fluctuations in R & D investments.

Relationship (12) is essentially a *nested* dynamic difference equation. The perturbation caused by the congestion effects may be neutralized or enforced by the R & D investments in the city, depending on the fine tuning of innovations to urban fluctuations. Thus the ultimate growth path may be a superimposition of two dynamic structures. Clearly, the above-mentioned fine tuning might again be achieved by an optimal control approach. In that case, however, one has to include additional constraints, as the amounts of money spent for productive investments, labor, energy, materials, public overhead investments and R & D investments have to be reserved from savings emerging from the income generated by the urban production value (see also Nijkamp, 1983). In addition, according to Figure 3, a balanced urban growth also requires a substantial amount of the urban production value to be earmarked for private and public consumption purposes (the so-called "demand-pull hypothesis").

6. OUTLOOK

The model described in this paper provides a simplified picture of a complex urban system driven by production and innovation effects. Despite its simplicity, it is able to encompass various mechanisms that act as driving forces for structural changes of a dynamic urban system. In addition, it also sets out

the conditions under which stable or non-stable urban growth patterns may emerge. Various ways are now open to extend the research presented above, such as the introduction of multiple conflicting objective functions for urban development policy, the introduction of spatial spillover effects in an open urban system so as to include also top-down impacts from a regional or national level (or central city-hinterland interactions), or the introduction of a set of separate difference (or differential) equations for specific urban sectors or markets (employment, housing, transportation, facilities, etc.).

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