PRICE-OUTPUT DYNAMICS AND RETURNS TO SCALE

Geoffrey Heal

September 1983
CP-83-43

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

FOREWORD

This Collaborative Paper is one of a series embodying the outcome of a workshop and conference on *Economic Structural Change: Analytical Issues*, held at IIASA in July and August of 1983. The conference and workshop formed part of the continuing IIASA program on Patterns of Economic Structural Change and Industrial Adjustment.

Structural change was interpreted very broadly: the topics covered included the nature and causes of changes in different sectors of the world economy, the relationship between international markets and national economies, and issues of organization and incentives in large economic systems.

There is a general consensus that important economic structural changes are occurring in the world economy. There are, however, several alternative approaches to measuring these changes, to modeling the process, and to devising appropriate responses in terms of policy measures and institutional redesign. Other interesting questions concern the role of the international economic system in transmitting such changes, and the merits of alternative modes of economic organization in responding to structural change. All of these issues were addressed by participants in the workshop and conference, and will be the focus of the continuation of the research program's work.

Geoffrey Heal
Anatoli Smyshlyaev
Ernö Zalai

ABSTRACT

I consider the dynamics of an economy where prices move in response to excess demand, and outputs change according to the difference between price and cost. If there are economies of scale in production, these adjustment processes lead the economy to one of two regimes. In one regime, output, productivity, and profits all rise, while prices fall. In the other, output, productivity, and profits all fall, while prices rise. Depending on initial conditions which are policy-amenable, the economy moves to self-reinforcing growth, or to stagflation. An exogenous shock, such as a rise in the price of an imported input or a sharp restriction of demand, may transfer a previously healthy economy from the growth-with-price-stability regime to the stagflationary regime.

PRICE-OUTPUT DYNAMICS AND RETURNS TO SCALE*

Geoffrey Heal**

1. INTRODUCTION

In this paper I study the dynamics of prices and outputs in an aggregative macroeconomic model. I show that the stability of the system is very sensitive to the nature of returns to scale in production. Diminishing returns guarantee the existence of a stable price-output combination. However, with increasing returns, the system studied may be unstable, and may be trapped either in a regime of falling output, falling profits, and rising prices, or in a regime of rising output, rising profits, and falling prices. One could think of these as the "vicious" and "virtuous" circles that have often been discussed in the context of macroeconomic performance.

The adjustment processes that give rise to this result are very classical in nature and were first studied by Samuelson (1947) and later by Arrow and Hurwicz (1963). I assume price to adjust in Walrasian fashion, rising in response to excess demand, and vice versa. The quantity adjustment rule is equally straightforward: firms increase output if price exceeds average cost, and vice versa. With diminishing or constant returns in production, these two processes lead the system to converge to an equilibrium at which demand equals supply, and price equals average cost. With increasing returns, however, another possibility emerges. This is that the system will converge to a state where markets almost clear. In this state, profits and output are rising and prices falling, or alternatively profits and output are falling and prices rising.

*I am grateful to Graciela Chichilnisky and Bill Nordhaus for valuable discussions of the material in this paper.
**Graduate School of Business, Columbia University, New York, USA.

The intuitive explanation of these results is simple. With economies of scale in production, an increase in output levels with prices constant will lead to higher profits. We can therefore even reduce prices slightly and still have higher profits. But lower prices will lead to higher demand, a further increase in output, and so on through the cycle again. To describe the vicious circle of falling profits and output and higher prices, the same argument can be made in reverse.

2. THE MODEL

The model to be studied could be thought of as an aggregative one-sector macroeconomic model, in the spirit of neoclassical growth models and many subsequent macrodynamic models. Alternatively, it could refer to the behavior of a single sector within a larger economy.

There is a single output, whose quantity is denoted by q and which is sold at a price p. The average cost of production is given by a smooth function $c(q)$, and demand is given by $D(p)$--again a smooth function.

The basic differential equations of the system are:

$$\dot{q} = a(p - c(q)) \tag{1}$$

$$\dot{p} = b(D(p) - q) \tag{2}$$

where a and b are positive constants. We define p^* and q^* as the equilibrium p and q values that make $\dot{q} = \dot{p} = 0$, i.e.

$$p^* = c(q^*) ; D(p^*) = q^* \tag{3}$$

The behavior of (1) and (2) can be studied graphically. Three cases may be distinguished. Figure 1 shows a case where $dc(q)/dq > 0$, corresponding to diminishing returns. (The demand curve is always assumed to be downward sloping.) Inspection of eqns. (1) and (2) establishes that the directions of motion in the four regions I, II, III, and IV of Figure 1 are as shown. This suggests that (p^*, q^*) is a globally stable equilibrium. This is readily verified by linearizing the system (1) and (2) about (q^*, p^*) and evaluating the roots of the resulting matrix. We have

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -a\,dc/dq & a \\ -b & b\,dD/dp \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} \tag{4}$$

If r is an eigenvalue of the matrix in (4), then r satisfies the following quadratic equation:

$$r^2 + r(ac' - bD') + ab(1 - c'D') = 0 \tag{5}$$

where $c' = dC/dq$ and $D' = dD/dp$. Equation (5) has two negative real roots if and only if $ac' - bD' > 0$ and $ab(1 - c'D') > 0$. But with $D' < 0, c' > 0$ implies that these conditions hold, confirming the motion shown in Figure 1 in the neighborhood of (q^*, p^*).

Figure 2 shows a case where there are mildly increasing returns in production, and the average cost curve cuts the demand curve from below: (q^*, p^*) is again a stable equilibrium.

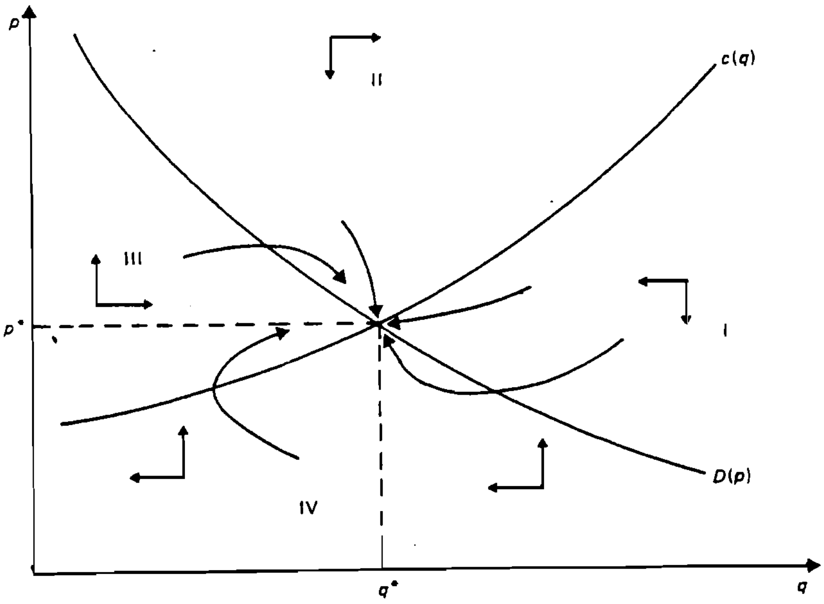

FIGURE 1 (q^*, p^*) is stable with a rising average cost curve.

More interesting is Figure 3, where returns to scale are sufficiently great for the average cost curve to cut the demand curve from above, i.e. the inequality

$$c' < 1/D' < 0 \qquad (6)$$

is satisfied. Now the roots of (5) are

$$r = -\frac{1}{2}(ac' - bD') \pm \frac{1}{2}((ac' - bD') - 4ab(1 - c'D'))^{\frac{1}{2}}$$

Since by (6) $c'D' > 1$, there are two real solutions in r, one positive and one negative. (q^*, p^*) is therefore now a saddle point. There is a stable manifold (shown by the broken line) along which motion is purely towards (q^*, p^*), and an unstable manifold (shown by the heavy line leading out of (q^*, p^*)) along which motion is exclusively away from (q^*, p^*). The slopes of these manifolds at (q^*, p^*) are given by the eigenvectors of the matrix (4) corresponding to the stable and unstable roots, respectively. From this one can readily verify that the unstable manifold has a negative slope at (q^*, p^*). It is clear from Figure 3 that the system converges to the arms Ex or Ey of the unstable manifold from almost every initial condition. In fact, for suitable choices of the speed-of-adjustment parameters a and b in eqns. (1) and (2), the system will spend most of its time very near one of these arms. Suppose that b, the constant controlling the speed with which price responds to excess demand, is very large. The price therefore adjusts very rapidly to eliminate excess demand. In this case the vertical component of the trajectories in Figure 3 will be large relative to the horizontal component, producing trajectories which, like those in Figure 4, move rapidly to the unstable manifold and then remain in the

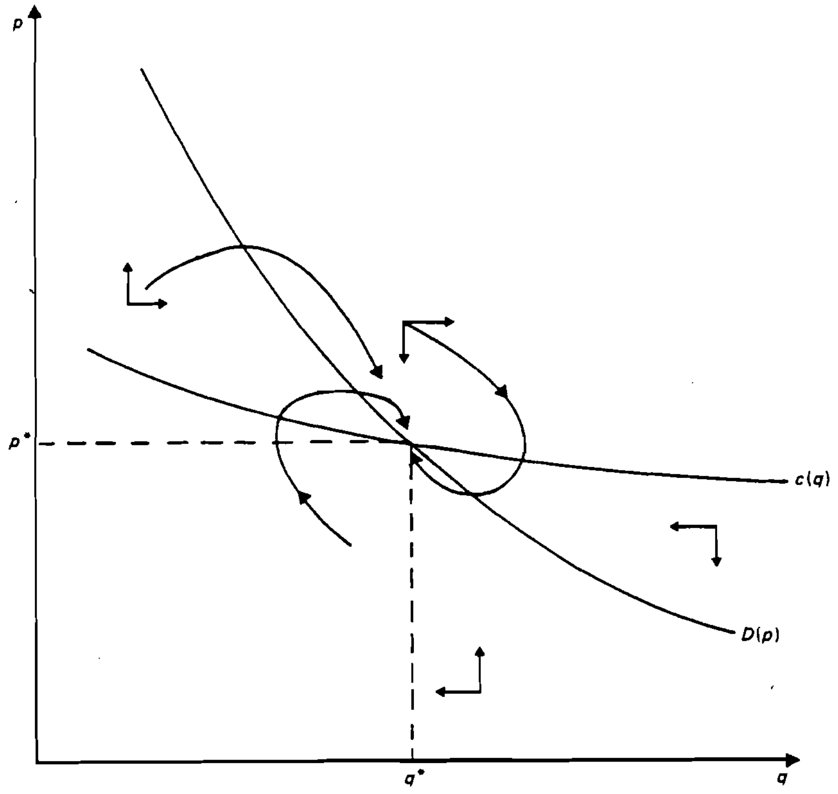

FIGURE 2 (q^*, p^*) is stable with a falling average cost curve that cuts $D(p)$ from below.

neighborhood of this.

The next step is to examine the economic properties of trajectories that lie in the neighborhood of the unstable manifold. Obviously, along Ex output is rising and price is falling; the opposite is true along Ey. It is also the case that along Ex profit per unit produced is rising, as the difference between price p and average production cost $c(q)$ is rising. Total profits are also rising, as output is rising. Conversely, along Ey there is an increasing loss on each unit sold. Our findings can be summarized as follows:

Proposition 1. *Consider the price and quantity adjustment system (1) and (2). Let returns to scale increase sufficiently that $c'D' > 1$. Then from almost any initial conditions the system will converge either to a regime of rising output, rising profits, and falling prices, or to a regime of falling output, falling profits, and rising prices. By making the rate b at which price responds to excess demand sufficiently large, demand can be made as near as desired to supply in either regime.*

Proof. The only point in Proposition 1 still requiring formal proof is the assertion that the market can be brought arbitrarily close to clearing by picking b large enough.

The slope of the unstable manifold yEx is, at (q^*, p^*), the slope of the eigenvector of the linear system (4) corresponding to the positive root. This eigenvector is the solution of

$$Ax = \bar{r}x \qquad (7)$$

where $\bar{r}$ is the positive root of the characteristic polynomial (5) and A is the

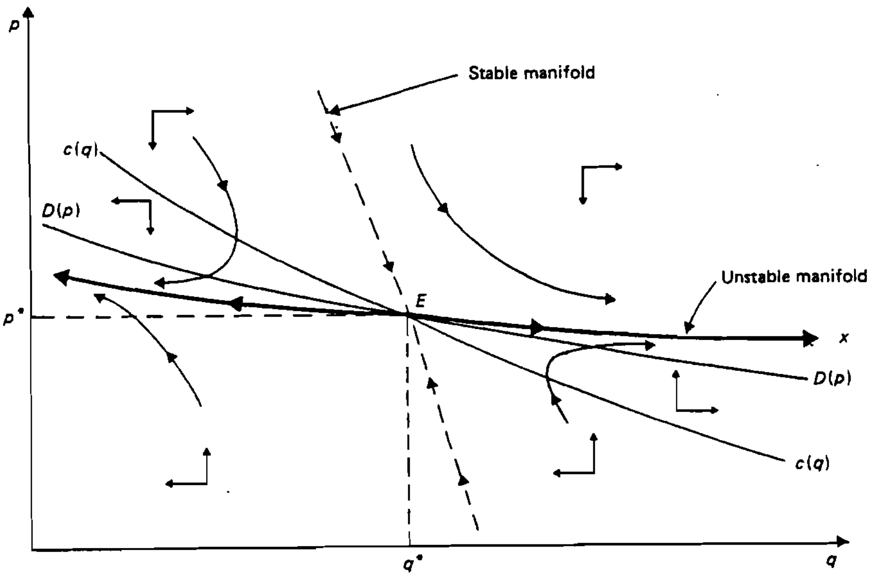

FIGURE 3 (q^*, p^*) is a saddle point when $c'D' > 1$. Representative trajectories are shown; these must cross $D(p)$ horizontally ($\dot{p} = 0$ if $D(p) = q$) and $c(q)$ vertically ($\dot{q} = 0$ if $p = c(q)$).

matrix in (4). Letting $x = (x_1, x_2)$, (7) can be expanded to give

$$-x_1(ac' + \bar{r}) + ax_2 = 0$$

$$-bx_1 + x_2(bD' - \bar{r}) = 0$$

and from the second of these equations,

$$x_1 / x_2 = D' - \bar{r} / b$$

so that as b becomes large, x_1/x_2 tends to D'. As x is measured in terms of deviations from the equilibrium (q^*, p^*), this proves that for large b the slopes of the graph of $D(p)$ and of the unstable manifold are the same at the equilibrium: hence in the neighborhood of (q^*, p^*) these curves coincide. But along the graph of $D(p)$, supply and demand are equal, as required.

3. CONCLUSIONS

I have analyzed the dynamics of an economy that adjusts in a very simple and intuitively straightforward way. Price responds to excess demand; output responds to the difference between price and cost. If there are increasing returns in production, then this very simple system produces rather rich results. The economy moves toward one of two regimes. In both, markets are "nearly" clearing. In one, all the policy-maker's goals are satisfied--outputs and profits rise, and prices fall. The other represents everything a policy-maker should dread--falling output, falling profits, and rising prices. One

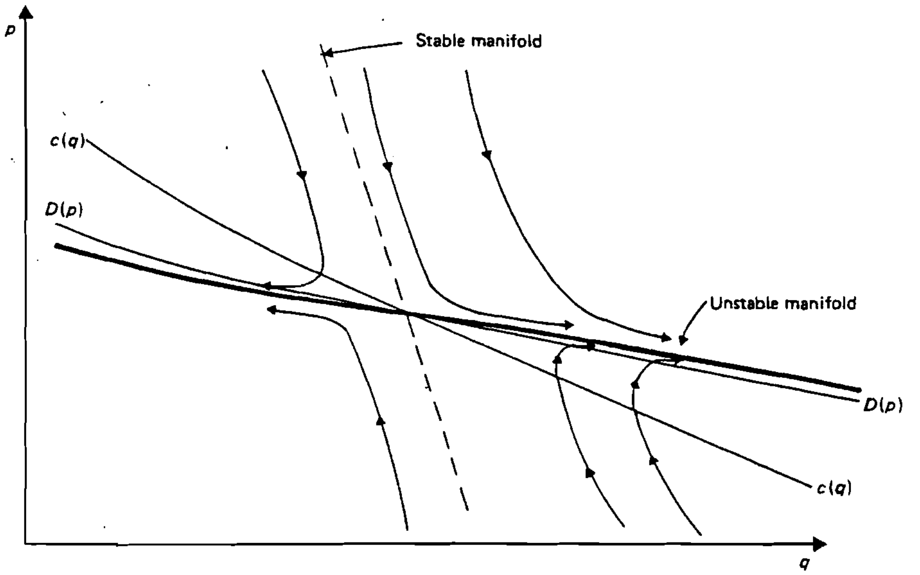

FIGURE 4 This reproduces the situation illustrated in Figure 3 when b is large. The trajectories spend all but an arbitrarily short time period in the neighborhood of the unstable manifold lying near the graph of $D(p)$.

regime represents growth with price stability, the other stagflation.

A number of earlier writers have commented on the apparent existence of "vicious" and "virtuous" circles of macroeconomic performance (Beckerman et al. 1965, Kaldor 1967). It is interesting to be able to provide a very simple theoretical model of the sources of this divergence. Kaldor (1967) and Cripps and Tarling (1973) speculated on the role of increasing returns in generating these self-reinforcing cycles; although our model is rather different from those that they had in mind, there is clearly a point of contact.

It is also interesting to consider the results of Houthaker (1979) in the light of the present model. In a cross-section study of a number of US industries, he found that those with above-average rates of output growth had above-average rates of productivity growth and below-average rates of price increase. Conversely, sectors whose outputs grew relatively slowly suffered from low relative rates of productivity growth and high relative inflation rates.

These findings are obviously quite consistent with the regimes described in Figures 3 and 4 and in Proposition 1: Some sectors were following the favorable part, and others the unfavorable part of the unstable manifold. Kaldor (1967) has reported a very similar set of findings in an international context; again, his findings are quite consistent with the regimes analyzed here.

I turn now to the policy implications of this analysis. (These are also discussed in general terms in Chichilnisky and Heal (1983).) In Figures 3 and 4 there are certain initial values of q and p for which the economy converges to the "virtuous" circle Ex, and others for which it converges to the less attractive regime along Ey. Consider now the effect of an exogenous increase in costs, which shifts the curve $c(q)$ to the right. This might, for example, be caused by a substantial increase in energy prices, or by a devaluation of the currency of an input-importing country. Then the set of initial conditions on which the system moves to the favorable outcome will be reduced, and that on which it moves to the unfavorable enlarged. An economy that was initially just in the favorable regime will be shocked by this cost increase into a regime of falling output and profits and rising prices. In other words, an exogenous cost increase could shift a previously healthy economy into a stagflationary regime.

A similar analysis can be applied to the consequences of a substantial drop in demand, represented by a downward shift of the $D(p)$ curve. Again the set of initial conditions leading to the favorable regime is reduced, and that leading to stagflation increases. Again, a previously healthy economy can be shifted from one regime to another. Such a drop in demand could be caused by domestic deflation, or by the growth of overseas competition.

Finally, let me comment on the limitations of this model as it stands, and on its possible extensions. There are three extensions that seem natural and obvious. One is to consider the adjustment processes (1) and (2) as applying to aggregate price and output dynamics in an open economy facing competition in the world market. In this case, the cycle of falling prices and rising output would lead to an increase in competitiveness in world markets, which would reinforce the tendencies already existing. Conversely, rising prices and falling output would reduce international competitiveness. Opening the model to world competition would therefore reinforce our conclusion that, if $c'D' < 1$, the system is unstable and converges either to a regime of growth and price stability, or to a regime of stagflation.

A second extension would be to link the demand side of the model to the production side, recognizing that higher output levels mean more employment of labor and higher profits, and so shift the demand curve to the right. This would clearly make the dynamics of the system more complex: for $c'D' < 1$ the intersection of the cost and demand curves would shift up to the left along the cost curve, causing the stable and unstable manifolds to move over time. In effect, once the economy entered the favorable (or unfavorable) regime, the set of initial conditions leading to that regime would be increased, but the regime itself would not change its character.

A third, and more substantial extension would be to link the level of output to the employment of labor, and to recognize that an increase in demand for labor would bid up the real wage and so raise the cost curve. It can be shown that this would establish forces that would tend to terminate or to moderate the periods of expansion or of contraction, and might even produce cyclical solutions.

REFERENCES

Arrow, K.J., and L. Hurwicz (1963). Decentralization and Computation In Resource-Allocation. In R. Pfouts (Ed.), *Essays in Economics and Econometrics in Honor of Harold Hotelling*. University of North Carolina Press, Chapel Hill, North Carolina.

Beckerman, W., et al. (1965). *The British Economy in 1975*. Cambridge University Press.

Chichilnisky, G., and G.M. Heal (1983). *Trade and Development in the Eighties*. Cambridge University Press, forthcoming.

Cripps, F., and R.J. Tarling (1973). *Growth in Advanced Capitalist Economies*. Cambridge University Press.

Houthaker, H. (1979). Growth and Inflation: Analysis by Industry. *Brookings Papers on Economic Activity*, I:240–257.

Kaldor, M. (1967). *The Causes of the Slow Rate of Growth of the U.K. Economy*. Cambridge University Press.

Samuelson, P.A. (1947). Market Mechanisms and Maximization. In J.E. Stiglitz (Ed.), *The Collected Papers of Paul Samuelson*, Vol. I. MIT Press, Cambridge, Massachusetts.