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# PRODUCTION TECHNOLOGIES AND THE PHILLIPS CURVE

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## FOREWORD

This Collaborative Paper is one of a series embodying the outcome of a workshop and conference on *Economic Structural Change: Analytical Issues*, held at IIASA in July and August of 1983. The conference and workshop formed part of the continuing IIASA program on Patterns of Economic Structural Change and Industrial Adjustment.

Structural change was interpreted very broadly: the topics covered included the nature and causes of changes in different sectors of the world economy, the relationship between international markets and national economies, and issues of organization and incentives in large economic systems.

There is a general consensus that important economic structural changes are occurring in the world economy. There are, however, several alternative approaches to measuring these changes, to modeling the process, and to devising appropriate responses in terms of policy measures and institutional redesign. Other interesting questions concern the role of the international economic system in transmitting such changes, and the merits of alternative modes of economic organization in responding to structural change. All of these issues were addressed by participants in the workshop and conference, and will be the focus of the continuation of the research program's work.

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**ABSTRACT**

In an economy with increasing returns to scale in production, wage changes and unemployment levels are shown to fluctuate systematically. Such fluctuations are part of the stable long-run configuration of the economy. They generate data sets in which wage changes show a negative Phillips-type correlation with the level of unemployment. This negative correlation is a reflection of the conventional Walrasian price adjustment process in the labor market, and does *not* imply that across equilibria there is a negative relation between wage changes and unemployment. In particular, it does *not* imply the existence of a trade-off between inflation and unemployment.

# PRODUCTION TECHNOLOGIES AND THE PHILLIPS CURVE

Graciela Chichilnisky* and Geoffrey Heal*

## 1. INTRODUCTION

The term "Phillips curve" has been widely used for more than a quarter of a century to describe a negative relationship between wage changes and the rate of unemployment. A.W. Phillips (1958) first noted that low rates of wage increase are associated with high rates of unemployment, and vice versa. While both the nature and the cause of this relationship have been the subject of extensive discussion, there is widespread agreement about the existence of such a relationship. (For a recent empirical study, see Sargan 1980.)

The policy implications of the Phillips curve have been an issue of contention, with some commentators arguing that it establishes the existence of a trade-off between inflation and unemployment, while others, mostly of a monetarist persuasion, argue that the curve is too much a short-run phenomenon for such conclusions to be drawn.

In spite of the extensive empirical and policy-related literature on the Phillips curve, there have been remarkably few attempts to give rigorous microeconomic foundations to this wage change-unemployment relationship. We show below that in fact one can formulate a very simple microeconomic model in which the Phillips curve is seen just as a reflection of the conventional Walrasian adjustment process; in this process prices increase with excess demand, and decrease with excess supply. Low levels of unemployment correspond to excess demand for labor, and high levels to excess supply; these have the normal Walrasian impact on wages. *However*, a substantial qualification is needed: a nontrivial Phillips curve can be guaranteed to emerge as the

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reflection of Walrasian adjustments in the labor market *only if there are economies of scale in production.* Otherwise the Phillips relation may be purely a disequilibrium response to an exogenous shock, showing no stability or persistence over time.

Our model has clear implications concerning the policy-relevance of the Phillips curve. These are akin to the monetarist's conclusions, though they emerge for very different reasons. Our model implies that a market equilibrium is characterized by a distribution of wage changes and unemployment levels. This is shown to be such that a sample of observations on the variables will yield a negative correlation between wage changes and levels of unemployment. The Phillips relationship between these variables is, however, not in any sense causal. Equilibrium values of wage changes and unemployment fluctuate, and are correlated in such a way as to lead to a negative observed relationship between these variables. A systematic attempt to change the level of unemployment produces a new equilibrium with another distribution of wage changes and unemployment: it does not lead to a shift along the Phillips curve obtaining at the first equilibrium. Hence this curve carries no implications about the change in mean equilibrium wage changes associated with a given change in mean equilibrium unemployment.

The model that we work with is a development of that presented in Heal (1982). It was established there that under certain conditions on demands, if there are increasing returns in production, then there may exist a price vector that is globally Walrasian stable, but which is not a market-clearing price vector. Thus the normal Walrasian adjustment mechanism leads the economy towards this price vector from any initial conditions; however, at this price vector, excess demands and supplies are nonzero. The basic intuition underlying this result is very simple, and is demonstrated in Figure 1. Figure 1a shows a production function with increasing returns to scale in part of its range; its input demand function is the broken curve shown in Figure 1b. If, as shown there, the supply curve SS passes through the gap in the demand curve, then there is no price at which the market clears. The price ratio $p^*$ is however stable, in that for $p > p^*$, supply exceeds demand, and vice versa. In Heal (1982) it is shown that this phenomenon can be formalized in a rigorous general equilibrium model, so that if demands are Hicksian or satisfy an extension of the gross substitutability condition, an economy with nonconvex production sets may have a Walrasian stable, non-market-clearing price vector.

This provides the point of departure of the present paper. We study an economy in which there exists such a stable non-market-clearing price vector. It is shown that in the long run the economy will be characterized by prices that oscillate around this price vector. These oscillations occur because excess demands are never zero: on the half-lines ending at $p^*$, the level of excess demand and so the rate of price adjustment will be bounded away from zero. As $p$ reaches $p^*$ from below, $dp / dt$ is still strictly positive, and as $p$ reaches $p^*$ from above, $dp / dt$ is still strictly negative. Hence, rather than converging smoothly to $p^*$, it can be shown that the economy will overshoot, with prices eventually oscillating in an interval about $p^*$. This behavior of excess demands, and so of $dp / dt$, is shown later in Figure 2.

## 2. THE MODEL

We shall work in the simplest possible framework, which is essentially that represented by Figure 1. A single input, referred to as labor, is used to produce a single output, consumption. $p$ is the price of labor relative to consumption, i.e. the real wage. The technology and demand structures are assumed to

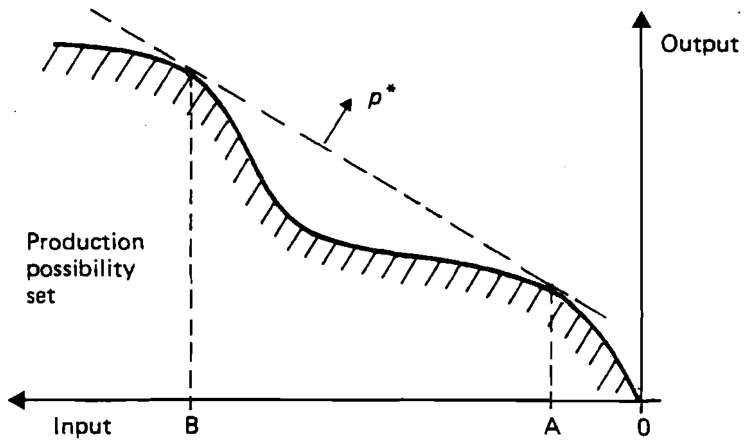


**FIGURE 1a** $p$ is the price of input relative to output. For $p \leq p^*$, the firm demands at least $0B$ of input; for $p \geq p^*$, it demands at most $0A$.

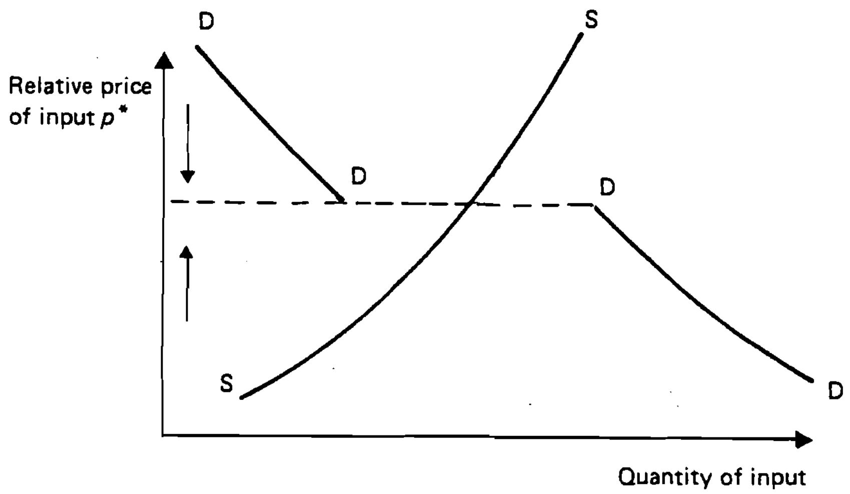


**FIGURE 1b** The input demand curve corresponding to Figure 1a is broken at $p^*$, with no price at which supply equals demand. However, $p^*$ is stable.

be such that there exists a relative price $p^*$ that satisfies two conditions. One is that $p^*$ is not a market-clearing price. The other is that excess demand for labor is positive for $p < p^*$ and negative for $p > p^*$. We formalize this:

(A1). There exists a relative price $p^*$ with $Z(p^*) \neq 0$, $Z(p) > 0$ for $p \leq p^*$ and $Z(p) < 0$ for $p \geq p^*$.

An example of such a situation is shown in Figure 1. General conditions for the existence of such a price vector are given in Heal (1982), though the conditions used to obtain global Walrasian stability there (demand being Hicksian or satisfying gross substitutability) are unnecessarily strong in the present context because of the one-dimensionality of the price space. The conditions for global stability in the one-dimensional case are considerably less restrictive than in the general case (Dierker 1973).

Letting the excess demand for labor at relative price $p$ be $Z(p)$, we consider the price adjustment process

$$p_{t+1} = p_t + KZ(p_t) \tag{1}$$

where $K > 0$ is a constant governing the speed of price adjustment. Assuming the firm to behave as a profit-maximizing price-taker, the excess demand function for labor $Z(p)$ has the form shown in Figure 2: it never assumes the value zero, $Z(p) > 0$ for $p \leq p^*$, and $Z(p) < 0$ for $p \geq p^*$.

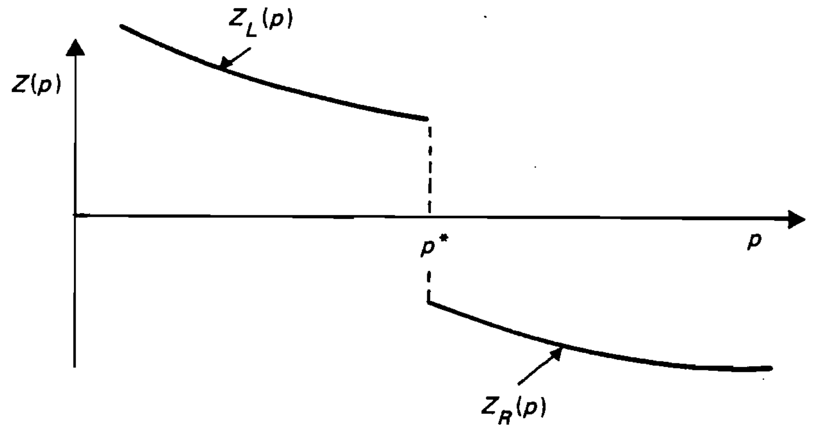


**FIGURE 2** The excess demand for labor $Z(p)$.

Our next step is to establish the behavior of prices under the adjustment process (1). We shall show that from any initial values they converge to within an interval denoted by $[\underline{p}, \bar{p}]$ containing $p^*$. Once in this interval, the relative price remains within it and oscillates about $p^*$.

To establish these results, it is first necessary to define several important values of the relative price. These definitions are interpreted geometrically in Figure 3. The excess demand function for labor in our economy is shown in Figure 2: the terms $Z_L(p)$ and $Z_R(p)$ are used to refer to the branches to the left and right of $p^*$, respectively, and so $Z_L(p^*)(Z_R(p^*))$ denote the values of $Z$ at $p^*$ on the left (right) branch.

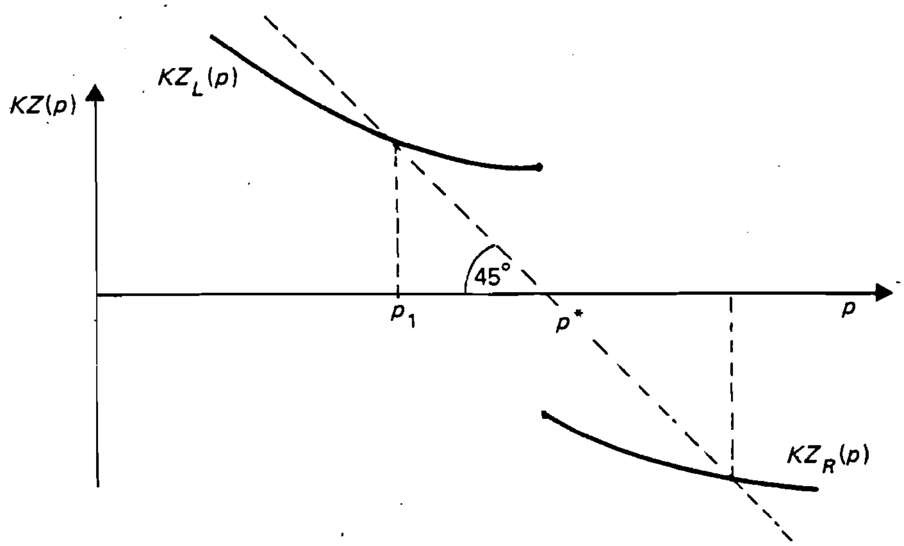


**FIGURE 3** If $p_1 \le p_t \le p^*$, then $p_{t+1} = p_t + KZ_L(p_t) \ge p^*$; and if $p^* \le p_t \le p_2$, then $p_{t+1} = p_t + KZ_R(p_t) \le p^*$. $p_1$ and $p_2$ define an interval within which overshooting of $p^*$ occurs. $p_3$ is the smallest number less than $p^*$ which can be reached in one step from above $p^*$.

Let $p_1$ be the solution of

$$KZ_L(p_1) = p^* - p_1$$

Let $p_2$ be the solution of

$$KZ_R(p_2) = p^* - p_2$$

Let $p_3 = \min\limits_{p \ge p^*} \left\{ p + KZ_R(p) \right\}$.

Let $p_4 = \max\limits_{p \le p^*} \left\{ p + KZ_L(p) \right\}$.

Intuitively, $p_1$ is the smallest $p$ less than $p^*$ from which the price can reach or overshoot $p^*$ in one step. $p_2$ plays a similar role above $p^*$. We assume the equations defining $p_1$ and $p_2$ to have unique solutions; as Figure 3 shows, this means that the curves $KZ_L(p)$ and $KZ_R(p)$ can each cut a 45° line through $p^*$ only once. We formalize this as:

(A2). $p_1$ and $p_2$ are the unique solutions of their defining equations.

$p_3$ is the smallest number less than $p^*$ that can be reached in one step from above $p^*$. Clearly $p_3$ is reached from the $p \geq p^*$ for which $KZ_R(p)$ is furthest below the 45° line. $p_4$ is defined symmetrically. Obviously, once within the interval $[p_3, p_4]$, the price cannot leave this interval.

We can now state and prove the following theorem.

**Theorem 1.** *Consider the price adjustment process (1). There exists $T < \infty$ such that for all $t \geq T$, $p_3 \leq p_t \leq p_4$. Furthermore, there is no $T'$ such that $p_t = p^*$ for all $t \geq T'$. For all $t \geq T$, sign $(p_t - p^*)$ remains constant for, at most, a finite number of periods.*[1]

**Proof.** Let the initial price $p_0$ be outside the interval $[p_1, p_2]$. It follows immediately, as $KZ(p)$ is bounded away from zero, that in a finite number of steps $p$ enters $[p_1, p_2]$. Let $T_1$ be the first $t$ for which $p_1 \leq p_t \leq p_2$.

Suppose $p_{T_1} \leq p^*$. Then by construction, for $t \geq T_1$, $p_t \leq p_4$. Now $p_{t+1} \geq p^*$. Hence for all $t \geq T_1+1$, $p_t \geq p_3$, and for all $t \geq T = T_1+1$, $p_3 \leq p_t \leq p_4$. A similar argument applies if $p_{T_1} \geq p^*$.

Note that as $Z_L(p^*) \neq 0$ and $Z_R(p^*) \neq 0$, $p = p^*$ cannot be a stationary point.

Finally, suppose $[p_3, p_4] \subset [p_1, p_2]$. Then for all $t \geq T$, sign $(p_t - p^*) = -$ sign $(p_{t+1} - p^*)$. Otherwise, sign $(p_t - p^*)$ remains constant, for, at most, the number of steps needed to move from the interval $[p_3, p_4]$ into the interval $[p_1, p_2]$. This completes the proof.

It should be noted that the oscillations in prices that emerge as the limiting behavior of the system may be very complex indeed. It is possible to establish the coexistence of limit cycles of several different frequencies. In the long run, the economy will be characterized by a relative price that oscillates about $p^*$. These fluctuations in $p$ will of course be associated with fluctuations in $Z(p)$. Define

$$\bar{Z} = \lim_{\substack{p \to p^* \\ p \leq p^*}} Z(p)$$

$$\underline{Z} = \lim_{\substack{p \to p^* \\ p \geq p^*}} Z(p)$$

Then the induced fluctuations in $Z(p)$ have the property that excess demand will never fall in the interval $(\underline{Z}, \bar{Z})$; $Z(p)$ is bounded away from zero. It immediately follows that the period-to-period price changes $(p_{t+1} - p_t)$ are bounded away from zero by $K$ times the same bound. This situation is summarized in Figure 4, where the excess demand for labor is plotted on the horizontal axis, and the price change $\Delta p$ on the vertical axis. The important point is that as $Z$ and $\Delta p$ are both bounded away from zero when there are nonconvexities, all observed $(Z, \Delta p)$ pairs must lie in the two shaded areas. They either satisfy $Z \geq \bar{Z}$, $\Delta p \geq K\bar{Z}$ or $Z \leq \underline{Z}$, $\Delta p \leq K\underline{Z}$. Note that in Figure 4, excess demand is shown increasing to the left, so that excess supply (unemployment, in the case of labor) increases to the right.

[1] It may be worth remarking that a result exactly analogous to Theorem 1 can be established for the continuous adjustment process $dp / dt = Z(p)$. This requires a more careful study of the properties of the discontinuity of $Z(p)$, using singularity theory.

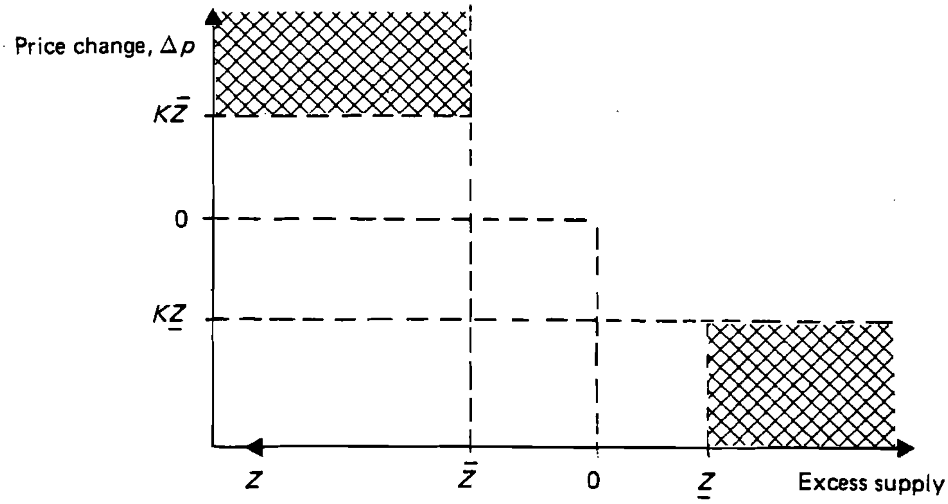


**FIGURE 4** Observable combinations of price change and excess supply.

Figure 4 makes it clear that the data generated by this economy on real wage changes $\Delta p$ and unemployment will be such that $\Delta p$ appears to be negatively correlated with unemployment. The observed wage-change-unemployment pairs, being drawn from the shaded quadrants, would reveal a negative correlation between wage changes and unemployment. These results can be conveniently summarized as follows.

**Theorem 2.** *Consider an economy using a single input (labor) to produce a single output (consumption). Let technologies, preferences, and endowments be such that the excess demand function for labor satisfies (A1) and (A2), and out of equilibrium let the dynamics of price adjustment be given by (1). Then in the long run this economy will generate data on real wage changes and on unemployment (or unfilled vacancies) exhibiting a Phillips-type negative relationship between real wage changes and unemployment.*

**Proof.** Theorem 1 established that in the long run the relative price oscillates about $p^*$ in the interval $[p_3, p_4]$. For all $t$, excess demand for labor lies outside the interval $\underline{Z} < 0 < \bar{Z}$, and price changes $p_{t+1} - p_t = \Delta p_t$ satisfy $\Delta p_t > K\bar{Z} > 0$ or $\Delta p_t < K\underline{Z} < 0$ and so lie outside the interval $K\underline{Z} < 0 < K\bar{Z}$. $\Delta p_t$ is positive for $Z > \bar{Z}$ and negative for $Z < \underline{Z}$. Hence observable combinations of $(\Delta p_t, Z_t)$ satisfy

$$\Delta p_t \geq K\bar{Z} > 0, \quad Z \geq \bar{Z} > 0$$

or

$$\Delta p_t \leq K\underline{Z} < 0, \quad Z \leq \underline{Z} < 0$$

Let us call these two regions $A$ and $B$, respectively. As $t \to \infty$, the system (1) generates an infinite number of data points in each region. Now note that in region $A$, wage changes are positive and there is excess demand for labor, whereas in region $B$ wage changes are negative and there is an excess supply of labor. This completes the proof.

Figure 4 makes it clear that the distribution of data on $\Delta p$ and on $Z$ is determined by $\underline{Z}$, $\bar{Z}$, and $K$. $\underline{Z}$ and $\bar{Z}$ determine the interval within which the excess demand $Z$ never falls, and this, given $K$, determines the interval within which $\Delta p$ never falls. The larger the discontinuity in $Z(p)$, i.e. the larger the nonconvexity in the production set, then the larger is the lower bound we can establish on price changes. *In general we would expect this to imply that the Phillips relationship would be more pronounced with larger nonconvexity.* Compare for example Figure 4 with Figure 5, where $\underline{Z}$ and $\bar{Z}$ are nearly zero. In Figure 4, the entire distribution of wage changes and excess demands could be very concentrated near (0,0), and scarcely observable as a statistical relationship. We note from this that increasing returns or nonconvexities are not strictly necessary for this phenomenon, but are likely to increase its significance.

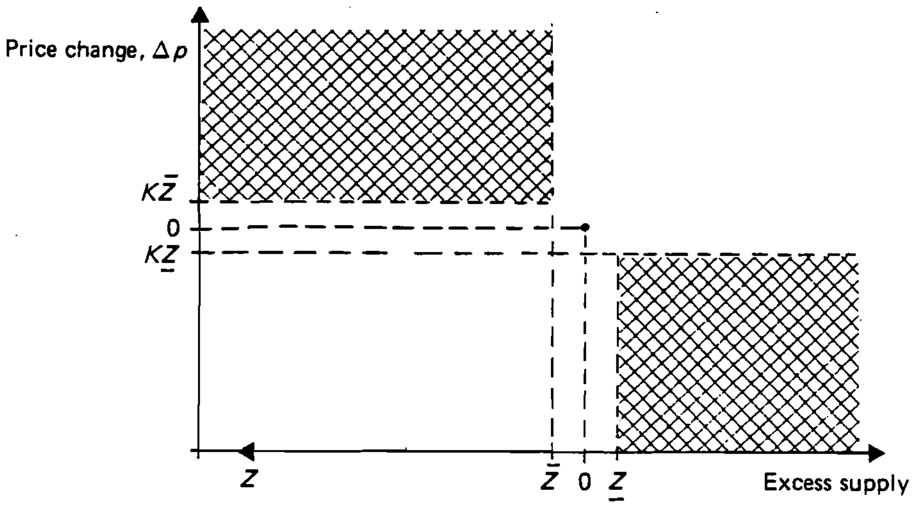


**FIGURE 5** Permissible $(\Delta p, Z)$ combinations when $\bar{Z}$ and $\underline{Z}$ are near zero.

As we are working in a general equilibrium framework, Figure 3 relates changes in the *real* wage, $\Delta p$, to the level of excess demand for labor. Obviously, if one were to assume a rate of inflation of, say, $i$ in the economy, then corresponding to an interval $(K\underline{Z}, K\bar{Z})$ within which real price changes never fall, would be an interval $(ip + K\underline{Z}, ip + K\bar{Z})$ from which nominal price changes would be blocked. In our rather simple framework, with a single homogeneous type of labor, we can speak of the excess demand for labor being unambiguously positive, zero, or negative. Obviously in practice labor markets are differentiated by skill type and by location, and any actual ata will be an aggregate across such groups. The precise measurement of excess demand would therefore pose some difficulties, but it seems clear that one could reasonably assume excess demand to be a decreasing function of unemployment minus vacancies, the statistic usually used in Phillips-type studies.

Finally, we comment on the policy implications of this derivation of the Phillips curve. It is clear that in our model, a negatively-sloped regression of wage changes on unemployment would not represent a locus of alternative equilibrium configurations. It would not be a curve along which the economy could move, but would simply be a reflection of the fluctuations in wage changes and unemployment associated with the stable long-run behavior of the economy. In the present model, one could think of policy variables as affecting either the demand or the supply curve, or both, in the labor market. This would clearly alter $\underline{Z}$ and $\bar{Z}$, thus changing the whole distribution of wage changes and unemployment. Mean wage changes and unemployment levels would be different, and a new Phillips curve would emerge. The Phillips curve is therefore a characteristic of a stable configuration of the economy, not a cross-equilibrium locus, and has no value in predicting how mean wage changes would respond to a new equilibrium, following a change in mean unemployment.

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