ON MUNICIPAL WATER PRICING PRACTICES IN THE SOUTHWESTERN SKÅNE REGION, SWEDEN

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resource management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

This paper was written as part of a collaborative study on water resources problems in Southwestern Skåne, Sweden, pursued by IIASA in collaboration with the Swedish National Environmental Protection Board and the University of Lund, and completed in 1982. The purpose of this paper was to examine the municipal water pricing system as it now exists in Skåne and to compare it with the two-part tariff system. The paper discusses the important relationship between the water pricing system and sudden shifts in municipal water demand.

Janusz Kindler
Leader
Impacts of Human Activities on
Environmental Systems Project

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ON MUNICIPAL WATER PRICING PRACTICES IN THE SOUTHWESTERN SKÅNE REGION, SWEDEN*

The problem of fixing the price for water has been defined by Reingardt as follows: The existing system of fixing prices according to the amount of water used, does not bear any relation to the cost of water delivery. Up to 90% of the annual costs of waterworks and waste treatment facilities are constant, but 90% of the revenues for water supply and waste evacuation are obtained, as a rule, from the price charged per cubic meter of water used (Reingardt, 1979).

This price-per-unit system proves to be of little use when there is an unexpected drop in water demand, because it results in an increase in the price of water, which in turn results in a further decrease of water demand and subsequent negative socio-economic consequences. Since an unexpected drop in water demand did occur in Sweden (Andersson, et al. 1979), this question attracted particular attention in connection with the Bolmen water transfer project in Skåne. Reingardt (1979) has proposed the two-part tariff system with a higher, fixed price for water.

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*The existence of this problem became apparent during discussions with Janusz Kindler, Lennart de Mare, Felix Ereshko and Sergei Orlovsky, when A. Lotov visited IIASA in April/May, 1980.

MATHEMATICAL DESCRIPTION OF THE SYSTEM CURRENTLY IN USE

The costs tied to water delivery can be divided into two parts:

(1) fixed costs, denoted by C_o;

(2) the cost per cubic meter of water used, described by linear function $C_1x(t)$, where C_1 is constant, and $x(t)$ is the volume of water delivered to consumers during a time period t.

Thus the total costs $C(t)$ of water supply during a time period t can be described as

$$C(t) = C_o + C_1x(t) \quad . \qquad (1)$$

In accordance with accepted practice in Sweden, we can suppose that in the process of water price setting, the waterworks, owned by municipalities, follow the principle that the source of the organization's revenue should be the sale of water, at the price set by it. In this case, the price chosen by the water supplier, in the time period t, should satisfy the condition

$$p(t)x(t) = C(t) \quad ,$$

or, taking account of (1),

$$p(t)x(t) = C_o + C_1x(t) \quad ,$$

or,

$$p(t) = \frac{C_o}{x(t)} + C_1 \quad . \qquad (2)$$

Instead of (2), a slightly different version of the price equation may be used, connecting price to the value of water consumption, during the preceding period of time, i.e.

$$p(t) = \frac{C_o}{x(t-1)} + C_1 \quad . \tag{3}$$

This equation may be used for an approximate description of the process of price setting by the municipal waterworks.

In addition to (3), it is necessary to have a mathematical model of consumer behaviour. The well-known consumption function suggested by Stone [3] has been used below:

$$x(t) = a_o + \frac{a_1}{p(t)} \qquad (a_o > 0,\ a_1 > 0) \quad , \tag{4}$$

where the constant a_o is minimum consumption, and a_1 is --

A system of two equations

$$\begin{aligned} p(t) &= \frac{C_o}{x(t-1)} + C_1 \\ x(t) &= a_o + \frac{a_1}{p(t)} \quad , \end{aligned} \tag{5}$$

describes approximately, the water tariff system currently in use in the Skåne region.

In the following pages, a mathematical model of the two-part tariff system has been constructed, based on the assumptions of L. Reingardt [2]. A comparison of the two systems is made subsequently.

MATHEMATICAL DESCRIPTION OF THE TWO-PART TARIFF SYSTEM

In the two-part tariff system, user payment is divided into two parts. Users have to pay a constant charge, p_o, for each time period, irrespective of the quantity of water they actually consume. The rest of the supplier's income is covered by the payment per cubic meter of water used. The price, p, is determined by balancing the income and expenditure of the organization, which is supposed to be a non-profit, self-supporting body,

$$p_o + p(t)x(t) = C(t) \quad . \tag{6}$$

Using (1), we obtain an equation for the price

$$p(t) = \frac{C_o - p_o}{x(t)} + C_1 \quad .$$

As in the case of the existing price system, let us suppose that the price is set according to water consumption during the preceding time period, i.e.

$$p(t) = \frac{C_o - p_o}{x(t-1)} + C_1 \quad . \tag{7}$$

Combining this equation with the demand function (4), we obtain a system of two equations, which describe the price dynamics in the case of the two-part tariff system

$$\begin{aligned} p(t) &= \frac{C_o - p_o}{x(t-1)} + C_1 \\ x(t) &= a_o + \frac{a_1}{p(t)} \quad . \end{aligned} \tag{8}$$

Since system (5) is a particular case of system (8) (p_o being zero), we shall examine both systems at once (see Figure 1). The thick line represents the demand curve (4), three dotted lines are the supply curves (7) with different relations between C_o and p_o (i.e. with different signs for $C_o' = C_o - p_o$). By applying economic principles, we obtain $C_o > 0$, $C_1 > 0$, $a_o > 0$, $a_1 > 0$, $p_o \geq 0$. The sign $C_o' = C_o - p_o$ as mentioned above, is not fixed a priori. In the case of $C_o < p_o$, model (8) is a commonly used cobweb model of market behaviour. Note that on

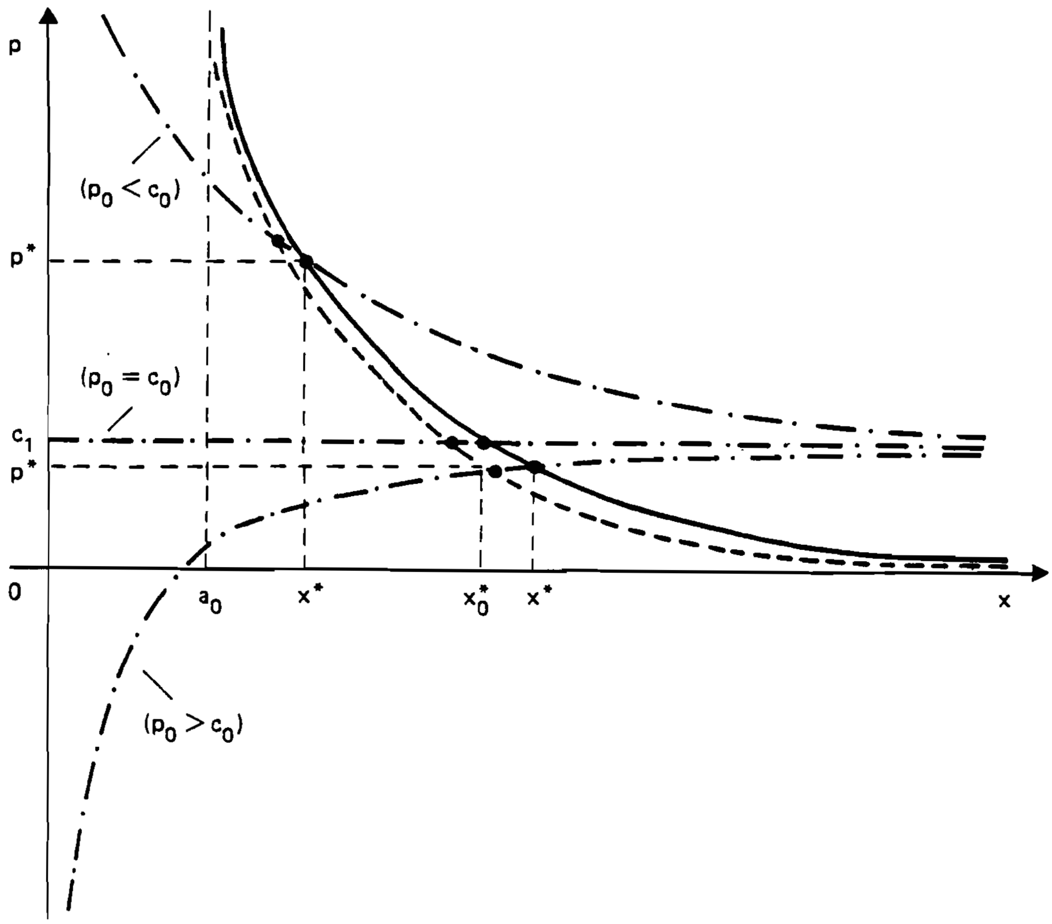

Figure 1.

the curve (7) (while $p_o > C_o$) the value of p(t) may appear to be negative, for sufficiently small values of x(t): (this fact has a simple economic interpretation: since income due to fixed prices appeared to be greater than expenditure, the non-profit nature of the organization made it necessary to return, in one way or another, part of the money taken in advance.) This situation is quite unnatural. To avoid it, we suppose that fixed charge p_o is not too great, so on the curve (7) the value of p(t) may appear to be negative, only in the case where $x(t-1) < a_o$.

Since $x(t-1) > a_o$, while $p(t-1) > 0$, in this situation p(t) is always positive, provided $p(o) > 0$. This requirement can be expressed as follows:

$$p(t) = \frac{C_o - p_o}{x(t)} + C_1 > 0 \qquad \text{while } x(t-1) > a_o \; .$$

This relation results in the restriction on the value of p_o:

$$p_o < C_o + C_1 a_o \quad . \tag{9}$$

MATHEMATICAL EXAMINATION OF THE
WATER PRICE MODEL

Properties of the mathematical model (8) will be investigated according to the following plan:

(1) The existence of the equilibrium point, i.e., a pair of (p^*, x^*) for which it holds

$$\begin{aligned} p^* &= \frac{C_o}{x^*} + C_1 \\ x^* &= a_o + \frac{a_1}{p^*} \quad ; \end{aligned} \tag{10}$$

(2) the stability of the point (p^*, x^*), i.e., the conditions of convergence of the trajectories of the system (8) with initial values $p(o) = p^o$, $x(o) = x^o$ to the point (p^*, x^*) for sufficiently wide set of p^o and x^o;

(3) the dependence of the coordinates of the equilibrium point, on the parameters of system (8);

(4) economic interpretation of the mathematical results obtained, and conclusions.

Existence of the Equilibrium Point

The existence of the unique equilibrium point seems to be obvious (Figure 1). Each of the supply curves (7), crosses a demand curve (4) at a unique point (p^*, x^*), and because of (9), at this point holds $p^* > 0$ and $x^* > a_o$. If a point of intersection between the curve (4) and the straight line $p(t) = C_1$ is denoted by x_o^*, it is obvious that

(1) $x^* < x_o^*$, $p^* > C_1$ while $C_o > p_o$;

(2) $x^* = x_o^*$, $p^* = C_1$ while $C_o = p_o$;

(3) $x^* > x_o^*$, $p^* < C_1$ while $C_o < p_o$.

The same results could be obtained formally. System (10) may be written as

$$p^* = C_1 + \frac{C_o'}{a_o + \frac{a_1}{p^*}} \quad . \qquad (11)$$

This equation can be transformed into a quadratic equation for p^*:

$$a_o(p^*)^2 + (a_1 - C_o' - C_1 a_o)p^* - C_1 a_1 = 0 \quad . \qquad (12)$$

The only positive root of this equation is

$$p^* = \left(\frac{C_1}{2} + \frac{C_o' - a_1}{2a_o}\right) + \sqrt{\left(\frac{C_1}{2} + \frac{C_o - a_1}{2a_o}\right) + \frac{c_1 a_1}{a_o}} \quad . \tag{13}$$

An analysis of this equation with respect to different signs of $(c_o - p_o)$ results in the same conclusions as the graphical analysis.

Stability Analysis

The system (8) gives the relations between the values of price in two successive periods:

$$p(t) = c_1 + \frac{c_o'}{a_o + \frac{a_1}{p(t-1)}} \quad . \tag{14}$$

To prove a convergence of p(t) to p* it is sufficient to show that for any t = 1,2,..., the relation holds

$$|p(t)-p^*|/|p(t-1)-p^*| < \alpha \quad , \tag{15}$$

where $\alpha < 1$ and α does not depend on t. According to (10) and (14) we get by elementary transformation

$$\frac{p(t) - p^*}{p(t-1)-p^*} = \frac{1 - \frac{C_1}{p^*}}{1 + \frac{a_o}{a_1} p(t-1)} \quad .$$

Let $C_o' > 0$. As mentioned above, $p^* > C_1$. Therefore

$$\left|\frac{p(t) - p^*}{p(t-1)-p^*}\right| = \left|1 - \frac{C_1}{p^*}\right| \cdot \left|\frac{1}{1 - \frac{a_o}{a_1}p(t-1)}\right| < \left|1 - \frac{C_1}{p^*}\right| \tag{16}$$

In the case $C_o' > 0$ we have $|1 - \frac{C_1}{p^*}| < 1$, i.e., we can take $\alpha = |1 - \frac{C_1}{p^*}|$. Since the estimation (16) holds for any $p(t-1) > 0$, (p^*,x^*) is a globally stable point for $C_o' > 0$.

When $C_o' = 0$, then $p(t) = p^* = C_1$ is not dependent on $p(t-1)$. Finally if $C_o' < 0$, the relation (15) follows from the equation (16) when $|1 - \frac{C_1}{p^*}| < 1$. Since $p^* < C_1$ for $C_o' < 0$, the condition $|1 - \frac{C_1}{p^*}| < 1$ is equivalent to

$$\frac{C_1}{p^*} < 2 \quad . \tag{17}$$

Using (13) we transform (17) to

$$C_o' > a_1 - \frac{a_o C_1}{2} \quad . \tag{18}$$

So when condition (18) holds, the trajectories of prices (14), uniformly converge to the equilibrium price p^* for any initial price $p(o)$ and the equilibrium point is globally stable.

The combination of conditions (18) and (9) results in the limitation on the value of $C_o' = C_o - p_o$, i.e., on the value of constant charge p_o,

$$p_o < \min \{C_o + a_o C_1, \ \ C_o + a_1 + \frac{a_o C_1}{2}\} \quad .$$

Marginal Analysis

Now we shall study the dependence of the equilibrium values p^* and x^* on the variations of demand, i.e., on the variations of the parameters a_o and a_1. The point (p^*,x^*) is the solution of the system of equations

$$F_1(p^*,x^*,a_o,a_1) = p^* - \frac{C_o'}{x^*} - C_1 = 0 \quad ,$$

$$F_2(p^*,x^*,a_o,a_1) = x^* - a_o - \frac{a_1}{p^*} = 0 \quad ,$$

where a_o and a_1 are independent variables, p^* and x^* are dependent ones. Functions F_1 and F_2 are continuously differentiable on their variables in this particular case.

If one estimates

$$D = \det\begin{pmatrix} \frac{\partial F_1}{\partial p^*} & \frac{\partial F_1}{\partial x^*} \\ \\ \frac{\partial F_2}{\partial p^*} & \frac{\partial F_2}{\partial p^*} \end{pmatrix} = \det\begin{pmatrix} 1 & \frac{C_o'}{(x^*)^2} \\ \\ \frac{a_1}{(p^*)^2} & 1 \end{pmatrix} = 1 - \frac{C_o'a_1}{(x^*p^*)^2}$$

in the case of $C_o' \leq 0$, $D \geq 1$, otherwise it is easy to deduce from equation (10) that $0 < \frac{C_o'a_1}{(x^*p^*)^2} < 1$, i.e. $0 < D < 1$. Note, that $\frac{\partial D}{\partial C_o'} = \frac{a_1}{(p^*x^*)^2} < 0$.

Thus we can apply here the theorem of partial derivatives of implicit function. As a result we have

$$\frac{\partial p^*}{\partial a_o} = - \frac{1}{D}\frac{C_o'}{(x^*)^2} \quad , \tag{19}$$

$$\frac{\partial p^*}{\partial a_1} = - \frac{1}{D}\frac{C_o'}{p^*(x^*)^2} \tag{20}$$

$$\frac{\partial x^*}{\partial a_o} = \frac{1}{D} \quad , \tag{21}$$

$$\frac{\partial x^*}{\partial a_1} = \frac{1}{p^*D} \quad , \tag{22}$$

The sudden drop in water demand (a_o or (and) a_1 decrease) always results in the decrease of the volume of water sold (this follows from (21) and (22)), but the magnitude of the decrease related to the drop in demand, depends on C_o' (i.e., on the price system). We have

$$\frac{\partial}{\partial C_o'}\left(\frac{\partial x^*}{\partial a_o}\right) = \left(\frac{\partial}{\partial C_o'}\frac{1}{D}\right) = -\frac{1}{D^2}\frac{\partial D}{\partial C_o'} = \frac{1}{D^2}\frac{a_1}{(x^*p^*)^2} > 0 \quad ,$$

and

$$\frac{\partial}{\partial C_o'}\left(\frac{\partial x^*}{\partial a_1}\right) = \frac{1}{p^*}\frac{\partial}{\partial C_o'}\left(\frac{1}{D}\right) = \frac{1}{p^*D^2}\frac{a_1}{(x^*p^*)^2} > 0 \quad .$$

So the value of the decrease (characterized by $\frac{\partial x^*}{\partial a_o}$ and $\frac{\partial x^*}{\partial a_1}$) increases together with the coefficient C_o', while 1) $\frac{\partial x^*}{\partial a_o} = \frac{1}{D} < 1$ for $C_o' < 0$, 2) $\frac{\partial x^*}{\partial a_o} = 1$ for $C_o' = 0$, 3) $\frac{\partial x^*}{\partial a_o} > 1$ for $C_o' > 0$. This means that the price system amplifies the drop in the demand due to a_o in the case of $C_o' < 0$. The drop in water consumption equals the drop in water demand for $C_o' = 0$ and the price system makes water consumption more stable in the case of $C_o' > 0$ (the decrease in water consumption is less than the drop in water demand).

The direction of the changes in the price level depends on the sign of the coefficient C_o'. The drop in water demand makes the prices decrease for $C_o' < 0$. On the contrary, the drop in water demand makes the prices increase for $C_o' > 0$. The greater the coefficient C_o' the greater the increase in prices. The price does not depend on the water demand for $C_o' = 0$. These fluctuations in water prices describe the pattern of water consumption: the rise or fall in prices, induced by the drop in the water demand, makes the decrease in water consumption greater or smaller. It is necessary to remember that the value of C_o' has to be limited by the relations (9) and (18).

CONCLUSIONS

The existing water price system, where a considerable portion of the income of the water supplier is obtained from the price per cubic meter of water used ($C_o' = C_o$), (because it is a non-profit organization), results in the growth of the price of water, if a sudden drop occurs in water demand. This in turn, results in the decrease of water consumption and so on. This process amplifies the initial drop in the water consumption.

The two-part tariff system ($C_o' = C_o - p_o$) with fixed charges p_o, partially ($p_o < C_o$) making up for fixed costs, reduces the price growth in the case of a sudden drop in water demand. Hence, the new price system makes water consumption more stable.

The two-part tariff system, where the fixed prices are equal to the fixed costs ($p_o = C_o$), results in the following: the water price does not depend on changes in the water demand. In this case, the decrease in water consumption parallels the drop in water demand.

An examination of the model shows the possibility of introducing the fixed prices p_o, which are greater than the fixed cost C_o ($C_o' < 0$). In this case, the price falls with the drop in the water demand and therefore stabilizes water consumption.

It should be noted, that in this case, fixed charges p_o should not be too great. To avoid breaking the relations (9) and (18), it is necessary first to construct the demand function and only then to set the value of fixed price p_o.

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