

IMPACTS OF PRICE VARIATIONS
ON THE BALANCE OF WORLD TRADE

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PREFACE

This report represents the first step in cooperation between IIASA and UNIDO exploring practical applications of systems analysis to problems of world economic development. Its main aim is to elaborate user-oriented software for investigating possible ways of reaching "acceptable" states of the world trade market. In addition, the report can also be considered as a users' manual for the software package developed for the purpose.

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Abstract

A standard mathematical model of a trade market is considered, and the concept of an acceptable state of the market is introduced, which takes into account the requirements of all the partners. To evaluate the 'distance' between the acceptable and current states of the market, a special mathematical approach is developed. This approach has been found useful for correcting the price vector to bring the states closer together.

Impacts of Price Variations on the Balance of World Trade

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Introduction

During recent years there has been constant growth in the ability of man to influence systems of global importance, such as climate, the natural environment, mineral resources, patterns of population, and so on. This potentially far-reaching influence has become increasingly available to groups of countries, individual countries, corporations, and even individual persons.

As a result, the responsibility for decisions made is now higher than ever before. Hence, decision makers must use all available tools to try to analyze the impacts of their actions, especially in cases when only a limited number of persons are authorized to make the decisions and to take responsibility for the

consequences.

The use of computers is undoubtedly one of these tools, whose potential is very far from exhausted. On the other hand, however, this use of computers is restricted by the need to build adequate mathematical models of the objects under investigation. Unfortunately, for most practical cases it is very difficult to determine the adequacy of such models *a priori*.

This article is devoted to the analysis of trade markets, which is an example of a situation where the adequacy of a mathematical model may be proved easily, and which demonstrates the great potential of computer-based methods.

1. A Description of a Trade Market

A system of partners (*e.g.* private persons, companies, countries, regions and so on) trading in a set of commodities within a given period of time is called a *trade market*. Knowing *volumes* and *prices* of commodities, it is possible to evaluate *export* , *import* and *balance* data, which characterize the state of the trade market.

Based on these data, one may evaluate the level of *acceptability* of the current state of the trade market from the viewpoint both of each of the partners and of the market as a whole.

The definition of desirable or acceptable states of the market permits us to formulate the following questions :

- Is the current state of the market a desirable one ?
- If not, how far is the current state from the desirable one ?
- What should we do to bring these two states nearer to one another ?

Let us start by describing a mathematical model of the trade market.

Let V_{ij}^k be volumes (measured in physical units) of the k th commodity sold by the i th partner to the j th one. If the price of a unit of the k th commodity is p^k , we may define export, import and balance for the trade between the partners as

$$exp_{ij} = \sum_{k=1}^K p^k V_{ij}^k \quad (1)$$

$$imp_{ij} = exp_{ji} \quad (2)$$

where K is the total number of commodities to be sold.

Total volumes of export and import for the i th partner will be

$$EXP_i = \sum_{j=1}^N exp_{ij} \quad (3)$$

$$IMP_i = \sum_{j=1}^N imp_{ij}, \quad (4)$$

where N is the number of partners, and finally

$$BALANCE_i = EXP_i - IMP_i \quad (5)$$

It is very easy to prove that the sum of all exports equals the sum of all imports due to the above relations.

2. Conditions of Acceptability of a Trade Market

Let us suppose that one may define lower and upper acceptable bounds for the export, import and balance indicators for each of the partners. Then

We will call a state of the trade market acceptable if the constraints

$$\underline{EXP}_i \leq EXP_i \leq \overline{EXP}_i \quad (6)$$

$$\underline{IMP}_i \leq IMP_i \leq \overline{IMP}_i \quad (7)$$

$$\underline{BALANCE}_i \leq BALANCE_i \leq \overline{BALANCE}_i \quad (8)$$

are valid, for all $i = [1, N]$.

The values of the lower and upper bounds may be decided by experts according to the scenario that is going to be considered. For example, data for the i th group of constraints may be defined by authorized representatives of the i th partner.

Besides the data characterizing the overall trade balance of each partner there may also exist constraints on volumes of commodities sold measured in physical units, due to limited industrial capacities, transport capabilities and so on. Therefore, the system of constraints describing the acceptable states should often be augmented by supplementary inequalities of the following type

$$V_{ij}^k \leq v_{ij}^k \leq \bar{V}_{ij}^k \quad (9)$$

for all k, i and j .

Cases where the conditions of acceptability are more complex will be described in Section 10. But these additional constraints do not change the mathematical statement of the problems to be solved, and therefore they are not considered here.

It should be emphasized here that the *expert* opinions expressed in constraints (6), (7), (8) and (9) may sometimes appear to be far from realistic or even downright inconsistent. Hence we must be ready to tackle cases where there is no acceptable state at all. On the other hand, it is also possible that we will have many acceptable states of the trade market for a given set of lower and upper bounding values.

3. Measurement of the "Imbalance" of the Trade Market

We can now use our definition of an acceptable state of the trade market to evaluate how far a given state is from an acceptable one.

Let us assume for the moment that conditions (6), (7), (8) and (9) are consistent. Then there exists an acceptable state for the model under consideration. We can transform a given state into this acceptable one by making appropriate changes in the volumes of commodities sold. Assume that this transformation involves adding to the volumes V_{ij}^k the values x_{ij}^k , respectively. The relative value of this change

$$\rho(x_{ij}^k) = \text{abs} \left[\frac{x_{ij}^k}{V_{ij}^k} \right]$$

characterizes the degree of imbalance for the flow of the k th commodity from partner i to partner j . The symbol of the absolute value must be used here because the values x_{ij}^k may be either positive and negative.

One measure of the "unacceptability" or "imbalance" of the state of the market as a whole could be formulated as

$$\rho(x) = \max_{k,i,j} \rho(x_{ij}^k)$$

This evaluation of the "distance" from the given state to the acceptable one only has any practical value if the acceptable state is unique. But usually the acceptable states are in fact nonunique and each has its own ρ value.

One way round this problem is to take just the minimum of these ρ values, thus eliminating the ambiguity in our definition. In other words, define the "distance" between the given and acceptable states as

$$\rho^* = \min_x \rho(x)$$

or

$$\rho^* = \min_x \left[\max_{k,i,j} \text{abs} \left[\frac{x_{ij}^k}{V_{ij}^k} \right] \right] \quad (10)$$

The value of ρ^ shows what minimum relative change is required to transform the given state of the trade market into an acceptable one.*

Mathematically the procedure for finding the imbalance ρ , which is in fact a special case of the Chebyshev approximation problem, can be reduced to the following *mathematical programming problem*

Minimize ρ

with respect to variables $\{ \rho, x_{ij}^k, \text{ for all } i, j, k \}$

subject to

$$\rho \geq 0 \quad (11)$$

$$-\rho V_{ij}^k \leq x_{ij}^k \leq \rho V_{ij}^k \quad (12)$$

$$V_{ij}^k \leq V_{ij}^k + x_{ij}^k \leq \bar{V}_{ij}^k \quad (13)$$

for all k, i, j .

$$\underline{EXP}_i \leq EXP_i \leq \overline{EXP}_i$$

$$\underline{IMP}_i \leq IMP_i \leq \overline{IMP}_i$$

$$\underline{BALANCE}_i \leq BALANCE_i \leq \overline{BALANCE}_i$$

where

$$BALANCE_i = EXP_i - IMP_i.$$

$$EXP_i = \sum_{j=1}^N \sum_{k=1}^K p^k (V_{ij}^k + x_{ij}^k) \quad (14)$$

$$IMP_i = \sum_{j=1}^N \sum_{k=1}^K p^k (V_{ji}^k + x_{ji}^k) \quad (15)$$

for all $i = [1, N]$.

4. An Illustrative Example: the World Trade Market in 1975

To demonstrate how this approach may be used in practice, let us analyze the state of the world trade market in the year 1975, using data from UNCTAD [1980].

The model used here represents a system of eleven trade partners, built on the principle of *regional association*. The list of the partners is given in Table 1.

Trade in seven aggregate commodities is considered; these commodities are listed in Table 2.

#	Trade partner	Identifier
1	USA and Canada	US
2	West Europe	EU
3	USSR and East Europe	SU
4	Japan	JA
5	Other developed count.	OD
6	Latin America	LT
7	Tropical Africa	AF
8	West Asia	WA
9	Indian subcontinent	IN
10	East Asia	EA
11	Asia with planned econ.	CN

Table 1.

#	Commodity description	Commodity identifier
1	Food products	AGRICULTURAL PRODUCTS
2	Raw materials	RAW MATERIALS
3	Energy products	ENERGY
4	Intermediate products	INTERMEDIATE PRODUCTS
5	Consumer nondurables	CONSUMER NON-DURABLES
6	Equipment	EQUIPMENT
7	Consumer durables	CONSUMER DURABLES

Table 2.

Since the model considers only aggregated commodities, evaluation of the

trade flows is only possible in monetary terms and not in physical units. This means that all the coefficients p^k are equal to one here.

Tables A.1.1 - A.1.7 (see Appendix 1) contain information about the volumes of export-import flows for all the partners and commodities considered for the year 1975 (in billions of US\$).

Remember that this model is considered here *only to demonstrate* how the approach can be used. Here we will use very simple additional constraints to describe the acceptable state of the market: namely, a state of the world trade market will be regarded as acceptable if the absolute value of the trade balance for each of the partners does not exceed, say, 10 % (or, for the second variant, 5 %) of the total volume of export. The acceptable bounds thus defined are shown in Table 3.

Partner	Acceptable range of balance: 10%		Acceptable range of balance: 5%	
	$\underline{BALANCE}_i$	$\overline{BALANCE}_i$	$\underline{BALANCE}_i$	$\overline{BALANCE}_i$
US	-14.	14.	-7.	7.
EU	-37.	37.	-18.5	18.5
SU	-8.	8.	-4.	4.
JA	-6.	6.	-3.	3.
OD	-2.	2.	-1.	1.
LT	-5.	5.	-2.5	2.5
AF	-2.	2.	-1.	1.
WA	-10.	10.	-5.	5.
IN	-0.7	0.7	-0.35	0.35
EA	-4.	4.	-2.	2.
CN	-1.5	1.5	-0.75	0.75

Table 3.

The solutions of the appropriate linear programming problem, as described in Section 3, are as follows:

- for an acceptable range of balance of 10 % ,
the value of the imbalance ρ equals 0.2413 ;
- for an acceptable range of balance of 5 % ,
the value of the imbalance ρ equals 0.2757 .

Tables A.2.1 - A.2.7 (see Appendix 2) show the values of the export-import flows that satisfy all the requirements of the acceptable state; these data are given only for the 10% acceptable range of balance .

Note that the acceptable state here is nonunique. For example, it is obvious that the state with zero flows will satisfy conditions (6)-(9), but the "distance" between the initial and zero states will be greater than the "distance" between the initial state and the state presented in Tables A.2.1 - A.2.7 .

In addition to the quantitative data, a graphical-analytical presentation (as shown in Figure 1) can also be useful. Here a graphical interpretation of the matrix of the trade flows is given. The character '+' denotes those flows that should be increased by $100 \rho^*$ % . Character '-' denotes those to be decreased by $100 \rho^*$ % . Character 'o' marks flows with nonmaximum changes. Finally, the dot '.' denotes zero flows. The columns of the table correspond to import flows, and the rows correspond to exports.

This picture says that two of the trade partners - "USA and Canada" and "West Asia" - were in a better position in 1975, from the viewpoint of the given definition of acceptability. This conclusion can be drawn because the solution of the linear programming problem involves the greatest possible increases in the import flows of both these partners while decreasing all the others.

5. Analysis of the Dynamics of the Development of a Trade Market

The approach described may also be used to evaluate trends in the development of a trade market, provided that expert forecasts of the dynamics of acceptable states can be formulated. The following procedure can be applied in this case.

Let the state of the trade market be sequentially considered for different periods of time, say, year-by-year, and let acceptable states also be defined for all these periods. Then, starting from an initial state, it is possible to calculate new states of the trade market step-by-step, each one satisfying the expert requirements and differing minimally from the previous state.

We will illustrate the use of this procedure with the following example. We consider the model sequentially for the years 1975, 1980, 1985, 1990, 1995 and 2000, and specify the dynamics of acceptable ranges for trade balance as shown in Table 4. Figure 2 shows these dynamics graphically.

The total volume of output data for this problem is too big to be presented here, so we will give only a graphical description of the results. Figures A.3.1 - A.3.11 (see Appendix 3) contain cumulative charts, which allow us to trace the trends of all export-import flows. Since the charts are cumulative, note that the uppermost curve in each case represents total volumes of either exports or imports.

In contrast to the static case, the dynamic formulation allows us to also take into account mutual payments for credits given, and can therefore be regarded as a mathematical model of a *payoff balance* between the partners in the trade market considered.

	1980		1985		1990		1995		2000	
	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>	<u>BALAN.</u>
US	10.3	10.4	7.5	7.8	4.3	4.5	2.2	2.3	0.0	0.0
EU	-17.4	-17.0	-13.0	-12.0	-5.6	-5.4	-2.3	-2.2	0.0	0.0
SU	-9.3	-9.0	-7.8	-7.5	-4.5	-4.3	-2.3	-2.2	0.0	0.0
JA	-1.2	-1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
OD	-1.8	-1.7	-1.3	-1.2	0.0	0.0	0.0	0.0	0.0	0.0
LT	-9.3	-9.0	-7.8	-7.5	-4.5	-4.3	-2.3	-2.2	0.0	0.0
AF	-7.9	-7.6	-6.5	-6.4	-3.4	-3.2	-1.8	-1.6	0.0	0.0
WT	38.0	41.0	31.0	33.0	15.0	18.0	7.0	9.0	0.0	0.0
IN	-2.9	-2.7	-1.9	-1.8	-1.2	-1.1	-0.6	-0.5	0.0	0.0
EA	-1.2	-1.1	-0.7	-0.6	0.0	0.0	0.0	0.0	0.0	0.0
CN	-1.5	-1.3	-0.9	-0.8	-0.4	-0.3	0.0	0.0	0.0	0.0

Table 4.

DEC 1991 10:14:28.1

100.57 10.7

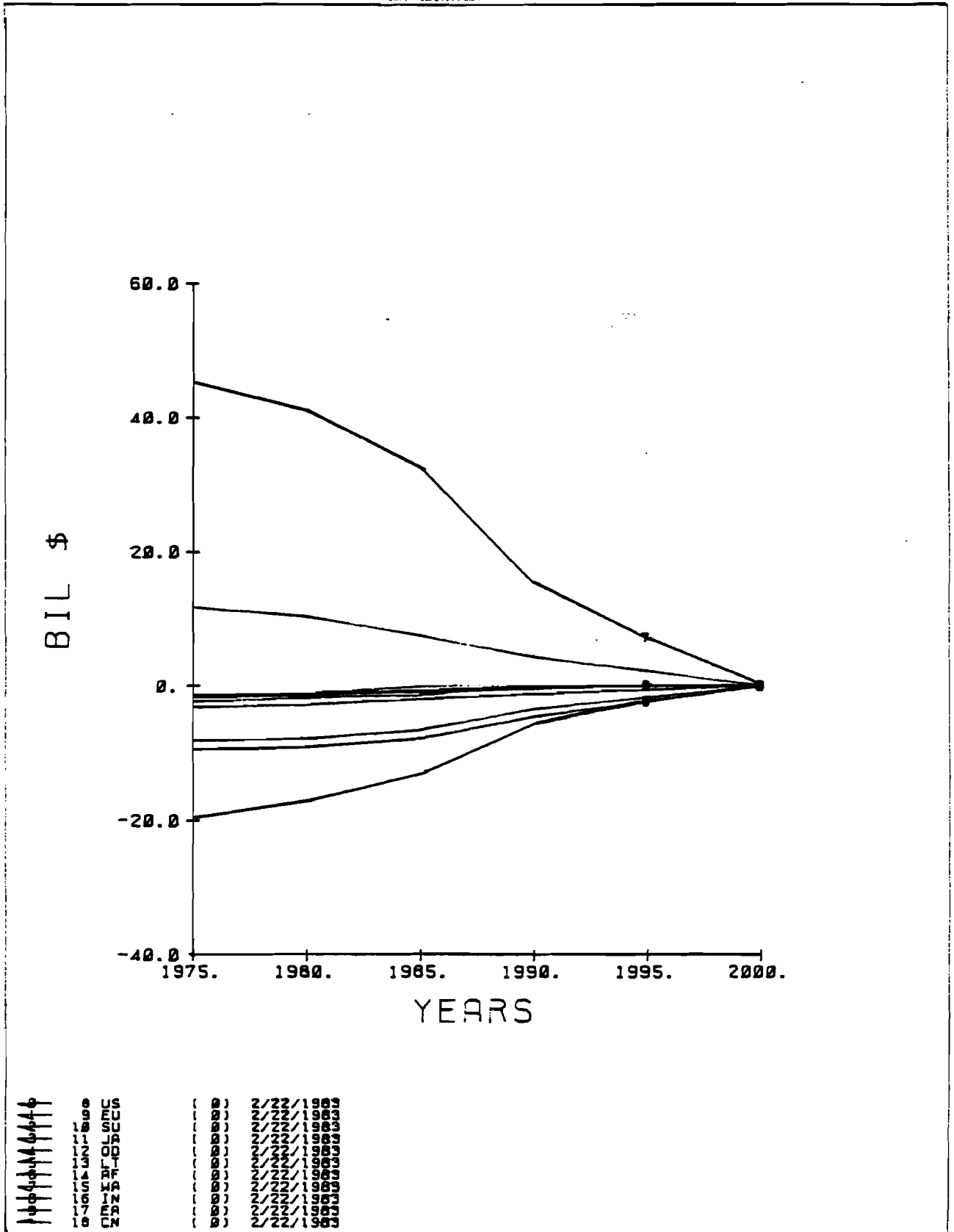


Figure 2.

6. The Dependence of the Imbalance on Model Parameters

One of the most interesting areas that can be explored using this approach is the dependence of the "unacceptability" or imbalance measure, ρ , on the parameters of the model.

As will become clear on further explanation, we may consider some concrete interdependences that are of practical value, without loss of generality of conclusions. In this model the dependence of the imbalance ρ on values of the price vector p is chosen for investigation, because this could prove useful in analyzing the stability of the trade market with respect to price variations.

To begin with, we will demonstrate how the imbalance value depends on prices of "energy" and "agricultural products". Price variations will be made with respect to the 1975 levels, which are set equal to 1. The definition of the acceptable states is the same as in Table 3.

Results of the calculations for two cases (with 10% and 5% acceptable ranges in trade balance, respectively) are given in Tables 5 and 6, and also shown in Figures 3 and 4.

Price level for energy	Value of ρ (%)	
	Acceptable range of balance: 10%	Acceptable range of balance: 5%
0.60	23.0318	25.5199
0.70	20.7145	23.1285
0.80	18.5312	20.8755
0.85	17.4862	20.3220
0.90	19.2325	22.8988
1.00	24.1324	27.5699

Table 5.

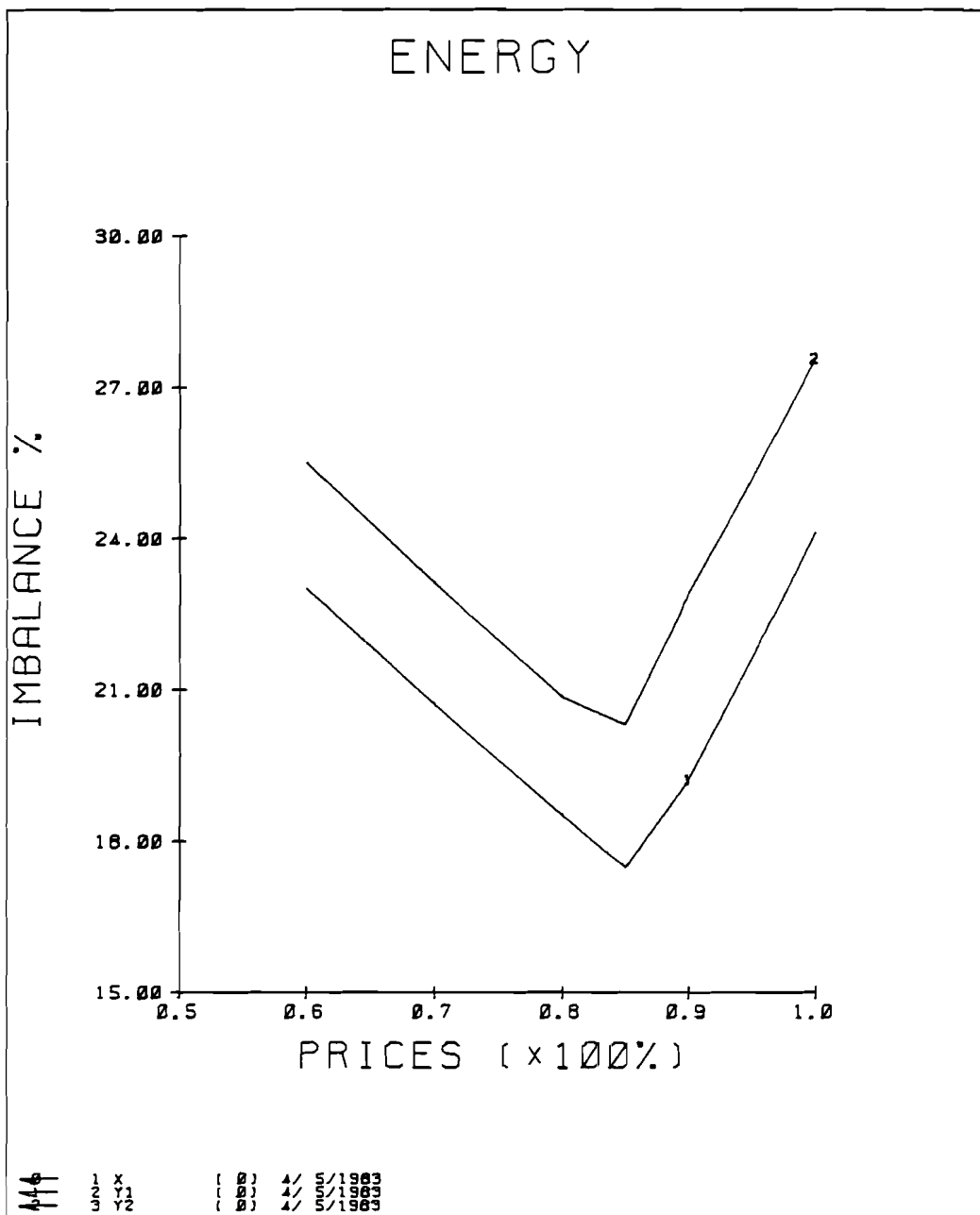


Figure 3.

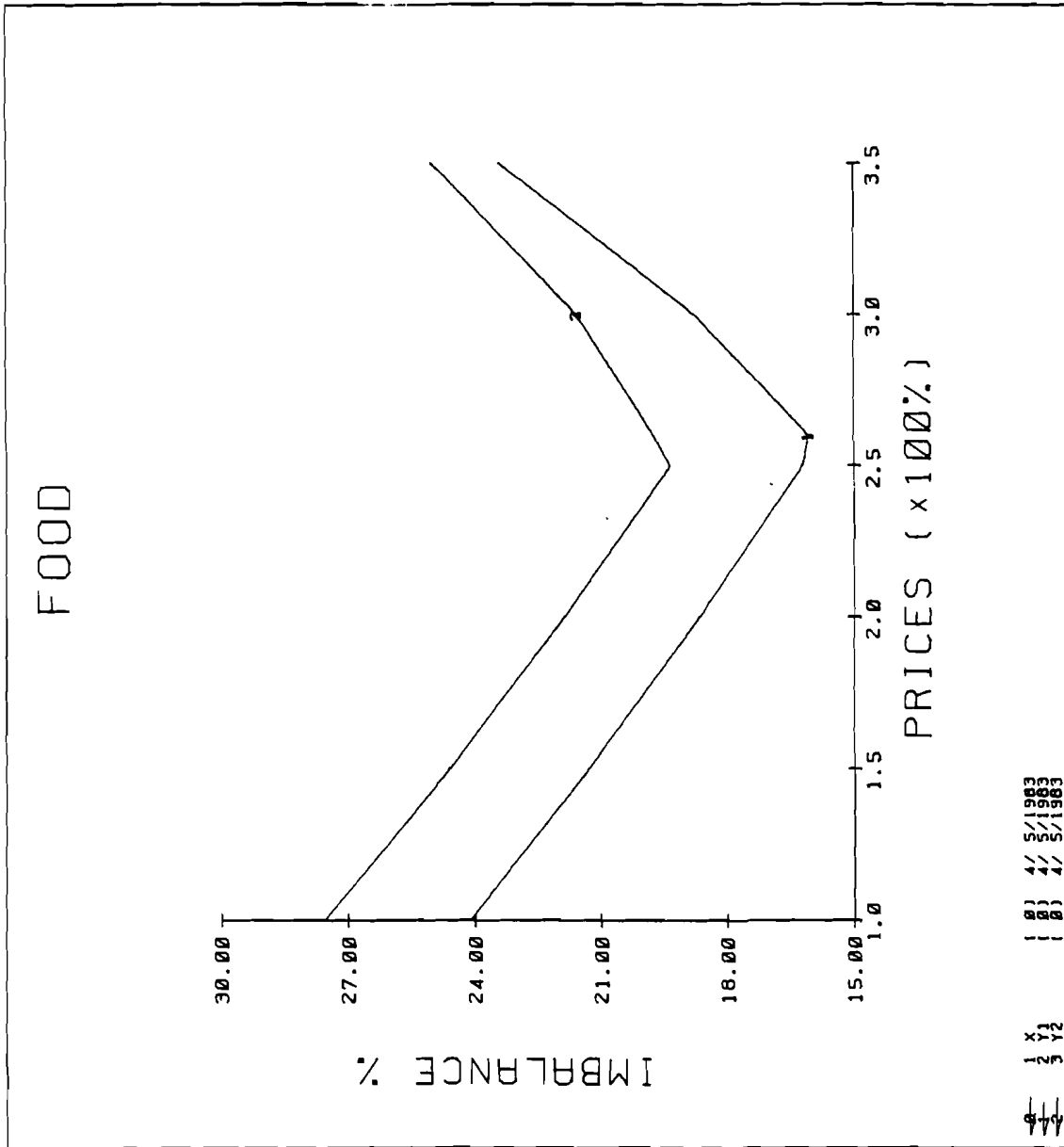


Figure 4.

Price level for agric.	Value of ρ (%)	
	Acceptable range of balance: 10%	Acceptable range of balance: 5%
1.00	24.1324	27.5699
1.50	21.3056	24.6208
2.00	18.6731	21.8744
2.50	16.2155	19.3104
2.60	16.0819	19.7516
3.00	18.7820	21.5839
3.50	23.4352	25.0442

Table 6.

These results imply that prices for energy products were relatively overinflated in 1975, but that prices for agricultural products were too low (low from the viewpoint of the optimal balance of the world trade market as a whole). It can also be seen that the state of world trade in 1975 was ten times more sensitive with respect to variations in energy prices, than to variations in the prices of agricultural products.

For example, using Table 5 it is possible to evaluate the level of oil price that would have "best" balanced the world trade market in 1975. Let the optimal level of the price for "energy" be $100\alpha\%$ of the actual 1975 level and let the share of oil in the total volume of energy products be β . Then, using Q to represent the total cost of energy products sold in 1975, we have the equation

$$\alpha Q = (1 - \beta) Q + \beta x Q,$$

where x is the optimal level of the oil price.

Therefore

$$x = \frac{\alpha + \beta - 1}{\beta}$$

Knowing that in 1975 β equaled approximately 0.5 and deducing from Figure 3 that α is about 0.85, we get $x = 0.7$. This means that the optimal 1975 oil price (optimal as regards total world trade) should have been 30% lower than the actual level.

It is of course clear that the analysis of the dependence of the imbalance value on the price vector p will not be complete until it is considered sequentially for different components of the vector. It may also be of great importance to consider the dependence for several components simultaneously and this may give quite different results to the sequential analysis.

For example, by considering the simultaneous variation of prices for "energy" and "agricultural products", we can find a state of the market with a "better" value of the imbalance than that in Tables 5 and 6. Figure 5 shows a piecewise linear approximation of the dependence of the imbalance on these two components. The trade balance was of the order of 5 % of the total exports here. It is easy to see that there exists a state with an imbalance of 14.9%, corresponding to energy prices at 80% and agricultural product prices at 150% of 1975 levels.

It should be noted here that analyzing of the value of the imbalance as a function of the components of the price vector p is a very difficult mathematical problem and that the analysis requires the development of special methods, which will be considered in the following sections.

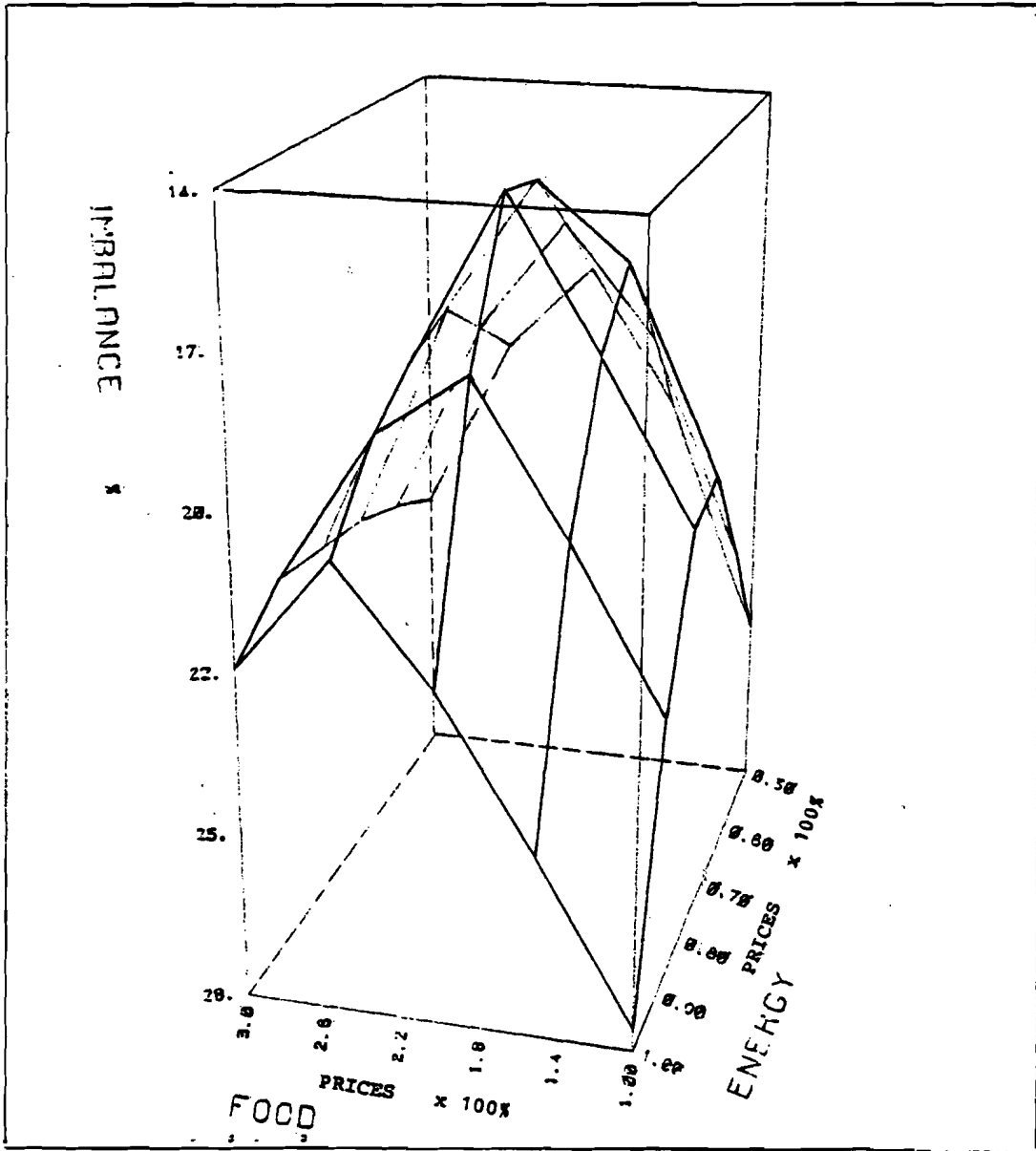


Figure 5.

7. Difficulties Arising in the Parametric Analysis of the Trade Market Model

Attempts to use classical mathematical methods to analyze the dependence of the imbalance on model parameters face serious difficulties, arising from various features of this dependence.

First of all, except for the most trivial problems, it is impossible to construct an analytical description of the dependence (*i.e.* one based on a system of equations or inequalities) that can be investigated by standard methods of mathematical analysis.

Secondly, the graphical-analytical method used earlier is of no use if more than two components are considered. Therefore, only methods of numerical analysis can be applied here.

Thirdly, classical numerical methods of mathematical analysis, such as Taylor-series expansion, also cannot be used here because of two specific properties of the functions to be analyzed:

- It is only possible to use the dependence of the imbalance on model parameters when one or more acceptable states exist. But this is not the case for all parameter values. In other words, the function under consideration is not defined for every set of its arguments. We must first establish whether or not there exists an acceptable state of the trade market.
- Even within the domain of the model definition, the use of Taylor-series expansion is not always feasible, because the function is nondifferentiable. Therefore, classical algorithms cannot be applied here to find, for example, a minimum value of the imbalance.

These analytically "difficult" properties - nonexistence and nondifferentiability - are due to the presence of inequalities in the mathematical statement of the problem ((11)-(15)) and are not, therefore, easily removable.

8. The Method of Compact Modelling

To enable highly reliable and effective standard software, based on methods of classical mathematical analysis, to be applied to the investigation of the interdependence of imbalance and model parameters, a special approach known as "*compact modelling*" has been developed.

This is based on the assumption that, even in case of a large-scale model, the user is really interested in the interdependence of certain input and output characteristics whose number is relatively low in comparison with the total number of internal degrees of freedom of the model (for example, the number of model variables).

From the methodological point of view the compact modelling approach is equivalent to reformulating the description of the model, in terms of only those data that are of interest for users, into a new form that permits the use of classical methods of mathematical analysis. This new description must be an adequate substitute for the original one in the sense that all the properties that significantly affect the behavior of the model are retained.

It is obvious that the effectiveness of the approach will depend on the volume of computer resources required for practical use. We do not suggest that this scheme will necessarily be effective for every model, but our experience has shown that the compact modelling approach can be used successfully for a fairly large class of finite-dimensional optimization and simulation models.

We will now present a formal description of the compact modelling approach.

Let us assume that there is a high-dimensional vector X , describing a state of a model M . Let vector X^* be a solution of a problem that was solved by means of model M and has the required properties. Then the process of solution may be formulated as

$$M(X^\circ) \rightarrow X^*,$$

where X° is an initial state of the model.

Let us assume that low-dimensional vectors p and D describe the input and output data, respectively. The user is actually interested in the interdependence between D and p . We will also define two conversion relations. The first permits the generation of the initial state of the model, using the initial input data

$$G(p) \rightarrow X^\circ$$

and the second converts the final state X^* into the output data

$$S(X^*) \rightarrow D.$$

Finally, the interdependence between the input and the output data can be written as

$$D = S(M(G(p))).$$

The operator $\tilde{M} = S(M(G(\bullet)))$ may be called the *compact image of model M*.

For a variety of reasons it is almost impossible to construct the explicit form of this compact image for the majority of mathematical models of practical value. Hence, whatever version of the approach is used, it should not be based on the use of the compact images in an explicit way.

However, the idea is useful, in that this image can be approximated locally for the immediate vicinity of a current vector p , rather than used globally for the whole set of vectors p under consideration. The construction of the compact image itself is generally not the aim of the investigation. Much more often

the user wants to determine values of the input data that will provide the required or desired values of the output data. For example, for the model of a trade market it is possible to formulate the problem of searching for components of the vector of relative prices p^k for which the value ρ^* of the imbalance is a minimum. To solve the problem by a numerical method, we do not need to have explicit formulae for the dependence of ρ^* on p . It is sufficient to be able to find local approximations of the function for any points along the way to the solution.

The compact modelling method can be compared with the procedure of local approximation of a function by a part of a power series. The part of the power series has a simpler description than the original function, but the description takes different forms for different points, because the coefficients of the series are themselves different for different points. In the compact modelling approach, the original high-dimensional description of the model is substituted by a sequence of low-dimensional local approximations of its compact image.

At the same time, this analogy has only restricted validity. In fact the dependence $D = M(p)$ cannot be correctly described by Taylor approximations, even if the model $X^* = M(X)$ is described by smooth enough functions.

There are three reasons for these "difficult" properties of the relation $D = M(p)$:

- D does not exist for all p ,
- the dependence of D on p is, generally speaking, nonunique,
- the dependence of D on p may be nondifferentiable, even if the functions describing the model M are themselves differentiable.

These properties are always present, even in the case of simple models

described in terms of linear programming problems or systems of linear inequalities. Therefore, the use of methods based on the Taylor-series approximations is ruled out for these models.

In order to make use of standard numerical techniques, the compact image of the model M used must

- have unique values defined for *all* vectors p considered,
- be close (in the sense of a metric) to the original dependence $D = \bar{M}(p)$ at all points where the latter exists,
- be smooth enough for the use of standard Taylor-series approximation.

To distinguish the compact image finally used from the original one, we will denote the former as $D = \bar{M}(p)$. This function $D = \bar{M}(p)$ may be approximated by a part of the Taylor-series in the vicinity of any point p and therefore standard numerical methods can be applied to investigate this dependence.

9. Practical Use of the Compact Modelling Approach

The main difficulty to be overcome in the compact modeling approach consists in choosing a method for the practical and effective construction of approximations of the compact image $\bar{D} = \bar{M}(p)$. The effectiveness of the method as a whole will depend on how quickly and exactly the Taylor approximations of the dependence $\bar{D} = \bar{M}(p)$ are calculated. We suggest the following scheme for constructing the approximations.

Let the original model M be formulated as a mathematical programming problem, or even as a system of equations and inequalities.

Instead of $X^(p)$, use the dependence $\bar{X}(p)$, which is the minimum point of*

the auxiliary function created for the mathematical programming problem according to the rules of a smooth version of the "Penalty Functions Method". Then we can use $\bar{D} = \bar{M}(p) = S(\bar{X}(p))$ as a compact image of the model M .

This auxiliary function is [see, for example, Fiacco and McCormick, 1968]

$$E = \rho + \sum_{s \in \Omega} P(\tau, \alpha_s) \quad (16)$$

where the so-called *penalty function* $P(\tau, \alpha)$ is defined as being smooth enough for all α and $\tau > 0$ and satisfies the following relation

$$\lim_{\tau \rightarrow +0} P(\tau, \alpha) = \begin{cases} 0, & \alpha \geq 0 \\ +\infty, & \alpha < 0 \end{cases}$$

and Ω is the set of active constraints of the mathematical programming problem.

If the auxiliary function (16) has its minimum at the point \bar{X} , *i.e.*

$$\bar{X} = \underset{X}{\operatorname{argmin}} E(\tau, X),$$

then it is possible to substitute $\bar{X}(p)$ into $D = S(X^*)$ instead of X^* .

Now we can demonstrate that the dependence $\bar{D} = \bar{M}(p)$ may be used as a compact image with the desired properties.

First, \bar{D} exists for any p , because the auxiliary function (16) has a minimum value independently of whether or not the mathematical programming problem is feasible.

Second, according to the known properties of the Penalty Functions Method, point \bar{X} is close enough (subject to appropriate choice of penalty function) to X^* for all p vectors, when the original model is feasible.

Third, the dependence $\bar{X}(p)$ is implicitly defined by the condition of stationarity of the auxiliary function (16)

$$\nabla_X E(\tau, \bar{X}(p)) = 0, \quad (17)$$

and the *implicit functions theorem* is applicable due to the smoothness of the

penalty function $P(\tau, \alpha)$. Therefore, the dependence $\bar{D} = \bar{M}(p)$ is also smooth enough for our purposes.

As can be concluded from the foregoing, the main problem with this approach is finding the vector $\bar{X}(p)$ for a given p . Direct optimization of the auxiliary function (16) is generally not possible, because the Penalty Functions Method is comparatively ineffective when used as an optimization algorithm. It is much better to build the X^* first and then to find \bar{X} , using X^* as an initial approximation, of course, only for those p where X^* exists.

We can now demonstrate the use of the approach for the mathematical model of trade markets (11)-(15) described in the previous sections, together with a quadratic penalty function P

$$P(\tau, \alpha) = \frac{1}{2\tau} \left[\frac{\alpha - \text{abs}(\alpha)}{2} \right]^2$$

We will consider the price vector p as the vector of input data and the value of imbalance ρ^* as the output one. As the compact image of the value ρ^* we will use the value of the auxiliary function

$$E = \rho + \frac{\tau}{2} \sum_{g=1}^{10} \sum_t \left[\frac{y_{gt} - \text{abs}(y_{gt})}{2} \right]^2 + \frac{\tau}{2} \sum_{g=11}^{19} \sum_t (y_{gt})^2, \quad (18)$$

calculated at its minimum point, for a small fixed value of the penalty parameter τ , where the internal sum is taken for all feasible t and the variables y_{gt} are defined by the following equations

$$-\tau y_{1t} + x_{ij}^k + \rho v_{ij}^k = 0, \quad (19)$$

$$-\tau y_{2t} - x_{ij}^k + \rho v_{ij}^k = 0, \quad (20)$$

$$-\tau y_{3t} + x_{ij}^k + v_{ij}^k = \underline{v}_{ij}^k, \quad (21)$$

$$\tau y_{4t} + x_{ij}^k + v_{ij}^k = \bar{v}_{ij}^k, \quad (22)$$

where $t = N^2(k-1) + N(i-1) + j$, for all k, i, j ,

$$-\tau y_{5t} + EXP_t = \underline{EXP}_t \quad (23)$$

$$\tau y_{6t} + EXP_t = \bar{EXP}_t \quad (24)$$

$$-\tau y_{7t} + IMP_t = \underline{IMP}_t \quad (25)$$

$$\tau y_{8t} + IMP_t = \overline{IMP}_t \quad (26)$$

$$-\tau y_{9t} + BALANCE_t = \overline{BALANCE}_t \quad (27)$$

$$\tau y_{10t} + BALANCE_t = \overline{BALANCE}_t \quad (28)$$

$$-\tau y_{11t} + EXP_t - IMP_t - BALANCE_t = 0 \quad (29)$$

$$-\tau y_{12t} + EXP_t - \sum_{j=1}^N \sum_{k=1}^K p^k (V_{ij}^k + x_{ij}^k) = 0 \quad (30)$$

$$-\tau y_{13t} + IMP_t - \sum_{j=1}^N \sum_{k=1}^K p^k (V_{jt}^k + x_{jt}^k) = 0, \quad (31)$$

for all $t = [1, N]$.

Sometimes the components of vector $\nabla_p \bar{D}$ may be necessary to build a local approximation of the compact image. Taking into account that, in the case considered, D is equivalent to E , and using the well-known "chain rule", we get

$$\nabla_p \bar{D}(\bar{X}(p), p) = \frac{\partial \bar{E}}{\partial p} + \nabla_X \bar{E} \frac{\partial \bar{X}}{\partial p}.$$

By virtue of (17), finally, we have

$$\nabla_p \bar{D} = \frac{\partial \bar{E}}{\partial p}. \quad (32)$$

For the specific case of the model (11)-(15), the components of the gradient $\nabla_p \bar{D}$ can be calculated by formulae

$$\frac{\partial \bar{D}}{\partial p^k} = \sum_{i=1}^N \sum_{j=1}^N \left[y_{12i} (x_{ij}^k + V_{ij}^k) + y_{13i} (x_{ji}^k + V_{ji}^k) \right],$$

for all $k = [1, K]$.

Note that linear local approximation of the compact image requires no components of the sensitivity matrix $\frac{\partial \bar{X}}{\partial p}$; it is sufficient just to consider the point \bar{X} .

The problem (18)-(31) can be solved by standard software routines for most dimensions of practical value. All the variables of this problem are free, all the constraints are equalities, and the objective function is separable and (considered piecewise) quadratic.

10. A Short Description of the Software for Analyzing the Balance of Trade Markets

A system of computer programs developed at IIASA [Issaev and Umnov, 1982, Lenko, 1983] for analyzing the balance of trade markets, referred to further as TMA, may be used in the following situations :

- for preparing descriptions of the given and acceptable states of the trade market,
- for solving the problem (11)-(16) to find the value of the current imbalance and to construct an acceptable state,
- to transform the output data into a convenient form.

In the *first* case, namely preparing descriptions of given and acceptable states, the user must prepare lists of the trade partners and the commodities to be sold. The total number of the partners must not be greater than m^2 , where m is the number of characters permitted in the computer. Each of the partners has its own name and identifier. The length of the name must not be longer than 32 characters, including blanks. The identifier has two characters in each case. An example of the form in which the data are prepared is shown in Table 1.

The user must also define the number and name of each of the commodities. The name has 32 characters, including blanks, while the number must be between 1 and 99. Note that one number is reserved for the monetary account. An example of such data is shown in Table 2.

Next, it is necessary to define nonzero values of the trade export-import flows for the given state. The TMA system makes it possible to input the elements of the cubic matrix *exporter-importer-commodity* in any order the user

likes. This input is made using a terminal in the dialogue mode.

Finally, the user should describe the constraints on the acceptable state, if such states exist, of course. An example of these data is shown in Table 3.

In the *second* case the TMA system prepares the MPS file required for problem (11) - (15) and runs solution procedures.

In the *third* case, depending on the specific user request, TMA prepares several types of output file, containing all the information about the given and acceptable states of the trade market. If the user wants it, a special graphical-analytical file can be prepared in the form described in Section 4.

When using the TMA system, the user should take into account that, although the upper limits N for the number of partners and K for the number of commodities are generally large enough, in practice it is only possible to consider relatively limited amounts of partners and commodities. This is because the dimensions of the problem (11) - (15) increase very rapidly with increasing N and K . The number of variables in this problem is $N^2(K+1)$, the number of constraints is not less than $3N^2(K+1)+N$ and the number of nonzero elements is not less than $8N^2(K+1)$. Experience has shown that these values more or less dictate the acceptable levels for N and K .

Based on practical experience using the approach described and the TMA system, we can make the following conclusions and recommendations.

The TMA system is relatively highly reliable and productive. Using IIASA's VAX 11-780 computer, working under the UNIX operating system, the whole cycle of calculations requires about 0.6 billion operations, or 20 mins of CPU time for the model described in the paper. If an initial basis were known (for example, from a previous run), the time required would be 6-7 times less.

There is no problem in specifying input information for the approach con-

sidered because all data can be measured as exactly as required.

Finally it is worth mentioning that the approach described can produce essentially the same results as traditional methods of trade market analysis. For example, if the constraint on the acceptable states consists only of a given import vector, the final state found has exactly the same structure as the initial one.

To conclude this paper let us consider the problem of minimizing the value of the imbalance of the world trade market in 1975. We need to find values of the components of the vector p corresponding to the minimum level of ρ^* . We set the level of prices in 1975 equal to one. The acceptable state of the trade market is the same as that described in Section 4.

Values of the components of the gradient of the dependence of ρ^* on p for the initial point are given in Table 7.

#	Commodity	$\frac{\partial \rho^*}{\partial p^k}$
1	Food products	-0.059
2	Raw materials	0.005
3	Energy products	0.460
4	Intermediate products	-0.101
5	Consumer nondurables	-0.022
6	Equipment	-0.185
7	Consumer durables	-0.029

Table 7.

The values of the relative prices p_k were varied within the following ranges

$$0.99 \leq p^1 \leq 3.00$$

$$0.10 \leq p^2 \leq 1.01$$

$$0.50 \leq p^3 \leq 1.01$$

$$0.99 \leq p^4 \leq 2.00$$

$$0.99 \leq p^5 \leq 4.00$$

$$0.99 \leq p^6 \leq 1.50$$

$$0.99 \leq p^7 \leq 4.00$$

Values of p^k , the auxiliary function and components of its gradient for different steps are shown in Table 8.

At the optimal point found the value of the imbalance equals $\sim 10.5\%$. Volumes of total export-import flows for this price vector are given in Figure 6.

In conclusion, it is necessary to mention the following points.

During practical use of the TMA system, it has been found that the description of an acceptable state may often be simplified by introducing some new variables, which, however, do not change the mathematical statement of the problem. These variables are :

the total export of the k th good by the i th partner

$$OUT_i^k = \sum_{j=1}^N p^k V_{ij}^k,$$

the total import of the k th good by the i th partner

$$IMP_i^k = \sum_{j=1}^N p^k V_{ji}^k,$$

the total cost of the k th good sold on the market

$$SUM^k = \sum_{i=1}^N OUT_i^k,$$

and also the total volume of the trade on the market

$$TOTAL = \sum_{i=1}^N EXP_i.$$

Further, the TMA system uses inequality constraints rather than upper and lower absolute bounds for all variables. This can be useful if it is necessary to predefine a given structure of trade flows. One example of this condition might be written

$$0.12EXP_i \leq exp_{ij} \leq 0.15EXP_i$$

This condition means that the share of the j th partner with respect to the total export of the i th partner must lie in the range 12 - 15 % .

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Iteration 0

$$\begin{aligned} \mathbf{p} &= ||1.000, 1.000, 1.000, 1.000, 1.000, 1.000, 1.000|| \\ \bar{E}(\mathbf{p}) &= 0.241 \\ \nabla_{\mathbf{p}} \bar{E} &= ||-0.059, 0.005, 0.460, -0.101, -0.022, -0.185, -0.029|| \end{aligned}$$

Iteration 1

$$\begin{aligned} \mathbf{p} &= ||1.014, 0.999, 0.887, 1.025, 1.006, 1.045, 1.007|| \\ \bar{E}(\mathbf{p}) &= 0.174 \\ \nabla_{\mathbf{p}} \bar{E} &= ||-0.069, -0.129, 0.027, 0.001, 0.016, 0.086, 0.006|| \end{aligned}$$

Iteration 2

$$\begin{aligned} \mathbf{p} &= ||1.221, 1.010, 0.846, 1.025, 0.990, 0.990, 0.990|| \\ \bar{E}(\mathbf{p}) &= 0.155 \\ \nabla_{\mathbf{p}} \bar{E} &= ||-0.069, -0.021, -0.191, 0.048, 0.033, 0.209, 0.022|| \end{aligned}$$

Iteration 3

$$\begin{aligned} \mathbf{p} &= ||1.929, 1.010, 0.827, 0.990, 0.990, 0.990, 0.990|| \\ \bar{E}(\mathbf{p}) &= 0.113 \\ \nabla_{\mathbf{p}} \bar{E} &= ||-0.053, -0.017, -0.157, 0.048, 0.031, 0.192, 0.021|| \end{aligned}$$

Iteration 4

$$\begin{aligned} \mathbf{p} &= ||2.162, 1.010, 0.809, 0.990, 0.990, 0.990, 0.990|| \\ \bar{E}(\mathbf{p}) &= 0.105 \\ \nabla_{\mathbf{p}} \bar{E} &= ||-0.000, -0.124, -0.006, 0.835, 0.047, 5.831, -0.136|| \end{aligned}$$

Table 8 .

APPENDIX 1. Values of the trade flows in the mathematical model of the world trade market for 1979 (initial state) .

YEAR 1975

AGRICULTURAL PRODUCTS

EXPORT from to	US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	3.592	2.704	0.200	0.230	1.327	4.165	0.686	0.065	0.276	1.692	0.042	14.979
EU	9.644	36.662	3.447	0.297	2.559	6.551	3.660	0.996	0.614	2.631	0.445	67.506
SU	2.315	1.525	4.685	0.025	0.498	3.798	0.400	0.982	0.488	0.271	0.459	15.446
JA	4.942	0.563	0.684	0.044	1.695	1.007	0.202	0.048	0.250	2.751	0.402	12.544
OD	0.250	0.341	0.007	0.044	0.147	0.035	0.077	0.004	0.061	0.232	0.014	1.212
LT	2.906	0.967	0.006	0.038	0.165	1.582	0.016	0.020	0.021	0.128	0.001	5.850
AF	0.393	1.226	1.206	0.037	0.231	0.041	0.266	0.037	0.064	0.146	0.585	4.232
WA	1.963	2.740	0.233	0.098	0.670	0.973	0.260	0.488	0.833	0.483	0.081	8.822
IN	1.602	0.523	0.018	0.008	0.222	0.036	0.052	0.021	0.294	0.278	0.052	3.106
EA	2.193	0.434	0.013	0.291	1.041	0.156	0.196	0.061	0.190	2.397	0.716	7.688
CN	0.461	0.025	0.217	0.011	0.271	0.158	0.055	0.163	0.093	0.161	0.	1.615
SUM	30.261	47.710	10.716	1.079	8.826	18.502	5.870	2.885	3.184	11.170	2.797	143.000

Table A.1.1

YEAR 1975

RAW MATERIALS

EXPORT from to US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	0.135	0.040	0.007	0.133	1.176	0.095	0.006	0.007	0.046	0.006	2.886
EU	3.620	0.742	0.003	0.767	1.197	0.707	0.818	0.039	0.289	0.060	10.321
SU	0.048	1.729	0.001	0.003	0.156	0.016	0.332	0.056	0.007	0.122	2.695
JA	0.884	0.085	0.006	1.466	0.574	0.089	0.046	0.239	0.754	0.051	4.214
OD	0.066	0.030	0.006	0.016	0.001	0.001	0.004	0.	0.007	0.002	0.133
LT	0.364	0.001	0.001	0.020	0.151	0.002	0.037	0.	0.006	0.	0.609
AF	0.005	0.038	0.	0.070	0.017	0.011	0.004	0.	0.	0.001	0.184
WA	0.047	0.007	0.001	0.005	0.007	0.001	0.024	0.011	0.006	0.002	0.184
IN	0.038	0.005	0.001	0.005	0.	0.001	0.027	0.003	0.016	0.001	0.102
EA	0.222	0.001	0.047	0.084	0.003	0.003	0.008	0.021	0.117	0.012	0.540
CN	0.020	0.017	0.	0.025	0.005	0.001	0.014	0.002	0.017	0.	0.104
SUM	5.008	2.665	0.067	2.594	3.287	0.927	1.320	0.378	1.265	0.257	21.972

Table A.1.2

YEAR 1975

ENERGY		EXPORT FROM											TOTAL
	US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN		
to													
US	7.613	0.743	0.195	0.001	0.030	10.550	3.113	8.955	0.	2.270	0.	33.470	
EU	1.630	16.076	6.615	0.038	0.223	1.547	4.732	43.418	0.008	0.271	0.009	74.567	
SU	0.029	0.252	6.672	0.004	0.	0.015	0.002	1.867	0.058	0.028	0.	8.927	
JA	2.131	0.065	0.311	0.	0.986	0.041	0.355	17.182	0.001	4.776	0.789	26.637	
OD	0.050	0.082	0.	0.003	0.116	0.012	0.005	2.450	0.	0.382	0.	3.100	
LT	0.573	0.083	0.199	0.043	0.009	4.684	1.310	6.232	0.	0.772	0.001	13.906	
AF	0.025	0.274	0.388	0.	0.011	0.263	0.295	1.745	0.	0.079	0.001	3.081	
WA	0.054	0.565	0.336	0.002	0.003	0.019	0.005	2.693	0.002	0.034	0.001	3.714	
IN	0.022	0.010	0.041	0.003	0.002	0.004	0.	1.187	0.029	0.050	0.003	1.351	
EA	0.066	0.027	0.042	0.126	0.081	0.013	0.023	6.672	0.	1.837	0.053	8.940	
CN	0.001	0.012	0.141	0.003	0.	0.001	0.	0.042	0.	0.110	0.	0.310	
SUM	12.194	18.189	14.940	0.223	1.461	17.149	9.840	92.443	0.098	10.609	0.857	178.003	

Table A.1.3

YEAR 1975

INTERMEDIATE PRODUCTS

EXPORT from to	US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	7.981	5.950	0.227	3.434	0.559	1.029	0.098	0.037	0.264	1.089	0.085	20.753
EU	5.857	72.934	3.354	1.959	1.055	1.764	1.401	0.364	0.689	1.127	0.285	90.789
SU	0.157	9.343	11.310	1.156	0.031	0.196	0.007	0.272	0.296	0.078	0.267	23.113
JA	0.920	1.057	0.287	0.	0.469	0.223	0.187	0.192	0.079	0.769	0.186	4.369
OD	0.780	2.008	0.021	0.936	0.312	0.020	0.010	0.065	0.040	0.240	0.036	4.468
LT	4.289	4.051	0.072	1.688	0.144	1.346	0.069	0.058	0.025	0.039	0.001	11.782
AF	0.360	2.797	0.840	0.499	0.171	0.038	0.205	0.032	0.081	0.108	0.227	5.358
WA	1.099	7.129	0.449	3.034	0.093	0.016	0.024	0.424	0.360	0.359	0.021	13.008
IN	0.412	0.807	0.230	0.661	0.035	0.017	0.017	0.082	0.080	0.133	0.003	2.477
EA	1.461	1.687	0.073	5.317	0.399	0.019	0.010	0.106	0.169	0.981	0.051	10.273
CN	0.153	0.966	0.343	1.626	0.079	0.048	0.026	0.044	0.011	0.025	0.	3.321
SUM	23.469	108.729	17.206	20.310	3.347	4.716	2.054	1.676	2.094	4.948	1.162	189.711

Table A.1.4

YEAR 1975

CONSUMER NON-DURABLE

EXPORT from to	US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	1.476	3.075	0.096	0.760	0.029	0.464	0.004	0.033	0.101	2.447	0.044	8.529
EU	1.211	23.328	0.940	0.490	0.043	0.216	0.032	0.152	0.117	2.345	0.150	29.024
SU	0.037	1.377	2.709	0.096	0.	0.007	0.	0.217	0.069	0.017	0.185	4.714
JA	0.345	0.552	0.010	0.	0.008	0.015	0.	0.	0.003	0.522	0.077	1.532
OD	0.228	0.686	0.011	0.145	0.075	0.011	0.001	0.	0.011	0.241	0.028	1.437
LT	0.921	1.023	0.011	0.135	0.004	0.516	0.	0.	0.002	0.052	0.004	2.668
AF	0.045	1.097	0.266	0.061	0.013	0.003	0.041	0.006	0.016	0.120	0.246	1.914
WA	0.204	1.773	0.105	0.400	0.004	0.005	0.003	0.164	0.060	0.250	0.070	3.038
IN	0.025	0.099	0.025	0.051	0.001	0.	0.	0.011	0.017	0.018	0.004	0.251
EA	0.205	0.478	0.005	0.334	0.070	0.013	0.001	0.003	0.010	0.315	0.133	1.567
CN	0.008	0.023	0.101	0.019	0.	0.001	0.	0.001	0.001,	0.007	0.	0.161
SUM	4.705	33.511	4.279	2.491	0.247	1.251	0.082	0.587	0.407	6.334	0.941	54.835

Table A.1.5

YEAR 1975

EQUIPMENTS		YEAR 1975											TOTAL
EXPORT from		EU	SU	JA	OD	LT	AF	WA	IN	EA	CN		
to	US												
US	21.120	10.390	0.099	5.622	0.087	0.311	0.010	0.008	0.012	1.001	0.002	38.662	
EU	12.230	70.449	1.870	4.146	0.166	0.168	0.026	0.082	0.054	0.510	0.008	89.709	
SU	0.874	8.420	20.241	0.850	0.002	0.001	0.	0.007	0.030	0.001	0.040	30.466	
JA	1.872	1.111	0.022	0.	0.063	0.051	0.001	0.	0.002	0.240	0.	3.362	
OD	2.364	4.612	0.016	1.455	0.232	0.031	0.001	0.001	0.008	0.151	0.001	8.872	
LT	8.240	6.708	0.210	2.441	0.042	1.021	0.003	0.	0.004	0.029	0.001	18.699	
AF	0.866	5.377	1.576	3.019	0.102	0.064	0.064	0.009	0.049	0.145	0.053	11.324	
WA	4.680	13.674	0.962	2.134	0.040	0.059	0.007	0.322	0.074	0.101	0.007	22.060	
IN	0.396	1.191	0.319	0.327	0.006	0.001	0.	0.003	0.040	0.032	0.003	2.318	
EA	3.409	3.244	0.063	4.517	0.292	0.026	0.001	0.001	0.075	0.859	0.015	12.502	
CN	0.166	1.026	0.948	0.816	0.	0.	0.	0.001	0.001	0.003	0.	2.961	
SUM	56.217	126.202	26.326	25.327	1.032	1.733	0.113	0.434	0.349	3.072	0.130	240.935	

Table A.1.6

YEAR 1975

CONSUMER DURABLES

EXPORT from	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
to											
US	2.034	0.046	2.370	0.326	0.246	0.066	0.006	0.064	0.967	0.015	8.758
EU	2.366	0.670	1.403	0.264	0.136	0.193	0.017	0.096	0.630	0.034	25.689
SU	0.050	2.279	0.068	0.007	0.002	0.001	0.022	0.009	0.009	0.155	3.058
JA	0.418	0.505	0.017	0.028	0.030	0.011	0.	0.024	0.218	0.034	1.285
OD	0.322	0.950	0.003	0.064	0.003	0.001	0.	0.006	0.102	0.005	1.870
LT	1.536	1.068	0.308	0.018	0.212	0.001	0.	0.006	0.050	0.019	3.224
AF	0.111	0.501	0.123	0.278	0.004	0.022	0.002	0.014	0.045	0.215	1.649
WA	0.429	0.041	0.569	0.006	0.005	0.005	0.110	0.031	0.103	0.066	3.576
IN	0.063	0.188	0.031	0.001	0.001	0.	0.001	0.004	0.013	0.004	0.358
EA	0.627	0.906	0.941	0.123	0.004	0.002	0.001	0.044	0.285	0.196	3.145
CN	0.011	0.051	0.040	0.	0.	0.001	0.	0.	0.005	0.	0.219
SUM	7.967	29.334	6.267	1.115	0.643	0.303	0.159	0.298	2.427	0.743	52.831

Table A.1.7

APPENDIX 2. Values of the trade flows in the mathematical model of the world trade market for 1979 (the "nearest" acceptable state) .

AGRICULTURAL PRODUCTS

EXPORT from to US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	2.725	0.248	0.174	1.007	5.170	0.852	0.049	0.343	1.284	0.032	15.241
EU	7.317	2.615	0.225	1.941	6.028	4.543	0.756	0.762	1.996	0.338	54.336
SU	2.874	3.554	0.019	0.378	2.881	0.303	0.745	0.606	0.206	0.348	13.071
JA	6.135	0.849	0.	1.286	0.764	0.153	0.036	0.190	2.087	0.305	12.232
OD	0.190	0.005	0.033	0.112	0.027	0.058	0.003	0.046	0.176	0.011	0.920
LT	3.607	0.007	0.029	0.125	1.200	0.012	0.015	0.026	0.097	0.001	5.853
AF	0.298	0.930	0.028	0.175	0.031	0.202	0.028	0.049	0.111	0.726	3.493
WA	2.437	0.289	0.122	0.832	1.208	0.323	0.370	1.034	0.600	0.101	10.717
IN	1.215	0.014	0.006	0.168	0.027	0.039	0.016	0.223	0.211	0.039	2.355
EA	2.430	0.010	0.221	0.790	0.118	0.243	0.046	0.144	1.819	0.543	6.693
CN	0.572	0.269	0.008	0.206	0.120	0.042	0.124	0.071	0.122	0.	1.553
SUM	29.800	8.775	0.865	7.020	17.574	6.770	2.188	3.494	8.709	2.444	126.464

Table A.2.1

RAW MATERIALS

EXPORT to	from US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	0.937	0.168	0.050	0.005	0.101	0.892	0.118	0.005	0.009	0.035	0.005	2.325
EU	1.577	2.746	0.563	0.002	0.582	0.908	0.878	0.621	0.048	0.219	0.046	8.190
SU	0.036	0.171	1.312	0.001	0.002	0.118	0.012	0.252	0.070	0.005	0.093	2.072
JA	0.671	0.020	0.106	0.	1.112	0.435	0.068	0.035	0.181	0.572	0.039	3.239
OD	0.050	0.023	0.	0.005	0.012	0.001	0.001	0.003	0.	0.005	0.002	0.102
LT	0.276	0.020	0.001	0.001	0.015	0.115	0.002	0.028	0.	0.005	0.	0.463
AF	0.004	0.029	0.029	0.	0.053	0.013	0.003	0.003	0.	0.	0.001	0.140
WA	0.058	0.091	0.009	0.001	0.006	0.009	0.001	0.018	0.014	0.007	0.002	0.216
IN	0.029	0.004	0.004	0.001	0.004	0.	0.001	0.020	0.002	0.012	0.001	0.078
EA	0.276	0.017	0.001	0.036	0.064	0.002	0.004	0.006	0.016	0.089	0.009	0.520
CN	0.015	0.002	0.021	0.	0.019	0.004	0.001	0.011	0.002	0.013	0.	0.088
SUM	3.929	3.291	2.096	0.052	1.970	2.497	1.094	1.002	0.342	0.962	0.198	17.433

Table A.2.2

ENERGY EXPORT		from	to	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
	US	US	US											
US	5.776	0.922	0.242	0.001	0.001	0.023	8.004	3.864	6.794	0.010	1.722	0.007	0.007	27.348
EU	1.237	12.196	5.019	0.029	0.029	0.169	1.174	4.144	32.940	0.072	0.206	0.021	0.021	57.131
SU	0.022	0.191	5.062	0.003	0.003	0.000	0.011	0.002	1.416	0.001	0.021	0.021	0.021	6.800
JA	1.617	0.049	0.386	0.000	0.000	0.748	0.031	0.269	13.036	0.000	3.623	0.599	0.599	20.359
OD	0.038	0.062	0.000	0.002	0.002	0.088	0.009	0.004	1.859	0.000	0.290	0.290	0.290	2.352
LT	0.435	0.063	0.151	0.033	0.033	0.007	3.554	0.994	4.728	0.000	0.586	0.586	0.586	10.552
AF	0.019	0.208	0.294	0.000	0.000	0.000	0.200	0.224	1.324	0.000	0.060	0.060	0.060	2.338
WA	0.067	0.701	0.417	0.002	0.002	0.004	0.024	0.006	2.043	0.002	0.042	0.042	0.042	3.309
IN	0.017	0.008	0.031	0.002	0.002	0.002	0.003	0.000	0.901	0.022	0.038	0.038	0.038	1.026
EA	0.050	0.020	0.032	0.096	0.096	0.061	0.010	0.029	5.062	0.000	1.394	0.040	0.040	6.794
CN	0.001	0.009	0.175	0.002	0.002	0.000	0.001	0.000	0.032	0.000	0.083	0.083	0.083	0.303
SUM	9.279	14.429	11.809	0.170	0.170	1.110	13.021	9.536	70.135	0.107	8.065	0.651	0.651	138.312

Table A.2.3

INTERMEDIATE PRODUCTS

EXPORT to	from US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	6.055	4.514	0.282	2.605	0.424	0.781	0.122	0.028	0.328	0.826	0.064	16.029
EU	4.444	55.333	2.545	1.486	0.800	1.338	1.063	0.276	0.584	0.855	0.216	68.940
SU	0.119	7.088	8.581	0.877	0.024	0.149	0.005	0.206	0.225	0.059	0.203	17.536
JA	0.698	0.802	0.356	0.	0.356	0.169	0.142	0.146	0.060	0.583	0.141	3.453
OD	0.592	1.523	0.016	0.710	0.237	0.015	0.008	0.049	0.030	0.182	0.027	3.589
LT	3.254	3.073	0.055	1.281	0.109	1.021	0.052	0.044	0.019	0.030	0.001	8.939
AF	0.273	2.122	0.637	0.379	0.130	0.029	0.156	0.024	0.061	0.082	0.282	4.175
WA	1.364	8.849	0.557	3.766	0.115	0.020	0.030	0.322	0.447	0.446	0.026	15.942
IN	0.313	0.612	0.174	0.501	0.027	0.013	0.013	0.062	0.061	0.101	0.002	1.879
EA	1.108	1.280	0.055	4.034	0.303	0.014	0.008	0.080	0.128	0.744	0.039	7.793
CN	0.116	0.733	0.260	1.234	0.060	0.036	0.020	0.033	0.008	0.019	0.	2.519
SUM	18.336	85.929	13.518	16.873	2.585	3.585	1.619	1.270	1.951	3.927	1.001	150.594

Table A.2.4

CONSUMER NON-DURABLE

EXPORT from to US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	1.120	0.119	0.577	0.022	0.352	0.005	0.025	0.125	1.856	0.033	6.567
EU	0.919	0.713	0.372	0.033	0.164	0.024	0.115	0.089	1.779	0.114	22.020
SU	0.028	2.055	0.073	0.006	0.005	0.000	0.165	0.052	0.013	0.140	3.576
JA	0.262	0.008	0.000	0.006	0.011	0.000	0.000	0.002	0.396	0.058	1.162
OD	0.173	0.008	0.110	0.057	0.008	0.001	0.000	0.008	0.183	0.021	1.089
LT	0.699	0.008	0.102	0.003	0.391	0.000	0.000	0.002	0.039	0.003	2.023
AF	0.034	0.832	0.046	0.010	0.002	0.031	0.005	0.012	0.091	0.187	1.452
WA	0.253	2.201	0.497	0.005	0.006	0.004	0.124	0.074	0.310	0.087	3.691
IN	0.019	0.075	0.039	0.001	0.000	0.000	0.008	0.013	0.014	0.003	0.191
EA	0.156	0.363	0.253	0.053	0.010	0.001	0.002	0.008	0.239	0.101	1.190
CN	0.006	0.017	0.014	0.000	0.001	0.000	0.001	0.001	0.005	0.000	0.122
SUM	3.669	26.279	2.083	0.190	0.950	0.066	0.445	0.386	4.925	0.747	43.083

Table A.2.5

EQUIPMENTS		EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
EXPORT	from											
to	US											
US	16.023	7.883	0.123	4.265	0.066	0.236	0.012	0.006	0.015	0.759	0.002	29.390
EU	9.279	53.448	1.419	3.145	0.126	0.127	0.020	0.002	0.041	0.387	0.006	63.060
SU	0.663	6.388	15.356	0.645	0.002	0.001	0.001	0.005	0.023	0.001	0.030	23.114
JA	1.420	0.843	0.017	0.	0.048	0.039	0.001	0.	0.002	0.182	0.	2.552
OD	1.794	3.499	0.012	1.104	0.176	0.024	0.001	0.001	0.006	0.115	0.001	6.733
LT	6.251	5.089	0.159	1.852	0.032	0.775	0.002	0.	0.003	0.022	0.001	14.186
AF	0.657	4.079	1.196	2.290	0.077	0.049	0.049	0.007	0.037	0.110	0.040	8.591
WA	5.809	16.974	1.194	2.649	0.050	0.073	0.009	0.244	0.092	0.125	0.009	27.228
IN	0.300	0.904	0.242	0.248	0.005	0.001	0.	0.002	0.030	0.024	0.002	1.758
EA	2.586	2.461	0.048	3.427	0.222	0.020	0.001	0.001	0.057	0.652	0.011	9.486
CN	0.126	0.778	0.719	0.619	0.	0.	0.	0.001	0.001	0.002	0.	2.246
SUM	44.908	102.346	20.485	20.244	0.804	1.345	0.095	0.329	0.307	2.379	0.102	193.344

Table A.2.6

CONSUMER DURABLES

EXPORT to	f.r.o.m US	EU	SU	JA	OD	LT	AF	WA	IN	EA	CN	TOTAL
US	1.543	1.986	0.057	1.798	0.247	0.187	0.082	0.005	0.079	0.734	0.011	6.729
EU	1.795	15.082	0.508	1.064	0.200	0.103	0.146	0.013	0.073	0.478	0.026	19.488
SU	0.038	0.346	1.729	0.052	0.005	0.002	0.001	0.017	0.007	0.007	0.118	2.322
JA	0.317	0.383	0.013	0.	0.021	0.023	0.008	0.	0.018	0.165	0.026	0.974
OD	0.244	0.721	0.002	0.314	0.049	0.002	0.001	0.	0.005	0.077	0.004	1.419
LT	1.165	0.810	0.005	0.234	0.014	0.161	0.001	0.	0.005	0.038	0.014	2.447
AF	0.084	0.380	0.253	0.093	0.211	0.003	0.017	0.002	0.011	0.034	0.163	1.251
WA	0.533	2.745	0.051	0.706	0.007	0.006	0.006	0.083	0.038	0.128	0.082	4.385
IN	0.048	0.143	0.039	0.024	0.001	0.001	0.	0.001	0.003	0.010	0.003	0.273
EA	0.476	0.687	0.012	0.714	0.093	0.003	0.002	0.001	0.033	0.216	0.149	2.386
CN	0.008	0.039	0.084	0.030	0.	0.	0.001	0.	0.	0.004	0.	0.166
SUM	6.251	23.322	2.753	5.029	0.848	0.491	0.265	0.122	0.272	1.891	0.596	41.840

Table A.2.7

APPENDIX 3. Graphical results of the forecasting procedure for
the development of the world trade market.

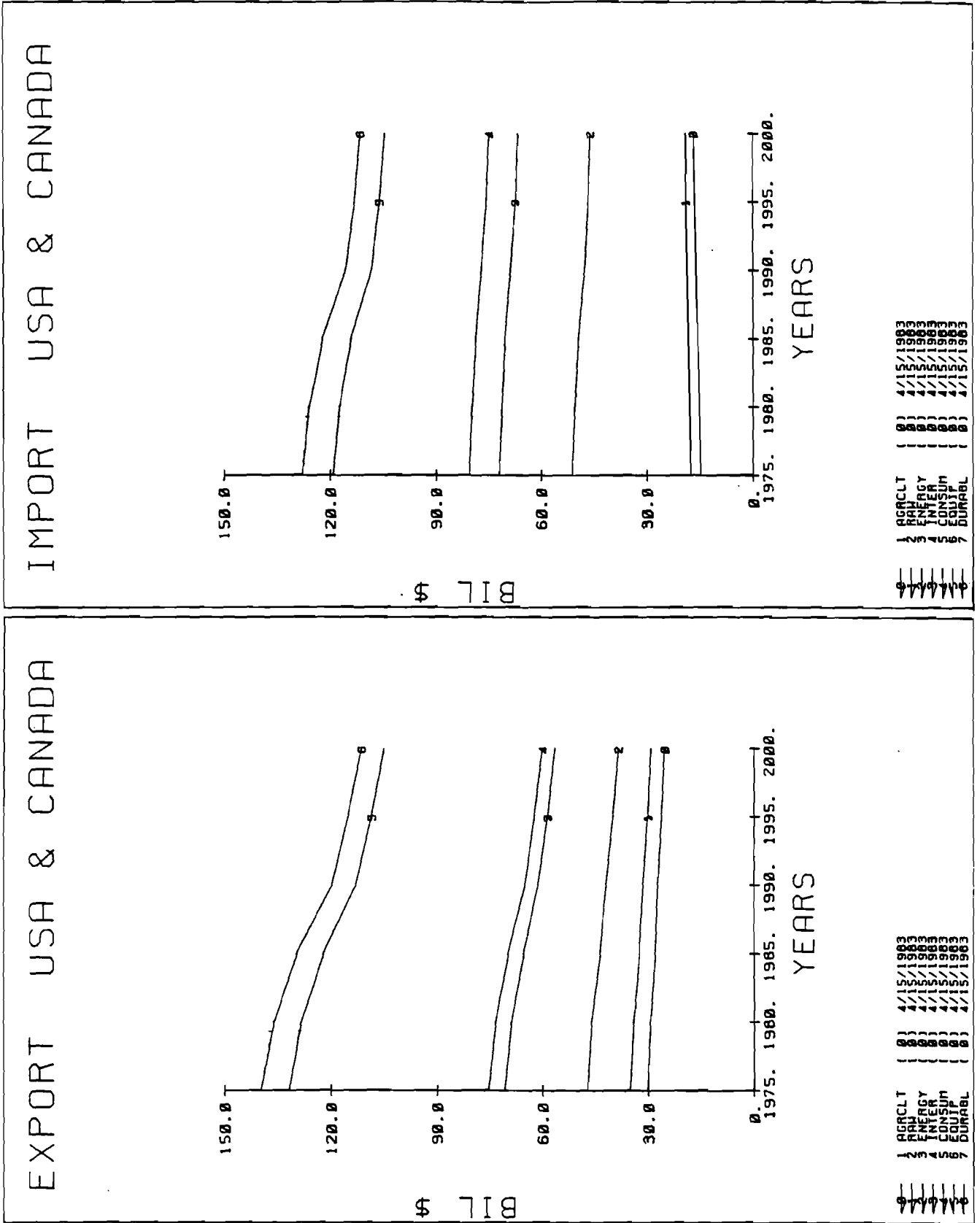


Figure A.3.1

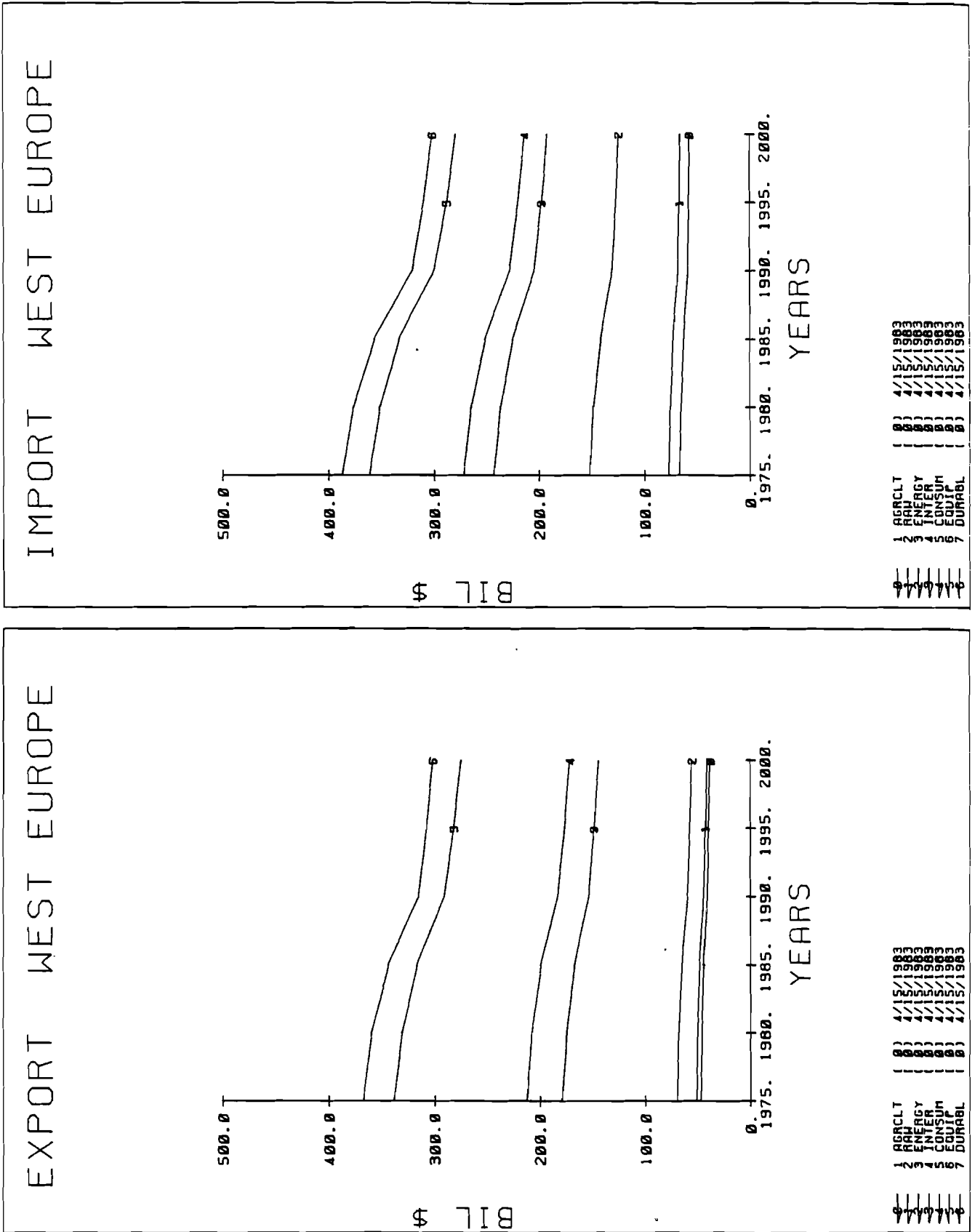


Figure A.3.2

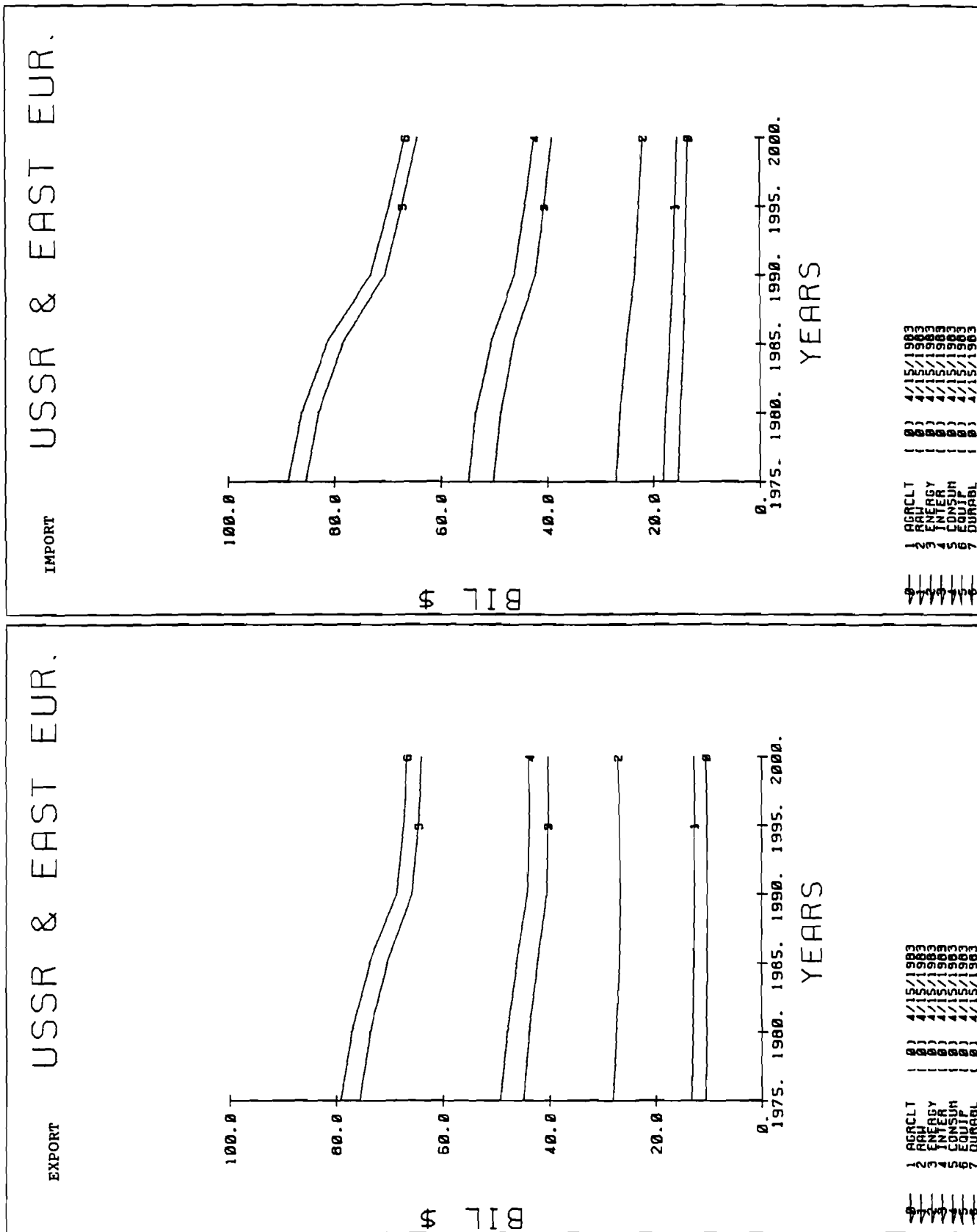


Figure A.3.3

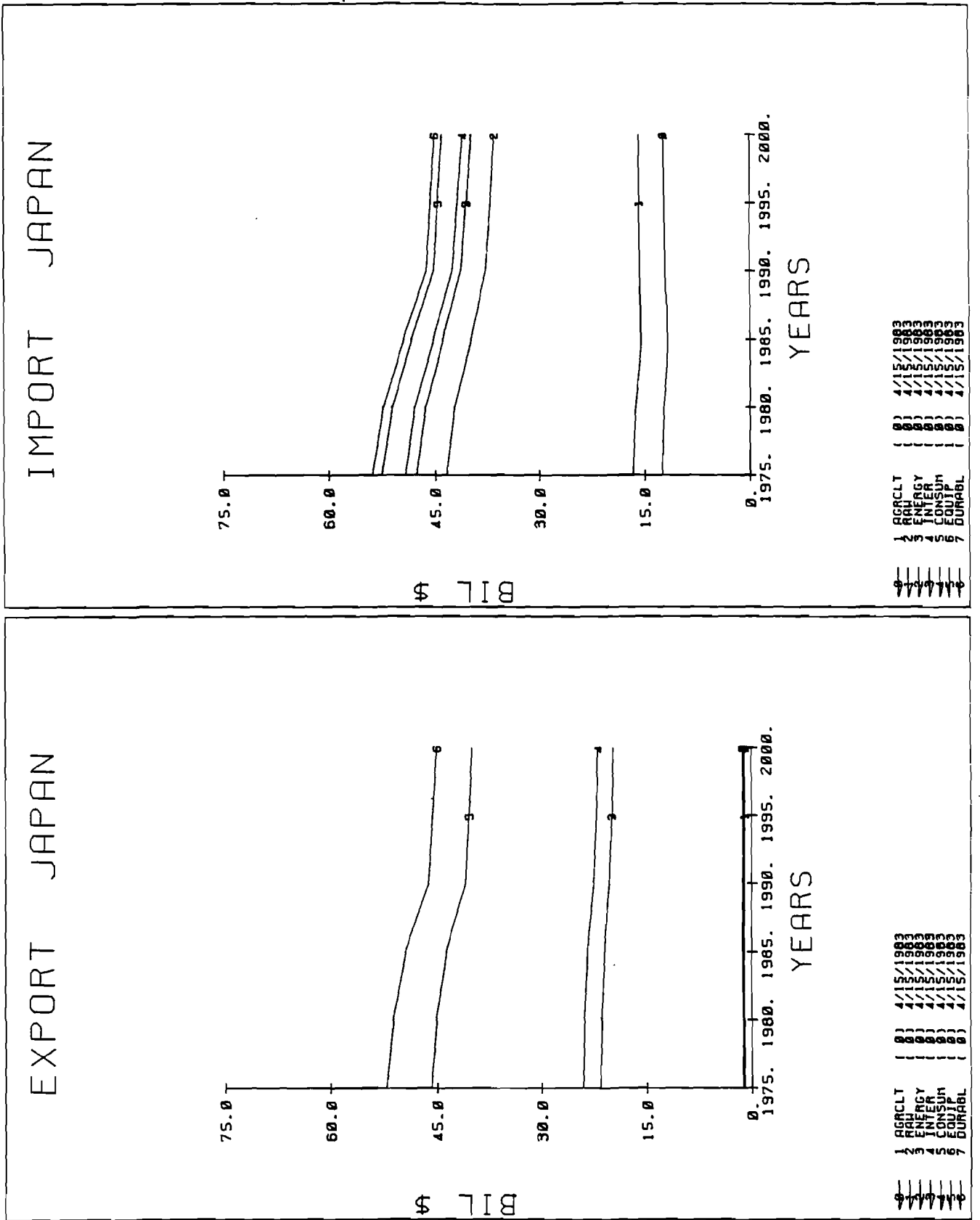


Figure A.3.4

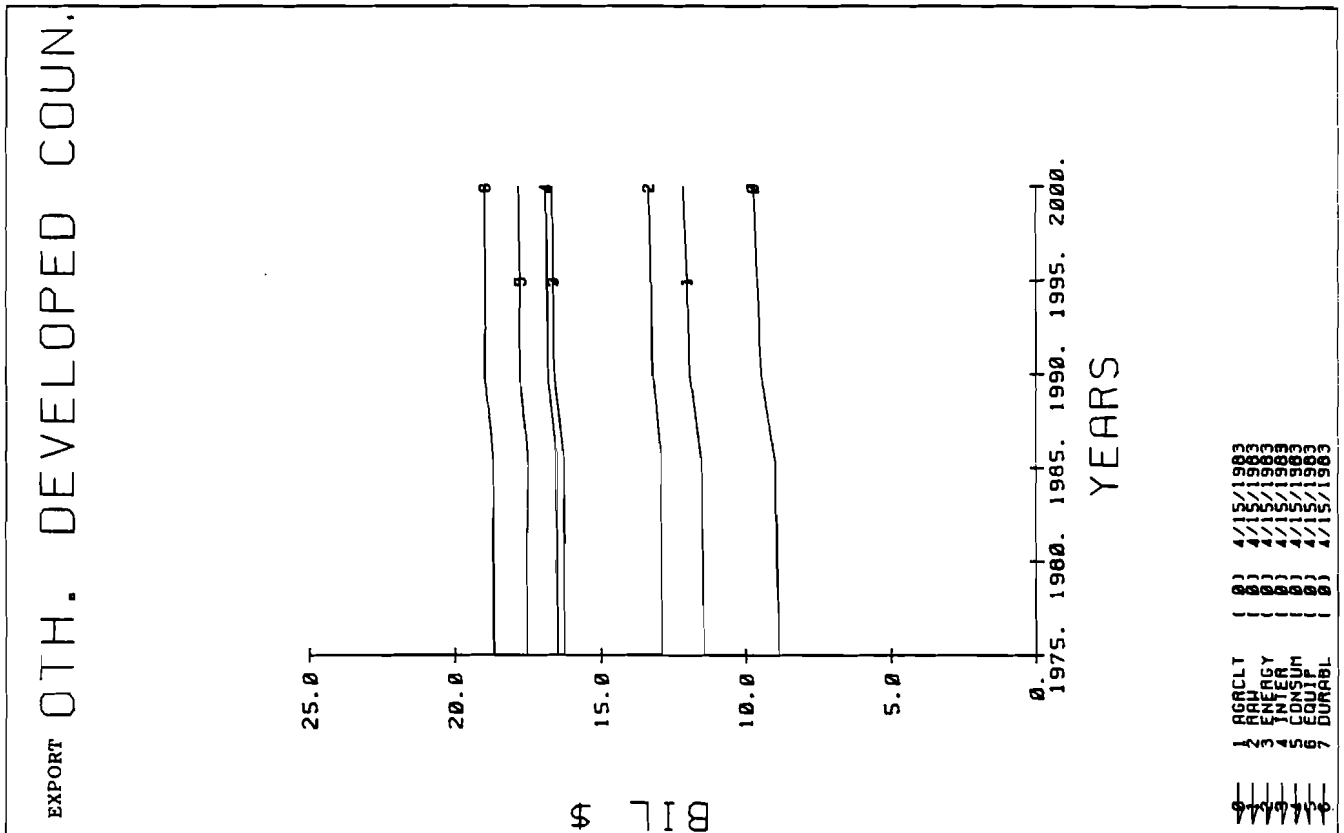
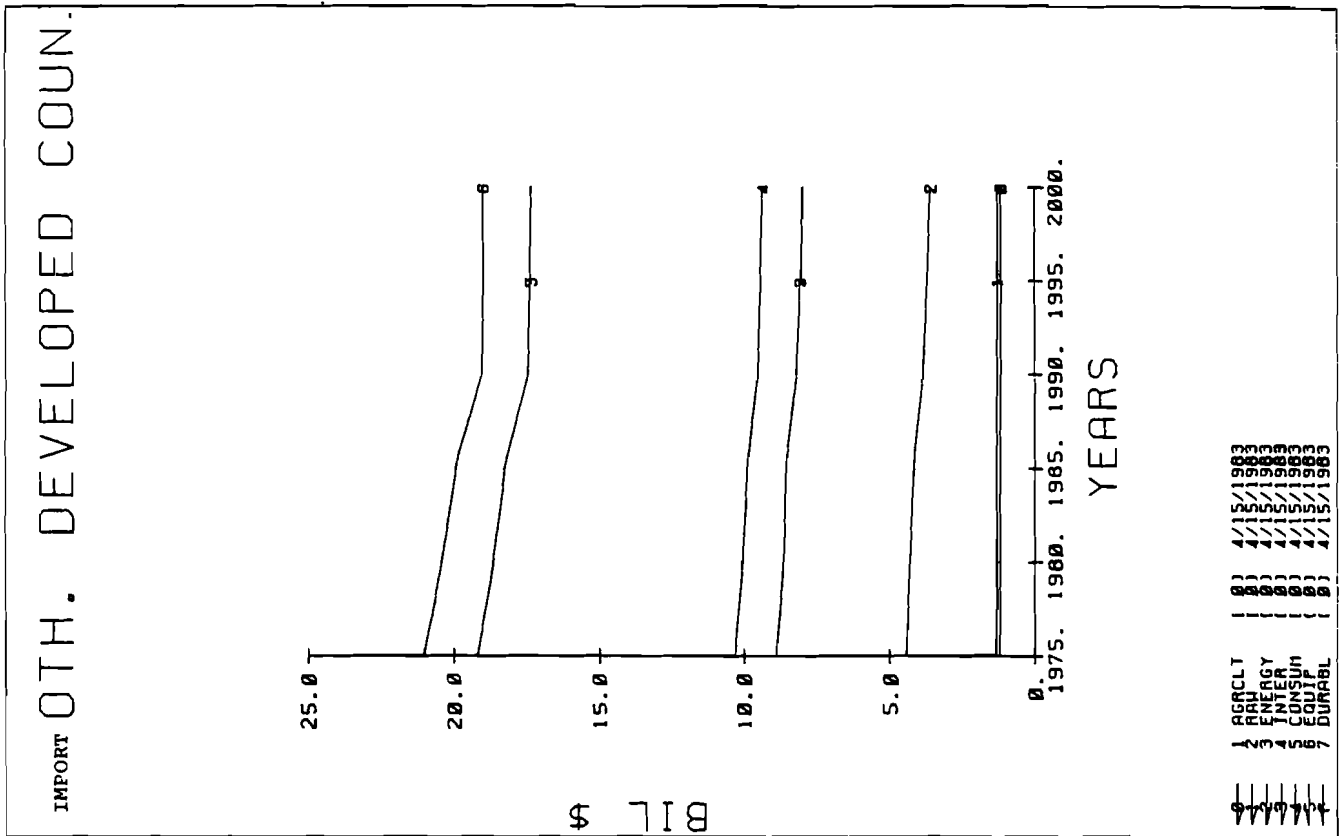


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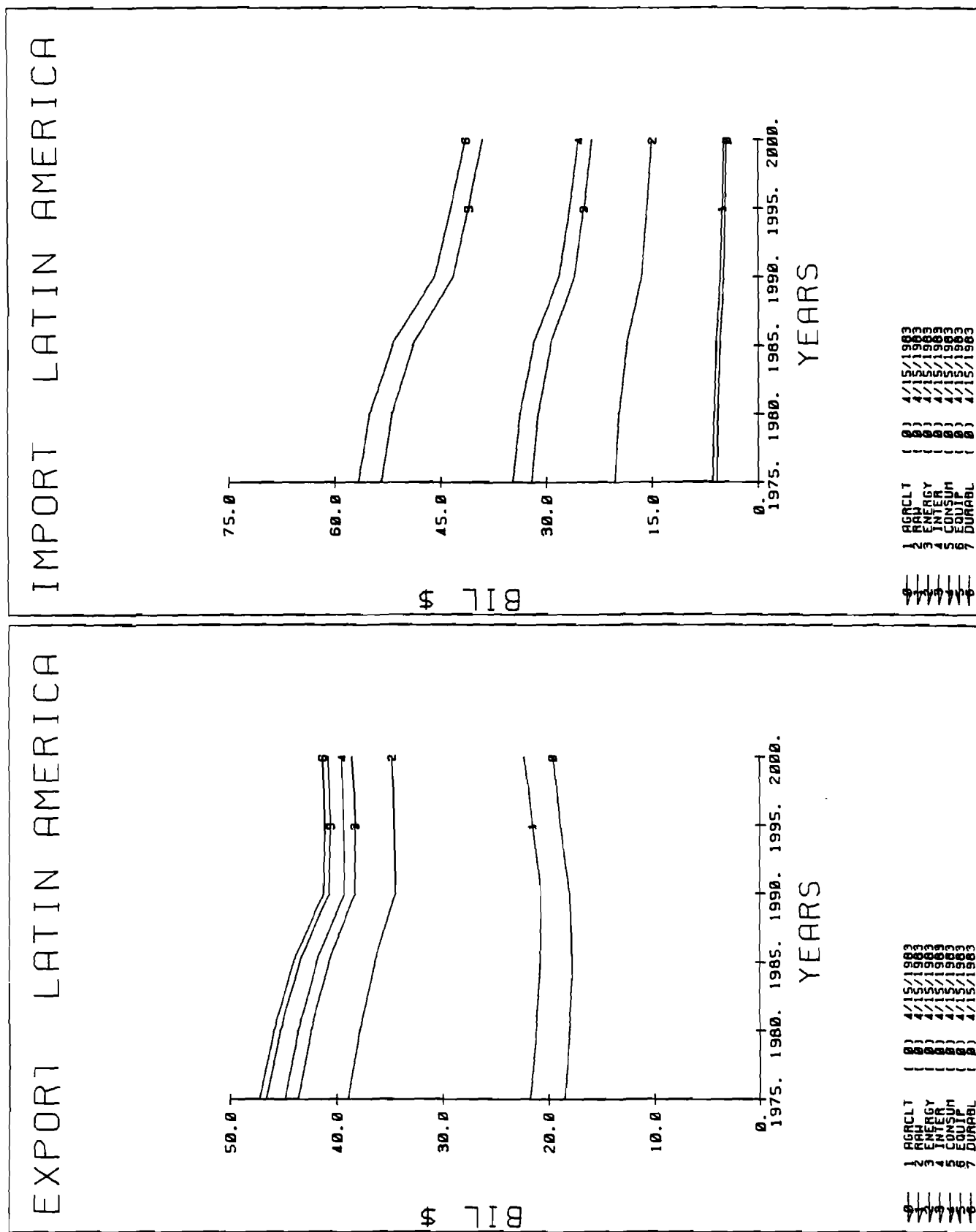


Figure A.3.6

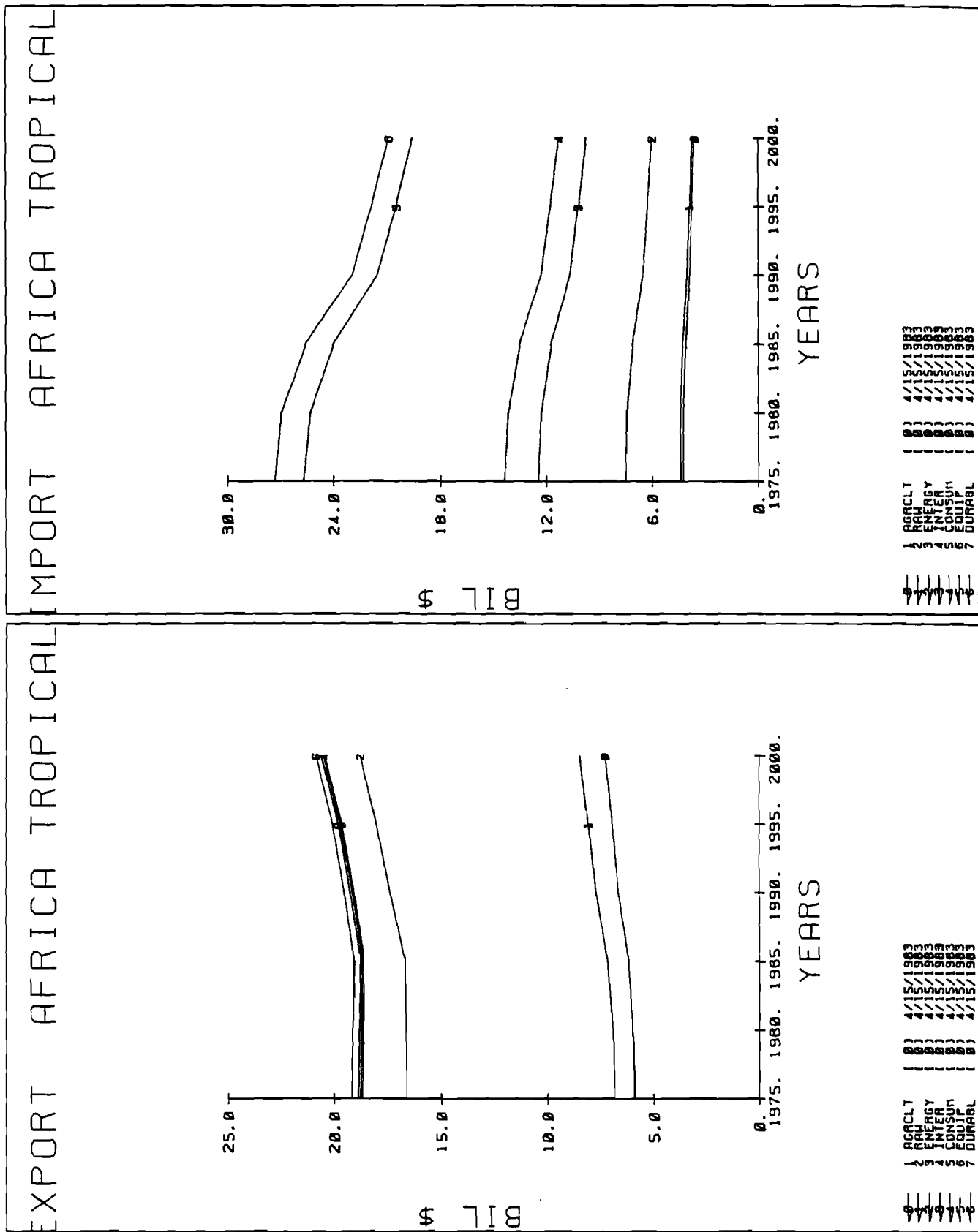


Figure A.3.7

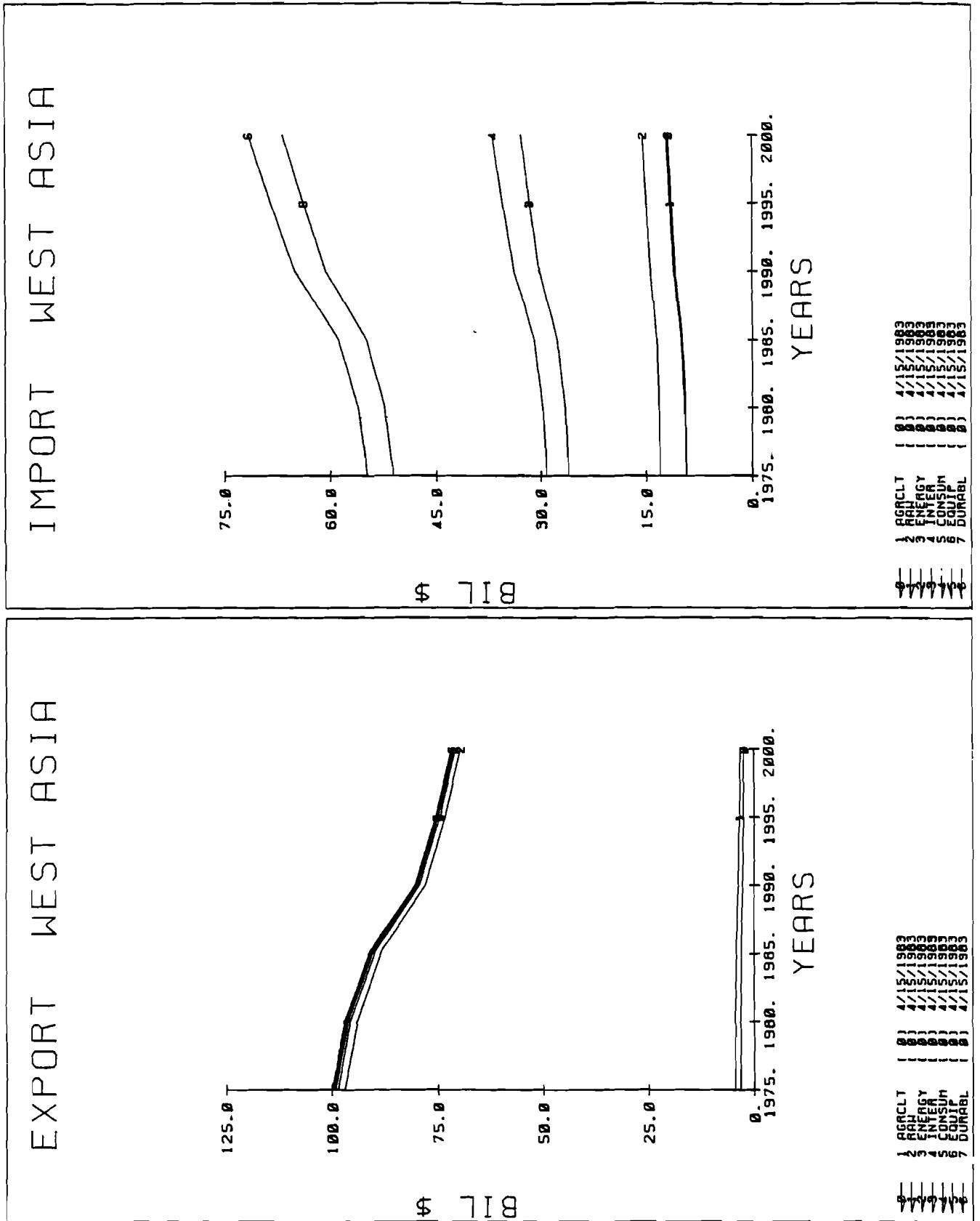


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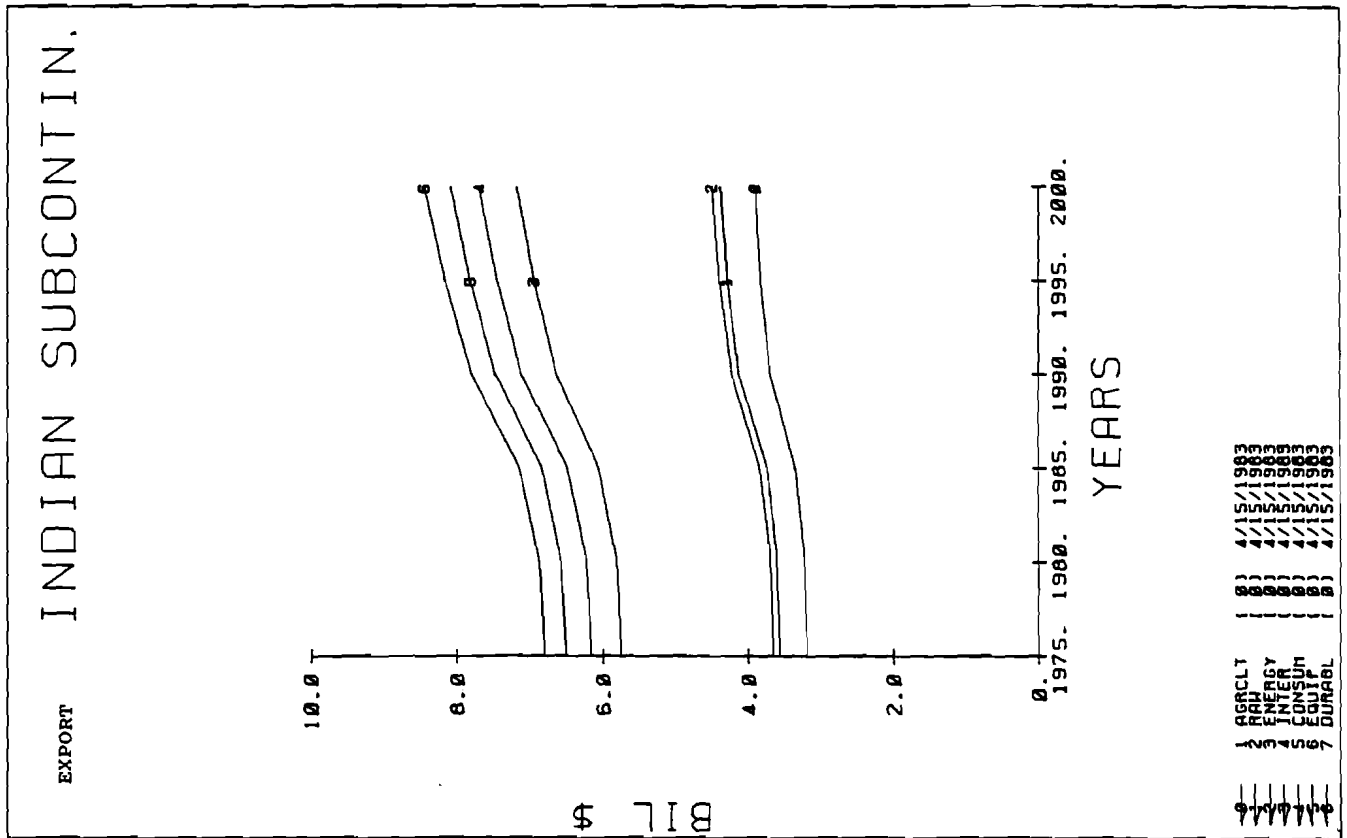
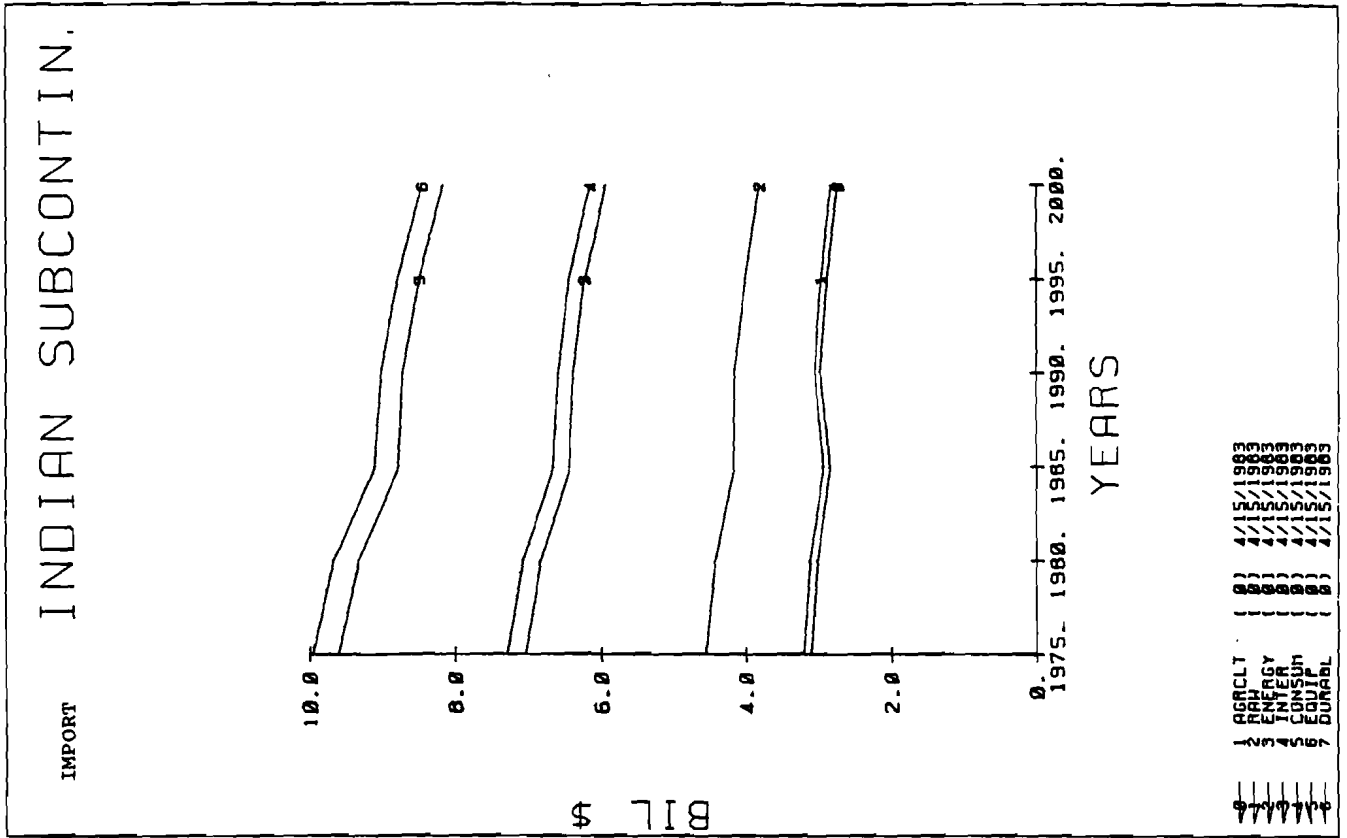


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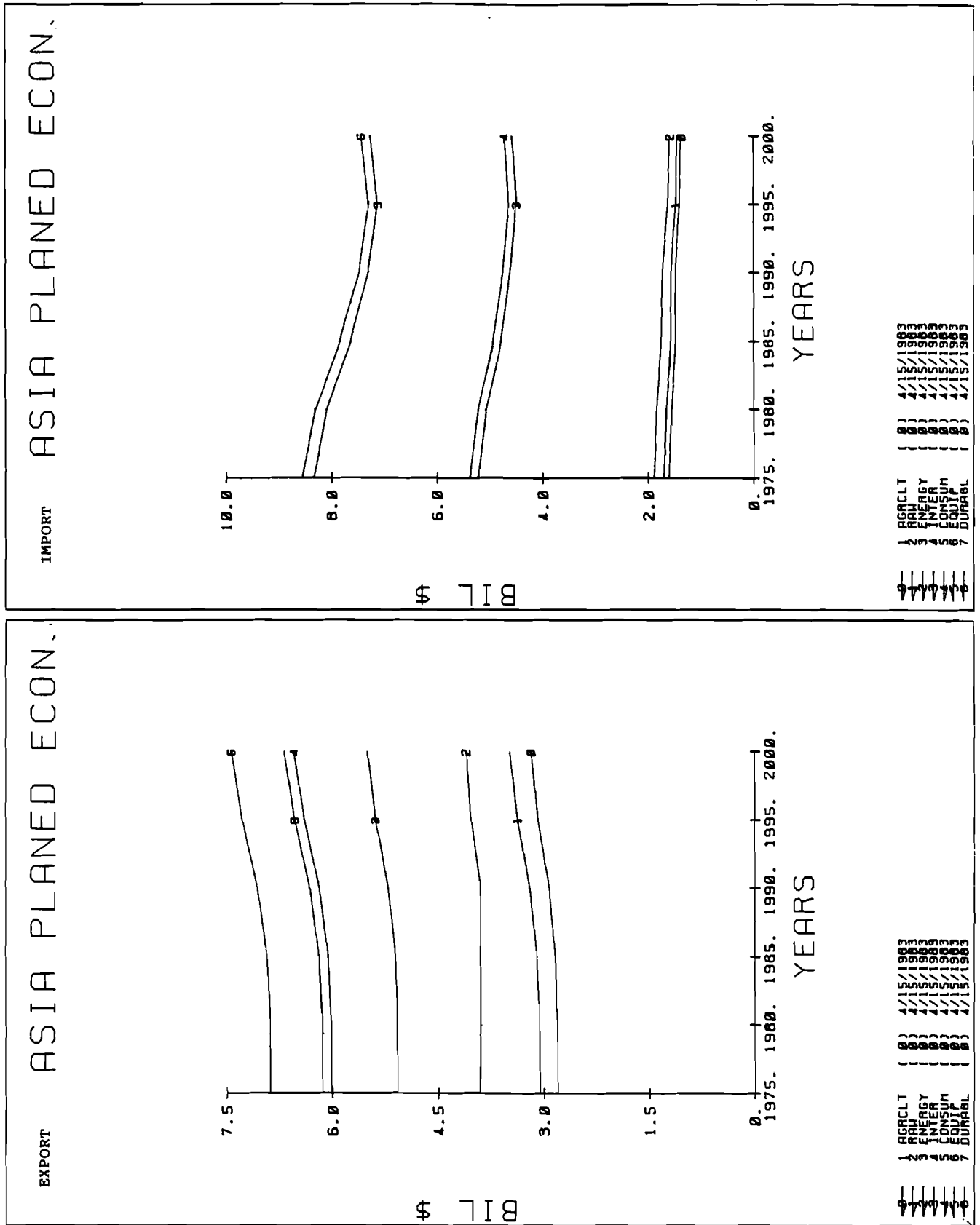


Figure A.3.10

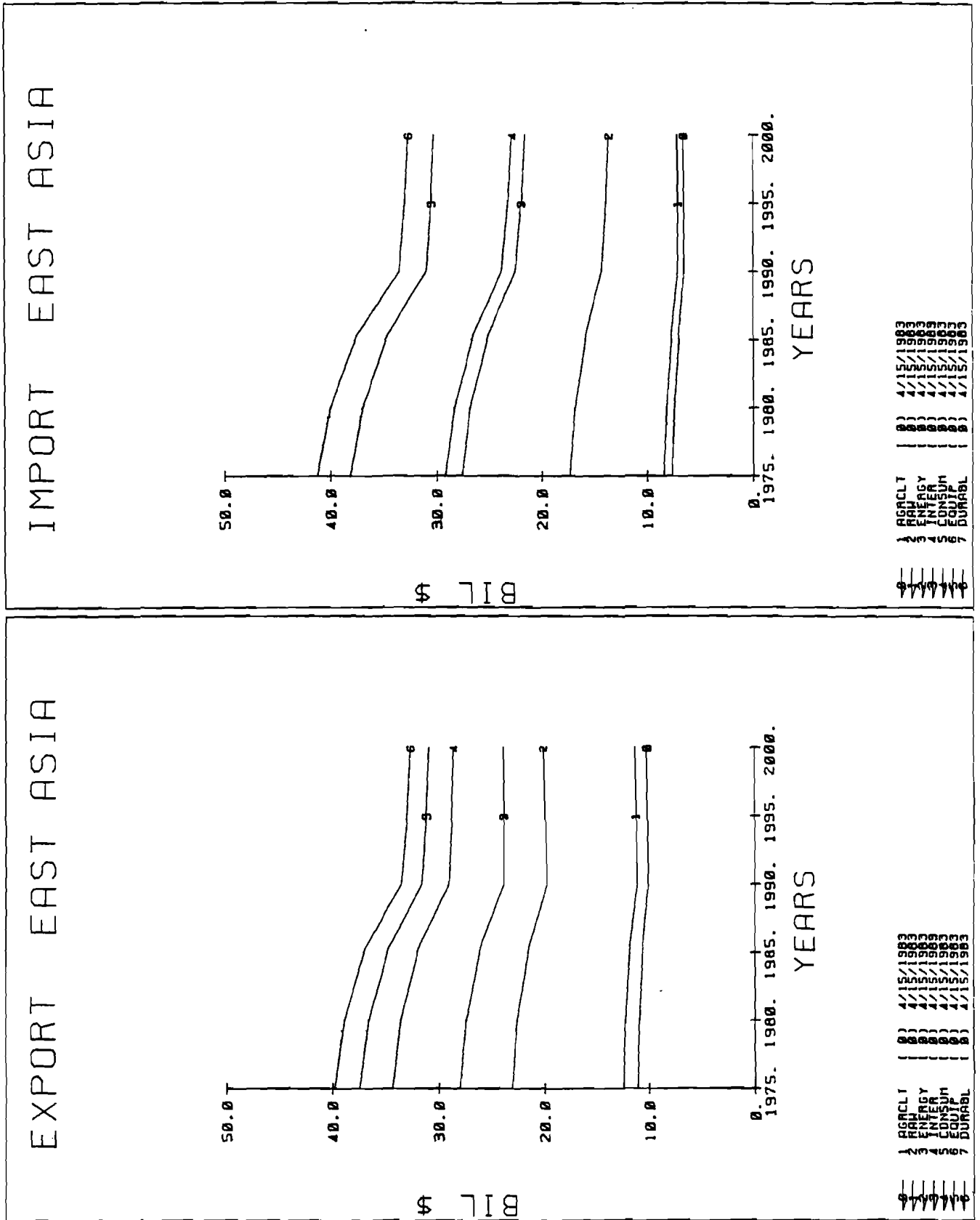


Figure A.3.11