WORKING PAPER

THE DEMAND FOR FOREST SECTOR PRODUCTS

Å E. Andersson
R. Brännlund
G. Kornai

November 1984
WP-84-87

IIASA
International Institute
for Applied Systems Analysis

THE DEMAND FOR FOREST SECTOR PRODUCTS

Å E. Andersson
R. Brännlund
G. Kornai

November 1984
WP-84-87

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forestry policy, forest industrial strategy, and related trade policies.

The key elements of structural change in the forest industry are related to a variety of issues concerning demand, supply, and international trade of wood products. Such issues include the development of the global economy and population, new wood products and substitution for wood products, future supply of roundwood and alternative fiber sources, technology development for forestry and industry, pollution regulations, cost competitiveness, tariffs and non-tariff trade barriers, etc. The aim of the Project is to analyze the consequences of future expectations and assumptions concerning such substantive issues.

In this paper the issue of demand analysis for the forest sector product is analyzed from a theoretical and an econometric points of view. An intermediary demand approach is advocated and applied. For the econometric estimates presented a database for Canada is used. The results indicate that the dual (cost) procedure *to intermediate demand* function estimation is preferable to the use of production functions to generate demand equations.

Markku Kallio
Project Leader
Forest Sector Project

CONTENTS

THE DEMAND FOR FOREST SECTOR PRODUCTS

Å E. Andersson, R. Brännlund and G. Kornai

1. INTRODUCTION

There are large variations between countries in per capita consumption of forest sector products. In 1979, the per capita consumption of paper was, for example, ten times larger in North America than in Latin America. This difference can to a large extent be explained by differences in standard of living. In the same year, the per capita income of Latin America was also not much more than one-tenth of the North American income per capita. For an international cross-section material, much of the variation of paper consumption follows the large variation in per capita income. A simple regression of paper consumption on income for an international sample of countries gives an income elasticity of 0.92. Among the high income countries, income per capita is not a good predictor. The consumption of paper was for instance almost twice as large in the USA in relation to Switzerland in a period when the per capita income was almost 40% above the US income per capita. The regression among the ten highest income countries reveals that in this group there is *practically no correlation* between general standard of living and per capita apparent consumption of paper. Differences in prices can neither explain these cross-section differences. On the whole, there is a marked difference in the use of forest sector products between USA and Western Europe, which cannot be explained by differences in standard of living or relative prices. In order to understand these differences, an analysis of differences in economic and technological structure is needed. It is, of course, anyhow possible to use a time series model involving the GNP for each country. This is, however, a valid procedure only if the economic structure underlying the growing GNP is changing in the same way in the prediction period as in the observation period.

Future demand for forest products can also be assumed to be characterized by changing response patterns in different regions of the world. Over the long term, impacts of a number of technological changes need to be evaluated. Examples such as advances in electronic information technologies, the development of super absorbent material and new packaging materials will affect the demand for forest products. This will probably be noticeable earlier and more strongly in some countries than in others. In some regions of the world, the impacts of the changing energy scene on both forest products and on competitive energy carriers need our consideration.

2. THE FINAL DEMAND FOR FOREST SECTOR PRODUCTS

Demand functions are stimulus-response function showing how a given vector of price-stimuli will trigger off demand responses among users of the commodities. In economic theory, consumers (households or governments) are assumed to choose a set of consumer goods (food, clothing, housing, literature, etc.) in a condition of a constraining budget. Such a budget constraint can be seen as a sum of quantities of different commodities consumed, weighted by the respective commodity prices, which has to be kept within the limits of the available income. The problem of the consumer is then often formulated as a maximization problem, in which the consumer chooses different commodities so as to maximize some utility indicator, while staying within the limits of the available budget.

Assume that consumers' preferences can be captured by a scalar utility function $u(x)$ and that they aspire a maximization of utility subject to their income constraint $\sum_i p_i x_i = y$, then we have the equivalent Lagrange-problem

$$\max_{\{x\}} L = u(x) - \lambda(p^T x - y) \tag{1}$$

which has as conditions of a maximum

$$\begin{aligned} &\frac{\partial u}{\partial x_i} - \lambda p_i = 0 \ ; \quad \{i = 1, \cdots, n\} \\ &p^T x - y = 0 \ ; \end{aligned} \tag{2}$$

and let

$$\text{Det}\begin{bmatrix} u_{11} & u_{12} & -p_1 \\ u_{21} & u_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{bmatrix}, \ldots, (-1)^n \ \text{Det}\begin{bmatrix} u_{11} & \cdots & u_{1n} & -p_1 \\ . & \cdots & . & . \\ . & \cdots & . & . \\ . & \cdots & . & . \\ u_{n1} & \cdots & u_{nn} & -p_n \\ -p_1 & \cdots & -p_n & 0 \end{bmatrix};$$

all larger than 0.

Denoting D_{ij} for the co-factor of row i and column j of the Hessian determinant given above and D for the determinant value, we also have

$$\sum_j S_{ij} p_j = 0; \quad (i = 1, \cdots, n) \tag{3}$$

where $S_{ij} = D_{ji} \lambda / D$

The implication of condition (3) is that some commodities ***must*** be substitutes for each other although some *can* be complements. This means that if the price of paper increases, the consumption of some other commodities must *increase* (i.e., theater performances, car services, etc.) although some might decrease (i.e., book reading, letter writing, etc.). The extent of such substitution is the major consideration of consumer demand theory. It can further be proved that

$$-\sum_j E_{ij} = E_{iy}, \qquad (i = 1, \cdots, n) \tag{4}$$

where $E_{ij} = \dfrac{\partial \ln x_i}{\partial \ln p_j}$ and $E_{ij} = \dfrac{\partial \ln x_i}{\partial \ln y}$, i.e., price- and income elasticities are constraining each other. A high income elasticity for paper thus ***requires*** high price elasticities on the average (if we have aggregated away complements).

Although consumer theory sheds some light on the problem of generating testable demand functions, it tends to be ***too general*** for econometric purposes. As can be seen from (2), (3), (4), these conditions do not generate enough structure for statistical testing and estimation purposes. The only consequence of the analysis is a stimulus-response function which is called the individual demand function

$D_{hi}(p_1, \cdots, p_i, \cdots, p_n, y_h)$ according to which each consumer h demand for commodity i is determined by *all* prices and his purchasing power as represented by income y_h. These demand functions $D_{ih}(p, y_h)$ where $p = [p_i]$, can be aggregated into a household consumption $D_i(p, y) = \sum_h D_{ih}(p, y_h)$ where $y = [y_h]$. This is illustrated by Figure 1. In general, we should thus expect a heterogeneous consumer population to generate a non-linear demand function even if each consumer is regulated by a linear demand function.

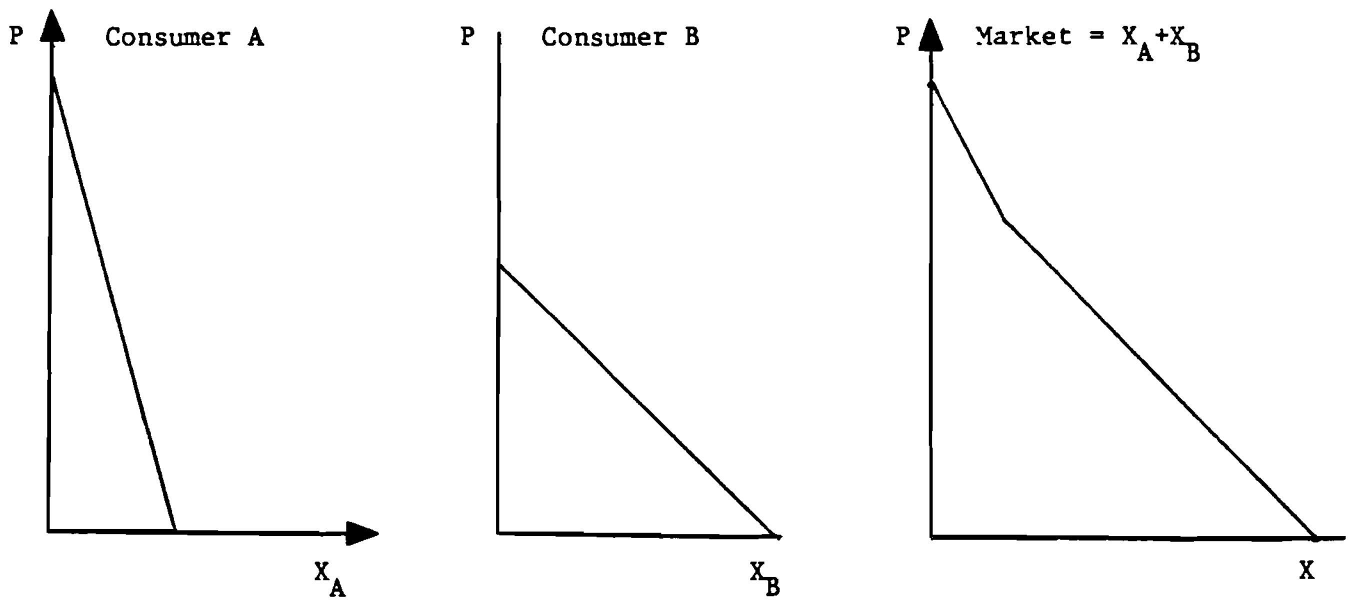

FIGURE 1.

Our arguments against the use of the demand theory offered above for an econometric study of forest sector demand are the following:

(1) As a general theory of demand in which $u(x)$ is the preference function of any decision maker (manager of a firm, government or household purchases), it is *too* general to generate statistical hypotheses.

(2) As a general theory of the consumer it uses too little of the structural information on individual consumers' behavior as developed by the behavioral sciences.

(3) If seen as a special theory of the consumer demand, it would cover a very limited share (10-20%) of the total forest sector demand.

3. THE THEORY OF INTERMEDIATE DEMAND

In the center of intermediate demand theory is the firm with one or many plants. A commodity like sawnwood is used as one of the many inputs $(\nu_1, \ldots, \nu_n)$ to be transformed by the use of the fixed equipment to generate a set of outputs $(q_1, \ldots, q_s)$, say, chairs, tables, beds and other pieces of furniture demanded by households, hotels, hospitals, and offices, etc. The model of the plant can thus be illustrated by Figure 2. The black box representation of the firm (or plant), can be summarized in

$$F(q_1, \ldots, q_s; \nu_1, \ldots, \nu_n) = 0; \tag{5}$$

which is a transformation function or an implicit production function according to which the inputs vector ν are transformed into the

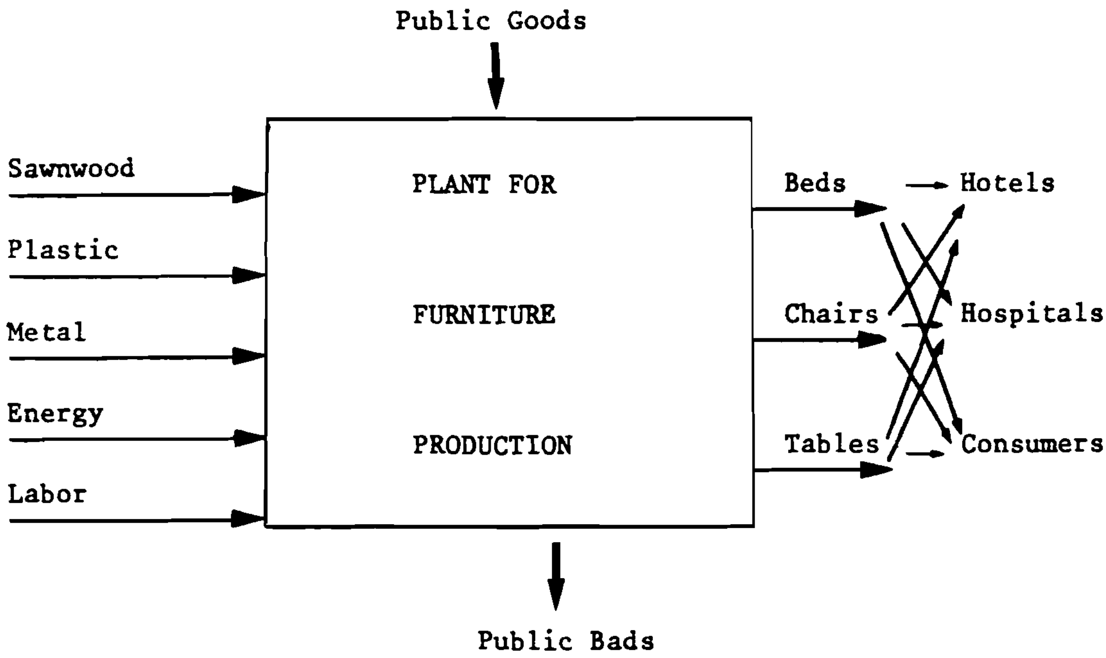

FIGURE 2.

outputs-vector q. The public goods and bads are suppressed. We assume that F is at least twice differentiable, increasing with q and decreasing with ν, and strictly quasi-convex.

The behavior of the firm or plant manager can under a capitalistic regime be assumed to be one of profit maximization (at the firm level) or cost minimization (at the plant level). The reasoning behind this assumption is one of social Darwinism. Any firm or plant operating under some other rule would not exist for long. Not maximizing profits in an environment where other similar firms maximize profits would imply a slower rate of capital accumulation and a relative decline of the non-optimizing units. At a general equilibrium where profits above market remuneration of management services are forced to zero, non-optimizing behavior would imply the bankruptcy, i.e., death, of all non-optimizing firms. No such argument can be raised in favor of optimizing behavior among consumers or monopolistic organizations, like government bodies.

The profit is defined as the difference between revenues from sales and costs of purchasing inputs:

$$V = \sum_i \pi_i q_i - \sum_j \omega_j \nu_j \tag{6}$$

where

π_i = output prices

ω_j = input prices

(The set of output *and* input prices is in the sequel symbolized by the vector p).

Profit V can be maximized subject to the transformation function constraint F.

$$\max_{\{q,\nu\}} L = \pi^T q - \omega^T \nu + \lambda F(q,\nu) \tag{7}$$

The necessary conditions of optimality of (7) are the following:

$$\pi_i / (\partial F / \partial q_i) = -\lambda; \qquad (i = 1, \ldots, s) \tag{8a}$$

$$\omega_j / (\partial F / \partial \nu_j) = \lambda; \qquad (j = 1, \ldots, n) \tag{8b}$$

$$F(q,\nu) = 0 \tag{8c}$$

The firm should thus adjust any pair of outputs until the ratio $[(\partial F/\partial q_i)/(\partial F/\partial q_k)]$ equals the given output price ratio $[\pi_i/\pi_k]$. It should furthermore adjust any pair of inputs until the ratio

$(\partial F/\partial \nu_j)/(\partial F/\partial \nu_l)$ equals the given relative input price (ω_j/ω_l).

Rational behavior of firms thus dictates that the firms must observe the parametrically changing price vector p and adjust through substitution among inputs or outputs until the "marginal rates of substitution". $(-dq_i/dq_k)$ and $(-d\nu_j/d\nu_l)$ correspond to the price ratios (π_k/π_i) and (ω_l/ω_j) or until $\pi_i/\omega_j = -[\frac{\partial F}{\partial q_i}/\frac{\partial F}{\partial \nu_j}]$.

Total differentiation of (8) is a means of determining the response pattern of a firm to any given small change of output- or input prices.

$$\begin{aligned} &\lambda F_{11} dq_1 + \cdots + \lambda F_{1m} d\nu_n + F_1 d\lambda = -d\pi_1 \\ &\lambda F_{m1} dq_1 + \cdots + \lambda F_{mm} d\nu_n + F_m d\lambda = d\omega_n \\ &F_1 dq_1 + \cdots + F_m d\nu_n = 0 \end{aligned} \tag{9a}$$

or in matrix form

$$\lambda H dx + f d\lambda = dp \tag{9b}$$

where

$$H = [F_{ij}] \; dx = \{dq_i, d\nu_j\}$$

$$f = \{F_1, \ldots, F_m\}, \text{ and } dp = \{-d\pi_1, \ldots, d\omega_n\}$$

The consequence of a change in the price structure on the adjustable input and output quantities can be determined by matrix inversion of (9b)

$$dx = (\lambda H)^{-1}(dp - f d\lambda) \tag{10}$$

It can be shown (Henderson-Quandt 1980) that

$$\frac{\partial q_j}{\partial p_j} > 0; \quad (j = 1, \cdots, s)$$

and

$$\frac{\partial \nu_k}{\partial \omega_k} < 0; \qquad (k = 1, \ldots, n)$$

i.e., if an output price increases (*ceteris paribus*), the firm will *increase* that output and if the price of an input increases the firm will also *decrease* the use of that input. This unique response pattern implies that slopes of demand and supply curves are clear cut to an extent not found in consumer theory.

Further specification of response patterns can be achieved only if we are prepared to make the transformation function $F(q,\nu)$ more specific. One way of specifying F is by assuming that the firm (or plant) produces one output (q) only. We can then write an explicit transformation function, called a ***production function***, i.e.:

$$q = q(\nu)$$

One can further specify $q(\nu)$ as, for example, the constant elasticity of substitution function, which is a special case of the Minkowski mean value function:

$$q = E(\sum_j \alpha_j \nu_j^{-\rho})^{-1/\rho} \tag{11}$$

with profit maximization we then get testable hypotheses of response as derived conditions of optimality:

$$\ln(\frac{\nu_j}{\nu_l}) = k + \sigma \ln(\frac{\omega_l}{\omega_j}) \tag{12}$$

where $\sigma = \frac{1}{1+\rho}$ and $k = \frac{1}{1+\rho} \ln \frac{\alpha_j}{\alpha_l}$.

4. TESTING FOR CONSTANT ELASTICITIES OF SUBSTITUTION

As can be seen from equation (12) the same elasticity of substitution between different inputs is assumed in this specification of the model. This assumption has been tested for three of the most important forest product user sectors, namely *non-residential*, *residential*, and *repair construction*. The data base used for this test is the constant price input-output series for Canada 1961-1974 as published by Statistics Canada.

The model used is equation (12) with a linear time effect and an added time effect and a stochastic error term with assumed mean value of zero and an assumed normal distribution with further assumed uncorrelated errors over time periods. The ordinary least squares method is used for estimating the regression equations. Table 1 gives the results. In the table only input pairs showing statistically significant parameters have been included. As can be seen from the table, statistical significance criteria could hardly make credible the assumption of equation (11) combined with a short term profit maximizing behavior of firms. This is further strengthened by the fact that a number of σ-values estimated, but not reported, are not significant from zero.

5. THE THEORY OF PRODUCTION AND INTERDEPENDENCIES BETWEEN FIRMS

The modern theory of production abstains from a simplistic two or three factor analysis of inputs conversion into outputs. Instead, a multitude of inputs, like wood, pulp, different types of paper products, chemicals, metals, labor, energy, etc., are interacting in the transformation

TABLE 1. Results for constant elasticity of substitution regressions for construction sectors of Canada based on data for period 1961-1974.

$$\ln\left[\frac{\text{Forestry prod.}}{\text{Fabr. wood prod.}}\right] = -9.2 \ln\frac{P_F}{P_W} - 0.1t \qquad R^2 = 0.6$$

(4) (3)

Repair constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Fabr. wood prod.}}\right] = -0.73 \ln\frac{P_{LT}}{P_{FW}} - 0.08t \qquad R^2 = 0.97$$

(3) (2.0)

Non res. constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Fabr. metal prod.}}\right] = -0.6 \ln\frac{P_{LT}}{P_{FM}} - 0.03t \qquad R^2 = 0.85$$

(2) (3.0)

Non res. constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Cement prod.}}\right] = -0.86 \ln\frac{P_{LT}}{P_{CP}} - 0.04t \qquad R^2 = 0.91$$

(3) (5)

Non res. constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Struct. Met. prod.}}\right] = -1.95 \ln\frac{P_{LT}}{P_{SM}} + 0.03t \qquad R^2 = 0.67$$

(3.7) (1.3)

Res. constr.

$$\ln\frac{[\text{Panels}}{\text{Iron \& steel prod.}}] = -1.84 \ln\frac{P_{Pan}}{P_{I\&S}} + 0.03t \qquad R^2 = 0.71$$

(4.1) (2.3)

Res. constr.

$$\ln\left[\frac{\text{Fabr. wood prod.}}{\text{Cement prod.}}\right] = -1.4 \ln\frac{P_{FW}}{P_{CP}} + 0.05t \qquad R^2 = 0.61$$

(2.3) (3.4)

Res. constr.

into one or many outputs. This procedure has two main advantages. Firstly, it better approximates the actual technological situation of modern firms. Secondly, it more clearly than in classical economic analysis of production by the use of land, labor, and capital inputs, puts the emphasis on *interdependencies* between different firms or aggregates of firms as users and producers of intermediary commodities.

Two economic research fields have emerged and developed from this emphasis on interdependencies between producers. One primarily microeconomic class of models is *activity analysis*. The other primarily macroeconomic class of models is *input-output* analysis. A bridging class of models is the von Neumann type of models of which input-output models and LP models are special cases.

One simplification of the von Neumann model made in input-output models is the assumption that each aggregate of firms produces one commodity only or that all firm information can be converted into a commodity by commodity framework.

In the production theory discussed in sections 3 and 4, we have shown that the inputs of commodities and primary factors like labor per output unit are generally influenced by the prices expected to be ruling. Avoidance to adapt input structures to a changing price structure would not be a viable behavior in a market economy unless all actors would be equally rigid. Such an assumption would obviously be at variance with common observations of firm behavior. Thus, we can safely assume that every a_{ij} is a function of the price vector p or equivalently

$$A = A(p); \text{where } A(p) \equiv \{a_{ij}(p)\} \text{ for } (i,j = 1,\ldots,n)$$

If this is the case, we can see the general interdependency problem for a linearly homogeneous economy as a *price* equilibrium problem, where

$$\lambda p = pA(p,\bar{\omega})+\omega a(p,\bar{\omega}) \equiv H^*(p,\bar{\omega}) \equiv H(p)$$

where

p = row vector of commodity prices

ω = row vector of primary input prices

$A(p)$ = price dependent square matrix of input-output coefficients

$a(p,\bar{\omega})$ = a price dependent rectangular matrix of primary input-output coefficients

λ = an endogenously determined multiplicative factor

At any given set of positive primary input prices $\bar{\omega}$, a set of prices of commodities and *chosen* A- and a-matrices can be endogenously determined. This is a special fixed point problem.

$$\lambda p = H(p); \text{ with } H(p>0) \geq 0; \text{ and } H(p) \text{ assumed continuous in } p \text{ and homogeneous.} \qquad (12)$$

For this problem, the following theorem is applicable.

THEOREM (cf. Nikaido, 1968, pp. 105-151; a proof is given on p. 152.):

Assume the following conditions hold

(a) $H(p) = (H_i(p))$ is defined for all non-negative p in R^n_+, with its values being also on non-negative vectors in R^n_+, $H(p) \geq 0$.

(b) $H(p)$ is continuous as a mapping H: $R^n_+ \to R^n_+$, except possibly at $p = 0$.

(c) $H(p)$ is positively homogeneous of order m, $1 \geq m \geq 0$ in the sense that $H(\alpha p)$ for $\alpha \geq 0$, $p \geq 0$.

Let $\Lambda = \{\lambda | H(p) = \lambda p$ for some $p \in P_n\}$, where $P_n = \{p | p \geq 0$, $\sum_{i=1}^{n} p_i = 1\}$ is the standard simplex.

Then, Λ contains a maximum which is denoted by $\lambda(H)$. Furthermore, if $m = 1$, $\lambda(H)$ is the greatest among all the eigenvalues of M. $\lambda - 1$ can be interpreted as the profitability of the economy.

When an equilibrium price vector p^* has been determined the relevant choice of technique has also been solved, and an equilibrium *implied* structure of production can be determined

$$x = A(p^*, \bar{\omega})x + f; \tag{13}$$

subject to $p^* f = \omega a(p^*, \bar{\omega})$. The theory of interdependent firms with substitution between all input commodities thus shows the existence of a general equilibrium with a simultaneous determination of techniques of production, production and price structures.

6. THE FOREST SECTOR IN THE FRAMEWORK OF INPUT OUTPUT ANALYSIS

It is of great importance for a consistent analysis of the forest sector to determine how each one of the users of the forest products would react to a changing input price structure. The analysis in the foregoing sections would indicate that *any* price change would lead to changing demand for all forest products. This is, however, a highly unlikely event in any real situation, even if the theory suggests such a result. A few sectoral outputs like electrical energy and transportation services enter every sector of production as inputs. As a contrast, other outputs like wood or paper products enter only a few sectors of production as inputs.

In these sectors of production (e.g., construction or printing and publishing) only a limited number of other inputs are used and are available for an adaptation in the form of substitution.

7. INPUT-OUTPUT ANALYSIS, PRODUCTION THEORY AND DUALITY

Demand for inputs is derived in the foregoing sections with assumptions of profit maximization and the existence of a mathematically well-behaved production function. Although profit maximization is a reasonable assumption at the level of firms, constrained cost minimization is an alternative and probably better assumption at the level of plants within corporations. Constrained cost minimization has certain advantages from a general theoretical point of view, as first demonstrated by Shephard (1953), and for econometric studies, as first demonstrated by Nerlove (1965). As a simple example, we first derive the factor demand equations if the demand for output is given, the production function is of Cobb-Douglas type and cost of production of this predetermined output is minimized.

In this example, the amount of capital and energy is assumed to be given by longer term arrangements. Only three input flows into the plant, say, chemicals, paper and labor, are assumed to be freely variable in the time period of analysis. The output q could be newspapers. The problem is thus:

$$\underset{\{\nu\}}{\text{minimize}} \ \text{Cost} \equiv \omega_1\nu_1 + \omega_2\nu_2 + \omega_3\nu_3 \qquad (14)$$

subject to

$$a\nu_1^{a_1}\nu_2^{a_2}\nu_3^{a_3} = \bar{q} \equiv \text{Projected Demand} \qquad (15)$$

$\alpha_1 + \alpha_2 + \alpha_3 = s \equiv$ economies of scale elasticity

The corresponding Lagrange optimization problem is given by the following expression:

$$\underset{\{\nu,\lambda\}}{\text{minimize}}\ L = \sum_i \omega_i \nu_i - \lambda(a \prod_i \nu_i^{\alpha_i} - \bar{q}); \tag{16}$$

with the following conditions of an optimum

$$\frac{\partial L}{\partial \nu_i} = \omega_i - \alpha_i a \prod_{j \neq i} \nu_j^{\alpha_j} \nu_i^{\alpha_i - 1} = 0; \tag{17}$$

Substituting factor demands as functions of prices (17) into the demand constraints (18), we arrive at the factor demand equations

$$\nu_i = c_i q^{1/s} \frac{\omega_1^{\alpha_1/s} \omega_2^{\alpha_2/s} \omega_3^{\alpha_3/s}}{\omega_i}; \tag{19}$$

where $c_i = \alpha_1[a\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3}]^{-1/s}$. If there are constant returns to scale the input-output coefficient $a_i \equiv \nu_i / q$ are price dependent only:

$$a_i = c_i(\omega_1^{\alpha_1}\omega_2^{\alpha_2}\omega_3^{\alpha_3})/\omega_i; \tag{20}$$

or

$$\ln a_1 = \ln c_1 - (1-\alpha_1) \ln\omega_1 + \alpha_2 \ln\omega_2 + \alpha_3 \ln\omega_3 \quad \text{etc.}$$

With increasing returns to scale input output coefficients would depend on $\bar{q}$ as well as on prices. The cost function that can be derived from a substitution of (19) into (14) is

$$c = cq^{1/s}\ \omega_1^{\alpha_1/s}\ \omega_2^{\alpha_2/s}\ \omega_3^{\alpha_3/s} \tag{21}$$

with $c = c_1 + c_2 + c_3$.

Shephard (1953) demonstrated that under the behavioral assumption made here, the minimized cost function and the production function are related by a property of duality.

Shephard's THEOREM:

$$\text{If } C(q,\omega) \equiv \min_{\nu} \{\omega^T \nu : f(\nu) \geq q\}$$

and that if f is continuous from above and that $C(q,\omega)$ is differentiable with respect to input prices at the point q^{*},ω^{*}, then

$$\nu(q^{*},\omega^{*}) = \nabla_p C(q^{*},\omega^{*});$$

where $\nu(q^{*},\omega^{*})$ is the vector of costs minimizing input flows needed to produce the output flow q^{*}, given the input prices ω^{*}.

As an example, we can use Shephard's theorem on cost equation (21):

$$\frac{\partial C(q^{*},\omega^{*})}{\partial \omega_1} = c_1 q^{*} \frac{1}{2} \omega_1^{*(\alpha_1 - 1)} \omega_2^{*\alpha_2} \omega_3^{*\alpha_3} = \nu_1$$

with c_1 defined as before. It is thus possible to develop a model of price and scale-dependent input demand ***either from production or from cost functions*** postulated at the plant level. Shephard's theorem indicates that is is in most cases easier to commence from cost functions.

8. COST FUNCTIONS AND INTERMEDIATE DEMAND FOR FOREST PRODUCTS

The generalized Leontief cost function was introduced by Diewert (1971) as a suitable second order approximation of a general input cost function. This cost function can be written as

$$C(q,\omega,t) = F(q,t) \sum_k \sum_l b\, \omega_k^{a}\, \pi_l^{1-a}; \quad (k,l = 1,\ldots,n)$$

with

C = minimized total cost

q = output

ω = vector of input prices = $\{\omega_l\}$

t = time period

$F(q,t)$ = scale and technology function

b_{kl} = coefficient of substitutability between input k and input l

A possible specification of $F(q,t)$ would be $F(q,t) = q^{1/s}e^{-ut}$. Some experiments in estimating α have indicated that the model is well behaved with α close to 0.5 (Frenger, 1982).

If we apply Shephard's Theorem, the input demand functions are

$$\frac{\partial C}{\partial \omega_k} = \nu = 0.5q^{1/s}l^{-ut} \sum_l b_{kl}\,\omega_k^{-0.5}\,\omega_l^{0.5}$$

Only if $b_{kl} = 0$ for all $k \neq l$ and $s=1$ would the fixed input-output coefficients be warranted. Partial price elasticities, E_{kl} can be defined as

$$E_{kl} = 0.5\left[\frac{b_{kl}\,\omega_l^{0.5}}{\sum_m b_{km}\,\omega_m^{0.5}} - \delta_{kl}\right] \equiv \frac{\partial \ln\left(\frac{\partial C}{\partial \omega_k}\right)}{\partial \ln \omega_l}$$

with δ_{kl} = Kronecker constant, E_{kl} can be used as one measure of the substitutability of two inputs, if the same level of production (q^{*}) is to be maintained.

The shadow (or dual) elasticity of substitution δ_{kl} was introduced by McFadden (see Frenger 1983) and is defined as

$$\delta_{kl} = -\frac{\partial \ln(\nu_k / \nu_l)}{\partial \ln \omega_k / \omega_l},$$

i.e., the elasticity of the cost minimizing ratio of inputs to a change in their relative price when cost, output and other prices are held constant.

Such elasticities have been measured for two forest product using sectors – *Manufacture of Wood and Wood Products* and *Printing and Publishing* by P. Frenger (1983). The results are shown in Table 2.

The elasticities of substitution are generally high for the inputs substitutable in the short run, and short and long run elasticities are close to each other in the Frenger study. On basis of this study, it seems reasonable to expect *intra-material substitution* elasticities to be high in comparison with capital labor substitution elasticities – an assumption at variance with the approach chosen in the so-called MSG-models (Johansen 1959, Bergman 1980, and Zalai 1980).

TABLE 2. Shadow elasticities of substitution 1975 of input demand equations 1969-1980 in Norway.

Input pairs	Manufact. of wood and wood products	Printing and publishing
Material – Energy	0.92	0.91
Material – Labor	1.27	1.24
Material – Capital	0.35	0.45
Energy – Capital	0.84	0.83
Labor – Capital	0.50	0.30
Energy – Labor	0.95	0.91

9. AN ECONOMETRIC STUDY OF INTERMEDIATE DEMAND

Our study is based on the duality approach to input demand analysis. In the study, we use the approach to estimate demand functions for the following forest sector products:

1. Raw material from forestry except primary processed lumber and timber
2. Lumber and Timber
3. Veneer and Plywood
4. Other fabricated wood materials
5. Pulp
6. Newsprint and paper stocks
7. Paper products

The data again are collected from input-output statistics for Canada 1961-1979, provided by *Statistics Canada* and by the *Comparative Economic Analysis Project* of IIASA.

The basic idea behind this stage of the study is to test the substitution hypothesis as econometrically specified by the Generalized Leontief Cost Function and the associated input demand functions derivable by the use of Shephard's theorem. If the hypothesis can be sustained, it forms the basis for specification of ***market demand functions***, with a determination of the ***minimal*** number of substitute prices to be included beside the own-price in such market demand functions.

Time series from 1961 to 1978, collected from input-output statistics for Canada, are used to estimate demand for forest products in four different user industries. Demand for sawnwood and panels are estimated for the construction and furniture industries. Demand for paper products are estimated for the printing and publishing industries and a sector which we call "the office sector." This means that we concentrate this presentation to a limited subset of the seven products mentioned above.

The econometric specification of the model is obtained as shown above by taking the derivatives of the *simplified* Diewert cost function with respect to the respective input prices

$$C(y,p) = y \sum_i \sum_j b_{ij}(p_i p_j)^{0.5} \quad \text{with the constraint} \quad b_{ij} = b_{ji}$$

$$\frac{\partial C(y,p)}{\partial p_i} = y \sum_j b_{ij}(p_j / p_i)^{0.5}, \quad b_{ij} = b_{ji}$$

Dividing through by y, we obtain the normalized demand for the ith product as a function of the input prices:

$$\frac{X_i^D}{y} = \sum_j b_{ij}(p_j / p_i)^{0.5}, \qquad b_{ij} = b_{ji}$$

This means that for each user sector there is a system of demand equations, i.e., one demand equation for each input.

The methods used to estimate those equations is Ordinary Least Squares (OLS) and Seemingly Unrelated Regression (SURE). Both methods yield unbiased and consistent estimates, but SURE is more efficient since we are dealing with systems of equations. The symmetric constraint, $b_{ij} = b_{ji}$, also implies that SURE is more accurate because one cannot easily incorporate the constraint when using OLS. A problem when using SURE is the short time series. This means that we have to choose a minimum of inputs because otherwise we will run out of degrees of freedom. If we, for example, use six inputs, the wood product included, the number of parameters to be estimated is 21 and our sample consists of 18 observations only, which means that we have insufficient degrees of freedom. Therefore, we have tried to aggregate inputs in a suitable way.

The results of the estimation are presented in Table 3, 4, 5, and 6 for each sector respectively. Cross-price and own-price elasticities are calculated for three different years, 1961, 1970, and 1978.

A positive value of the parameter indicates substitute for wood and a minus sign indicates a complementary relationship. The conclusion is that the estimated elasticities seem to be reasonable in size and sign except for the printing and publishing sectors. Another conclusion is that labor seems to be a complement to wood in the wood using sectors and substitute to paper in paper using sectors.

TABLE 3. Parameter estimates, construction sector, SURE.

	Wood	Services	Cement	Labor
Wood	0.0094 (0.82)	0.0583** (2.46)	0.0061 (0.36)	−0.0216* (−1.96)
Services		0.0043 (0.042)	0.109 (1.39)	−0.0269* (−1.98)
Cement			−0.04 (−0.60)	−0.03 (−1.75)
Labor				0.376*** (22.2)
Cross-price elasticities	**Services**	**Cement**	**Labor versus wood**	
1960	0.50	0.053	−0.17	
1970	0.60	0.062	−0.22	
1978	0.48	0.050	−0.14	
Own-price Elasticities				
Wood				
1961	−0.38			
1970	−0.44			
1978	−0.39			

* Significant at the 10% probability level

** Significant at the 5% probability level

*** Significant at the 1% probability level

TABLE 4. Parameter estimates, furniture sector (SURE).

	Wood	Oth.Mat.	Labor
Wood	−0.04* (−2.48)	0.139** (11.12)	−0.2 (−1.70)
Oth.Mat		0.099** (6.56)	−0.0168 (−1.39)
labor			0.359** (22.74)
Cross-price elasticities	**Oth.Mat.**	**Labor versus wood**	
1961	0.76	−0.084	
1970	0.87	−0.12	
178	1.04	−0.12	
Own-price elasticities			
Wood			
1961	−0.68		
1970	−0.75		
1978	−0.92		

* Significant at the 5% probability level

** Significant at the 1% probability level

TABLE 5. Parameter estimates, office sector, OLS.

	Paper	Oth.Mat.	Labor
Paper	−0.05***	0.084***	0.006
	(−3.4)	(5.2)	(1.1)
R^2 = 0.73		$D-W$ = 1.92	
Cross−price elasticities	Oth.Mat.	Labor versus Paper	
1961	0.93	0.061	
1970	0.97	0.072	
1978	1.03	0.062	
Own−price elasticities			
Paper			
1961	−0.99		
1970	−1.04		
1978	−1.09		

*** Significant at the 1% probability level.

TABLE 6. Parameter estimates, printings and publishing, OLS.[a]

	Paper	Services	Labor
Paper	0.61***	0.77***	0.34***
	(9.75)	(8.0)	(9.3)
R^2 = 0.88		$D-W$ = 2.17	

[a] This sector is only estimated for the period 1961-1974.

*** Significant at the 1% probability level.

10. CONCLUSIONS AND DIRECTIONS FOR FURTHER STUDIES

Our analysis and the econometric study based on it clearly shows that an intermediate demand approach is the only consistent procedure to be used in demand analysis for the forest sector. This is also observed partially in the construction of the global trade model, where, e.g., the demand for pulp is determined within the model itself and not by exogenous demand forecasts. The next step to be taken is to formulate macro-demand functions in which the use of e.g. wood is related to the level of construction and furniture output and to the prices of the substitute and complement inputs to wood in these sectors. A possible input–output equation would then be, e.g.,:

$$\left[\frac{\text{sawnwood input}}{\text{construction} + \alpha \text{ furniture output}}\right] = \sum_i b_{i,\text{wood}} \left[\frac{\text{price of } i\text{–th input}}{\text{price of sawnwood}}\right]^{0.5} + c_\omega \left[\frac{1}{t}\right]$$

where t = time.

Such a macro approach would avoid the need for data from time series of I–O tables.

REFERENCES

Bergman, L. and A. Por. 1980. A Quantitative General Equilibrium Model of the Swedish Economy. WP-80-04. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Diewert, W.E. 1971. An Application of the Shepard Duality Theorem: A Generalized Leontief Production Function. *Journal of Political Economy*, 79(3).

Doran, H.E. and D.F. Williams. 1982. The Demand for Domestically Produced Sawntimber: An Application of the Diewert Cost Function. *Australian Journal of Agricultural Economics*, August.

Frenger, P. 1978. Factor Substitution in the Interindustry Model and the use of Inconsistent Aggregation. Chapter V.2. in Fuss and MacFadden, *Production Economics: A Dual Approach to Theory and Applications*. Amsterdam: North-Holland, 1978.

Frenger, P. 1983. The Generalized Leontief Cost Function in the Short- and in the Long-Run. Oslo: Central Bureau of Statistics Norway.

Henderson, J.M. and R.E. Quandt. 1980. *Microeconomic Theory, A Mathematical Approach*. 3rd Edition. New York: McGraw-Hill.

Johansen, L. 1959. *A Multisectoral Study of Economic Growth*. Amsterdam: North-Holland.

Nerlove, M. 1965. *Estimation and Identification of Cobb-Douglas Production Functions*. Amsterdam: North-Holland.

Shephard. 1953. *Cost and Production Functions*, Princeton: Princeton University Press.

Zalai, E. 1980. A Non-Linear Multisectoral Model for Hungary. WP-80-148. Laxenburg, Austria: International Institute for Applied Systems Analysis.