THE USE OF FLOW MODEL ANALYSIS (FMA) IN THE CASE OF INCOMPLETE MATHEMATICAL MODELS

*Oleg Burdakov, Anatoli Golovin, Klim Kim, Alexander Umnov, Igor Shompolov**

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Dr. Anatoli Golovin and Dr. Alexander Umnov, IIASA
Dr. Oleg Burdakov, Computing Center of the Academy of Sciences of the USSR, Moscow.
Dr. Klim Kim, Central Economic-Mathematical Institute, Moscow.
Dr. Igor Shompolov, Moscow Physico-Technical Institute, Moscow.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

Preface

This paper describes part of an investigation made by IIASA in cooperation with UNIDO and a number of institutions of the Academy of Sciences of the USSR. The main purpose of this collaboration is to develop new methodologies for analyzing mathematical models and to test them on applied problems of practical value.

Vitali Kaftanov

Deputy Director of IIASA

Summary

The main aim of the 'System for Analyzing Mathematical Flow Models' (FMA system) described in this paper is to supply the decision-maker with a computerized tool for quantitative investigation of problem that can be described, at least partially, in terms of standard mathematical models of the flow type.

The FMA system permits the decision-maker to find states of the considered model that satisfy all introduced constraints, are close to the desirable structure, and are optimal with respect to single- or multiobjective evaluation.

1. Introduction

The most important problem in mathematical modelling consists in evaluating the reliability of the results obtained with the models. Usually the reliability of the results is assumed to be guaranteed by the adequacy of the model, i.e. by taking into account all essential relations defining the behavior of the object being modelled. Therefore, it is reasonable to attempt to solve simulation, optimization, or forecast problems where the models are considered adequate.

It would be naive to believe, however, that the mathematics that has been developed intensively during the last hundred years enables us to describe adequately all properties of objects (e.g. socio-economic phenomena) whose essential nature differs from that of the objects of physics or engineering. Practice indeed confirms that our mathematical culture is well developed for describing physical phenomena and engineering operations.

Disregarding the problem of constructing adequate mathematical models, we shell consider here the possibility of using incomplete models, which take into account only some of the essential relations and properties describing the object; that is, in cases where construction of an adequate model is impossible or extremely difficult. The main purpose of this paper is to show for a sufficiently wide class of decision-making problems that the completeness of a model is not necessary for results obtained with the model to be correct.

Such a model may, for example, be used not for seeking the "best" decision but rather for establishing whether some decision is acceptable or not from the viewpoint of the decision-maker. Indeed, if an incomplete model identifies some decision as unacceptable (i.e., the decision does not satisfy some of the formal conditions of acceptability included in the model), then this decision will be also unacceptable for any more complete model. At the same time it is possible that an acceptable decision obtained from an incomplete model may be unacceptable for a more complete model. When we say that one decision is more acceptable than another, it means that the first decision satisfies all of the acceptability conditions met by the second as well as some additional acceptability conditions.

Thus, in the approach that we propose for utilization of incomplete models, the mathematical model, in conjunction with the computer, is used in decision-making only as a tool for picking out a set of acceptable solutions. The decision-maker, using informal, empirical, or intuitive criteria, chooses the decision that is the best from his point of view.

As well as leading to satisfactory results, an advantage of this approach over traditional ones consists of the possibility for the decision-maker to have a more active role. On the other hand, as will be seen below, in the proposed approach it is necessary to avoid certain difficulties arising in its practical realization.

2. A Typical Problem

This section is devoted to the analysis of trade markets, for which adequacy of a mathematical model may be proved easily, and which demonstrates the great potential of the described approach.

A system of partners (e.g. private persons, companies, countries, regions and so on) trading in a set of commodities within a given period is called a *trade market*. If the *volumes* and *prices* of the commodities are known, it is possible to evaluate *export*, *import*, and *balance* data characterizing the state of the trade market.

Using these basic data, one may evaluate the level of *acceptability* of the current state of the trade market from the viewpoint both of each of the partners and of the market as a whole.

The definition of desirable or acceptable states of the market permits us to formulate the following questions :

- Is the current state of the market a desirable one ?

- If not, how far is the current state from the desirable one ?

- What should we do to bring these two states nearer to one another ?

Let us start by describing a mathematical model of the trade market.

Let v_{ij}^k be the volume (measured in physical units) of the kth commodity sold by the ith partner to the jth one. If the unit price of this commodity is p^k , we may define the export, import, and balance for the total trade between the partners as

$$exp_{ij} = \sum_{k=1}^{K} p^k v_{ij}^k$$
$$imp_{ij} = exp_{ji}$$

where K is the total number of commodities traded.

The total volumes of exports and imports for the ith partner will be

$$EXP_i = \sum_{j=1}^{N} exp_{ij}$$
$$IMP_i = \sum_{j=1}^{N} imp_{ij}.$$

where N is the number of partners, and finally

$$IMBALANCE_i = EXP_i - IMP_i$$

It is very easy to prove that the sum of all exports equals the sum of all imports using to the above relations.

Let us suppose that one may define lower and upper acceptable bounds for the export, import and balance indicators for each of the partners.

We will call a state of the trade market acceptable if the constraints

$$\underline{EXP}_i \le EXP_i \le \overline{EXP}_i$$
$$\underline{IMP}_i \le IMP_i \le \overline{IMP}_i$$
$$\underline{IMBALANCE}_i \le IMBALANCE_i \le \overline{IMBALANCE}_i$$

are valid, for all $i=[1,N]$.

The values of the lower and upper bounds may be decided by experts according to the scenario that is going to be considered. For example, data for the ith group of constraints may be defined by authorized representatives of the ith partner.

Besides the data characterizing the overall trade balance of each partner, there may also exist constraints due to limited industrial capacity, transport capabilities, and so on. Therefore, the system of constraints describing the acceptable states may often be augmented by supplementary inequalities, for example, of the following type:

$$\underline{v}_{ij}^k \le v_{ij}^k \le \bar{v}_{ij}^k$$

for all k, i and j .

It should be emphasized here that the *expert* opinions expressed in the constraints described above may sometimes appear to be far from realistic, or even inconsistent. Therefore we must be ready to tackle cases where there is no acceptable state at all. On the other hand, it is also possible that there will be many acceptable states of the trade market for the same set of constraints.

We can now use our definition of an acceptable state of the trade market to evaluate how far a given state is from an acceptable one.

Let us assume for the moment that all conditions are consistent. Then there exists an acceptable state for the model under consideration. We can transform a given state into this acceptable one by making appropriate changes in the volumes of commodities sold. If this transformation involves adding x_{ij}^k to the volumes v_{ij}^k , then the relative value of this change,

$$\rho(x_{ij}^k) = abs \left[\frac{x_{ij}^k}{v_{ij}^k}\right]$$

characterizes the degree of imbalance for the flow of the kth commodity from partner i to partner j. The absolute value must be used here because x_{ij}^k may be either positive or negative.

One measure of the "unacceptability" or "imbalance" of the state of the market as a whole could be formulated as

$$\rho(x) = \max_{k,i,j} \rho(x_{ij}^k)$$

This evaluation of the "distance" from the given state to the acceptable one only has practical value if the acceptable state is unique. But usually the acceptable states are in fact nonunique and a different ρ value is associated with each.

One way a round this problem is to take just the minimum of these ρ values, thus eliminating the ambiguity in our definition of "acceptable". In other words, we define the difference between the given and acceptable states as

$$\rho^* = \min_x \rho(x)$$

or

$$\rho^* = \min_x \left[\max_{k,i,j} abs \left[\frac{x_{ij}^k}{v_{ij}^k}\right]\right]$$

The value of ρ^* shows what minimum relative change is required to transform the given state of the trade market into an acceptable one.

Mathematically, the procedure for finding the imbalance ρ, which is in fact a special case of the Chebyshev approximation problem, can be reduced to the following *mathematical programming problem.*

Minimize ρ

with respect to $\{ \rho, x_{ij}^k$, for all i,j,k $\}$

subject to

$$\rho \geq 0$$
$$-\rho v_{ij}^k \leq x_{ij}^k \leq \rho v_{ij}^k$$
$$\underline{v}_{ij}^k \leq v_{ij}^k + x_{ij}^k \leq \overline{v}_{ij}^k$$

for all i,j,k .

$$\underline{EXP}_i \leq EXP_i \leq \overline{EXP}_i$$
$$\underline{IMP}_i \leq IMP_i \leq \overline{IMP}_i$$
$$\underline{IMBALANCE}_i \leq IMBALANCE_i \leq \overline{IMBALANCE}_i$$

where

$$IMBALANCE_i = EXP_i - IMP_i ,$$
$$EXP_i = \sum_{j=1}^{N} \sum_{k=1}^{K} p^k (v_{ij}^k + x_{ij}^k)$$
$$IMP_i = \sum_{j=1}^{N} \sum_{k=1}^{K} p^k (v_{ji}^k + x_{ji}^k)$$

for all $i = [1,N]$.

3. The General Flow Model

We shell now consider a not completely connected graph of *nodes* linked by oriented *flows*. All nodes are numbered and each of the nodes can be a source, a drain, or simultaneously both . Each of the flow may consist of different *types of flow* or components. Figure 1 shows an example of this graph.

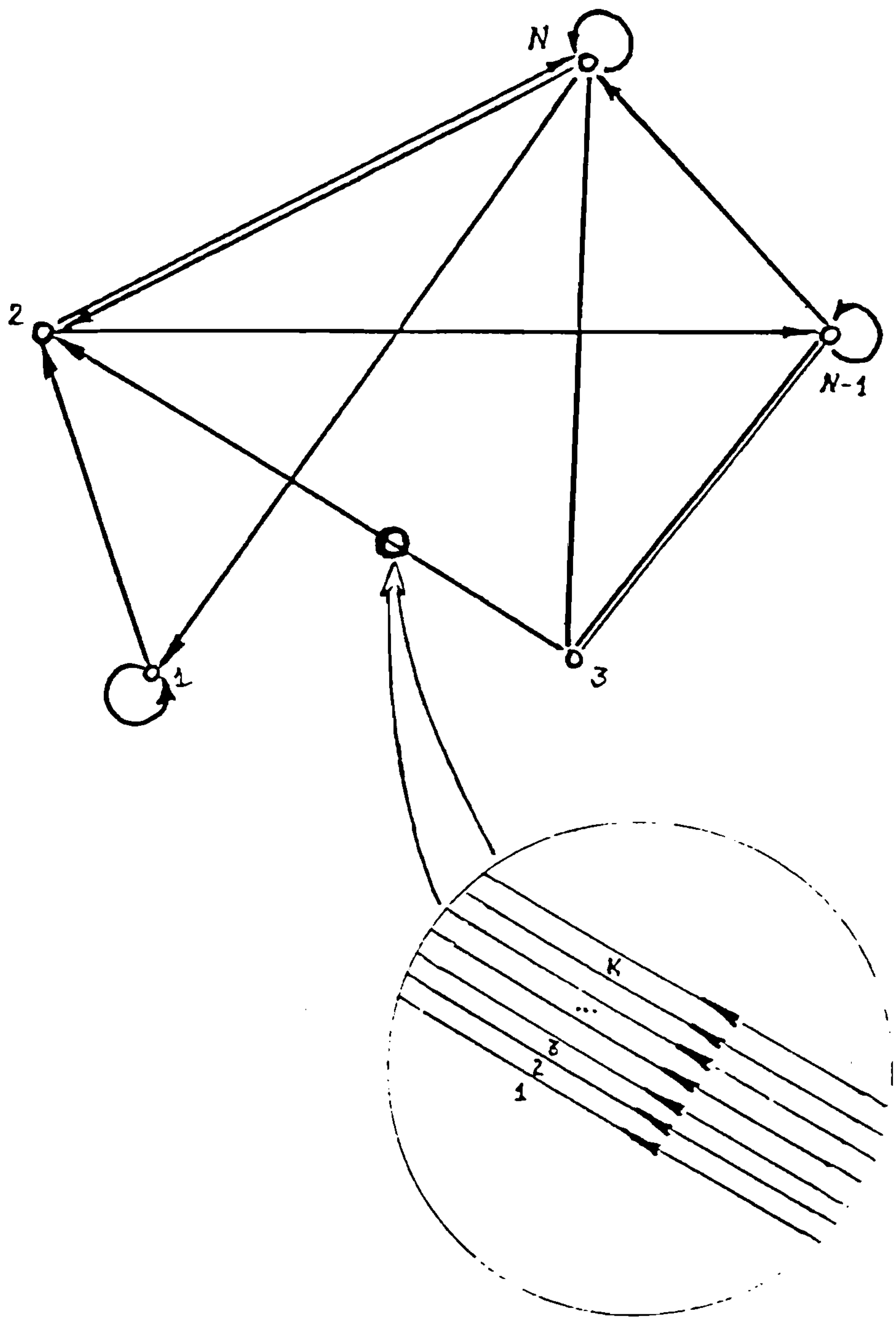

Figure 1.

Let the model have N nodes and K types of flow. The main quantitative characteristic of a flow is its *value*. Generally, each type of flow has its own unit measure of value. There may also exist a common measure of the values of all flow types, which is called the equivalent value or simply *equivalent*.

If the flow from the ith node to the jth node of the kth type is v_{ij}^k, then its equivalent will be

$$e_{ij}^k = p_{ij}^k v_{ij}^k,$$

where p_{ij}^k is a given positive constant.

The flow model analysis system uses the values v_{ij}^k and e_{ij}^k to describe both input and output data of the flow model.

Each state of the model, i.e. nonnegative cube matrix with elements v_{ij}^k, can be specified as *unacceptable* or *acceptable* or *desirable*. The unacceptable set consists of those states of the model for which at least one of the necessary conditions of acceptability is violated. The complement of this set is the set of acceptable states. The set of desirable states of the model is described by conditions that are not necessary. Therefore, this set can have intersections with both the acceptable and unacceptable sets.

To simplify the description of the definition of the acceptable or desirable states of the model in the FMA system, we can use the following *auxiliary* variables :

$$\sum_{i=1}^{N} e_{ij}^k,$$

$$\sum_{j=1}^{N} e_{ij}^k,$$

$$\sum_{k=1}^{K} e_{ij}^k,$$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} e_{ij}^k.$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K} e_{ij}^{k},$$
$$\sum_{j=1}^{N}\sum_{k=1}^{K} e_{ij}^{k},$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{K} e_{ij}^{k}.$$

Sometimes we may have the same values of p_{ij}^{k} for all feasible indices i, j, k. In these cases the following variables can be also used in the statement of the problem :

$$\sum_{i=1}^{N} v_{ij}^{k},$$
$$\sum_{j=1}^{N} v_{ij}^{k},$$
$$\sum_{k=1}^{K} v_{ij}^{k},$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N} v_{ij}^{k},$$
$$\sum_{i=1}^{N}\sum_{k=1}^{K} v_{ij}^{k},$$
$$\sum_{j=1}^{N}\sum_{k=1}^{K} v_{ij}^{k}.$$

Summation for a subset of feasible indices is not permitted in an explicit way here, but it is always possible to split any node into a system of new ones and extract desirable subsets of indices.

Finally, the user can formulate the problem in terms of the *imbalancing* variables. They are :

$$e_{ij}^{k} - e_{ji}$$
$$\sum_{j=1}^{N} e_{ij}^{k} - \sum_{j=1}^{N} e_{ji}^{k},$$
$$\sum_{k=1}^{K} e_{ij}^{k} - \sum_{k=1}^{K} e_{ji}^{k},$$
$$\sum_{j=1}^{N}\sum_{k=1}^{K} e_{ij}^{k} - \sum_{j=1}^{N}\sum_{k=1}^{K} e_{ji}^{k}.$$

Imbalancing variables may have both positive and negative values.

4. The Conditions of Acceptability

The user of the FMA system may define the conditions of acceptability of a state of the model by introducing constraints on the variables v_{ij}^k, e_{ij}^k and all auxiliary variables.

The constraints on the absolute value of a variable may be of the following types:

- the variable must be equal to a given value,
- the variable must be not less than a given value,
- the variable must be not greater than a given value,

and (subject to the initial value is given)

- the variable cannot be changed,
- the variable must not decrease in value,
- the variable must not increase in value.

It is also possible to introduce constraints for the ratio of a pair of variables :

- the ratio must be equal to a given value,
- the ratio must be not less than a given value,
- the ratio must be not greater than a given value.

In terms of the equations and inequalities these constraints may be written for a variable v as

$$v = C, \quad v \geq C, \quad \text{or} \quad v \leq C,$$

where C is a given constant.

Analogously, for a pair of variables v^1 and v^2

$$\frac{v^1}{v^2} = C, \quad \frac{v^1}{v^2} \geq C, \quad \text{or} \frac{v^1}{v^2} \leq C.$$

Finally, in the FMA system we can use the most general affine dependence between two variables

$$v^1 = Av^2 + R,$$
$$v^1 \geq Av^2 + R,$$

or

$$v^1 \leq Av^2 + R,$$

where A and R are arbitrary constants.

All variables described in Section 3 can be included in these relations. The total number of constraints is limited only by the available computer resources.

5. The Conditions of Desirability

The simplest way to introduce a desirable state into this model is to describe it explicitly. The user may define the desired value for any subset of flows in the model. The FMA system proves whether this definition is acceptable one or not. If the definition is acceptable, the system will calculate appropriate values of the remaining flows to grant the acceptability of the state as a whole.

Normally the desired state is unacceptable, i.e. the constraints describing the conditions of acceptability are incompatible with the desired values of the flows. In this case the FMA system builds a new state of the model that is acceptable and is the closest to the desired state in the sense of the Chebyshev metric.

Let a considered acceptable state be v and the given desired state be v^*.

Then we may define the distance between these two states as

$$\rho = \min_{v} \max_{i,j,k} \text{abs} \frac{v_{ij}^{k*} - v_{ij}^{k}}{v_{ij}^{k*}},$$

subject to v satisfying all of the above-defined constraints.

This minimax objective permits us to avoid ambiguity in the solution.

In the FMA system a special modification is used. Very often the chosen metric depends only on a subset of flows, which we call *leading* flows. The remaining flows may have arbitrary values, which have no practical meaning. It is reasonable to try to continue the minimax procedure, fixing all the leading flows at the optimal levels.

In practice, this means that all leading flows are removed from consideration as variables in the optimization problem and a new set of leading elements (with a new value of ρ) is built. This step may be repeated until all flows are fixed or ρ becomes zero.

This procedure of *sequential fixation* produces a ranking of the whole set of flows in the model. Let ρ^t be the solution of the optimization problem in the tth step of the fixation process and Ω_t be the corresponding subset of leading flows. Then ρ^t may be treated as a relative measure of the required *relative* change of the flows in the subset Ω_t to bring them to the given desirable state.

In the FMA system the user can control the sequential fixation procedure, limiting the number of steps in the fixation or terminating it as soon as the required level of ρ is reached. The fixations are of course made simultaneously for all defined constraints.

Finally, the FMA system permits us to use *weight coefficients* to correct the dependence of ρ on the flows if necessary. In the general case the distance between the acceptable and desirable states is

$$\rho = w_{ij}^{k} \min_{v} \max_{i,j,k} \text{abs} \frac{v_{ij}^{k*} - v_{ij}^{k}}{v_{ij}^{k*}},$$

where w_{ij}^{k} is a nonnegative constant.

6. Optimization in the FMA System

The FMA system with the features described permits us to solve optimization problems, maximizing or minimizing any of the variables introduced in Section 3. To do this it is sufficient to include in the considered model a new formal flow that equals the optimized function. We shall call this the *objective* flow.

By giving the desirable objective flow a very large value (for maximization) or a very small value (for minimization) and choosing an appropriate weight coefficient for the flow, we shall find that the resulting value of the objective flow is optimal.

In the same manner we can carry out a multiobjective optimization procedure, by introducing several objective flows and supplying them with equal weight coefficients. A point of the Pareto set will be the solution in this case.

7. Analyzing Structural Change with the FMA System.

Any feasible combination of the above procedures, which manipulate the weight coefficients and objective variables, may be used in the FMA system. One of the most important procedures in practice is to insert *null* flows. Of course, direct use of a flow with zero value is not possible, but there can be no objection to do this if the flow has zero weight. This makes it possible to reserve a new element in the considered model.

The reservation of null flows may be useful in improving the model in the case of infeasibility. An unacceptable, but desirable, state of the model may be approximated by an acceptable state that is found by the FMA system minimizing the 'distance' between these states.

Finally, the FMA system can be used to analyze dynamic models. In this case the desirable state of the model is considered as the initial state. The conditions of acceptability describe the final state. If necessary, the dynamic procedure may consist of several steps, each of which has an independent description of the conditions of acceptability. The final state for one step is used as the initial state for the next step.

8. An Example of Using the FMA System for Energy Development Projections

We shall now to demonstrate how the FMA system can be used to analyze the development of the CMEA energy market until the year 2000. Usually, such an analysis involves detailed considerations of the fuel-energy balances and the energy-economy interactions in each of the CMEA countries. These subjects are described by means of models that take into account a large number of parameters. Generally, there is a great deal of uncertainty attached to these parameters with respect to future developments. One possible way of describing all the essential features of the modelled system is to use analytical techniques to define more or less realistic trajectories of future energy developments. Nevertheless, there will still be problems of model verification, data reliability, and the like. Besides, the more parameters that are used in the model, the more difficult it becomes to run the model, to analyze the results, and to eliminate the errors.

Another approach has been found suitable for assessing future developments of energy systems, which takes into account the acceptability of the future situation of the modelled system. This approach will now be described. The FMA system was used to assess some of the boundaries of an acceptable structure of primary energy consumption for the European countries of the CMEA, based on estimates of likely trends in the production of primary energy sources and on assumptions regarding future rates of economic growth and energy elasticities for these countries over the period 1985-2000. Feasible values (or ranges of values) were then identified for energy exports from the CMEA countries (mainly from the Soviet Union) to the rest of the world.

The following assumptions were made:

- Future energy imports to the CMEA will not exceed existing ones and have to be minimized.

- Assuming that a set of feasible solutions exists, the process of finding an acceptable solution has to take into account the criterion of minimizing a maximal change in the structure of energy consumption; that is, of finding a feasible structure for the future that is as close as possible to the existing structure.

- Energy flows between CMEA countries have to be as stable as possible but, at the same time, export of energy from these countries to the rest of the world has to be maximized.

The process of assessing the acceptability of the future structure of primary energy consumption has to include two main procedures:

- assessing the existence of a feasible structure,

- defining possible boundaries of acceptability.

These procedures were performed for each five-year interval of the period considered, and the results of one step were used as the initial conditions for the next step. The first procedure consisted of finding a feasible structure of energy flows subject to the criteria mentioned above; the second procedure involved the investigation of variables with values close to the previously found solutions for the structure of primary energy consumption for each country. One of the important results of the second procedure was a definition of possible ranges of energy exports from the CMEA countries to the rest of the world.

A substantial feature of the acceptable structures of primary energy consumption in the CMEA is the changes in the shares of solid fuels. For most countries, these shares have to be decreased. But in the case of Hungary, it is possible to have the same share of solid fuels in 2000 as in 1980; and in the case of Roumania this share has to be increased.

The share of liquid fuels has to be decreased almost everywhere (except in Poland). For gaseous fuels, the share has to be incrrased in every country (except Hungary and Roumania). The substantial growth of the share of primary electricity in total energy consumption is caused by the development of nuclear energy programs in the USSR and the East European countries.

Under these changes in the structure of primary energy consumption, the possible amount of energy exported from the CMEA countries was assessed. It was found that the main energy source that could be exported is natural gas.

The results of the study are only preliminary and are based on data and assumptions about future energy development that had been made by the authors themselves. Therefore the results serve only to show the feasibility of the approach proposed for the assessment of acceptable structures of primary energy consumption and exchange within the CMEA region and also for estimation of possible primary energy export levels by the CMEA, including exports of natural gas.

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