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THE SPATIAL GROWTH OF TOKYO METROPOLITAN AREA*

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Contributions to the Metropolitan Study:7

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LIST OF CONTRIBUTIONS TO THE METROPOLITAN STUDY:

1. Anas, A., and L.S. Duann (1983) Dynamic Forecasting of Travel Demand. Collaborative Paper, CP-83-45, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria.

2. Casti, J. (1983) Emergent Novelty and the Modeling of Spatial Processes. Research Report, RR-83-27, IIASA, Laxenburg, Austria.

3. Lesse, P.F. (1983) The Statistical Dynamics of Socio-Economic Systems. Collaborative Paper, CP-83-51, IIASA, Laxenburg, Austria.

4. Haag, G., and W. Weidlich (1983) An Evaluable Theory for a Class of Migration Problems. Collaborative Paper, CP-83-58, IIASA, Laxenburg, Austria.

5. Nijkamp, P., and U. Schubert (1983) Structural Change in Urban Systems. Collaborative Paper, CP-83-57, IIASA, Laxenburg, Austria.

6. Leonardi, G. (1983) Transient and Asymptotic Behavior of a Random-Utility Based Stochastic Search Process in Continuous Space and Time. Working Paper, WP-83-108, Laxenburg, Austria.

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FOREWORD

Contributions to the Metropolitan Study:7

The Project "Nested Dynamics of Metropolitan Processes and Policies" was initiated by the Regional & Urban Development Group in 1982, and the work on this collaborative study started in 1983. The series of contributions to the study is a means of conveying information between the collaborators in the network of the project.

This study has been prepared by Professor Masahisa Fujita, who was awarded the Palander prize 1983 for his path-breaking studies of regional and metropolitan growth and structural change.

In this paper Professor Fujita examines the optimality of the equilibrium development pattern of the Tokyo metropolitan region. He concludes that the metropolitan region of Tokyo is growing in a *non-optimal* way, primarily as a consequence of "tulipmania expectations" in the land market. He advocates a strong land taxation policy correcting the prices of land into a pattern reflecting real values of different land areas in the region. The major advantage of such a land taxation scheme would be increasing intensity of land use at commuting distances of 40 to 70 minutes from the CBD. This would decrease the current intensive housing construction in the extremely distant suburbs, thus reducing the growth of average commuting time in the region.

Åke E. Andersson
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THE SPATIAL GROWTH OF TOKYO METROPOLITAN AREA

M. Fujita

1. INTRODUCTION

As a location theorist, it is a great honour to receive the Erik Kempe-prize in memory of Tord Palander.

Tord Palander is a towering figure in the history of location theory. Beginning with Von Thünen, the field of location theory was long dominated by German scholars. Tord Palander is the first major location theorist to originate outside Germany, and he will always remain one of the giants in the field.

Location theory is an economic theory devoted to the study of human space. If we look around this room, we perceive one kind of human space, namely that of a modern auditorium inside a larger building. As location theorists, however, we leave this particular domain to the architect. But should we take one step outside this building, we move into the territory of location theory, which is complex, hierarchical and all-embracing. From the doorstep of the building, we would first notice the huge campus of the University, and if we could see far enough, the community surrounding the University. Beyond this, the city of Umeå looms as yet another human space, functioning as the regional capital of Norrland. And Norrland, of course, is only a small space within Sweden itself, which in turn in embedded in the European community, and so on.

As location theorists, we ask questions about these various human spaces, we try to explain the structures we observe, and finally, we try to come up with ways for improving on the imperfections we observe. For example, if we looked at Umeå from the air, we would notice a fairly typical pattern: there is a downtown between the railway station and old harbor, downtown is surrounded by a residential area, and in the far suburbs are located highly land-consuming activities, such as paper mills, and an airport. A diligent location theorist would try to understand this land-use pattern, asking, ultimately, questions such as "is this the optimal land-use pattern?", and if not, "how can we improve it?". As an additional example, consider the northern region of Sweden which is relatively undeveloped. Location theorists, and others as well, might be interested in knowing how to foster the development of this reigon. One answer, given about 25 years ago, was to create the Umeå Univeristy at the center of Norrland, which, I think, was an excellent idea. A region can develop only if it is attractive to industries and people, in particular, to young people. Today, the Umeå University is playing a great role in the development of this region as the educational, research and intellectual center of Norrland.

Space provides for the richness and diversity of human society. It is apparent that the culture of Sweden is very different from that of the US or Japan. In large part, this is because distance intervenes with irregular, natural conditions, allowing for unique cultural developments. But space, too, can cause problems. Different locations are synonymous with different climates, different natural endowments and different accessibilities to other parts of the world, and hence different economic advantages or disadvantages. For example, today's North-South problem is in large part a locational problem. Hence the question of how to equitably and efficiently coordinate the development of various parts of the world is the biggest challenge to today's location theorists, and indeed, to the world at large.

The famous book, *Beiträge zur Standortstheorie*, was written by Tord Palander in 1935; it is a monumental work in the history of location theory. According to Martin Beckmann, who himself is one of the great location theorists, it is a book that truly belongs to the MEISTER KLASSE (Beckmann 1981, p.1). In a book of 420 pages, he gives a very condensed treatment of many topics in location theory, and suggests a number of new ideas which greatly influenced the later development of location theory. Each location theorist may be attracted by a different part of this enormous book. I myself, however, have been most influenced by Chapter 9, where he insists on the necessity of depicting the economic development process. That is, he insists on the necessity of dynamic location theory, which he himself worked toward developing. In our recent study of the spatial growth of Tokyo (Fujita and Kashiwadani 1982) which is the main topic of my speech, we tried to follow this suggestion by Palander. Until very recently, the theory of urban land-use has exclusively been a static theory. Everyone knows however, that urban infrastructures such as buildings and transport facilities are among the most durable objects we make and it is very costly to adjust them. It is obvious, therefore, that the static theory of urban land-use is intrinsically limited in its usefulness because it completely neglects the durability and adjustment costs of urban infrastructure. What we tried to do in our work was to demonstrate that, by introducing the dimension of time, as well as space, we could provide location theory with additional practical utility.

2. LAND PROBLEMS OF TOKYO

Tokyo is a huge city, one of the biggest in the world. Tokyo Metropolitan Area (TMA) is about 50 km in radius, and its population is about 20 million (more than twice that of all Sweden). Since such a huge population is concentrated in a small area, the land price is naturally quite high. At some locations, the land price is more than $40,000 (about 300,000 Krona) per square meter.

It is interesting to note that the total asset price of all Japanese land is about the same as that of the US. That is, the sale price of Japanese territory is about the same as the US. If we recall that the area of the US is about 25 times the area of Japan, and the population of the US is about twice Japan's, this surprising fact suggests how important land is in the Japanese economy.

The high land price in Tokyo contributes to a very poor housing condition. For example, in Tokyo city, the average size of living-floor-space is at present about 4 m^2 per person, which is by far smaller than the European and American standard. You may reason that since so many people are concentrated in such a small area, each person can naturally consume only a small housing space. This is partly true. However, if we take a closer look at the land-use pattern of Tokyo, we realize it is very dispersed. Presently, most flat houses are being built at locations about 40 km from the center of Tokyo. By railway, it takes about 90 minutes per trip. Within this 90-minute commuting distance, approximately one half of the available land remains undeveloped or used for agriculture. In other words, one half of the present city area is retained for the purpose of land speculation. Moreover, the construction frontier of housing is moving outward at considerable speed passing over much vacant land. In Tokyo then, land is being developed in a typical, urban sprawl fashion, and in that the land price is very high, the housing conditions are poor. Furthermore, the commuting time for the average worker is painfully long. Today, 2 hours' commuting time per trip is not exceptional for the CBD workers of Tokyo. Some people are even spending 3 hours commuting in one direction.

Given this situation, many people, particularly those who live in Tokyo, argue that the land-use of Tokyo is highly inefficient, since a fair number of people are commuting long distances, past vacant land. There is also a pervasive feeling that today's land prices are unnecessarily high, a condition which is assumed responsible for the extensive, unoccupied land.

On the other hand, some economists argue that the current land-use pattern in Tokyo is not necessarily inefficient from a long-run point of view. This is because some land in the suburbs should be left for high-density developments in the future. In other words, at each time, we need to construct both low-density housing and high-density housing since people with different incomes and different household structures demand different types of housing. If we occupy all the land in the suburbs by low-density housing today, then at some point in the future only two options will remain for the construction of high-density housing. One is that construction can occur on vacant land in very distant suburbs, while the other is that construction may take place in the wake of demolition of low-density housing. It is not difficult to see that neither way is very efficient in the long run. The efficient way is to leave some vacant land for the construction of high-density housing in the future, while constructing low-density housing in the present. This means efficient land development in the long-run will naturally follow a dispersed growth pattern. And, in order to save the necessary amount of land for the future high-density development, land prices today must be sufficiently high so as to discourage a dense, radial development out from the CBD.

Who is right in this case? The cool-headed economist who finds the long run inviting, or the citizens of Tokyo presently suffering from the difficult housing situation and tiresome commuting. We may put the question as follows: in order to ensure the efficient spatial growth of Tokyo in the future, how much vacant land should be left at each distance today and how high should the land price be set at each location? This is the question that I asked in our study of Tokyo.

3. MODELING THE SPATIAL GROWTH OF TOKYO

How do we set out to investigate such a question, which sounds almost impenetrable when one thinks of the scale of the problem? It is no simple matter to confront the dynamics of a city whose population approaches 20 million. Fortunately, we were able to proceed in a straightforward and fruitful manner.

First, we constructed a dynamic model of optimal spatial growth for Tokyo. By using this model, we calculated the optimal spatial growth process for the time period 1955-2020. Then the actual spatial growth of Tokyo was compared with the optimal spatial growth for the time period 1955-1980, for which actual data are available. In addition, the actual land prices of Tokyo at various locations during 1980 were compared with those efficient prices associated with the optimal spatial growth (i.e., shadow prices of land).

Let me explain certain features of the approach in more detail. We were primarily concerned with the spatial pattern of the area 10 km out from the center of Tokyo. That is, *our study area* was exterior to the CBD of Tokyo. As it turns out, land-use in the suburbs is more controversial than the land-use of the CBD, so this focused our study on real issues. In Tokyo, the dominant means of commuting for most workers is the rapid railways which extend radially from the CBD. We therefore divided the study area into 21 zones according to time distance by rapid railway from the center of Tokyo (in the base year 1965). In order to be able to describe the structural aspects of space, we classified all the buildings in the study area into five types: high-rise apartments (more than 5 stories), low-rise apartments, flat houses, buildings for manufacturing firms, and buildings for service firms. The total demand for buildings of each type was exogenously given as a function of time: for each year from 1955 to 1975, the total demand was fixed to the actual demand value, and for each year after 1975, the total demand was estimated under various scenarios. The problem then became one of determining the construction processes for buildings of each type so as to accommodate the streams of given building demands in the most efficient manner. The objective function we stated in formulating the optimization problem was the present value of net revenues from development in the study area. Under a few added assumptions, this objective function was seen to be equivalent to minimization of the present value of the sum of all transport costs, including the disutility costs of commuting.

The specification of our model was not arbitrary. It was formulated in such a way that, if all market participants exercised perfect foresights about future prices of land and buildings, the competitive land market of Tokyo would realize the solution of the model, namely, the optimal spatial growth path.

To be perfectly honest, I would like to say that our model has an obvious limitation. The efficiency test was essentially ex post facto in nature; for each year from 1955 to 1975, the total demand for buildings of each type was fixed to the actual demand. Hence, our study dealt only with efficiency in the *spatial allocation* of given total land-use demand in Tokyo. An efficiency test for the global land market and means for determining total demand are left to future work.

Now, how do we solve such an optimization problem? If we adopt the discrete time unit of one year, the problem has more than 100 thousand variables. Thus, if we had blindly attacked this problem via computer, an enormous amount of computation would have been required. Fortunately, we succeeded in solving the problem with a minimal amount of computation by using optimal control theory in a continuous framework. Several theoretical results from the work of colleagues were also useful (among others, the result by Schweizer and Varaiya 1976). As it turns out, the sollution can be obtained in almost purely analytic fashion. Hence, despite the magnitude of the problem, the computational costs were minimal. On computer, it took only about 20 seconds for the calculation of the optimal growth path under each scenario.

4. COMPUTATIONAL RESULTS AND THE ACTUAL PROCESS

Let me next compare the computational results with the actual growth process of Tokyo. Before that, however, one note is in order. The computational results, of course, depend on the parameter values of the model, in particular, the time discount rate and the growth rate of the total number of households in the study area after 1975. It turned out, however, that within a reasonable range of parameter values, the optimal spatial growth path is relatively stable. Therefore, let me now compare the actual process with the optimal process under the standard set of parameter values, namely, a 10% discount rate and a household-increase of 60 thousand per year[1].

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[1]This is the number of our study area which is about one fourth of the land area of TMA. Hence, for the whole TMA, the household-increase is about 240 thousand per year.

It is immediately apparent that the actual growth process is very similar to the optimal growth process, at least qualitatively. In particular, both actual and optimal growth processes take the typical form of *urban sprawl*. This means that initially both processes exhibit *leap-frog developments*, whereby low-density buildings are constructed at some discontinuous distance from the high-density buildings. Both processes then exhibit *scattered development* such that the construction of low-density buildings does not exhaust all available land in those areas undergoing low-density development. Finally, both processes exhibit *mixed development;* all this means is that in the end most areas show a mixture of many different building types.

On the basis of our solution and these observations, we can conclude that the very scattered developmental process of actual Tokyo reflects at least a certain degree of efficiency in the long run. If we make a closer comparison of the two processes, however, we notice that, quantitatively, the actual development process is considerably more dispersed than the optimal process. For example, if we compare the two land-use patterns in 1975, we can see that, within the area of a 60-minute commutation, the vacant land in the actual process is approximately twice that of the optimal process.

Regarding land prices, we can observe that, in zones near the CBD, actual land prices are very close to the efficient land prices of the optimal process. This is true both for 1975 and 1980. Thus it seems that the current, extremely high land prices near the CBD of Tokyo are not without economic rationale. On the other hand, in the middle suburbs, 40 to 70 minutes from the city center, the actual land prices are about twice the efficient land prices. This is true both for 1975 and 1980. We suspect this to be the major reason for the expanses of land left unoccupied in the middle suburbs, a condition which promotes development in the distant suburbs.

You may ask why such high land prices can persist in the middle suburbs. I suspect these high land prices are largely due to so called *tulip-mania expectations*. Since land is an indefinitely durable good, and since there is no definite end to the time, such high land prices can persist without collapse as long as people continue to hold very strong expectations of future land prices. Tulip-mania expectation, of course, causes inefficient urban sprawl.

5. CONCLUSION

In summary, our study indicates that in the middle suburbs of Tokyo today, more than the efficient amount of land is left vacant, and that land prices there are too high, thereby failing to reflect the future, efficient land development of Tokyo.

Of course, our model is relatively crude, and our conclusions may therefore be less than unimpeachable. But I believe them to be at least qualitatively true.

If this is the case, how can we recover the efficiency of the land market in Tokyo? Theoretically, the answer is simple. In order to correct the inefficiency due to tulip-mania expectations, a strong land tax policy is necessary so that the land market will be dominated by bid land prices based on real values of land. And, the necessary information for this land tax policy can be obtained by using a model such as ours. Of course, land tax policy is an extremely sensitive political issue in Japan. In order to determine the actual land tax rate, we need a much more sophisticated study. But, the basic approach would remain unchanged.

In conclusion, Tord Palander emphasized, in his famous book, the importance of dynamics in location theory. My study was formulated in the light of this very worthwhile emphasis. I hope we have contributed to demonstrating his basic point that location theory will become more useful when time and space are considered simultaneously.

APPENDIX

We briefly summarize here the model of optimal spatial growth for TMA and the optimality conditions for the problem (for detail, refer to Fujita and Kashiwadani 1982).

If we exclude the Tokyo Bay area, the land-use pattern in TMA exhibits a circular symmetry with respect to the center of Tokyo (the Tokyo Station). Most of the land within 10 km of the center of Tokyo was well developed before 1955. Considering these facts, we chose to study a fan-shaped area in the western part of TMA more than 10 km from the center of Tokyo. *The study area* is about one fourth the land area of TMA. We call the area of TMA which is within 10 km of the center of Tokyo the CBD. The study area was divided into 21 zones according to time distances by rapid railways from Tokyo station in the base year 1965. Because this study does not address land-use change in the CBD, the number of employees in the CBD is exogenously given for each year from 1955. Therefore, those land-using activities whose locations are endogenously determined in the study area are households, local service firms and manufacturing firms.

We classify all residential houses into the following three categories: high-rise apartments (H-apartments, more than five stories), apartments and flat houses. Considering actual housing conditions in TMA, we assume that all households whose members work in the CBD can be classified a priori as residing in H-apartments, apartments or flat houses; all households whose members work in local service firms or manufacturing firms can be classified a priori as residing in apartments or flat houses. Based on this assumption, we classify all households by their residential types. The indices are h=1: households living in H-apartments, h=2: households living in apartments, and h=3: households living in flat houses. Each household is assumed to consume a unit of local services per unit of time.

We assume that local services are produced with a constant-coefficient technology: one unit of local service is produced by E_s^h workers of type h(h=2,3) using one unit of building for service production and other inputs costing $C_s(t)$. Similarly, we aggregate all manufacturing goods into a composite manufactured good, which is produced with a constant-coefficient production technology: each unit is produced by E_m^h workers of type h(h=2,3) using one unit of building for manufacturing production and additional inputs costing $C_m(t)$, where $E_m^2 + E_m^3 = 1$. The outputs of each manufacturing firm are either exported from the port or the railway terminal located close to the center of TMA, or distributed uniformly among consumers who are symmetrically located with respect to the center of TMA.

Based on the above assumptions, we next formulate the model of optimal spatial growth for TMA. The problem is to determine the construction processes for buildings of five types in the study area from 1955 to 2020 (or 2000) so as to accommodate the streams of given building demands in the most efficient manner. As noted in the text, the objective function minimized is the present value of the sum of transport costs (including disutility costs of commuting). Hence, our optimal planning problem can be formulated as follows (for notation, see the end):

Choose construction process $u_\ell^i(t)$ and activity allocation processes $y_{\ell r}^h(t)$, $N_\ell^s(t)$ and $N_\ell^m(t)$ ($i=1, 2, 3, s, m$; $h=1, 2, 3$; $\ell \varepsilon NC$, $r \varepsilon L$, $t \varepsilon [t_0, \infty)$) so as to minimize

$$\int_{t_0}^{\infty} e^{-\gamma(t-t_0)} \Big[\sum_{h,\ell,r} (T^h_{\ell r}(t) + p_z(t)\beta_h(t) d_{\ell r}) y^h_{\ell r}(t) + \sum_{\ell} T^m_{\ell}(t) N^m_{\ell}(t) \Big] dt$$

subject to the following constraints:

(a) variation of building stock

$$\dot{x}^i_{\ell}(t) = u^i_{\ell}(t),\ i = 1,\ 2,\ 3,\ s,\ m,\ \ell \varepsilon NC$$

(b) building demand-supply

$$\sum_r y^h_{\ell r}(t) \leqq x^h_{\ell}(t),\ N^s_{\ell}(t) \leqq x^s_{\ell}(t),$$

$$N^m_{\ell}(t) \leqq x^m_{\ell}(t),\ h = 1,2,3,\ \ell \varepsilon NC$$

(c) labor demand-supply

$$N^h_r(t) \leqq \sum_{\ell} y^h_{\ell r}(t),\ h = 1,2,3,\ r \varepsilon C,$$

$$N^s_r(t)\ E^h_s + N^m_r(t)\ E^h_m \leqq \sum_{\ell} y^h_{\ell r}(t),\ h = 2,\ 3,\ r \varepsilon NC,$$

(d) local service demand-supply

$$\sum_{h,r} y^h_{\ell r}(t) \leqq N^s_{\ell}(t),\ \ell \varepsilon NC$$

(e) land

$$\sum_i k_i x^i_{\ell}(t) \leqq L_{\ell},\ \ell \varepsilon NC$$

(f) activity-unit number

$$\sum_{\ell r} y^h_{\ell r}(t) = N_h(t),\ \sum_{r \varepsilon C} N^h_r(t) = N^h_C(t),$$

$$\sum_{\ell} N^m_{\ell}(t) = N_m(t),\ h = 1,2,3$$

(g) initial condition

$$x^i_{\ell}(t_0) = \bar{x}^i_{\ell},\ i = 1,\ 2,\ 3,\ s,\ m,$$

where all variables are nonnegative.

The solution of the above problem specifies the most socially efficient development plan of the study area as a function of the following exogenous parameters: $T^h_{\ell r}(t)$, $P_z(t)\beta_h(t)$, $T^m_{\ell}(t)$, $N_h(t)$, $N_m(t)$, $N^h_r(t)$ and γ .

In order to formulate the optimality conditions for the problem, let us define the *bid (building) rent function* Ψ^i_e for each activity type i (i = 1, 2, 3, s, m) as follows:

households, i = 1, 2, 3

$$\Psi_\ell^h(r, P_\ell^s, W_r^h, U_h, t) = Y_n^h(t) + W_r^h - T_{\ell r}^h(t) = P_\ell^s$$

$$- P_z(t)(z_h(U_h, 0, t) + \beta_h(t) d_{\ell r})$$

service producers

$$\Psi_\ell^s(P_\ell^s, \{W_\ell^h\}_h, \Pi_s, t) = P_\ell^s - \sum_{h=2,3} W_\ell^h E_s^h - C_s(t) - \Pi_s,$$

manufacturing firms

$$\Psi_\ell^m(\{W_\ell^h\}_h, \Pi_m, (t)) = P_m(t) - \sum_{h=2,3} W_\ell^h E_m^h - C_m(t) - \Pi_m - T_\ell^m(t),$$

where $z_h(u_h, o, t)$ is the inverse function of the utility function, $U_h = u_h(z, d_{\ell r} = o, t)$, with respect to z, and z is the amount of composite consumption good. Then, the optionality conditions for the problem can be obtained from the maximum principle of optimal control theory as follows.

For a set of functions $u_\ell^i(t)$, $y_{\ell r}^h(t)$, $N_\ell^s(t)$, $N_\ell^m(t)$ (i = 1, 2, 3, s, m, h = 1, 2, 3, $\ell \in NC$, $r \in L$, $t \in [t_0, \infty)$) to be an optimal solution, it is necessary and sufficient that there exist $x_\ell^i(t)$ and a set of multiplier functions $P_\ell^s(t)$, $W_r^h(t)$, $R_\ell^i(t)$, $Q_j(t)$, $P_{i\ell}(t)$ and $P_\ell(t)$ (i = 1, 2, 3, s, m, j = 1, 2, 3, m, h = 1, 2, 3, $r \in L$, $\ell \in NC$, $t \in [t_0, \infty)$) which satisfy the following set of conditions:

(i) building rental market equilibrium conditions

residential houses: h = 1, 2, 3

$$R_\ell^h(t) = \max \{\max_r \Psi_\ell^h[r, P_\ell^s(t), W_r^h(t), \bar{U}_h(t), t] + Q_h(t), 0\},$$

$$R_\ell^h(t) = \Psi_\ell^h[r, P_\ell^s(t), W_r^h(t), \bar{U}_h(t), t] + Q_h(t) \text{ if } y_{\ell r}^h(t) > 0,$$

$$\sum_r y^h_{\ell r}(t) \leqq x^h_\ell(t),$$

$$\sum_r y^h_{\ell r}(t) = x^h_\ell(t) \quad \text{if} \quad R^h_\ell(t) > 0,$$

buildings for local service production

$$R^s_\ell(t) = \max \{\Psi^s_\ell[P^s_\ell(t), \{W^h_\ell(t)\}_h, \pi_s = 0, t], 0\},$$

$$R^s_\ell(t) = \Psi^s_\ell[P^s_\ell(t), \{W^h_\ell(t)\}_h, \pi_s = 0, t] \quad \text{if} \quad N^s_\ell(t) > 0,$$

$$N^s_\ell(t) \leqq x^s_\ell(t),$$

$$N^s_\ell(t) = x^s_\ell(t) \quad \text{if} \quad R^s_\ell(t) > 0,$$

buildings for manufacturing firms

$$R^m_\ell(t) = \max \{\Psi^m_\ell[\{W^h_\ell(t)\}_h, \bar{\pi}_m(t), t] + Q_m(t), 0\},$$

$$R^m_\ell(t) = \Psi^m_\ell[\{W^h_\ell(t)\}_h, \bar{\pi}_m(t), t] + Q_m(t) \quad \text{if} \quad N^m_\ell(t) > 0,$$

$$N^m_\ell(t) \leqq x^m_\ell(t),$$

$$N^m_\ell(t) = x^m_\ell(t) \quad \text{if} \quad R^m_\ell(t) > 0,$$

(ii) labor market equilibrium conditions

(ii-1) commuting pattern equilibrium condition

$$\Psi^h_\ell[r, P^s_\ell(t), W^h_r(t), \bar{U}_h(t), t] = \max_r \Psi^h_\ell[r, P^s_\ell(t), W^h_r(t), \bar{U}_h(t), t]$$

$$\text{if } y^h_{\ell r}(t) > 0,$$

(ii-2) local labor market equilibrium conditions

$$\left.\begin{aligned} &N_r^h(t) \leqq \sum_\ell y_{\ell r}^h(t), \qquad W_r^h(t) \geqq 0 \\ &N_r^h(t) = \sum_\ell y_{\ell r}^h(t) \quad \text{if} \quad W_r^h(t) > 0 \end{aligned}\right\} \quad \begin{aligned} &r \in C \\ &h = 1, 2, 3, \end{aligned}$$

$$\left.\begin{aligned} &N_r^s(t)E_s^h + N_r^m(t)E_m^h \leqq \sum_\ell y_{\ell r}^h(t) \\ &N_r^s(t)E_s^h + N_r^m(t)E_m^h = \sum_\ell y_{\ell r}^h(t) \quad \text{if} \quad W_r^h(t) > 0 \end{aligned}\right\} \quad \begin{aligned} &r \in NC \\ &h = 2, 3, \end{aligned}$$

(iii) local service market equilibrium conditions

$$\sum_{r,h} y_{\ell r}^h(t) \leqq N_\ell^s(t), \quad P_\ell^s(t) \geqq 0,$$

$$\sum_{r,h} y_{\ell r}^h(t) = N_\ell^s(t) \quad \text{if} \quad P_\ell^s(t) > 0,$$

(iv) building construction market equilibrium conditions

$$\left.\begin{aligned} &P_{i\ell}(t) \leqq k_i P_\ell(t) + B_i(t) \\ &P_{i\ell}(t) = k_i P_\ell(t) + B_i(t) \quad \text{if} \quad u_\ell^i(t) > 0 \end{aligned}\right\} \quad i = 1, 2, 3, s, m,$$

(v) asset market equilibrium conditions

(v-1) building stock

$$\dot{P}_{i\ell}(t) = \gamma P_{i\ell}(t) - R_{i\ell}(t),$$

(v-2) land

$$\dot{P}_\ell(t) \leqq \gamma P_\ell(t) - R_A(t),$$

$$\dot{P}_\ell(t) = \gamma P_\ell(t) - R_A(t) \quad \text{if} \quad \sum_i k_i x_\ell^i(t) < L_\ell$$

(vi) variation of building stocks and land constraints

$$\dot{x}^i_\ell(t) = u^i_\ell(t),\ u^i_\ell(t) \geqq 0,$$

$$\sum_i k_i x^i_\ell(t) \leqq L_\ell$$

(vii) activity-unit number constraints

$$\sum_{\ell,r} y^h_{\ell r}(t) = N_h(t),\ h = 1,\ 2,\ 2$$

$$\sum_\ell N^m_\ell(t) = N_m(t),$$

$$\sum_{r\varepsilon C} N^h_r(t) \equiv N^h_C(t),\quad h = 1,\ 2,\ 3$$

(viii) transversality conditions

(viii-1) initial

$$x^i_\ell(t_0) = \bar{x}^i_\ell,$$

(viii-2) terminal

$$\lim_{t\to\infty} e^{-\gamma t} P_{i\ell}(t) = 0,\quad \lim_{t\to\infty} e^{-\gamma t} P_\ell(t) = 0.$$

The economic meaning of these optimality conditions becomes clear when we compare them with the equilibrium conditions for the associated competitive market problem. It is not difficult to show that these optimality conditions can be viewed as the equilibrium conditions in a competitive market through which the public authority tries to realize the optimal solution by using an appropriate income or profit subsidy (or tax) policy. Under this market interpretation of the above optimality conditions, the economic meanings of dual variables (i.e., multipliers) are as explained in the NOTATION.

NOTATION

Indices

ℓ,r: zone, district

C: the set of districts in CBD

NC: the set of districts outside CBD

L: the set of all districts

h: household type

h=1: households in H-apartments

h=2: households in apartments

h=3: households in flat houses

i: building type

i=1: H-apartments

i=2: apartments

i=3: flat houses

i=S: buildings to services

i=m: buildings for m-firms

t: time

Parameters

γ: time discount rate

$T^h_{\ell r}(t)$: monetary commuting cost per h-type household from ℓ to r.

$P_z(t)$: price of the composite consumption good

$P_z(t)\beta_h(t)$: disutility cost of commuting per distance for type h household

$T^m_\ell(t)$: average transport cost per manufactured goods in ℓ

k_i: lot size per building i

$R_A(t)$: agricultural land rent

$N^h_r(t)$: demand for labor type h in zone $r \varepsilon C$

$N^h_c(t)$: total number of type h households working in CBD

$N_m(t)$: total outputs of m-farms

E^h_j: labor-input coefficients, j = s,m

L_ℓ: area of district ℓ

U_n: utility level

π_s: profit level for s-firms per output

π_m: profit level for m-firms per output

$P_m(t)$: price of m-output

z: amount of composite consumption good

$y_n^h(t)$: non-wage income for h

Variables

$u_\ell^i(t)$: number of type i buildings constructed in zone ℓ at time t

$y_{\ell r}^h(t)$: number of type h households residing in zone ℓ and commuting to zone r

$N_\ell^s(t)$: output level of service in ℓ

$N_\ell^m(t)$: output level of m-firms in ℓ

Dual variables

$P_\ell^s(t)$: price of local service in zone ℓ

$W_r^h(t)$: wage for a type h worker in r

$R_\ell^i(t)$: rent for a building of type i in ℓ

$Q_i(t)$: income-subsidy or prifit-subsidy for a unit activity of type i (i = 1, 2, 3, m)

$P_{i\ell}(t)$: price of a type i building in ℓ

$P_\ell(t)$: price of land in ℓ

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