A MODEL FOR THE FOREST SECTOR

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SUMMARY

This report describes a dynamic linear programming model for studying long-range development alternatives for forestry and forest-based industries at the national and regional level. The Finnish forest sector is used to illustrate the process of implementation and to provide numerical examples. The model is composed of two subsystems, the forestry and the industrial subsystems, which are linked through the timber supply. The forestry submodel describes the development of the volume and age distribution of different tree species within the nation or regions concerned. The industrial submodel considers various production activities, such as the saw mill industry, the panel industry, and the pulp and paper industry, as well as secondary processing of primary products. For each individual product, alternative technologies may be employed. Thus, the production process is described by a small Leontief model with substitution. In addition to constraints related to the supply of timber and the demand for forest products, production is restricted through the availability of labor, production capacity, and financial resources. The production activities are grouped into financial units and investments are made within the financial resources of such units.

The model is designed for multicriteria analysis. Objective functions related to GNP, balance of payments, employment, wage income, stumpage earnings, and industrial profit have been formulated. End conditions are proposed, which can be determined through an optimal solution of a stationary model for the whole forest sector.

The structure of the integrated forestry–forest industry model is given in the canonical form of dynamic linear programs for which special solution techniques may be employed. A version of the Finnish forest sector model has been implemented using SESAME, an interactive mathematical programming system, and a number of numerical runs are presented to illustrate possible uses of the model.

FOREWORD

Dynamic linear programming models have been extensively used at IIASA to examine the long-term consequences of various economic policies and scenarios. Applications have included energy supply prospects, alternative paths for regional development, and the planned improvement of agricultural production. This report develops the methodology further and presents an application to the study of forestry and the forest-based industries; the Finnish forestry sector is used as an illustrative example.

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1. INTRODUCTION

As is the case with several other natural resources, wood is undergoing a transition from being an ample to a scarce resource in many regions of the world. In addition, the forest sector plays an important role in the economies of a number of countries. Therefore, long-term policy analysis for the forest sector, i.e. forestry and the forest industries, is becoming an important issue for those countries.

We concentrate here on two basic approaches to the analysis of the long-range development of the forest sector: simulation and optimization. Simulation techniques (e.g. system dynamics) help us to understand and quantify basic relationships influencing the development of the forest sector (see Randers 1976, Jeger *et al.* 1978, Seppälä *et al.* 1980). Using a simulation technique we can evaluate the consequences of a specific policy. However, if *only* simulation is used it can be difficult to identify an "appropriate" (or in some sense efficient) policy. This is because the forest sector is in fact a large-scale dynamic system, calling for policies that satisfy a large number of conditions and requirements. To solve this problem we need an optimization technique, or more specifically, multiobjective optimization. Because of the complexity of the system in question, linear programming (Dantzig 1963) is considered to be the most appropriate technique. It is worth noting that the optimization technique itself should be used on some sort of simulation basis; i.e. different numerical runs based on different assumptions and objective functions should be carried out to help in the selection of an appropriate policy. Specific applications of such an approach to the planning of an integrated system of forestry and forest industries have been reported, for instance, by Jackson (1974) and Barros and Weintraub (1979).

Because of the nature of forest growth, any model used must necessarily be dynamic. In this report we consider a dynamic linear programming (DLP) model for the forest sector. Using this approach the overall planning horizon (e.g. a 50-year period) is partitioned into a finite number of shorter periods (e.g. of five years each); for each of these shorter periods we consider a static linear programming model. A dynamic linear program thus comprises a number of static models, interlinked via various state variables (i.e. different types of "inventories," such as the amount of wood in the forests, production capacity, assets, liabilities, etc., at the end of a given period are equal to those at the beginning of the following period).

In our forest-sector model, each static model comprises two basic submodels: a forestry submodel and an industrial submodel of production, marketing and financing. The forestry submodel also describes ecological and land availability constraints for the forest, as well as labor and machinery constraints on harvesting and planting activities. The industrial submodel is a small input–output model of both mechanical (e.g. sawmill and plywood) and chemical (e.g. pulp and paper) production activities. Secondary processing of the primary products is also included in this submodel, particularly because of the expected importance of such activities in the future.

The rate of production is constrained by wood supply (which is one of the major links between the submodels), by final demand for forest products, by labor supply, by the availability of production capacity, and finally, by financial considerations.

The evaluation of alternative policies for the forest sector is very much a multiobjective procedure: while selecting a reasonable long-term policy, the preferences or needs of different interest groups (such as government, industry, labor, and forest owners) have to be taken simultaneously into account. It should also be noted that the forestry and forest industry submodels have different transient times: a forest normally requires a growing period of at least 40–60 years, whereas a major structural change in the forest industry may occur within a much shorter period. Because of the complexity of the system, it is sometimes desirable first to consider forestry and the forest industries fairly independently, and only then to analyze an integrated model (see Kallio *et al.* 1979).

It is also important to take into account all the uncertainties that can affect the forest and related sectors over such a long period. A stochastic model might at first seem to be more appropriate for this purpose. However, the high input-data requirements of such a model and the difficulties of applying stochastic solution techniques led us to the conclusion that the dynamic linear programming approach suggested would be more realistic.

This report consists of two main parts. In the first (Sections 2–4), we describe the methodological approach followed. In the second (Section 5), we present a specific implementation for the Finnish forest sector and illustrate this with several hypothetical, numerical examples.

2. THE FORESTRY SUBSYSTEM

Mathematical programming has been widely applied in forestry for operations management and planning (e.g. Wardle 1965, Navon 1971, Dantzig 1974, Newnham 1975, Williams 1976, Kilkki *et al.* 1977). In this section we follow a traditional method of formulating forest tree population as a dynamic linear programming (DLP) system. We describe the forestry submodel, where the decision variables (control activities) are harvesting and planting activities, and where the state of the forests is represented by the volume of trees (m^3) in different species and age groups. Because the model is formulated in the DLP framework, we will examine in detail the following elements: (i) state equations, which describe the development of the system, (ii) constraints, which restrict the feasible trajectories for forest development, and (iii) the planning horizon.

2.1. State Equations

Each tree in the forest is assigned to a class specifying its age and species. A tree belongs to age group a ($a = 1,2,\ldots,N-1$) if its age is at least $(a-1)\,\Delta$ but less than $a\,\Delta$, where Δ is a given time interval (for example, five years). In the highest age group, $a = N$, all trees are included that have an age of at least $(N-1)\,\Delta$. (Instead of age groups, we might alternatively assign trees to size groups specified by diameter or volume.) We denote by $w_{sa}(t)$ the number of trees of species s (e.g. pine, spruce, birch, etc.) in age group a at the beginning of period t ($t = 0,1,\ldots,T$). Index s might also be extended to refer to region, soil class, or forest management strategy.

Let $\alpha^s_{aa'}(t)$ denote the proportion of trees of species s and age group a that will proceed to age group a' during period t. We shall consider a model formulation where the length of each period is Δ. Therefore, we may assume that $\alpha^s_{aa'}(t)$ is independent of t and equal to zero, unless a' is equal to $a+1$ (or a for the highest age group). We then denote $\alpha^s_{aa'}(t) = \alpha^s_a$ with $0 \le \alpha^s_a \le 1$. The ratio $1 - \alpha^s_a$ may be viewed as the attrition rate corresponding to time interval Δ and species s in age group a. We introduce a subvector $w_s(t) = \{w_{sa}(t)\}$, specifying the age distribution of trees (number of trees) for each species s at the beginning of period t. Assuming neither harvesting nor planting take place at the end of the period, the age distribution of trees at the beginning of the next period, $t+1$, will then be given by $\alpha^s w_s(t)$, where α^s is the square $N \times N$ growth matrix, which describes the removals due to thinning and death of trees from natural causes. From our definition, this takes the form

$$\alpha^p = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \alpha^s_1 & 0 & \cdots & 0 & 0 \\ 0 & \alpha^s_2 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \alpha^s_{N-1} & \alpha^s_N \end{bmatrix}$$

Introducing a vector $w(t) = \{w_s(t)\} = \{w_{sa}(t)\}$, describing tree species and age distribution, and a block-diagonal matrix α with submatrices α^s on its diagonal, the species and age distribution at the beginning of period $t+1$ will be given by $\alpha w(t)$.

We denote by $u^+(t)$ and $u^-(t)$ the vectors of planting and final harvesting activities for period t. The state equation describing the development of the forest will then be

$$w(t+1) = \alpha w(t) + \eta u^+(t) - \omega u^-(t) \tag{1}$$

where matrices η and ω specify activities in such a way that $\eta u^+(t)$ and $-\omega u^-(t)$ are the incremental changes in numbers of trees resulting from planting and harvesting activities, respectively.

It should be noted that the growth rate of the trees is in fact dependent upon the number of trees growing per unit area. Hence, instead of (1) we should really use a nonlinear equation, but this would complicate the problem considerably. Therefore, it seems more practical to use the linear approximation (1) with some average figures in the matrix α. It is possible to check these figures after the solution has been obtained and to update them if necessary.

A planting activity n may be specified to mean the planting of one tree of species s that enters the first age group ($a = 1$) during period t. Thus, matrix η has one unit column vector for each species. The nonzero element of such a column falls in the row of the first age group for species s in equation (1).

A harvesting activity h is specified by variables $u_h^-(t)$, which determine the level of this activity. The coefficients ω_{ah}^s of matrix ω are defined so that $\omega_{ah}^s u_h^-(t)$ is the number of trees of species s from age group a harvested when activity h is applied at level $u_h^-(t)$. Thus, these coefficients show the age and species distribution of trees harvested when activity h occurs.

2.2. Constraints

Land. Let $H(t)$ be the vector of total acreage of the different soil types d available for forests at period t. Let G_{ad}^s be the area of soil type d required by one tree of species s and age group a. We assume that each species uses only one type of soil; i.e. only one of the elements G_{ad}^s, $d = 1,2,\ldots$, is nonzero. Thus, if we consider more than one land type, then the index s may also refer to the soil. Defining the matrix $G = (G_{ad}^s)$, we have the land availability restriction

$$Gw(t) \le H(t) \tag{2}$$

In this formulation we assume that the land area $H(t)$ is exogenously given. Alternatively, we may endogenize vector $H(t)$ by introducing activities and a state equation for changing the areas of different types of land. Such a formulation would be justified if changes in soil type over time were considered or if other land-intensive activities, such as agriculture, were included in the model.

Besides straightforward land availability constraints, specific requirements for land allocation (such as preserving the forest as a watershed or recreational area) may be stated in the form of inequality (2). In such cases (the negative of) a component of $H(t)$ would define a lower bound on such an allocation, while the left-hand side would yield (the negative of) the land allocated in a solution of the model.

Sometimes constraints on land availability may be given in the form of equalities requiring that all land made available through harvesting in a given period be used during the same period for planting new trees of a type appropriate for the soil. Forest laws in many countries even require explicitly that this type of procedure be followed.

Labor and Other Resources. Harvesting and planting activities require resources such as machinery and labor. Let $R_{gn}^+(t)$ and $R_{gh}^-(t)$ be the usage of resource g at unit levels of planting activity n and harvesting activity h, respectively. Defining the matrices $R^+(t) = \{R^+_{gn}(t)\}$ and $R^-(t) = \{R^-_{gh}(t)\}$, and also the vector $R(t) = \{R_g(t)\}$ of available resources during period t, we may write the resource availability constraint as follows

$$R^+(t)\,u^+(t) + R^-(t)\,u^-(t) \le R(t) \tag{3}$$

A more satisfactory way of representing resource availability is to use supply functions for which piecewise-linear approximations may be employed.

Wood Supply. The requirements for wood supplied by forestry to the forest industries can be given in the form

$$Vw(t) + S(t)u^-(t) = y(t) \tag{4}$$

where vector $y(t) = \{y_k(t)\}$ specifies the requirements for different timber assortments k (e.g. pine logs, spruce pulpwood, etc.); matrix $S(t)$ transforms quantities of harvested trees of different species and ages into the volumes of different timber assortments, while $Vw(t)$ accounts for thinning activities. Note that the volume of any given tree being harvested is assigned in (4) between logs and pulpwood in a proportion that depends on the species and age group of the tree. For each species, several classes of log size may be specified. The possibility of using any size of log as pulpwood may also be included in the model: in this way the size distinction between logs and pulpwood actually becomes endogenous.

2.3. Planning Horizon

The forest system has a very long transient time: one complete rotation of the trees in the forest may, in extreme conditions, require more than one hundred years. Naturally, various uncertainties make it difficult to plan for such a long time horizon. On the other hand, if the planning horizon chosen is too short, it is impossible to take into account all the longer-term consequences of activities implemented at the beginning of the planning horizon. In order to set an end condition for the forests, i.e. a condition to be attained by the end of the period or planning horizon in question, we shall employ the standard concept of sustainable yield.

To analyze a stationary regime for the forests, we set $w(t+1) = w(t) = w$, for all t. The state equation (1) can then be restated as

$$w = \alpha w + \eta u^+ - \omega u^- \tag{5}$$

Imposing constraints (2) through (4) on variables w, u^+, and u^-, we can solve a static linear programming problem to find an optimal stationary state w^* of the forest (and corresponding harvesting and planting activities). The solution of a dynamic linear program with end constraints

$$w(T) = w^* \tag{6}$$

yields the optimal transition to this sustainable yield state.

Another way of introducing sustainable yield is to consider an infinite-period formulation and to impose constraints $w(t) = w(t+1)$, $u^-(t) = u^-(t+1)$, and $u^+ = u^+(t+1)$, for all $t \ge T$. If the model parameters for period t are assumed independent of time for all $t \ge T$, then the dynamic, infinite-horizon, linear programming model may be formulated as a $(T+1)$-period problem, where the last period represents a stationary solution for periods $t \geq T$ and the first T periods represent the transition from the initial state to the stationary solution.

There is a certain difference between these two approaches to handling the stationary state. In the first, we initially find the optimal stationary solution independently of the transition period, and thereafter we determine the optimal transition to this stationary state. In the second approach, we link the transition period with the period corresponding to the stationary solution. The linkage takes place in the stationary-state variables, which are determined in an optimal way taking into account both periods simultaneously.

3. THE INDUSTRIAL SUBSYSTEM

We will now consider the industrial subsystem of the forest sector. Again the formulation used is that of a dynamic linear programming model. We discuss first the section related to the production of and final demand for forest products, after which we move to financial considerations.

3.1. Production and Demand

Let $x(t)$ be the vector (levels) of production activities i for period t, for $t = 0,1,\ldots,T-1$. Such activities may include the production of sawnwood, panels, pulp, paper, converted products, etc. For each single product j there may exist several alternative production activities i, which may be specified in terms of alternative uses of raw material, technology, etc. Let U be the matrix of wood usage per unit of production activity, so that the wood processed by industries during period t is given by vector $Ux(t)$. Note that matrix U has one row for each timber assortment k (corresponding to the components of the timber-supply vector $y(t)$ in the forestry model). Some of the elements of U may be negative. For instance, saw milling consumes logs but produces raw material (industrial residuals) for pulp mills; this by-product then appears as a negative component in matrix U. We denote by $r(t) = \{r_k(t)\}$ the vector of wood raw-material inventories at the beginning of period t (i.e. wood harvested but not yet processed by industry). As above, let $y(t)$ be the amount of wood harvested in different timber assortments, and $z^+(t)$ and $z^-(t)$, respectively, the (vectors of) import and export of different assortments of wood during period t. Then we have the following state equation for the wood raw-material inventory

$$r(t+1) = r(t) + y(t) - Ux(t) + z^+(t) - z^-(t) \tag{7}$$

In other words, the wood inventory at the *end* of period t is the inventory at the *beginning* of that period plus wood harvested and imported less wood consumed and exported *during* that period. Note that if there is no storage (change), and neither import nor export of wood, then (7) reduces to $y(t) = Ux(t)$; i.e. wood harvested equals wood consumed. For imports and exports of wood we assume upper limits $Z^+(t)$ and $Z^-(t)$, respectively

$$z^+(t) \leq Z^+(t) \quad \text{and} \quad z^-(t) \leq Z^-(t) \tag{8}$$

The production process may be described by a simple input–output model with substitution. Let $A(t)$ be an input–output matrix having one row for each product j and one column for each production activity i, so that $A(t)x(t)$ is the vector of net production when production activity levels are given by $x(t)$. Let $m(t) = \{m_j(t)\}$ and $e(t) = \{e_j(t)\}$ be the vectors of import from and export to the forest sector, respectively, for products j. Then, ignoring any possible changes in the product inventory, we have

$$A(t)x(t) + m(t) - e(t) = 0 \tag{9}$$

Domestic consumption is included in $e(t)$. For exports and imports we assume externally given bounds, $E(t)$ and $M(t)$, respectively

$$e(t) \leq E(t) \tag{10}$$

$$m(t) \leq M(t) \tag{11}$$

Such constraints may be substituted by piecewise-linear export and import functions in the model. The same applies to trade in timber.

Production activities are further restricted through labor and mill capacities. Let $L(t)$ be the vector of different types of labor available for the forest industries. Labor may be classified in different ways taking into account, for instance, types of production and types of responsibility in the production process (e.g. work force, management, etc.). Let $\rho(t)$ be a coefficient matrix such that $\rho(t)x(t)$ is the vector of demand for different types of labor given production activity levels $x(t)$. Thus we have

$$\rho(t)x(t) \leq L(t) \tag{12}$$

Again, such a constraint may be substituted by a piecewise-linear supply function for labor.

We consider the production (mill) capacity here as an endogenous state variable. Let $q(t)$ be the vector of the amount of different types of such capacity at the beginning of period t. Such types may be distinguished by region (where the capacity is located), by product group for which the capacity is used, and by the different technologies required or available to produce a given product. Let $Q(t)$ be a coefficient matrix such that $Q(t)x(t)$ is the vector of demand for these types of capacity. Such a matrix has nonzero elements only when the region—product—technology combination of a certain production activity matches that of the type of capacity available. The production capacity restriction is then given as

$$Q(t)x(t) \leq q(t) \tag{13}$$

The development of the capacity over time is given by a state equation

$$q(t+1) = (I - \delta)q(t) + v(t) \tag{14}$$

where δ is a diagonal matrix accounting for (physical) depreciation and $v(t)$ is a vector of investments (in physical units). Capacity expansions are restricted through financial resources. We do not consider possible constraints arising from other sectors, such as machinery or construction, whose capacity may be employed in forest-sector investment.

3.2. Finance

We now turn our attention to the financial aspects. We partition the set of production activities i into financial units (so that each activity belongs uniquely to one financial unit). Furthermore, we assume that each production capacity is assigned to a financial unit so that each production activity employs only capacities assigned to the same financial unit as the activity itself.

Production capacity in (14) is given in physical units. For financial calculations (such as determining taxation) we define a vector $\bar{q}(t)$ of fixed assets. Each component of this vector determines fixed assets (in monetary units) for a financial unit related to the capacity assigned to that unit. Thus, fixed assets are aggregated according to the grouping of production activities into financial units, for instance, by region, by industry, or by group of industries.

Financial and physical depreciation may differ from one another, for instance when the former is specified by law. We define a diagonal matrix $(I - \bar{\sigma}(t))$ such that $(I - \bar{\sigma}(t))\bar{q}(t)$ is the vector of fixed assets left at the end of period t when investments are not taken into account. Let $K(t)$ be a matrix in which each component determines the increase in fixed assets of a certain financial unit per (physical) unit of an investment activity. Thus the components of vector $K(t)v(t)$ determine the increase in fixed assets (in monetary units) for the financial units when investment activities are applied (in physical units) at a level determined by vector $v(t)$. Then we have the following state equation for fixed assets

$$\bar{q}(t+1) = (I - \bar{\sigma}(t))\bar{q}(t) + K(t)v(t) \tag{15}$$

For each financial unit we consider external financing (long-term debt) as an endogenous state variable. Let $l(t)$ be the vector of initial balance of external financing for different financial units in period t. In this notation, the state equation for long-term debt is as follows

$$l(t+1) = l(t) + l^{+}(t) - l^{-}(t) \tag{16}$$

We restrict the total long-term debt through a measure that may be considered as the market value of a financial unit. This measure is a given percentage of the total assets, less short-term liabilities. Let $\mu(t)$ be a diagonal matrix of such percentages, and let $b(t)$ be the endogenous vector of total stockholders' equity (including cumulative profit and stock). Then the upper limit on loans is given as

$$(I - \mu(t))\,l(t) \le \mu(t)\,b(t) \tag{17}$$

Alternatively, external financing may be limited, for instance, to a percentage of a theoretical annual revenue (based on available production capacity and on assumed prices of products). Note that no repayment schedule has been introduced in our formulation, because an increase in repayment can always be compensated by an increase of withdrawals in the state equation (16).

Next we will consider the profit (or loss) over period t. Let $p^{+}(t)$ and $p^{-}(t)$ be vectors whose components indicate profits and losses, respectively, for the financial units. By definition, profit and loss cannot be simultaneously nonzero for any financial unit. For any given solution of the model, this condition is usually fulfilled through the choice of objective function.

Let $P(t)$ be a matrix of prices for products (with one column for each product and one row for each financial unit) such that the vector of revenue (for different financial units) from sales $e(t)$ outside the forest industry is given by $P(t)e(t)$. Let $C(t)$ be a matrix of direct unit production costs, including, for instance, timber, energy, and direct labor costs. Each row of $C(t)$ refers to a financial unit and each column to a production activity. The vector of direct production costs for financial units is then given by $C(t)x(t)$.

The fixed production costs may be assumed proportional to the (physical) production capacity. We define a matrix $F(x)$ such that the vector $F(t)q(t)$ yields the fixed costs over period t for each of the financial units. According to the notation introduced above, (financial) depreciation is given by the vector $\bar{\sigma}(t)\bar{q}(t)$. We assume that interest is paid on the initial balance of debt. Thus, if $\varepsilon(t)$ is the diagonal matrix of interest rates, then the vector of interest paid (by the financial units) is given by $\varepsilon(t)l(t)$. Finally, let $D(t)$ be a vector of exogenously given cash expenditure covering all other costs. Then the profit (loss) before tax is given as follows

$$p^+(t) - p^-(t) = P(t)e(t) - C(t)x(t) - F(t)q(t) \\ - \bar{\sigma}(t)\bar{q}(t) - \varepsilon(t)l(t) - D(t) \tag{18}$$

The stockholder equity $b(t)$, which was introduced above, now satisfies the following state equation

$$b(t+1) = b(t) + (I - \tau(t))p^+(t) + B(t) \tag{19}$$

where $\tau(t)$ is a diagonal matrix for taxation and $B(t)$ is the (exogenously given) amount of stock issued during period t.

Finally, we consider cash (and receivables) for each financial unit. Let $c(t)$ be the vector of cash at the beginning of period t. The change in each during period t is due to the profit (loss) after tax, depreciation (i.e. noncash expenditure), debts incurred, repayments, and investments. Thus we assume that the possible change in cash due to changes in accounts receivable, in inventories (wood, end products, etc.), and in accounts payable cancel each other out (or that these quantities remain unchanged during the period). Alternatively, such changes could be taken into account by assuming, for instance, that the accounts payable and receivable, and the inventories, are proportional to the annual sales of each financial unit.

Using our earlier notation, the state equation for cash is now

$$c(t+1) = c(t) + (I - \tau(t))p^+(t) - p^-(t) + \bar{\sigma}(t)\bar{q}(t) \\ + l^+(t) - l^-(t) - K(t)v(t) + B(t) \tag{20}$$

3.3. Initial and Final State Conditions

In our industrial model, we now have the following state vectors: wood raw material inventory $r(t)$, (physical) production capacity $q(t)$, fixed assets $\bar{q}(t)$, long-term debt $l(t)$, cash $c(t)$, and total stockholders' equity $b(t)$. For each of them we have an initial value and possibly a limit on the final value. We shall refer to these initial and final values by superscripts 0 and *, respectively; thus we write the given initial state as

$$\left.\begin{array}{lll} r(0) = r^0\,, & q(0) = q^0\,, & \bar{q}(0) = \bar{q}^0 \\ l(0) = l^0\,, & c(0) = c^0\,, & b(0) = b^0 \\ r(T) \geq r^*\,, & q(T) \geq q^*\,, & \bar{q}(T) \geq \bar{q}^* \end{array}\right\} \tag{21}$$

$$l(T) \leq l^*\,, \quad c(T) \geq c^* \tag{22}$$

The initial state is determined by the state of the forest industries at the beginning of the planning horizon. The final state may be determined as a stationary solution, in a manner similar to that described above for the forestry model.

4. THE INTEGRATED SYSTEM

We now consider the integrated forestry–forest industries model. First we present a general discussion on possible formulations of objective functions for such a model. Then we summarize the model in the canonical form of dynamic linear programming. A tabular representation of the structure of the integrated model is also given.

4.1. Objectives

The forest sector may be viewed as a system controlled by several interest groups or parties. Any given party may have several objectives that are in conflict with each other. Furthermore, the objectives of any one party may clearly be in conflict with those of another party. For instance, the following parties might reasonably be taken into account: representatives of industry, government, labor, and the forest owners. Among the objectives of industry, a major concern may be the development of the profitability of different financial units. Government may be interested in the contribution of the forest sector to gross national product, the balance of payments, and employment. The labor unions are interested in employment and total wages earned in forestry and in different industries within the sector. The objectives of the forest owners may be to increase their income from selling and harvesting wood. Such objectives refer to different periods t (within the overall planning horizon) and possibly also to different product lines. We will now give simple examples of how such objectives may be formulated as linear objective functions. Nonlinear objectives may also be employed.

Industrial Profit. The vector of profits for the industrial financial units was defined above as $(I - \tau(t))p^+(t) - p^-(t)$ for each period t. If we wanted to distinguish between different financial units, then actually each component of such a vector could be considered as an objective function. However, for practical purposes we frequently aggregate such objectives, for instance summing discounted profits over all periods, summing over financial units, or summing over both periods *and* financial units.

Contribution to Gross National Product. To define the contribution of the forest sector to GNP, we consider the sector as a "profit center," where no wage is paid to the employees within the sector, where no price is paid for raw materials originating from this sector, and where neither interest nor taxes are paid. The contribution to GNP is then simply the profit for such a unit.

Contribution to the Balance of Payments. The contribution of the forest sector to the balance of payments may be expressed by an amended version of the expression for GNP. The necessary changes are as follows: first, multiply the components of the price vector by the share of exports in total sales; second, multiply the components of the cost vectors by the share of imported inputs in each cost term; third, multiply by the share in foreign debts (among all long-term debts) of the financial unit concerned; and finally, replace the depreciation function by investment expenditures on imported goods.

Employment. Total employment (in man-years) for each period t, for different types of labor in different activities and regions, has already been included in the left-hand sides of inequalities (3) and (12).

Wage Income. For each subgroup of the work force, the wage income for period t is obtained by multiplying the expressions for employment (given above) by the average annual salary within each such subgroup.

Stumpage Earnings. Besides the wage income for forestry (which we already defined above), and an aggregate profit (as expressed in (6)), we may also wish to take into account stumpage earnings, i.e. the income related to the price of wood prior to harvesting. This income may be readily obtained for a given timber assortment if the components of the harvest yield vector $y(t)$ are multiplied by the respective prices of the different types of wood.

4.2. The Integrated Model

The integrated forestry–forest industry model is illustrated in Figure 1. The interactions between the forestry and industrial subsystems occur via roundwood consumption. Both subsystems also interact with the general economy (which in our model is exogenous). This latter subsystem determines the supply (in terms of prices and quantities) of capital, labor, energy, and land. Domestic consumption and export demand (import supply) are also determined by the general economy subsystem.

We will now summarize the integrated forestry–forest industry model in the canonical form of dynamic linear programming (see e.g. Propoi and Krivonozhko 1978). Denote by $X(t)$ the vector of all state variables (defined above) at the beginning of period t. The components of this vector include the number of trees of each type in the forest, different types of production capacity in the industry, wood inventories, external financing, etc. Let $Y(t)$ be the nonnegative vector of all control activities for period t, that is, the vector of all decision variables, such as levels of harvesting or production activities. An upper bound vector for $Y(t)$ is denoted by $\hat{Y}(t)$ (some of whose components may be infinite). We assume that the objective function to be maximized is a linear function of the state vectors $X(t)$ and the control vectors $Y(t)$, and we denote by $\gamma(t)$ and $\lambda(t)$ the coefficient vectors for $X(t)$ and $Y(t)$, respectively, for such an objective function. This function may be, for instance, a linear combination of the objectives defined above. For multicriteria analysis $\gamma(t)$ and $\lambda(t)$ are matrices. The initial state $X(0)$ is denoted by X^0, and the final requirement for $X(T)$ by X^*. Let $\Gamma(t)$ and $\Lambda(t)$ be the coefficient matrices for $X(t)$ and $Y(t)$, respectively, and let $\xi(t)$ be the exogenous

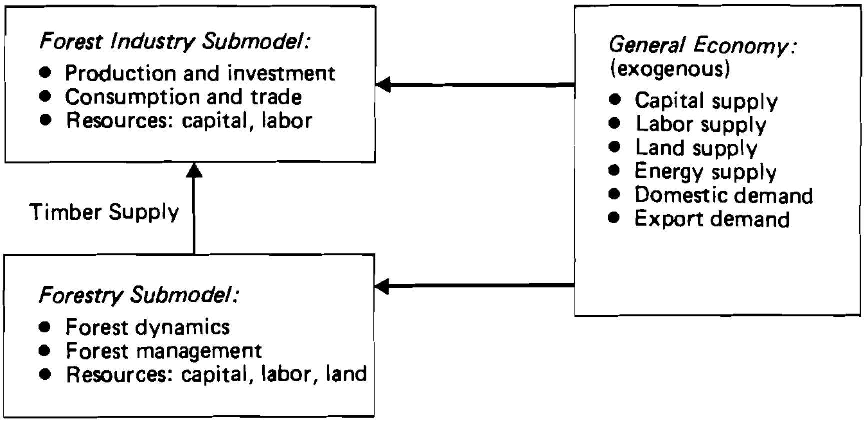

FIGURE 1 Integration of forest-sector submodels.

right-hand side vector in the state equation for $X(t)$. Let $\Phi(t)$ and $\Omega(t)$ be the corresponding matrices and $\psi(t)$ the right-hand side vector for the constraints. Then the integrated model can be stated in the canonical form of dynamic linear programming as follows:

find $Y(t)$ for $0 \le t \le T-1$, and $X(t)$ for $1 \le t \le T$, such as to

$$\text{maximize} \sum_{t=0}^{T-1} (\gamma(t)X(t) + \gamma(t)Y(t)) + \gamma(T)X(T)$$

subject to

$$X(t+1) = \Gamma(t)X(t) + \Lambda(t)Y(t) + \xi(t) \qquad \text{for } 0 \le t \le T-1$$

$$\Phi(t)X(t) + \Omega(t)Y(t) \mathrel{\hat{=}} \psi(t) \qquad \text{for } 0 \le t \le T-1$$

$$0 \le X(t), \quad 0 \le Y(t) \le \hat{Y}(t) \qquad \text{for all } t$$

with the initial state

$$X(0) = X^0$$

and the final requirement

$$X(T) \mathrel{\hat{=}} X^*$$

The notation $\hat{=}$ for the constraints and the final requirement refers either to $=$, to $\le$, or to $\ge$, separately for each constraint. The coefficient matrix (corresponding to variables $X(t)$, $Y(t)$, and $X(t+1)$) and the right-hand side vector of the integrated forestry–forest industry submodel of period t are given as

$$\begin{bmatrix} -\Gamma(t) & -\Lambda(t) & I \\ \Phi(t) & \Omega(t) & 0 \end{bmatrix} \text{ and } \begin{bmatrix} \xi(t) \\ \psi(t) \end{bmatrix}$$

respectively. Their structure is illustrated in Figure 2, using the notation introduced in Sections 2 and 3.

	w	r	q	$\bar{q}$	ℓ	b	c	u^+	u^-	y	x	z^+	z^-	m	e	v	ℓ^+	ℓ^-	p^+	p^-	
	$-\alpha$							$-\eta$	ω												$= 0$
		$-I$								$-I$	U	$-I$	I								$= 0$
			$\delta - I$													$-I$					$= 0$
				$\bar{\delta} - I$												$-K$					$= 0$
					$-I$												$-I$	I			$= 0$
						$-I$													$\tau - I$		$= 0$
				$-\delta$			$-I$									K	$-I$	I	$\tau - I$	I	$= B$
	G																				$\leqslant H$
								R^+	R^-												$\leqslant R$
	V								S	$-I$											$= 0$
											A			I	$-I$						$= 0$
											ρ										$\leqslant L$
			$-I$								Q										$\leqslant 0$
					$\mu - I$											μ					$\geqslant -\mu B$
			F	$\bar{\delta}$	ϵ						C				$-P$				$+I$	$-I$	$= -D$
Upper bounds:												Z^+	Z^-	M	E						

FIGURE 2 The constraint matrix $\begin{bmatrix} -\Gamma(t) & -\Lambda(t) \\ \Phi(t) & \Omega(t) \end{bmatrix}$, the right-hand side vector $\begin{bmatrix} \xi(t) \\ \psi(t) \end{bmatrix}$, the state vector $X(t)$, the control vector $Y(t)$, and the upper bound vector $\bar{Y}(t)$ for the submodel of period t of the integrated model.

5. APPLICATION TO THE FINNISH FOREST SECTOR

5.1. Implementation

A version of the integrated model was implemented using an interactive mathematical programming system called SESAME (Orchard-Hays 1978). The model generator is written using SESAME's data management extension, called DATAMAT. Each particular model is specified by the data tableaux of the generator programs. The example described here was specifically designed for the Finnish forest sector. This model may have, at most, ten periods, each of which represents a five-year interval. The whole country is considered here as a single region. Table 1 shows the dimensions of the model.

TABLE 1 Dimensions of the Finnish forest-sector model.

Characteristic	Value
Number of periods[a]	10
Length of each period in years[a]	5
Number of regions	1
Number of tree species	1
Number of age groups for trees[a]	21
Harvesting activities[a]	2
Soil types	1
Harvesting and planting resources	1
Timber assortments	2
Production activities	7
Types of labor in industry	7
Types of production capacity	7
Number of financial units	1
Number of rows in a ten-period LP	520
Number of columns in a ten-period LP	612

[a]These values may be specified by the model data. The numbers show the actual values used in the Finnish model.

The seven product groups considered are sawnwood, panels, further processed (mechanical) wood products, mechanical pulp, chemical pulp, paper and board, and converted paper products. For each product group we specify a separate type of production capacity and labor force. All production has been aggregated into one financial unit. Just one type of tree represents all the tree species in the forests. The trees are classified into 21 age groups. Thus, the age increment being five years, the oldest group contains trees more than 100 years old. Two harvesting activities are possible within the model framework, and two timber assortments are considered – logs and pulpwood.

The data for the Finnish model were provided by the Finnish Forest Research Institute. They are partially based on the official forest statistics (Yearbook of Forest Statistics 1977/1978) published by the same institute. Validation runs (which eventually resulted in the current formulation of the system) were carried out by contrasting model solutions with experience gained in the earlier simulation study of the Finnish forest sector carried out by Seppälä *et al.* (1980).

5.2. Scenario Examples

For illustrative purposes we will now describe a few test runs. Most of the data used in these experiments correspond approximately to those for the Finnish forest sector. This is the case, for instance, with the initial state; i.e. numbers of trees in the forests, different types of production capacity, etc. More hypothetical scenarios were used, however, for certain key quantities, such as final demand, and price and cost development. Thus, the results obtained do not necessarily reflect any specific, real situation but are rather presented in order to illustrate possible uses of the model.

For each test run a ten times five-year period model was constructed. Labor constraints for both industry and forestry were relaxed. In this stage, just one activity for converted paper products was considered, while both imports and exports of roundwood were excluded. The assumed demand for wood products is given in Table 2. Mechanical pulp is assumed not to be exported. At the end of the planning horizon, we require that, in each age group, there exist at least 80 percent of the number of trees initially in those groups. For production capacity a similar final requirement was set at 50 percent. Initial production capacity is given in Table 3 and the initial age distribution of the trees is illustrated in Figure 3.

TABLE 2 Assumed annual demand for forest products.

Period	Sawn-wood ($10^6 m^3$)	Panels ($10^6 m^3$)	Chemical pulp (10^6 ton)	Paper and board (10^6 ton)	Converted paper (10^6 ton)
1980–84	7.0	1.7	1.2	4.8	0.5
1985–89	7.5	2.0	1.1	5.8	0.7
1990–94	8.0	2.2	1.0	7.0	0.9
1995–99	8.8	2.5	0.9	8.3	1.2
2000–04	9.3	2.8	0.8	9.8	1.6
2005–09	9.7	3.2	0.7	11.6	2.1
2010–14	10.2	3.6	0.7	13.2	2.9
2015–19	10.7	4.1	0.6	15.1	3.8
2020–24	11.2	4.6	0.6	17.1	5.1
2025–29	11.6	5.2	0.6	19.2	6.9

TABLE 3 Annual production capacity at the beginning of the planning horizon in 1980 and in 2010 according to Scenario A.

Product	1980	2010
Sawnwood ($10^6 m^3$)	7.0	10.2
Panels ($10^6 m^3$)	1.7	3.6
Mechanical pulp (10^6 ton)	2.2	1.9
Chemical pulp (10^6 ton)	4.0	4.3
Paper and board (10^6 ton)	6.2	6.2
Converted paper products (10^6 ton)	0.5	2.9

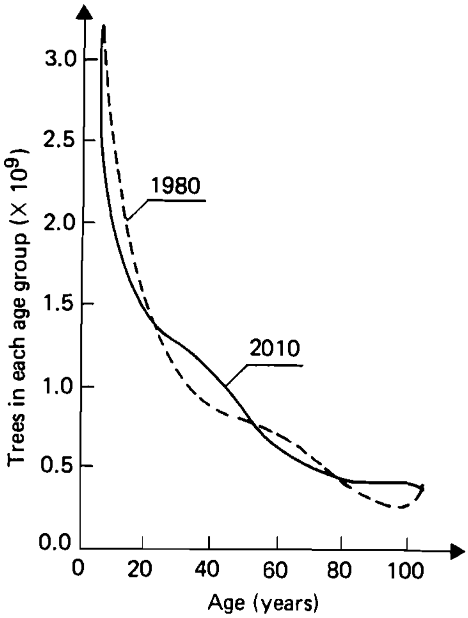

FIGURE 3 Age distribution of trees in 1980 and in 2010 according to Scenario B.

5.2.1. Scenario A: Base Scenario

For the first run the discounted sum of industrial profits (after tax) was chosen as an objective function. Such an objective should reflect at least approximately the forest industry's behavior in response to changes in cost structure and price developments. The resulting production trajectory is illustrated in Figure 4. Mechanical processing activities are limited almost exclusively to the assumed demand for sawnwood and panels. The same is true for converted paper products. However, chemical pulp produced is almost entirely used in paper mills, and therefore the potential demand for export has not been tapped. Neither have the possibilities for exporting paper been fully exploited. Paper exports decline sharply from an initial level of 5 million ton yr^{-1} and approach zero toward the end of the planning horizon. This is due to the strongly increasing production of converted paper products. The corresponding structural change in the production capacity of the forest industry over the 30-year period from 1980 to 2010 is shown in Table 3.

5.2.2. Scenario B: GNP Potential

For the second run we chose as an objective function the discounted sum of the contributions of the forest sector to gross national product. Compared with Scenario A, there is no significant difference in the production of sawnwood, panels, or converted paper products, for which export demand once again sets limits on production. However, there is a significant difference in pulp and paper production. Pulp is now produced to satisfy fully the demand for export. Paper production now steadily increases from 5 million ton yr^{-1} to nearly 9 million ton yr^{-1} by the end of the planning horizon. Paper exports still decline, again due to

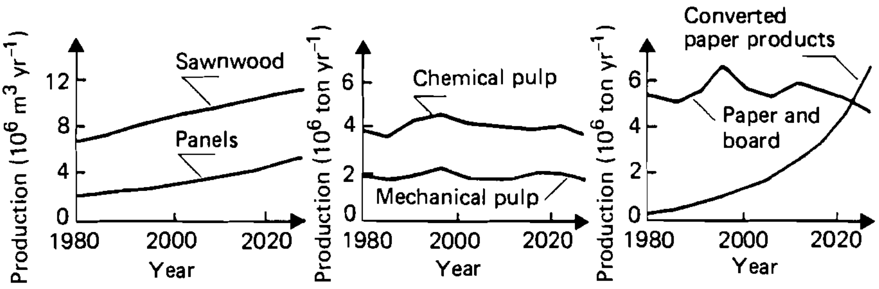

FIGURE 4 Production in the Finnish forest industry under Scenario A.

the increasing uses for converted paper products. Therefore, the export demand for paper is not fully exploited.

The bottleneck for paper production is now imposed by the biological capacity of the forests to supply wood. The annual use of roundwood increases from about 40 million m^3 to 65–70 million m^3 (see Figure 5a). The increase in the yield of the forests may be explained in terms of the change in the age structure of the forests during the planning horizon (see Figure 3 for the period up to 2010).

Notice that there is a significant difference in wood use between Scenarios A and B. In Scenario A (based on profit maximization), national wood resources are being used inefficiently; i.e. under the assumed price and cost structure, the poor profitability of the forest industry results in investment behavior that does not make full use of the forest resources.

5.2.3. Scenarios C and D: Variations in Product Price and Demand

The prices of forest products in Scenario C represent average world market prices. The prices used in Scenario A are 10% above this level (taking into account possible quality differences). Investment is now unprofitable, and therefore, under the profit-maximization criterion, production declines at the same rate as capacity depreciation. As illustrated in Figure 5a, by the year 2020 wood resource utilization has fallen to 50% of its level in 1980. The same is also true for GNP.

In Scenario D, demand for all forest products is double that of Scenario A. Because of poor profitability, only part of this demand potential is, however, exploited. The resulting wood consumption is shown in Figure 5a. Compared with Scenario A, it is about 10 percent higher.

5.2.4. Scenario E: Inflation

Next, we modified the base scenario by an eight-percent annual real inflation of all cost and price figures. This applies to production cost factors, such as wages, energy and wood costs, investment costs, and the rate of interest as well as to forest product prices. In this scenario, the inflation losses due to taxation and the gains due to a real reduction in loan repayments approximately outweigh each other, so that profit maximization results in roughly the same production figures as in Scenario A (see Figure 5b).

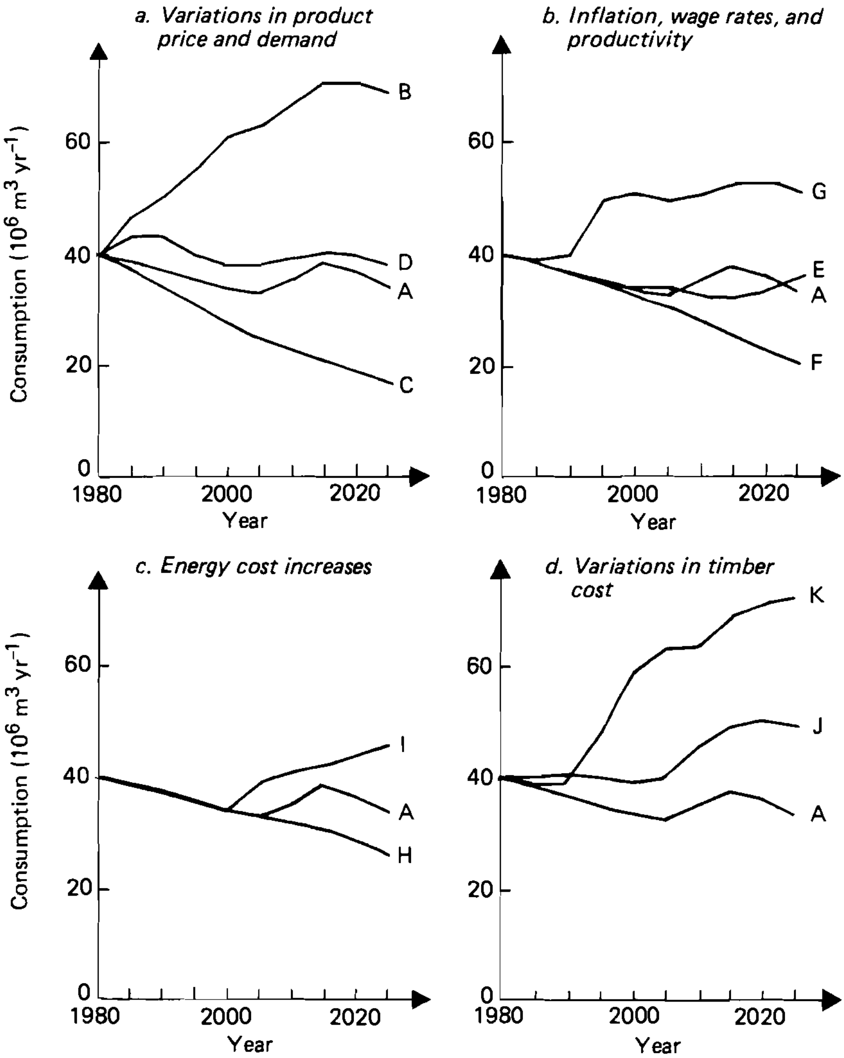

FIGURE 5 Industrial consumption of roundwood for Scenarios A–K.

5.2.5. Scenarios F and G: Wage Rates and Productivity

For Scenario F, an annual real increase of 2% was assumed in all wage costs. The near-term effect is minor in comparison with Scenario A, whereas over the longer term such an increase results in a significant reduction in production (see Figure 5b). On the other hand, a change in the opposite direction, i.e. a 2% annual increase in labor productivity, influences production significantly earlier. As shown by Scenario G in Figure 5b, wood consumption increases to a level of 50 million m^3 annually only 10–15 years after the beginning of the period studied.

5.2.6. Scenarios H and I: Energy Cost Increases

In the next two scenarios, the energy costs of Scenario A were increased by 2.7% annually (i.e. by 15% over five years). Scenario H, illustrated in Figure 5c, shows the resulting decrease in production, which is significant only in the long term. In Scenario I we assume, in addition, that the price of wood as a primary energy source increases by the same rate of 2.7% annually. Around the year 2000 the use of wood for energy becomes competitive with industrial wood processing and from then on the total consumption of wood starts to increase. The difference between Scenarios I and H in Figure 5c represents this use of wood as a source of energy.

5.2.7. Scenarios J and K: Variations in Timber Cost

Finally, two experiments were carried out to study the sensitivity of consumption to the price of wood. In Scenario J a 20% decrease was imposed as compared to the wood costs in Scenario A; in Scenario K the corresponding decrease was 2% annually. The impacts on wood consumption are shown in Figure 5d. The 20% decrease results in better profitability and consequently a steady and immediate growth of the industry. The annual decrease of 2% results in significant change only after 10 years. However, thereafter forest industry growth is fast and reaches the biological limits of the forests (i.e. 60–70 million m^3 of wood harvested annually) around the year 2000.

6. SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

We have formulated and presented a dynamic linear programming model of the forest sector. Such a model may be used to study long-range development alternatives for forestry and the forest-based industries at both national and regional levels. Our model comprises two subsystems, the forestry and the forest industry subsystems, which are linked together through the supply of roundwood from forestry to industry. We also have corresponding static, temporal submodels of each subsystem for each interval (e.g. each five-year period) considered within the planning horizon. The dynamic model is then composed of a series of these static submodels, coupled together through a number of inventory-type variables, i.e. state variables.

The forestry submodel describes the development over time of the volume and the age distribution of different tree species within a nation or its regions. Among the factors explicitly considered are the land available for timber production and the labor available for harvesting and planting activities. Ecological constraints, such as preserving land for use as a watershed, may also be taken into account.

In the industrial submodel we consider various production activities, such as saw milling, panel production, and pulp and paper milling, as well as the further processing of primary products. For each individual product, alternative production activities employing, for instance, different technologies, may be included. Thus, the production process is essentially described by a small Leontief model with substitution. For end-product demand an exogenously given upper limit is assumed. Some products, such as pulp, may also be "imported" into the forest sector for further processing. Apart from the biological limits on wood supply and the demand for wood-based products, production is restricted through the availability of labor, production capacity, and financial resources. The availability of different types of labor (by region) is assumed to be given. The development of different types of production capacity depends on the initial situation in the country and on investments, which are endogenous decisions within the model. The production activities are grouped into financial units to which the respective production capacities belong. Investments are made within the financial resources of each such unit. External finance is made available to each unit, up to a limit determined by the market value of that unit. Income tax is assumed to be proportional to the net income of each financial unit.

The structure of the integrated forestry–forest industry model is given in the canonical form of dynamic linear programs for which special solution techniques may be employed (see, for instance, Propoi and Krivonozhko 1978, Kallio and Orchard-Hays 1979). Objective functions related to gross national product, and employment and profit for the forest industry as well as for forestry itself, have been formulated. Final conditions (i.e. values for the state variables at the end of the planning horizon) have been proposed, and these are determined via an optimal solution of a stationary model for the forest sector.

The Finnish forest sector model has been implemented using the interactive mathematical programming system SESAME (Orchard-Hays 1978). It is a ten-period model, with each period five years in length. The complete model comprises 520 rows and 612 columns.

A number of numerical scenarios have been presented to illustrate possible uses of the model. Both the discounted industrial profit and the discounted contribution to GNP have been used as objective functions. The basic Scenario A illustrates a case where the internal wood price and wage structure results in rather poor profitability for the forest industries. This, in turn, leads to investment behavior that provides insufficient capacity for making full use of the wood resources. However, because somewhat hypothetical data were used for various key parameters, no conclusions should be drawn from these runs for the real case of Finland.

Several scenario runs have been presented to illustrate the use of the model for studying the influence of various demand and price assumptions concerning forest products, as well as the potential impact of a number of production cost factors.

The aim of this work has been to formulate, implement, and validate the Finnish forest-sector model. One natural continuation of the research would be to use the model for studying other important features of the forest sector. For instance, the influence of alternative scenarios regarding world market prices for wood products would be of great interest. Further studies could examine employment and wage-rate questions, labor availability restrictions and productivity, the effects of new technologies for harvesting and wood processing, the influence of inflation, exchange-rate changes, and alternative taxation schemes, competition for land use between forestry and agriculture, site improvement, ecological constraints, the use of wood as a source of energy, and so on. Given the required data, such studies could be carried out relatively easily.

Further research requiring a larger modeling effort might concentrate on regional economic aspects, on linking the forest-sector model to a national economic model for greater consistency, and on studying the inherent group decision problem underlying any attempts at controlling the development of the forest sector. The first of these three topics would require a complete revision of our model generation program and, of course, the regionalized data. The second task might be carried out by building into the existing model a simple input–output model for the entire economy, with the nonforest sectors aggregated into only a few sectors. Alternatively, our current model could be linked to an existing national economic model. Finally, the group decision problem might usefully be analyzed, for instance, using multicriteria optimization, based on the use of reference-point optimization (Wierzbicki 1979, Kallio *et al.* 1980).

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APPENDIX: MODEL NOTATION

Indices

a, a'	age group of trees in years $(1, 2, \ldots, N)$
d	type of forest land
g	type of resource for forestry activities
h	harvesting activity
i	production activity (in the forest industries)
j	industrial product
k	timber assortment
n	planting activity
s	tree species
t	period in years $(1, 2, \ldots, T)$

State and Control Variables

$b(t)$	stockholders' equity at the beginning of period t
$b^0 = b(0)$	initial level of stockholders' equity
$c(t)$	cash (and receivables) at the beginning of period t
$c^0 = c(0)$	initial amount of cash
c^*	final requirement for cash
$e(t) = \{e_j(t)\}$	export (and sales outside the forest sector) of forest products during period t
$l(t)$	balance of external financing at the beginning of period t
$l^0 = l(0)$	initial balance of external financing
l^*	final requirement for external financing
$l^+(t)$	debts incurred during period t
$l^-(t)$	repayments made during period t
$m(t) = \{m_j(t)\}$	import of forest products during period t
$p^+(t)$	profit during period t
$p^-(t)$	financial loss during period t
$q(t)$	production capacity at the beginning of period t
$q^0 = q(0)$	initial level of production capacity
q^*	final requirement for production capacity
$\bar{q}(t)$	fixed assets at the beginning of period t
$\bar{q}^0 = \bar{q}(0)$	initial value of fixed assets

$\bar{q}^*$	final requirement for fixed assets
$r(t) = \{r_k(t)\}$	timber assortments inventory at the beginning of period t
$r^0 = r(0)$	initial level of timber assortments inventory
r^*	final requirement for timber assortments inventory
$u^-(t) = \{u_h^-(t)\}$	level of harvesting activities during period t
u^-	level of harvesting in a stationary solution
$u^+(t) = \{u_n^+(t)\}$	level of planting activities during period t
u^+	level of planting in a stationary solution
$v(t)$	level of investments (in physical units) during period t
$w(t) = \{w_s(t)\} = \{w_{sa}(t)\}$	number of trees at the beginning of period t
$w^0 = w(0)$	initial number of trees
w^*	final requirement for the number of trees
w	number of trees in a stationary solution
$x(t)$	level of production activities during period t
$X(t)$	state vector at the beginning of period t
$X^0 = X(0)$	initial state
X^*	requirement for final state
$y(t) = \{y_k(t)\}$	supply of timber assortments during period t
$Y(t)$	level of control activities during period t
$z^+(t)$	import of timber assortments during period t
$z^-(t)$	export of timber assortments during period t

Parameters

$\alpha_{aa'}^s(t)$	proportion of trees of species s and age group a that will proceed to age group a' during period t
α, α^s	matrices of coefficients $\alpha_{aa'}^s(t)$
$\gamma(t)$	objective function coefficients for the state vector $X(t)$
$\Gamma(t)$	coefficient matrix for the state vector $X(t)$ in the state equation
σ	physical depreciation rates
$\bar{\sigma}(t)$	financial depreciation rates
Δ	age interval in an age group of trees (e.g. five years)
$\varepsilon(t)$	interest rates for external financing
$\psi(t)$	right-hand side vector of constraints for period t
$\Phi(t)$	coefficient matrix for the state vector $X(t)$ in constraints for period t
μ	matrix relating planting activities to the increase in the number of trees
$\lambda(t)$	objective function coefficients for the control vector $Y(t)$

$\Lambda(t)$	coefficient matrix for the control vector $Y(t)$ in the state equation
ω	matrix relating harvesting activities to the decrease in the number of trees
$\Omega(t)$	coefficient matrix for the control vector $Y(t)$ in constraints for period t
$\rho(t)$	labor requirement for different production activities
$\tau(t)$	tax factors for the forest industries during period t
$\mu(t)$	upper bound on external financing as a percentage of total assets less short-term liabilities
$\xi(t)$	right-hand side vector for the state equation of period t
$A(t)$	input–output matrix for the forest industries
$B(t)$	stock issued during period t
$C(t)$	variable production costs
$D(t)$	exogenously given costs
$E(t)$	upper bound on demand for forest products
$F(t)$	fixed costs (per unit of production capacity)
$G = (G_{ad}^{s})$	land requirement of each species in various age groups
$H(t)$	land available for forests
I	identity matrix
$K(t)$	investment costs per unit of capacity
$L(t)$	labor available for forest industries
$M(t)$	upper limit on import of forest products
N	number of age groups for trees
$P(t)$	prices of forest products
$Q(t)$	matrix of capacity requirements for production activities
$R(t) = \{R_g(t)\}$	resources available for forestry activities
$R^+(t) = \{R_{gn}^+(t)\}$	resource usage for planting activities
$R^-(t) = \{R_{gh}^-(t)\}$	resource usage for harvesting activities
$S(t)$	matrix transforming the numbers of trees harvested into volumes of timber assortments
T	number of periods
U	usage of timber assortments by various production activities
V	wood yield matrix for thinning activities

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