Contour Maps of Population Surfaces

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FOREWORD

At one time the limits of demographic research were set by the data available. Some results could be obtained by analysis of birth and death registrations and censuses, for the few countries for which these were available, and for the few breakdowns that were permitted by the tabulating equipment in existence. With so little data available, analysis by hand methods was perfectly feasible.

Now there are masses of data, and the problem has shifted: the due consideration and interpretation of data is a major difficulty. Under the guidance of James Vaupel and Anatoli Yashin methods have been developed that provide interpretation at a glance. In respect of Sweden, for instance, mortality data are available for some 85 ages for more than two centuries. To look at 20,000 numbers and draw out their meaning is a major research enterprise in itself. Yet on the methods used in this book all that information is contained in a single contour map.

Of course the method has antecedents, that go all the way back to Descartes' insight that mathematical functions are represented on a plane. From that it is only a step to showing empirical curves on a plane, and then to go on to three dimensions, as these contour maps do. Stages in this progression were diagrams by Perozzo and Lexis that show a population as lines on the age-time plane. Arthur and Vaupel (1984), building on Preston and Coale (1983), generalize the Lexis diagram to a Lexis surface.

The material that follows makes ingenious use of computation, and takes advantage of many antecedent forms of diagram. It is presented in this research report neither for its originality as a method of representation, nor for the exhibition of programming skill by Bradley Gambill. The drawings here appeal by reason of their substantive interest. Whether mortality improvement takes place by cohorts or by periods; at what historical periods has the fall in mortality been most striking; was the baby boom after World War II due to more births to women in the prime ages, say the twenties, or was it due to longer continuance of childbearing by women in their thirties? These and many other issues are raised in the text, but that text is suggestive rather than exhaustive, and the maps can well be used as a basis for further research.

Nathan Keyfitz
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SUMMARY

Contour maps are useful for displaying demographic surfaces, including surfaces of population levels and fertility, marriage, and mortality rates. Most often the surfaces are defined over age and time, but such other dimensions can be used as life expectancy or population growth rate. This paper presents a bouquet of contour maps to suggest the broad potential of their use in demographic studies. The maps presented range from maps of Italian mortality, French population levels, and U.S. birth rates, to maps of Coale and Demeny's and Brass's model life tables. The value of the maps lies in their substantive import: by giving demographers visual access to population surfaces, the maps can help demographers uncover and understand population patterns. The text of the paper adumbrates some of these patterns and discusses the use of contour maps in exploratory data analyses and model building, including the use of maps of residuals in fitting models to data.

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INTRODUCTION

Contour maps, which are widely used in depicting spatial patterns, can be readily adapted to represent any surface that is defined over two dimensions. In particular, an array of demographic data can often be pictured in an intelligible and graphically striking way by a contour map. The data might pertain to population levels or to rates of fertility, marriage, divorce, migration, morbidity, or mortality. Most often the data are structured by age and time--e.g., age-specific mortality rates over time--but in some cases other dimensions might be used, such as life expectancy or population growth rate.

Contour maps permit visualization of population surfaces and offer a panoramic view impossible to obtain from the usual graphs of levels or rates at selected ages over time or at selected times over age. Furthermore, a contour map is often superior to a three-dimensional perspective plot in providing a clear, yet rich representation of a demographic surface; it is usually difficult on a three-dimensional plot to discern the exact position of the surface above the age and time dimensions and three-dimensional plots become confusing if made too detailed, especially when displayed on a moderately-priced monitor or printer. Contour maps are particularly effective in highlighting patterns in the interaction of age, period, and cohort effects.

Contour maps have been used only occasionally by demographers, in part because of the computational effort required, in part because of the lack of detailed data over long stretches of age and time, and in part because the advantages of working with population surfaces have not been fully appreciated. In their influential study of changes in death rates over time, Kermack, McKendrick, and McKinlay (1934) superimpose on three of their tables some rough lines that are, in effect, contours of relative mortality. The pioneering study by Delaporte (1941) includes a set of contour maps that summarize mortality patterns in several European countries; Federici (1955) directs attention to Delaporte's contour maps in her survey of demographic methods.

Three recent developments should lead to greater use of demographic contour maps in the future. First, advances in computers, including the proliferation of inexpensive microcomputers, are providing demographers with convenient computational power. Second, extensive arrays of population statistics are being collected, published, and put on computer tapes and other storage devices; some of this data is by single year of age and time (e.g., Natale and Bernassola (1973), Vallin (1973), Heuser (1976, 1984) and Veys (1983)). Finally, there is growing recognition among demographers of the usefulness of population surfaces. Demographers have drawn three-dimensional representations of population densities at least since Perozzo (1880) and Lotka (1926 and 1931), but it was only recently that an article by Arthur and Vaupel (1984), building on important research by Bennett and Horiuchi (1981) and Preston and Coale (1982), focused attention on the conceptual and analytical advantages of population surfaces.

Arthur and Vaupel introduce the phrase "Lexis surface" to describe a surface of population densities and we will continue that usage here. Other kinds of surfaces will be called mortality surfaces, fertility surfaces, and so on, as appropriate, and we will interchangeably use the equivalent phrases "population surface" or "demographic surface" as generic terms to describe any kind of surface pertaining to some population parameter.

This paper presents a bouquet of contour maps to suggest the broad potential of their use in population studies. The maps are of interest to demographers not because the computer programming required considerable virtuosity and creativity, especially to make it all work in 64K of memory on an ordinary IBM PC, nor because the technique of contour mapping is new (the technique is widely used in other applications and was systematically employed by Delaporte (1941) to analyze mortality patterns). Rather, the value of the maps lies in their substantive import--they provide demographers with visual access to data that sheds light on significant patterns and trends in population levels, mortality rates, fertility rates, and other demographic parameters. In the following text we point to some of the more interesting substantive features of the maps, to suggest the worth of such maps in population studies.

Every picture presented deserves a thousand words or more of explanation and analysis and we hope that demographers will exploit the maps as a rich lode for research; a first step, in the analysis of Italian mortality, is taken by Caselli, Vaupel, and Yashin (1985). As a compilation of data, the demographic maps displayed in this paper might be compared with the demographic tables presented in Keyfitz and Flieger (1968) or Preston, Keyfitz and Schoen (1972), and we hope that the maps will prove to be useful complements to such tabular compendiums. The typical map in this paper is based on more than 6000 data points and there are about 70 distinct maps altogether, so that the maps collectively display roughly 400,000 values. Each page of tables in Keyfitz and Flieger contains about 625 statistics and there are about 650 pages of tables, amounting again to roughly 400,000 values. Similarly, each page of tables in Preston, Keyfitz, and Schoen contains about 575 figures, on average, and there are more than 700 tables, for a grand total, once again, of roughly 400,000 values.

THE EVOLUTION OF ITALIAN MALE MORTALITY

Figure 1a displays the contours of mortality for Italian males from age 0 to 79 and for years 1870 to 1979. The map is based on mortality rates, q, for single years of age and time assembled by Natale and Bernassola (1973) and Caselli (forthcoming)*. Thus the map, in the space of half a page or so, summarizes 80 times 110, or some 8800, mortality rates.

Data are discrete, but a surface is continuous: the surface $q(x,y)$ can be defined by linearly interpolating between adjacent data points. The values of $q(x,y)$ give the height of the mortality surface over age x and time y. The lines on a contour map connect adjacent points on a surface that are of equal height; these lines are sometimes called level lines or isograms. In Figure 1a, one of the level lines represents a mortality rate of about 11 percent: the line starts in 1870 at age 35 and ends in 1979 at age 56, indicating that 56-year-old Italian men in recent years faced the same chance of mortality that 35-year-olds faced about a century ago.

*These mortality rates are calculated on a cohort basis. That is, the data pertain to the probability that a person born in a particular calendar year and alive at exact age x will die before his or her $x+1$-st birthday. This type of probability refers to events that affect each single-year cohort at each age in two successive calendar years; for a discussion of this, see Vallin (1973) or Wunsch and Termote (1978). Standard calculations yield period life tables for two consecutive years from a diagonal reading of the cohort life tables. For our data, this allowed computation of period life tables from 1869-70 to 1978-79. For convenience in constructing mortality surfaces, we assumed that each $_1q$ calculated by this method described the height of the mortality surface at age x and a time y halfway through the two-year period which we took to be January 1, 1870 for the 1869-1870 period and so on. Thus on our maps of Italian mortality, and on other maps that we based on cohort rates, the point on the horizontal axis labeled 1890, say, refers to January 1, 1890. The mortality rate given at age 10, say, in 1890 is the mortality rate suffered by the cohort born not in 1880 but in 1879.

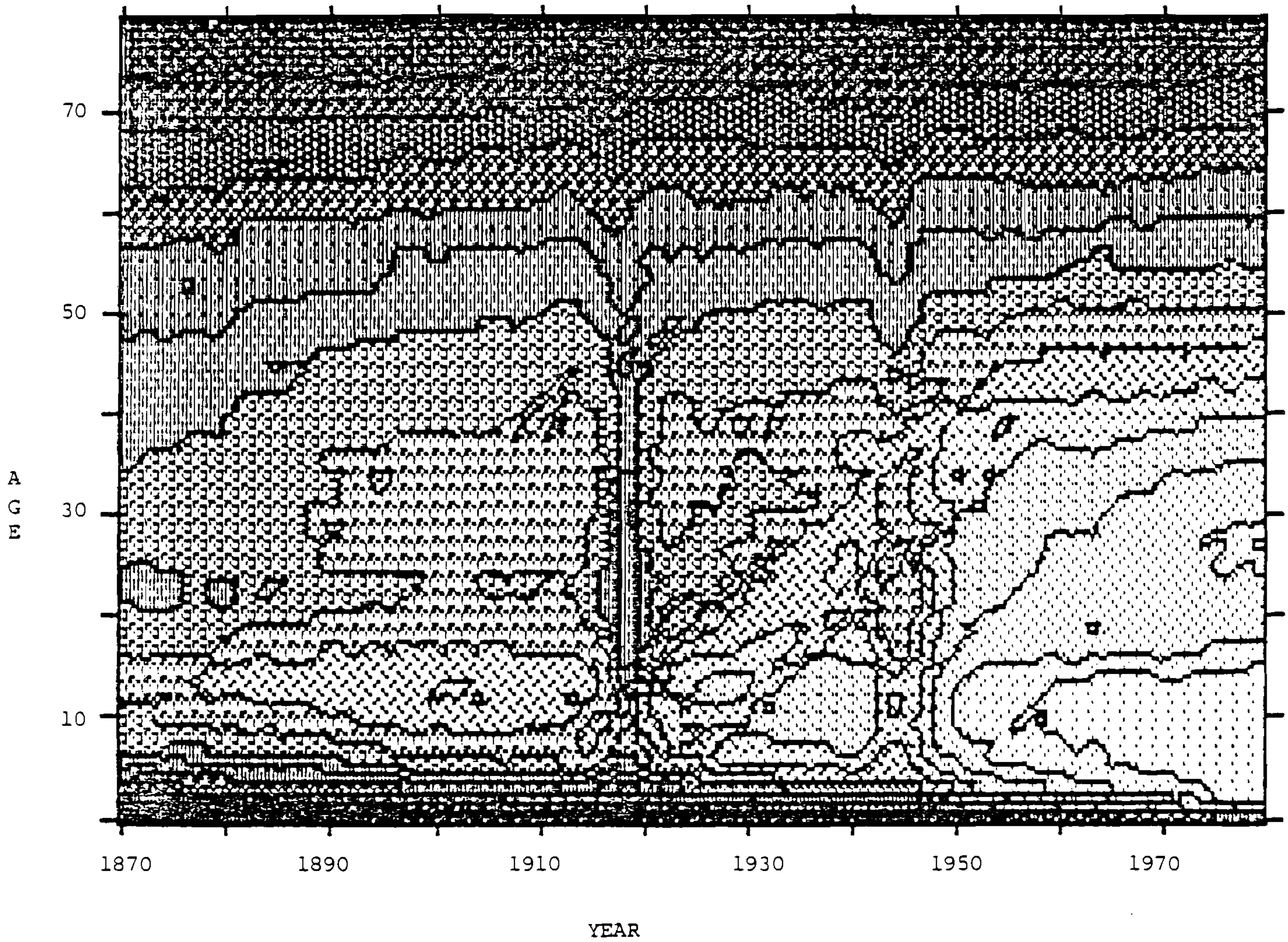
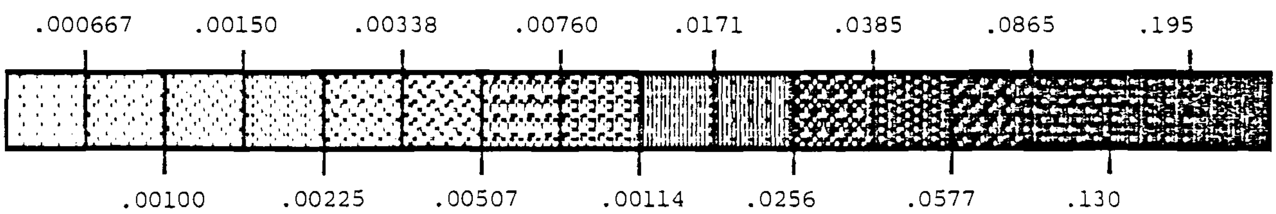


Figure 1a: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From age 0 to 79 and year 1870 to 1979



The major features of the evolution of Italian male mortality are apparent on the map. The devastation of World War I and the Spanish Influenza appears as a sharp ridge of high mortality that interrupts the map around 1918. A lower ridge shows the effects of World War II. The general pattern over time is one of progress against mortality, rapid at younger ages and slow at advanced ages. The general pattern over age is equally clear: high mortality in infancy and again among the elderly. The intriguing diagonal patterns suggest possible cohort effects, most notably during the 1920's and 1930's among the cohorts born around 1900: males in these cohorts, who were in their late teens and early twenties during World War I, may have been particularly debilitated by the war and its aftermath. Also notable are the various islands and peninsulas of high mortality that run across the map between ages 20 and 25: as discussed by Caselli, Vaupel, and Yashin (1985), these reflect various disruptive socio-economic and political events in Italian history as well as the tendency for younger men to engage in reckless behavior.

LEVELS, SHADES, AND GRIDS

An important consideration when designing a contour map is how many different levels to use. The computer program that we employed to draw the maps, which was developed by Gambill under Vaupel's direction at Duke University and at the International Institute for Applied Systems Analysis, allows lines to be drawn at up to 15 levels, separating the surface into 16 tiers. Use of fewer lines sacrifices detail, whereas use of more lines tends to make the map less intelligible: 15 levels is a reasonable compromise, although use of 10 or 20 levels might be considered. Delaporte draws lines at 19, 20, or 21 levels on his various maps of European mortality; a number of the figures in this paper, including Figures 15 and 16, use fewer than 15 levels. Figure 1b(i) presents the contours of Italian male mortality using 10 levels rather than the 15 levels used in Figure 1a.

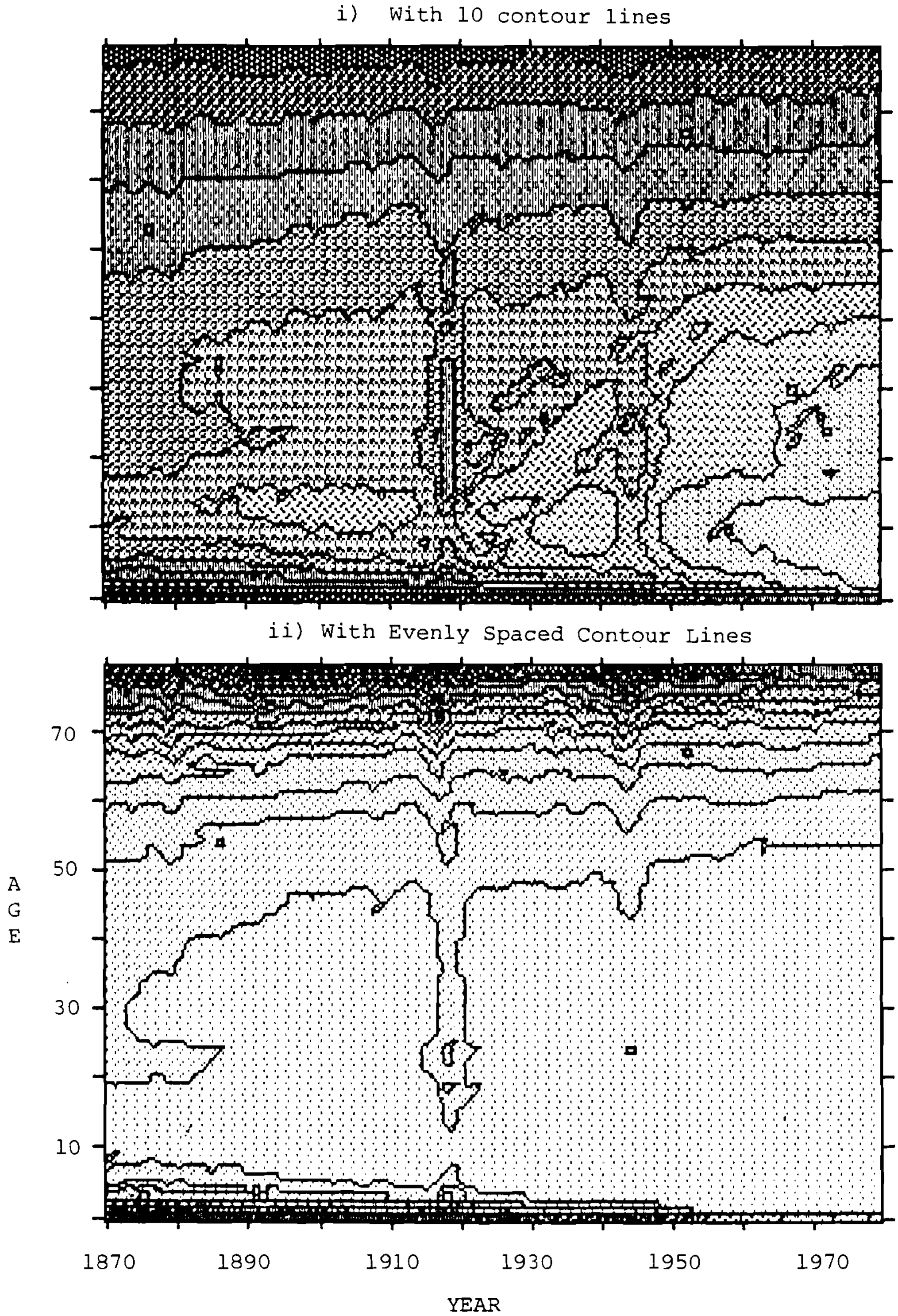
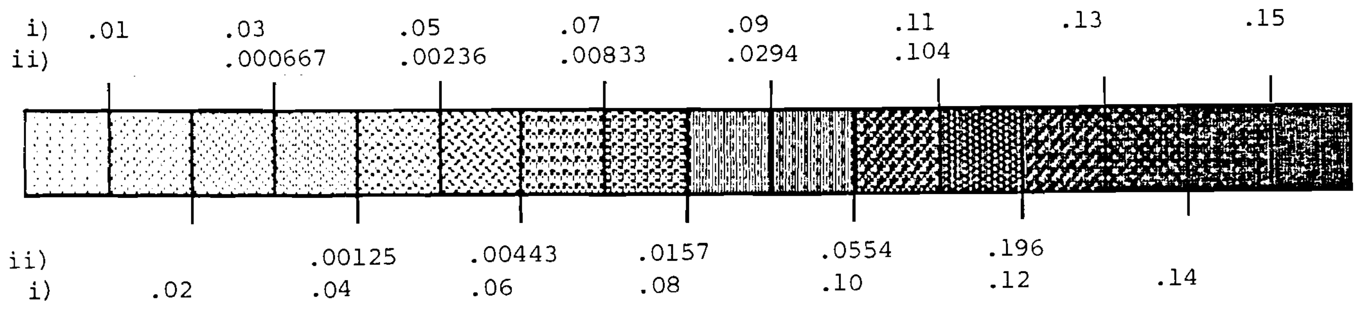


Figure 1b: Italian Male Mortality Rates - with differing contour lines
From age 0 to 79 and year 1870 to 1979



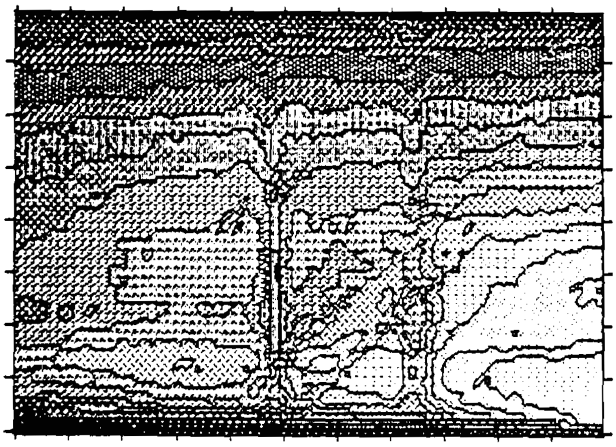
Which specific elevations the contour lines should connect is a second important design decision. On mortality surfaces, where mortality rates might approach a minimum of the order of magnitude of 0.0001 and a maximum of 1, use of equally spaced lines--say at 0.01, 0.02, and so on up to 0.15--results in a map where the contours are clumped together at the youngest and oldest ages, with a largely empty expanse in-between. Figure 1b(ii) illustrates this for Italian male mortality. The map is far more informative when the lines are spread out at constant multiples--e.g., each line representing a level 50 percent higher than the previous line, as in Figure 1a. Alternatively, a convenient scale can be used: Delaporte places his lines at levels of mortality of 1, 2, 3, ..., 9, 10, 12, 15, 20, 30, 50, 100, 150, 200, 250, 300, 350, and 400 per thousand, and in several figures in this paper, including Figures 5 and 17, level lines are selectively placed at convenient levels.

Demographers often work with transformations such as the log or logit, so it might seem reasonable to transform the surface $q(x,y)$ into the surface of, say, log $q(x,y)$ and then to draw level lines at equal intervals on the transformed surface. If the transformation is monotonic, like the log or logit transformation, an identical contour map can be drawn by spacing the level lines at appropriately unequal intervals on the original surface. In the case of logarithms, the level lines should be at multiples of each other rather than being equally spaced. Thus, the map in Figure 1a can also be interpreted as depicting log mortality rates.

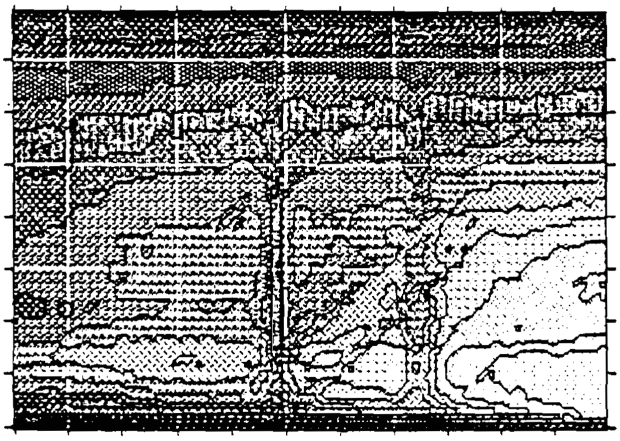
A key feature of the computer program we developed is the shading of regions according to the height of the surface. The shading varies from light to dark as the surfaces rise from low to high levels of mortality. Such shading, which is time-consuming to do by hand but easy with the help of a computer, makes the overall pattern of a mortality surface more immediately comprehensible, especially if the map is viewed at a distance. At the same time, the details of small peaks and pits and of the twists and turns of the contours lines are still there to be scrutinized at close range. Literature, critics note, can be profitably read at different levels of understanding; we suggest the reader try viewing Figure 1a and perhaps some of the other figures in this paper at levels of 25 centimeters and 5 meters.

Sometimes it is useful to draw a grid on a contour map so that the coordinates of various points can be conveniently located. In Figure 1c(ii) the map in Figure 1a is redrawn with a superimposed grid every twenty years of time and age; for ease of comparison the original map is also presented, at the same scale, in Figure 1c(i). The grid detracts a bit from the underlying pattern--that is the price of adding additional information. Grids are also included in Figures 2a and b, 18 and 28.

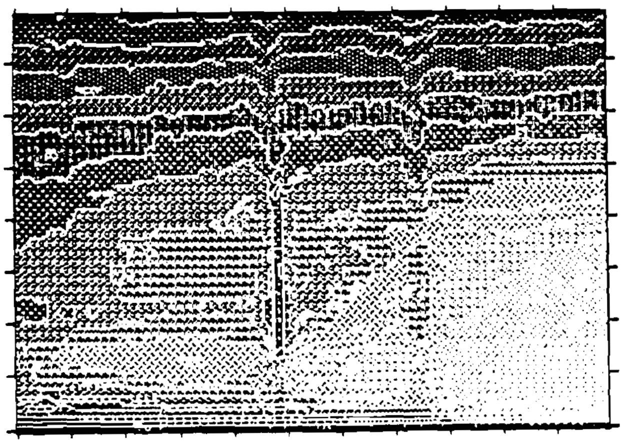
i) With Contour Lines and Shading



ii) With a Grid



iii) Without Contour Lines



iv) Without Shading

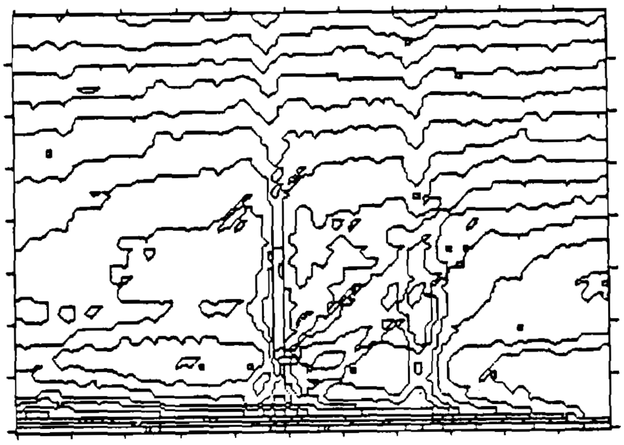
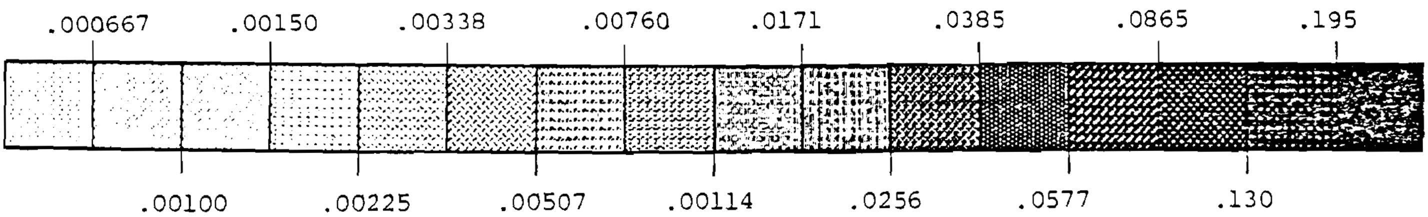


Figure 1c: Italian Male Mortality Rates - with contour lines from .000667 to 1.95 at multiples of 1.5



To see general trends it may be helpful to suppress the contour lines in a map of a population surface. In Figure 1c(iii) the map in Figure 1a is redrawn with shading but without lines. Alternatively, one could draw a traditional contour map with lines but without shading. Figure 1c(iv) displays such a map for Italian male mortality. The lines in this figure are not labeled but they could be.

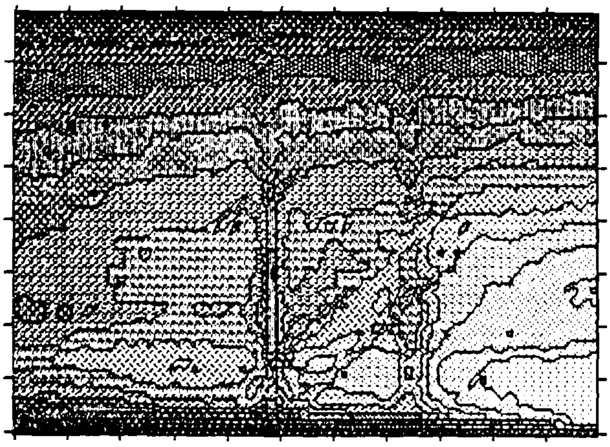
SMOOTHED MAPS

It is useful to take a close look at the small blemishes isolated from contour lines on a contour map, because these spots indicate outliers-- very localized peaks or pits-- that might be due to erroneous data values. Consider, for instance, the spot in Figure 1a at about age 53 and year 1877: it turned out that this blemish was indeed produced by an error made in transcribing the Italian mortality data to a computer tape. (The error was corrected, but we left the spot as an illustration). On the other hand, the mark at about age 19 in 1962 represents a point where the mortality surface barely crosses a contour level, like the top of a sea mount that appears as a small island just rising above the level of the surrounding ocean.

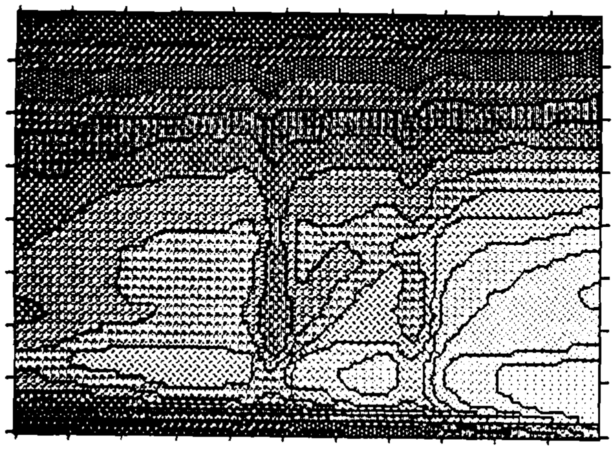
In addition to these blemishes, some cluttered areas appear in Figure 1a. These represent virtual plateaus where the mortality surface is repeatedly crossing and recrossing a level line, or cliffs where mortality rates are rising or falling rapidly. To reduce this kind of noise and to suppress the details of local fluctuations so that global patterns can be more clearly perceived, it may be useful to smooth a surface. Delaporte presented both raw and smoothed contour maps of mortality rates in various European countries: on his "adjusted" maps, Delaporte drew smooth contour lines based on his feeling for the data. We used a mechanistic, computer algorithm to produce the smoothed map shown in Figure 1d(ii); to aid comparison, Figure 1a is reproduced, at reduced scale, in Figure 1d(i). In the smoothed map the height of the surface at age x in year y was replaced by the average of the 25 heights in the 5 by 5 square of points from $x-2$ to $x+2$ and from $y-2$ to $y+2$. On the edges of the map, where a full 5 by 5 array of data points is not available, the smoothing procedure averages the available data.

Instead of smoothing by averaging over a 5 by 5 square, a larger (or smaller) square might be used. In Figure 1d(iii) Italian male mortality is smoothed on an 11 by 11 square. Global patterns in this map are somewhat clearer than in Figure 1d(ii) but some interesting local detail is lost and effects that are concentrated in

i) no smoothing



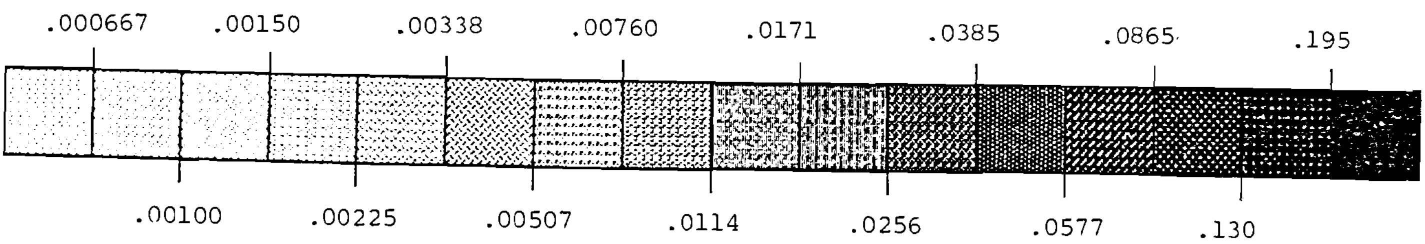
ii) smoothed on a 5 by 5 square



iii) smoothed on an 11 by 11 square

iv) smoothed on a weighted 5 by 5 square

Figure 1d: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5



time or age, such as infant mortality and mortality during the 1918 Spanish influenza epidemic, are smeared out.

A variety of alternative smoothing procedures might be used, including procedures that replace points by a weighted average of adjacent points, the weights diminishing with distance. Figure 1d(iv) presents a map of Italian male mortality smoothed by an algorithm in which the weights given to the points in a 5 by 5 square were proportional to:

1	4	6	4	1
4	16	24	16	4
6	24	36	24	6
4	16	24	16	4
1	4	6	4	1

Thus the points in the corners of the square were given weights of 1/256, whereas the point in the center received a weight of 36/256. The theoretical advantages of such weighted smoothing algorithms (see Tukey (1977) for an introductory discussion) have to be balanced against the conceptual simplicity and computational convenience of the kind of straightforward averaging illustrated in Figures 1d(ii) and (iii). Note that in Figure 1d(iv) the surface is reduced by two years along each edge because the smoothing procedure used requires a full 5 by 5 array of data; special modification of the procedure could be made, analogous to the modification of the smoothing procedure used to produce Figures 1d(ii) and (iii), so that a weighted smoothing could handle data points up to the edges of the surface.

CLOSE-UPS

As mentioned above and as discussed by Caselli, Vaupel and Yashin (1985), the patterns of male mortality in Italy from ages 10 to 49 for years 1910 to 1969 reveal some interesting cohort effects. Figures 2a and b present contour maps of this restricted age and time area: the maps can be considered enlargements or close-ups of a section of the map in Figure 1a. Thus contour maps can be used both to display a large data array and also to focus in on selected portions of the array. Note that because in Figure 2a the contours are at drawn at the same levels as in the original Figure 1a, only half of the possible levels are utilized. In Figure 2b, twice as many contours are drawn as in Figure 2a, every other contour in the

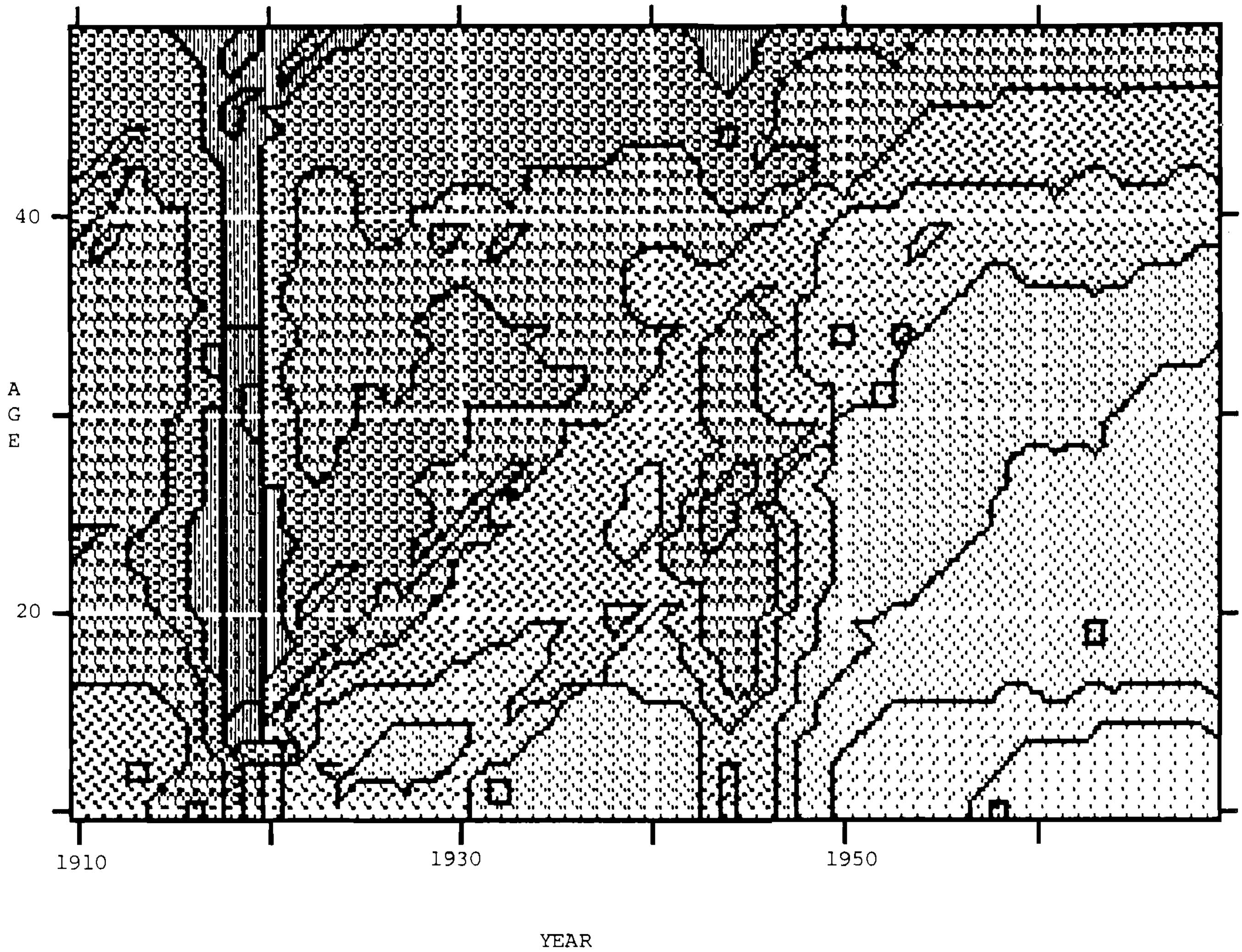
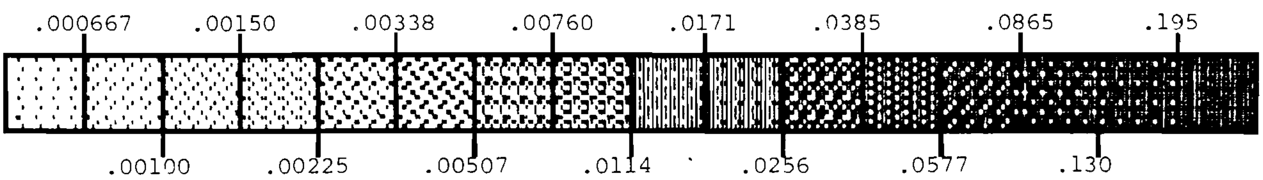


Figure 2a: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From age 10 to 49 and year 1910 to 1969



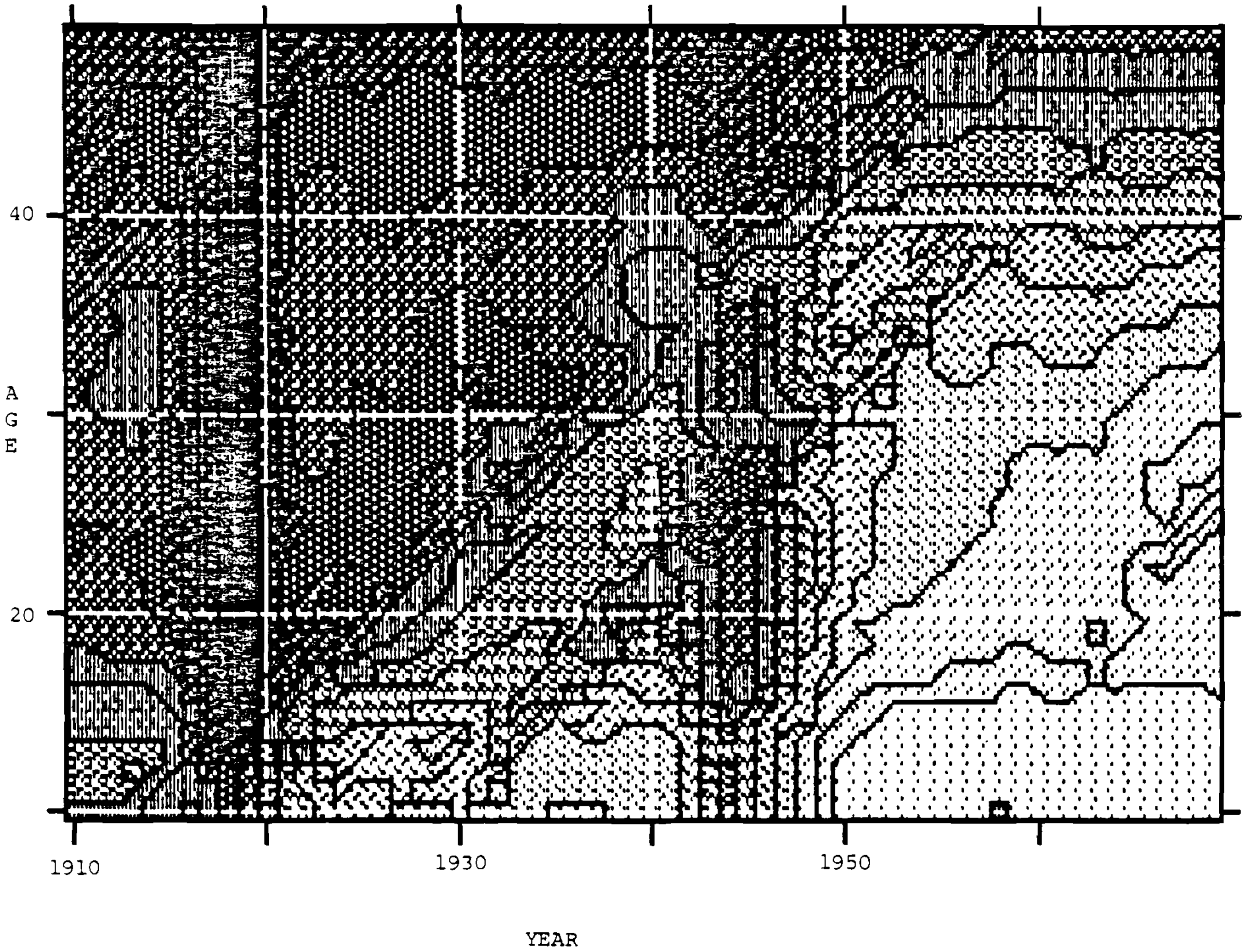
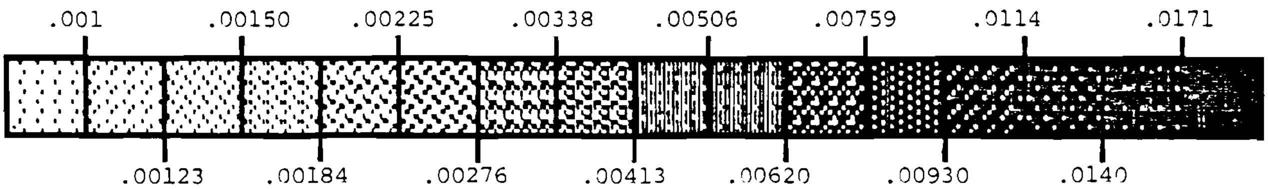


Figure 2b: Italian Male Mortality Rates - with contour lines from .001 to .0171
at multiples of the square root of 1.5
From age 10 to 49 and year 1910 to 1969



second figure being at the same level as a contour in the first figure. Part of the advantage of a close-up is that if the height of the surface varies less in the restricted region being scrutinized, then the level lines can be located at closer intervals to reveal more local detail.

Figures 2a and b make the diagonal patterns on Figure 1a more apparent; the added grid makes it clear that the patterns do indeed run along cohort lines. Consider one of the most striking differences on the map, the difference between the mortality rates suffered by the cohort born in 1903, which appears on the map at age 10 in the year labelled 1914, and the cohort born in 1908, which appears on the map at age 10 in the year labelled 1919. (Please see previous footnote for an explanation of why the year of birth is one less than the current year minus age.) These two cohort diagonals are separated at most ages between 10 and 30 by two contour levels, indicating a rough mortality differential of about 50 percent; after age 30 the discrepancy appears to sharply diminish and then disappear.

To explore this intriguing differential and similar differentials among nearby cohorts, we produced Figure 2c, which plots contour lines of mortality for the cohorts born between 1894 and 1924; these cohorts are followed from age 0 to 54. Note that in Figure 2c, year of birth runs along the horizontal axis, not current year. The figure reveals some strong cohort differences, especially among cohorts born between 1903 and 1909 and among cohorts born between 1917 and 1920. It is interesting to see how the period effects of World Wars I and II appear on this figure, as backward diagonals.

To more closely scrutinize the differences among the cohorts born between 1903 and 1909, we produced Figure 2d, which plots the contours of the mortality rates experienced by the 1904 to 1909 cohorts, from age 0 to age 54, relative to the mortality rates experienced by the 1903 cohort. Thus Figure 2d facilitates comparisons of the cohorts, with the 1903 cohort serving as the standard for comparison. In the case of the 1908 cohort, for instance, the map reveals that the 1903 cohort suffered substantially higher mortality at all ages except ages 8, 9, and 10— the 1903 cohort experienced these ages just prior to the First World War, whereas the 1908 cohort passed through these ages during the final years of the war and the Spanish Influenza epidemic. At ages 14 and 15, which the 1903 cohort lived through from 1917 to 1919, the 1903 cohort experienced more than five times the mortality experienced at these same ages by the 1908 cohort. Rough calculations indicate that between ages 16 and 34, mortality rates for the 1908 cohort averaged about 50 percent of those for the 1908 cohort, and between ages 35 and

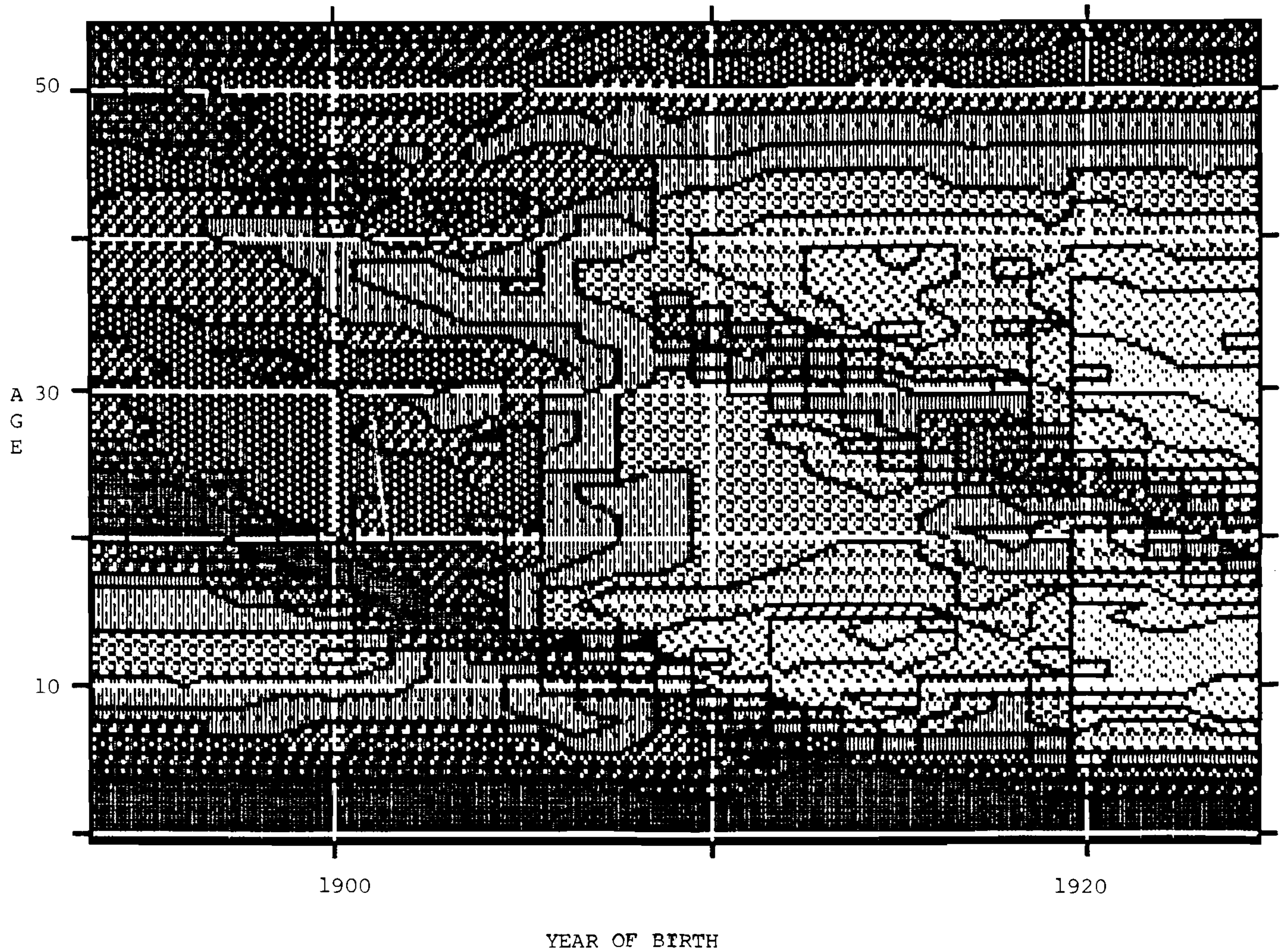
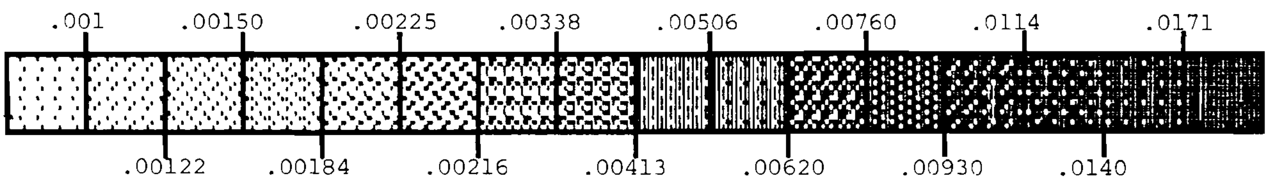


Figure 2c: Italian Male Cohort Mortality Rates - with contour lines from .001 to .0171 at multiples of the square root of 1.5 From age 0 to 54 and year of birth 1894 to 1924



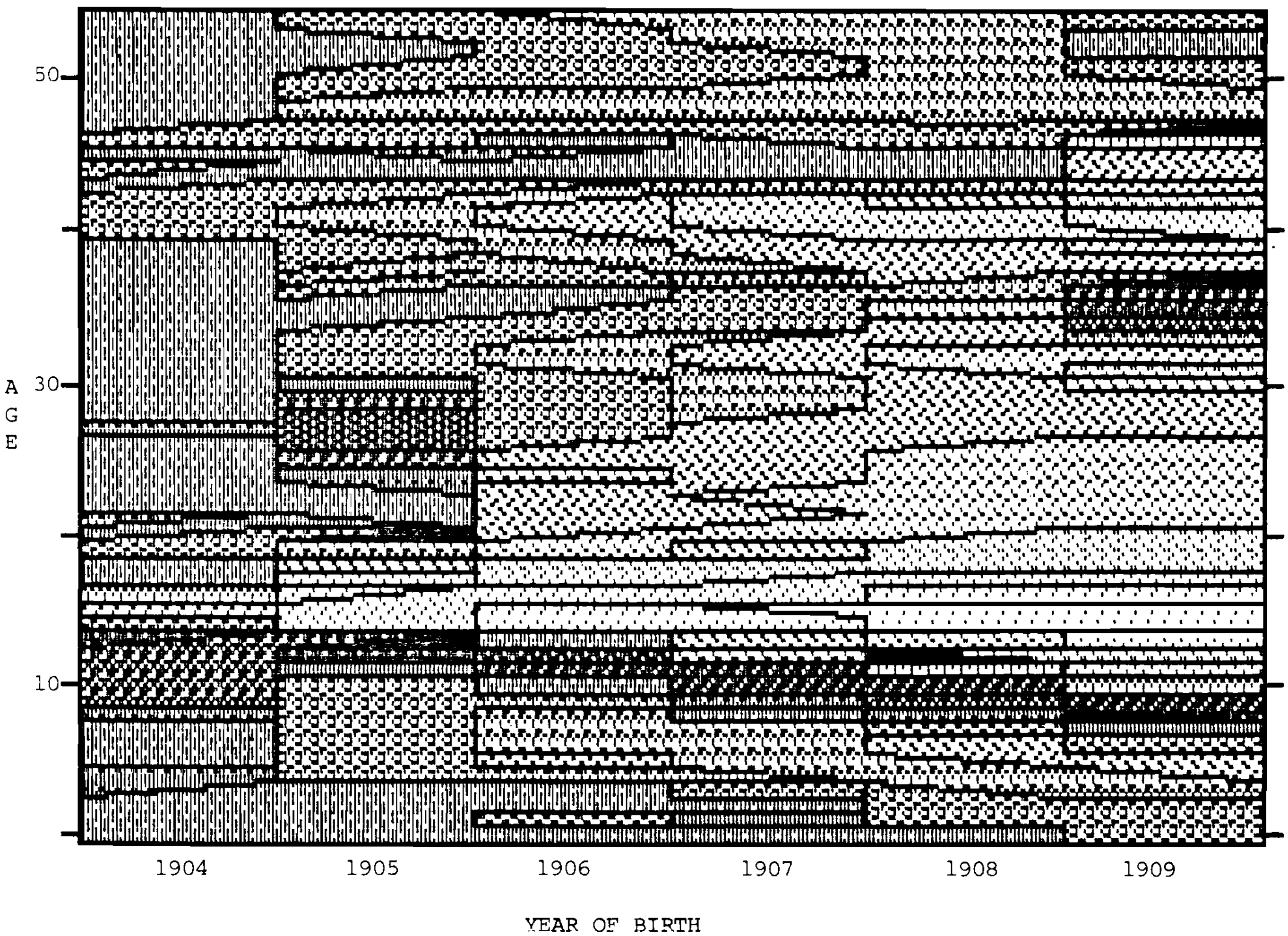
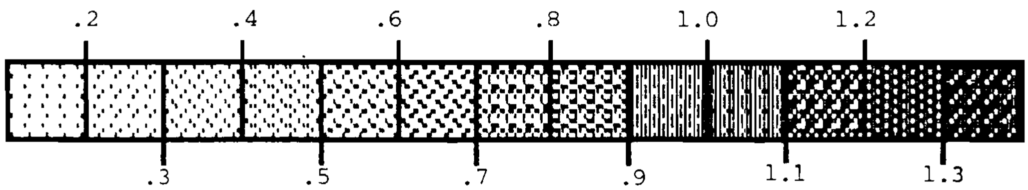


Figure 2d: Italian Male Cohort Mortality Relative to 1903 Cohort Age-Specific Levels - with contour lines from .2 to 1.3 at intervals of .1 From age 0 to 54 and year of birth 1904 to 1909



54 mortality rates were about 30 percent lower on average.

Thus the general tendency was for the risk differential to diminish, but local fluctuations complicate the pattern. Careful scrutiny of the details of the pattern of mortality differential between the 1903 and 1908 cohorts, and of similar differentials between other cohorts, might lead to better understanding of the interaction of age, period and cohort effects. More generally, our use of Figures 1a and 2a, b, c, and d, provides a suggestive illustration of the value of contour maps in exploratory analyses of population surfaces.

MAPS FROM INTERPOLATED DATA

The mortality rates for Italian males used in Figures 1 and 2 are available by single year of age and single year of time. Frequently demographers have to work with less finely-spaced data; mortality rates, for instance, may be available every decade or so, by five-year age classes. Figure 3 displays the evolution of Italian male mortality based on data published in Preston, Keyfitz, and Schoen (1972). Data sets from this source were available for 1881, 1891, 1901, 1910, 1921, 1931, 1960, and 1964. Death rates were given for five-year age categories from age 5 up to age 80, as well as for age zero and the four-year category from age 1 to 5. We converted the n-year death rates into single-year death rates such that the resulting mortality curve followed a piece-wise linear trajectory; we then used simple linear interpolation between the available data points over time to estimate the height of the mortality surface at intermediate points in time. Comparison of Figures 1a and 3 reveals the difference it makes to work with detailed data as opposed to interpolated data. The global patterns of mortality over age and time are apparent in Figure 3, but all the interesting local features, including the effects of the two World Wars, are lost.

The longest time series of mortality rates are available for Sweden. We used mortality rates based on interpolations made by Vaupel, Manton, and Stallard (1979) and for recent years by ourselves, of data from Keyfitz and Flieger (1968) for 1778 to 1882 and from various editions of the Swedish Statistical Yearbook for 1881 through 1981. These data were available for the most part for five year periods; before 1880 the data were given for five year age categories but thereafter data by single year of age were available. It is apparent from Figure 4, which shows the evolution of Swedish female mortality from 1779 to 1981, that systematic progress against mortality at all ages began in Sweden around 1830. A re-

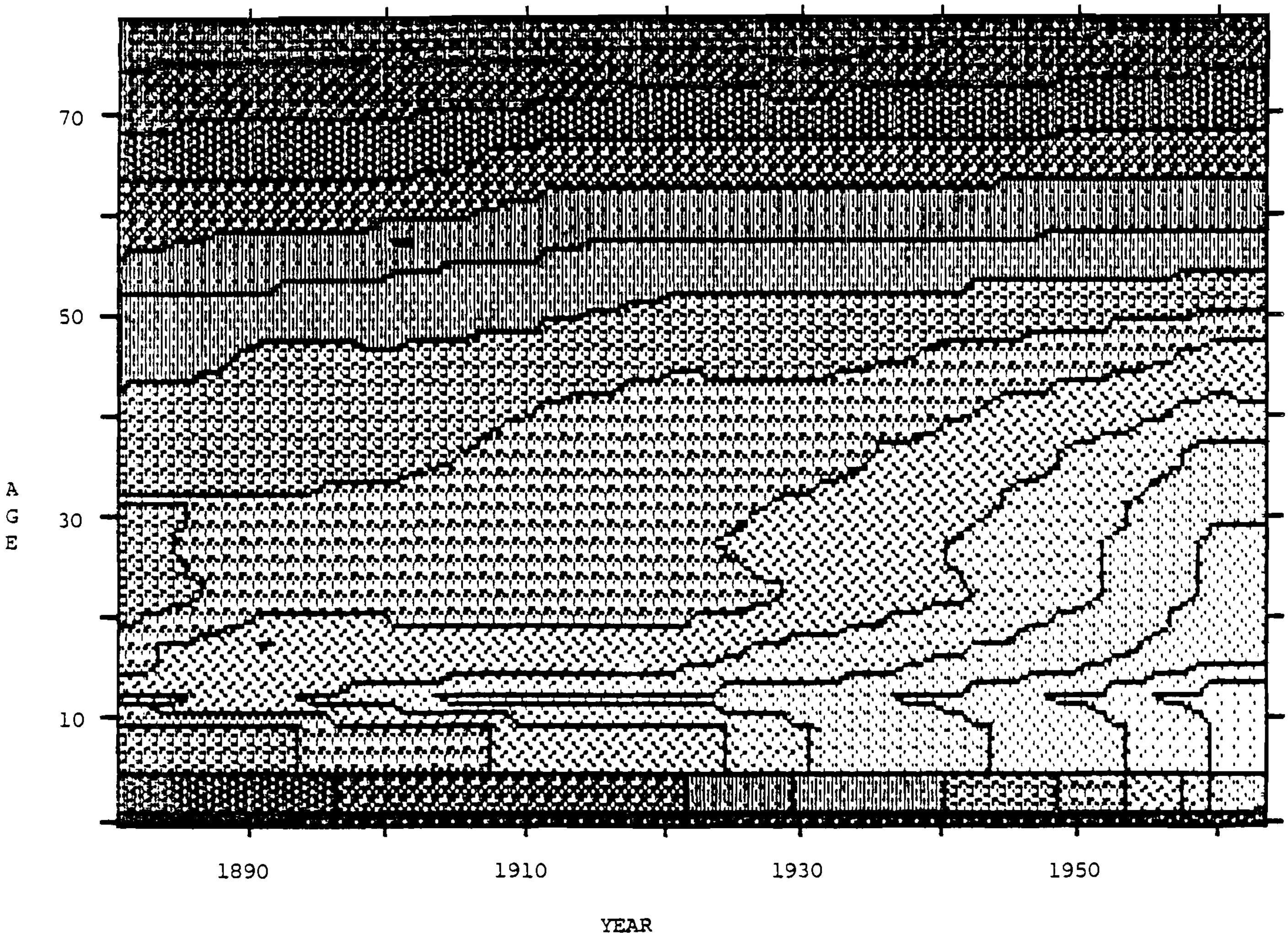
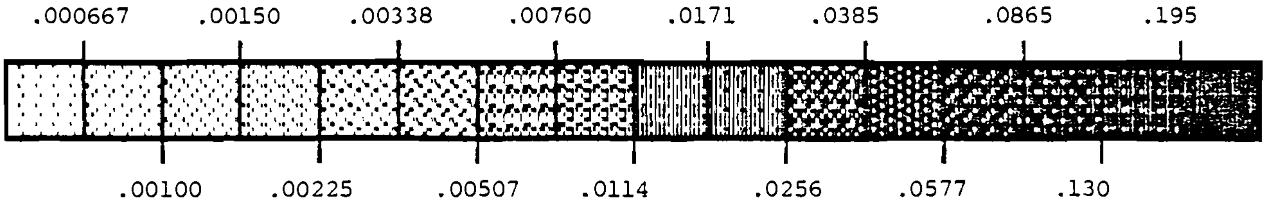


Figure 3: Italian Male Mortality Rates (Interpolated Data) - with contour lines from .000667 to .195 at multiples of 1.5
From age 0 to 79 and year 1881 to 1964



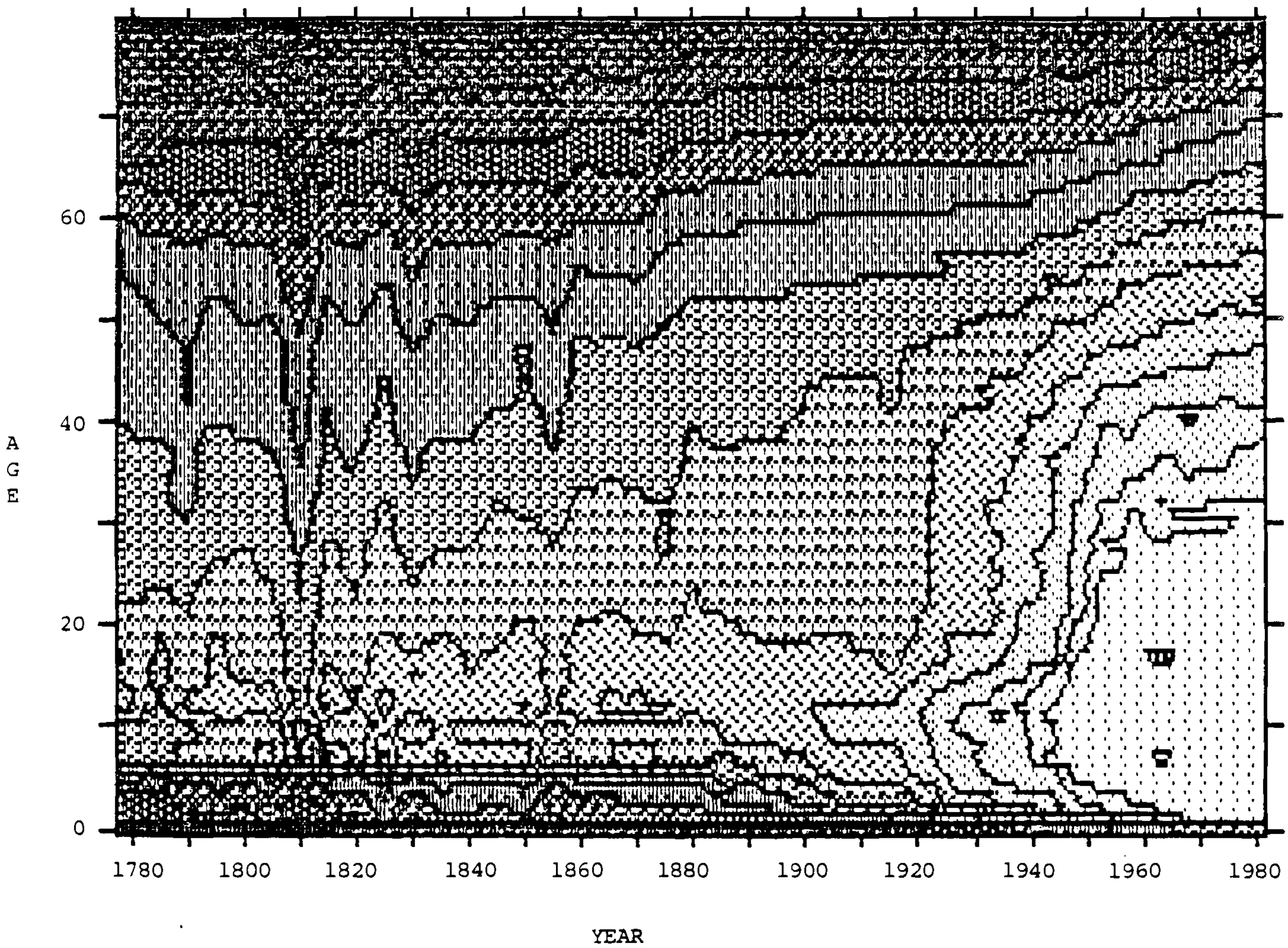
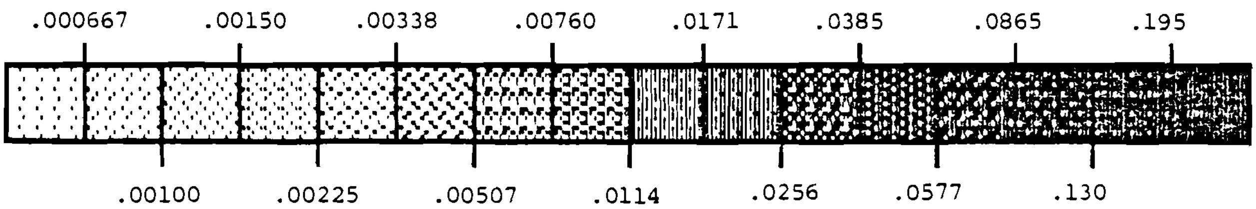


Figure 4: Swedish Female Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From age 0 to 79 and year 1778 to 1981



markable acceleration of progress, especially at younger ages, starts after 1920. In early years, strong fluctuations are evident, especially the destructive effects of the Swedish war with Russia in 1808-9.

AGE-PERIOD, AGE-COHORT, AND COHORT-PERIOD MAPS OF U.S. FEMALE FERTILITY

Figure 5 displays the contours of U.S. birth rates from 1917 to 1980 for women from age 14 to 49; the figure is based on data compiled by Heuser (1976, 1984). In the dark center of the baby boom, for women around age 23 around 1960, fully a quarter of women gave birth each year. The concentration of high birth rates among women in their early and mid twenties and the cycles of high and low birth rates that characterize baby booms and busts are strikingly revealed on the map.

Figure 5 is a standard map in which current year runs along the horizontal axis and age runs up the vertical axis. Other coordinates help reveal cohort effects. In particular, because the eye can follow vertical and horizontal lines more easily than diagonals, it may be useful to twist a contour map so that year of birth, rather than current year, runs along the horizontal axis. Figure 6 illustrates this approach. Alternatively, as shown in Figure 7, year of birth may run along the horizontal axis and current year along the vertical axis. We only used five contour lines on Figure 7 because the lines were otherwise too closely spaced to be intelligible.

Taken together, Figures 5, 6, and 7 indicate that the age effect in fertility is very strong, that period fluctuations are also strong, but that cohort effects appear to be much less prominent. Perhaps more refined methods of presentation will reveal persistent cohort patterns; some relevant analysis is presented later in this paper in conjunction with Figures 8 through 11, 15, 18 through 20, 27, and 45. Note that the period effects shown in Figures 5, 6, and 7 can be separated into three parts. Before age 18, fertility rates have remained low, and after age 35 or so, there is a general pattern of declining fertility. It is between ages 18 and 35, and especially around age 23, where the most dramatic absolute swings in fertility rates have occurred. In conjunction with Figure 15, we will consider relative fluctuations in fertility rates, in contrast with absolute fluctuations shown in Figures 5, 6, and 7.

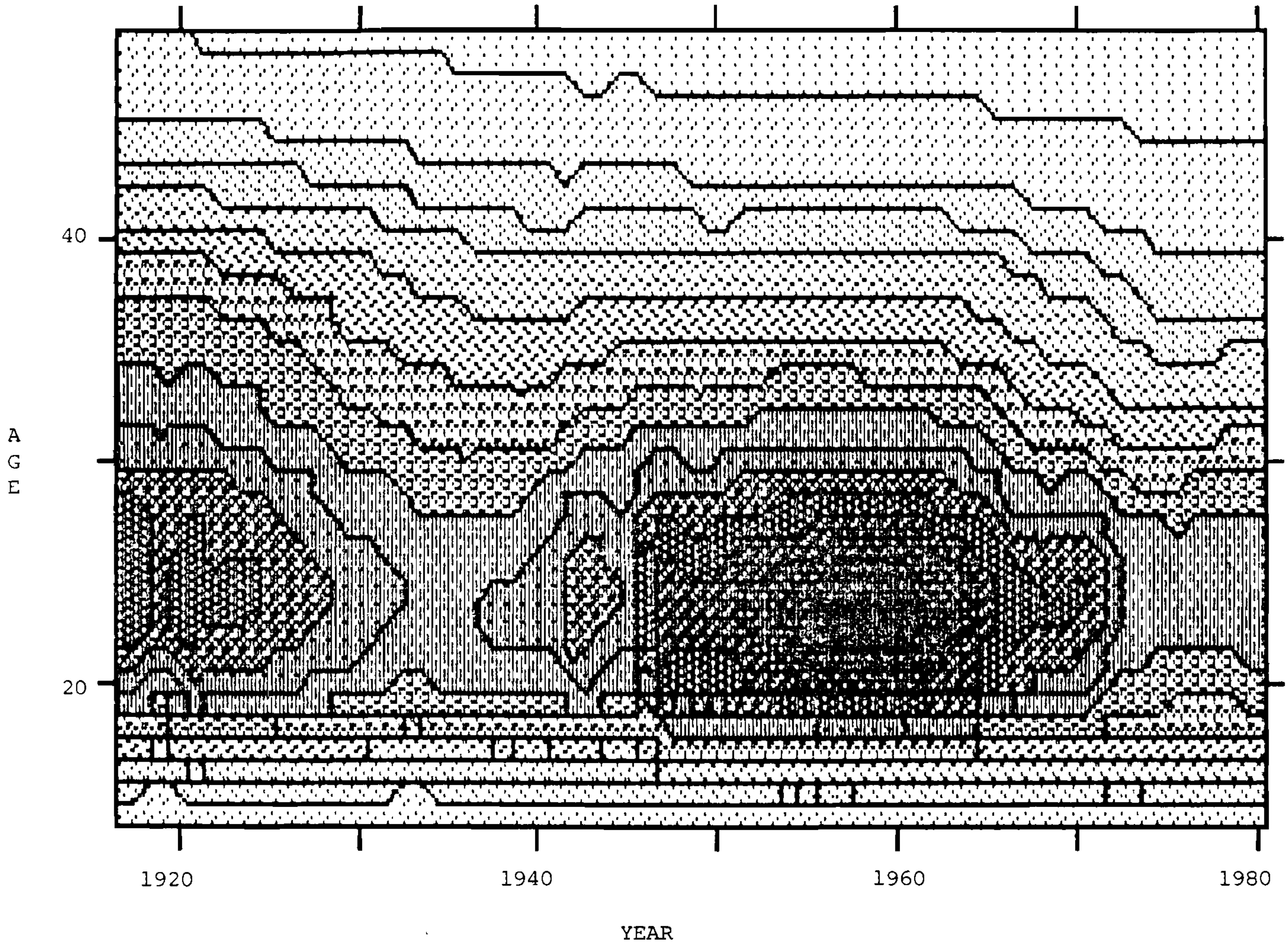
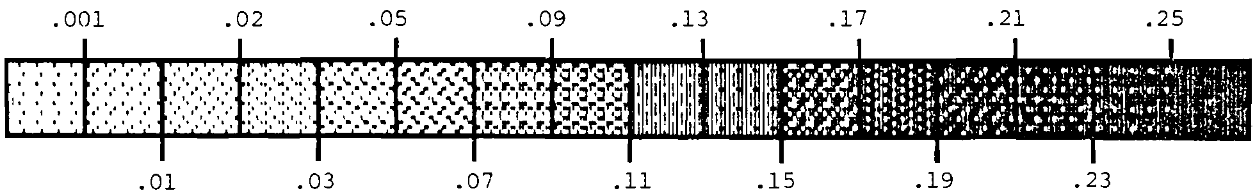


Figure 5: U.S. Fertility Rates - with contour lines selectively placed from .001 to .25
From age 14 to 49 and year 1917 to 1980



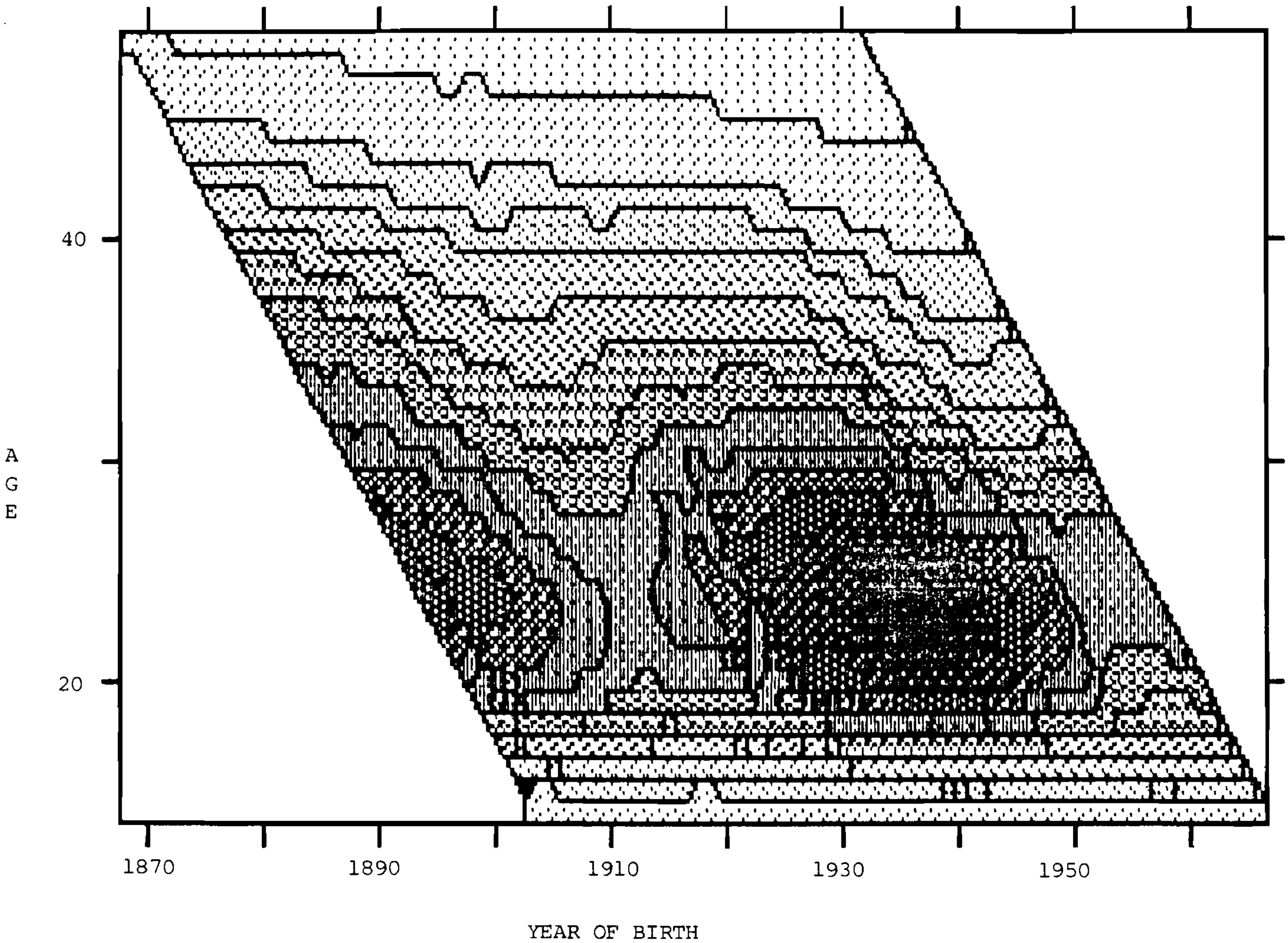
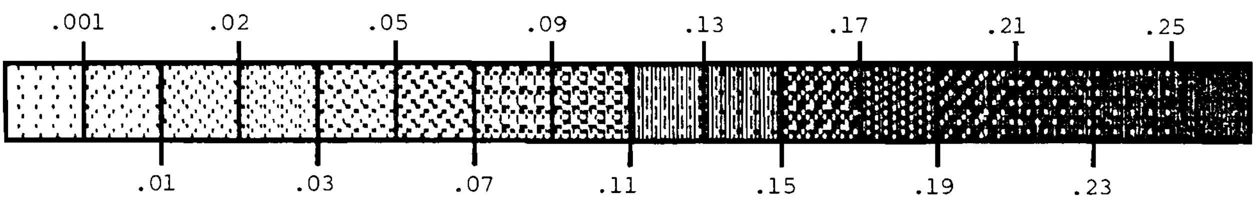


Figure 6: U.S. Cohort Fertility Rates - with contour lines selectively placed from .001 to .25
From age 14 to 49 and year of birth 1868 to 1966



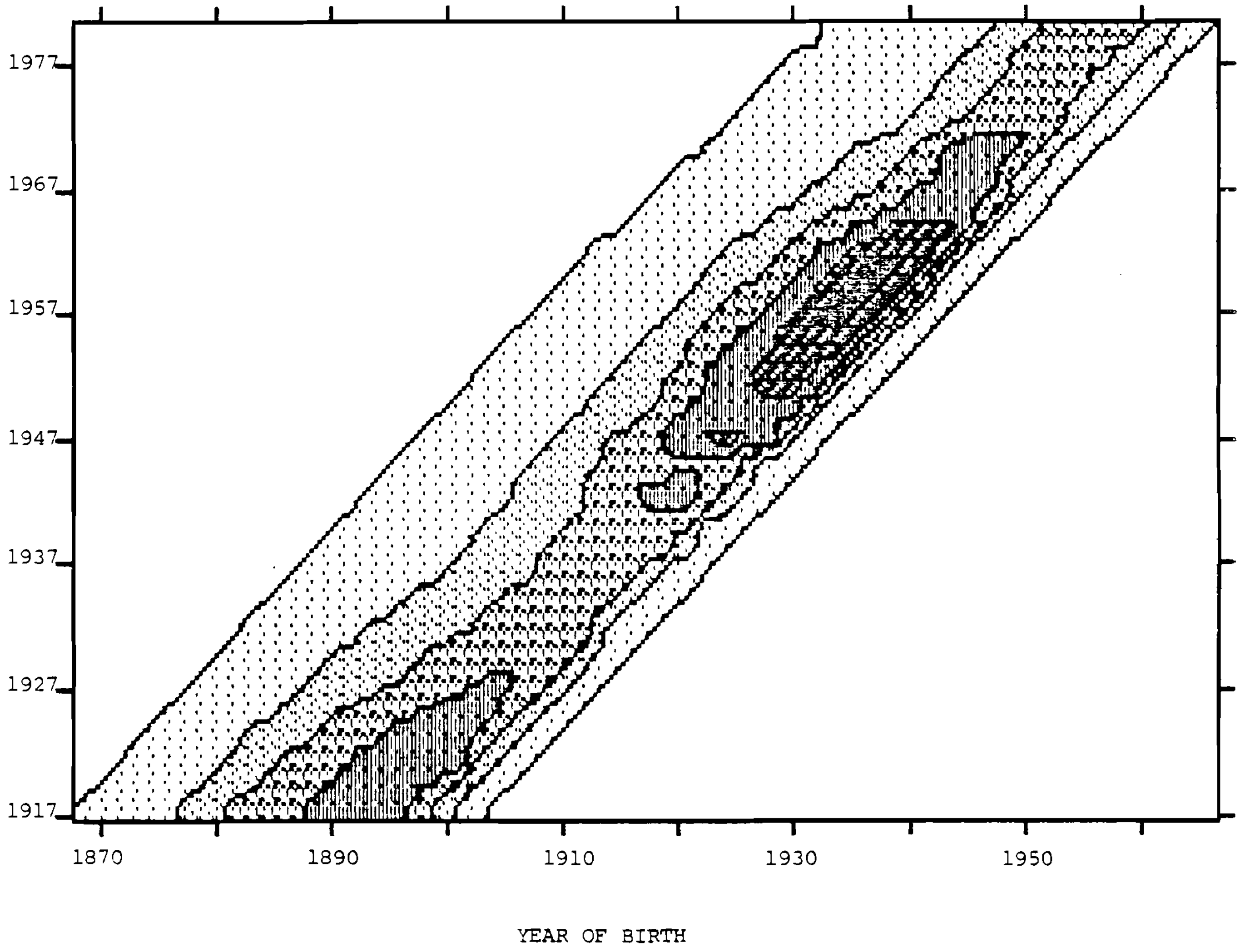
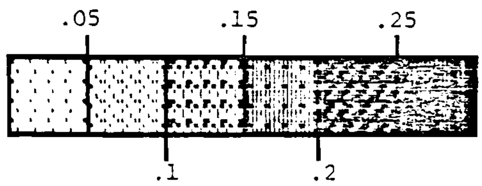


Figure 7: U.S. Fertility Rates by Current Year and Year of Birth - with selected contour lines from .05 to .25
From current year 1917 to 1980 and year of birth 1868 to 1966



ALTERNATIVE GRAPHIC DISPLAYS OF U.S. FEMALE FERTILITY

The most commonly used method for displaying demographic rates over age and time is to plot the rates over time for selected ages or over age for selected times. In Figure 8, for instance, U.S. birth rates are graphed over time at ages 18, 23, and 28 and in Figure 9 the birth rates are graphed from age 14 to 49 for years 1920, 1950, and 1980. Comparison of Figures 8 and 9 with the contour maps presented in Figures 5, 6, and 7 reveals some of the strengths and weaknesses of these alternative graphic displays. The contour maps present far more information and give an overview of the entire surface. The plots over age and time focus attention on trends and fluctuations in those two directions. The contour maps might be compared to a lavish Chinese banquet, whereas the the graphs over age and time are more like a delicate Japanese dinner.

Figures 10 and 11 show two plots of the U.S. birth rate data drawn from a three-dimensional perspective. Figure 10, like all the contour maps in this paper, was produced using a simple IBM PC (with 64K of memory) and an inexpensive printer; Figure 11 was drawn using a mainframe computer and high-quality plotter. With appropriate software, it is possible not only to produce but also to rotate such three-dimensional pictures on a computer monitor so that they can be viewed from various angles: the three perspectives shown in Figure 11 suggest how informative this kind of rotation can be. Clearly, three-dimensional plots will be an important tool for demographers.

Three-dimensional plots, however, will complement, and not supercede, contour plots. A three-dimensional plot sacrifices some of the richness of detail that is clearly portrayed on the corresponding contour map; furthermore, it is difficult on a three-dimensional plot to relate a point on the surface to the exact age and year underlying the point. Just as architects, surveyors, and engineers rely on contour maps for site planning and cartographers use contour and topographical maps in depicting a variety of terrains and surfaces, demographers will also undoubtedly find that for some purposes contour maps are the most appropriate means for representing a population surface. Fisher's (1982) comprehensive comparison of an array of different methods for "mapping information" reveals the relative advantages, for many purposes, of "the combined use of contour lines and tones".

Any graphical method has its strengths and weaknesses. Our point is simply that demographers should consider adding contour maps to their toolkit, to supplement graphs of rates over age and time, three-dimensional plots, age-distribution

Figure 8: U.S. Birth Rates from 1917 to 1980
Curves, from lightest to darkest, are drawn at ages 18, 23, and 28

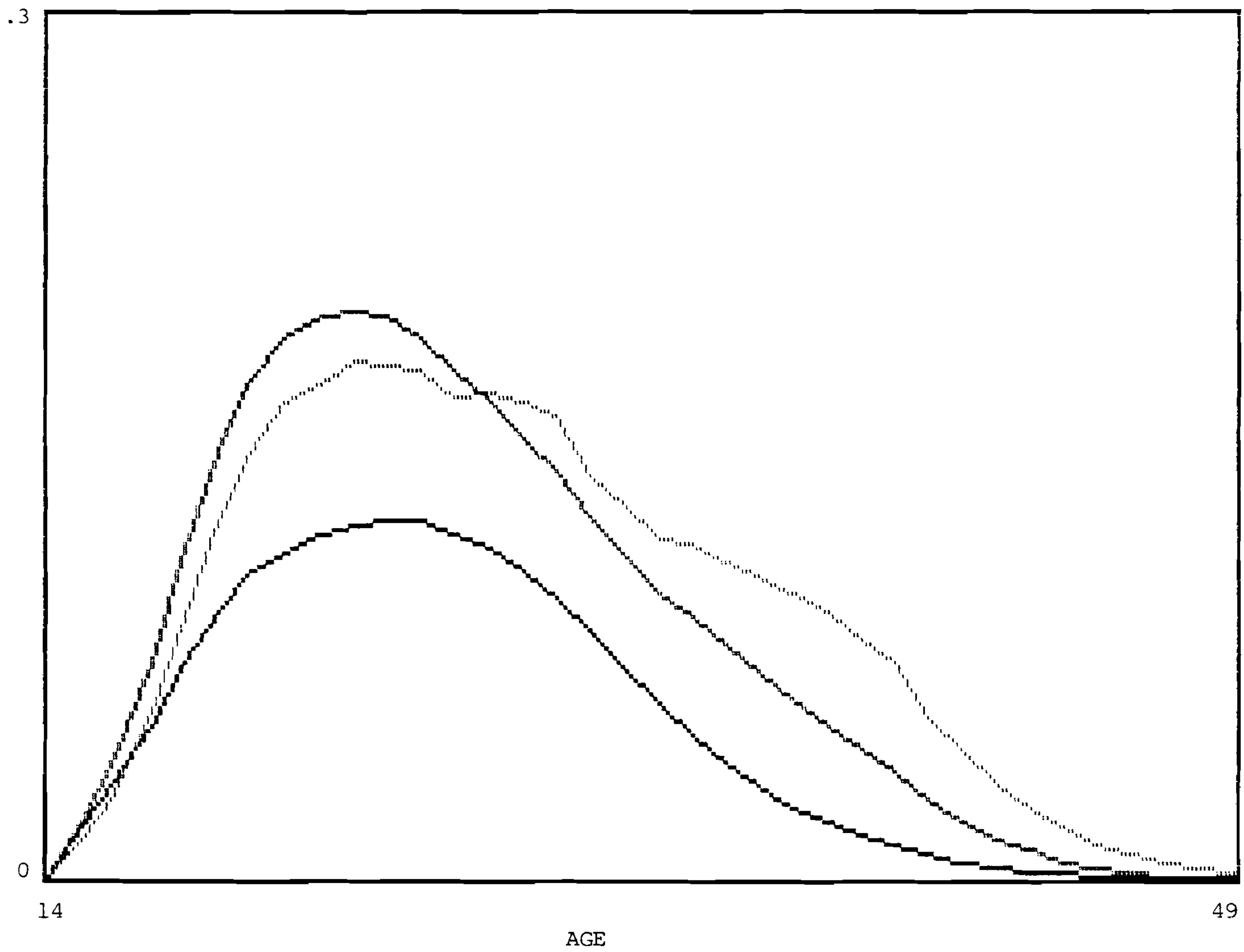


Figure 9: U.S. Birth Rates from Age 14 to 49. Curves are drawn, from lightest to darkest, at Years 1920, 1950 and 1980

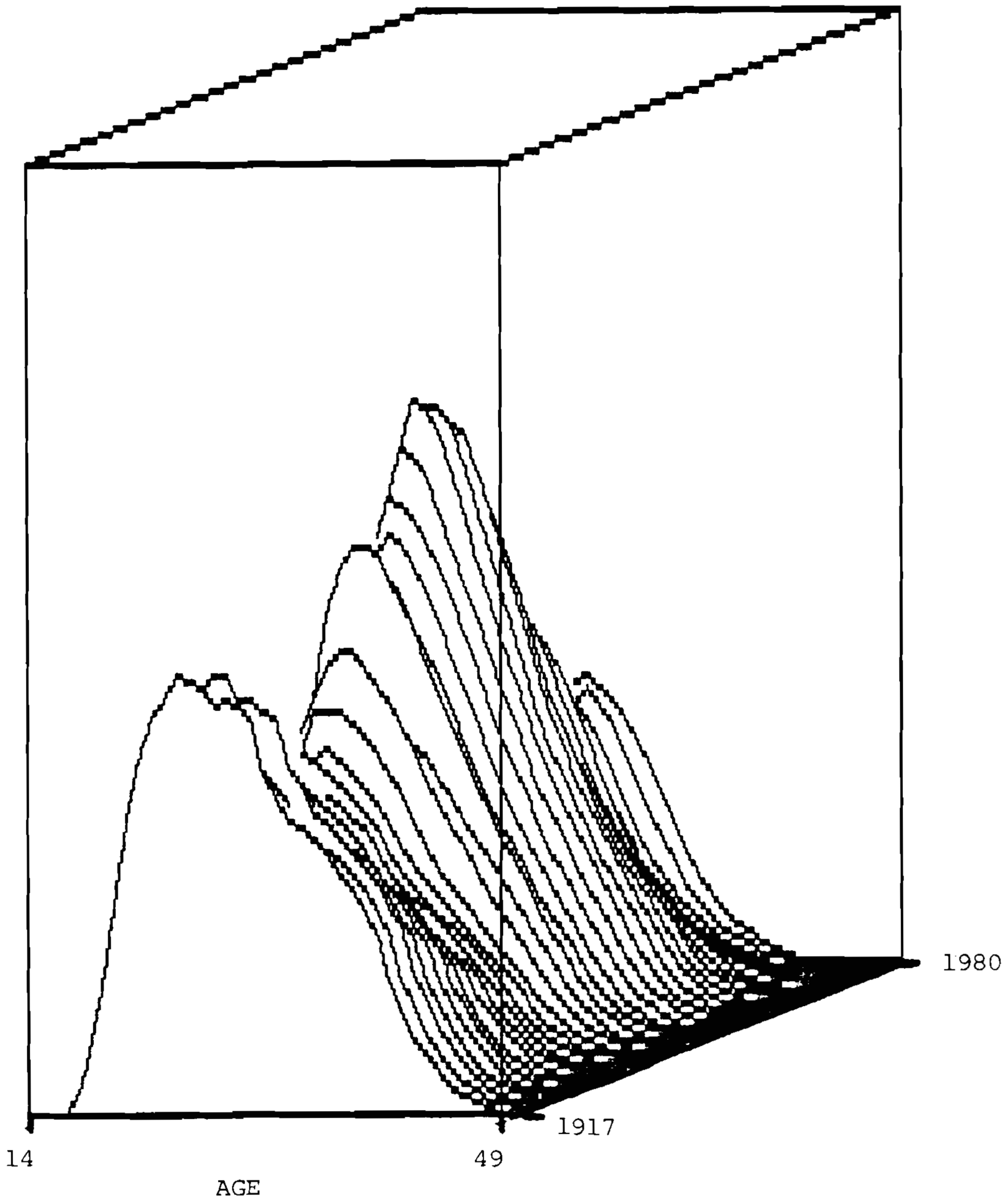


Figure 10: Three-dimensional Perspective of U.S. Fertility Rates over Ages 14 to 49 and Years 1917 to 1980.

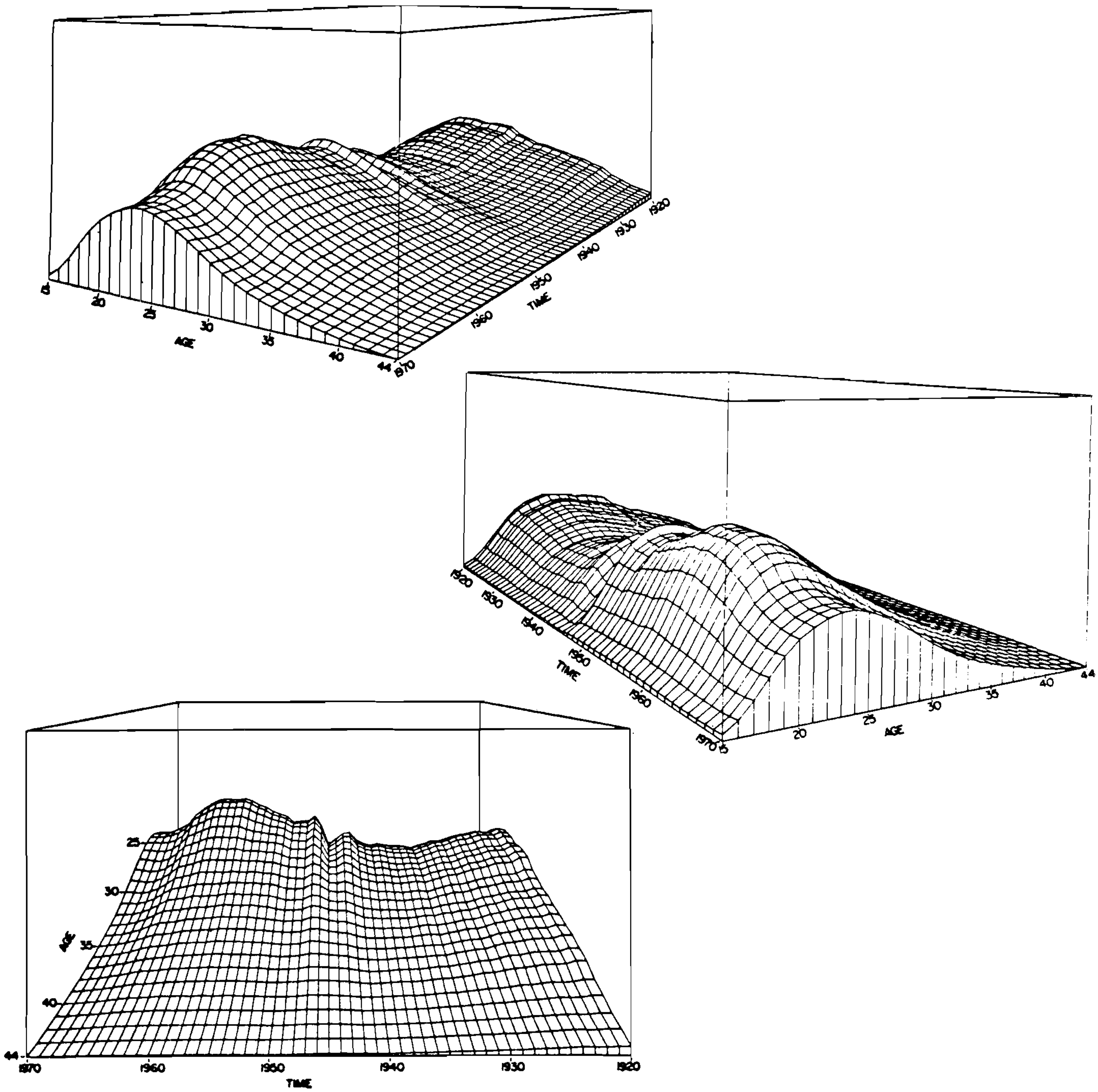


Figure 11: High Quality Three-dimensional Plots of U.S. Fertility Rates*.

* Pullum, Thomas W., "Separating Age, Period and Cohort Effects in White U.S. Fertility, 1920-1970". <u>Social Science Research</u> 9, pp 225-224; Academic Press. Inc., 1980.

pyramids, and other techniques.

FRENCH POPULATION

Surfaces of population levels, so-called Lexis surfaces, are especially important in the theory of population dynamics, as discussed by Arthur and Vaupel (1984). Figure 12 presents such a surface, of French population levels from birth to age 79 from years 1851 to 1965, based on interpolations of data in Keyfitz and Flieger (1974). Births run across the bottom of the map. As suggested by the figure, and confirmed by looking at the underlying data values, the annual number of births in France has hovered around a level of three-quarter of million, with a low of under 600 thousand and a peak of just over 850 thousand. Total population increased only by about ten percent from 1851 to 1945, but then it jumped by twenty percent from 1945 to 1965; this again is suggested by map and confirmed by looking at Keyfitz and Flieger's tables.

In a population closed to migration, a Lexis surface has to continually fall off along any cohort line, as the members of the cohort die. A number of French cohorts, however, grew in size with age, not only because of the usual kind of in-migration but also because of effective in-migration caused by the inclusion of Nice and Savoy (starting with the 1861 table) and the re-inclusion of Alsace and Lorraine (starting with the 1920 table; Alsace and Lorraine were excluded in earlier tables, back through 1871). Note, in particular, the diagonal displaying the fluctuating population levels of the cohorts born between 1915 and 1920.

To depict the change that has occurred over time in age-specific population levels, or such other demographic statistics as fertility or mortality rates, it is useful to draw contour maps of relative surfaces on which the value of the statistic at each point is calculated relative to value of the statistic in some base year. Figure 13, for example, presents French population levels relative to the population level at the various ages in the first year, 1851. In conjunction with Figure 12, this figure reveals the general trends in French population levels as well as some interesting local fluctuations. The cycles in numbers of birth are apparent on Figure 13, as is the great relative increase in population at the oldest ages, with about three and a half times as many people age 79 in 1964 compared with the number of people at this age in 1851. Note the area of the map with a value of about 1: this area shows the ages and times when population was at the same level as it was at the corresponding age in 1851. Thus, for example, twenty-four-year-

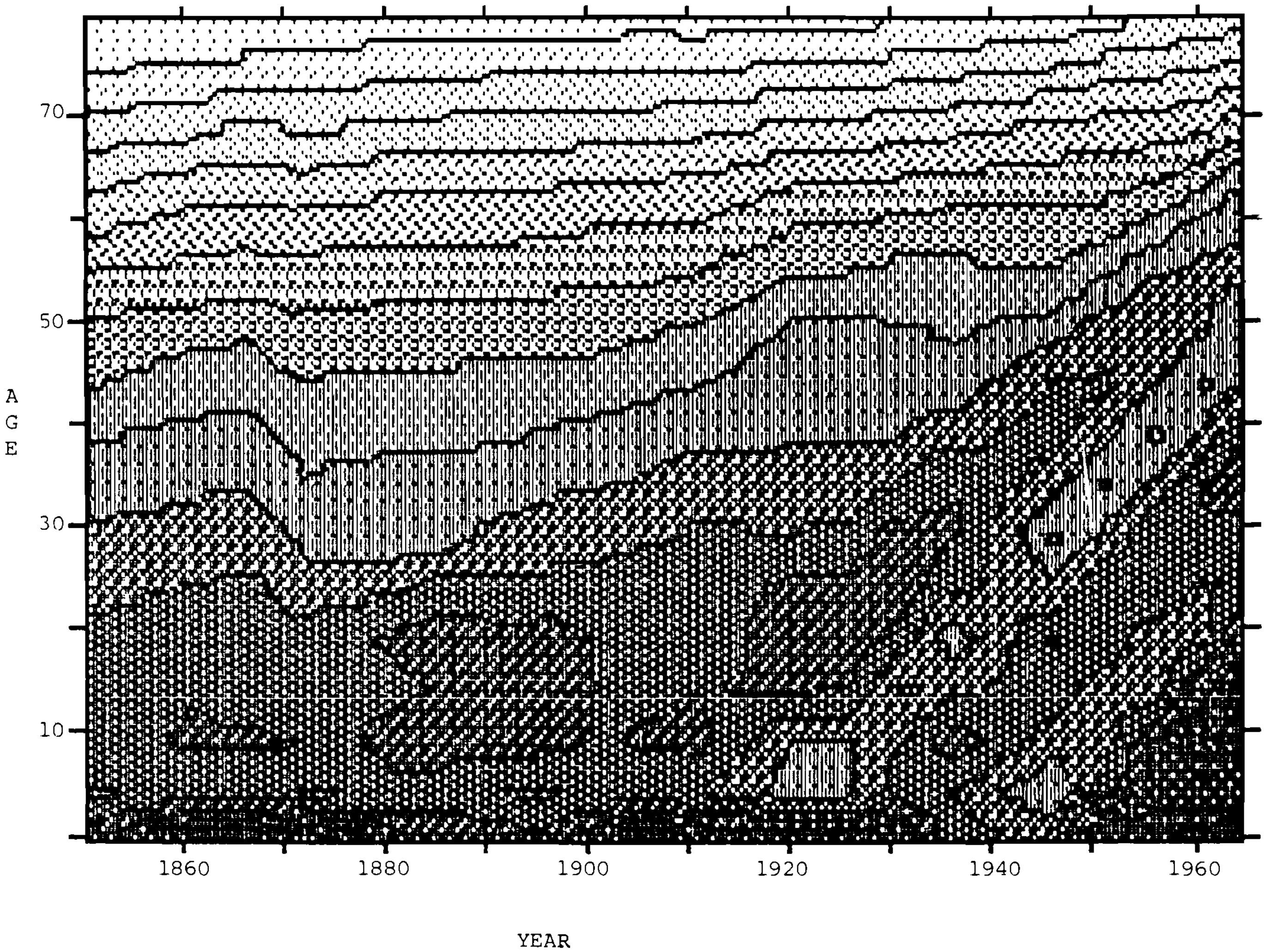
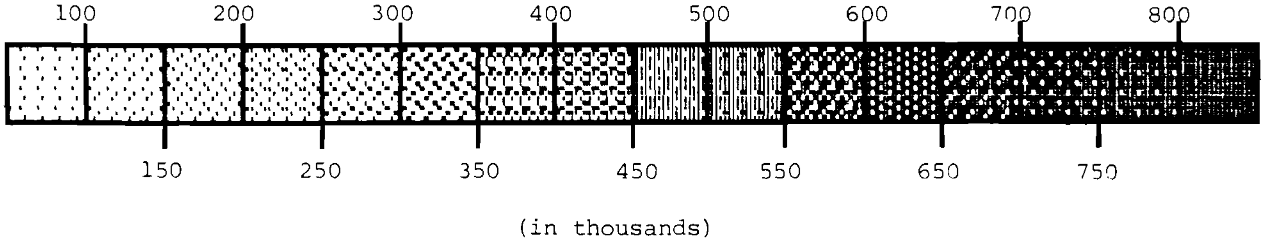


Figure 12: French Population at Each Age and Time - with contour lines from 100,000 to 800,000 at intervals of 50,000
From age 0 to 79 and year 1851 to 1964



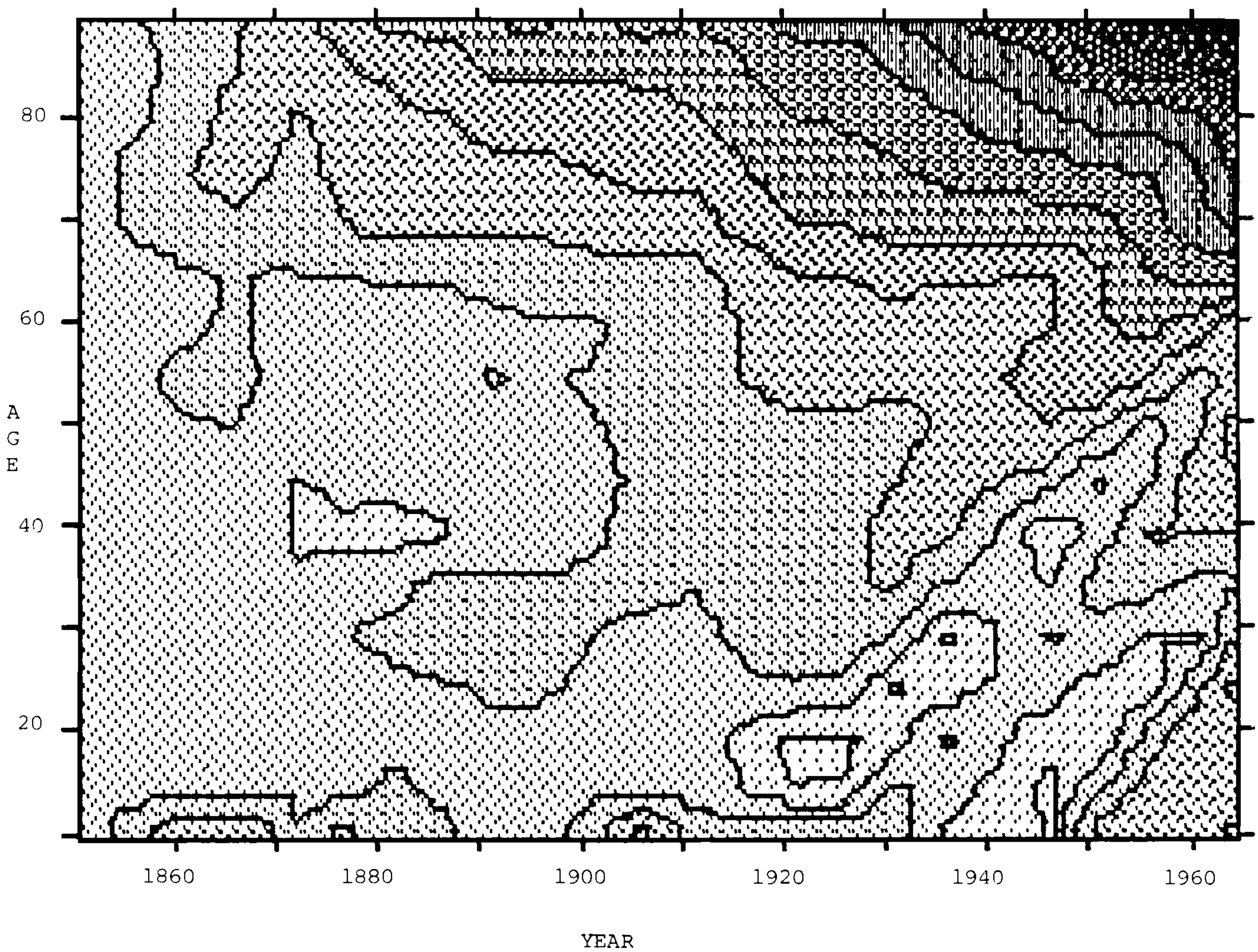
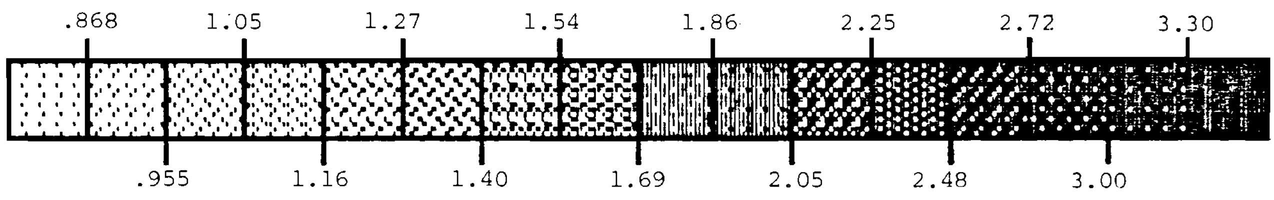


Figure 13: French Male Population Relative to 1851 Age-specific Levels - with contour lines from .868 to 3.30 at multiples of 1.1 From age 0 to 79 and year 1851 to 1964



olds were about as numerous in the mid 1960's as they were in the 1850's.

RELATIVE SURFACES OF ITALIAN MORTALITY, U.S. FERTILITY, AND BELGIUM POPULATION

Two other applications of relative contour maps are shown in Figures 14 and 15. Figure 14 displays age-specific mortality rates for Italian males relative to their levels in 1925, a year roughly halfway through the period studied. The map clearly reveals the great progress that has been made in reducing mortality at the youngest ages compared with the slow progress at the oldest ages. The map also puts the devastation of World War I in perspective: the war essentially erased a half century of progress, but the setback was temporary and prewar mortality rates at most ages were re-reached and surpassed within a decade or so.

Figure 15 presents age-specific birth rates for U.S. females relative to their level in the final year, 1980. The map highlights the dramatic reduction in birth rates above age 35, compared with the less radical (relative) changes at younger ages. Even the baby boom pales in significance when viewed from this perspective.

Instead of dividing a demographic array by the age-specific statistics for a particular year, the array could be divided by the period-specific statistics for a particular age. For example, Figure 16 shows Italian male mortality rates at various ages relative to the infant mortality rate in the appropriate year. The falling contours on the top half of the map emphasize a trend that was less apparent in the original Figure 1a, namely that progress against mortality at older ages has been slower than that at younger ages. In 1870 mortality rates at age 65 were roughly a fifth the level of infant mortality; a century later, mortality rates at age 65 were about a quarter higher than the prevailing infant rates.

Demographic statistics can also be expressed relative to some composite age-specific or period-specific measure. Figures 17 and 18 provide two examples. To produce Figure 17, Belgian age-specific female population levels (from Veys 1983) were divided by the total Belgium female population in each year. Thus, the map gives contours of the age-distribution of the population, i.e., the percentage of the population in each year that are at various ages. The diagonal traces of the small cohorts born during World Wars I and II are apparent, as is the general trend of the age composition of the population to shift upwards to older ages. As a proportion of the population, 70-year-olds were as important in 1970 as 40-year-olds were in 1892.

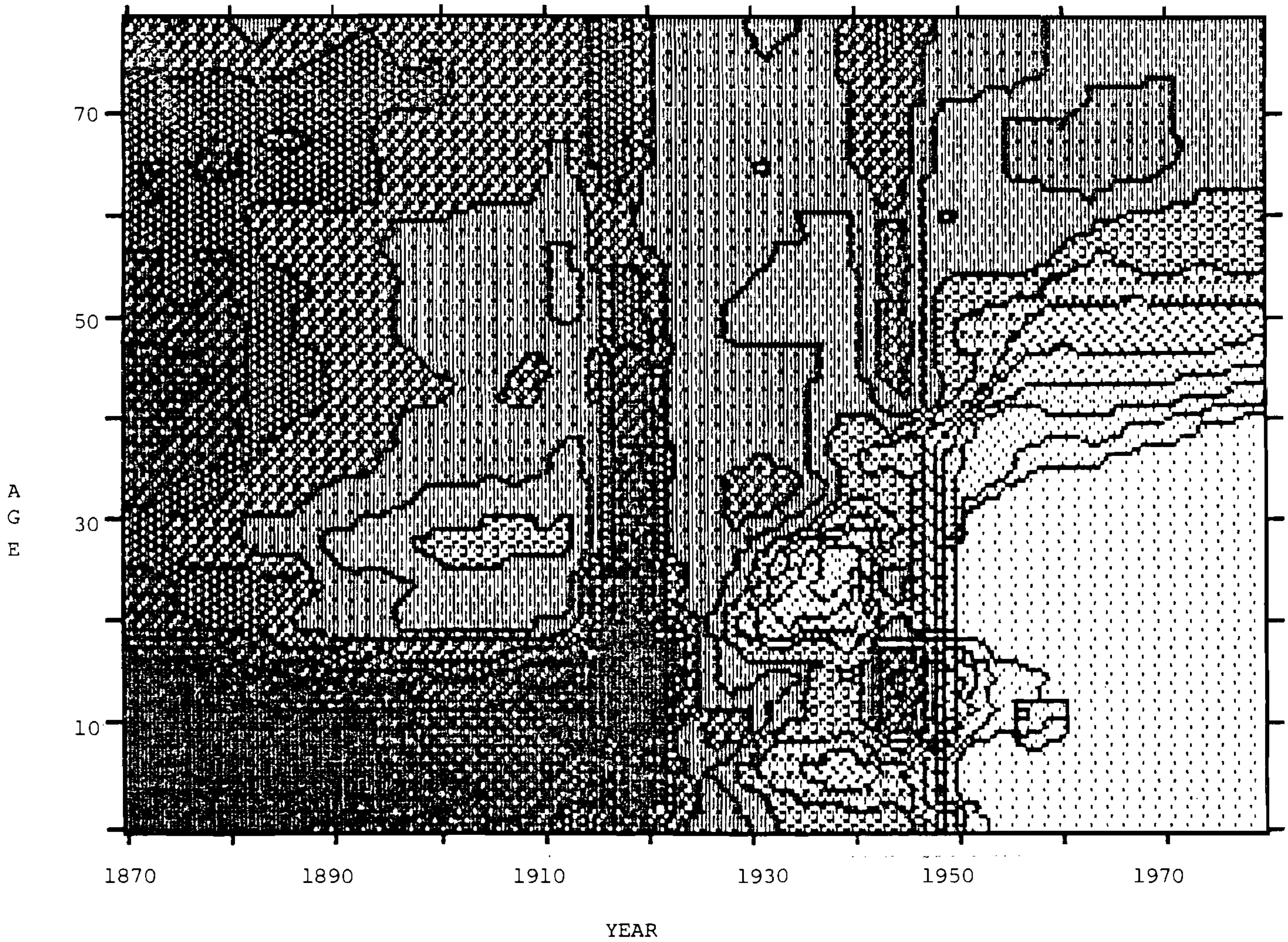
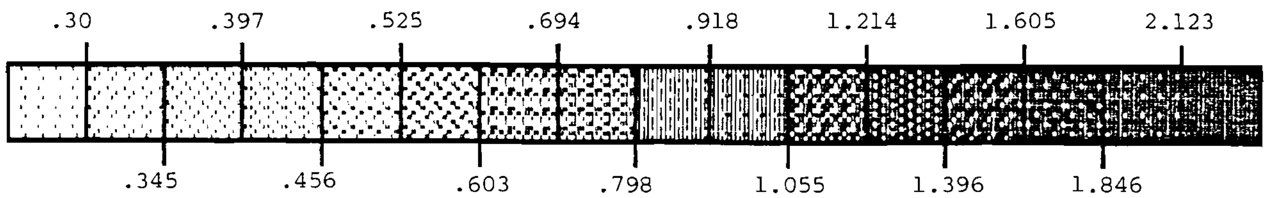


Figure 14: Italian Male Mortality Rates Relative to Age-specific 1925 Levels - with contour lines from .3 to 2.12 at multiples of 1.15, smoothed on a 5 by 5 square
From age 0 to 79 and year 1870 to 1979



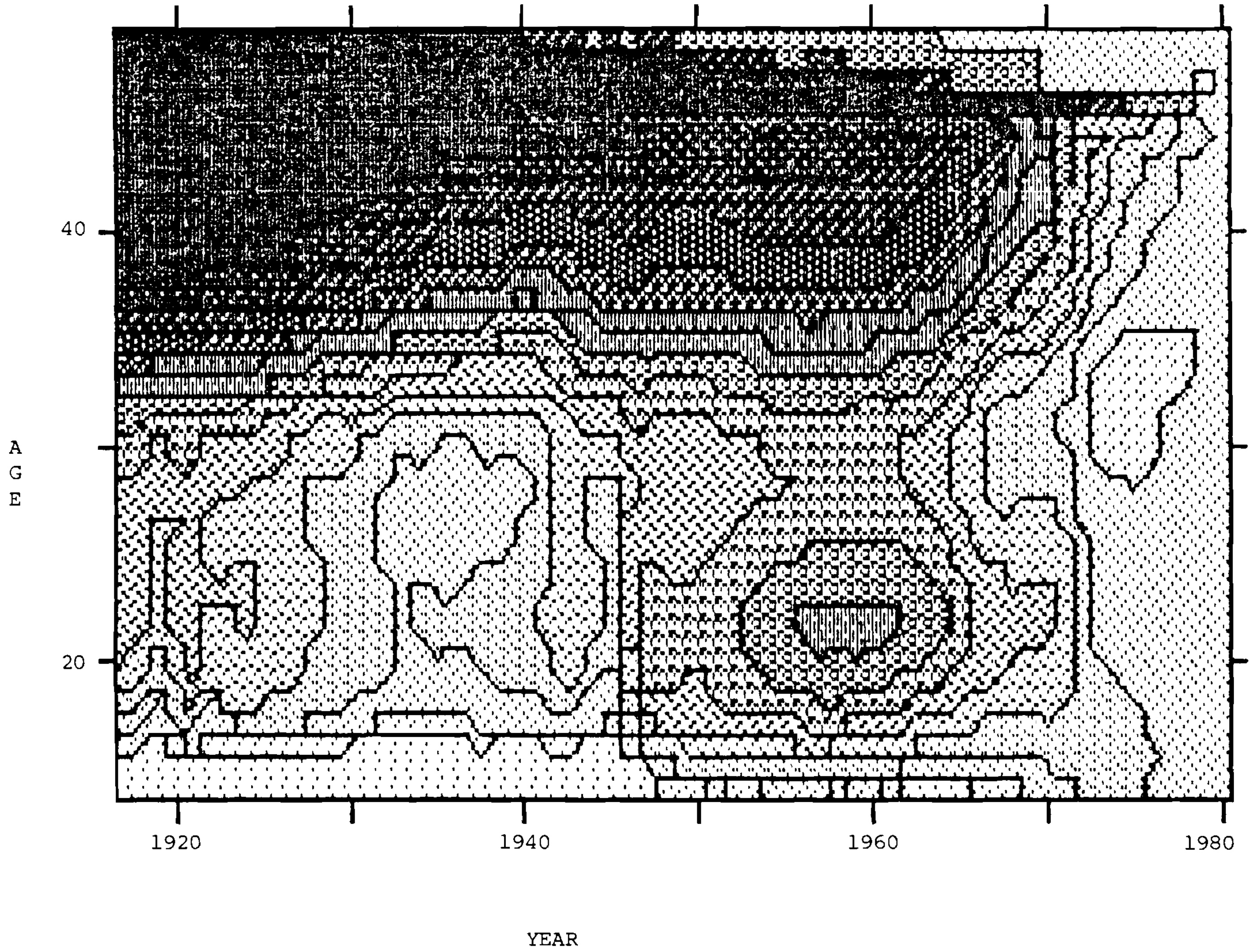
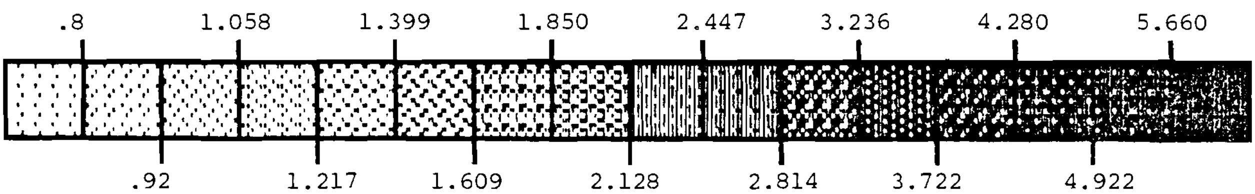


Figure 15: U.S. Birth Rates Relative to Age-specific 1980 Levels - with contour lines from .8 to 5.66 at multiples of 1.15
From age 14 to 49 and year 1917 to 1980



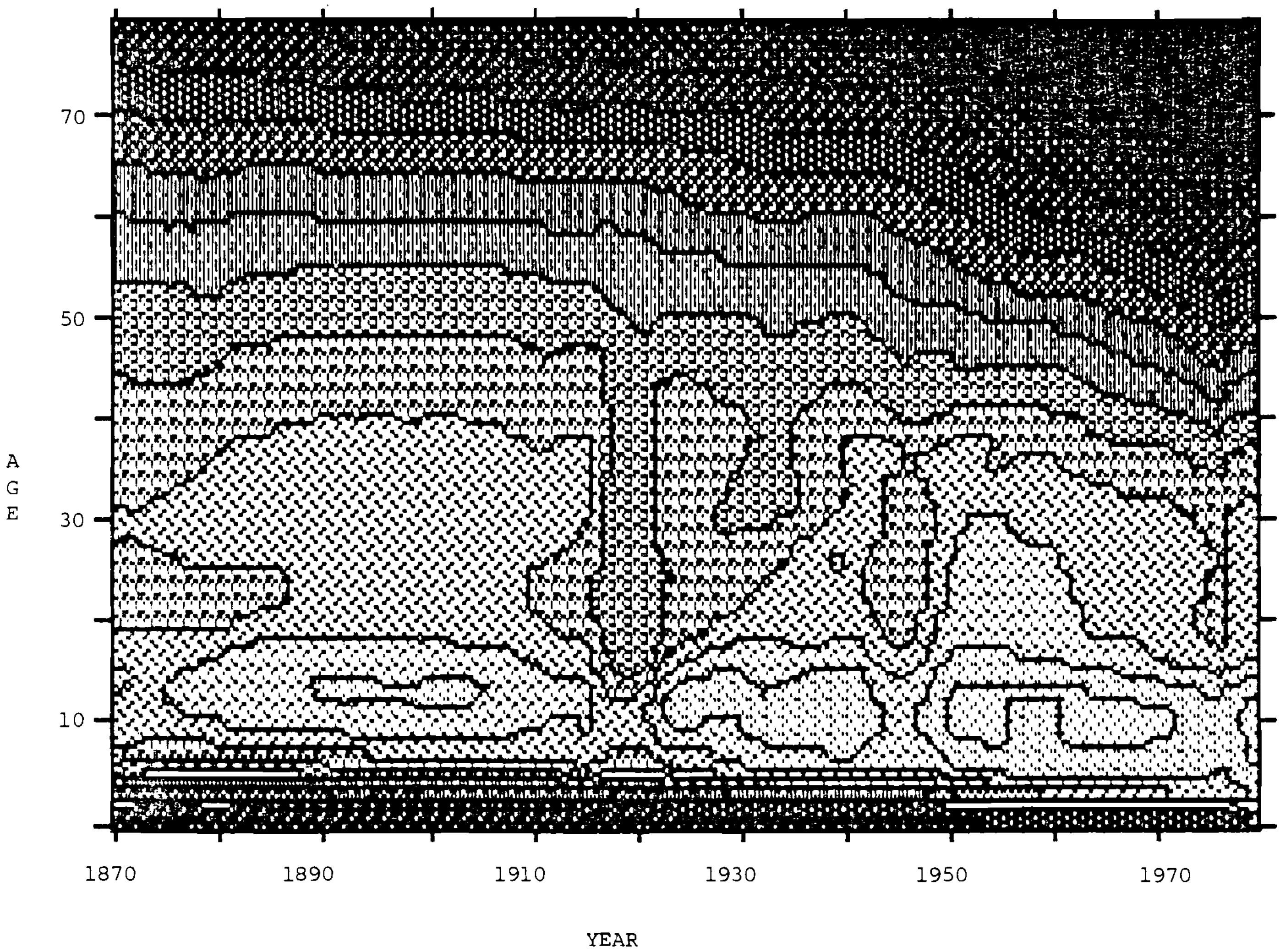
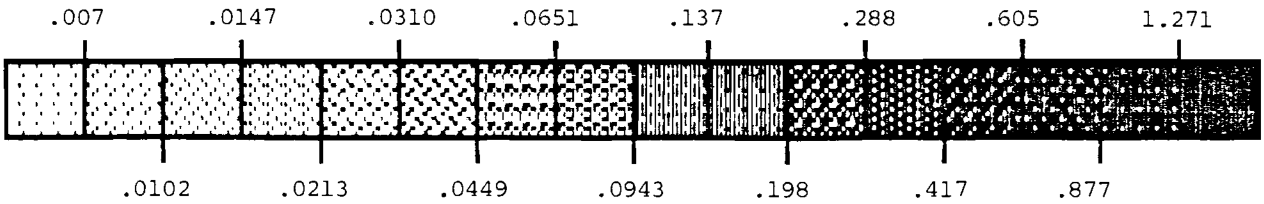


Figure 16: Italian Male Mortality Rates Relative to Infant Mortality - with contour lines from .007 to 1.27 at multiples of 1.5, smoothed on a 5 by 5 square
From age 0 to 79 and year 1870 to 1979



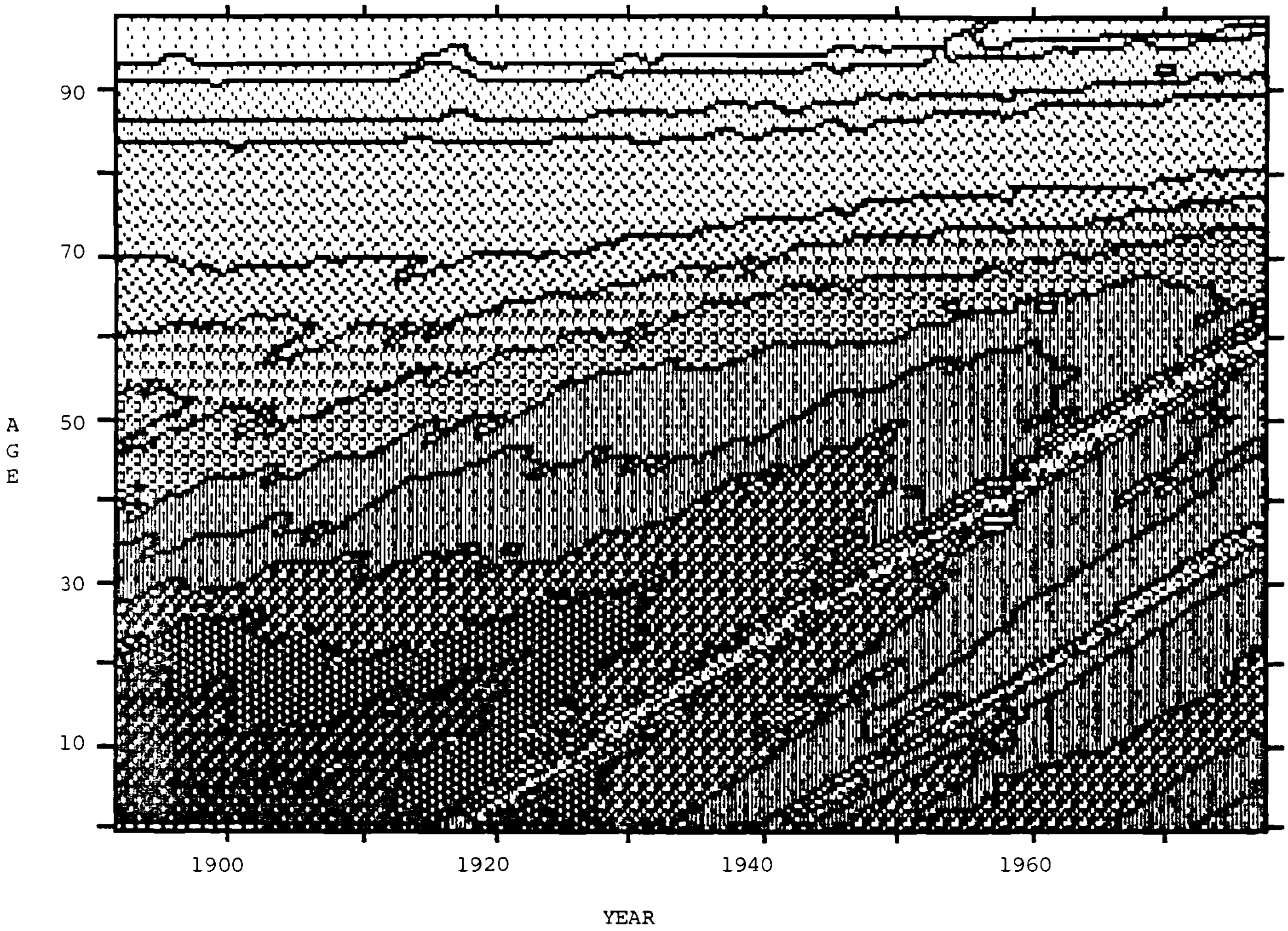


Figure 17: Age-distribution of Belgian Female Population - with contour lines selectively placed from .00005 to .027
From age 0 to 99 and year 1892 to 1977

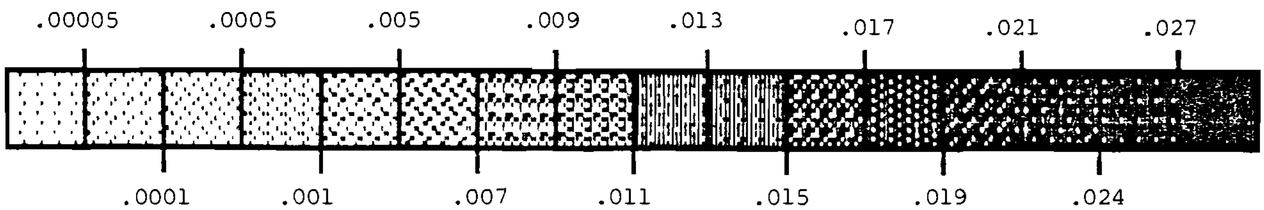


Figure 18, which is based on U.S. fertility data, is similar in nature except that the contours pertain to cumulative levels up through age 49 relative to the total level over all ages. The map can be interpreted as showing the proportion of all births in a given year that occurred to women of some age or less—in a synthetic population in which there were equal numbers of women at each age. The general trend is downward, especially at older ages: a greater cumulative proportion of children are being born each year to younger women. This trend runs through the periods of baby boom and bust.

Finally, it may sometimes be useful to examine contour maps based on statistics relative to a cohort-specific measure rather than either an age-specific or period-specific measure. Consider, for instance, Figure 19, which is similar to Figure 18 except that cumulative fertility is computed relative to total *cohort* fertility up through age 39. Here too the general trend is downward: a greater cumulative proportion of the children born in each successive cohort are born to younger women. For the cohorts born around 1915, half the children were born by age 27; whereas for the 1940 and 1941 cohorts, half the children were born by age 23.

SMALL MULTIPLES

To compare global patterns among several population surfaces it may be useful to shrink contour maps down in size and present several of them on the same page: Tufte (1983) calls this the "small multiples" approach. Figure 20 presents maps of U.S. birth rates at various parities: in Figure 20a, first-birth rates (i.e., the proportion of all women of some age in some year who have their first child) are displayed; in Figure 20b, second-birth rates are displayed, and so on. In each of the small multiples, the same contour levels are used. Because the birth rates shown in these figures are all based on the same denominator at any particular age and time, namely, the total number of women of that age in that year, the maps can in effect be added together to approximate the total birth rate at any age and time; the result is approximate because seventh and higher birth orders are omitted. In other words, the maps decompose total fertility into the contribution made at different parities.

This decomposition indicates that the absolute fluctuations in numbers of first and second children were more significant, in creating the waves of baby booms and busts, than the absolute fluctuations in higher-order births: for both first and

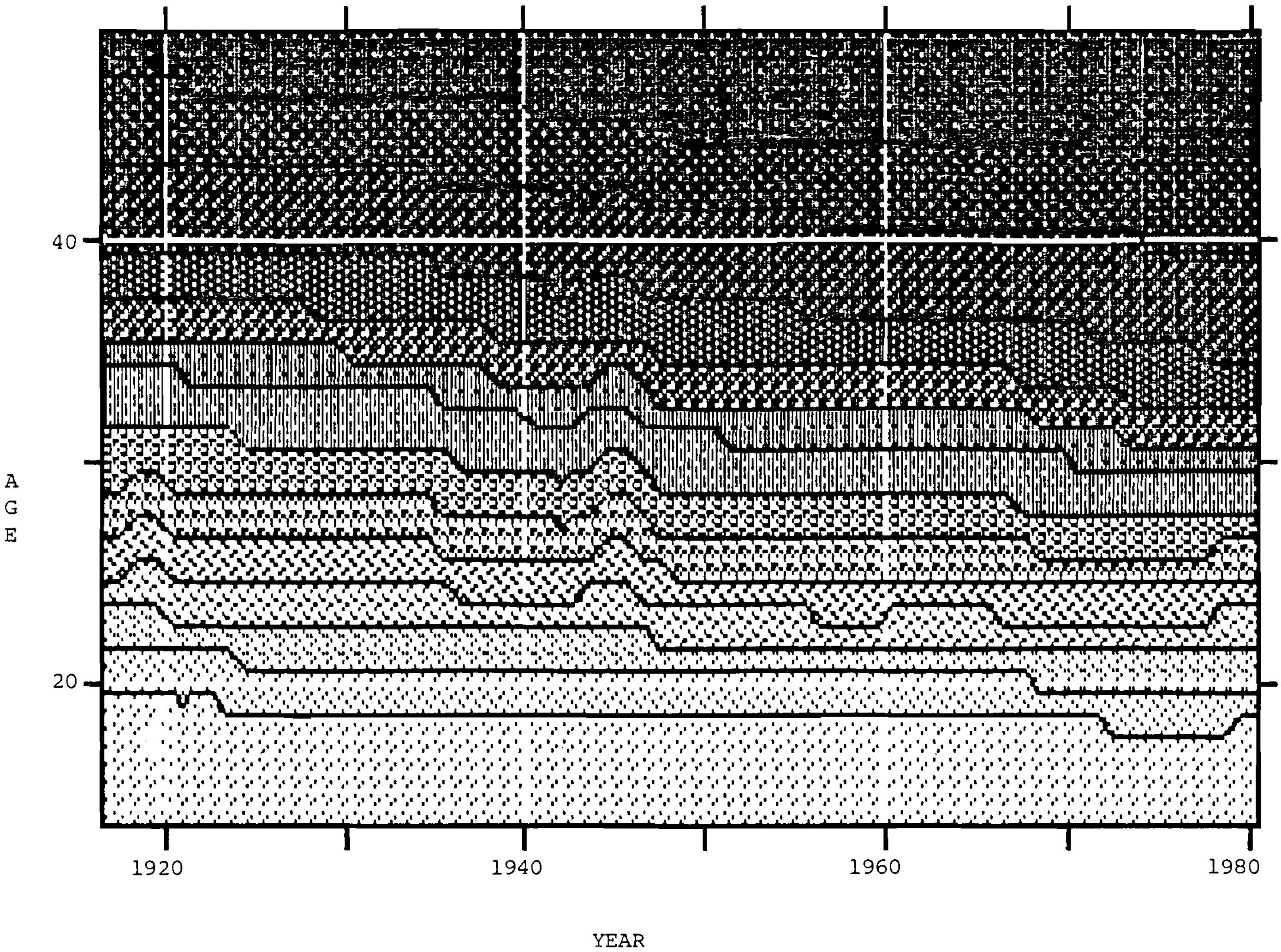
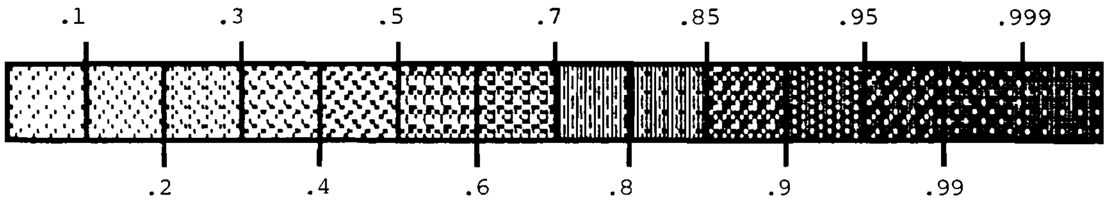


Figure 18: Cumulative Distribution of U.S. Births by Age of Mother and Year with contour lines selectively placed from .1 to .999
From age 14 to 49 and year 1917 to 1980



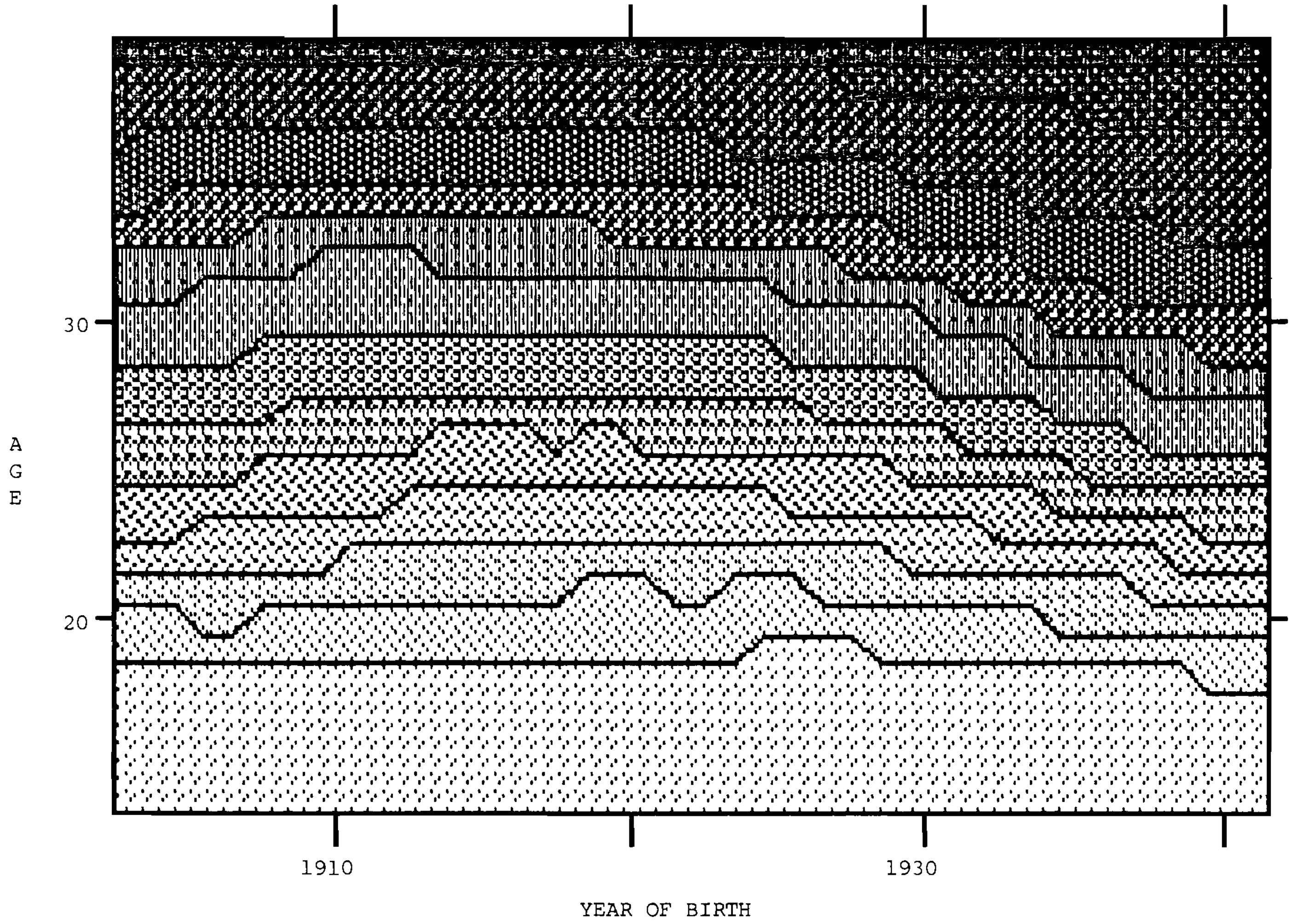
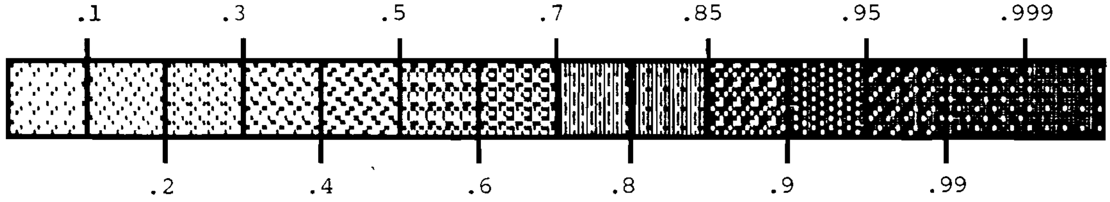


Figure 19: Cumulative Distribution of U.S. Births by Age and Year of Birth of mother - with contour lines selectively placed from .1 to .999 From age 14 to 39 and year of birth 1903 to 1941



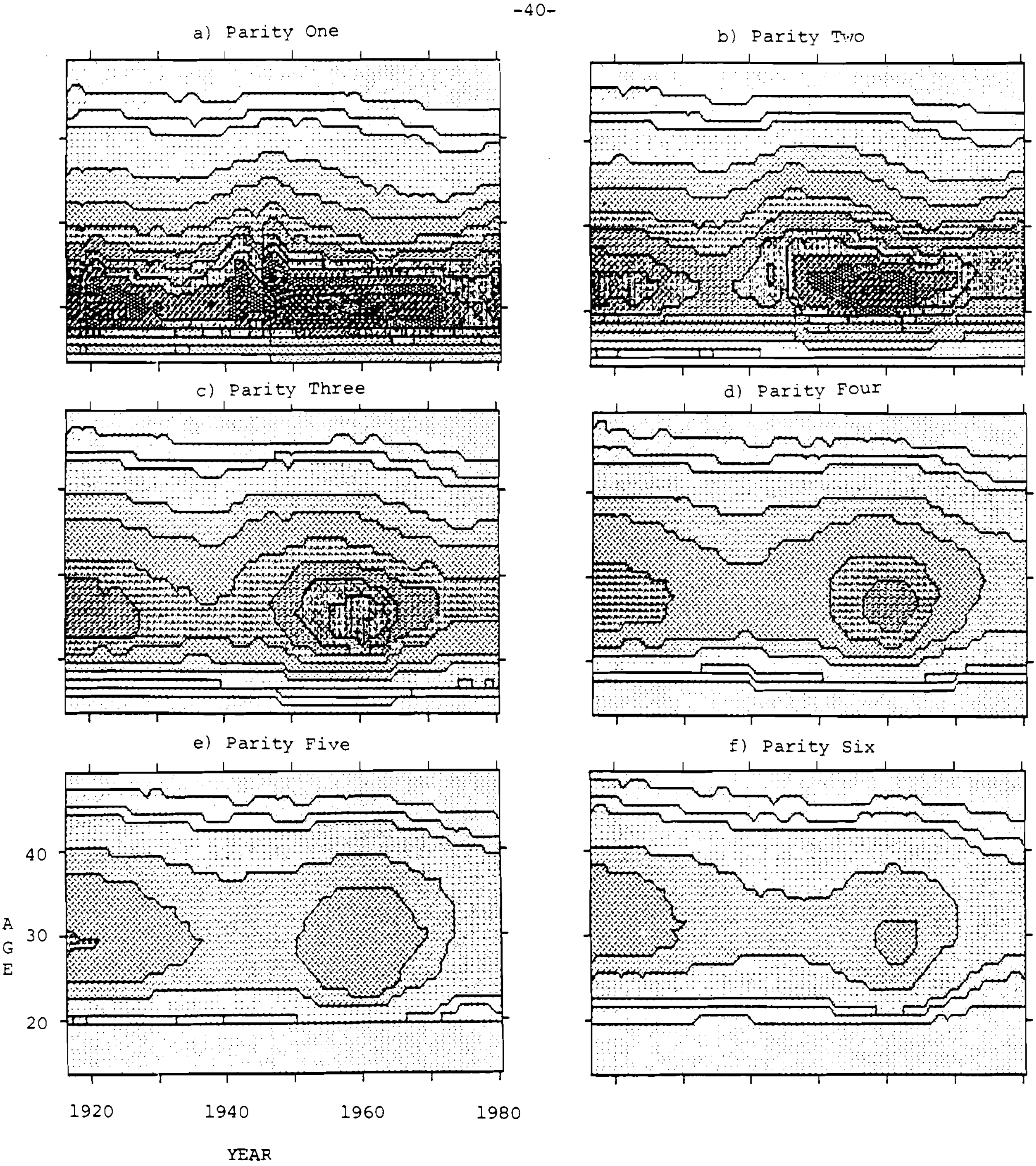
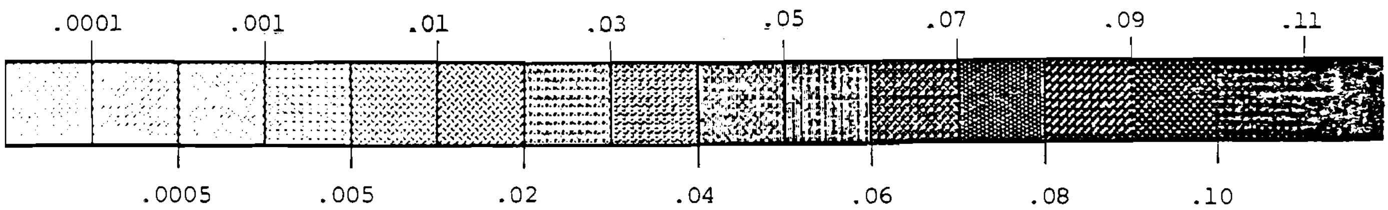


Figure 20 : U.S. Birth Rates at Parities 1 to 6 with contour lines selectivelv placed from .0001 to .1
From age 14 to 49 and year 1917 to 1980



second children, fertility rates at the peak age (around 20 for first births and 22 for second births) were about five percentage points higher around 1960 than they were around 1935, whereas for fourth and higher birth orders, the absolute difference was only one or two percentage points. As shown on Figure 5, the absolute difference between the total fertility rate at age 22 between 1935 and 1960 was about 14 percentage points, so that the change in first and second birth rates accounted for roughly two-thirds of the total change.

One aspect of the recent baby boom that is strikingly revealed by the maps in Figure 20 is that the peak of the boom occurred around 1960 at all six parities, at ages that drift upward from about 20 in the case of first births to about 30 in the case of sixth births. The maps also show the recent trend toward greater first and second birth rates among women in their late 20's and in their 30's, although fourth and higher-order birth rates for these women (as well as women at other ages) continue to decline. In general, the maps provide further evidence of the importance of age and period effects but substantial cohort effects are not apparent.

Figure 21 presents another illustration of the use of small multiples, this time to compare mortality rates from age 15 to 49 for years 1910 to 1965 for Italian, French, and Belgian males and females. The effects of the First and Second World Wars are most prominent in this period and for these ages and the maps graphically depict the different impact of the two wars among the two sexes in the three countries. The diagonal swath of high mortality for Italian males born around the turn of the century, discussed earlier, does not show up on any of the other five maps in Figure 21. The prevalence of many small blocks on the Belgian maps indicates some scattered noise or local fluctuations, perhaps attributable to the smaller size of the Belgian population compared with that of Italy or France or to irregularities in the available mortality data.

Figure 22 displays relative mortality rates for males and for females in England and Wales, Sweden, and Italy from age 5 to 79 for years 1870 to 1978. In each case, the mortality rate for a given age is relative to the rate at that age in 1870. Thus the maps provide a picture of the pattern of progress made in reducing mortality rates since 1870. The maps are analagous to the tables with rough contour lines in Kermack, McKendrick, and McKinlay (1934); Preston and van der Walle (1978) and Coale and Kisker (1985) present similar tables. These analysts ascribe the diagonal contours in their tables to cohort effects.

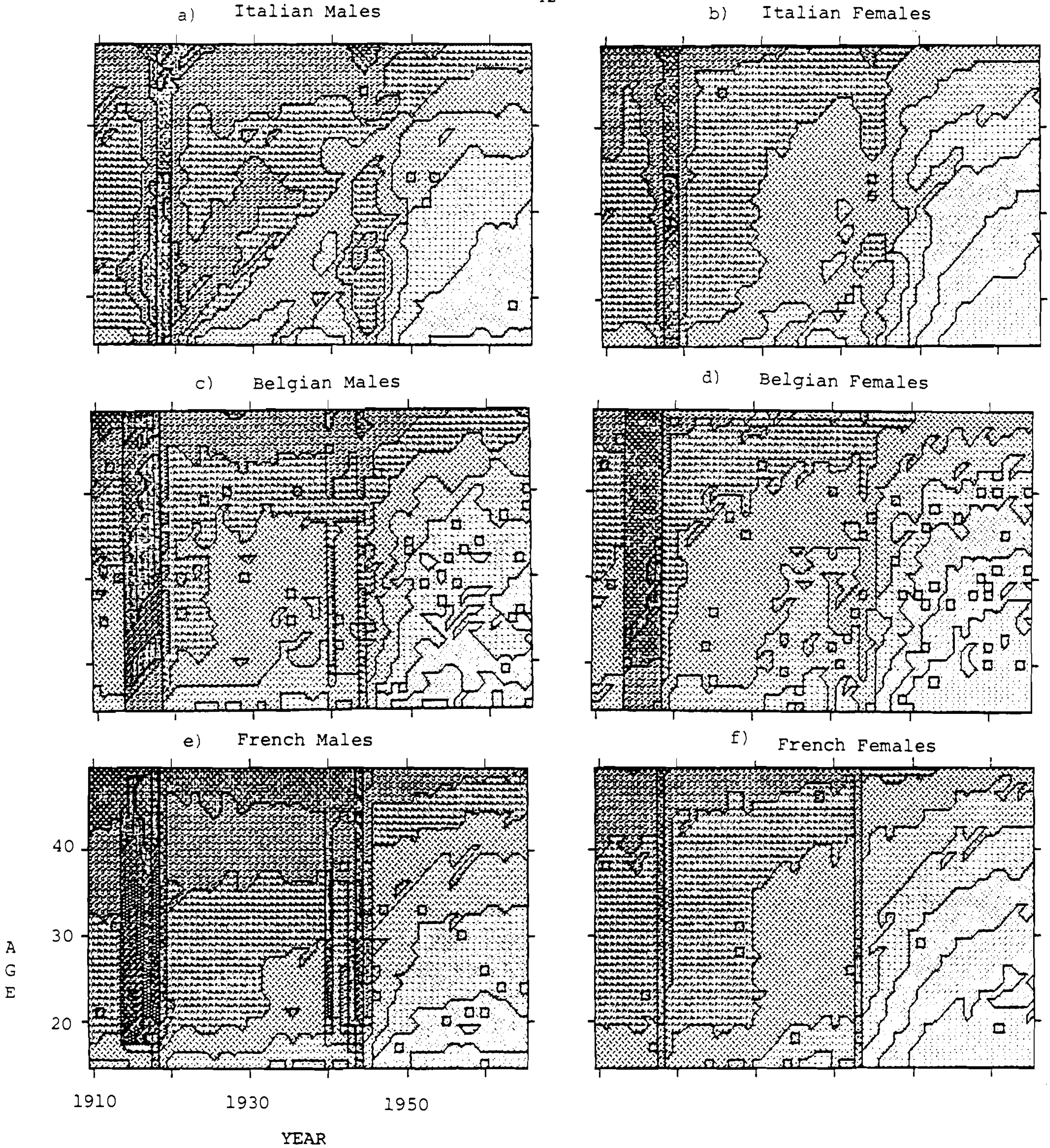
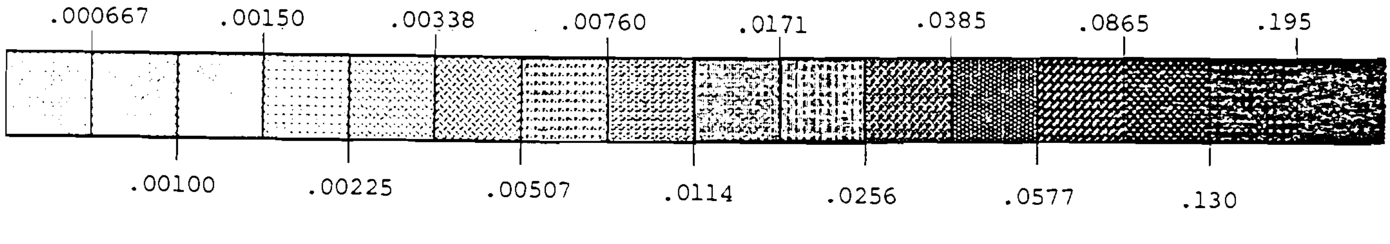


Figure 21: Mortality Rates in Italy, Belgium and France - with contour lines from .000667 to .195 at multiples of 1.5
From age 15 to 49 and year 1910 to 1965



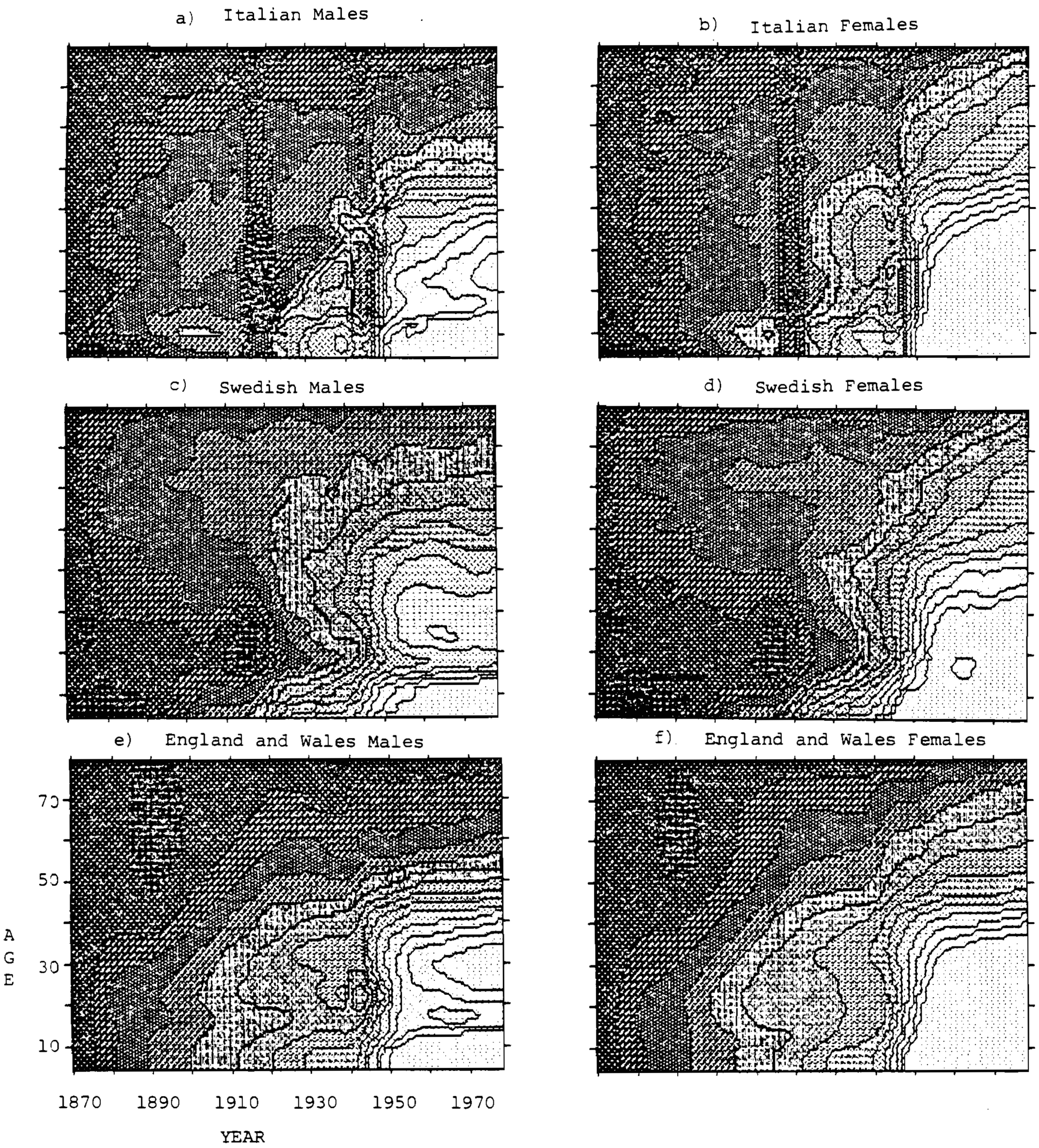
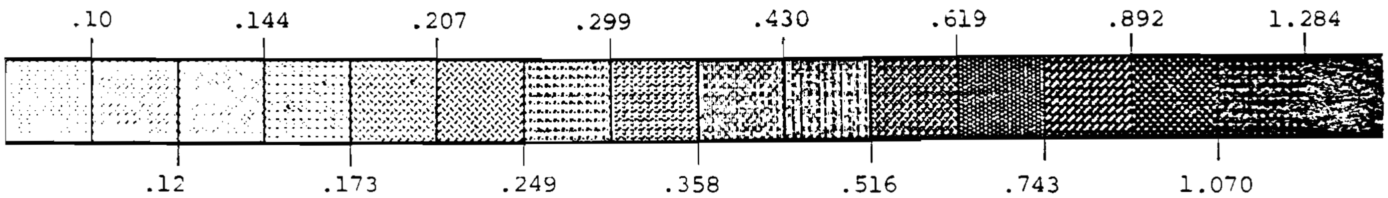


Figure 22 : Mortality Rates Relative to 1870 Age-specific Levels with contour lines from .1 to 1.28 at multiples of 1.2, smoothed on a 5 by 5 square From age 5 to 79 and year 1870 to 1978



The maps in Figure 22 provide a richer, more detailed picture of the various local and global patterns in the changes in mortality rates, in vertical, horizontal, and diagonal directions. Some diagonal trends are evident, especially for females in England and Wales, but it is also evident that age and period effects play a substantial role in the evolution of mortality. The light rectangles in the lower right corners of all six maps show the rapid progress in reducing mortality at younger ages since World War II; the darkness of the remaining three quadrants of the maps displays the slow rate of progress at older ages and during earlier years. Note that the light rectangles in the lower right corners are divided, for males but not for females, by a a darker stripe, indicating relatively slow progress against mortality, at roughly ages 16 to 20.

The maps in Figure 22 for the three countries were produced using different kinds of data. As noted earlier, mortality rates for Italy were available for single years of time and age. For Sweden, the rates were generally available for five year periods; before 1880 the rates were for five year age categories and afterwards for single years of age. Finally, for England and Wales, the rates were available for five year age categories for single years of time about once per decade. Differences in the smoothness of the maps, especially for England and Wales compared with Italy, are probably largely attributable to these differences in data richness.

In the analyses of Kermack, McKendrick, and McKinlay, Preston and van der Walle, and Coale and Kisker, mortality rates were taken relative to a period earlier than 1870, the underlying assumption being that at an early enough period there would have been no systematic pattern of progress against mortality. Figure 23 shows mortality rates for Swedish males and females relative to the average levels at each age in the period from 1778 to 1799. The figure reveals the fluctuating pattern of mortality before the middle of the nineteenth century and the general pattern of progress against mortality subsequently. The pattern is clearly more complex than a pure cohort-effect model would suggest.

A direct way of considering the hypothesis that "a cohort carries its mortality level with it" is to examine mortality surfaces that are calculated relative to a cohort's mortality levels. In Figure 24, for instance, in each of the six surfaces shown the mortality rate at each age and year was divided by the mortality rate at that age for the cohort born in 1870. The pattern that emerges shows some strong diagonals, but it is apparent that there are also important effects in horizontal and vertical directions. Interpretation of these and similar surfaces should also be

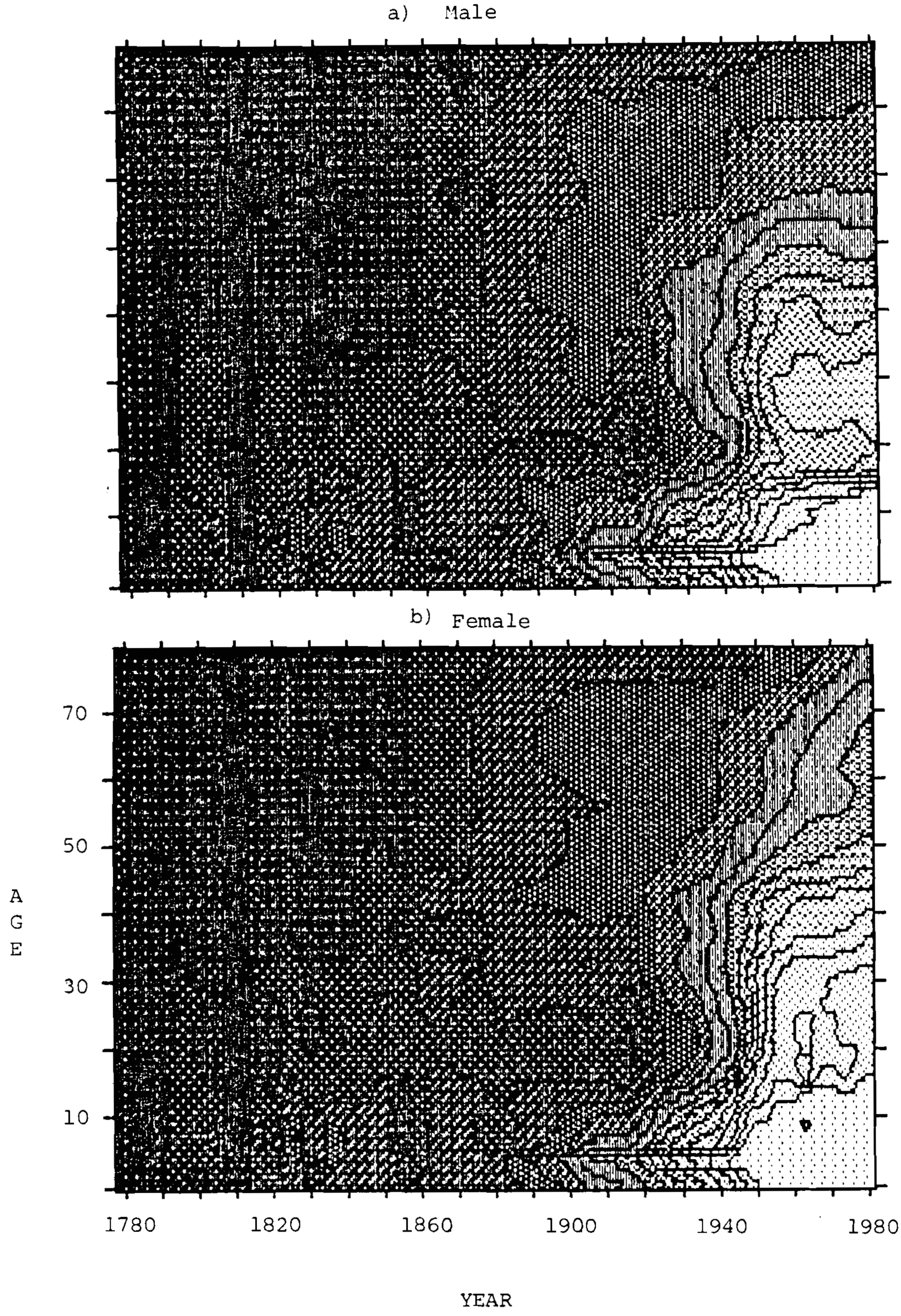
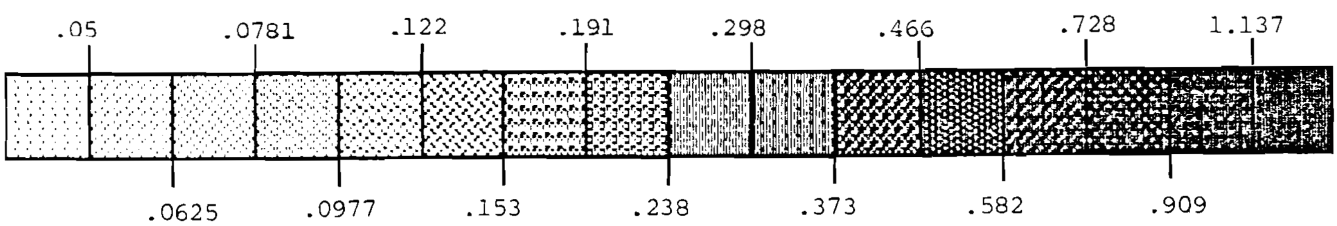


Figure 23: Swedish Mortality Rates Relative to Age-specific Levels from 1778 to 1799 with contour lines from .05 to 1.137 at multiples of 1.15, smoothed on a 5 by 5 square From age 0 to 79 and year 1778 to 1981



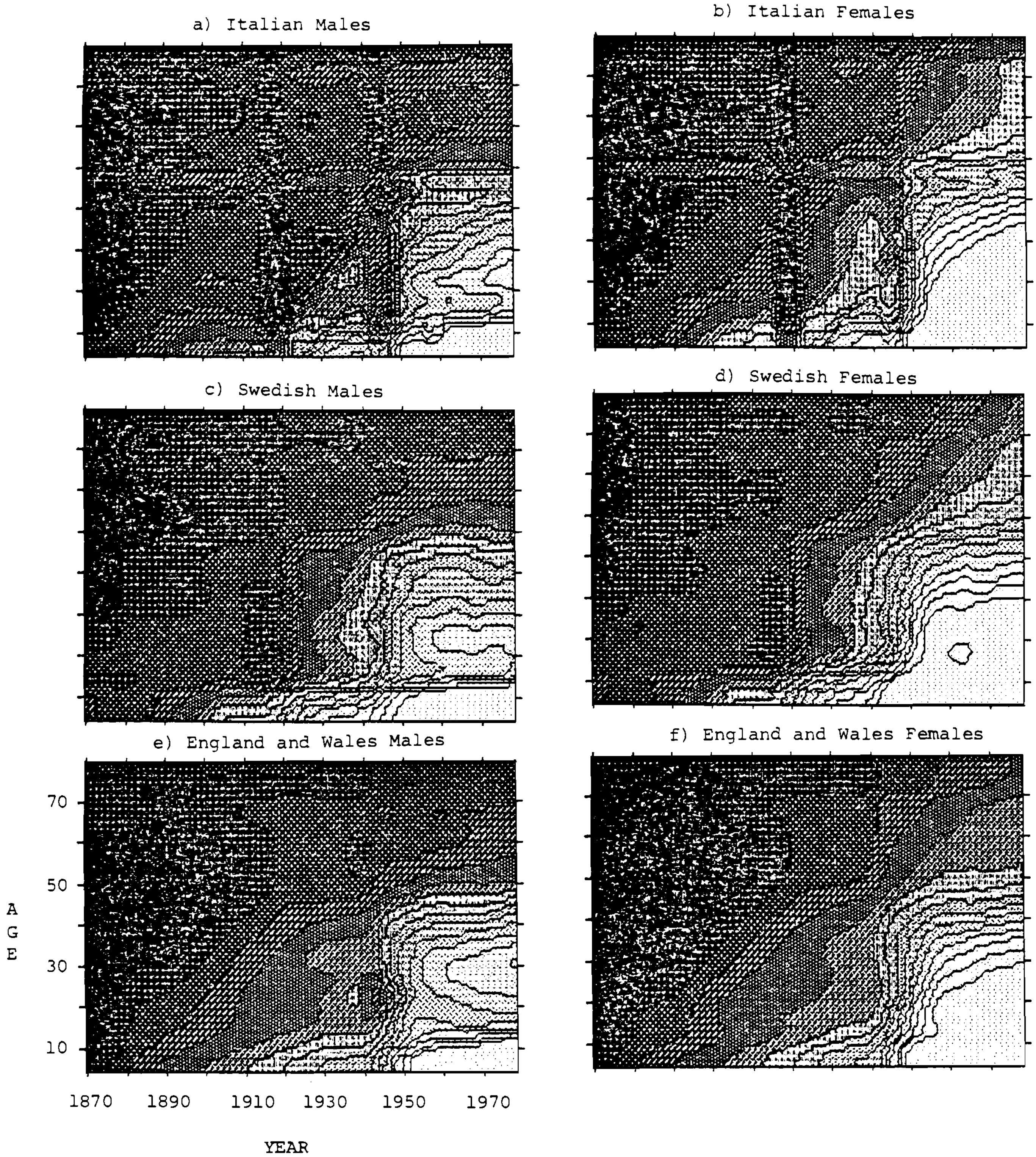
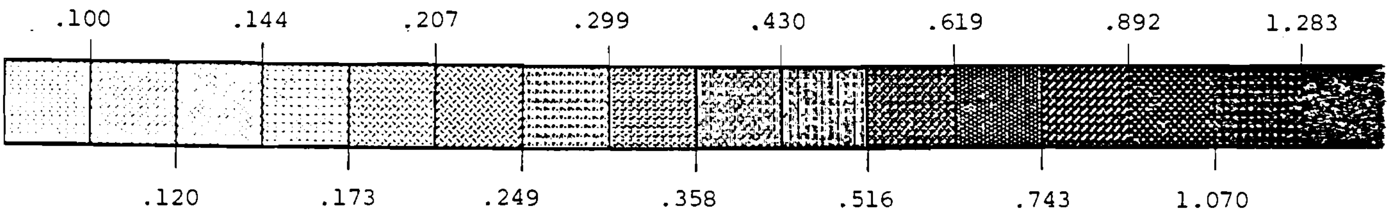


Figure 24 : Mortality Rates Relative to 1870 Age-specific Cohort Levels - with contour lines from .1 to 1.28 at multiples of 1.2, smoothed on a 5 by 5 square From age 5 to 79 and year 1870 to 1978



tempered by realization that diagonal patterns can emerge not only as a result of cohort effects but also as the result of the interaction of period and age effects.

RATIO SURFACES

Instead of using small multiples, another approach to comparing two or more demographic surfaces is to compute some new surfaces that represent at every point either the difference or the ratio of the height of one of the original surfaces to another. Figure 25a, for instance, shows the ratio of male to female mortality rates in Italy, smoothed on a 5 by 5 grid. To highlight the ages and periods when Italian male and female mortality rates were roughly equal, Figure 25b presents a modified version of this map in which only three contours lines are drawn, for equal male and female death rates and for levels ten percent above and below equality. The two figures reveal the worsening discrepancy between male and female mortality. In 1870, male and female mortality rates were roughly comparable at most ages, female mortality being somewhat more than 10 percent higher than male mortality around age 30 and male mortality being somewhat more than 10 percent higher than female mortality around age 50. By 1979, female mortality was substantially less than male mortality at all ages, being only half as high as male mortality between ages 15 and 30 and again between ages 40 and 70.

Two further ratio surfaces are presented in Figures 26 and 27. Figures 26a and b display the ratio of Italian male mortality to French male mortality from age 10 to 70 for years 1900 to 1960. The ratio fluctuates busily, producing the large number of contour lines shown in Figure 26a; in Figure 26b the contour lines are suppressed. Like two hollow eyes in a skull, the black rectangles reveal the greater toll of death among young French males relative to young Italian males during the two World Wars. The lighter shades on the bottom of the maps, under age 20 or so before the First World War and after the Second, and under about age 35 between the wars, show when French males were experiencing less mortality than their Italian counterparts; at older ages, it is the Italians who have the advantage. Note that the lighter shades on Figure 26a are somewhat masked by the contour lines; Figure 26b, by suppressing these lines, makes the region of French advantage more apparent.

Figure 27 shows the difference, in the United States, between first and second birth rates, i.e., the proportion of all women of some age at some time who are having their first child minus the proportion who are having their second child. The

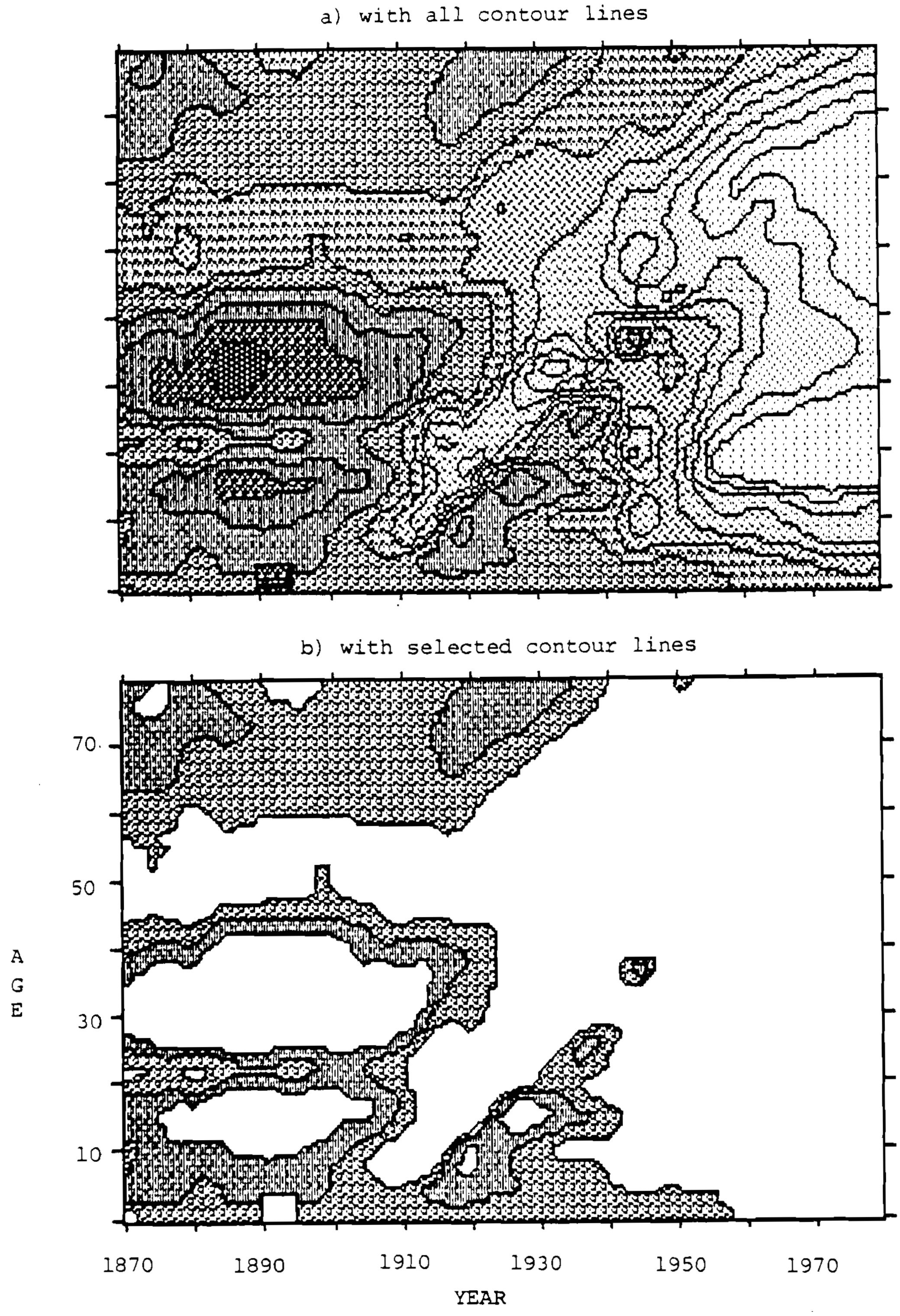
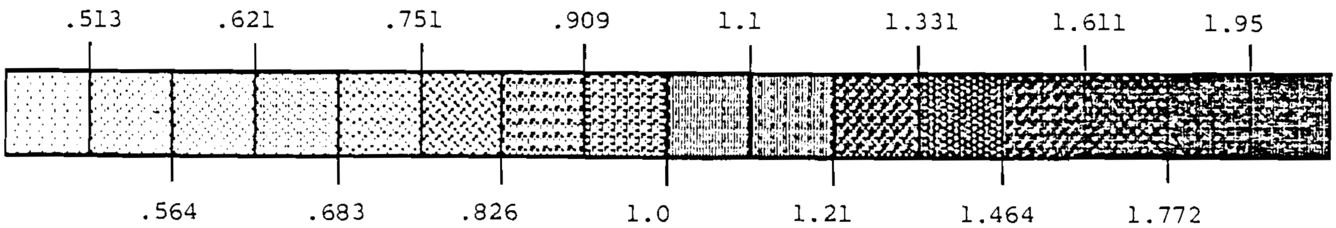


Figure 25: Italian Female Mortality Rates Divided by Italian Male Mortality Rates with contour lines from .51 to 1.95 at multiples of 1.1
From age 0 to 79 and year 1870 to 1979



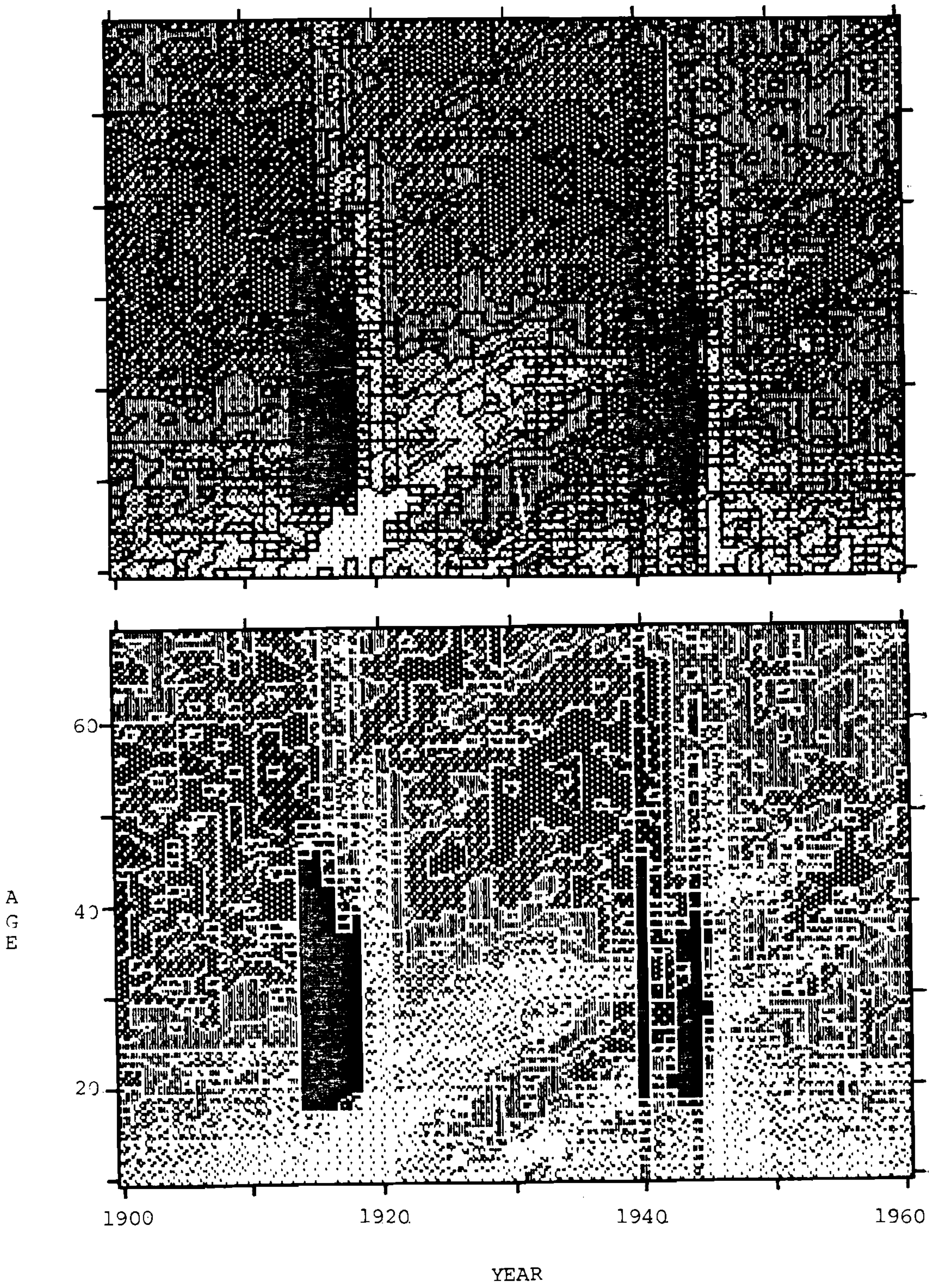
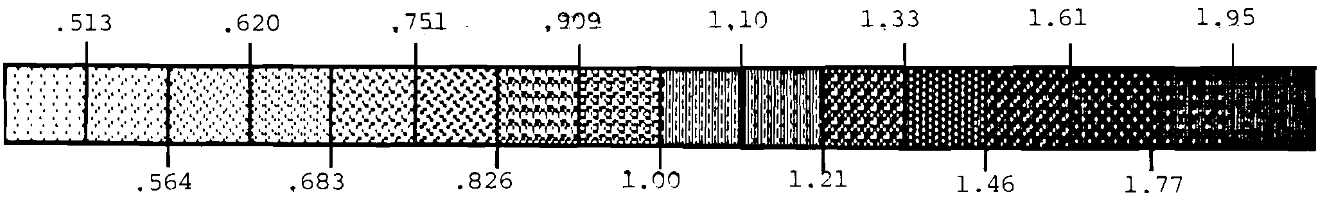


Figure 26: French Male Mortality Rates Divided by Italian Male Mortality Rates - with contour lines from .51 to 1.95 at multiples of 1.1
From age 10 to 70 and year 1900 to 1960



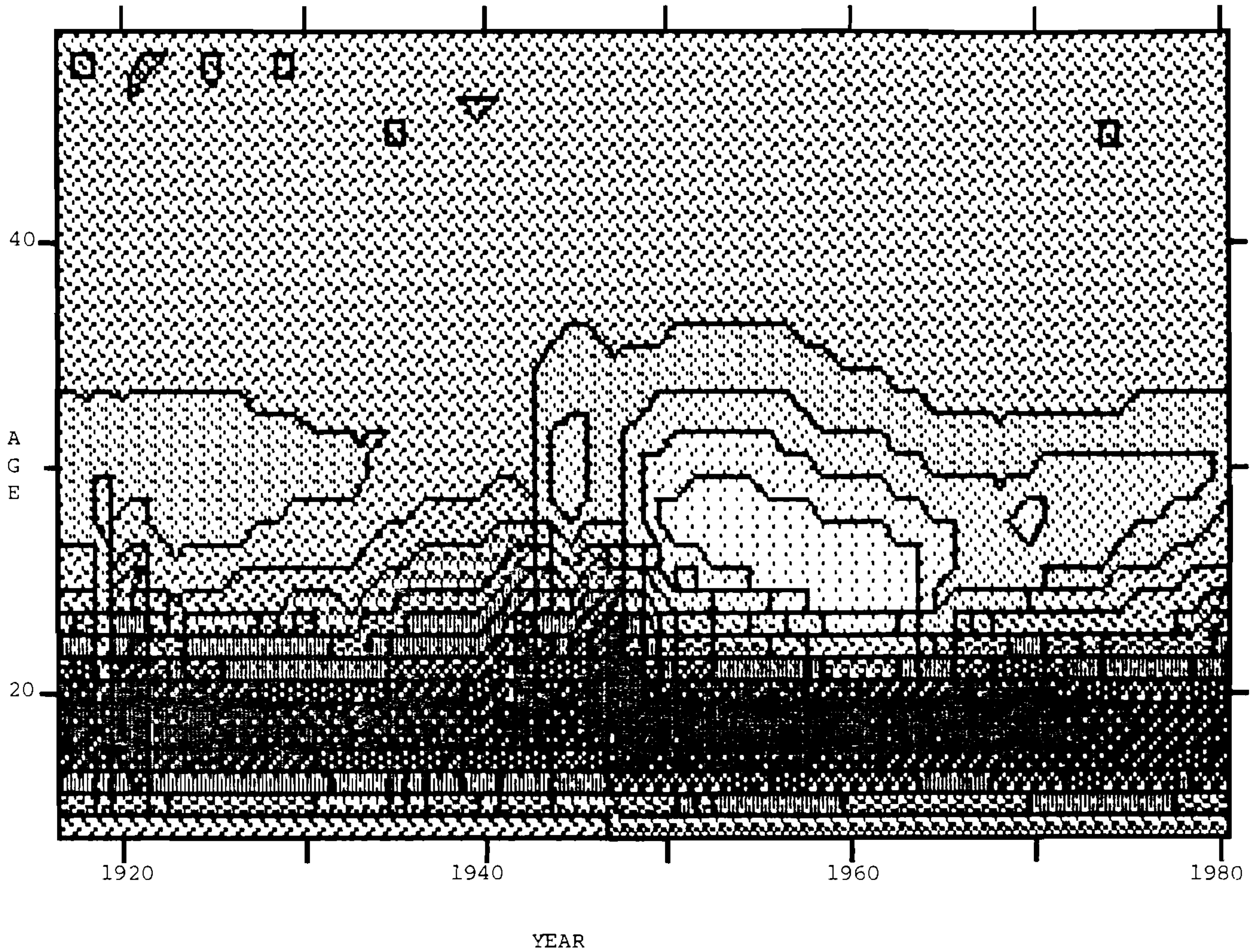
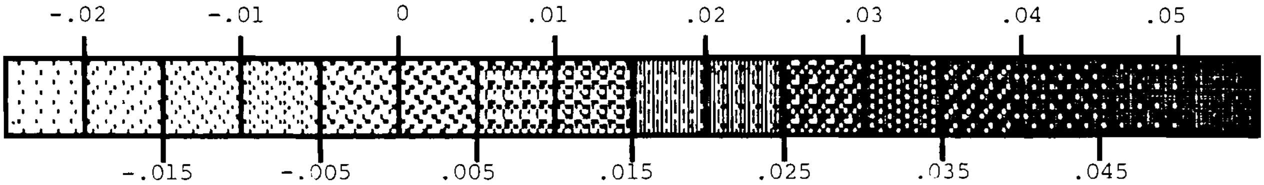


Figure 27: U.S. Fertility Parity One Minus Parity Two - with contour lines from -.02 to .05 at intervals of .005
From age 14 to 49 and year 1917 to 1980



most striking feature of the figure is the rapidity of the shift from the predominance of first births to the predominance of second births, as indicated by the dark horizontal swath giving way to a light swath. The mid-point of this shift is described by the contour at level zero running across the map at roughly age 25. The contour reaches a peak at age 30 in 1940 and falls to age 22 in 1960. In 1960, at the center of the baby boom, the shift from first births to second births is not only early, but also especially dramatic.

SEX-RATIOS, NUPTIALITY, AND CAUSE-SPECIFIC MORTALITY

In addition to maps of death rates and birth rates and of population levels, contour maps can be drawn based on any other kind of data that is structured along two dimensions. Figures 28, 29 and 30 suggest three possibilities. Figure 28 displays the ratio of males to females in Belgium, based on Veys' (1983) data. An interesting contour to follow on the map is the contour at level 1: at ages below this line, males outnumber females, and at older ages females predominate. The line starts at age 43 in 1892, rapidly falls to age 20 during the First World War, gradually rises to back to age 43 or so by the mid 1940's, and then remains at roughly this age up through 1977. A striking feature of the map as a whole is the increasing predominance of females at older ages. Even in 1892, there were 25 percent more females than males at age 85, but by 1977, there were close to twice as many females at this age than males.

Figure 29 shows age-specific marriage rates for Italian females, based on preliminary, unpublished data supplied to us by Dr. Viviana Egidi of the Department of Demographic Science at the University of Rome. The rates give the proportion of women at different ages and times who marry; the denominator is not the number of unmarried women, but the total number of women. The most noticeable feature of the map is the stability of the contours over the period studied, from 1952 to 1980, especially before age 30. The concentration of marriage in a narrow band of ages is apparent: at age 22, roughly 8 or 10 percent of women are marrying, whereas by age 34 the proportion marrying has fallen off to under 1 percent.

Contour maps can also be used to display surfaces of cause-specific mortality. In Figure 30, for instance, rates of male mortality in England and Wales from tuberculosis are displayed for 80 years of age over about a century of time; the map is based on interpolations of data from Keyfitz and Flieger (1968). The high toll of mortality exacted by tuberculosis in the middle ages of life, especially from 30 to

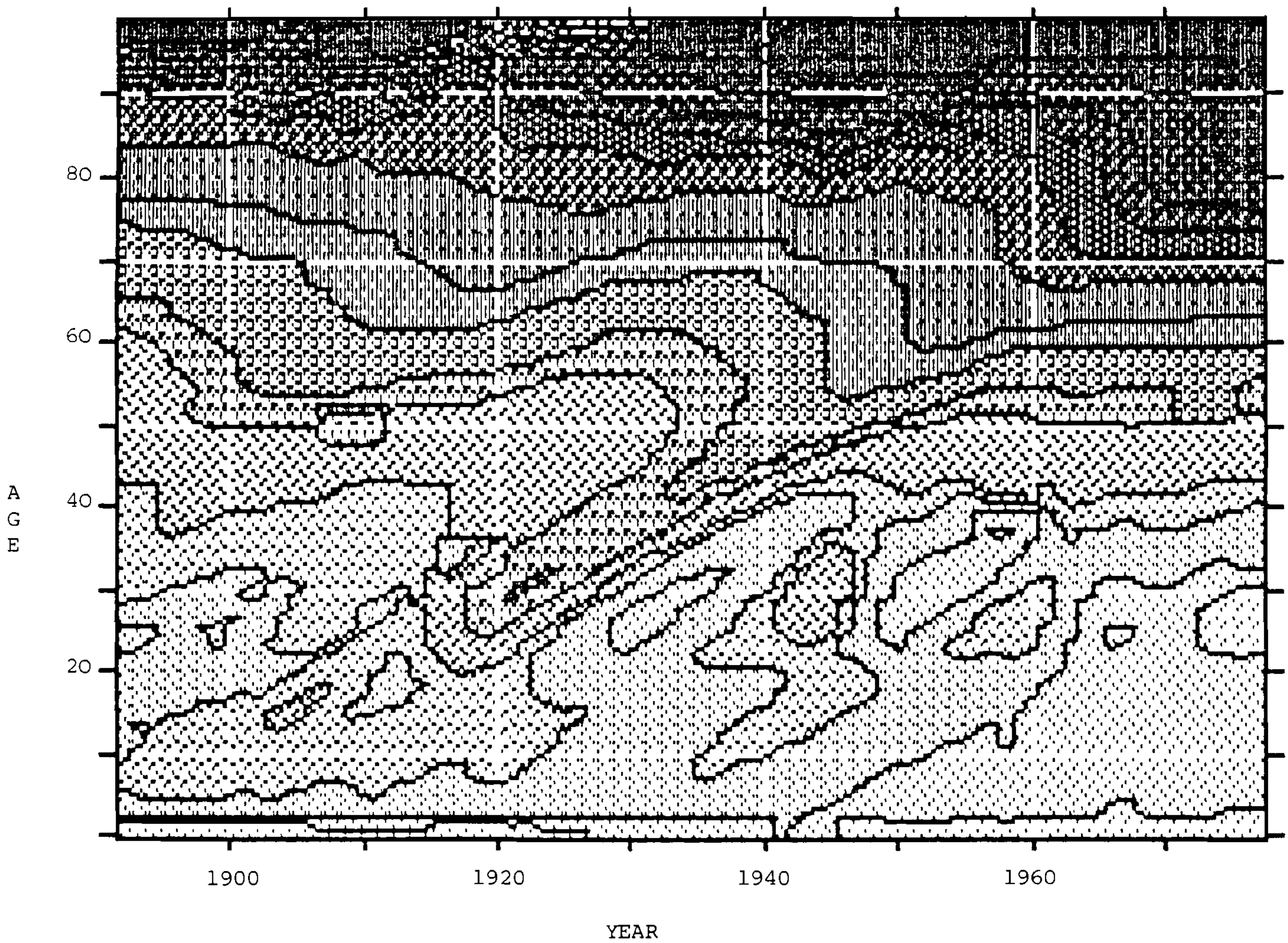
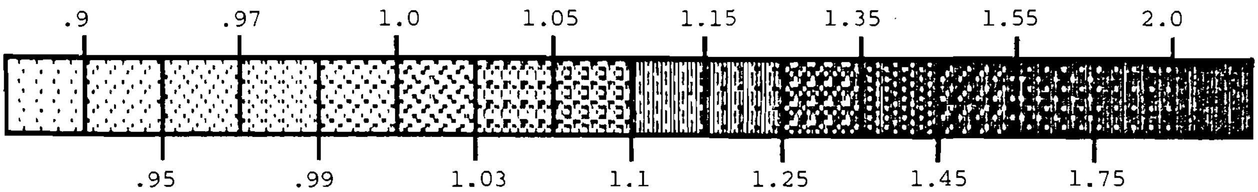


Figure 28: Belgian Female Population Divided by Belgian Male Population - with contour lines selectively placed from .85 to 2.0, smoothed on a 5 by 5 square
From age 0 to 99 and year 1892 to 1977



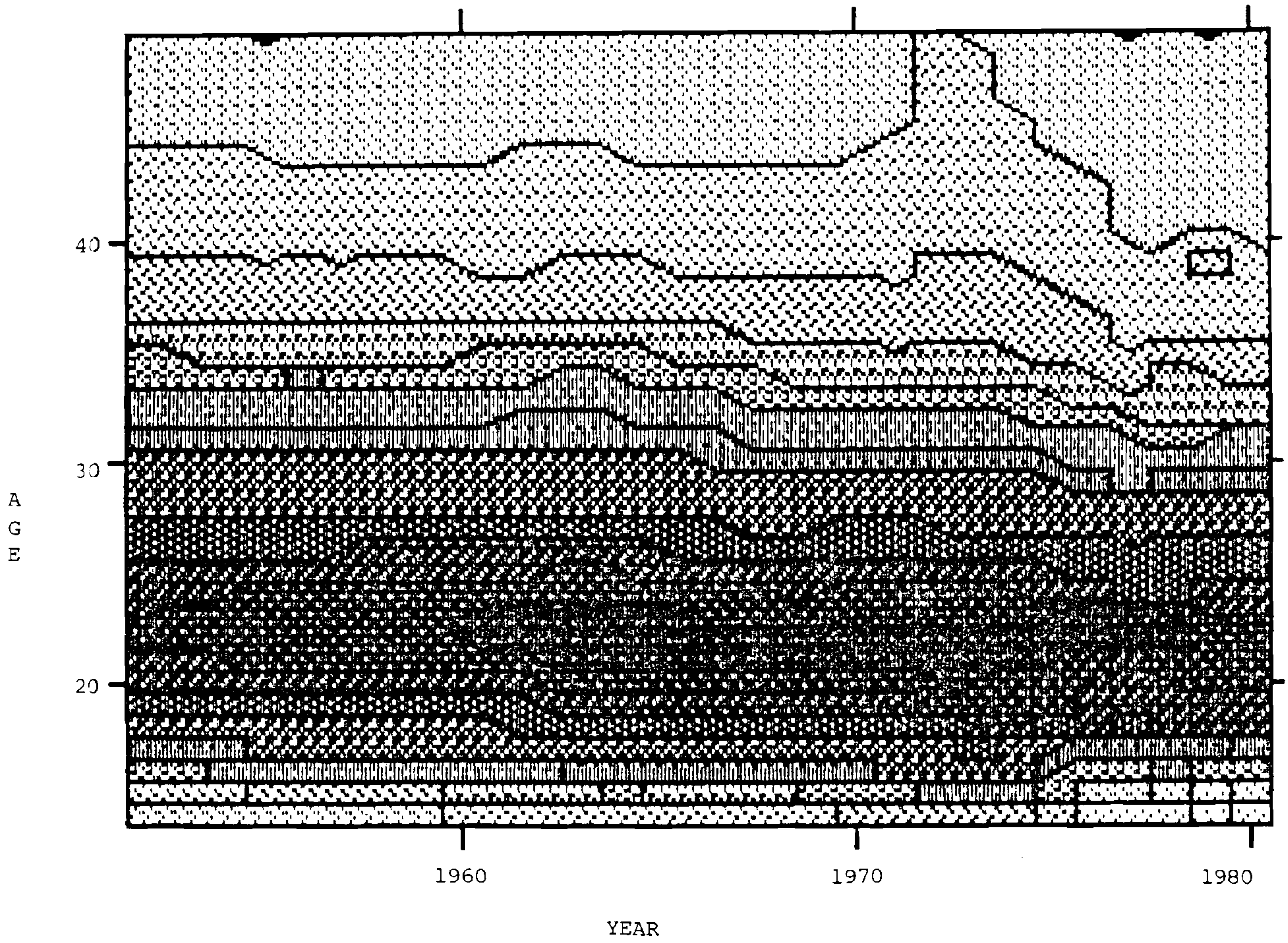
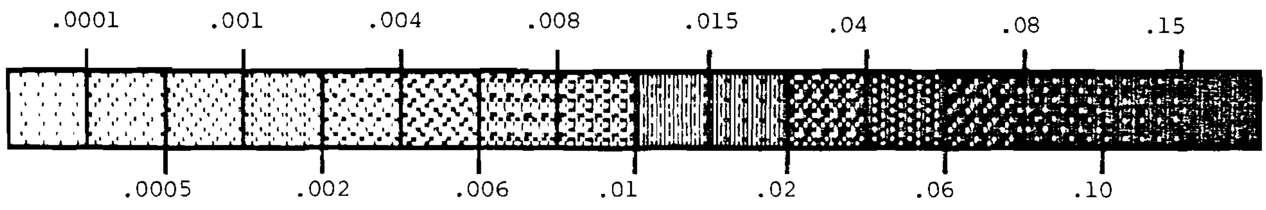


Figure 29: Age-specific Nuptiality of Italian Females - with contour lines selectively placed from .0001 to .15
From age 14 to 49 and year 1952 to 1980



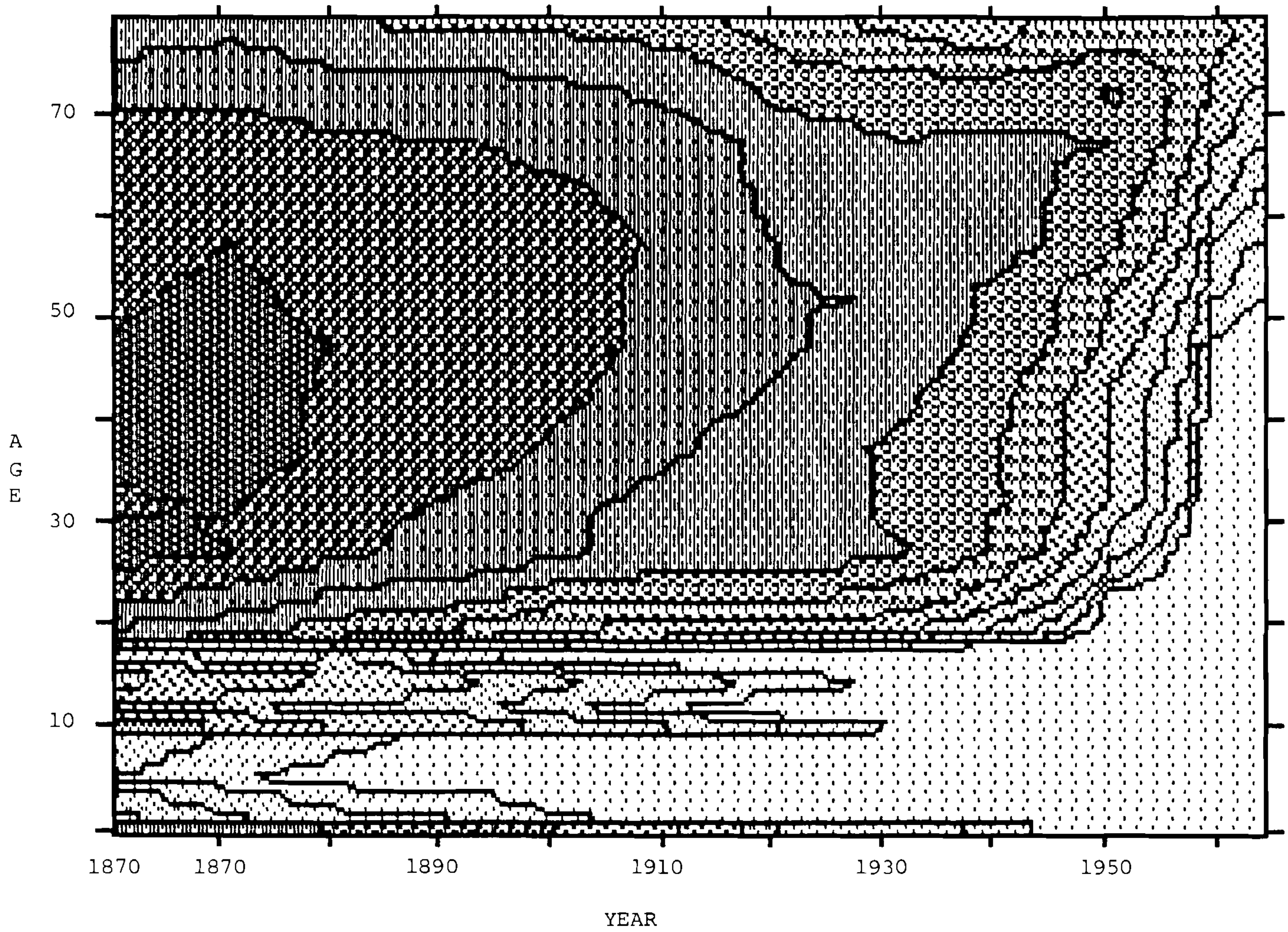
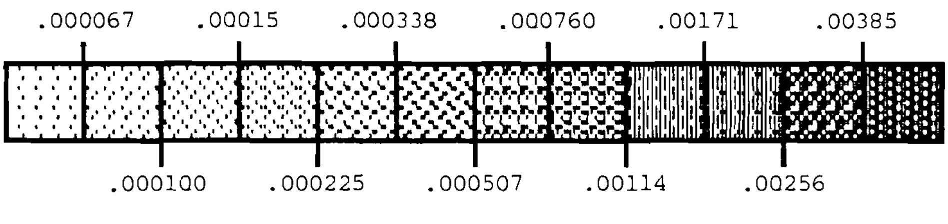


Figure 30: England and Wales Male Tuberculosis Mortality Rates - with contour lines from .000067 to .00385 at multiples of 1.5
From age 0 to 79 and year 1861 to 1964



60, is apparent on the map, as is the acceleration of progress against tuberculosis after the Second World War. Each contour line is 50 percent higher than the next, so this progress was very rapid indeed. Note also that there seems to be some evidence of cohort effects interacting with the period trend.

LIFE TABLE STATISTICS FOR BELGIAN FEMALES

Life tables often provide statistics by age and over time on population size, number of deaths, death rates, period survivorship, period life expectancy, and sometimes cohort survivorship. All six of these statistics are available, for example, in Veys' (1983) compilation of Belgian life tables from age 0 to 99 for years 1892 to 1977. Figures 31 through 36 use Vey's data for Belgian females to illustrate the different kinds of contour map patterns produced by different kinds of life table statistics. The differences in the patterns are quite striking, each kind of population statistic producing what might be called its characteristic pattern, its fingerprint:

- Lexis surfaces of population level tend to show strong diagonals at younger ages, bending over toward horizontal lines at advanced ages.
- Surfaces of deaths tend to be marked by two ridges running over time, one ridge (a cliff, really) highest in infancy, the other highest around age 70 or 80. As progress is made against mortality, the first ridge declines, but the second rises: deaths become increasingly concentrated around age 80. Perpendicular ridges mark periods of devastation, like World War I and its aftermath.
- Mortality rates lie in a valley between two steep walls of high mortality in infancy and at advanced ages. Again, perpendicular ridges mark disasters. Over time, as progress is made against mortality, the valley slopes off, most rapidly as younger ages, producing contours that sometimes look like flattened s's and sometimes like u's turned sideways.
- Survivorship and life expectancy falls off with age, more slowly in recent years because of the progress that has been made against mortality. Depending on the axes, there are either strong diagonals, gradually becoming less steep with age (when cohort survivorship is plotted over current year), or flatter diagonals, again becoming even flatter with age. The maps of survivorship and life expectancy tend to be smoother than the maps of deaths and mortality rates, the life expectancy maps being the smoothest. This is to be ex-

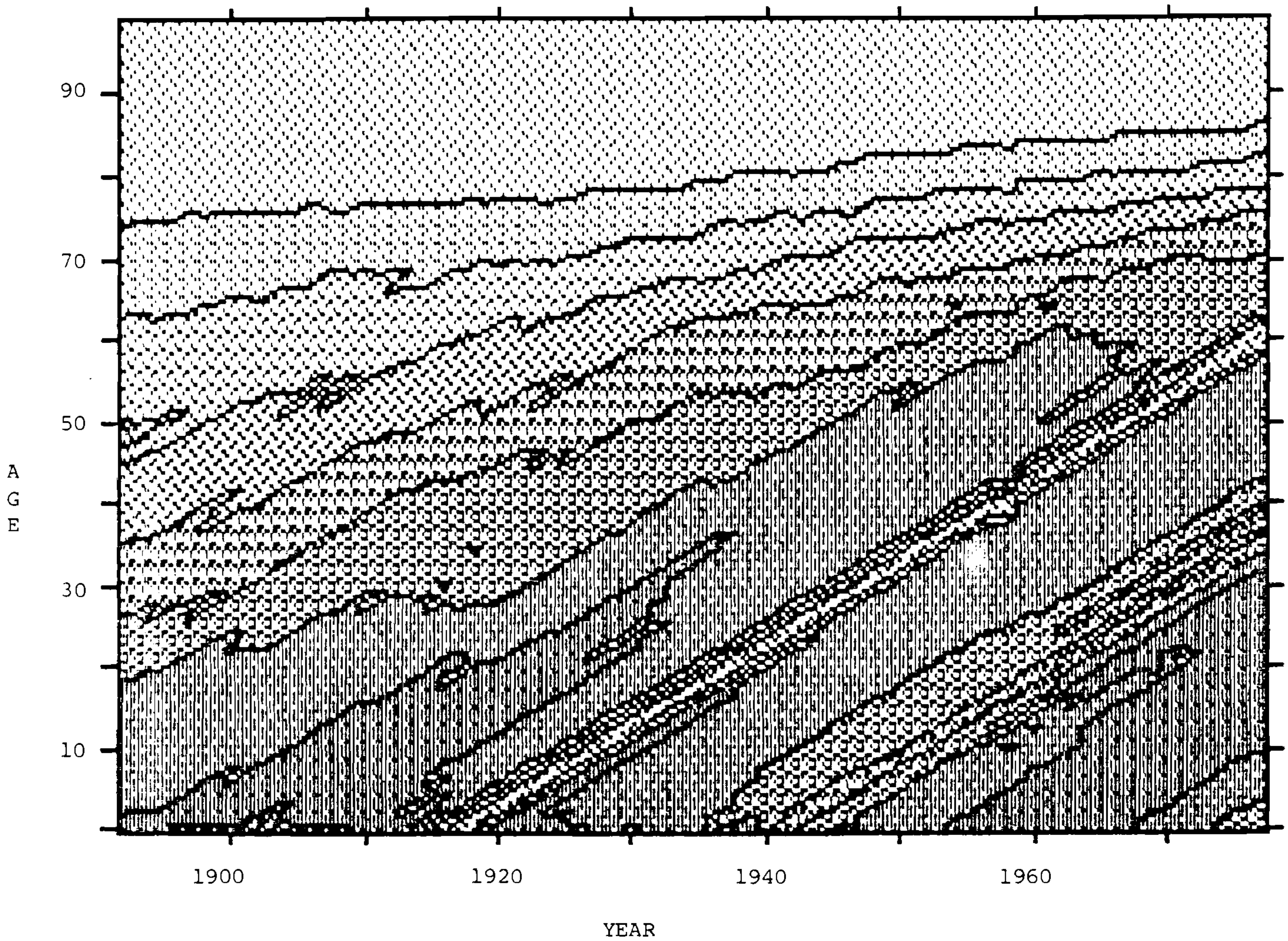
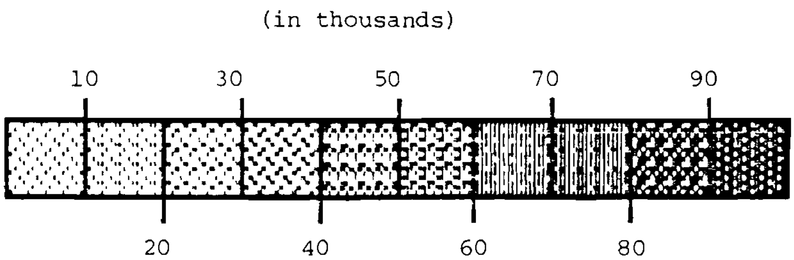


Figure 31: Belgian Female Population for Single Year of Age and Time - with contour lines from 10000 to 90000 at intervals of 10000
From age 0 to 99 and year 1892 to 1977



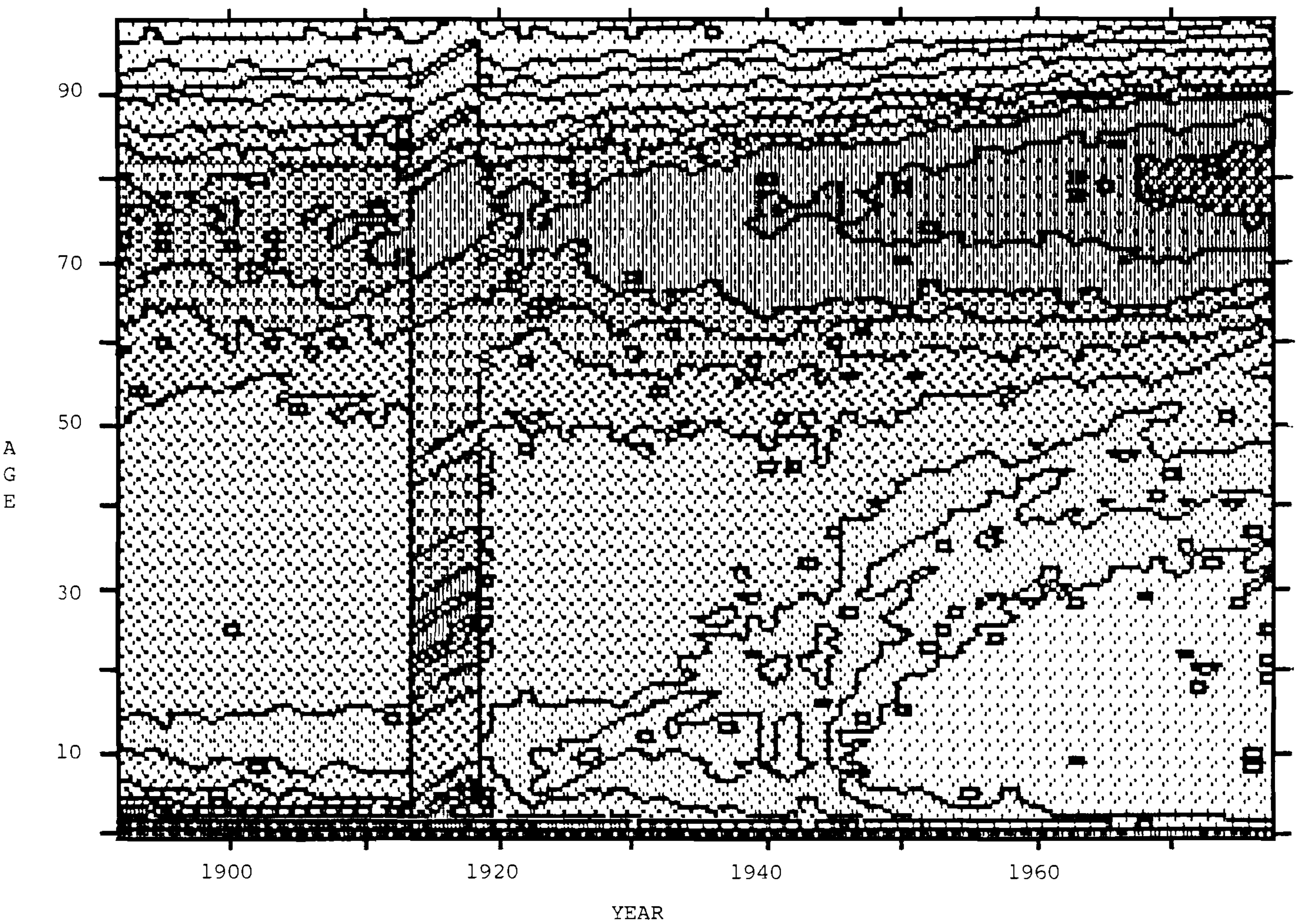
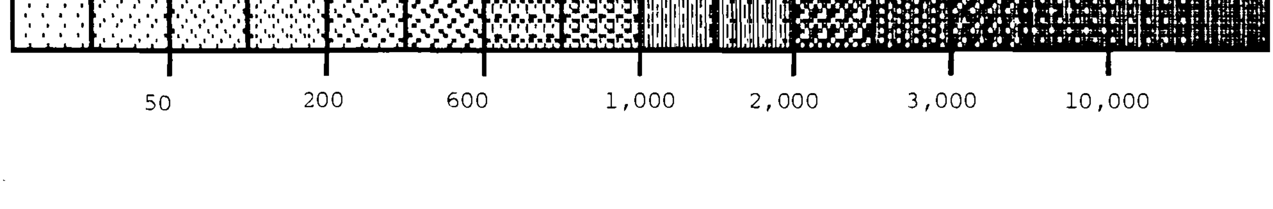


Figure 32: Belgian Female Deaths - with contour lines selectively placed from 10 to 15,000
From age 0 to 99 and year 1892 to 1977



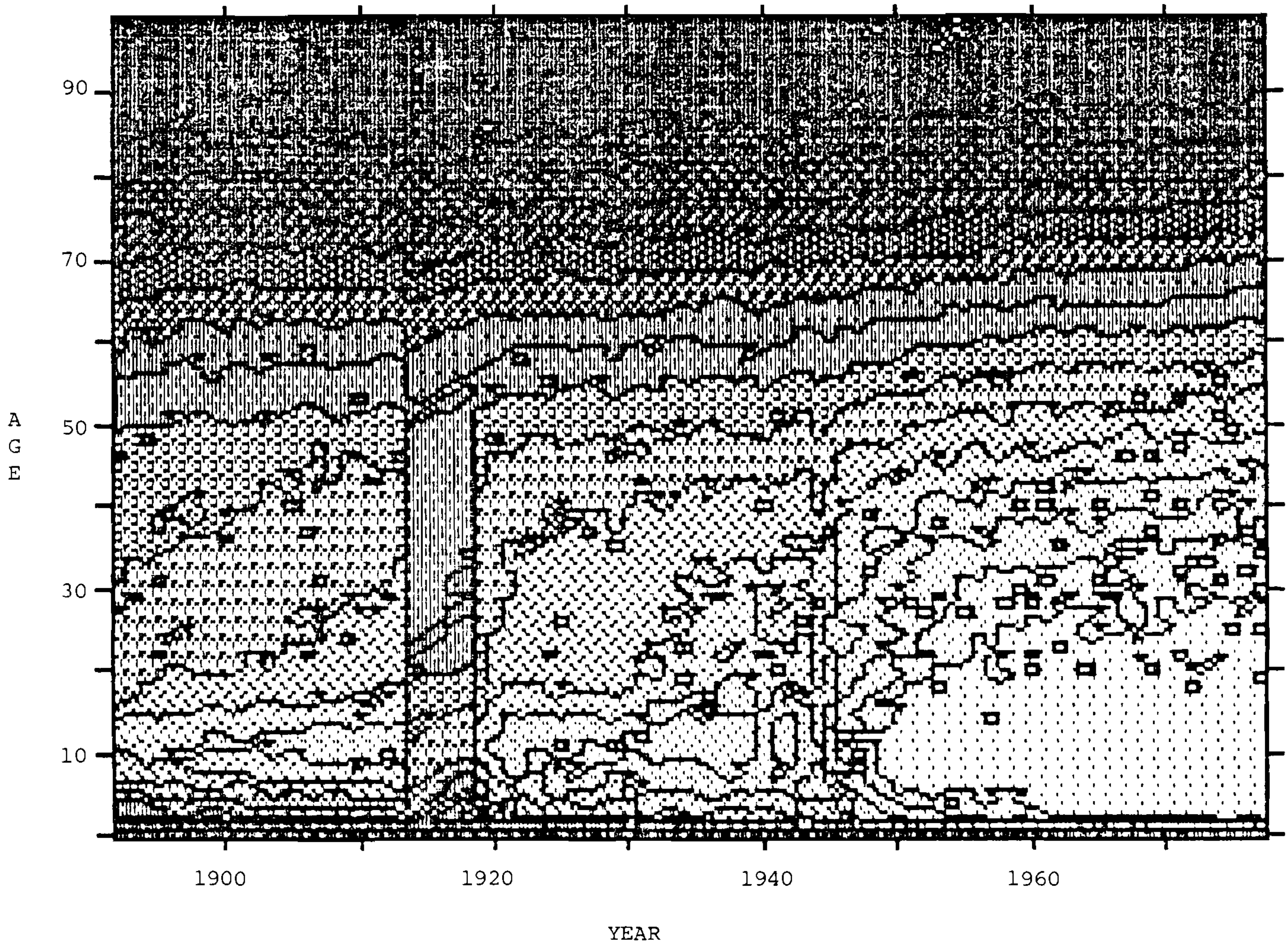
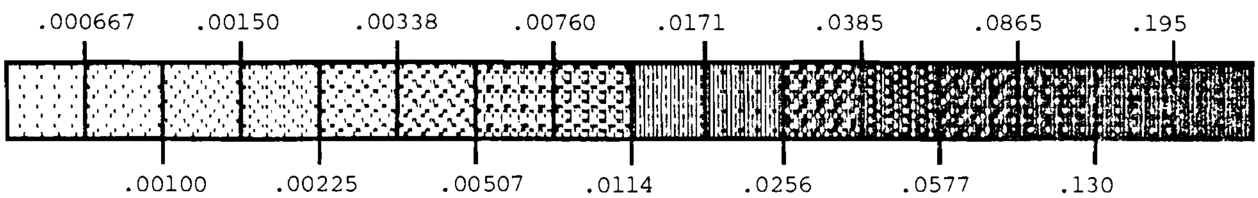


Figure 33: Belgian Female Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From age 0 to 99 and year 1892 to 1977



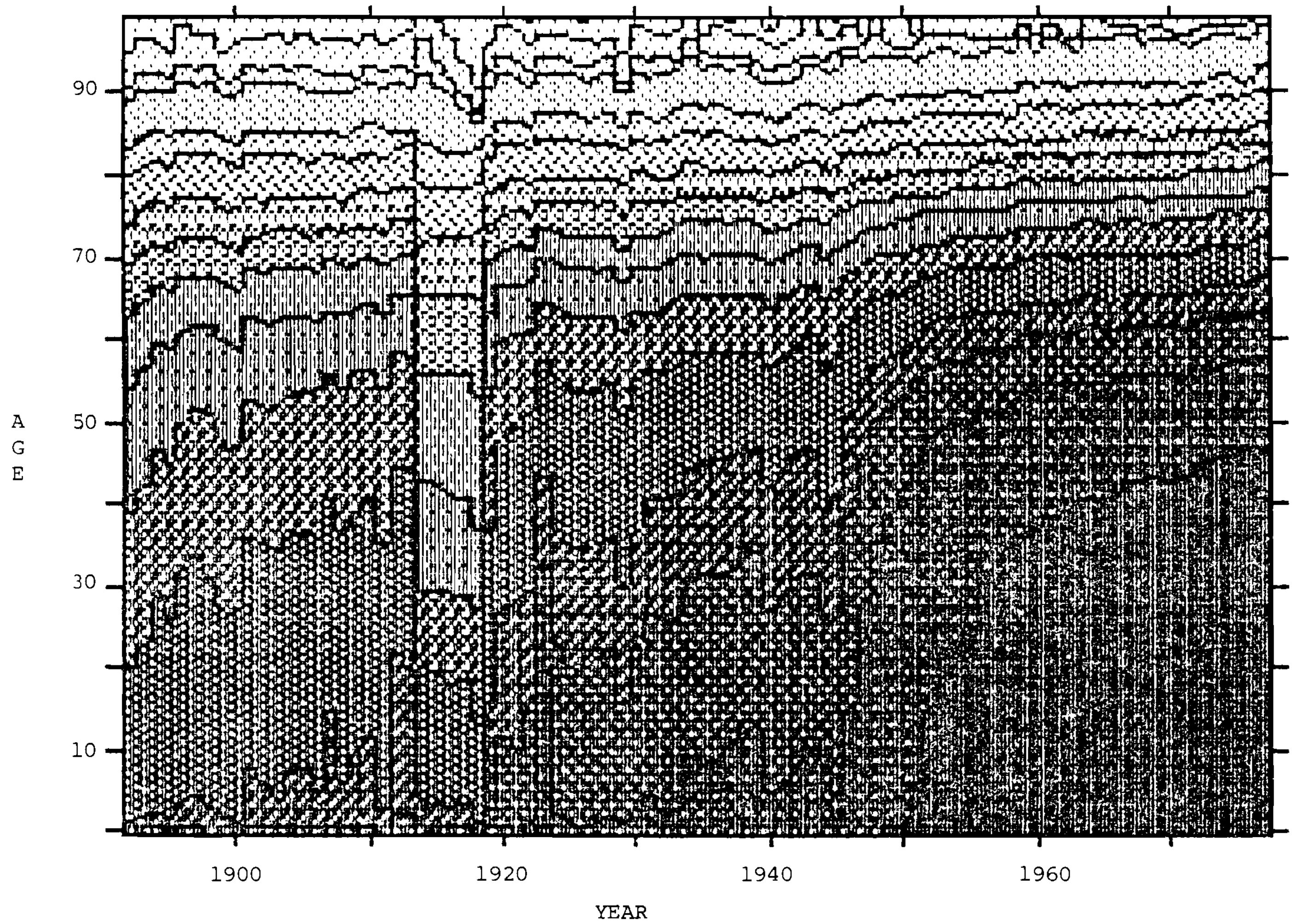
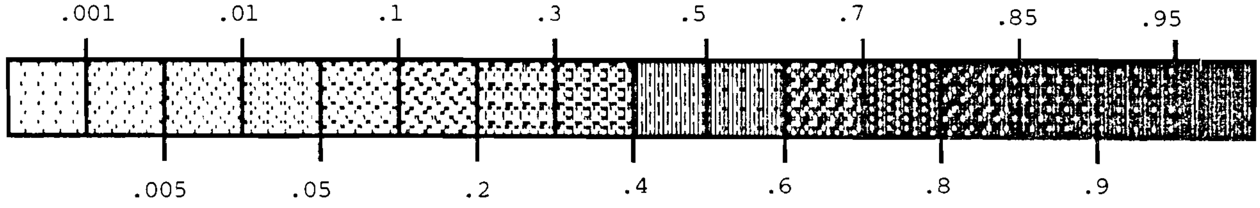


Figure 34: Belgian Female Period Survivorship - with contour lines selectively placed from .001 to .95
From age 0 to 99 and year 1892 to 1977



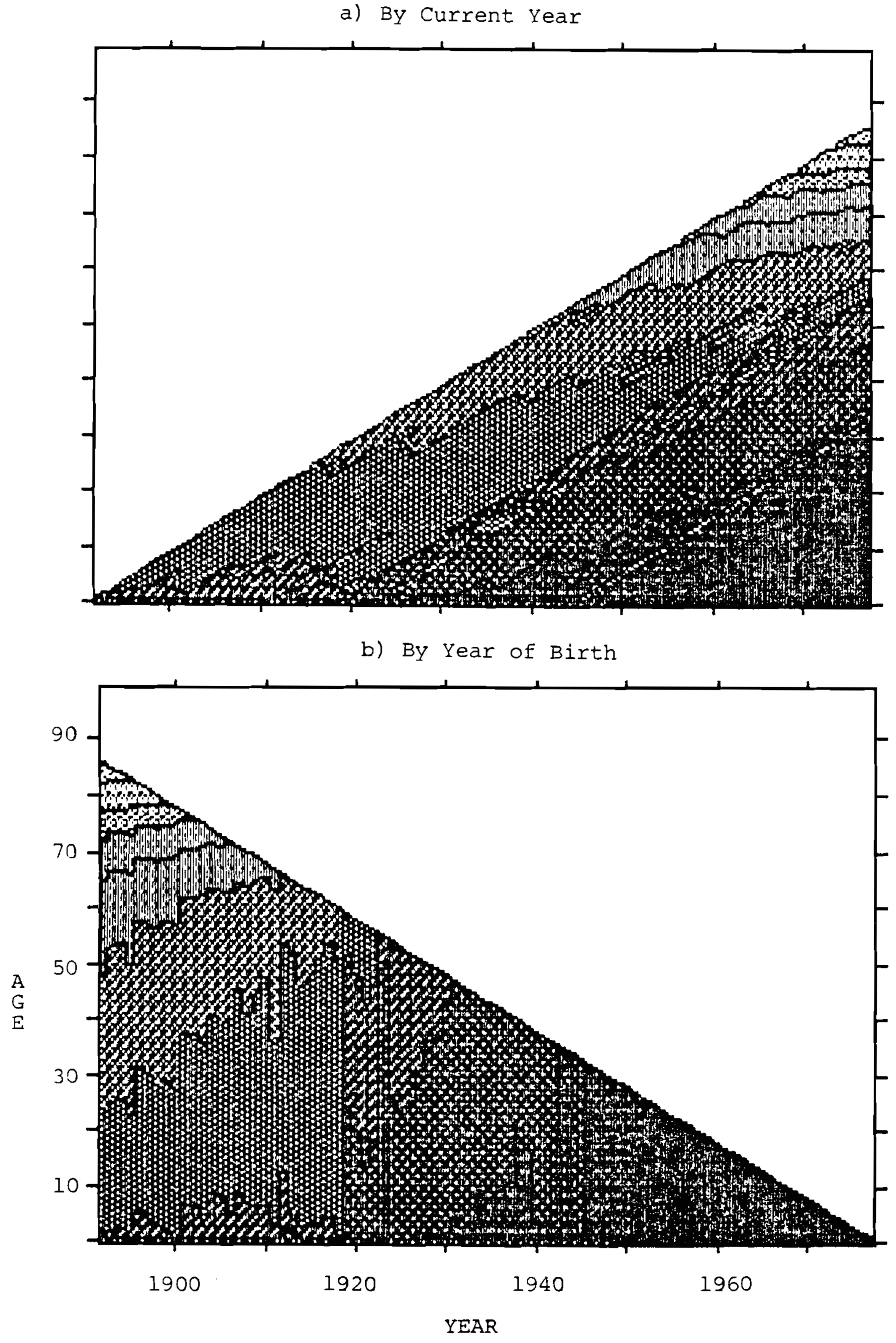
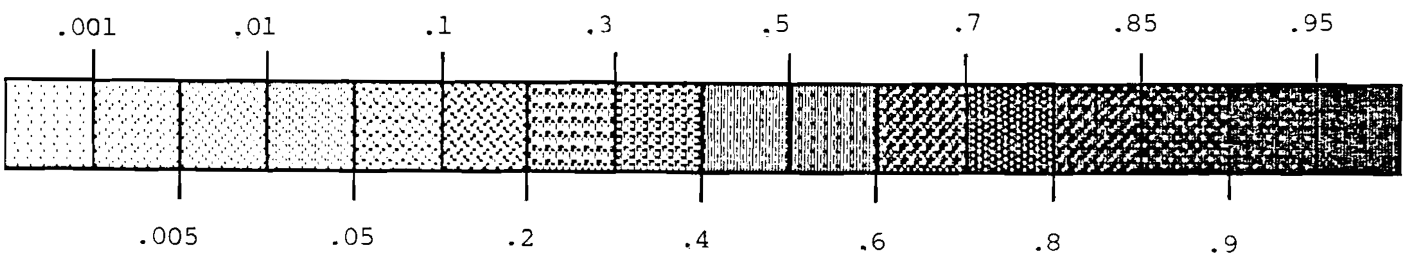


Figure 35: Belgian Female Cohort Survivorship by Current Year and Age, and Year of Birth and Age - with contour lines selectively placed from .001 to .95



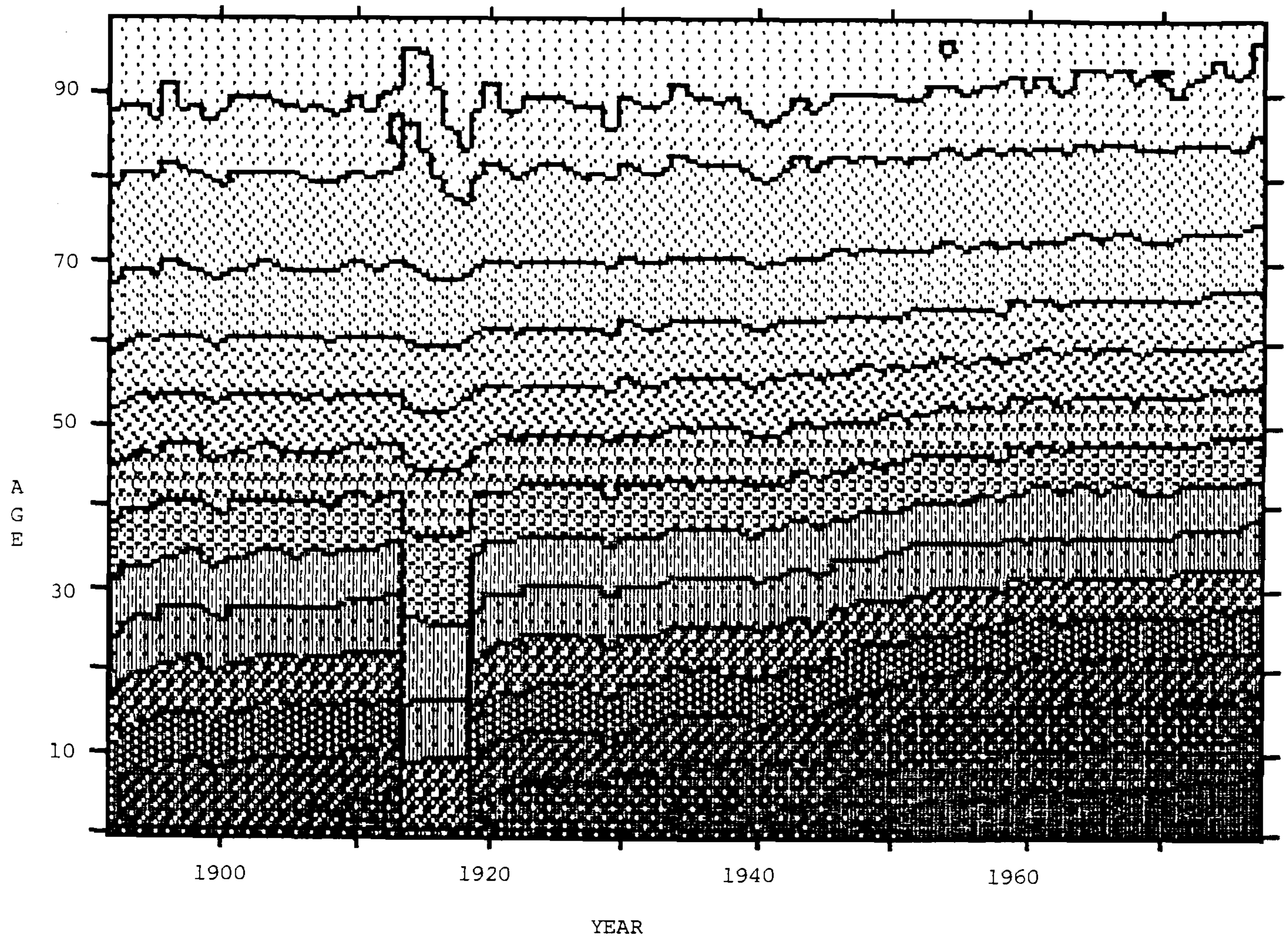
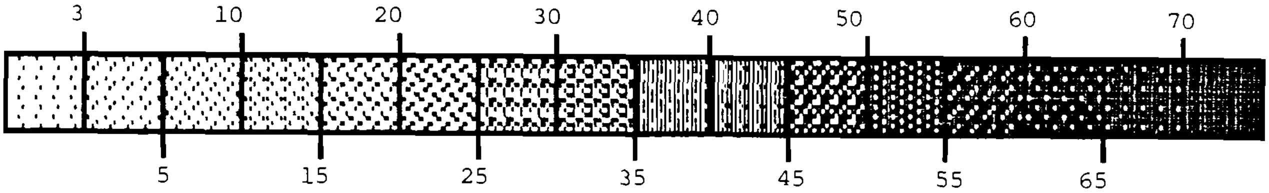


Figure 36: Belgian Female Period Life Expectancy - with contour lines selectively placed from 3 to 70
From age 0 to 99 and year 1892 to 1977



pected: life expectancy can be considered a kind of average of survivorship figures, and survivorship an average of mortality rates. Moreover, the cohort maps tend to be smoother than the period maps, since the cohort maps average out period effects.

U.S. FEMALE MORTALITY RATES FROM 1900 TO 2050

Faber (1982) published life tables for U.S. males and females by single year of age from birth to 119 for every tenth year from 1900 through 2050. The mortality rates at advanced ages and after 1980 are based on extrapolations. Figure 37a displays the surface of annual death rates (i.e., of q) for U.S. females and Figure 37b displays the surface of the force of mortality. The two surfaces are very similar, showing what every demographer knows: unless mortality rates are very high, q and the force of mortality are roughly equal. Note that the maps graphically reveal Faber's underlying assumption that progress against mortality will slow down in the future.

The nature of this assumption is presented even more clearly in Figure 38, which displays the force of mortality over age relative to the force of mortality in 1980. The right side of Figure 38 is neither a mirror image of the left side nor a smooth continuation. The left side shows the rapid historical progress that has been achieved against mortality at most ages and the recent acceleration of progress at some ages, especially some of the older ages. The right side portrays a lackluster future with slow and slowing rates of progress. Perhaps Faber's forecasts can be described as "conservative" in some sense, but they are certainly radically different from the performance of the past.

MODEL LIFE TABLES

Demographers frequently make use of model life tables, especially those developed by Coale and Demeny (1984). In the Coale and Demeny tables, death rates in various age categories are given by the life expectancy of the population, for males and for females, and for four different kinds of hypothetical populations, labelled East, West, North, and South. Figure 39 presents contour maps for females of these four types. Note that the horizontal axis gives life expectancy rather than time. Thus Figure 39 illustrates not only the use of small multiples to portray synthetic data, but also the use of a variable other than age or time as one of the dimensions on a contour map.

a) Mortality Rates

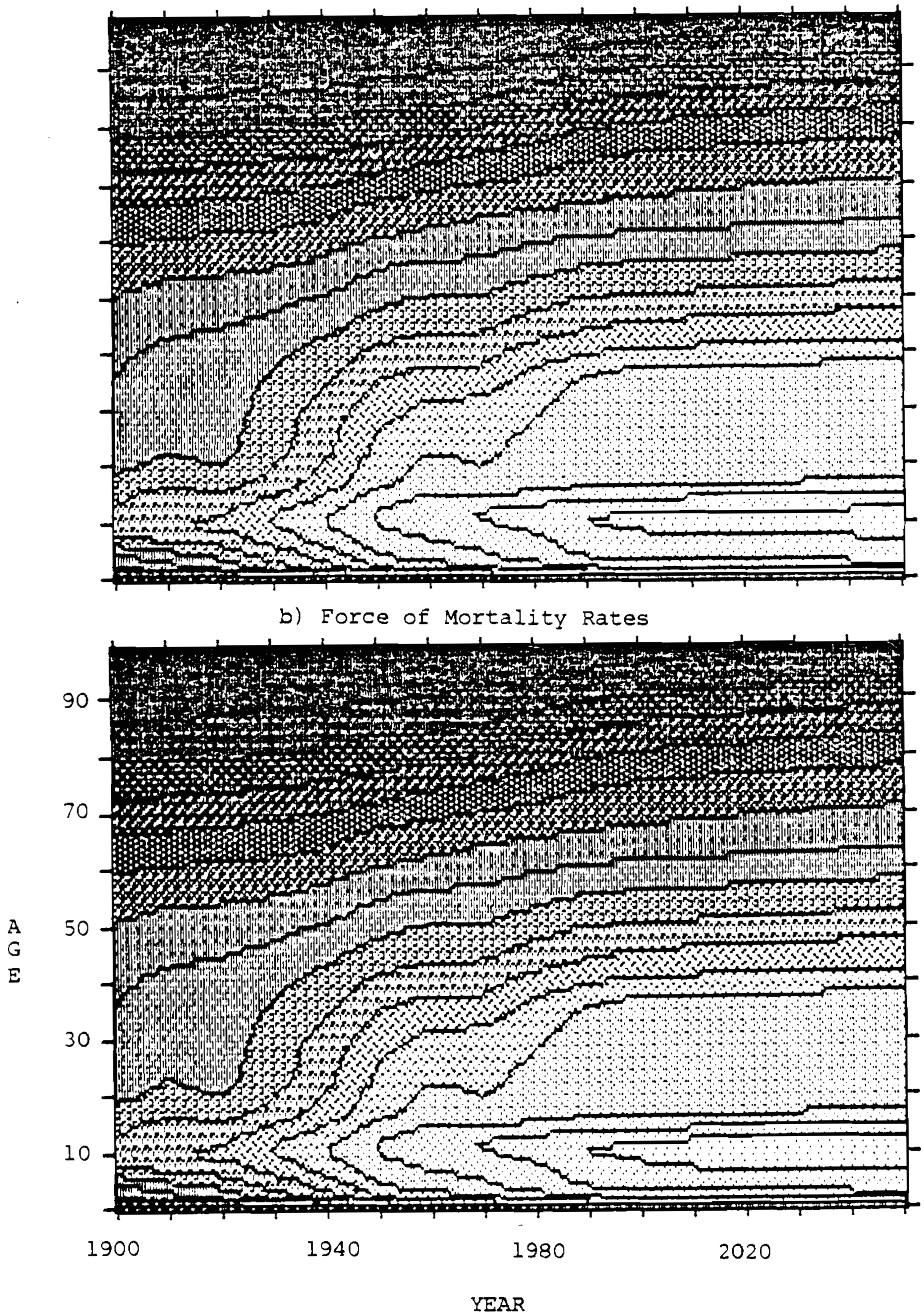
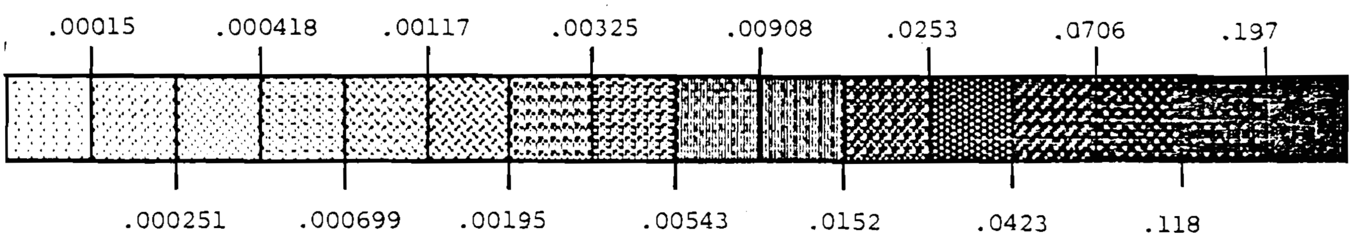


Figure 37: U.S. Female Mortality and Force of Mortality Based on Faber Life Tables - with contour lines from .00015 to .197 at multiples of 1.67
From age 0 to 99 and year 1900 to 2050



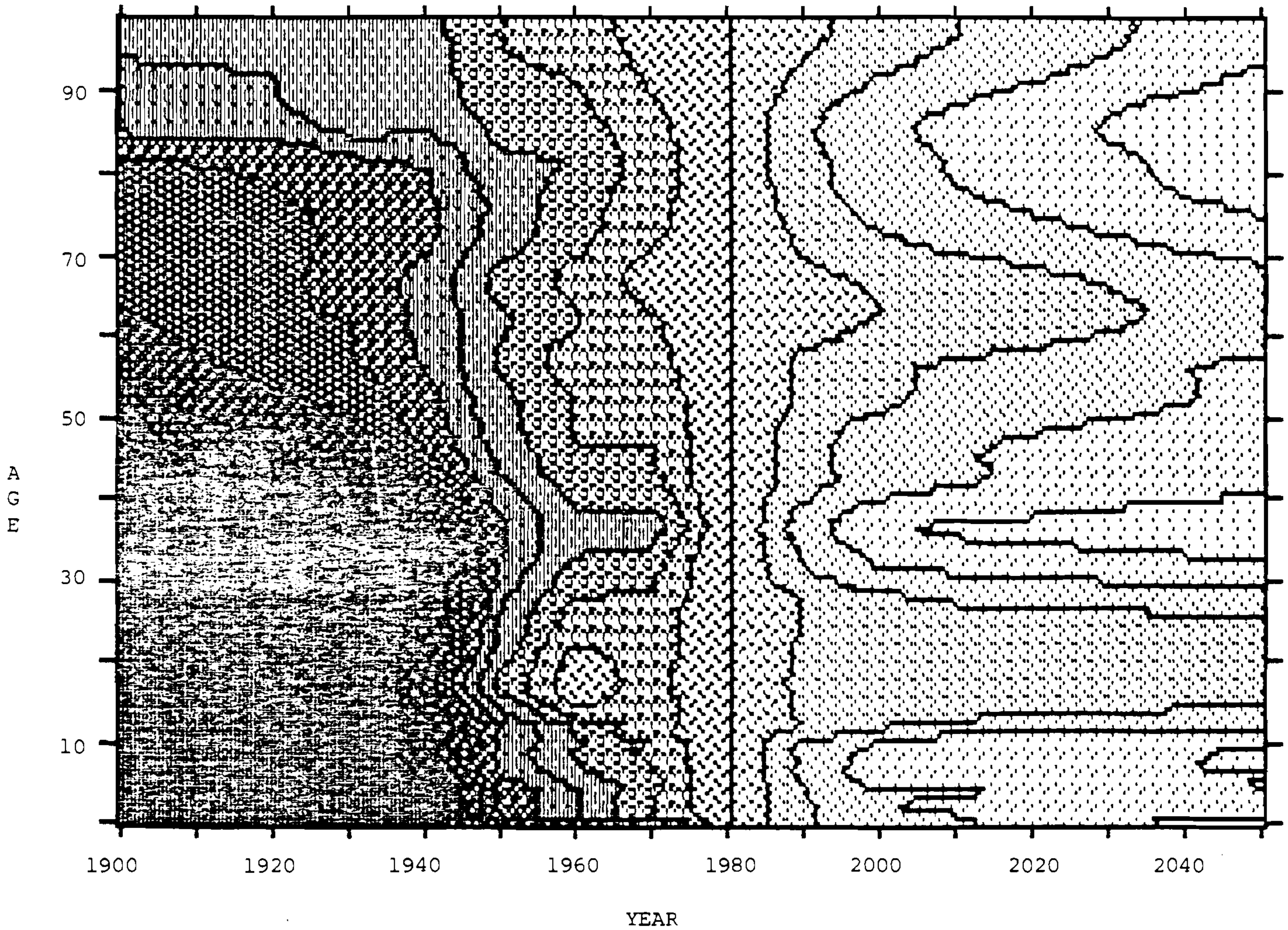
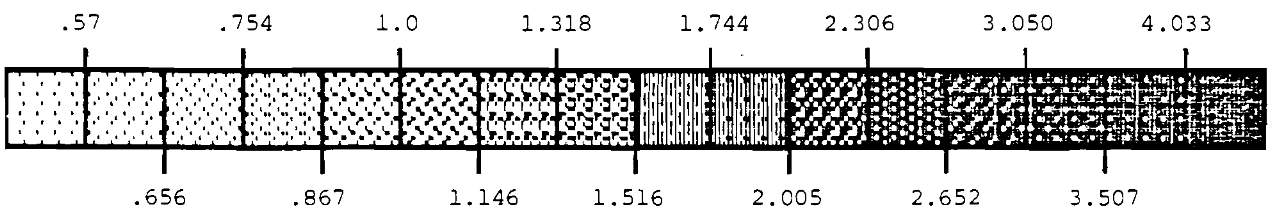


Figure 38: U.S. Mortality Rates (as Given by Faber) Relative to 1980 Age-specific Levels - with contour lines from .57 to 4.033 at multiples of 1.146 From age 0 to 99 and year 1900 to 2050



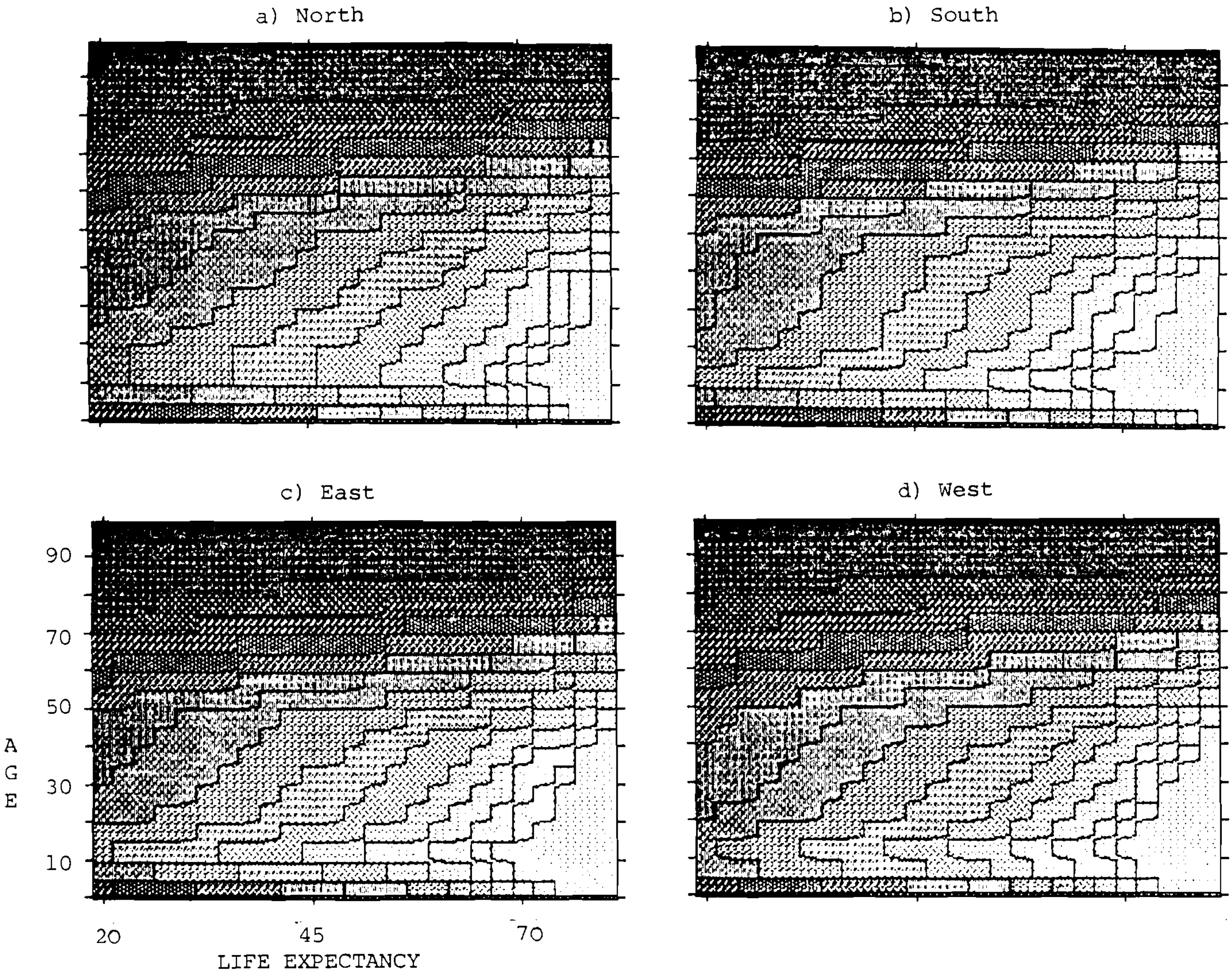
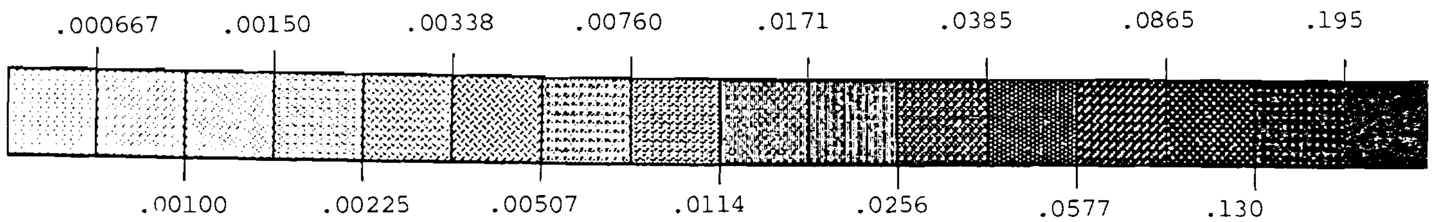


Figure 39: Death rates from Coale and Demeny Model Data for North, South, East and West Regions - with contour lines from .000667 to .195 at multiples of 1.5

From age 0 to 99 and life expectancy 20 to 80



The most immediately striking feature of the four maps in Figure 39 is their similarity: the differences among regions seem relatively small compared with the enormous difference resulting from change in life expectancy. It takes careful scrutiny of the maps to reveal the differences among regions at the same level of life expectancy. But note that the contour lines are drawn at multiples of 1.5: mortality rates at some age and life expectancy on two maps that appear to be similar might differ by as much as 50 percent. As age increases from 10 to 99 and as life expectancy grows from 20 years to 80 years, mortality rates change by several factors of 1.5; indeed, the highest contour on each of the maps is almost 300 times greater than the lowest contour. Among the four regions, at some specific age and life expectancy, mortality rates rarely differ by more than a single factor of 1.5. A factor of 1.5 looks small compared with a factor of 300; but as users of the Coale and Demeny tables know, a factor of 1.5 may be of crucial significance.

Another approach to construction of model life tables was developed by Brass (1971). In this approach, a standard trajectory of survivorship proportions, $p(x)$ where x stands for age, is modified by parameters a and b to produce alternative trajectories, $p'(x)$, such that

$$logit(p'(x)) = a + b \ logit(p(x)) \quad ,$$

where the logit function is given by

$$logit(p) = .5 \ log((1-p)/p) \quad .$$

Given a trajectory of survivorship proportions, a trajectory of forces of mortality (and hence mortality rates) can be readily calculated. Figure 40 illustrates how the values of the parameters a and b affect the resulting age-specific force of mortality: the parameter a runs along the horizontal axis of each map and the five maps represent different values of b. The maps are consistent with the interpretation of a and b offered by Brass (1971; see also Ewbank, Gomez de Leon, and Stoto 1983 and Zaba 1979), namely that the parameter a essentially determines the level of mortality, whereas the parameter b changes the rate at which mortality increases with age: the higher b is, the sooner a particular level of mortality is reached (at any value of a).

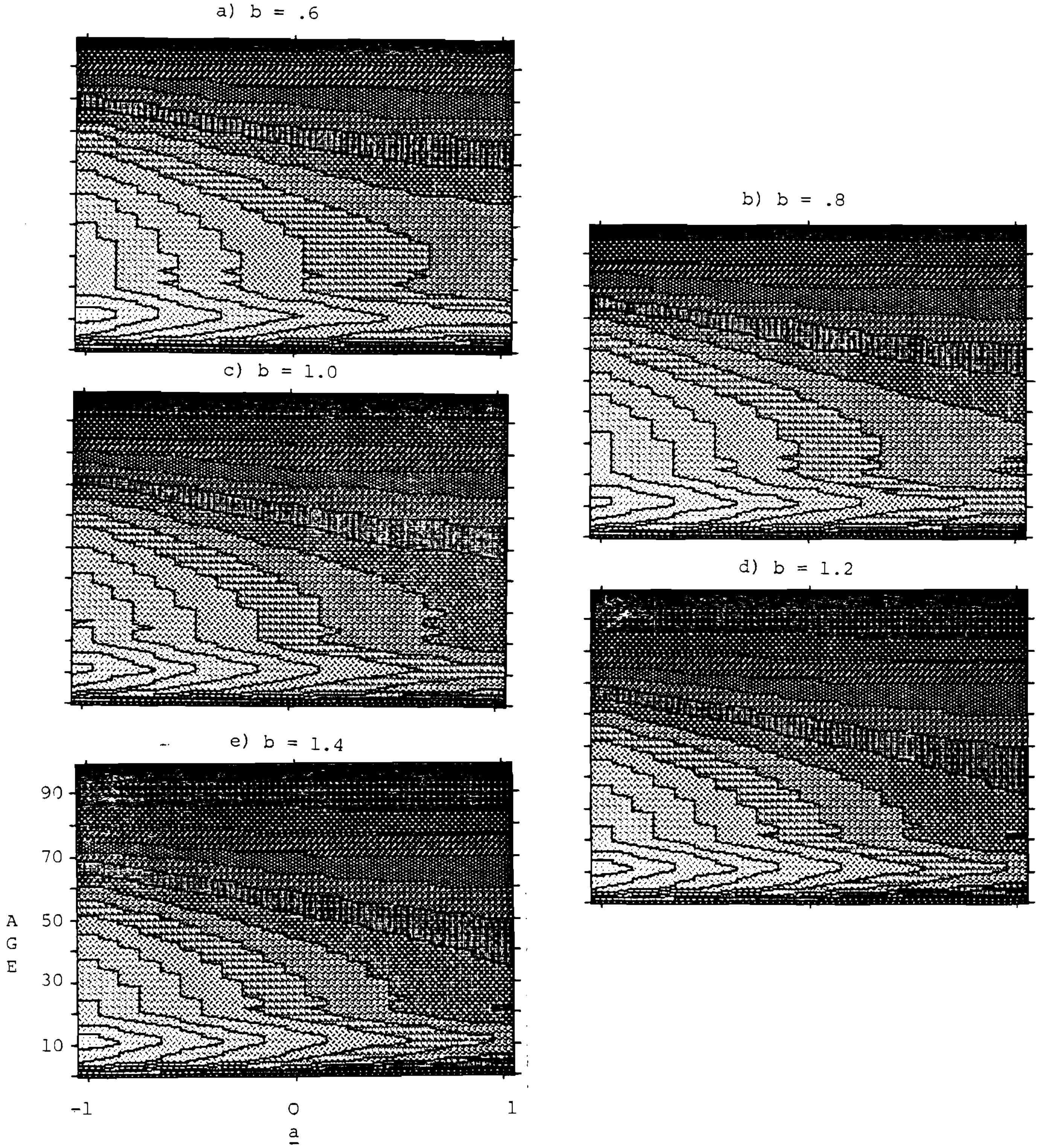
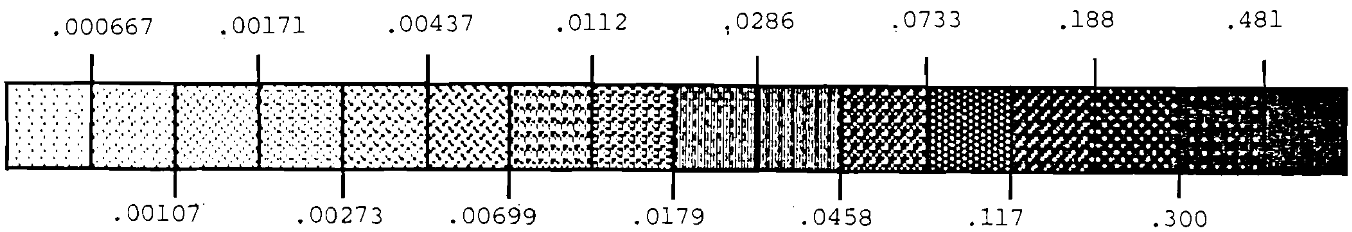


Figure 40: Force of Mortality From Brass's Model as **a** varies from -1 to 1 at intervals of .1, and b changes with each map - with contour lines from .000667 to .48 at multiples of 1.6
From age 0 to 99 and a -1 to 1.



MAPPING RESIDUALS TO SHOW GOODNESS OF FIT

How well does a model fit some empirical data? If the data are defined over two dimensions, then a contour map can be used to display the residuals, i.e., the differences between the actual values and the values predicted by the model. By scrutinizing the pattern of the residuals, an analyst may glean some clues as to how to improve the model. (Tukey (1977) and Mosteller and Tukey (1977) provide clear discussions of the use of residuals in data analysis and model building.)

As an illustration of this general method, Figures 41a, b, c, and d show how well a modified form of Brass' model fits Italian female mortality data. The modification made involves the use of 1926 Italian female mortality rates as the standard rather than Brass' original standard; these mortality rates were used because 1926 seemed to be an average year, roughly halfway through the transition from high mortality to lower mortality. Figure 41a displays the actual values of Italian female mortality rates and Figure 41b displays the values estimated by the modified Brass model. Figure 41c displays the surface of residuals, i.e., the surface of d equal to q minus q', where q is the observed mortality rate and q' is the mortality rate predicted by the modified Brass model. Different values of a and b in the model were estimated for each year of time. The values used for a and b are displayed in the graph in Figure 44d.

The values of a and b were calculated to minimize the sum of the squared deviations of Brass's model from the transformed data, using the logit formulas given above. As shown in Figure 41d, the values of a have increased almost linearly over time, the trend being interrupted only during the First and Second World Wars. The values of b, on the other hand, were fairly stable until the Second World War, with a notable peak during the First World War, and then increased substantially. The value of a, as indicated earlier in conjunction with Figure 40, can be interpreted as describing the overall level of mortality: as a increases, the level of mortality decreases. The value of b can be interpreted as determining the rate at which mortality increases with age: the higher b, the higher mortality rates at later ages are compared with the rates at younger ages. This interpretation of a and b is consistent with the pattern of Italian female mortality shown in Figure 41a. Note in particular that after 1945 mortality rates at younger ages, age 30 say, increasingly diverge from mortality rates at advanced ages.

As shown in Figure 41c, the residuals of the fitted model are in general fairly small, in nearly all cases between -0.005 and 0.005. The goodness of the fit can also be seen be comparing Figures 41a and b: the patterns of contours on the two

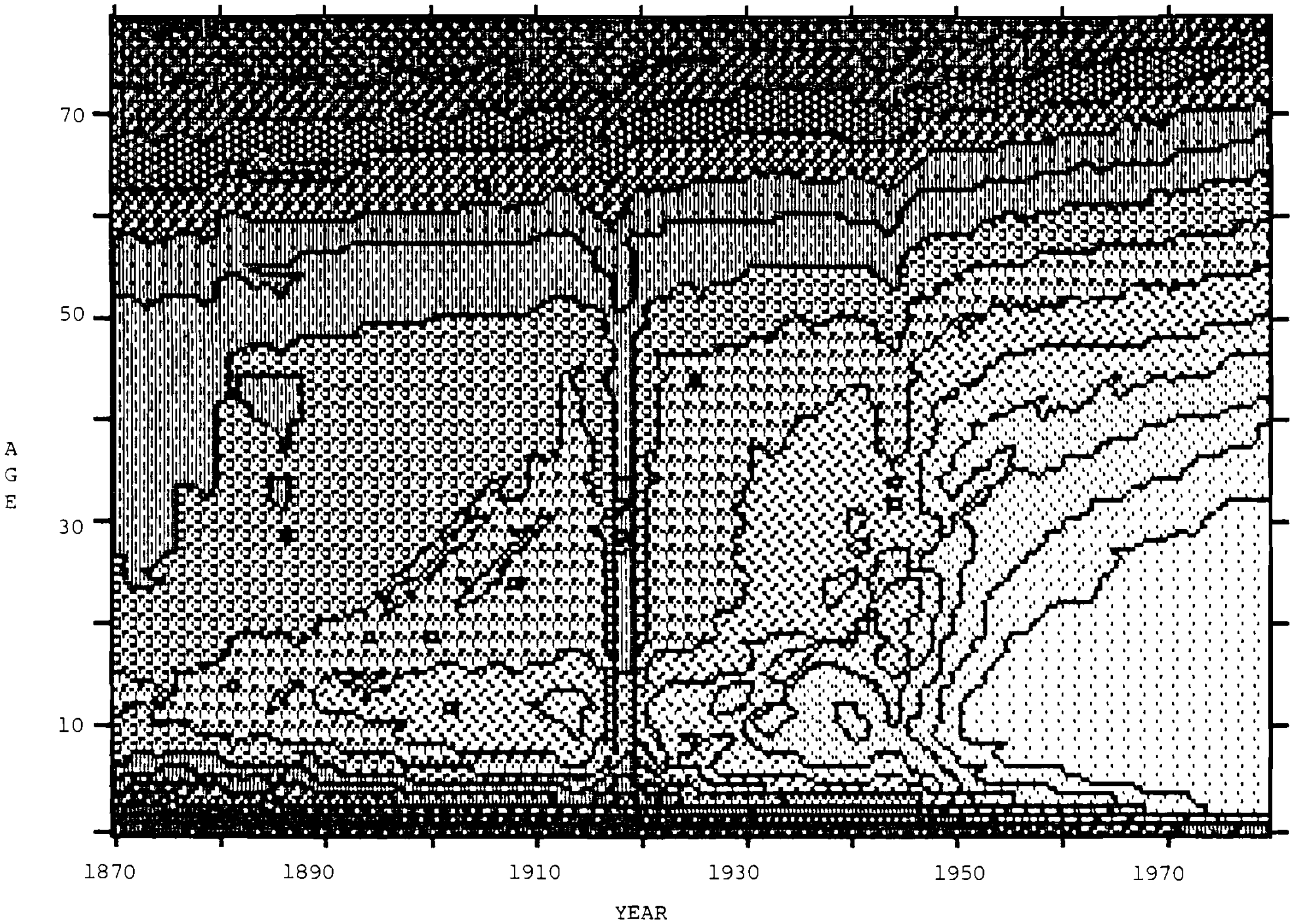
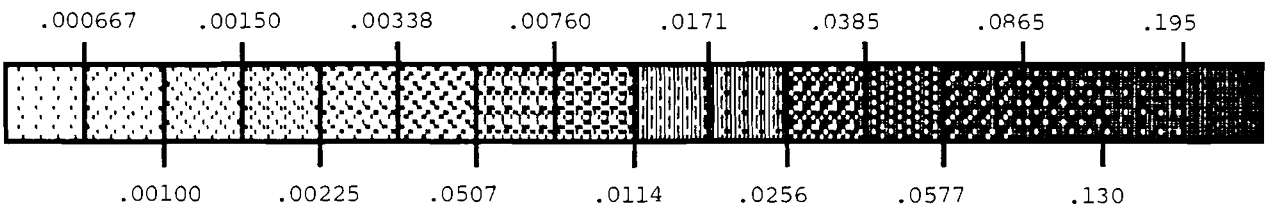


Figure 41a: Italian Female Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From age 0 to 79 and year 1870 to 1979



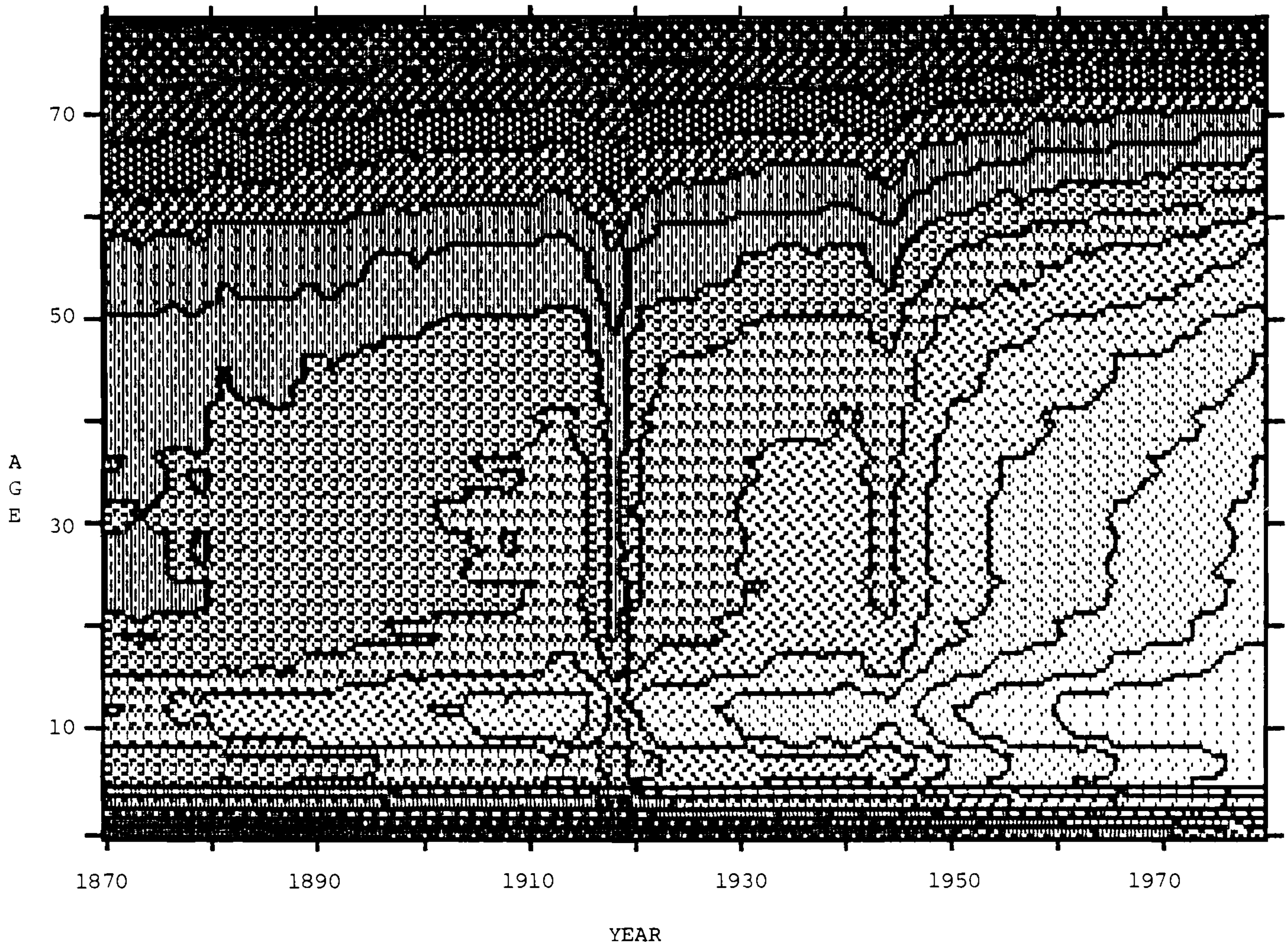
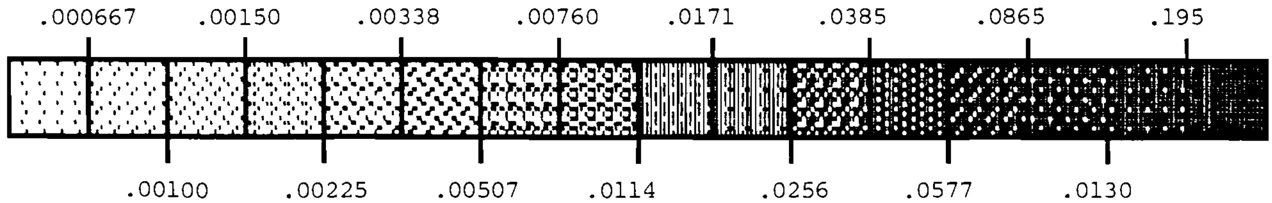


Figure 41b: Italian Female Mortality Rates Given by Modified Brass's Model - with contour lines from .000667 to .195 at multiples of 1.5 From age 0 to 79 and year 1870 to 1979



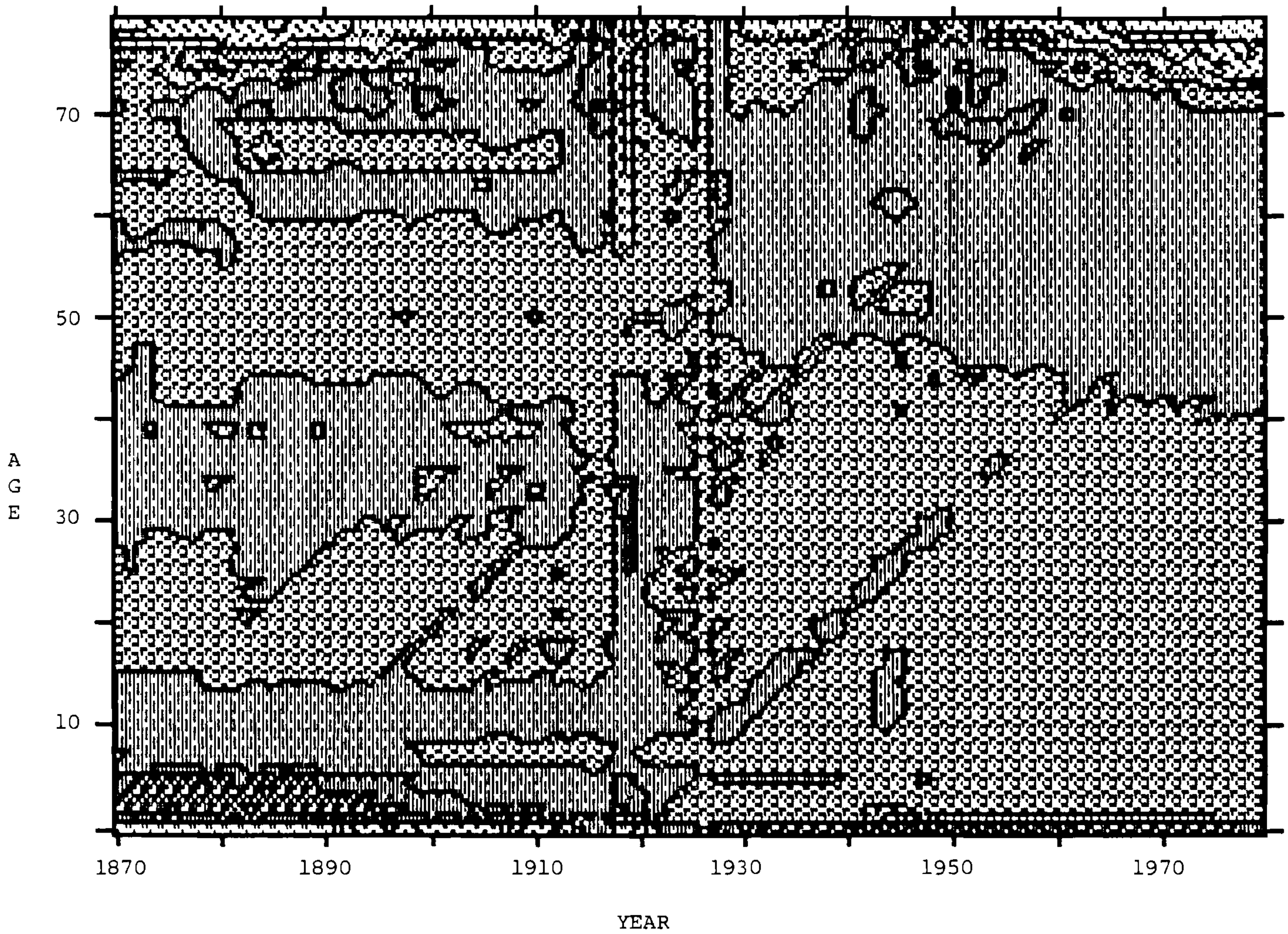
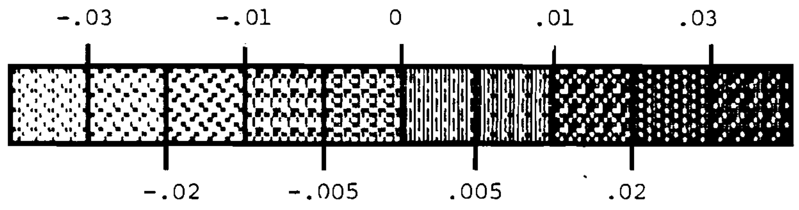


Figure 41c: Residuals from Modified Brass's Model - with contour lines selectively placed from -.03 to .03
From age 0 to 79 and year 1870 to 1979



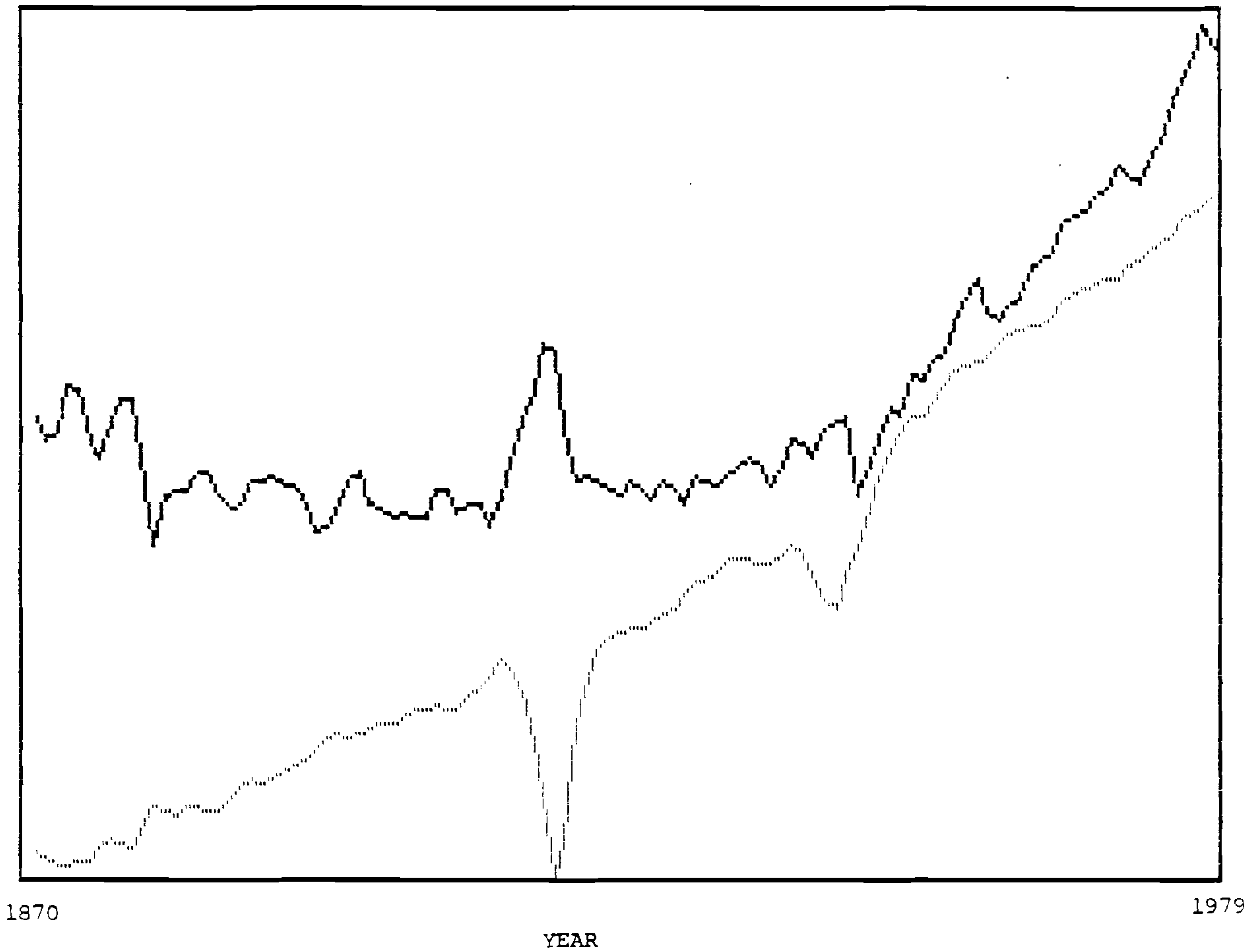


Figure 41d: Plot of a's and b's Used in Producing Italian Female Mortality Rates with Brass's Model - Curve b is the darker curve

maps are quite similar. Careful comparison of Figures 41a and b reveals that even though the absolute deviations of the fitted model from the actual data are small, the relative deviations are sometimes sizeable, especially when mortality levels are low. For instance, the actual mortality rate at about age 25 in about 1965 falls is about 0.000667; the estimated mortality rate at this age and time, however, is about 0.00150, or more than twice the actual rate. To minimize different measures of goodness of fit, different estimation procedures could be used.

The pattern of residuals in Figure 41c provides some clues as to how to extend the model to fit the data better. After 1926 the estimated values tend to be too large at younger ages and too small at older ages. Before 1926 the picture is more complex, but there is some rough tendency toward the reverse pattern: the estimated values tend to be too small at younger ages and too large at older ages. Thus there appears to be some interaction between the age and period effects on the map of residuals. Also note that between ages 15 and 25 there is a swath running across the map of residuals that indicates the estimated values tend to be too large: the 1926 "standard" apparently does not appropriately reflect the general pattern of mortality between ages 15 and 25. Similarly, there is a darker swath, indicating underestimated values, running across the map between ages 60 and 70 or so. These systematic deviations suggest some ways the model might be improved.

The map of residuals includes several intriguing diagonal patterns, most notably two diagonals running between roughly ages 10 and 30, the first in the period 1890 to 1910 and the second in the period 1927 to 1950. The Brass model estimates "standard" age effects on a period by period basis; the residuals, by showing what is left after these effects are removed, provide some leads as to where cohort effects occur.

As a general strategy in exploratory data analysis and model building, it is often useful to remove age and period effects (or, more generally, the main effects along the x and y dimensions) from the data and then look at the residuals. One way of doing this is to fit a model, such as the Brass model or some other model of mortality such as the four-parameter extended Brass models proposed by Zaba (1979) and by Ewbank, Gomez de Leon, and Stoto (1983) or the eight-parameter model developed by Heligman and Pollard (1980). Another, more direct approach is to simply subtract or divide out of a surface the average values for different ages and different years. Thus, the relative surfaces shown earlier in Figures 13 through 18 can be interpreted as residual surfaces for which either a period effect or an age effect has been removed.

Figure 42a displays two surfaces of residuals calculated by removing both a period and an age effect: the original surface, which presents U.S. fertility rates, was shown in Figure 5. Figure 42a(i) shows the residuals when age and period effects are divided out; Figure 42a(ii) shows the residuals when these effects are subtracted out. In both cases, the period effect in any year was calculated as the average fertility rate in that year over the ages 14 through 49 (i.e., total fertility divided by 36) and the age effect was similarly calculated as the average fertility rate at some age over the period from 1917 through 1980. In the multiplicative model, estimated fertility rates are simply given by the product of the two effects, divided by the overall average fertility over all ages and years. In the additive model, the estimated fertility rates are given by the sum of the two effects, minus the overall average fertility. (The division and subtraction of overall average fertility normalizes the estimates so that the residuals center around 1 for the multiplicative model and around zero for the additive model.)

The residuals indicate that the multiplicative model is more informative than the additive model. The multiplicative model fits the data especially well during the ages, from 20 to 35, at which most births occur. The model also fits well during the period from about 1940 to 1955. The regular pattern of the residuals, indicating overestimates and underestimates in opposite corners, suggests that a simple extension of the model might substantially improve the fit. One way of adjusting the model to correct the errors in the corners would be to add an interaction effect of age and time, given say by the product of age minus 30 and year minus 1940. Such an interaction term would have an positive value in the lower left and upper right corners and a negative value in the lower right and upper left corners.

The values of the age and period effects are shown in Figures 42b and 42c. The age effect follows a remarkably regular pattern that looks much like the probability density function of the Gamma, Weibull, or Lognormal distributions. Thus, it might be possible to closely fit this curve by a a two-parameter function. The period effect shown in Figure 42c displays the waves of the baby booms and busts, as well as the historically low levels of fertility reached in the late 1970's. This curve is fairly regular and possibly might also be approximated by a two or three parameter function, perhaps one that incorporated information about prevailing social, economic, or demographic conditions.

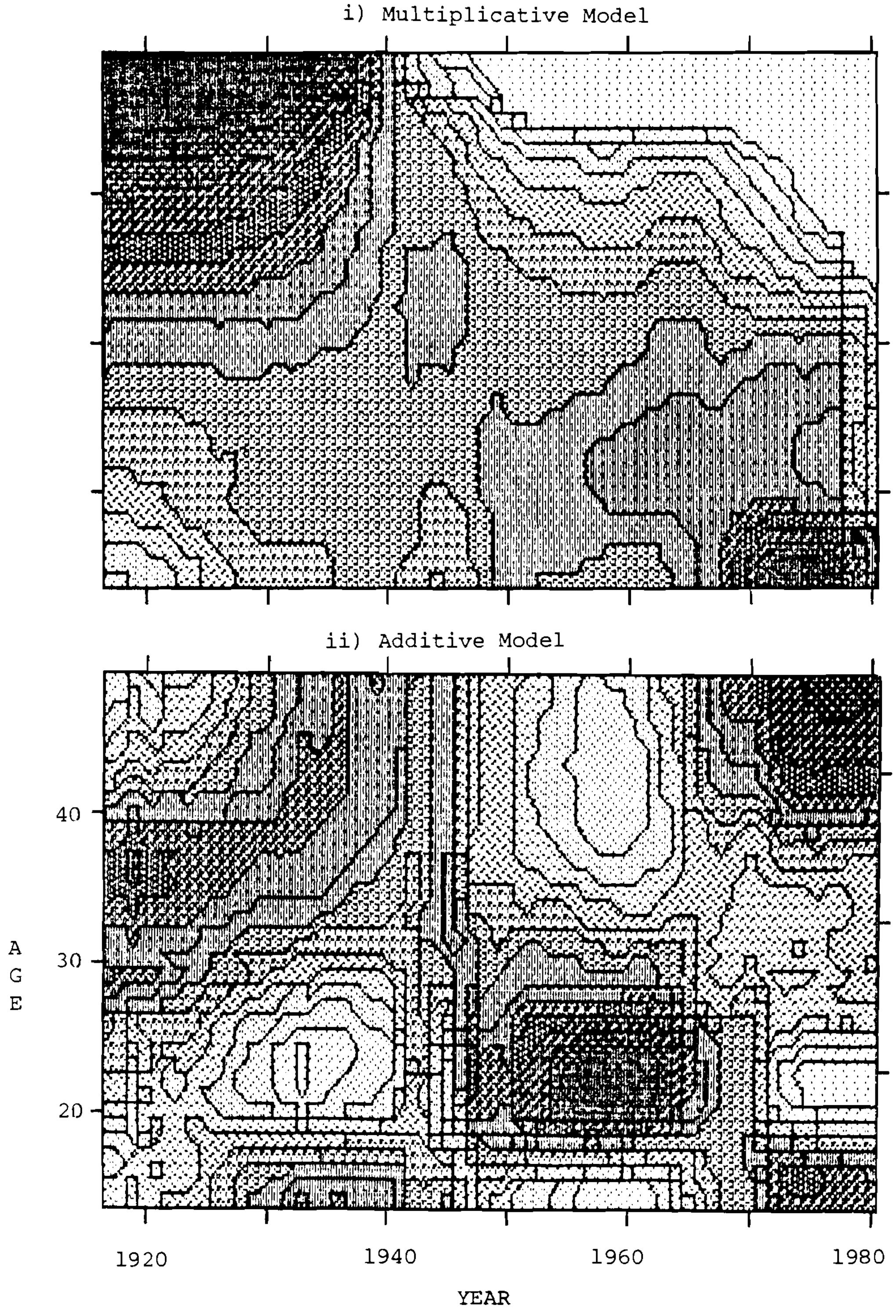
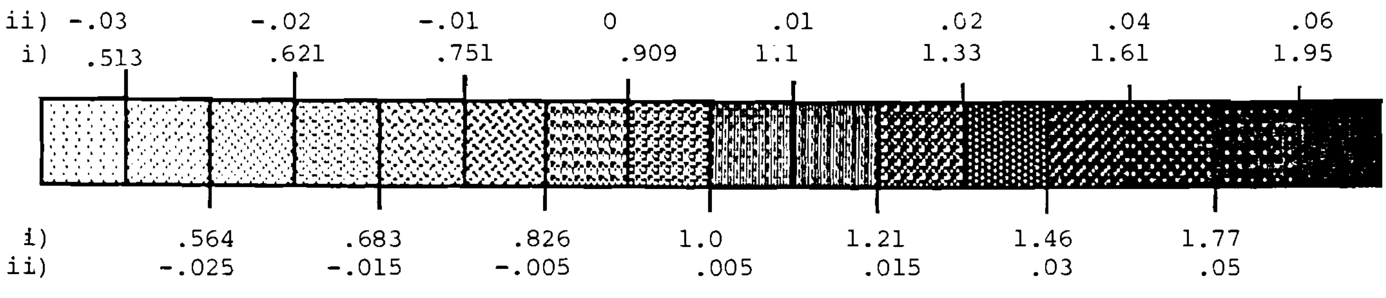


Figure 42a: Residuals from Multiplicative and Additive Models of U.S. Fertility
From age 14 to 49 and year 1917 to 1980



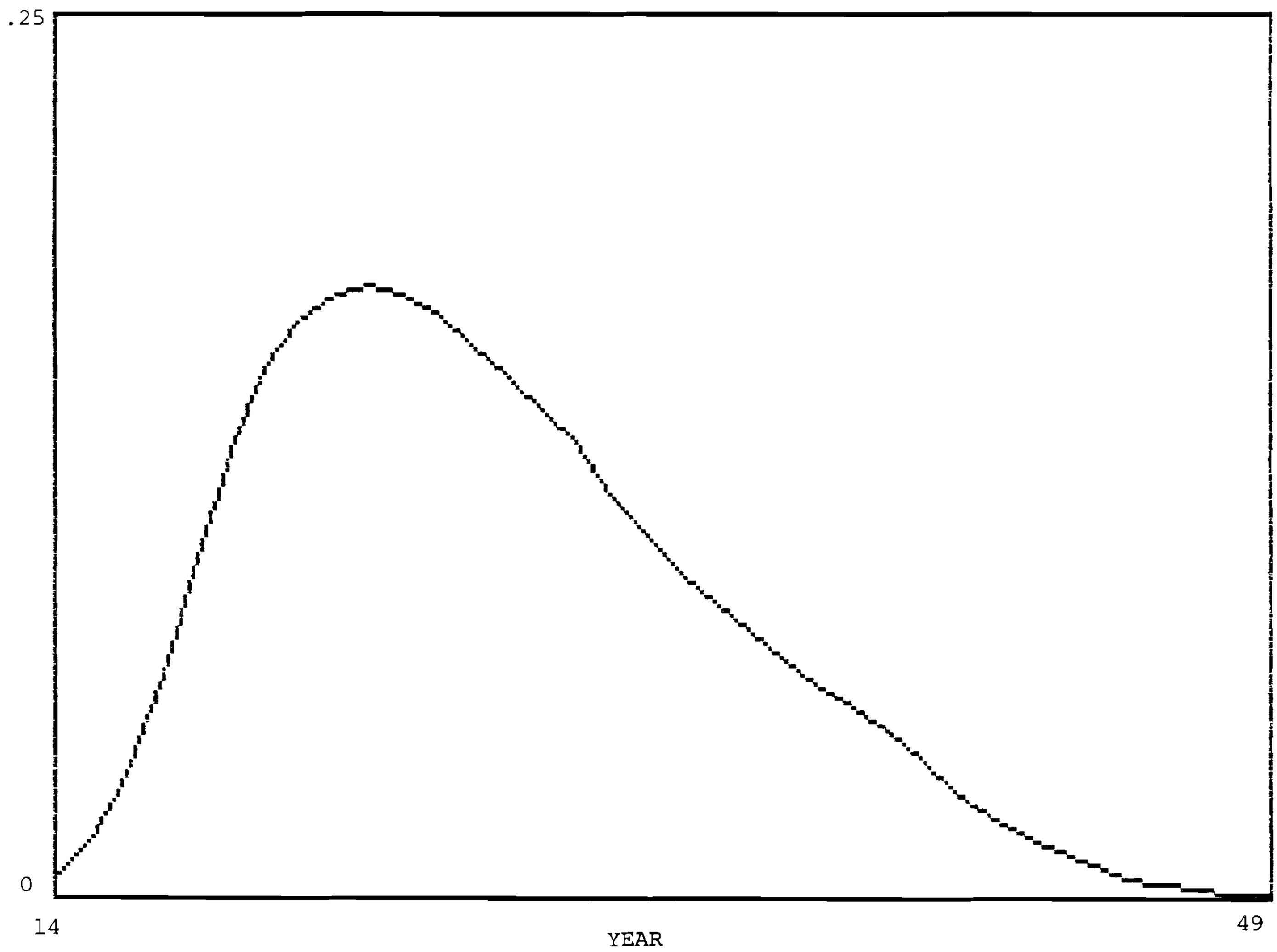


Figure 42b: Average U.S. Fertility Rates from 1917 to 1980 over Age 14 to 49

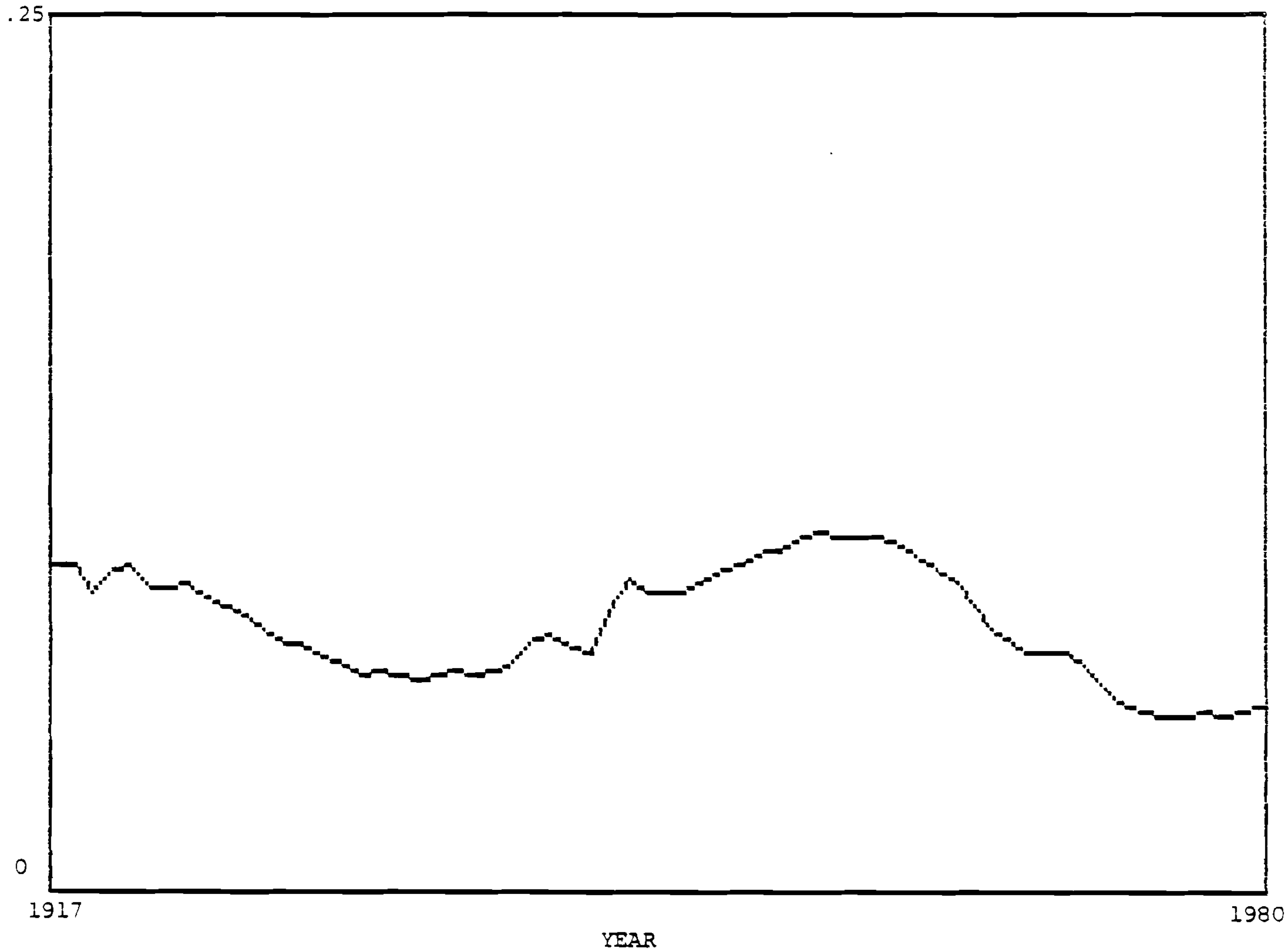


Figure 42c: Average Fertility Rates for Each Year from 1917 to 1980

MAPS OF THEORETICAL DEMOGRAPHICAL MODELS

To understand actual population phenomenon, demographers often analyze simplified, theoretical models that capture some aspect of reality (Keyfitz (1977)). Contours maps can be used to show how some variable of interest in such models responds to changes in two of the parameters. Figures 43 and 44 provide such illustrations.

Suppose mortality rates follow the female model West schedule of Coale and Demeny. Further suppose that a population is stable and is governed by these mortality trajectories (which can be classified by the single mortality measure $\overset{o}{e}_0$, life expectancy at birth) and by some growth rate r. What proportion of the population will be above age 65? The contour map in Figure 43 displays the answer, for various values of $\overset{o}{e}_0$ and r. The two general effect are clear: the longer life expectancy, and the slower the rate of population growth, the more people above age 65. The map shows how these two effect interact. For instance, the map reveals that about ten percent of a stable population will be above age 65 if either (1) life expectancy is about 23 years and the population is shrinking at two percent per year or (2) life expectancy is 80 years and the population is growing at two percent per year. If life expectancy is 80 years and the population is shrinking at two percent per year, then over a third of the population will be above 65.

As a second example, suppose that the force of mortality at any age is given by a Gompertz curve such that $\mu(x) = a\ e^{bx}$. How will life expectancy at birth change as a and b vary? Figure 44 graphically answers this question. Consider, for instance, the situation described on the middle of the right margin of the map when a is 0.00050 and b is 0.10, yielding a life expectancy of about 48 years. Cutting the mortality level by a factor of ten, i.e., reducing a to 0.00005, would increase life expectancy to 70 years. Alternatively, holding a constant but slowing the rate of aging by reducing b from 0.10 to 0.06 would have the same effect. Clearly, the effect on life expectancy of a small proportional change in the rate of mortality increase is much greater than a similar change in the level of mortality.

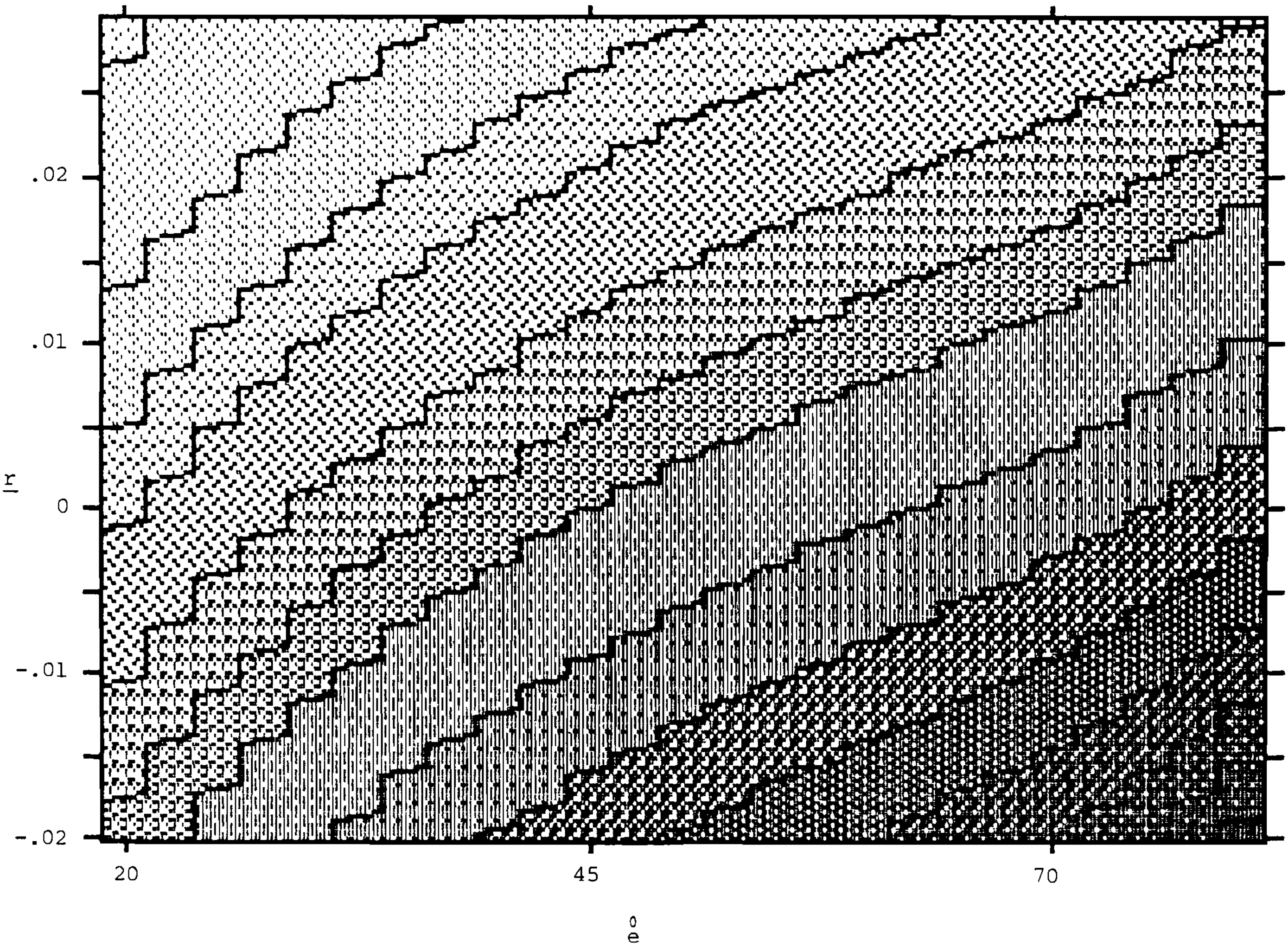
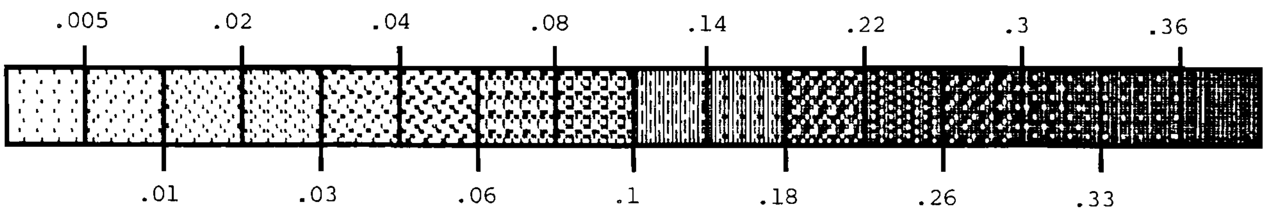


Figure 43: Proportion of People Above Age 65 - with contour lines selectively placed from .005 to .36
From $\underline{r}$ = -.02 to .0295 and $\overset{0}{\underline{e}}$ = 20 to 80



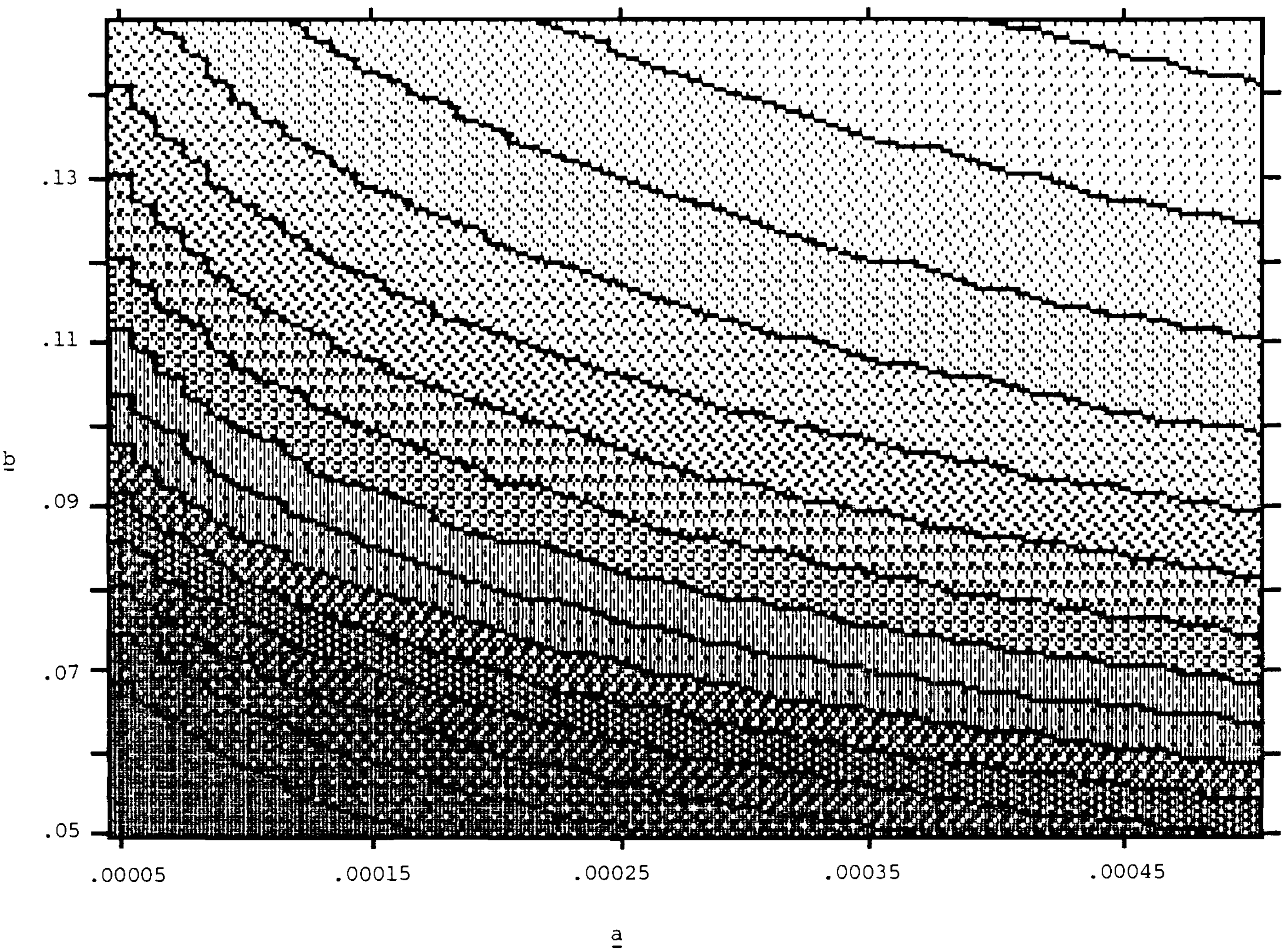
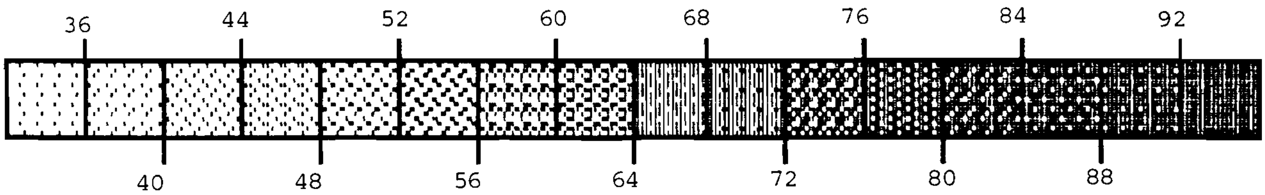


Figure 44: Changes in Life Expectancy at Birth Given by the Gompertz Curve as a and b vary - with contour lines selectively placed from 36 to 92 years of life expectancy
From b = .05 to .149 and a = .00005 to .00050



CONCLUSION

The figures presented in this paper suggest just a few of the numerous ways that demographers can use contour maps to clearly, efficiently, and simultaneously display both persistent global and prominent local patterns in population rates or levels over two dimensions. In particular, contour maps can strikingly reveal the interaction between age, period, and cohort patterns.

Even in cases where some demographic data already have been carefully scrutinized by perceptive analysts who have uncovered most of the interesting patterns, contour maps may be useful in highlighting these patterns in a visually revealing manner. With contour maps, what was before understood now can be *seen*. Furthermore, the maps, by giving demographers a new perspective on data, may focus attention on some neglected aspects and patterns in even thoroughly-studied data.

Beyond efficient description, contour maps can help demographers with exploratory data analysis and with model building. Surfaces can be computed relative to some part of the surface or to another surface; and different surfaces can be placed next to each other and compared. The patterns produced by a model can be displayed as can the fit of the model to some empirical data.

The resulting contour maps can be displayed not only as printed output but also on a computer monitor. The shades used in the maps presented in this paper range from black to light grey, but the maps can be produced in glowing colors, on a color computer monitor or using a color printer. The effects are dramatic, as is the speed with which a computer can draw a map. A large computer is not needed--we have used a standard IBM PC with 64K of memory.

Tufte, in his lucid exposition of *The Visual Display of Quantitative Information* (1983), concludes that graphic designs should give "visual access to the subtle and difficult, that is, the revelation of the complex". Demographic surfaces can be particularly complex. A mortality surface, for example, might be defined over nearly a century of age and more than a century of time, comprising close to ten thousand data points that may vary over four orders of magnitude. Contour maps are a striking, efficient, and clear means of giving demographers visual access to such surfaces.

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