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**THE EXCHANGE COMPONENT OF IASA'S  
FOOD AND AGRICULTURE MODEL FOR  
THE EUROPEAN ECONOMIC COMMUNITY**

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## FOREWORD

Understanding the nature and dimensions of the world food problem and the policies available to alleviate it has been the focal point of IIASA's Food and Agriculture Program (FAP) since it began in 1977.

National food systems are highly interdependent, and yet the major policy options exist at the national level. Therefore, to explore these options, it is necessary both to develop policy models for national economies and to link them together by trade and capital transfers. Over the years FAP has, with the help of a network of collaborating institutions, developed and linked national policy models of twenty countries, which together account for nearly 80 percent of important agricultural attributes such as area, production, population, exports, imports and so on. The remaining countries are represented by 14 somewhat simpler models of groups of countries.

The European Community (EC) is a major actor on the world market. An aggregate food and agriculture model of the EC, in which the EC is treated as one nation has been developed by the FAP, as part of the IIASA/FAP basic linked system.

In addition, development of a detailed model of the EC, in which the member nations of the EC are represented by separate models which interact among themselves within the framework of the common agricultural policy (CAP) of the EC, was initiated in 1978. This was begun in collaboration with the University of Göttingen, which received a grant from the Volkswagen Foundation. The work on model development was transferred to IIASA in 1982, where it continued until the end of 1984, under a grant from the Centre for World Food Studies (CWFS) of the Netherlands.

In this paper, which is one of a series of papers reporting the work on the development of the detailed EC model, Erik Geyskens describes the exchange component of the model.

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# The Exchange Component of IIASA's Food and Agriculture Model for the European Economic Community

*Erik Geyskens*

## 1. Introduction

The Food and Agriculture Model for the European Economic Community (EC) describes dynamically supply, demand, income and price formation in the EC member countries with special reference to the agricultural sector and the food situation. It is a member of a family of linkable models constructed to be a representation of the world food system, and hence it satisfies linkage requirements with respect to commodity classification, time increment (one year) and methodology. Within these linkage requirements there is enough scope to represent specific issues such as the Common Agricultural Policy (CAP). The work is coordinated by the Food and Agriculture Program at the International Institute for Applied Systems Analysis (IIASA).

This paper describes the exchange component of the EC-model. The other components of the EC-model, such as the demand component, the supply component and the policy component, are described elsewhere.

The paper is structured as follows. Section 2 contains an overview of the EC-model.\* In section 3 a simplified version of the open exchange model used in the modeling approach of IIASA's Food and Agriculture Program is formulated.\*\* The comparative statics of the simplified version are discussed in section 4. In section 5 the main classifications, i.e. the countries and commodities, of the EC-model are defined. Section 6 describes the exchange component of the EC model, and in section 7 the EC policy setting is discussed.\*\*\* The model solution is presented in section 8.

## 2. Overview of the Model

The EC model can be logically subdivided into an exchange component and a supply component, see Figure 1. In the supply component, given one year's price realizations on the national market, next year's supply is planned and factor services as well as intermediate inputs are bought in accordance with the plans. At the beginning of the next period when the production has been generated, each EC country possesses (possibly negative) ownership entitlements for commodities.

Then the execution of the second model component, the *exchange* component is started. Here for each country income formation and demand are

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\* For a more detailed overview of the EC-model, see U. Färber et al (1984).

\*\* For an overview of the IIASA/FAP approach see Parikh and Rabar (1981). The open exchange model is described in Keyzer (1981a, 1983), and Keyzer and Rebelo (1982).

\*\*\* Sections 6 and 7 are based on the paper of M. Keyzer (1981 b)

described taking into consideration government intervention at the national as well as at the EC level. In the exchange component commodity balances and overall financial balance interconnect the markets of the individual commodities and the individual countries according to a general equilibrium approach.

### 3. A Simplified Version of the Open Exchange Model

#### 3.1. Supply - Gross Domestic Product - Taxes - Trade

Supply is assumed to be determined prior to exchange taking place. Supply is defined as net output, i.e., gross supply minus intermediate demand. Denoting the supply vector by  $y$  and the corresponding price vector by  $p$ , gross domestic product (GDP)  $Y$  is given by:

$$Y = p'y \quad (1)$$

(*value added definition* of GDP). We note that in this and the next section all prices  $p$  are expressed in the same international currency.

The net import vector  $z$  is defined as the difference between demand  $x$  and supply  $y$ :

$$z = x - y \quad (2)$$

(*commodity balance equation*). Defining the net import value at domestic prices to be  $Z$ :

$$Z = p'z, \quad (3)$$

and total consumption expenditures to be  $C$ :

$$C = p'x, \quad (4)$$

we get the *aggregate demand definition* of GDP:

$$Y = C - Z \quad (5)$$

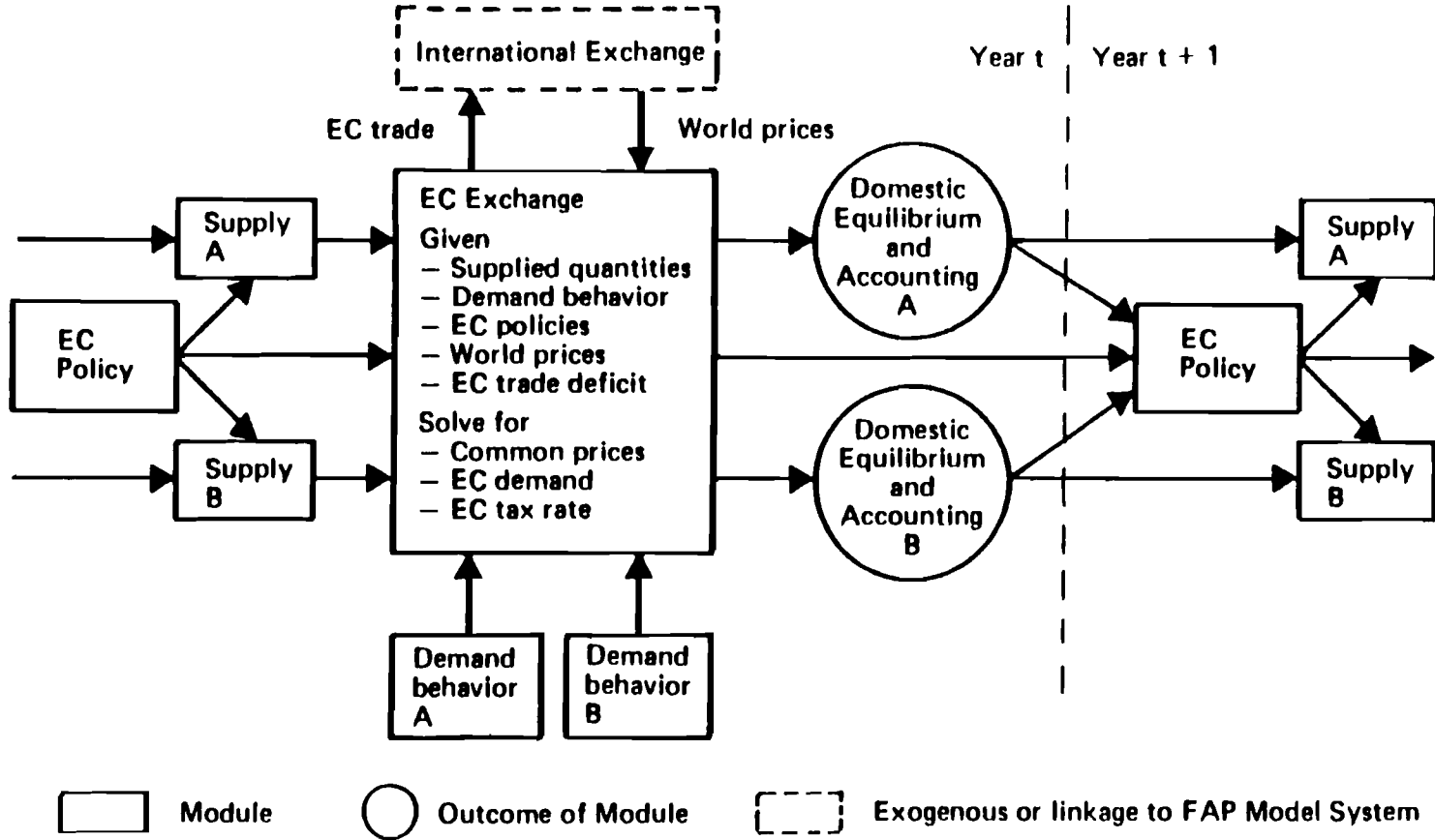
The *income definition* of GDP states that GDP equals disposable income  $m$  plus taxes  $f$ :

$$Y = m + f. \quad (6)$$

In the simplified version of the model it is assumed that consumption  $C$  equals disposable income  $m$ :

$$C = m \quad (7)$$

Figure 1. A Schematic Outline of the EC Model (Simplified for two countries)



Equations (5), (6) and (7) imply the *tax equation*:

$$f = -Z = -p'z . \quad (8)$$

Thus taxes  $f$  is part of GDP used to finance exports  $-z$  evaluated at domestic prices  $p$ . Assuming taxes proportional to GDP, denoting the tax rate by  $t$  and defining  $\varphi = (1 - t)$ , one has:

$$m = \varphi Y = \varphi p'y . \quad (9)$$

Suppose, for the moment, total consumption  $C$  would not be equal to disposable income  $m$ , but be determined by the following consumption function:

$$C = \alpha m \quad (10)$$

One then would get the following tax equation:

$$f + (1-\alpha)m = -Z, \quad (11)$$

showing that savings perform the same role in the model as do taxes. Thus the model results will not change for different values of  $\alpha$ . This is due to the fact that the simplified version of the open exchange model does not distinguish between different income classes.

The income definition of GDP (equation 6) does not say how disposable income is arrived at. However, by writing this equation in the form:

$$m = Y - f = p'y - f, \quad (12)$$

it says that disposable income equals revenues from production activities minus taxes. Instead of value added or actual endowments  $y$  one could work with pseudo-endowments  $\tilde{y}$  which are equal to actual endowments plus transfers. Disposable income  $m$  is then given by:

$$m = p'\tilde{y} - f , \quad (13)$$

and the tax equation becomes:

$$f = p'(\tilde{y} - x) = p'(\tilde{y} - y) + p'(y - x) = p'(\tilde{y} - y) - p'z . \quad (14)$$

Thus taxes now serve to finance transfers plus exports. Of course the model solution does not change by the introduction of pseudo-endowments. However, its use allows one to satisfy the mathematical conditions for convergence of the algorithm.

### 3.2. Demand

Demand is modeled according to the Linear Expenditure System (LES):

$$p'\hat{x} = p'\hat{c} + b(m - p'c), \quad \iota'b = 1, \quad (15)$$

where:

$c$  = committed demand vector

$b$  = marginal budget shares

$\iota$  = summation vector

$\hat{x}$  = diagonal matrix with the elements of the vector  $x$  on its diagonal.

Committed demand  $c$  is the sum of private committed demand, investment and government demand. Both investment and government demand are exogenously determined.

Making use of equation (9):  $m = \varphi p'y$  and defining matrices  $A = yb'$ ,  $M = \hat{c} - cb'$ , the expenditure system above can be written as

$$p'\hat{x} = \varphi p'A + p'M. \quad (16)$$

This system is called the canonical form of the model. In the canonical form the matrices  $A$  and  $M$  are predetermined, but only the matrix  $A$  is affected by the tax variable  $\varphi$ .

In the algorithm the canonical form is solved for prices  $p$ , demand  $x$  and tax variable  $\varphi$ , taking into account the balance of trade equation and the price formation equation. It should be noted that, since in the algorithm the canonical form (16) is solved, the model allows for more general demand systems than the LES. One of the conditions for the convergence of the algorithm to a unique equilibrium solution is:  $A_{ij} + M_{ij} > 0$ , for each  $i$  and each  $j$ , see Keyzer and Rebelo (1982). In the case of the LES, this implies the inequality  $y_i > c_i$ , assuming  $b_i > 0$ . If the inequality does not hold, the solution is to work with pseudo-endowments.

### 3.3. Balance of Trade Equation

A balance of trade equation is imposed as an overall budget equation. It states that net imports  $z$ , evaluated at given world market prices  $p_w$ , should be equal to a prespecified trade deficit  $k$ :

$$p_w'z = p_w'(x - y) = k. \quad (17)$$

This equation, together with the tax equation (8):  $-p'z = f$ , implies the following government budget equation:

$$(p_w - p)'z = k + f. \quad (18)$$



Trade subsidies are thus financed by trade deficit plus taxes. If disposable income  $m$  is defined in terms of pseudo-endowments instead of endowments, then the relevant tax equation is equation (14) and the government budget equation becomes:

$$(p_w - p)'z + p'(\tilde{y} - y) = k + f. \quad (19)$$

Thus, trade subsidies and transfers are financed by trade deficit and taxes.

### 3.4. Price Formation

For each commodity the government is thought to pursue a price policy through a tariff. This price policy is implemented as follows. Targets and bounds for the price as well as the net trade are specified, as illustrated in Figure 2. The government would like the  $i$ -th price to have the value  $\dot{p}_i$  and the  $i$ -th net trade to be  $\dot{z}_i$ . If price and net trade cannot be kept at target simultaneously, it wants the outcome to lie on the heavy line in Figure 2. This figure illustrates the policy where net trade should be at target as long as the price is within bounds (net trade has a priority over price), while Figure 3 illustrates the policy where price should be at target as long as net trade is within bounds (price has a priority over net trade). The adjustment schemes may be extended to a chain of priorities to include more than two target elements (for example, price, net trade and stocks), or to represent nested priorities between two variables.

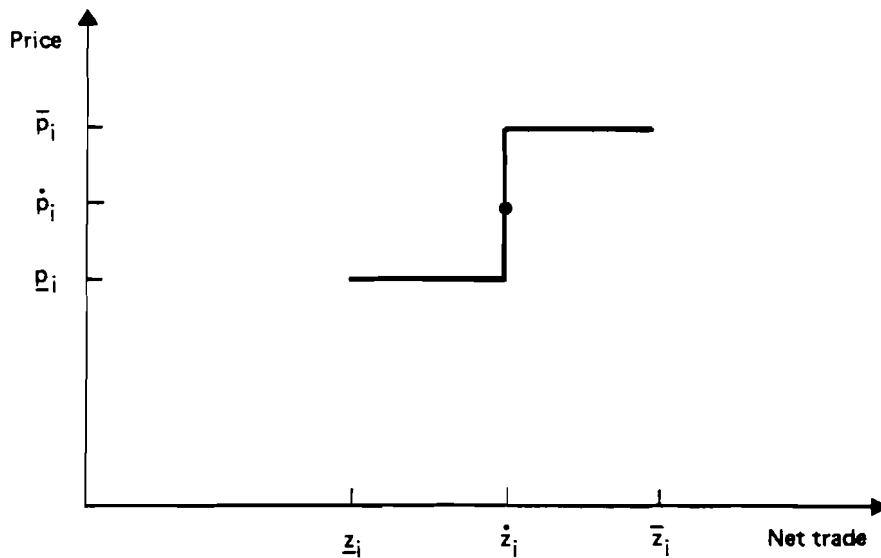


Figure 2. Net Trade Has Priority Over Price

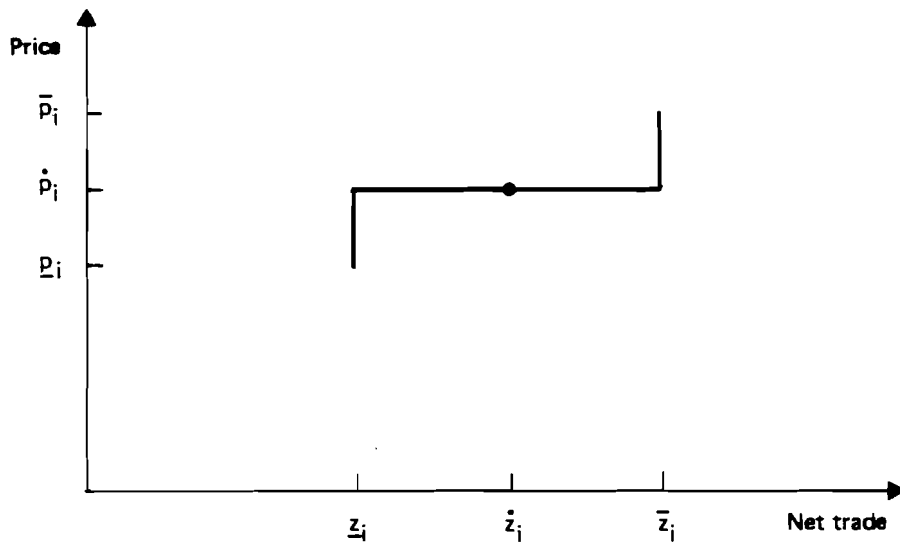


Figure 3. Price Has Priority Over Net Trade

#### 4. The comparative statics of the simplified version

In this section we momentarily drop the assumption that supply or net output is predetermined. The comparative static analysis can then more conveniently be carried out by means of net trade equations instead of expenditure equations.

The country's economy is assumed to have a production possibility set  $T$  which consists of all net output vectors  $y$  which are technically feasible given the quantities of the fixed production factors. If the net outputs are predetermined at levels  $\bar{y}$  as in the simplified version, then  $T = \{y \mid 0 \leq y \leq \bar{y}\}$ .

The trade utility function  $g$  is derived from the utility function  $u$  and the production possibility set  $T$ :

$$g(z) = \max_{\mathbf{x}} \{u(\mathbf{x}) \mid \mathbf{x} \in T + z\}. \quad (20)$$

For each net trade vector  $z$ , the trade utility function  $g(z)$  indicates the maximum level of utility attainable for the country.

The country's budget is denoted by  $f$ , since in the simplified version the budget equals taxes. In general, however, the budget may consist, for example, of tariff revenues or foreign aid. By analogy with the theory of the consumer, an indirect trade utility function  $h$  is defined as

$$h(p, f) = \max_z \{g(z) \mid p'z \leq -f\}. \quad (21)$$

For each price vector  $p$  and budget  $f$ , the indirect trade utility function  $h$  indicates the maximum level of utility attainable for the country. The indirect trade utility function  $h$  is positively homogeneous of degree one in prices  $p$  and budget  $f$ .

The concept of an indirect trade utility function was introduced by Woodland (1980). He showed that, under certain conditions on the utility function  $u$  and the production possibility set  $T$ , the derived net trade equations  $z(p, f)$  can be obtained by applying Roy's Identity to the indirect trade utility function  $h$ :

$$z(p, f) = - \frac{\partial h(p, f)}{\partial p} / \frac{\partial h(p, f)}{\partial f}. \quad (22)$$

According to the simplified version, the country will, for given domestic prices  $p$ , maximize the value of the indirect trade utility function  $h$  with respect to taxes  $f$  subject to the balance of trade equation. The equilibrium conditions thus consist of the derived net trade equations and of the balance of trade equation:

$$z = z(p, f), \quad (23)$$

$$p_w' z = k. \quad (24)$$

Of course, since the price formation equation is not differentiable everywhere, the comparative static results will only apply to equilibrium prices where the price formation equation is differentiable, and thus, to (infinitesimal) small changes in the independent variables which do not cause regime switches.

The system of derived net trade equations (23) will be examined first without taking into account the balance of trade equation. It is well known that the total differential of such a system can be written as

$$dz = z_f [df - z'dp] + Kdp, \quad (25)$$

where  $z_f$  is the vector of derivatives of net trade  $z$  with respect to taxes  $f$ , and where  $K$  is the substitution matrix. Under certain regularity conditions on the trade utility function  $g$ , the substitution matrix  $K$  has the following properties:

$$\begin{array}{ll} K = K' & \text{(symmetry)} \\ p'K = 0 & \text{(adding-up)} \\ Kp = 0 & \text{(homogeneity)} \\ x'Kx < 0, \text{ for all } x \neq \alpha p, \alpha \text{ a real scalar} & \text{(negativity)*} \end{array} \quad (26)$$

Also,  $p'z_f = 1$  (see Barten, 1977). The negativity condition implies that the diagonal elements of  $K$  are negative. It should be noted that the matrix  $K$  is the difference between the proper substitution matrix obtained from the derived

\* Barten (1977) showed that the negativity condition is implied by the assumption that the (trade) utility function  $g$  is strong quasi-concave, i.e.,

$$x' \frac{\partial g(z)}{\partial z \partial z'} x < 0 \text{ for all non-zero } x \text{ such that } \frac{\partial g(z)}{\partial z} x = 0.$$

The negativity condition could be relaxed to the weaker condition:

$$x' Kx \leq 0 \text{ for all } x \neq \alpha p, \alpha \text{ a real scalar}$$

However, as is clear from the discussion in Barten (1977), the (trade) utility function  $g$  then cannot be twice differentiable. Since we want to interpret the infinitesimal changes as small finite changes, the negativity condition as stated seems to be appropriate.

demand equations and the matrix relating changes in net outputs to changes in prices obtained from the derived net output equations. If net outputs are predetermined, the latter matrix reduces to the zero matrix.

In the simplified model taxes are not predetermined but endogenous. Therefore, the changes in taxes must be consistent with the balance of trade equation. Taking the total differential of both sides of the balance of trade equation (24) gives

$$p_w' dz = dk - z' dp_w \quad (28)$$

Premultiplying of both sides of equation (25) by  $p_w'$  and substitution of the resulting expression for  $p_w' dz$  in (28) gives

$$df = z' dp + \frac{1}{p_w' z_f} [dk - z' dp_w - p_w' K dp]. \quad (29)$$

In this equation changes in taxes are related to changes in the exogenous variables such that the balance of trade equation remains satisfied. From (29) it follows that  $\frac{\partial f}{\partial k} = 1/p_w' z_f$ .

Substitution of  $df$  in (25) by the expression for  $df$  in (29) leads to the following system:

$$dz = z_k [dk - z' dp_w - p_w' K dp] + K dp, \quad (30)$$

where  $z_k$  denotes the vector of partial derivatives of net trade  $z$  with respect to the balance of trade  $k$ :  $z_k = z_f/p_w' z_f$ . In this system changes in net trade are related to changes in the exogenous variables such that the balance of trade equation remains satisfied. For

$$K^* = [I - z_k p_w'] K, \quad (31)$$

the system above can be written as

$$dz = z_k [dk - z' dp_w] + K^* dp. \quad (32)$$

The matrix  $K^*$  will be called the domestic-price-effect matrix.

From (32) one sees that changes in domestic prices affect net trade only via the domestic-price-effect matrix  $K^*$ . The domestic-price-effect  $K^* dp$  can be split up into the two components:  $K dp$  and  $-z_k p_w' K dp$ . The first component, the substitution component, represents the effect of domestic price changes on net trade in the absence of a balance of trade restriction and with taxes hold constant. The effect on the trade deficit situation of the country is  $p_w' K dp$ . This amount can, of course, be positive, negative or zero. For the sake of interpretation only, assume  $p_w' K dp$  is negative. The country then has a surplus equal to  $-p_w' K dp$ . This surplus is spent as if it was an additional increase in the trade deficit. The effect on net trade is then given by  $-z_k p_w' K dp$ . Note that the

decomposition of the restricted substitution matrix  $K^*$  is invariant under order-preserving transformations of the utility function  $v$ , since the substitution matrix  $K$  itself is invariant under such transformations. The second effect can therefore be identified as the balance of trade effect of domestic price changes. The total balance of trade effect is given by  $z_k [dk - z' dp_w - p_w' Kdp]$  as can be seen from equation (30). It is made up of three components: the component  $z_k dk$  is the proper balance of trade effect, the component  $-z_k z' dp_w$  represents the balance of trade effect of changes in world market prices, and the third component  $-z_k p_w' Kdp$  is the balance of trade effect of domestic price changes.

The domestic-price-effect matrix  $K^*$  has the following properties:

$$\begin{aligned} p_w' K^* &= 0 && \text{(adding-up)} \\ K^* p &= 0 && \text{(homogeneity)} \end{aligned}$$

Also,  $p_w' z_k = 1$ . The adding-up property, which reflects the balance of trade restriction, follows from the definition of  $K^*$  itself (equation 31). The homogeneity property, which shows that the decisions of the consumer and producer are based on domestic prices, follows from the homogeneity property of the substitution matrix  $K$ . The homogeneity property says that a proportional change in all domestic prices will leave net trade unchanged. Of course, taxes will increase by the same proportion, as can be seen from equation (29). Because of the homogeneity property, domestic prices can be normalized by restricting the price of a base commodity, the last commodity for example, to be equal to the world market price. This approach also implies that tariffs are normalized by setting the tariff of the last commodity equal to zero.

We turn now to the discussion of the welfare effects of changes in the exogenous variables. The total differential of the indirect trade utility function  $h$  is given by:

$$dh = \frac{\partial h(p,f)}{\partial p'} dp + \frac{\partial h(p,f)}{\partial f} df. \quad (33)$$

Making use of equation (22) and equation (29), one gets

$$dh = \frac{\lambda}{p_w' z_f} [dk - z' dp_w - p_w' Kdp], \quad (34)$$

with  $\lambda$  the marginal utility of the budget:  $\lambda = \frac{\partial h(p,f)}{\partial f} > 0$ .

The marginal utility of the trade deficit is given by

$$\frac{\partial h}{\partial k} = \frac{\lambda}{p_w' z_f} > 0. \quad (35)$$

It is equal to the marginal utility of the budget multiplied by  $1/p_w' z_f$ , which is the derivative of the budget with respect to the trade deficit.

The effect on utility of an increase in the world market price of a single commodity  $i$  is equal to:

$$\frac{\partial h}{\partial p_{i_w}} = - \frac{\lambda}{p_w' z_f} z_i . \quad (36)$$

The effect is positive if the country is an exporter of the commodity, negative if it is an importer of the commodity. If all world market prices change, the change in utility is equal to  $-\frac{\lambda}{p_w' z_f} z' dp_w$ , and thus of the same sign as the magnitude  $-z' dp_w$ . This magnitude can be interpreted as changes in the terms of trade (see Woodland, 1980, pp. 912-913). Thus small changes in world market prices  $p_w$  will cause an increase in utility if, and only if, the terms of trade improve.

The effect of domestic price changes on the level of utility is given by

$$\frac{\partial h}{\partial p} = - \frac{\lambda}{p_w' z_k} p_w' K dp . \quad (37)$$

In general, nothing can be said a priori on the sign of this effect. However, if the change in domestic prices is positively (negatively) proportional to the world market prices translated by any scalar multiple of the domestic prices, then the level of utility does increase (decrease). To see this, let:

$$dp = (p_w - \beta p) d\alpha \quad , \beta \text{ a real scalar } , \quad \alpha > 0 . \quad (38)$$

Because of the homogeneity property and the negativity property of the substitution matrix  $K$ , one then has

$$\frac{\partial h}{\partial p} = - \frac{\lambda}{p_w' z_k} p_w' K (p_w - \beta p) d\alpha > 0 . \quad (39)$$

It should be stressed that in general nothing can be said about the effect on the level of utility of a change in one domestic price. Since  $p'K = 0$ , equation (37) can be written as

$$\frac{\partial h}{\partial p} = \frac{\lambda}{p_w' z_k} (p_w - p)' K dp . \quad (40)$$

Thus, even in the case that all domestic prices except two are equal to the world market prices and that one of these two domestic prices moves into the direction of the world market price, can the level of utility decrease.

In this section the comparative static results of the simplified version of the open exchange model have been presented. One should keep in mind that these results are derived under the assumption of the existence of one (social) utility function. Therefore the results may differ if there are many socio-economic groups with different utility functions.

## 5. Countries, Commodities

### 5.1. Countries

The EC model comprises 8 national models for:

1. Federal Republic of Germany
2. France
3. Italy
4. The Netherlands
5. Belgium - Luxemburg
6. United Kingdom
7. Ireland
8. Denmark

Development of a model for Greece is being considered.

### 5.2. Commodities

Inter EC supply and demand are expressed at raw material level according to a 15 commodity classification. The EC commodity list is somewhat more disaggregated than the so-called IIASA commodity list.

EC Commodity List	IIASA Commodity List
1. wheat	1. wheat
2. coarse grain	2. coarse grain
3. rice	3. rice
4. bovine + ovine meat	4. bovine + ovine meat
5. dairy	5. dairy
6. pork,poultry,eggs } 7. fish	6. other animals
8. protein feed	7. protein feed
9. oilseeds	8. other food + beverages
10. sugar	
11. fruit	
12. vegetables	
13. beverages + resid. other foods } 14. nonfood agriculture	
15. nonagriculture	9. nonfood agriculture
	10. nonagriculture

## 6. The EC-exchange

### 6.1. Introduction

This section describes the exchange component of the EC model. Here supply and demand of the individual EC countries are interconnected through market clearing and policy setting. The exchange rates are supposed to be exogenously given. At a later stage a version with endogenous exchange rates interlinked through a "snake" agreement under the European Monetary System may be implemented (see M. Keyzer, 1981b). A variable now may have two subscripts. The first subscript refers to commodities, the second subscript refers to countries or the world market. The commodity index is  $i$ , the index for countries is  $j$  and the index for the world market is  $w$ .

In this section and the next sections national prices are expressed in national currencies, EC prices in European Currency Units (ECU), and world market prices in US dollars.

### 6.2. Demand

Private demand,  $q_{ij}$ , follows a Linear Expenditure System, expressed at raw material level:

$$P_{ij} q_{ij} = P_{ij} v_{ij} + b_{ij}(C_j - \sum_h P_{hj} v_{hj}) \quad , \quad \sum_i b_{ij} = 1, \quad (41)$$

where:

$v_{ij}$  = private committed demand

$b_{ij}$  = marginal budget share

$C_j$  = total private consumption

Government demand,  $g_{ij}$ , is determined exogenously to the model. The value of government demand  $G_j$  is:

$$G_j = \sum_i P_{ij} g_{ij}. \quad (42)$$

Gross investment,  $i_{ij}$ , is also exogenously determined. The investment value  $I_j$  is:

$$I_j = \sum_i P_{ij} i_{ij}. \quad (43)$$

Total demand  $x_{ij}$  is the sum of private demand, government demand and gross investment.

$$x_{ij} = q_{ij} + g_{ij} + i_{ij}. \quad (44)$$

Total demand can be written in the form of a Linear Expenditure System:

$$P_{ij} x_{ij} = P_{ij} \gamma_{ij} + b_{ij}(E_j - \sum_h P_{hj} \gamma_{hj}), \quad (45)$$

where

$$\gamma_{ij} = v_{ij} + g_{ij} + i_{ij},$$

$$E_j = C_j + G_j + I_j.$$



### 6.3. Supply - Gross Domestic Product - Taxes - Trade

Gross domestic product at market prices  $Y_j$  is given by

$$Y_j = \sum_i p_{ij} y_{ij} . \quad (46)$$

Private consumption  $C_j$  is taken as a linear function of disposable income  $m_j$  and commodity prices  $p_{ij}$ :

$$C_j = \alpha_j m_j + \sum_i p_{ij} \beta_{ij} . \quad (47)$$

This can be looked at as a specification with real consumption as a linear function of real income. Disposable income  $m_j$  is defined as gross domestic product  $Y_j$ , plus private transfer from abroad  $F_j$  minus transfer to EC budget  $T_j$ , minus government consumption:

$$m_j = Y_j + F_j - T_j - G_j . \quad (48)$$

Clearly, (47) and (48) imply:

$$Y_j = C_j + S_j + (T_j - F_j) + G_j , \quad (49)$$

where  $S_j$  stands for savings.

Tax contribution to the EC is, in accordance with EC regulation, taken to be proportional to the "tax base", i.e. the GDP at market prices:

$$T_j = tY_j . \quad (50)$$

Observe that  $t$  is the same for all countries in the EC. Transfer from abroad is treated as a given commodity bundle:

$$F_j = \sum_i p_{ij} \phi_{ij} . \quad (51)$$

The commodity balance equation now becomes:

$$z_{ij} = x_{ij} + g_{ij} + i_{ij} - y_{ij} . \quad (52)$$

At EC level net import  $z_i$  obviously is

$$z_i = \sum_j z_{ij} . \quad (53)$$

and the net import value at domestic market prices,  $Z_j$ , is given by

$$Z_j = \sum_i p_{ij} z_{ij} . \quad (54)$$

The aggregate demand definition of GDP becomes:

$$Y_j = C_j + G_j + I_j - Z_j \quad (55)$$

The trade deficit  $k_j$  in international currency is given by

$$k_j = \sum_i p_{iW} z_{ij} , \quad (56)$$

where  $p_{iW}$  is the given international price of commodity  $i$ .

### 7. EC Policy Setting

We first describe the modeling of the price policy for the CAP commodities, i.e. the commodities subject to the CAP.

For the CAP commodities, the EC sets an import price (threshold prices)  $\bar{p}_i$ , and an export price (intervention price)  $\underline{p}_i$ , such that  $\underline{p}_i \leq \bar{p}_i$ , expressed in terms of ECU's.

Let  $\Pi_{\$}$  be the exogenously prescribed exchange rate between US\$ and the ECU (\$ per ECU) and let world market prices  $p_{iW}$  be measured in US\$. The levy on import then is

$$l_i^+ = \bar{p}_i - p_{iW} / \Pi_{\$} ,$$

and the refund on export is:

$$l_i^- = \underline{p}_i - p_{iW} / \Pi_{\$} .$$

The levy charged (i.e. restituted) becomes prohibitive as soon as the intra-EC price  $p_i$  drops below  $\bar{p}_i$ . Thus import is zero if  $p_i < \bar{p}_i$ . Analogously, export outside EC cannot be performed without a loss unless  $p_i = \underline{p}_i$ . In terms of complementarity conditions this can be specified as:

$$\begin{aligned} \underline{p}_i \leq p_i \leq \bar{p}_i & \quad \text{price within bounds} \\ (\bar{p}_i - p_i) z_i^+ = 0 & \quad \text{no imports unless price is at upper bound} \\ (p_i - \underline{p}_i) z_i^- = 0 & \quad \text{no export unless price is at lower bound} \\ z_i = z_i^+ - z_i^- & \\ z_i^+ , z_i^- \geq 0 & \end{aligned} \quad (57)$$

where  $z_i^+$  refers to imports and  $z_i^-$  to exports.

Figure 4 illustrates the internal effect of the CAP. The net trade of the EC with the world is indicated on the horizontal axis. Exports are measured negatively to the left of the zero point, and imports positively to the right of this point. Prices are measured vertically. Instead of using separately a total EC demand and a total (fixed) EC supply curve, an EC net demand curve is used which gives for every price the difference between total EC demand and total (fixed) EC supply. The EC net demand curve can vary because of shifts in the demand curves of individual countries (due to a higher income, for example). According to the price policy of the EC, the upper price (threshold price)  $\bar{p}_i$  is valid when the EC is an importer of the agricultural product, while the lower price (intervention price)  $\underline{p}_i$  is valid when the EC is an exporter of the product. Only if the trade of the EC with the world is zero may the price lie between the two bounds. The actual price (equilibrium price) will then depend on the position of the EC net demand curve. Clearly, the greater the difference between the upper price  $\bar{p}_i$  and the lower price  $\underline{p}_i$ , the higher the probability of self-sufficiency of the EC.

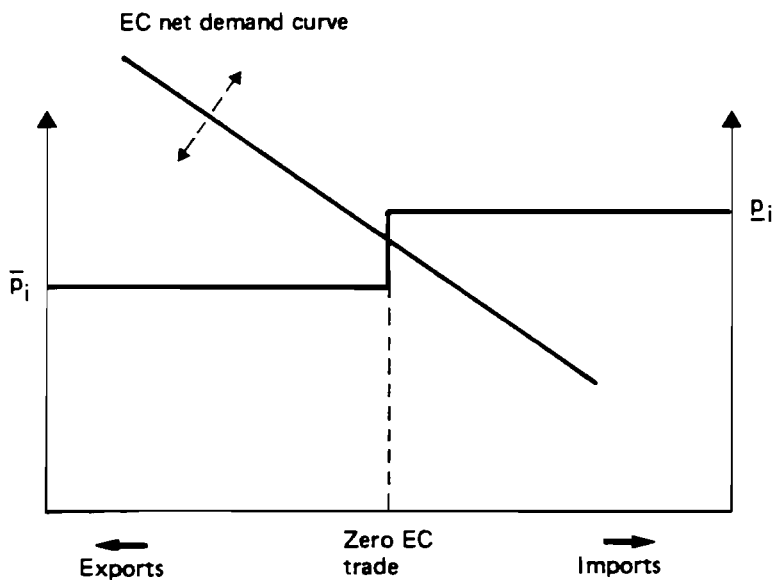


Figure 4. The Variable Import/Export Levy and Refund Policy.

Importers and exporters in the EC do not trade in terms of European Units of Account, but in national currencies. Therefore, the prices ( $\underline{p}_i$ ,  $\bar{p}_i$ ) have to be converted into national currencies. A very special feature of the CAP is that the exchange rates which are used to convert Units of Account into national currencies, the so-called "green" rates, may (temporarily) lag behind the official rate and deviate from the market rate.

Thus market prices  $p_{ij}$  of CAP commodities in country  $j$  are determined as:

$$p_{ij} = \Pi_j^g p_i, \quad (58)$$

where  $\Pi_j^g$  is the green exchange rate. Obviously the prices between two EC countries, say  $j$  and  $k$ , will differ at market exchange rates. The difference is compensated by a kind of tariff rate, the so-called Monetary Compensatory Amount (MCA),  $\mu_{jk}$ .

$$\mu_{jk} = \left( \frac{\Pi_j^g}{\Pi_j} \right) / \left( \frac{\Pi_k^g}{\Pi_k} \right) - 1, \quad (59)$$

where  $\Pi_j$  is the market exchange rate. The price relation then is:

$$p_{ij} = \frac{\Pi_j}{\Pi_k} (1 + \mu_{jk}) p_{ik}. \quad (60)$$

We observe that the system ensures that whenever  $\underline{p}_{ij} < p_{ij} < \bar{p}_{ij}$ , it does not pay for any country to trade with non-EC members, but that intra-EC trade is possible at all prices  $\underline{p}_i \leq p_i \leq \bar{p}_i$ .\*

For commodities not subject to the CAP, a straight common import tariff is imposed:

$$p_{ij} = \frac{\Pi_j}{\Pi_{\$}} (1 + \theta_i) p_{i\$}. \quad (61)$$

Although (57) shows the main principles of the CAP, the import and export prices ( $\underline{p}_i, \bar{p}_i$ ) are, for several commodities, determined as functions of international prices and are not fixed within a year. Moreover, buffer stocks also play a role of their own, especially in commodities for which the EC has a large share of the international market (such as dairy).

Let stocks of commodity  $i$  be denoted by the variable  $w_i$ . The EC defines a lower bound  $\underline{w}_i$ , an upper bound  $\bar{w}_i$ , and a target quantity  $\dot{w}_i$ , such that  $\underline{w}_i \leq \dot{w}_i \leq \bar{w}_i$ . The price bounds at which the stock policy becomes active are denoted by  $\underline{p}_i$  and  $\bar{p}_i$ . These price bounds may differ from the price bounds  $\underline{p}_i$  and  $\bar{p}_i$  at which the variable import/export levy and refund policy becomes active. Since the stock policy (if any) becomes active first, one has  $\underline{p}_i \leq \underline{p}_i \leq \bar{p}_i \leq \bar{p}_i$ . The stock policy can be represented in terms of the following complementarity conditions:

\* We must assume that this relation also holds for specifying price bounds at the national level. Otherwise the relations (57) would not hold, since a levy might be prohibitive in Country A without being prohibitive in Country B. In actual fact the bounds at the national level are specified as  $\bar{p}_{ij} = \Pi_j^g \bar{p}_i$  and  $\underline{p}_{ij} = \Pi_j^g \underline{p}_i$ . This, however, is an inconsistency in the CAP which cannot be represented in this model.

$$\begin{aligned}
 p_i &\leq p_i \leq \bar{p}_i && \text{price within bounds} \\
 \underline{w}_i &\leq w_i \leq \bar{w}_i && \text{stock within bounds} \\
 (w_i - \bar{w}_i) &= w_i^+ - w_i^- && \text{stock deviation from target} \\
 w_i^+ (p_i - \underline{p}_i) &= 0 && \text{no buying of stock unless price is at lower bound} \\
 w_i^- (\bar{p}_i - p_i) &= 0 && \text{no selling of stock unless price is at upper bound.} \\
 w_i^+, w_i^- &\geq 0 && 
 \end{aligned}
 \tag{62}$$

where  $w_i^+$  refers to increase of stocks and  $w_i^-$  to decrease of stocks.

Clearly, commodity balance with stocks is:

$$z_i = \sum_j z_{ij} + w_i \tag{63}$$

The stock policy is illustrated in Figure 5.

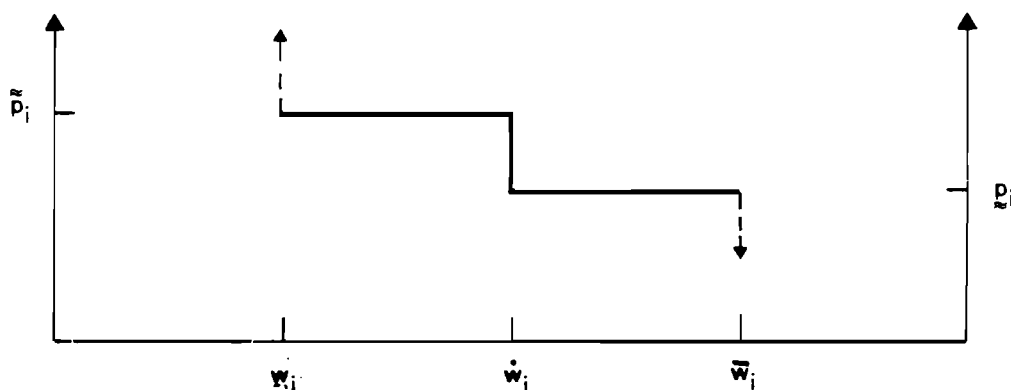


Figure 5. Stock policy

The combination of the two sets of complementarity conditions (57) and (62) defines the complementarity conditions representing the combination of the stock policy with the variable import/export levy and refund policy.

### 8. The EC-canonical form

Starting from the total demand system (45) and substituting back to exogenous variables leads to the following system:

$$p_{ij} x_{ij} = p_{ij} \gamma_{ij} + \sum_h p_{hj} [(1-t) \alpha_j b_{ij} y_{hj} + \alpha_j b_{ij} (\phi_{hj} - g_{hj}) + b_{ij} (\beta_{hj} - v_{hj})] \tag{64}$$

Define matrices  $A_j$  and  $M_j$  with typical elements

$$a_{hi} = \alpha_j b_{ij} y_{hj}, \tag{65}$$

$$m_{hi} = b_{ij} [\alpha_j (\phi_{hj} - g_{hj}) + \beta_{hj} - v_{hj}] + \delta_{hi} \gamma_{ij}, \tag{66}$$

where  $\delta_{hi} = 1$  if  $h=i$  and zero otherwise. The expenditure system (64) can then in matrix notation be written as

$$p_j' \hat{x}_j = (1-t)p_j' A_j + p_j' M_j, \quad (67)$$

where  $\hat{x}_j$  is a diagonal matrix with the elements of the vector  $x_j$  on its diagonal. This system is the canonical form for country  $j$  expressed in national prices  $p_j$ .

Since the levels of prices within the EC are determined by the demand of all countries, the country specific canonical forms have to be added up. To do this, they first have to be expressed in the same base price vector. Let  $I^{cap}$  be the set of commodities subject to the CAP. Define the base price vector  $p_b$  as

$$p_{ib} = \begin{cases} p_i & \text{if } i \in I^{cap}, \\ (1 + \theta_i) p_{iw} / \Pi_s & \text{if } i \notin I^{cap}, \end{cases} \quad (68)$$

and define the matrix  $D$  as

$$d_{ij} = \begin{cases} \Pi_j^s & \text{if } i \in I^{cap}, \\ \Pi_j & \text{if } i \notin I^{cap}. \end{cases} \quad (69)$$

The matrix  $D$  is determined exogenously. The price equations (58) and (61) can then be expressed as

$$p_{ij} = d_{ij} p_{ib}, \quad \text{for each } i, \quad (70)$$

or in the usual vector notation as

$$\hat{p}_j = \hat{d}_j \hat{p}_b. \quad (71)$$

Making use of the latter equation, the expenditure system (67) can, after post-multiplication of both sides by  $\hat{d}_j^{-1}$ , be written as

$$p_b' \hat{x}_j = (1-t)p_b' \hat{d}_j A_j \hat{d}_j^{-1} + p_b' \hat{d}_j M_j \hat{d}_j^{-1}. \quad (72)$$

Defining

$$A_j^* = \hat{d}_j A_j \hat{d}_j^{-1}, \quad (73)$$

$$M_j^* = \hat{d}_j M_j \hat{d}_j^{-1}, \quad (74)$$

one can write

$$p_b' \hat{x}_j = (1-t)p_b' A_j^* + p_b' M_j^*. \quad (75)$$

which is the canonical form for country  $j$  expressed in the base price vector  $p_b$ . The EC-canonical form is obtained by adding up all country canonical forms expressed in base price vector  $p_b$ :

$$p_b' \sum_j \hat{x}_j = (1-t)p_b' \sum_j A_j^* + p_b' \sum_j M_j^* . \quad (76)$$

The EC-canonical form is solved for base price  $p_b$ , total EC demand  $\sum_j x_j$  and EC-tax rate  $t$ , subject to the rules of the common price policy, the stock policy and the following EC trade deficit equation:

$$p_w' z = k , \quad (77)$$

where  $k$  is the prespecified trade deficit at EC level evaluated at world market prices. One notes that the EC tax rate is endogenously determined.

The approach described above is not fully consistent. First, there is an inconsistency in the calculation of taxes. Because of the existence of MCA's, the amount of taxes country  $j$  pays according to the solution of the model may differ from the taxes  $t Y_j$  the country has to pay according to the EC-rule. In fact, from equation (72) one derives that taxes  $T_j$  calculated by the model are given by:

$$\begin{aligned} T_j &= \Pi_j t p_b' d_j A_j \hat{d}_j^{-1} \iota \\ &= \Pi_j t p_j' y_j b_j' \hat{d}_j^{-1} \iota \\ &= t Y_j \sum_i b_{ij} a_{ij} . \end{aligned} \quad (78)$$

(one makes use of equation 65 and 71) where  $\iota$  is the summation vector, and where  $a_{ij} = \Pi_j / \Pi_i^g$  if  $i \in I^{cap}$  and one otherwise. If  $\sum_i b_{ij} a_{ij} \neq 1$ , then  $T_j \neq t Y_j$ .

The inconsistency can be solved by working with pseudo-endowments  $\tilde{y}_j = y_j / \sum_i b_{ij} a_{ij}$ .

A second inconsistency may arise since the cost of the MCA's is not included in the model. The model solution does not ensure that the computed cost of the price support for a CAP-commodity equals its real cost. Hence, the model solution does not ensure that the EC-budget is in equilibrium. The calculated cost in ECU's of the price support for CAP-commodity  $i$  is

$$\left( \frac{p_{iw}}{\Pi_s} - p_{ib} \right) z_i , \quad (79)$$

while the real cost is

$$\frac{p_{iw} z_i}{\Pi_s} - \sum_j \frac{p_{ij}}{\Pi_j} z_{ij} . \quad (80)$$

Making use of the relations:  $p_{ij} = \prod_j^g p_i$ ,  $i \in I^{cap}$  (equation 58) and  $p_i = p_{ib}$ ,  $i \notin I^{cap}$  (equation 58), one sees that both cost amounts are equal if, and only if

$$\sum_j \frac{\prod_j^g}{\prod_j} z_{ij} = z_i . \quad (81)$$

If the equality above does not hold, the EC-tax rate is corrected ex post so as to ensure that the EC-budget is in equilibrium. This method is also used to correct for the omission of the cost of stocks. Ex post correction of the EC-tax rate almost does not affect the net-trade figures of the CAP-commodities, since the consumption of these commodities largely exists of committed consumption. The trade deficit now becomes the final adapting variable.

It might be that the difference between the computed cost and the real cost of the EC budget is considered too high to apply the ex post correction of the EC tax rate. In that case the expected difference between computed cost and real cost is added to the prespecified trade deficit figure before running the model. The trade deficit figure obtained after the correction of the EC-tax rate will then be equal to the prespecified trade deficit. The expected difference between computed cost and real cost is calculated, either by using last year's net trade figures, or the net trade figures corresponding to expected or average prices, or the net trade figures obtained by a first run of the model.



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