WORKING PAPER

CONTOUR MAPS OF DEMOGRAPHIC SURFACES

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INTRODUCTION

Contour maps, which are widely used in depicting spatial patterns, can be readily adapted to represent any surface that is defined over two dimensions. In particular, an array of demographic data can often be pictured in an intelligible and graphically striking way by a contour map. The data might pertain to population levels or to rates of fertility, marriage, divorce, migration, morbidity, or mortality. Most often the data are structured by age and time--e.g., age-specific mortality rates over time--but in some cases other dimensions might be used, such as life expectancy or crude birth rate. Contour maps permit visualization of demographic surfaces and offer a panoramic view impossible to obtain from the usual graphs of levels or rates at selected ages over time or at selected times over age. Furthermore, a contour map is often superior to a three-dimensional perspective plot in providing a clear, yet rich representation of a demographic surface; it is usually difficult on a three-dimensional plot to discern the exact position of the surface above the age and time dimensions and three-dimensional plots become confusing if made too detailed, especially when displayed on a moderately-priced monitor or printer. Contour maps are particularly effective in highlighting patterns in the interaction of age, period, and cohort effects.

Contour maps have been used only occasionally by demographers, perhaps because of the computational effort required or because of the lack of detailed data over long stretches of age and time. In their influential study of changes in death rates over time, Kermack, McKendrick, and McKinlay (1934) superimpose on three of their tables some rough lines that are, in effect, contours of relative mortality. The pioneering study by Delaporte (1941) includes a set of contour maps that summarize mortality patterns in several European countries; Federici (1955) directs attention to Delaporte's contour maps in her survey of demographic methods. Recent advances in computers, including the development of powerful microcomputers, as well as the collection and publication of extensive arrays of demographic statistics for single years of age and single years of time (e.g., Natale and Bernassola (1973), Vallin (1973), Heuser (1976) and Veys (1983)), should lead to greater use of demographic contour maps in the future.

This paper presents a bouquet of contour maps to suggest the broad potential of their use in demographic studies. Every picture presented could serve as the basis for a thousand words or more of explanation and analysis, but here we merely serve up the maps as illustrations of the method. For an example of how such maps can be used in demographic analysis, see the study by Caselli, Vaupel, and Yashin of Italian mortality (1985).

LEVELS, SHADES, AND GRIDS ILLUSTRATED BY ITALIAN MALE MORTALITY

Figure 1a displays the contours of mortality for Italian males from age 0 to 79 and for years 1870 to 1979. The map is based on mortality rates, q, for single years of age and time assembled by Natale and Bernassola (1973) and Caselli (forthcoming). Data are discrete, but a surface is continuous: the surface $q(x,y)$ can be defined by linearly interpolating between adjacent data points. The values of $q(x,y)$ give the height of the mortality surface over age x and time y.

The lines on a contour map connect adjacent points that are of equal height; these lines are sometimes called level lines or isograms. In Figure 1a, one of the level lines represents a mortality rate of about 11 percent: the line starts in 1870 at age 35 and ends in 1979 at age 56, indicating that 56-year-old Italian men in recent years faced the same chance of mortality that 35-year-olds faced about a century ago.

An important consideration when designing a contour map is how many different levels to use. The computer program that we employed to draw the maps, which was developed by Gambill under Vaupel's direction at Duke University and at the International Institute for Applied Systems Analysis, allows lines to be drawn at up to 15 levels, separating the surface into 16 tiers. Use of fewer lines sacrifices detail, whereas use of more lines tends to make the map less intelligible: 15 levels is a reasonable compromise, although use of 10 or 20 levels might be considered. Delaporte draws lines at 19, 20, or 21 levels on his various maps of European mortality; a number of the figures in this paper, including Figures 15 and 16, use fewer than 15 levels. Figure 1b presents the contours of Italian male mortality using 10 levels rather than the 15 levels used in Figure 1.

Which specific elevations the contour lines should connect is a second important design decision. On mortality surfaces, where mortality rates might approach a minimum of the order of magnitude of 0.0001 and a maximum of 1, use of equally spaced lines--say at 0.01, 0.02, and so on up to 0.15--results in a map where the contours are clumped together at the youngest and oldest ages, with a largely empty expanse in-between. Figure 1c illustrates this for Italian male mortality. The map is far more informative when the lines are spread out at constant multiples--e.g., each line representing a level 50 percent higher than the previous line, as in Figure 1a. Alternatively, a convenient scale can be used: Delaporte places his lines at levels of mortality of 1, 2, 3, ..., 9, 10, 12, 15, 20, 30, 50, 100, 150, 200, 250, 300, 350, and 400 per thousand, and in several figures in this paper, including Figures 5 and 33, level lines are selectively placed at convenient levels.

Demographers often work with transformations such as the log or logit, so it might seem reasonable to transform the surface $q(x,y)$ into the surface of, say, log $q(x,y)$ and then to draw level lines at equal intervals on the transformed surface. If the transformation is monotonic, like the log or logit transformation, an identical contour map can be drawn by spacing the level lines at appropriately unequal intervals on the original surface. In the case of logarithms, the level lines should be at multiples of each other rather than being equally spaced. Thus, the map in Figure 1a can also be interpreted as depicting log mortality rates.

An innovation in the computer program we used is the shading of regions according to the height of the surface. The shading varies from light to dark as the surfaces rise from low to high levels of mortality. Such shading, which is time-consuming to do by hand but easy with the help of a computer, makes the overall pattern of a mortality surface more immediately comprehensible, especially if the map is viewed at a distance. At the same time, the details of small peaks and pits and of the twists and turns of the contours lines are still there to be scrutinized at close range. Literature, critics note, can be profitably read at different levels of understanding; we suggest the reader try viewing Figure 1a and perhaps some of the other figures in this paper at levels of 5 meters and 25 centimeters.

Sometimes it is useful to draw a grid on a contour map so that the coordinates of various points can be conveniently located. In Figure 1d the map in Figure 1a is redrawn with a superimposed grid every twenty years of time and age. The grid detracts a bit from the underlying pattern--that is the price of adding additional information. Grids are included in some of the maps presented later in this paper.

To see general trends it may be helpful to suppress the contour lines in a map of a population surface. In Figure 1e the map in Figure 1a is redrawn with shading but without lines. Alternatively, one could draw a traditional contour map with lines but without shading. Figure 1f displays such a map for Italian male mortality. The lines in Figure 1f are not labelled but they could be.

SMOOTHED MAPS

It is useful to take a close look at the small black blemishes isolated from contour lines on a contour map, because these dark spots indicate outliers--very localized peaks or pits--that might be due to erroneous data values. Consider, for instance, the black spot in Figure 1a at about age 54 and year 1878: it turned out that this blemish was indeed produced by an error made in transcribing the Italian mortality data to a computer tape. (The error was corrected, but we left the spot as an illustration.) On the other hand, the mark at about age 20 in 1962 represents a point where the mortality surface barely crosses a contour level, like the top of a sea mount that appears as a small island just rising above the level of the surrounding ocean.

In addition to these blemishes, some rough black blots are smeared across level lines in Figure 1a. These represent virtual plateaus where the mortality surface is repeatedly crossing and recrossing a level line. To eliminate this kind of noise and to suppress the details of local fluctuations so that global patterns can be more clearly perceived, it may be useful to smooth a surface. Delaporte presented both raw and smoothed contour maps of mortality rates in various European countries: on his "adjusted" maps, Delaporte drew smooth contour lines based on his feeling for the data. We used a mechanistic, computer algorithm to produce the smoothed map shown in Figure 1g. In this map the height of the surface at age x in year y was replaced by the average of the 25 heights in the 5 by 5 square of points from $x-2$ to $x+2$ and from $y-2$ to $y+2$. On the edges of the map, where a full 5 by 5 array of data points is not available, the smoothing procedure averages the available data.

Instead of smoothing by averaging over a 5 by 5 square, a larger (or smaller) square might be used. In Figure 1h we smoothed Italian male mortality on an 11 by 11 square. Global patterns in this map are somewhat clearer than in Figure 1g but some interesting local detail is lost and effects that are concentrated in time or age, such as infant mortality and mortality during the 1918 Spanish influenza epidemic, are smeared out.

A variety of alternative smoothing procedures might be used, including procedures that replace points by a weighted average of adjacent points, the weights diminishing with distance. Figure 1i presents a map of Italian male mortality smoothed by an algorithm in which the weights given to the points in a 5 by 5 square were proportional to:

1	4	6	4	1
4	16	24	16	4
6	24	36	24	6
4	16	24	16	4
1	4	6	4	1

Thus the points in the corners of the square were given weights of 1/256, whereas the point in the center received a weight of 36/256. The theoretical advantages of such weighted smoothing algorithms (see Tukey (1977) for an introductory discussion) have to be balanced against the conceptual simplicity and computational convenience of the kind of straightforward averaging illustrated in Figures 1g and h. Note that in Figure 1i the surface is reduced by two years along each edge because the smoothing procedure used requires a full 5 by 5 array of data; special modification of the procedure could be made, analogous to the modification of the smoothing procedure used to produce Figures 1g and h, so that a weighted smoothing could handle data points up to the edges of the surface.

CLOSE-UPS

As discussed by Caselli, Vaupel and Yashin (1985), the patterns of male mortality in Italy from ages 10 to 49 for years 1910 to 1969 reveal some interesting cohort effects. Figures 2a and 2b present contour maps of this restricted age and time area: the maps can be considered an enlargement or close-up of a section of the map in Figure 1a. Thus contour maps can be used both to display a large data array and also to focus in on selected portions of the array. Note that in Figure 2a the contours are drawn at different, more narrowly spaced levels than the contours in Figure 1a, but in 2b they are drawn at the same intervals as in 1a: part of the advantage of a close-up is that if the height of the surface varies less in the restricted region being scrutinized, then the level lines can be located at closer intervals to reveal more local detail.

MAPS FROM INTERPOLATED DATA

The mortality rates for Italian males used in Figures 1 and 2 are available by single year of age and single year of time. Frequently demographers have to work with less finely-spaced data; mortality rates, for instance, may be available every decade or so, by five-year age classes. Figure 3 displays the evolution of Italian male mortality based on data published in Preston, Keyfitz, and Schoen (1972). Data sets from this source were available for 1881, 1891, 1901, 1910, 1921, 1931, 1960, and 1964. Death rates were given for five-year age categories from age 5 up to age 80, as well as for age zero and the four-year category from age 1 to 5. We converted the n-year death rates into single-year death rates using a standard method (described in Vaupel, Manton, and Stallard 1979) and then used simple linear interpolation between the available data points over time to estimate the height of the mortality surface at intermediate points in time. Comparison of Figures 1a and 3 reveals the difference it makes to work with detailed data as opposed to interpolated data.

The longest time series of mortality rates are available for Sweden: we used data, from Keyfitz and Flieger (1968) for 1778 to 1882 and from various editions of the Swedish Statistical Yearbook for 1881 through 1981, which was available for the most part for five year periods and for five year age categories before 1880 but by single year of age thereafter. Figure 4 shows the evolution of Swedish female mortality from 1778 to 1981 based on interpolations (Vaupel, Manton, and Stallard 1979) we made using this data.

AGE-PERIOD, AGE-COHORT, AND COHORT-PERIOD MAPS OF U.S. FEMALE FERTILITY

Figure 5 displays the contours of U.S. birth rates from 1917 to 1980 for women from age 14 to 49; the figure is based on data compiled by Heuser (1976; 1984). In the dark center of the baby boom, for women around age 23 around 1960, fully a quarter of women gave birth each year. The concentration of high birth rates among women in their early and mid twenties and the cycles of high and low birth rates that underly Easterlin's theory are strikingly revealed on the map.

Figure 5 is a standard map in which current year runs along the horizontal axis and age runs up the vertical axis. Other coordinates help reveal cohort effects. In particular, because the eye can follow vertical and horizontal lines more easily than diagonals, it may be useful to twist a contour map so that year of birth, rather than current year, runs along the horizontal axis. Figure 6 illustrates this approach. Alternatively, as shown in Figure 7, year of birth may run along the horizontal axis and current year along the vertical axis. We only used five contour lines on Figure 7 because the lines were otherwise too closely spaced to be intelligible: even with five lines there are two black splotches where fertility rates are increasing so rapidly that the contour lines run together.

ALTERNATIVE GRAPHIC DISPLAYS OF U.S. FEMALE FERTILITY

The most commonly used method for displaying demographic rates over age and time is to plot the rates over time for selected ages or over age for selected times. In Figure 8, for instance, U.S. birth rates are graphed over time at ages 18, 23, 28, 33, 38, and 43 and in Figure 9 the birth rates are graphed from age 14 to 49 for years 1920, 1930, 1940, 1950, 1960, 1970, and 1980. Comparison of Figures 8 and 9 with the contour maps presented in Figures 5, 6, and 7 reveals some of the strengths and weaknesses of these alternative graphic displays.

Figures 10 and 11 show two plots of the U.S. birth rate data drawn from a three-dimensional perspective. Figure 10, like all the contour maps in this paper, was produced using an ordinary printer; Figure 11 was drawn using a computer plotter. The three-dimensional plot sacrifices some of the richness of detail that is clearly portrayed on the corresponding contour maps; furthermore, it is difficult on the three-dimensional plot to relate a point on the surface to the exact age and year underlying the point.

What graphic method should be used to display a demographical surface? Clearly, each method has its strengths and weaknesses, with contour maps having special advantages in some circumstances. Demographers should consider adding contour maps to their toolkit of graphical techniques.

RELATIVE SURFACES OF SWEDISH POPULATION, ITALIAN MORTALITY, AND U.S. FERTILITY

To depict the change that has occurred over time in age-specific fertility rates, mortality rates, population levels, or other demographic statistics, it is useful to draw contour maps of relative surfaces on which the value of the statistic at each point is calculated relative to value of the statistic in some base year. Consider, for example, Figures 12 and 13. Figure 12 displays Swedish population levels from birth to age 79 from years 1780 to 1963, based on interpolations of data in Keyfitz and Flieger (1971); Figure 13 presents these population levels relative to the population level at the various ages in the first year, 1780.

Two other applications of this approach are shown in Figures 14 and 15. Figure 14 displays age-specific mortality rates for Italian males relative to their levels in 1925, a year roughly halfway through the period studied. Figure 15 presents age-specific birth rates for U.S. females relative to their level in the final year, 1980.

Instead of dividing a demographic array by the age-specific statistics for a particular year, the array could be divided by the period-specific statistics for a particular age. For example, Figure 16 shows Italian male mortality rates at various ages relative to the infant mortality rate in the appropriate year.

Demographic statistics could also be expressed relative to some composite age-specific or period-specific measure. Figures 17 and 18 provide two examples. To produce Figure 17, Belgium age-specific female population levels (from Veys 1983) were divided by the total Belgium female population in each year. Thus, the map gives contours of the age-distribution of the population, i.e., the percentage of the population in each year that are at various ages. The black blotches and streaks are due to rapidly changing or fluctuating population levels, causing several contour lines to run together. Figure 18, which is based on U.S. fertility data, is similar in nature except that the contours pertain to cumulative levels up through age 49 relative to the total level over all ages. The map can be interpreted as showing the proportion of all births in a given year that occurred to women of some age or less--in a synthetic population in which there were equal numbers of women at each age.

Finally, it may sometimes be useful to examine contour maps based on statistics relative to a cohort-specific measure rather than either an age-specific or period-specific measure. Consider, for instance, Figure 19, which is similar to Figure 18 except that cumulative fertility is computed relative to total cohort fertility up through age 39.

SMALL MULTIPLES

To compare global patterns among several population surfaces it may be useful to shrink contour maps down in size and present several of them on the same page: Tufte (1983) calls this the "small multiples" approach. Figure 20 presents maps of U.S. birth rates at various parities: in Figure 20a, first-birth rates (i.e., the proportion of childless women of some age in some year who have their first child) are displayed; in Figure 20b, second-birth rates are displayed, and so on. In each of the small multiples, the same contour levels are used.

Figure 21 presents another illustration of the use of small multiples, this time to compare mortality rates from age 10 to 49 for years 1910 to 1965 for Italian, French, and Belgian males and females. The effects of the First and Second World Wars are most prominent in this period and for these ages.

Figure 22 displays relative mortality rates for males and for females in England and Wales, Sweden, and Italy from age 5 to 80 for years 1870 to 1978. In each case, the mortality rate for a given age is relative to the rate at that age in 1870. Thus the maps provide a picture of the pattern of progress made in reducing mortality rates since 1870. The maps are analagous to the tables with rough contour lines in Kermack, McKendrick, and McKinlay (1933); Preston and van der Walle (1978) and Coale and Kisker (1985) present similar tables. These analysts ascribe the diagonal contours in their tables to cohort effects. The maps in Figure 22 provide a richer, more detailed picture of the various local and global patterns in the changes in mortality rates, in vertical, horizontal, and diagonal directions.

The maps in Figure 22 for the three countries were produced using different kinds of data. As noted earlier, mortality rates for Italy were available for single years of time and age. For Sweden, the rates were generally available for five year periods; before 1880 the rates were for five year age categories and afterwards for single years of age. Finally, for England and Wales, the rates were available for five year age categories for single years of time about once per decade. Differences in the smoothness of the maps, especially for England and Wales compared with Italy, are probably largely attributable to these differences in data richness.

In the analyses of Kermack, McKendrick, and McKinlay, Preston and van der Walle, and Coale and Kisker, mortality rates were taken relative to a period earlier than 1870, the underlying assumption being that at an early enough period there would have been no systematic pattern of progress against mortality. Figure 23 shows mortality rates for Swedish males and females relative to the average levels at each age in the period from 1778 to 1799. The figure reveals the fluctuating pattern of mortality before the middle of the nineteenth century and the general pattern of progress against mortality subsequently. The pattern is clearly more complex than a pure cohort-effect model would suggest.

A direct way of considering the hypothesis that "a cohort carries its mortality level with it" is to examine mortality surfaces that are calculated relative to a cohort's mortality levels. In Figure 24, for instance, in each of the six surfaces shown the mortality rate at each age and year was divided by the mortality rate at that age for the cohort born in 1870. The pattern that emerges shows some strong diagonals, but it is apparent that there are also important effects in horizontal and vertical directions. Interpretation of these and similar surfaces should also be tempered by realization that diagonal patterns can emerge not only as a result of cohort effects but also as the result of the interaction of period and age effects.

RATIO SURFACES

Instead of using small multiples, another approach to comparing two or more demographic surfaces is to compute some new surfaces that represent at every point either the difference or the ratio of the height of one of the original surfaces to another. Figure 25a, for instance, shows the ratio of male to female mortality rates in Italy, smoothed on a 5 by 5 grid. To highlight the ages and periods when Italian male and female mortality rates were roughly equal, Figure 25b presents a modified version of this map in which only three contours lines are drawn, for equal male and female death rates and for levels ten percent above and below equality.

Two further ratio surfaces are present in Figures 26 and 27. Figure 26 displays the ratio of Italian male mortality to French male mortality from age 10 to 70 for years 1900 to 1960. Figure 27 gives the difference between first and second birth rates in the U.S.

SEX-RATIOS AND NUPTIALITY

In addition to maps of death and birth rates and of population levels, contour maps can be drawn based on any other kind of data that is structured along two dimensions. Figures 28 and 29 suggest two possibilities. Figure 28 displays the ratio of males to females in Belgium, based on Veys' (1983) data. And Figure 29 shows age-specific marriage rates for Italian females, based on preliminary, unpublished data supplied to us by the Department of Demographic Science at the University of Rome.

LIFE TABLE STATISTICS FOR BELGIAN FEMALES

Life tables often provide statistics by age and over time on population size, number of deaths, death rates, period survivorship, period life expectancy, and sometimes cohort survivorship. All six of these statistics are available, for example, in Veys' (1983) compilation of Belgian life tables from age 0 to 99 for years 1892 to 1977. Figures 30 through 35 use Veys' data for Belgian females to illustrate the different kinds of contour map patterns produced by different kinds of life table statistics. The differences in the patterns are quite striking; note in particular the difference between the period and cohort patterns of survivorship.

U.S. FEMALE MORTALITY RATES FROM 1900 TO 2050

Faber (1982) published life tables for U.S. males and females by single year of age from birth to 119 for every tenth year from 1900 through 2050. The mortality rates at advanced ages and after 1980 are based on extrapolations. Figure 36 displays the surface of annual deaths rates (i.e., of q) for U.S. females and Figure 37 displays the surface of the force of mortality. The two surfaces are similar except at advanced ages when mortality is very high. Note that the maps graphically reveal Faber's underlying assumption that progress against mortality will slow down in the future. The nature of this assumption is revealed even more clearly in Figure 38, which displays the force of mortality over age relative to the force of mortality in 1980.

MODEL LIFE TABLES

Demographers frequently make use of model life tables, especially those developed by Coale and Demeny (1983). In the Coale and Demeny tables, death rates in various age categories are given by the life expectancy of the population, for males and for females, and for four different kinds of hypothetical populations, labelled East, West, North, and South. Figure 39 presents contour maps for females of these four types. Note that the horizontal axis gives life expectancy rather than time. Thus Figure 39 illustrates not only the use of small multiples to portray synthetic data, but also the use of a variable other than age or time as one of the dimensions on a contour map.

Another approach to construction of model life tables was developed by Brass (1971). In this approach, a standard trajectory of survivorship proportions, $p(x)$ where x stands for age, is modified by parameters a and b to produce alternative trajectories, $p'(x)$, such that

$$logit(p'(x)) = a + b\ logit(p(x)) \quad ,$$

where the logit function is given by

$$logit(p) = .5\ log(p\ /\ (1-p)) \quad .$$

Given a trajectory of survivorship proportions, a trajectory of forces of mortality (i.e., death rates) can be readily calculated. Figure 40 illustrates how the values of the parameters a and b affect the resulting age-specific force of mortality: the parameter a runs along the horizontal axis of each map and the five maps represent different values of b.

MAPPING RESIDUALS TO SHOW GOODNESS OF FIT

How well does a model fit some empirical data? If the data are defined over two dimensions, then a contour map can be used to display the residuals, i.e., the differences between the actual values and the values predicted by the model. By scrutinizing the pattern of the residuals, an analyst may glean some clues as to how to improve the model. (Tukey (1977) and Mosteller and Tukey (1977) provide clear discussions of the use of residuals in data analysis and model building.) As an illustration of this general method, Figure 41 shows how well a modified form of Brass' model fits Italian female mortality data. The modification made involves the use of 1925 Italian female mortality rates as the standard rather than Brass' original standard. Figure 41a displays the actual values of Italian female mortality rates. Figure 41b displays the values estimated by the modified Brass model. Figure 41c displays the surface of residuals, i.e., the surface of d equal to q minus q', where q is the observed mortality rate and q' is the mortality rate predicted by the modified Brass model. Different values of a and b in the model were chosen for each year of time. The values used for a and b are displayed in the graph in Figure 41d.

As a first step in exploratory data analysis and model building, it may be useful to remove age and period effects (or, more generally, the main effects along the x and y dimensions) from the data and then look at the residuals. The relative surfaces shown in Figures 13 through 18 can be interpreted as residual surfaces for which either a period effect or an age effect has been removed. Figure 42a displays a surface of residuals calculated by removing both a period and an age effect: the original surface, which presents U.S. fertility rates, was shown in Figure 5; from this surface the average fertility rate at each age and the average fertility in each year was divided out and then, to normalize the resulting values, the values were multiplied by the overall average fertility rate. The values of these average rates are shown in Figures 42b and 42c. The contour line at the level of one in Figure 42a shows when fertility rates can be exactly explained by a simple multiplicative model of age and period averages; the dark and light areas on the map show when fertility rates are higher or lower than the rates predicted by the simple age-period model.

MAPS OF THEORETICAL DEMOGRAPHIC MODELS

To understand actual population phenomenon, demographers often analyze simplified, theoretical models that capture some aspect of reality (Keyfitz 1977). Contours maps can be used to show how some variable of interest in such models responds to changes in two of the parameters. Figures 43 and 44 provide such illustrations. Suppose mortality rates follow the female model West schedule of Coale and Demeny. Further suppose that a population is stable and is governed by these mortality trajectories (which can be classified by the single mortality measure e_0) and by some growth rate r. What proportion of the population will be above age 65? The contour map in Figure 43 displays the answer, for various values of e_0 and b.

As a second example, suppose that the force of mortality at any age is given by a Gompertz curve such that $u(x) = ae^{bx}$. How will life expectancy at birth change as a and b vary? Figure 44 graphically resolves this question.

CONCLUSION

The figures presented in this paper suggest just a few of the numerous ways that demographers can use contour maps to clearly, efficiently, and simultaneously display both persistent global and prominent local patterns in population rates or levels over two dimensions. In particular, contour maps can strikingly reveal the interaction between age, period, and cohort patterns.

Even in cases where demographic data already has been carefully scrutinized by perceptive analysts who have uncovered most of the available interesting patterns, contour maps may be useful in highlighting these patterns in a visually revealing manner. With contour maps, what was before understood now can be seen. Furthermore, the maps, by giving demographers a new perspective on this data, may focus attention on some neglected aspects and patterns in even thoroughly-studied data.

Beyond efficient description, contour maps can help demographers with exploratory data analysis and with model building. Surfaces can be computed relative to some part of the surface or to another surface; and different surfaces can be placed next to each other and compared. The patterns produced by a model can be displayed as can the fit of the model to some empirical data.

The resulting contour maps can be displayed not only as printed output but also on a computer monitor. The shades used in the maps presented in this paper range from black to light grey, but the maps can be produced in glowing colors, on a color computer monitor or using a color printer. The effects are dramatic, as is the speed with which a computer can draw a map. A large computer is not needed--as described in the Appendix, we have used an IBM P.C.

Tukey, in his lucid exposition of *The Visual Display of Quantitative Information* (1983), concludes that graphic designs should give "visual access to the subtle and difficult, that is, the revelation of the complex". Demographic surfaces can be particularly complex. A mortality surface, for example, might be defined over nearly a century of age and more than a century of time, comprising close to ten thousand data points that may vary over four orders of magnitude. Contour maps are a striking, efficient, and clear means of giving demographers visual access to such surfaces.

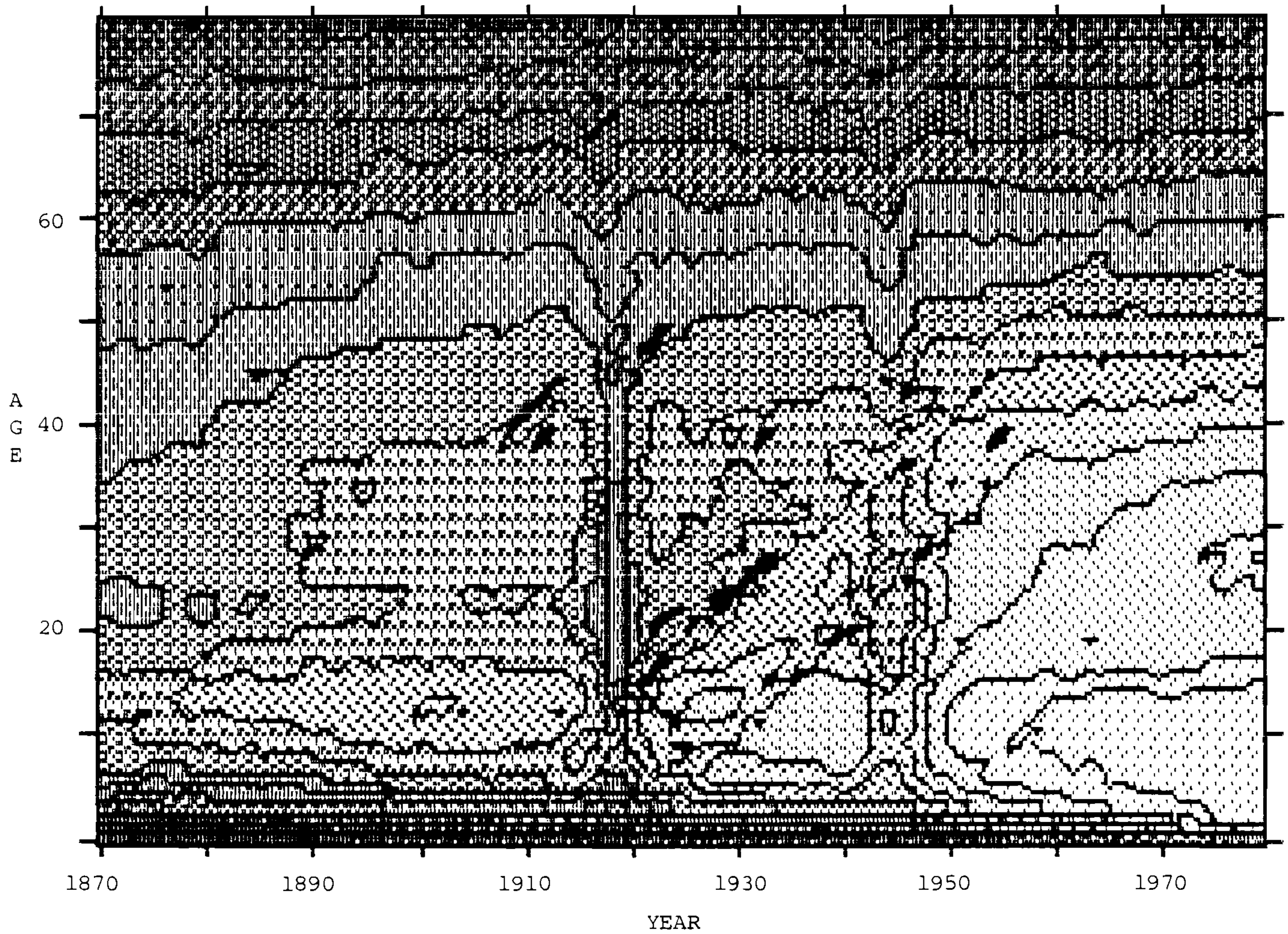
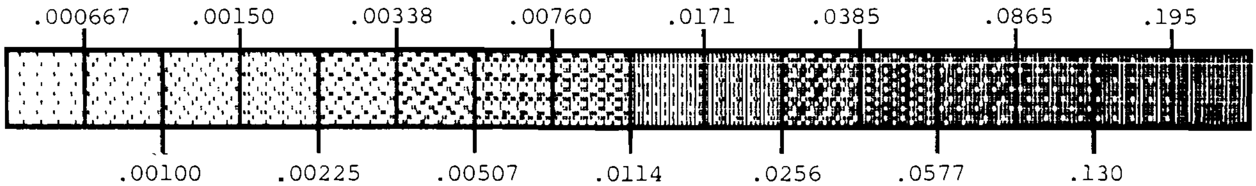


Figure 1a: Italian Male Mortality Rates - with contour lines from .000667 to .195
at multiples of 1.5
From age 0 to 79 and Year 1870 to 1979



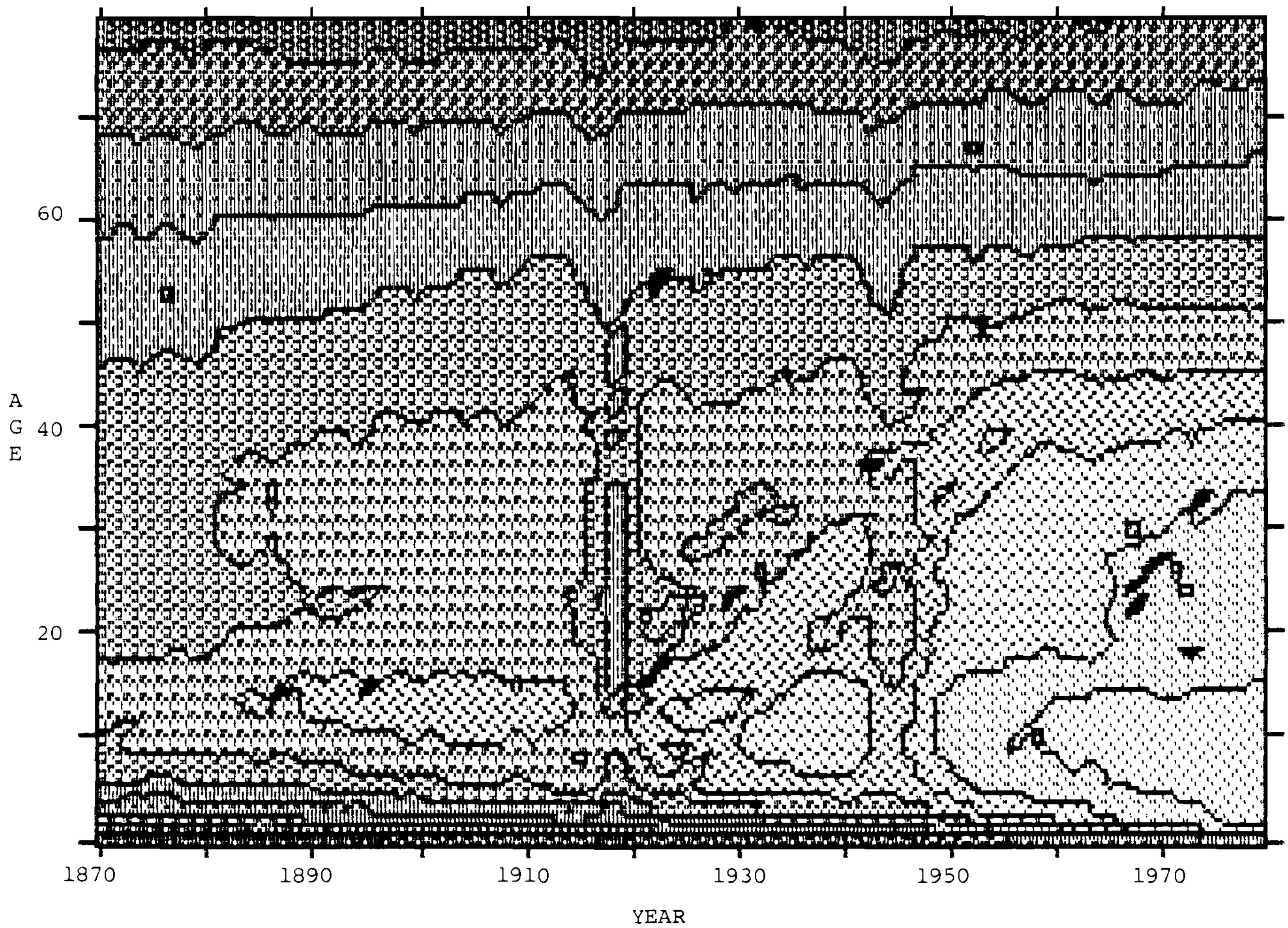
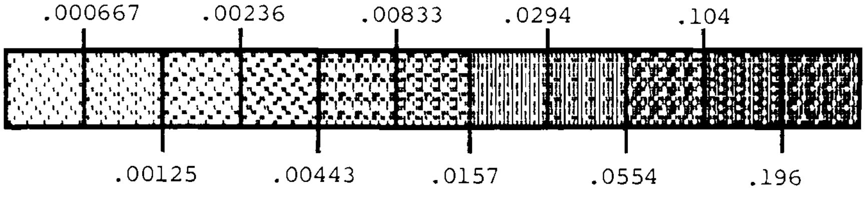


Figure 1b: Italian Male Mortality Rates - with contours from .000667 to .196
at multiples of 1.88
From Age 0 to 79 and Year 1870 to 1979



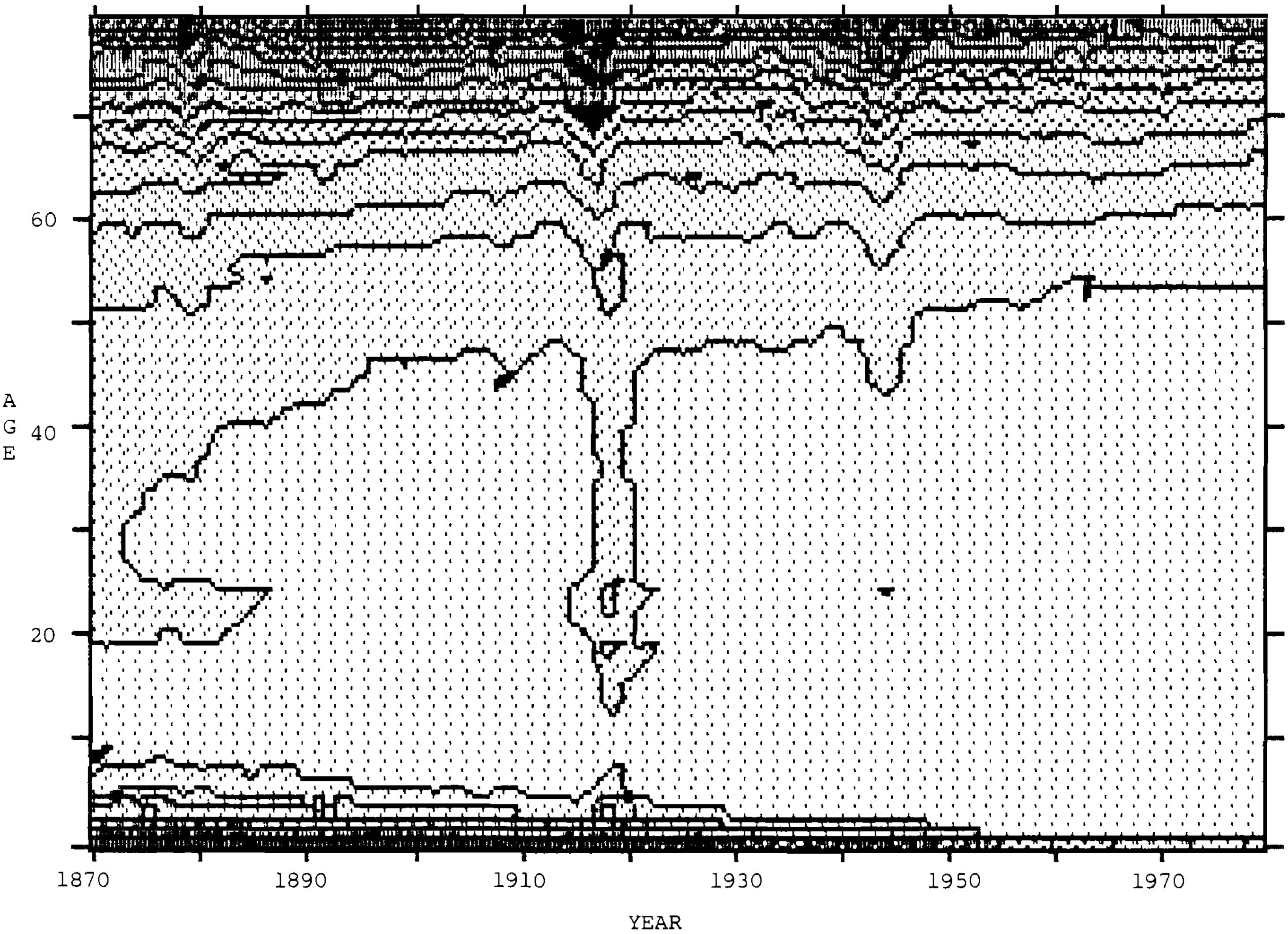
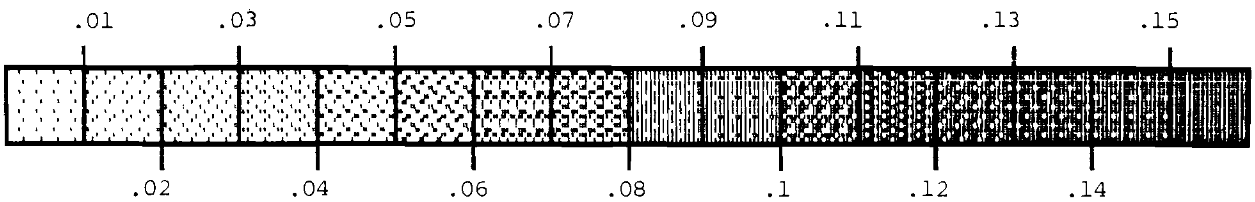


Figure 1c: Italian Male Mortality Rates - with contour lines from .01 to .15 at even intervals
From Age 0 to 79 and Year 1870 to 1979



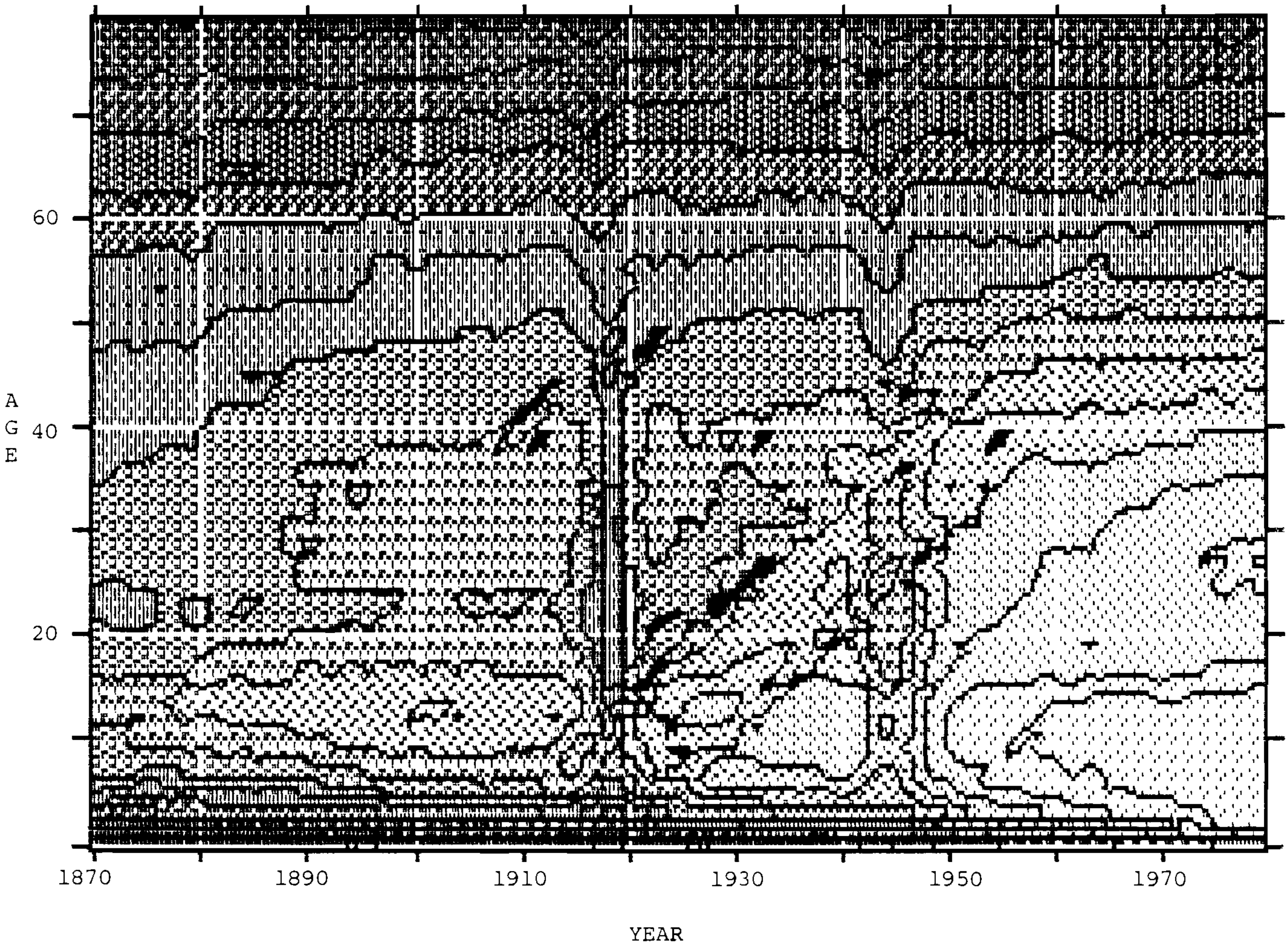
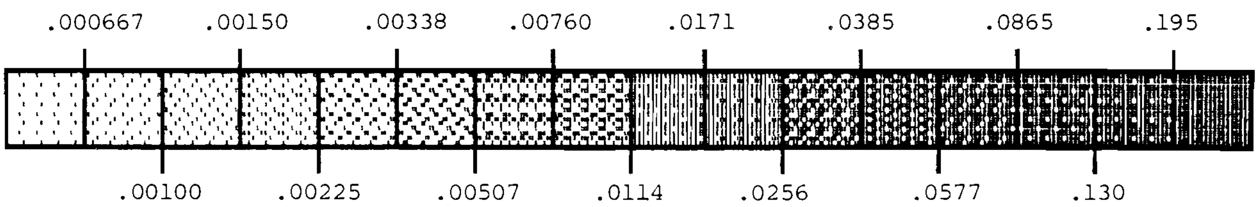


Figure 1d: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5, with a grid
From Age 0 to 79 and Year 1870 to 1979



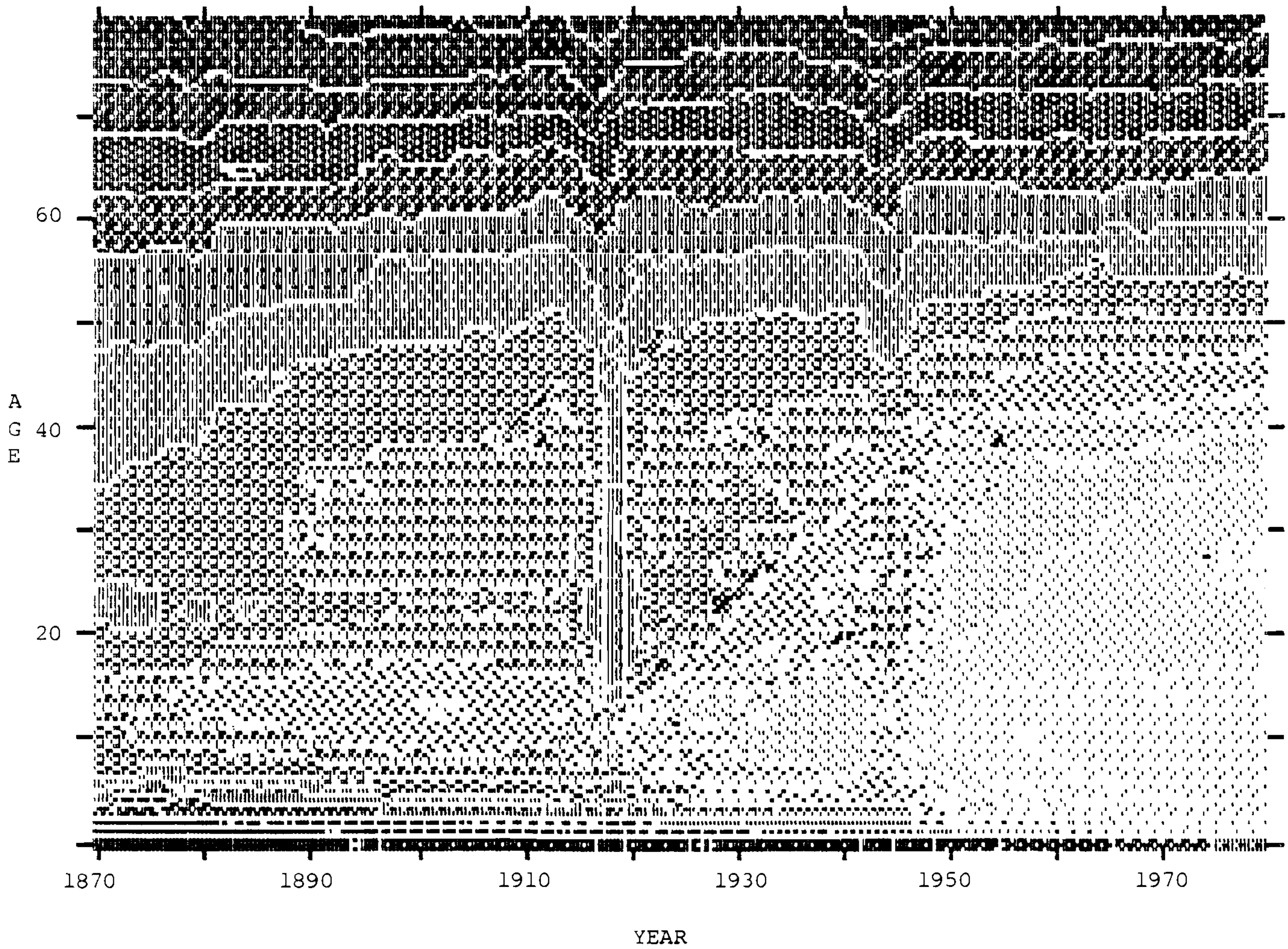
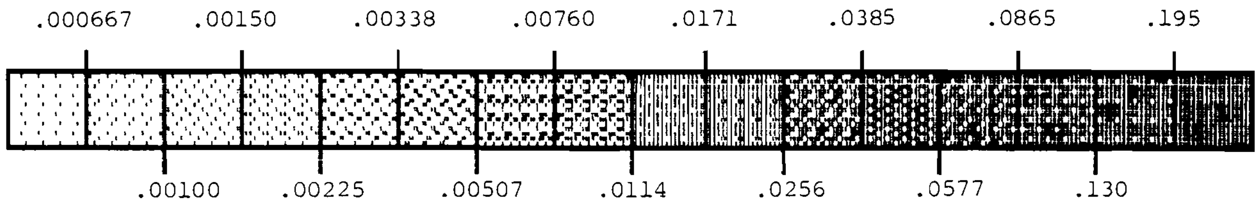


Figure 1e: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5, with contour lines drawn in white
From Age 0 to 79 and Year 1870 to 1979



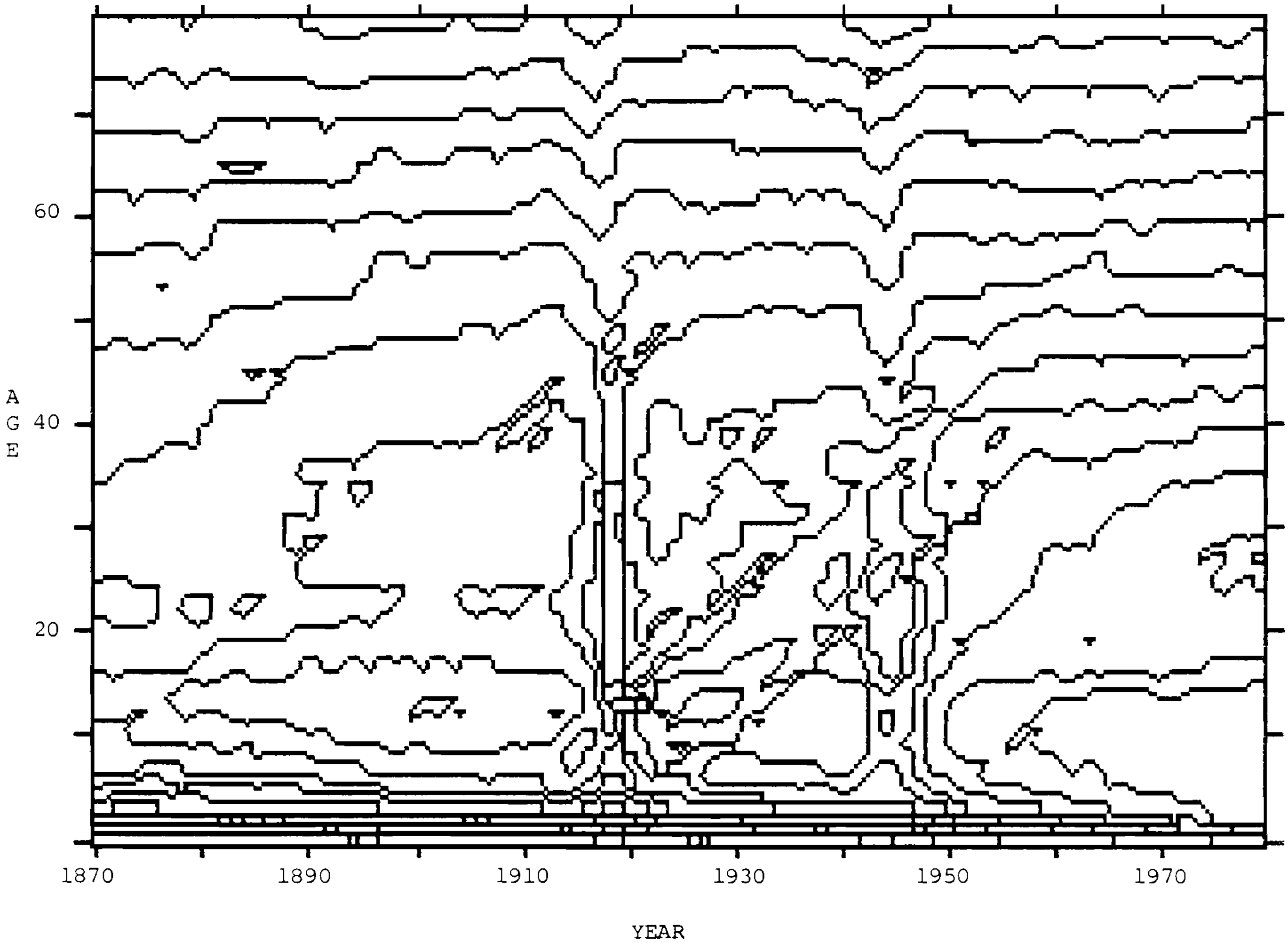


Figure 1f: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5, with no shading
From Age 0 to 79 and Year 1870 to 1910

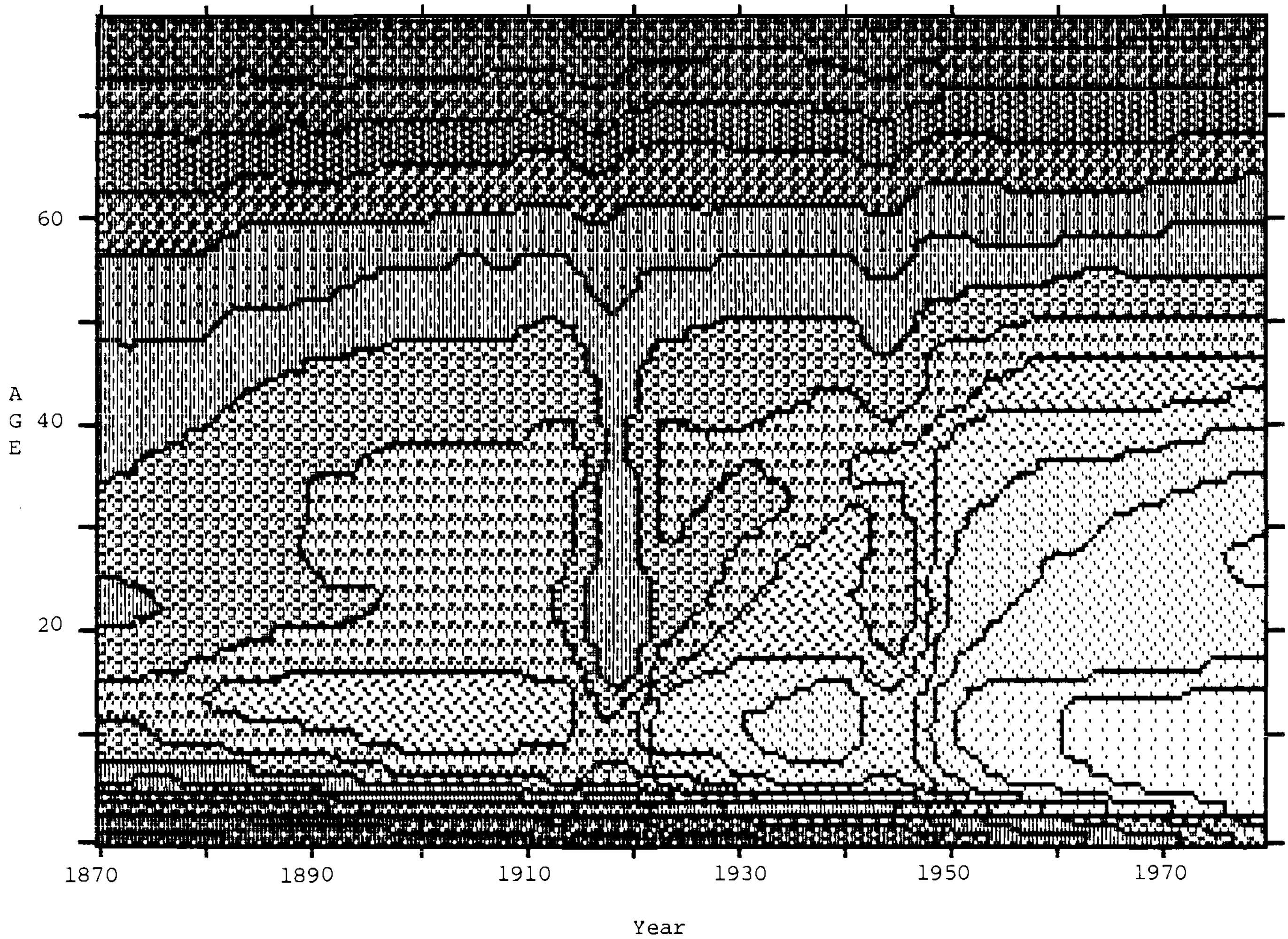
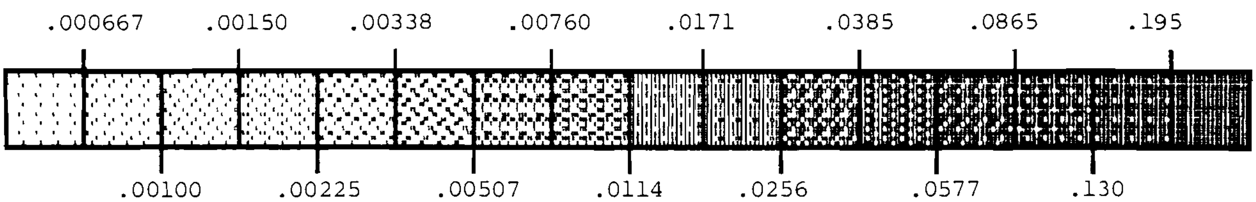


Figure 1g: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5, smoothed on a 5 by 5 sqaure From Age 0 to 79 and Year 1870 to 1979



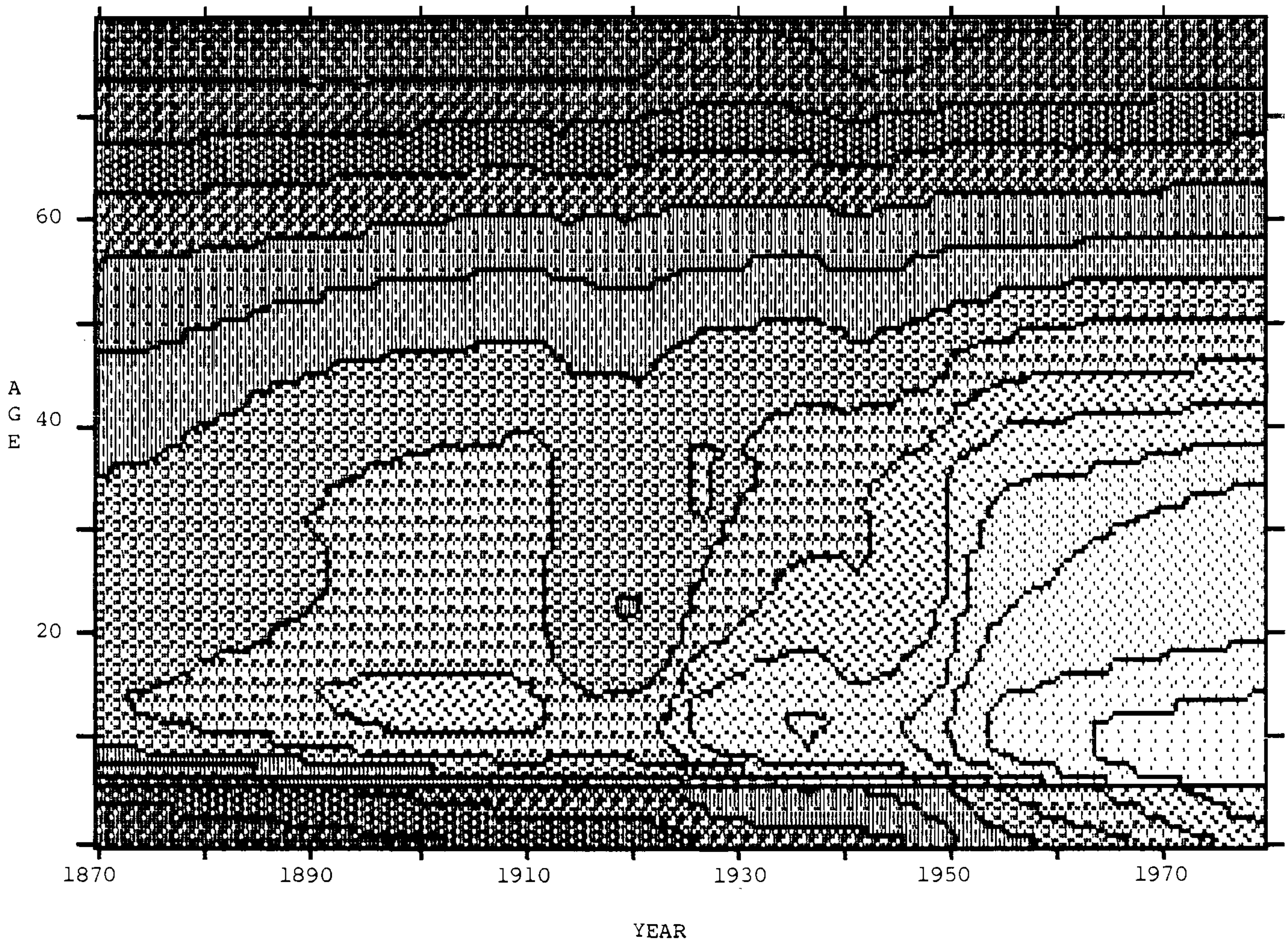
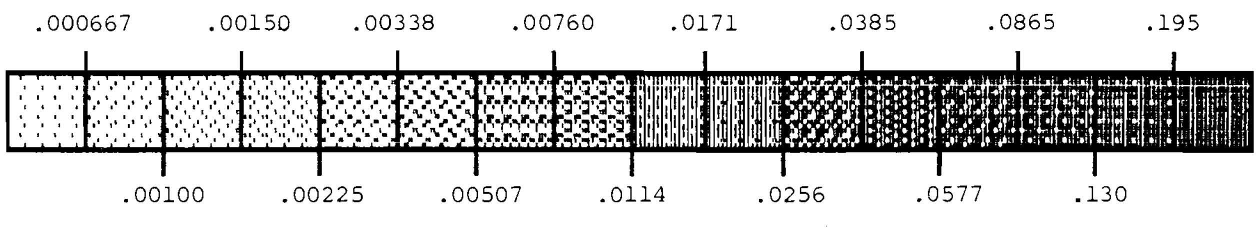


Figure 1h: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5, smoothed on an 11 by 11 square From Age 0 to 79 and Year 1870 to 1979



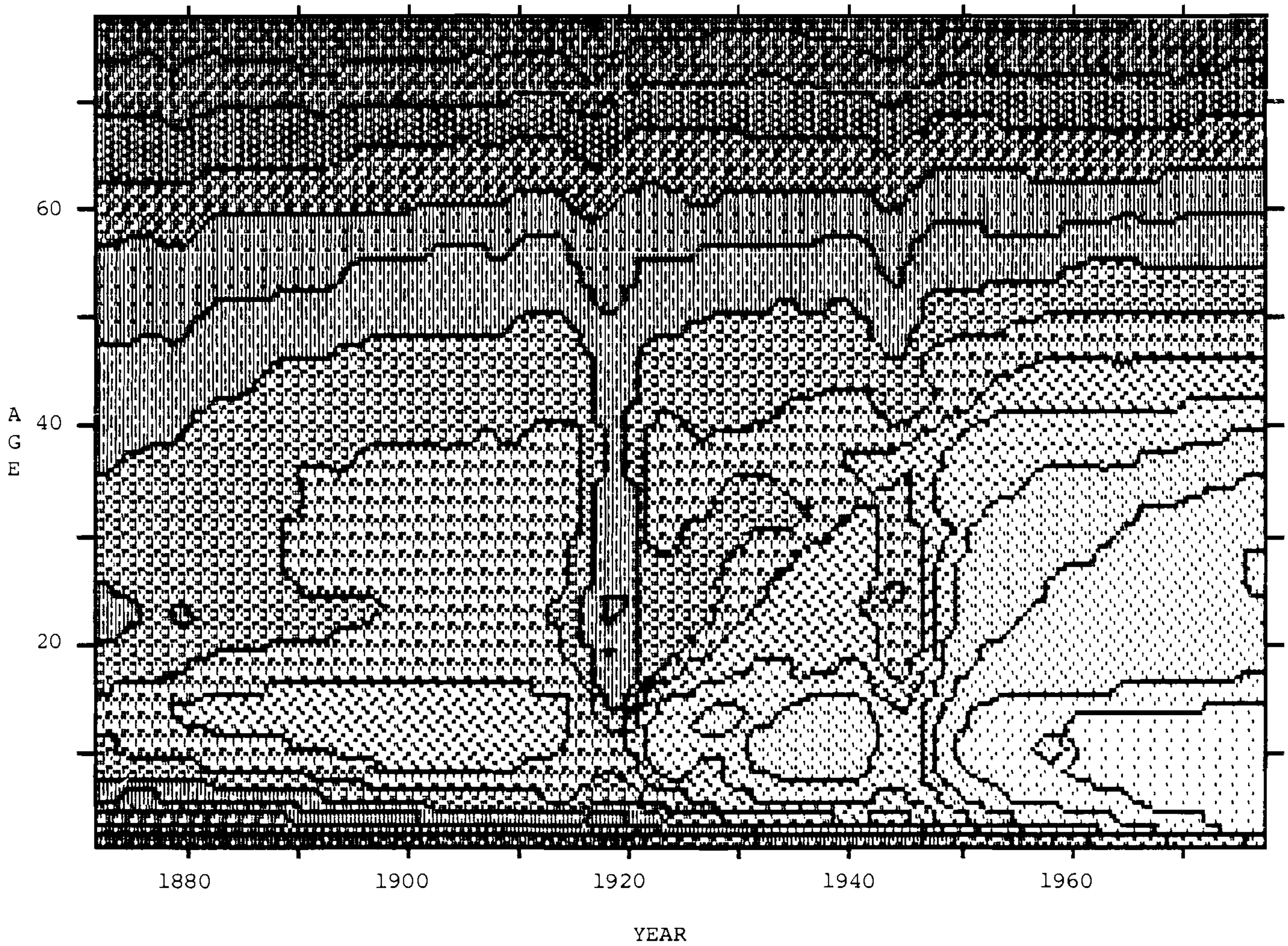
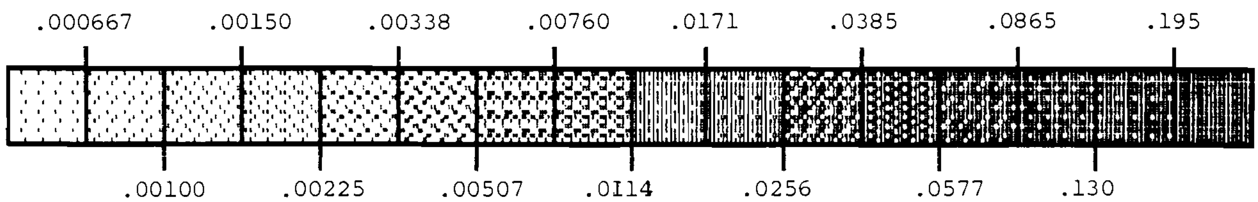


Figure 1i: Italian Male Mortality Rates - with contours from .000667 to.195
at multiples of 1.5, smoothed on a weighted 5 by 5 sqaure
From Age 2 to 77 and Year 1872 to 1977



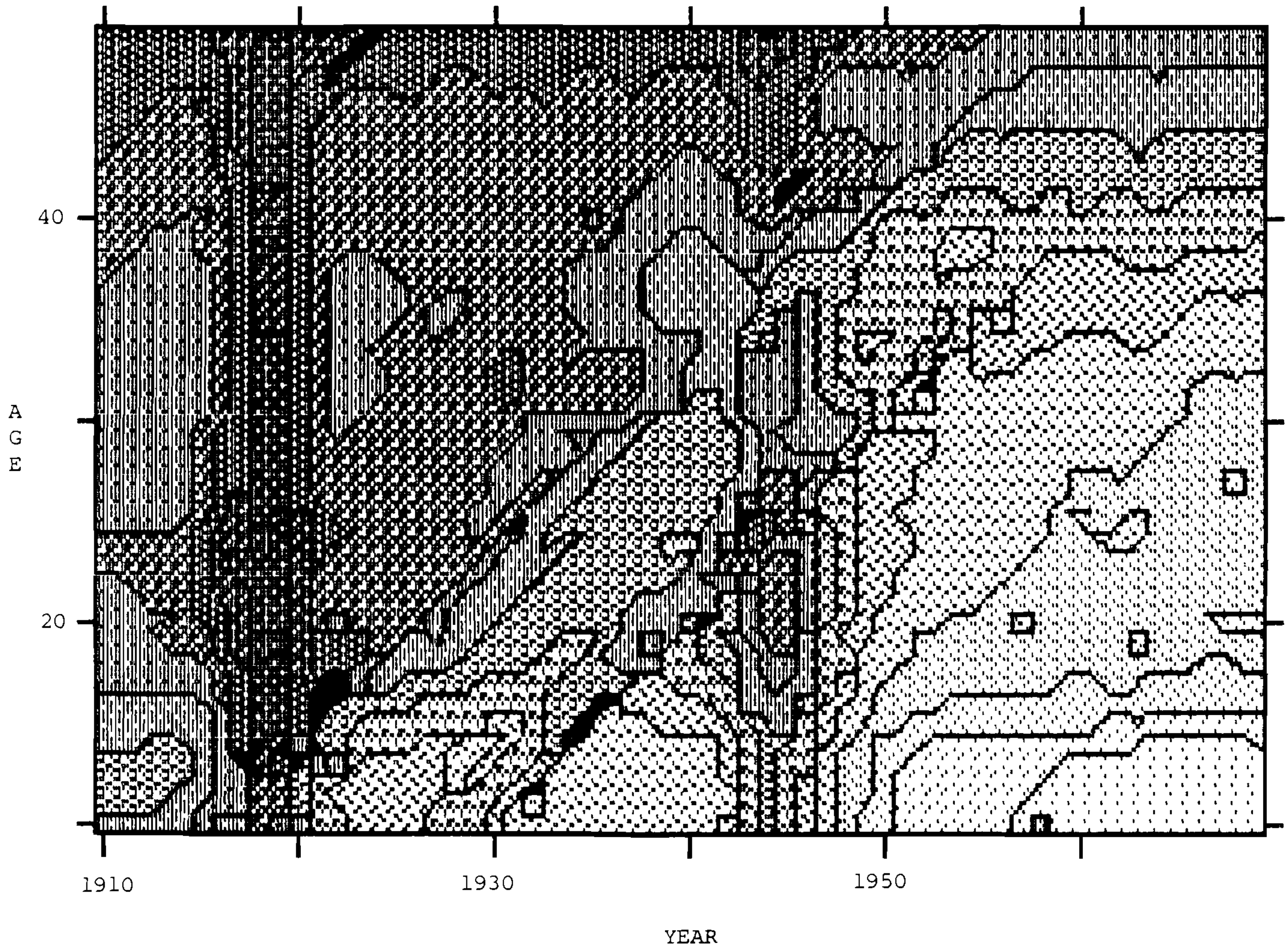
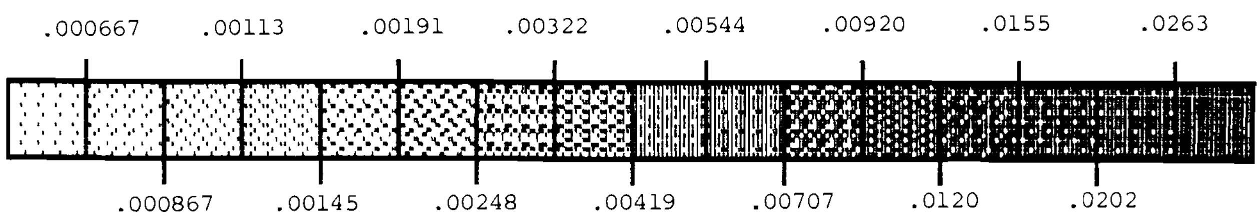


Figure 2a: Italian Male Mortality Rates - with contour lines from .000667 to .0263 at multiples of 1.3
From Age 10 to 49 and Year 1910 to 1969



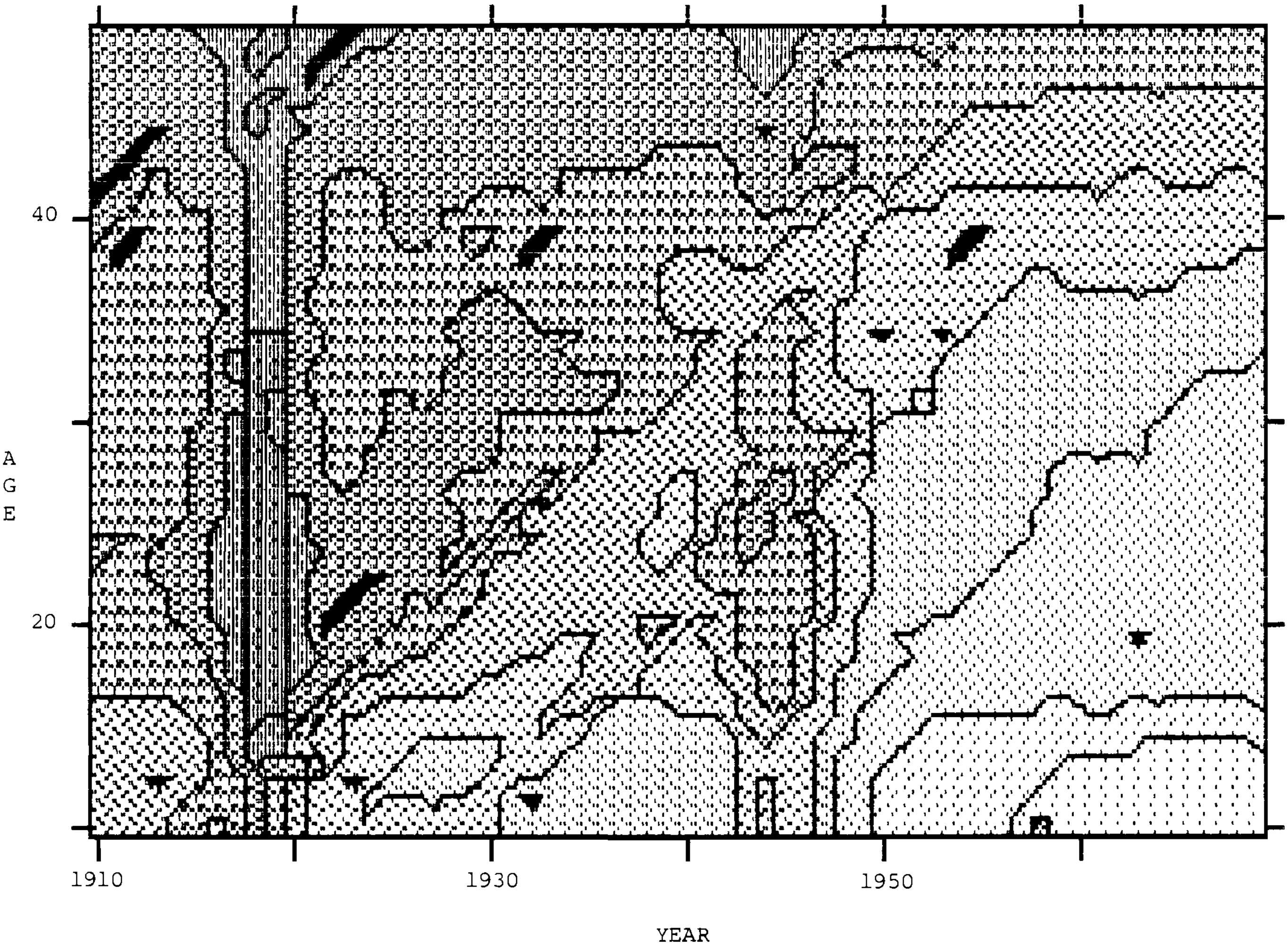
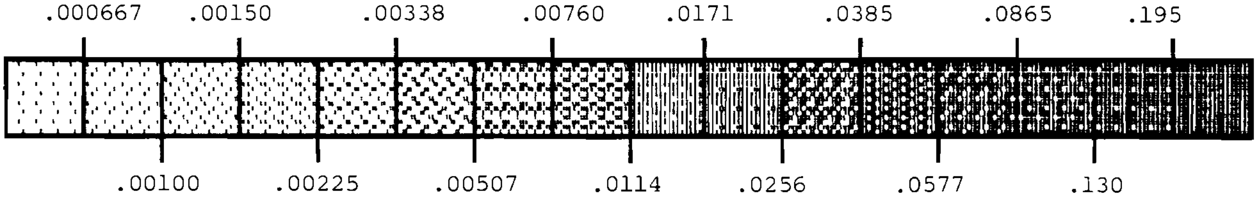


Figure 2b: Italian Male Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From Age 10 to 49 and Year 1910 to 1969



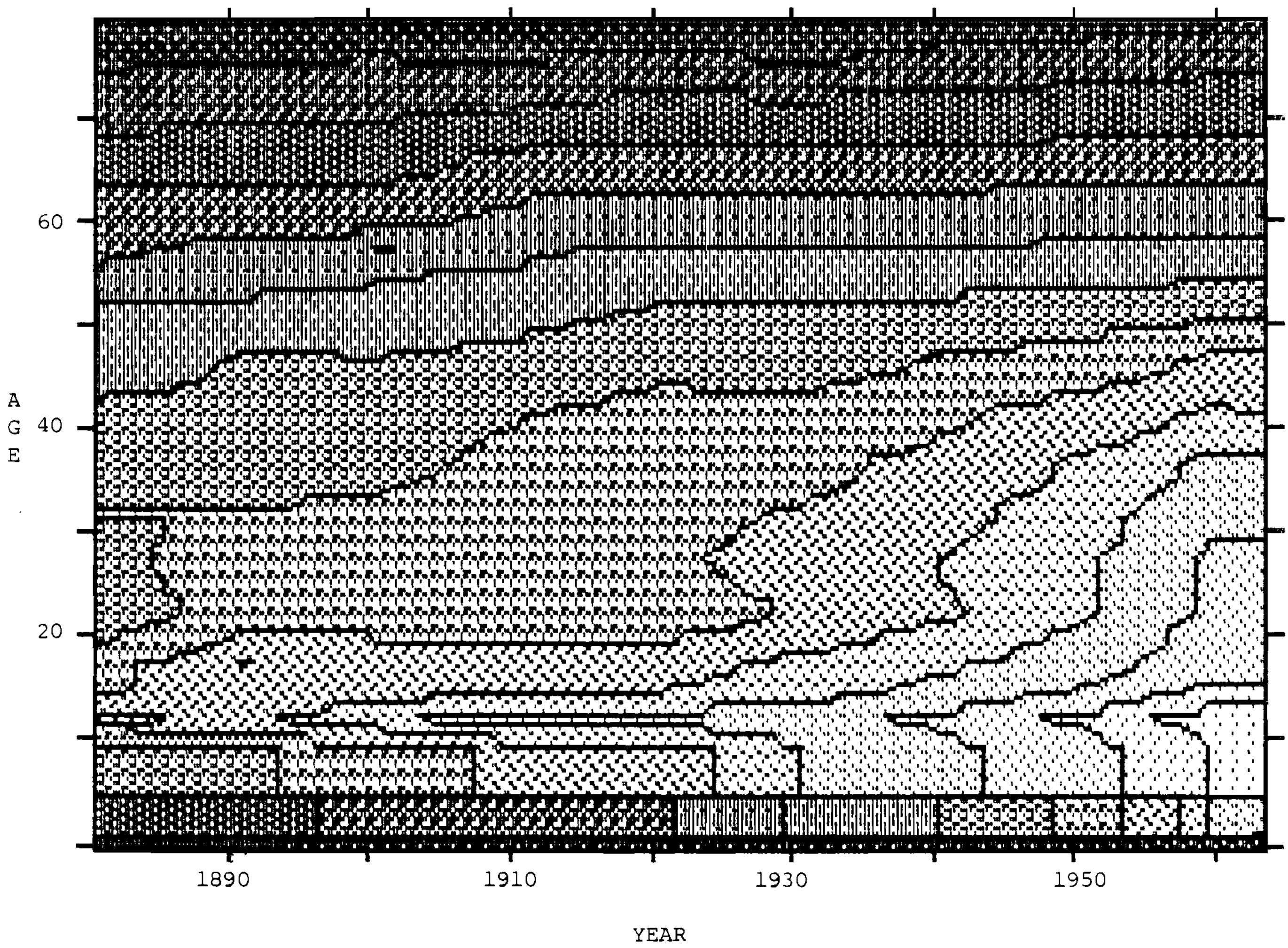
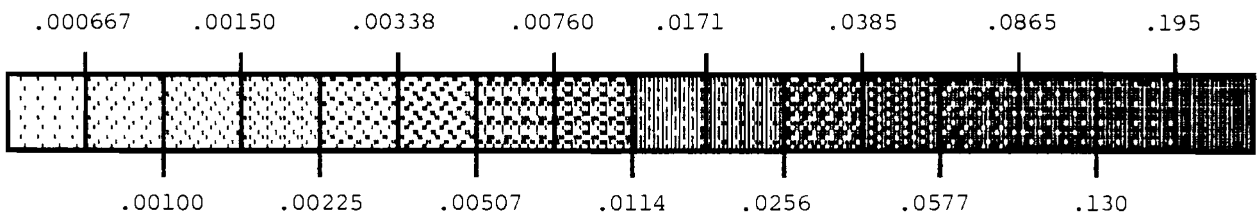


Figure 3: Italian Male Mortality Rates (interpolated data) -- with contour lines
from .000667 to .195 at multiples of 1.5
From Age 0 to 79 and Year 1881 to 1963



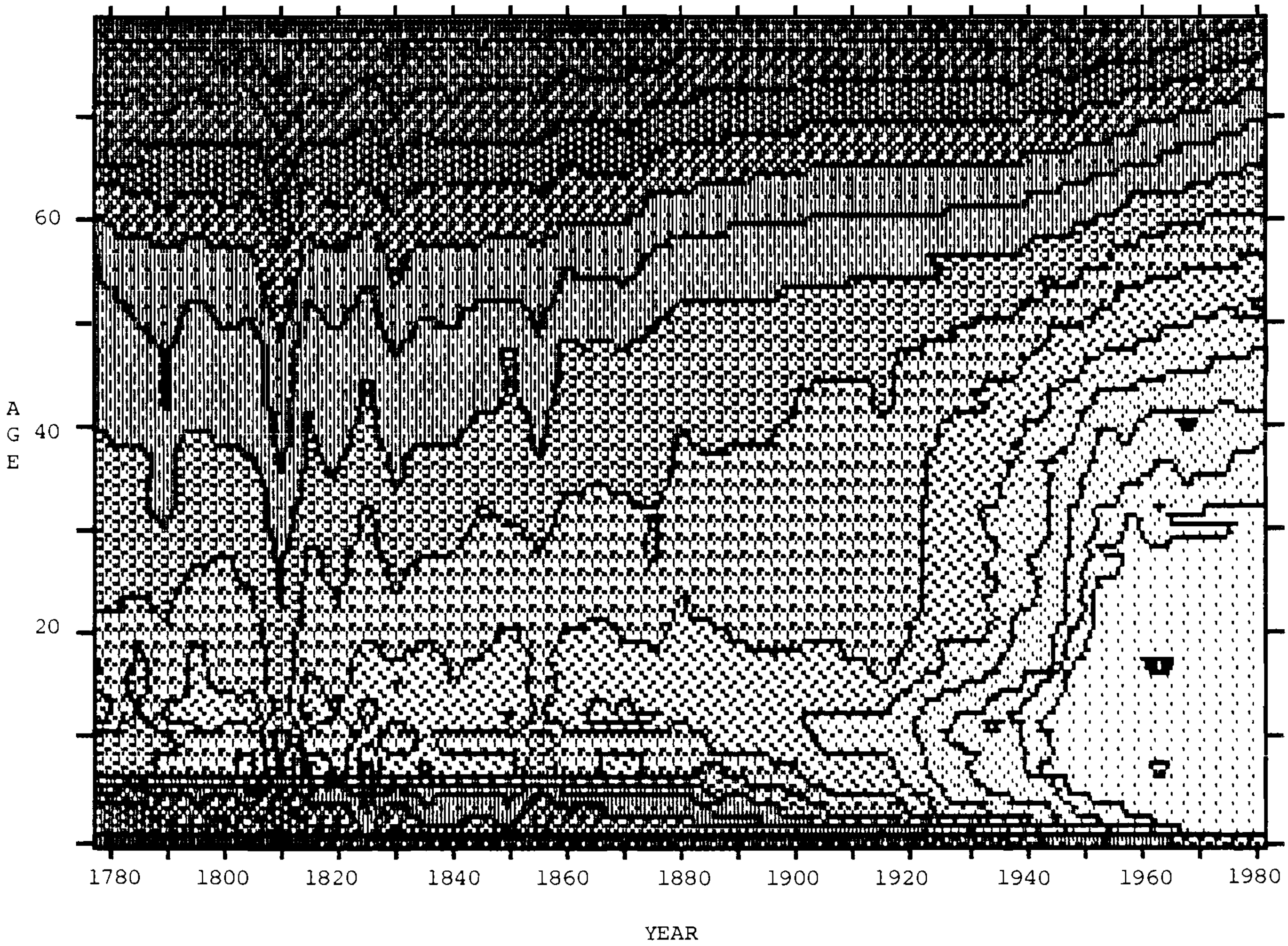
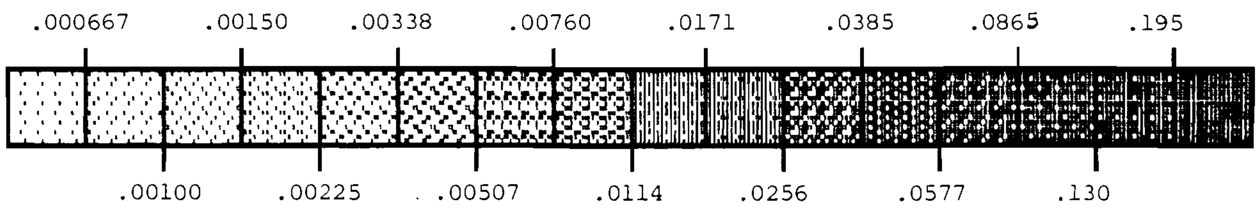


Figure 4: Swedish Female Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From Age 0 to 79 and Year 1778 to 1981



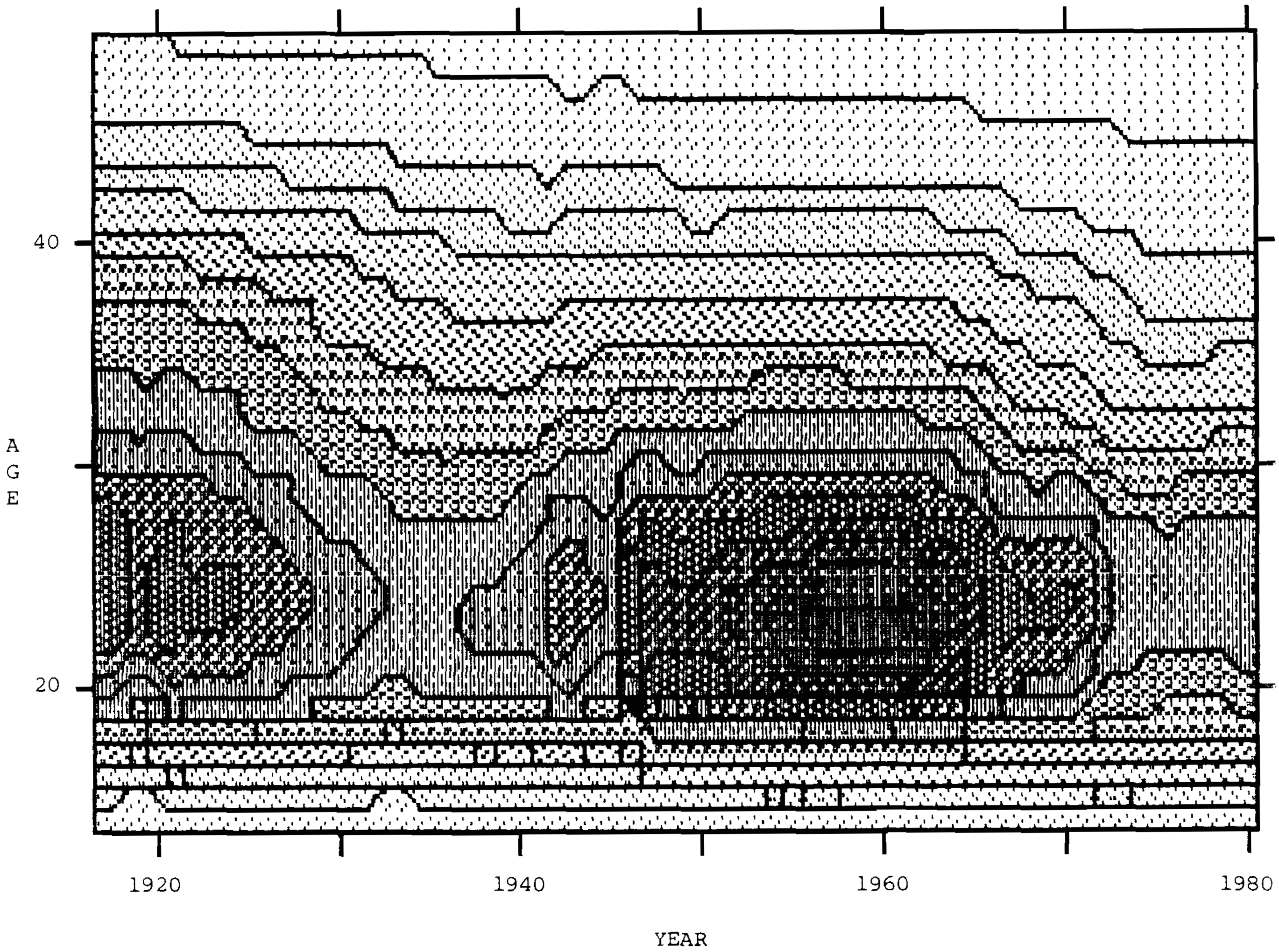
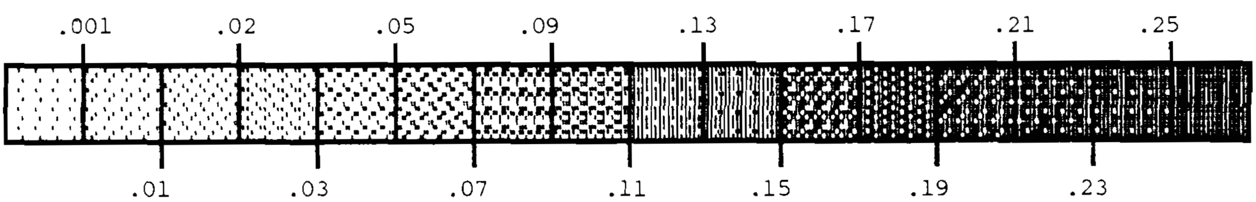


Figure 5: U.S. Birth Rates - with contour lines selectively placed from .001 to .25
From Age 14 to 49 and Year 1914 to 1980



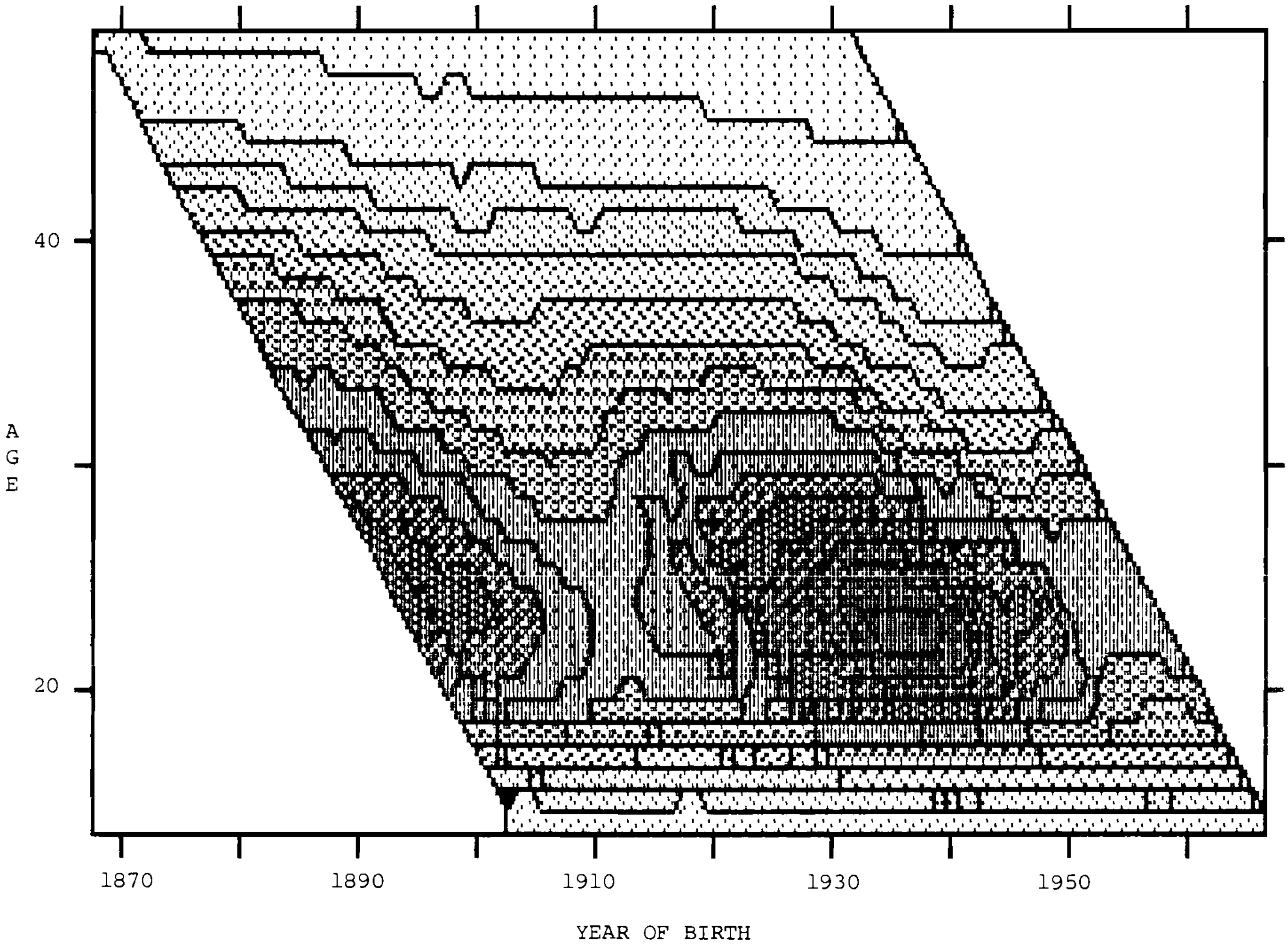
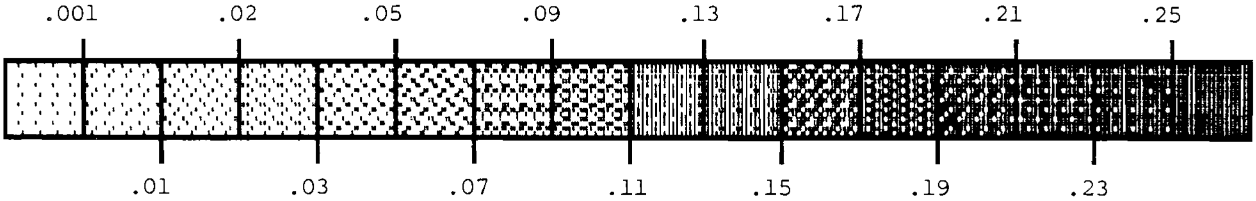


Figure 6: U.S. Cohort Fertility - with contours selectively placed from .001 to .25
From Age 14 to 49 and Year of Birth 1868 to 1966



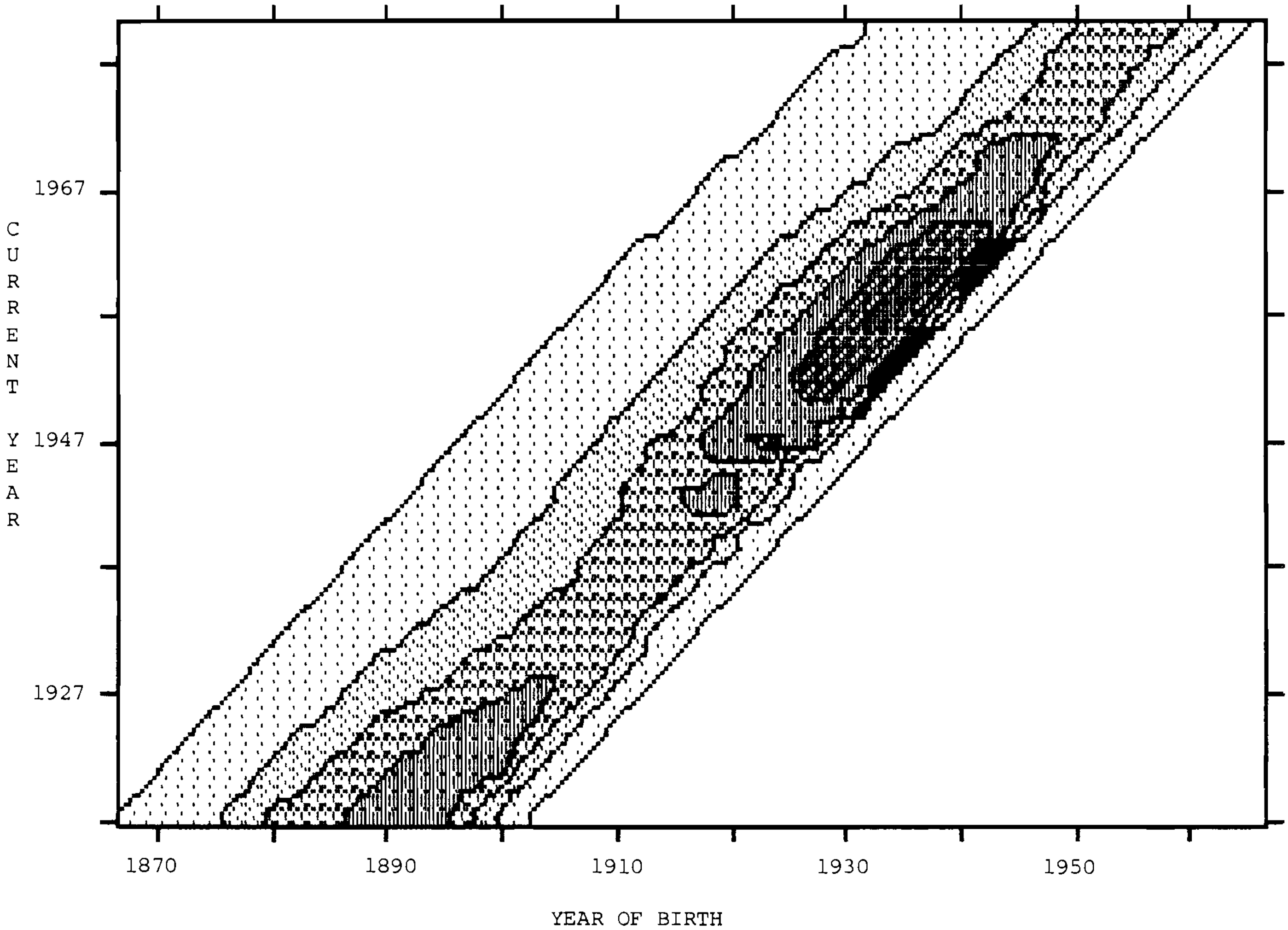
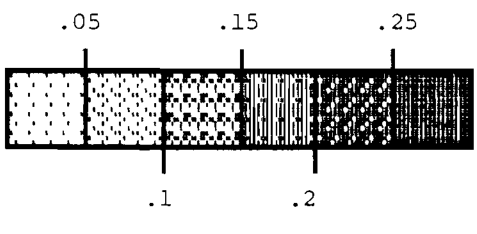


Figure 7: U.S. Fertility by Year of Birth and Current Year - with contours from .05 to .25 at evenly spaced intervals
From Current Year 1917 to 1980 and Year of Birth 1867 to 1966



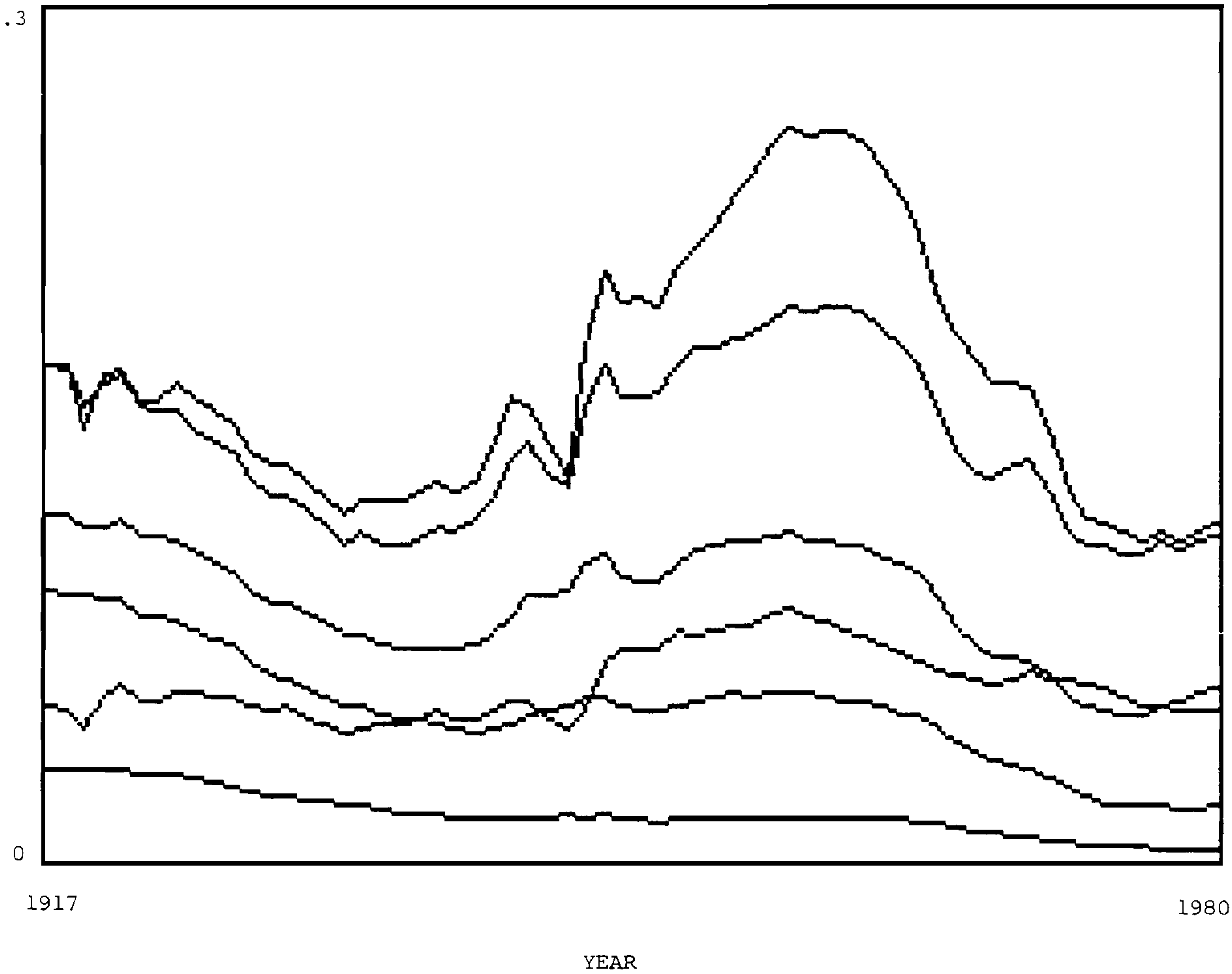


Figure 8: U.S. Birth Rates Over Time From 1917 to 1980 at Ages 18, 23, 28, 33, 38, 43

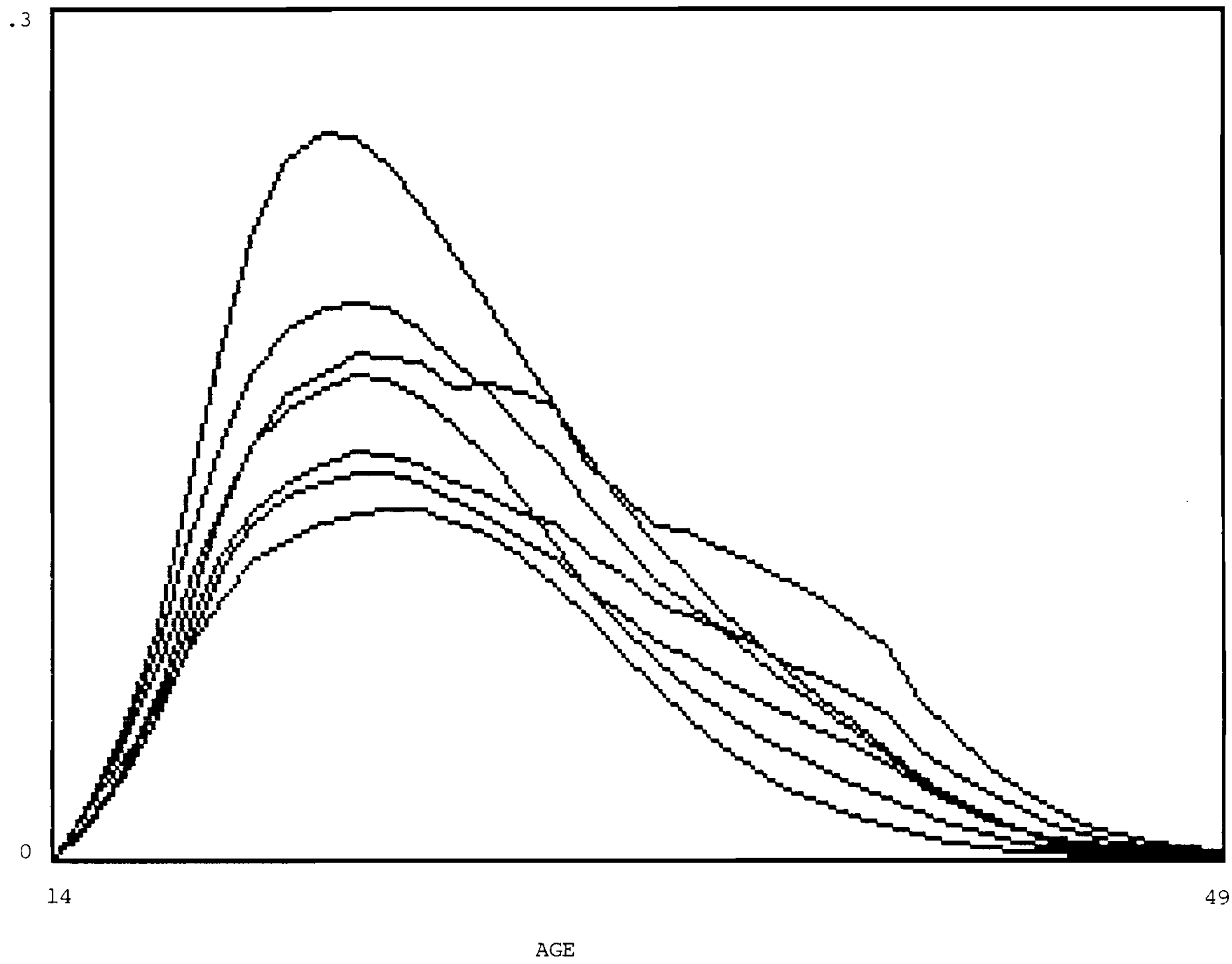


Figure 9: U.S. Birth Rates From Age 14 to 49 at Years 1920, 1930, 1940, 1950, 1960, 1970, 1980

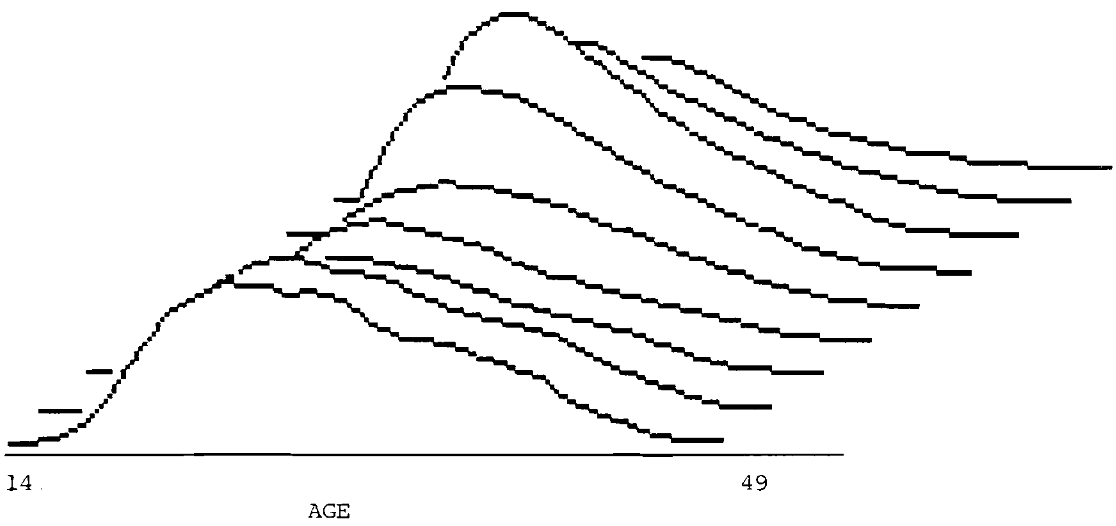


Figure 10: Three-dimensional perspective of U.S. Fertility Rates

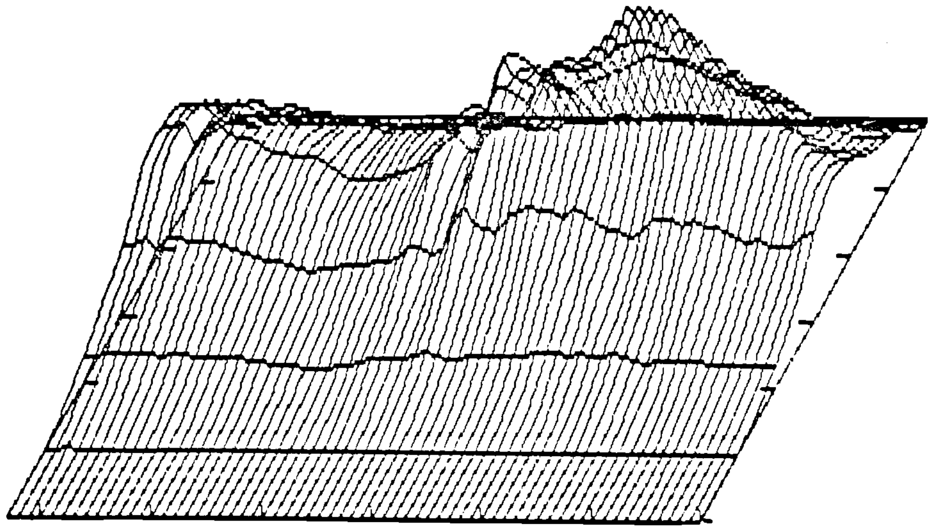


Figure 11: Three-dimensional perspective of U.S. Fertility Rates - by a computer plotter *

* From Fertility in America: Heterogeneity and the Effect of Birth Order, by William Hodges

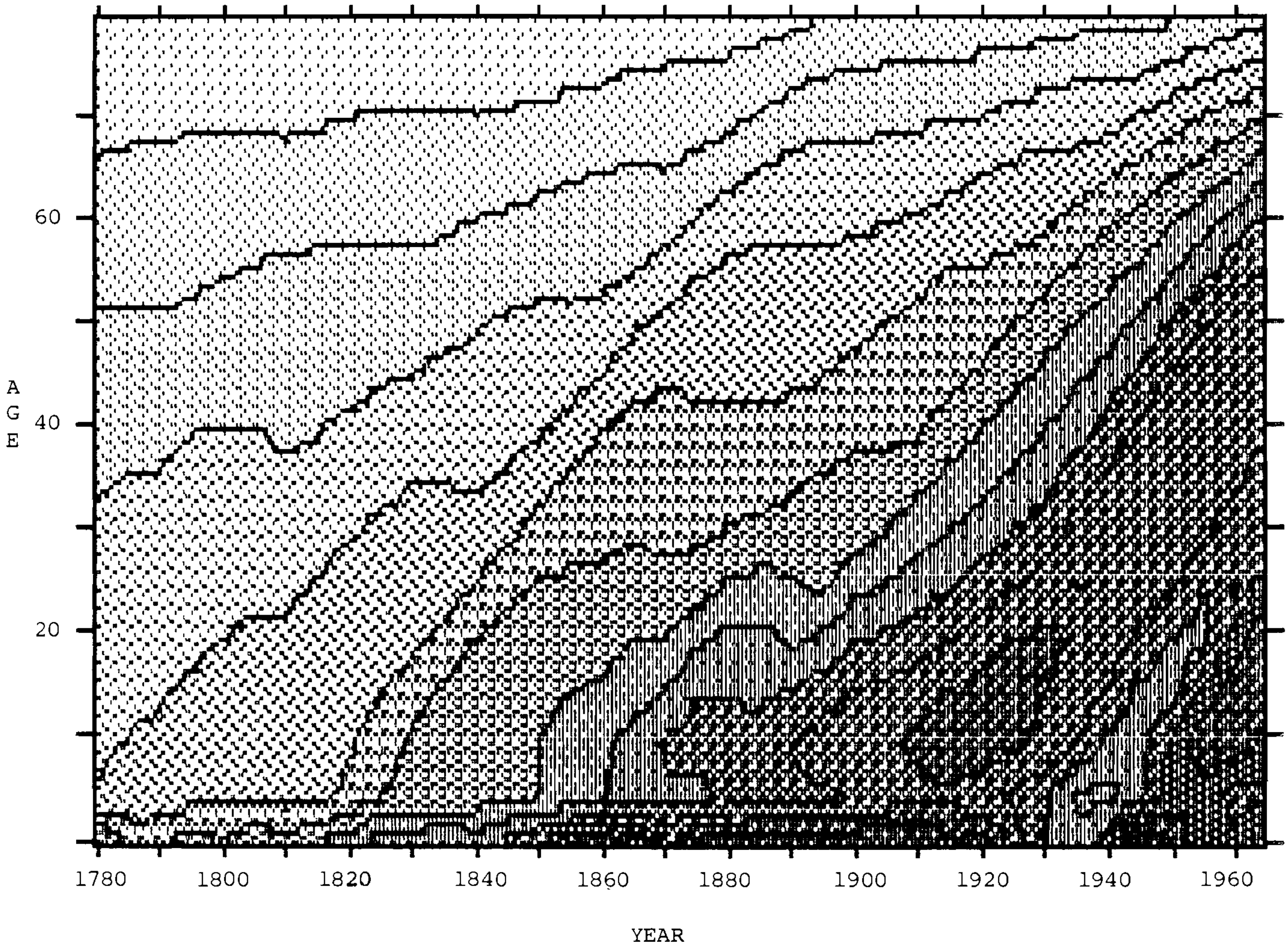
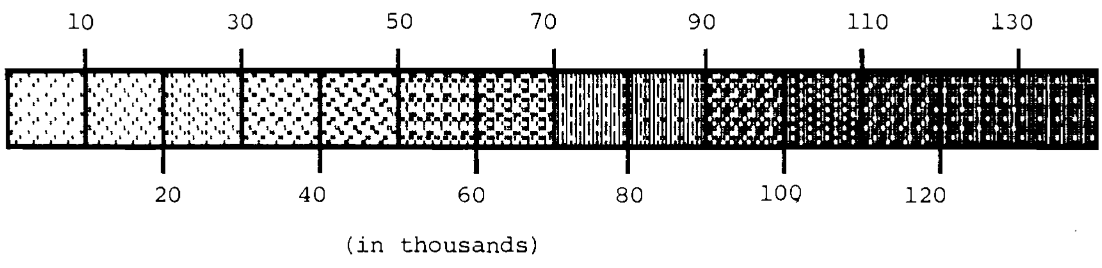


Figure 12: Swedish Population - with contours lines placed evenly from 10,000 to 130,000
From Age 0 to 79 and Year 1780 to 1963



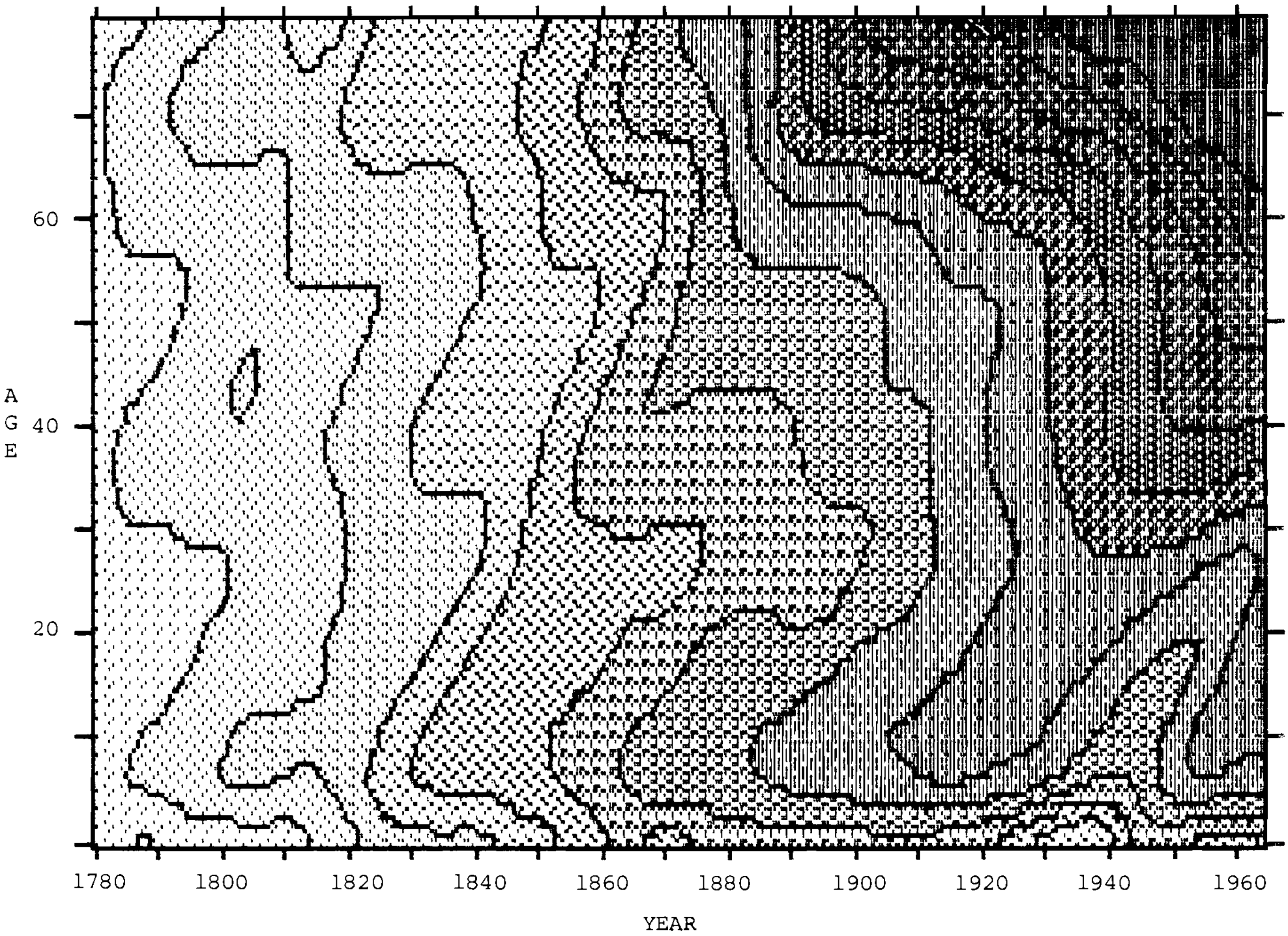
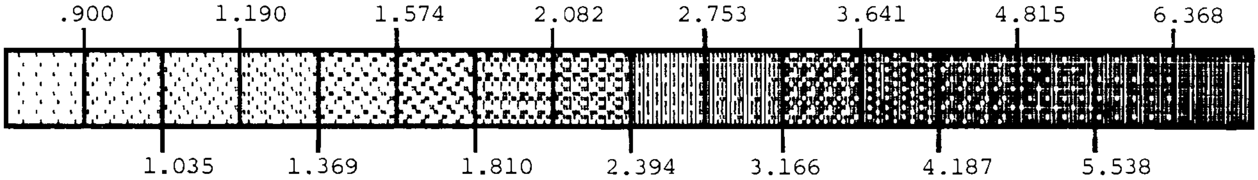


Figure 13: Swedish Population Relative to Age Specific 1780 Levels - with contour lines from .9 to 6.37 at multiples of 1.15, smoothed on a 5 by 5 square From Age 0 to 79 and Year 1780 to 1963



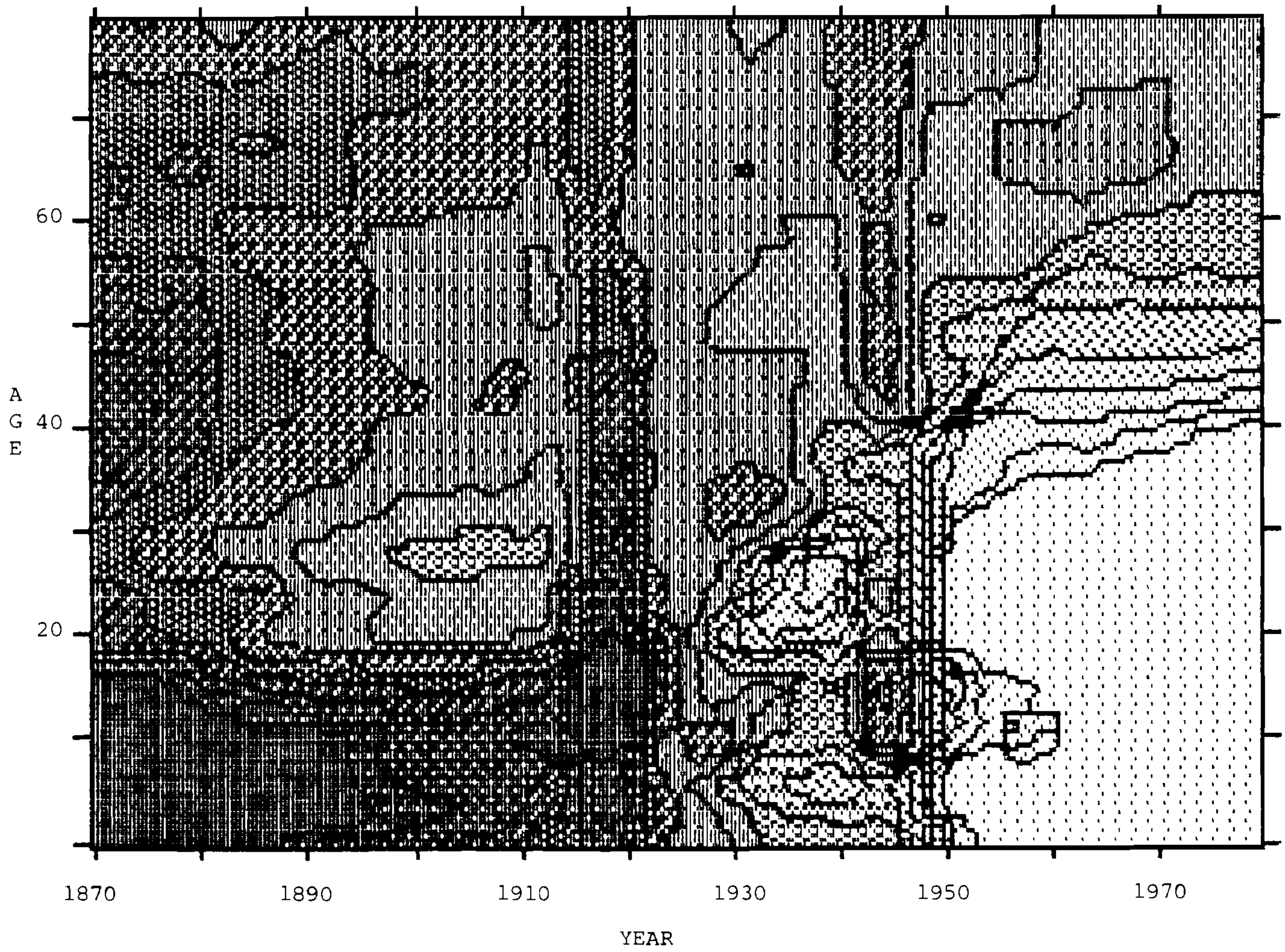
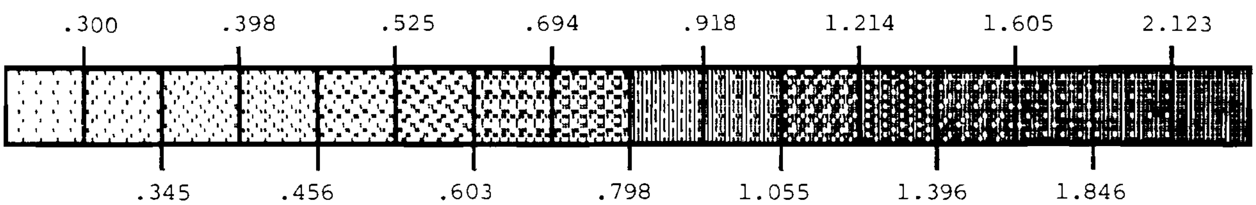


Figure 14: Italian Male Mortality Rates Relative to Age Specific 1925 Levels - with contour lines from .3 to 2.12 at multiples of 1.15, with 5 by 5 smoothing From Age 0 to 79 and Year 1870 to 1979



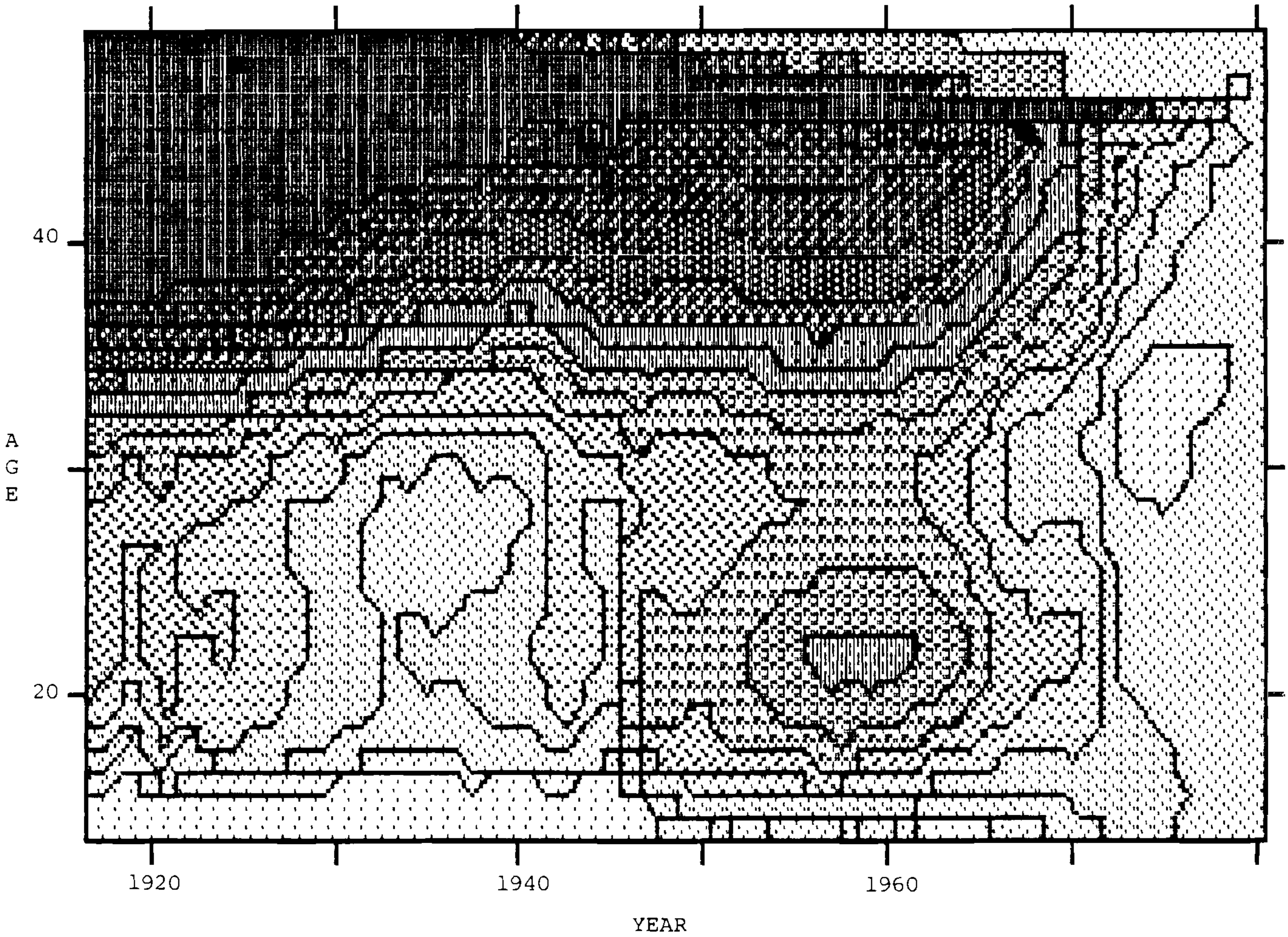
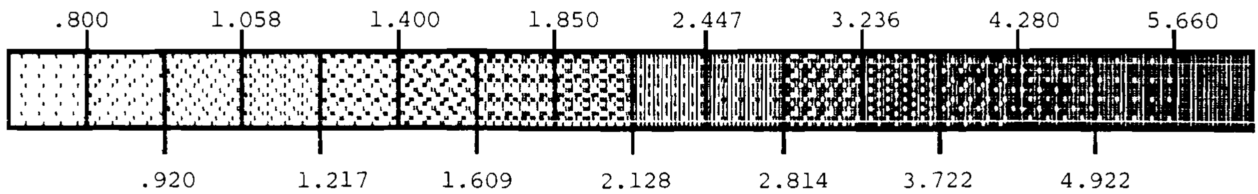


Figure 15: U.S. Birth Rates Relative to Age Specific 1980 Levels - with contour lines from .8 to 5.66 at multiples of 1.15
From Age 14 to 49 and Year 1917 to 1980



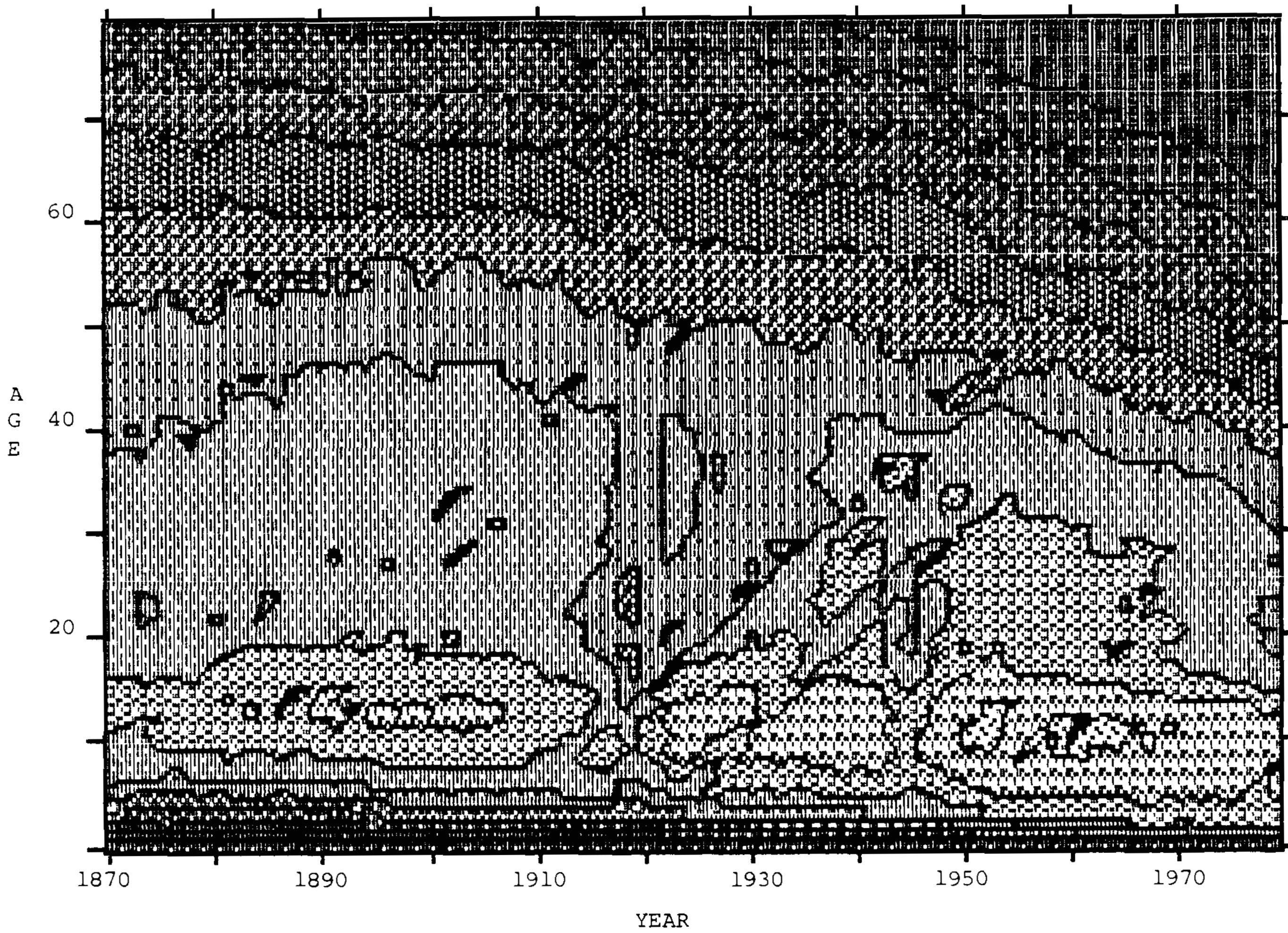
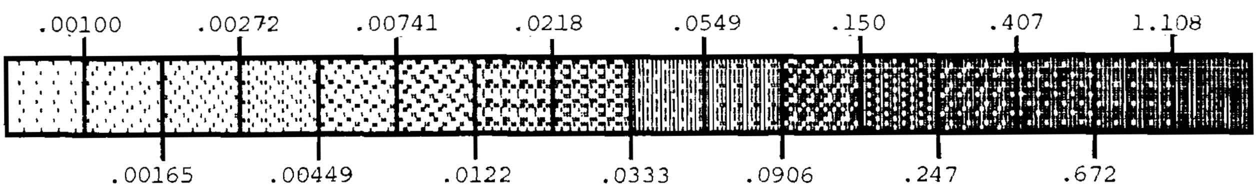


Figure 16: Italian Male Mortality Rates Relative to Infant Mortality - with contour lines from .001 to 1.11 at multiples of 1.65, 5 by 5 smoothing From Age 0 to 79 and Year 1870 to 1979



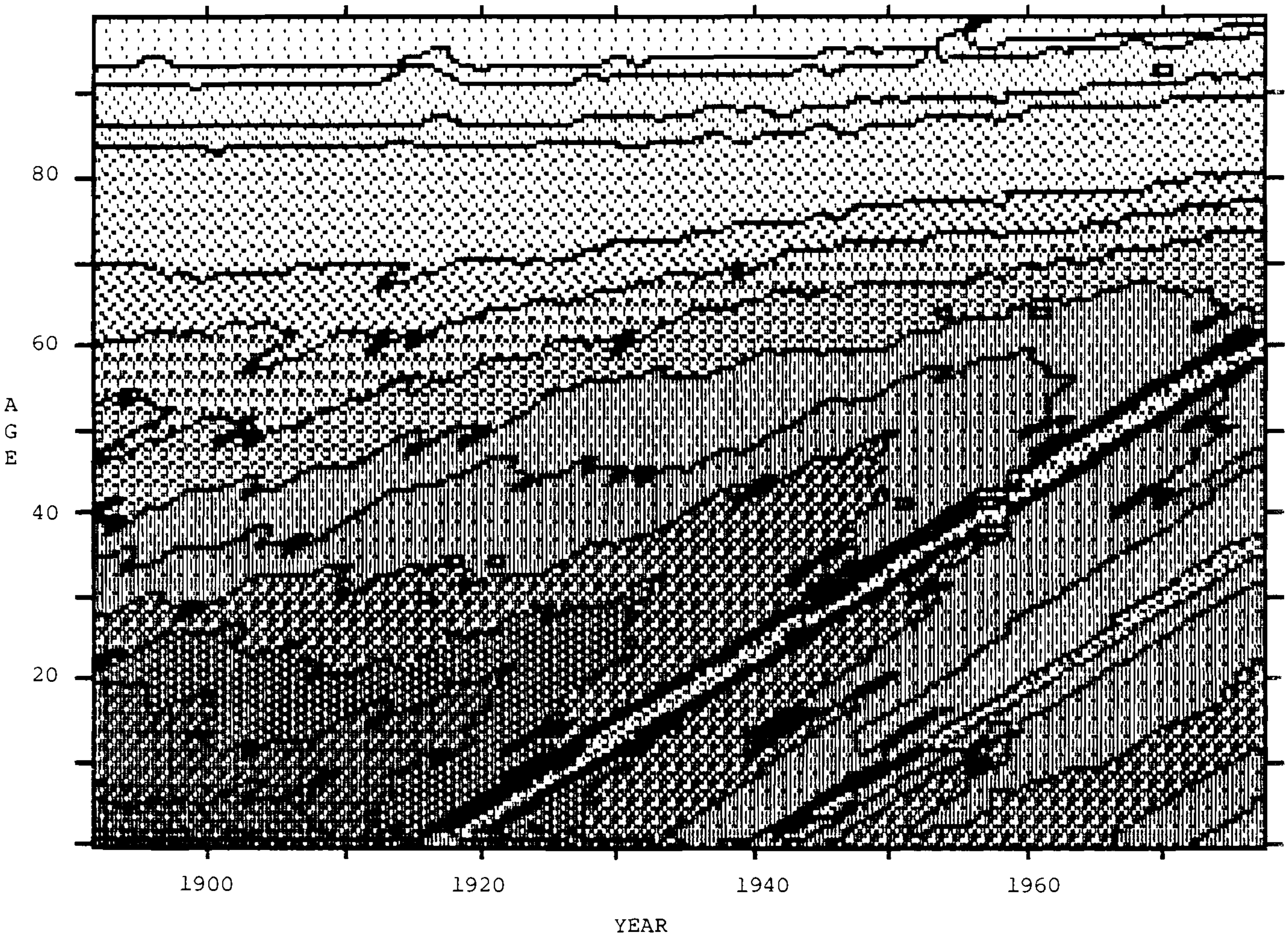
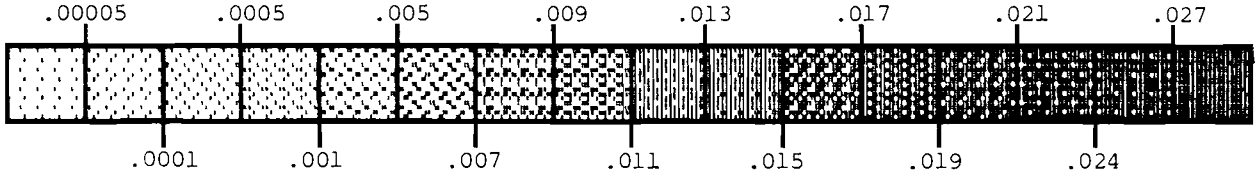


Figure 17: Age-distribution of Belgian Female Population - with contour lines selectively placed from .00005 to .027
From Age 0 to 99 and Year 1892 to 1977



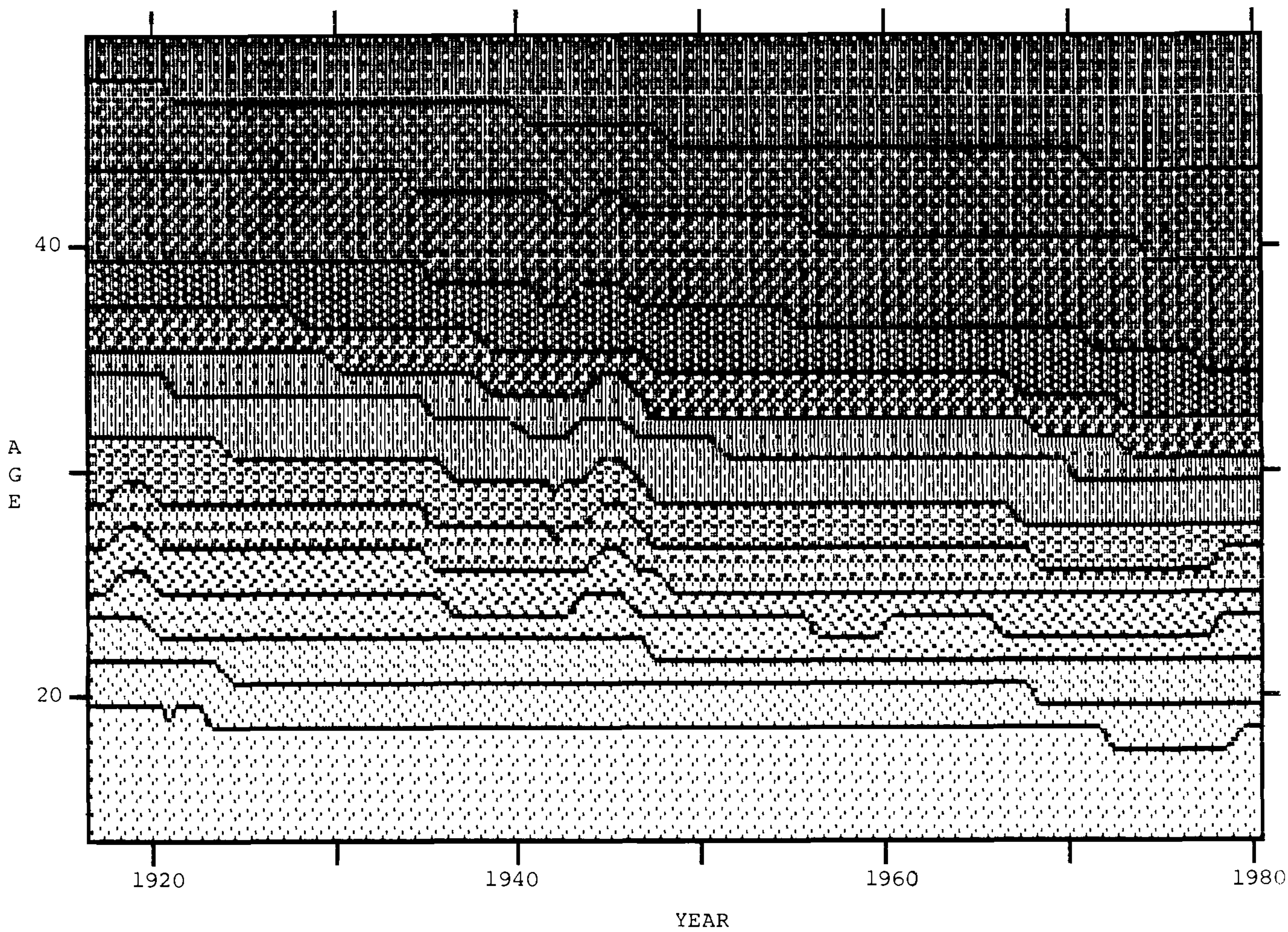
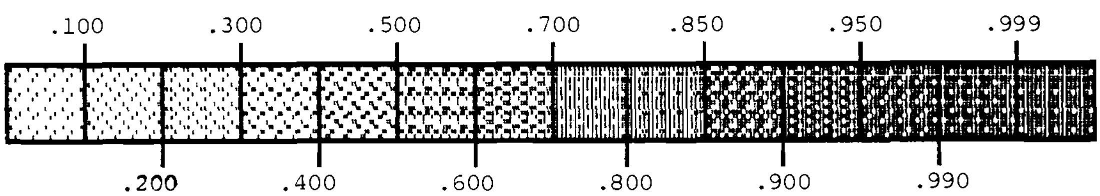


Figure 18: Cumulative U.S. Fertility Rates - with contour lines selectively placed from .1 to .999
From Age 14 to 49 and Year 1917 to 1980



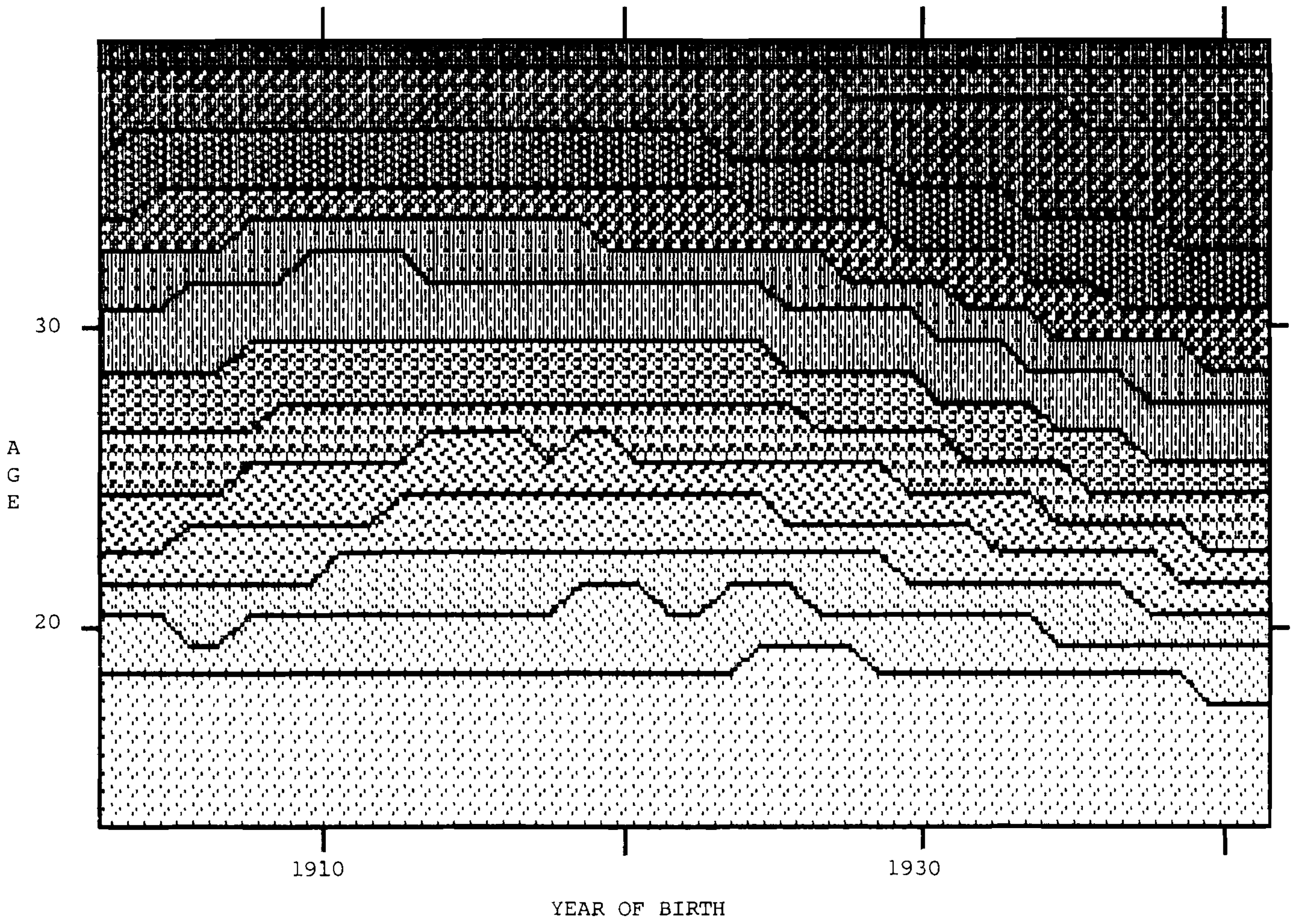
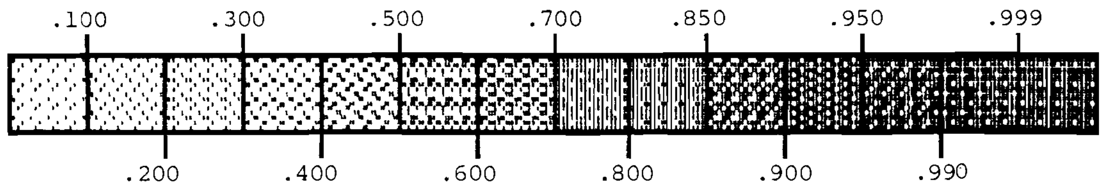
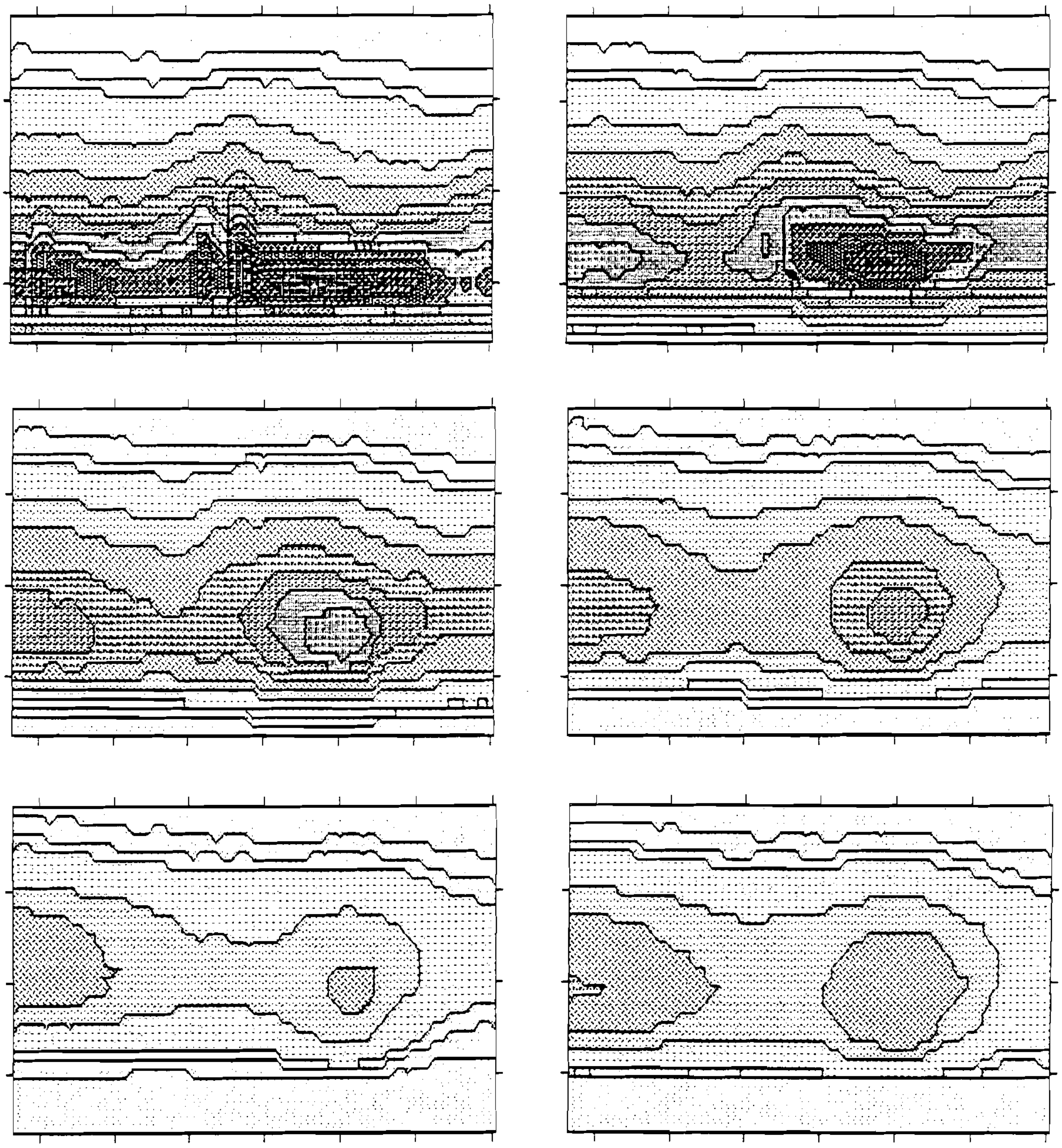
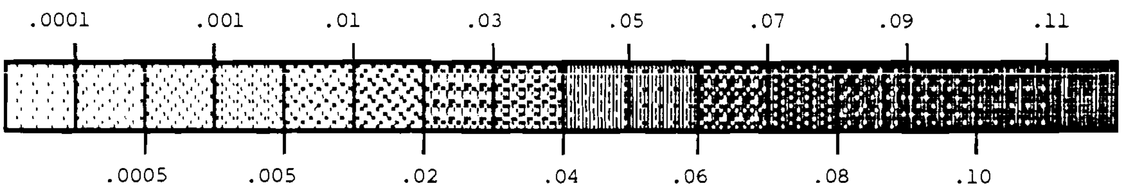


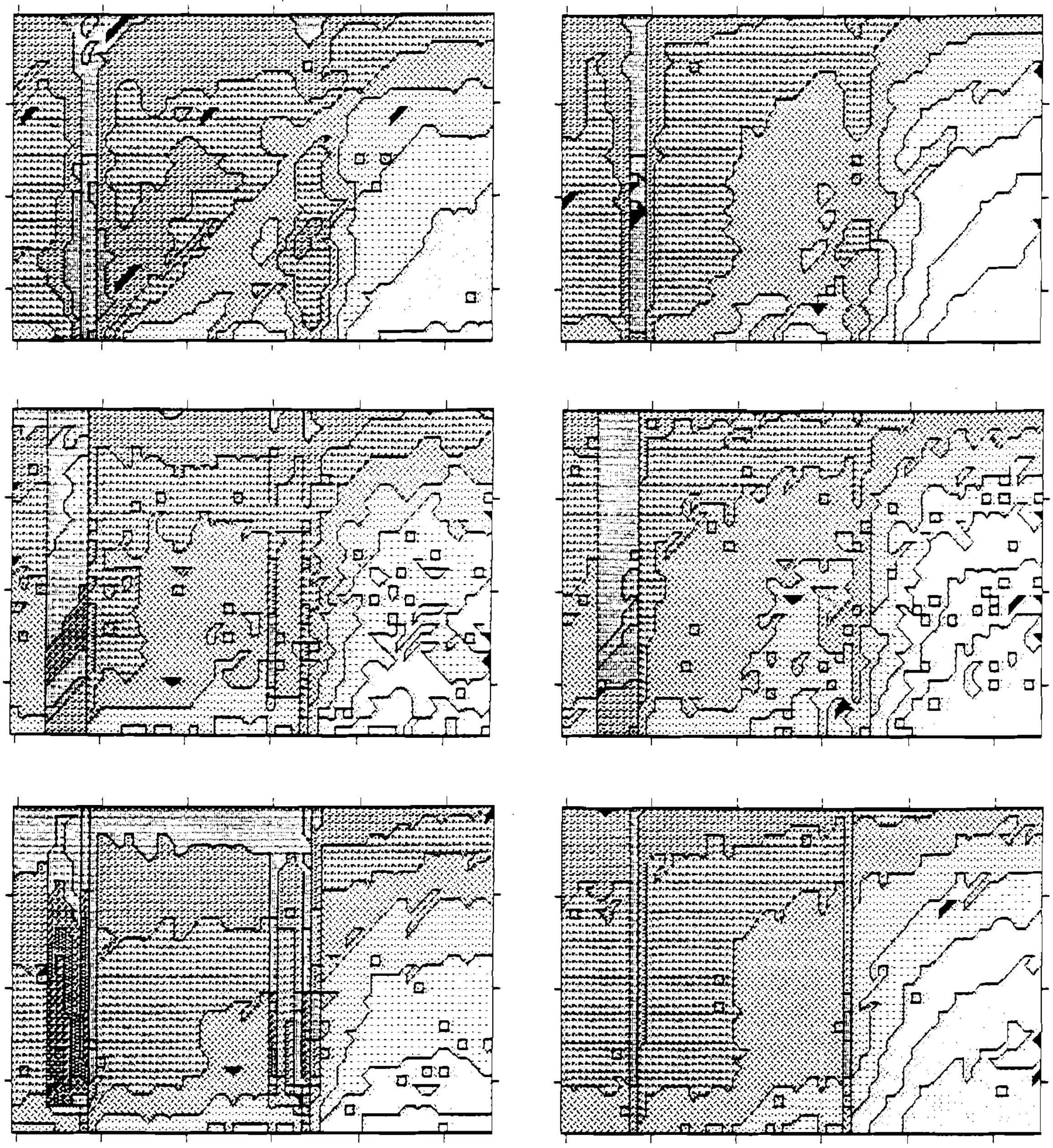
Figure 19: Cumulative U.S. Cohort Fertility - with contour lines selectively placed from .1 to .999
From Age 14 to 39 and Year of Birth 1903 to 1941



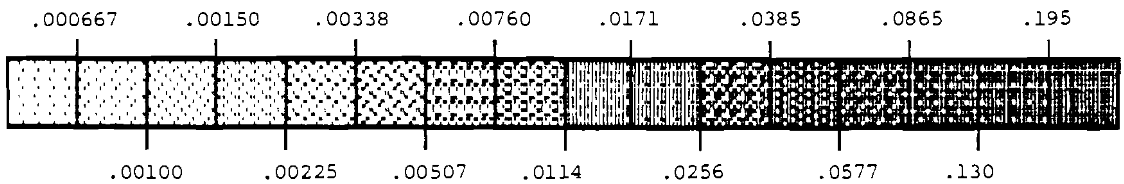


Figures 20a-f: U.S. Birth Rates at Parities 1 through 6 - with contour lines
selectively placed from .0001 to .1
From Age 14 to 49 and Year 1917 to 1980





Figures 21a-f: Mortality Rate Comparison: Male Mortality in the Left Column, Female Mortality in the Right Column, and Italian, Belgian, and French Mortality From Top to Bottom - with contours from .000667 to .195 From Age 15 to 49 and Year 1910 to 1965



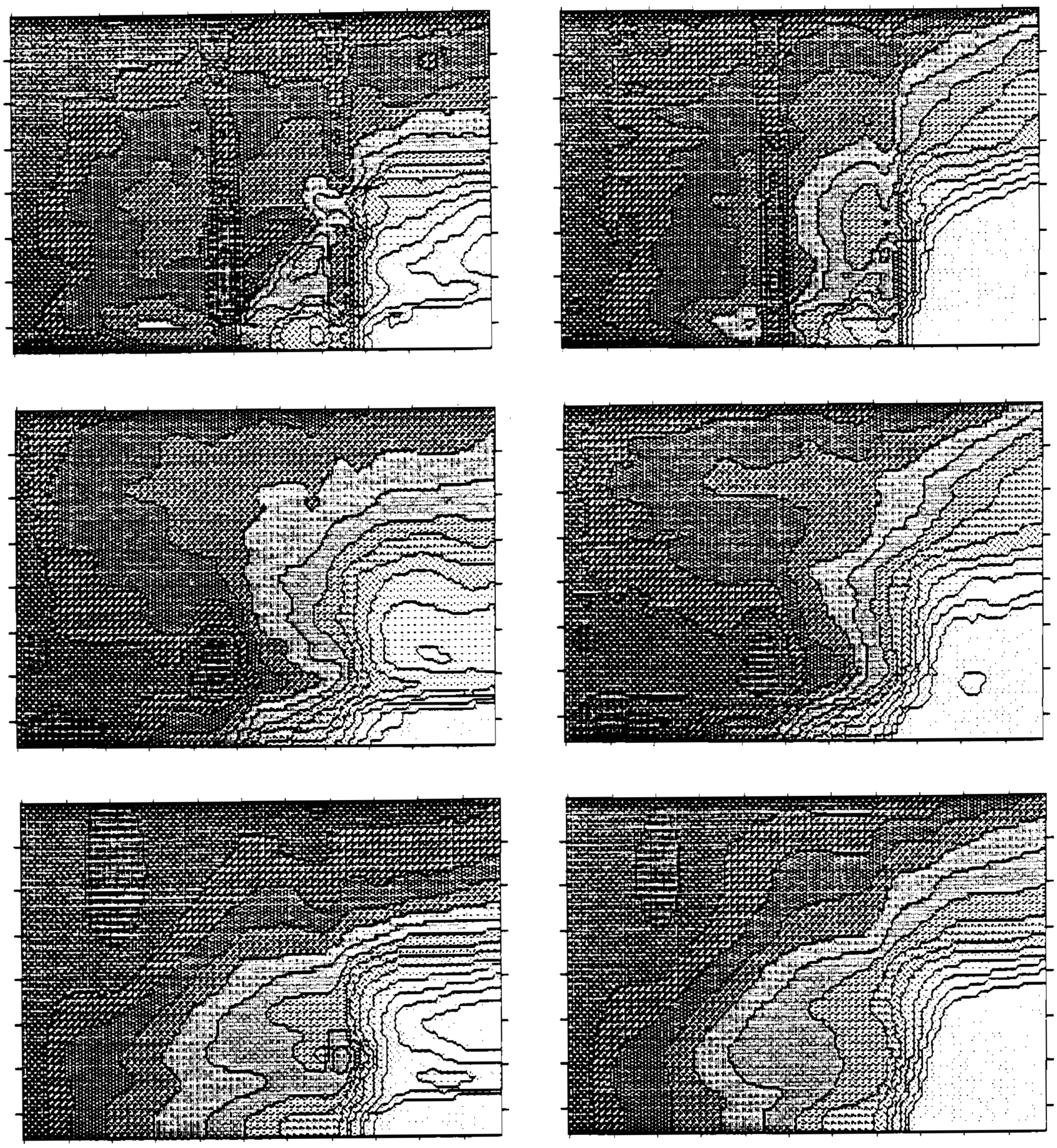
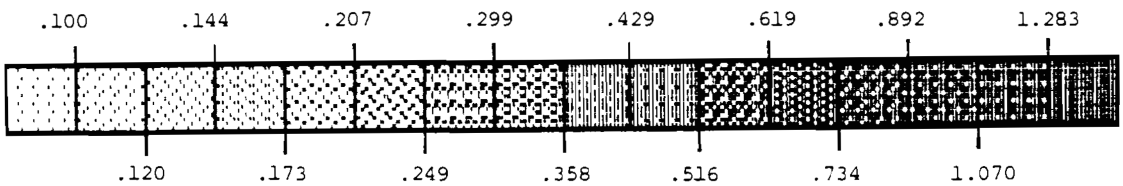


Figure 22: Mortality Rates Relative to 1870 Age Specific Levels: Male Mortality in the first column, female mortality in the second column and Italian, Swedish, and England and Wales from top to bottom - with contours from .1 to 1.28 at multiples of 1.2 , smoothed on a 5 by 5 square From Age 5 to 79 and Year 1870 to 1978



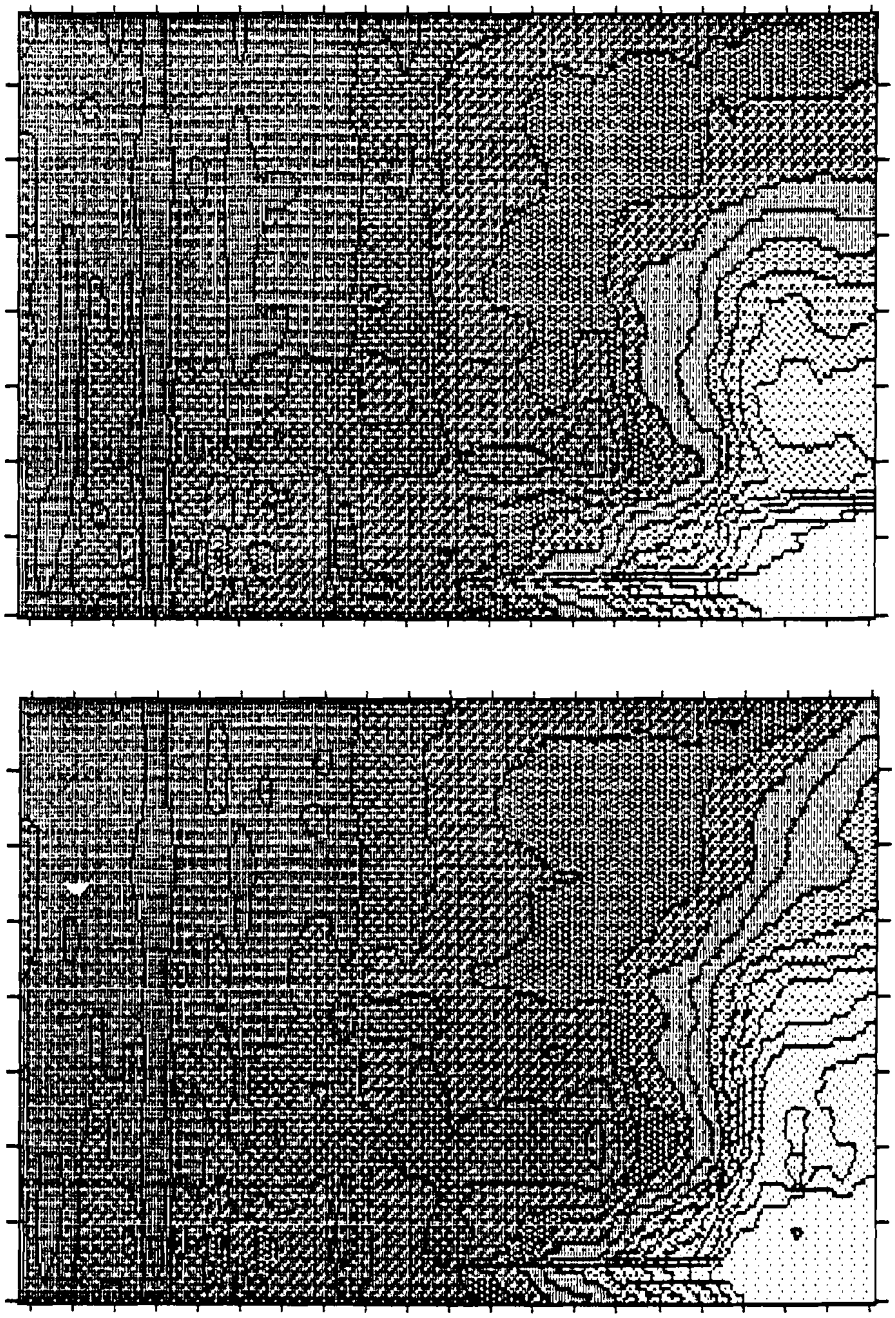
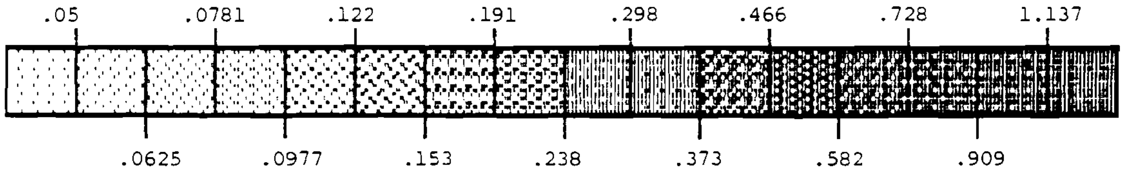


Figure 23: Swedish Mortality Relative to Age Specific Mortality from 1780 to 1799
- with contour lines from .05 to 1.137, smoothed on a 5 by 5 square
From Age 0 to 79 and Year 1780 to 1981



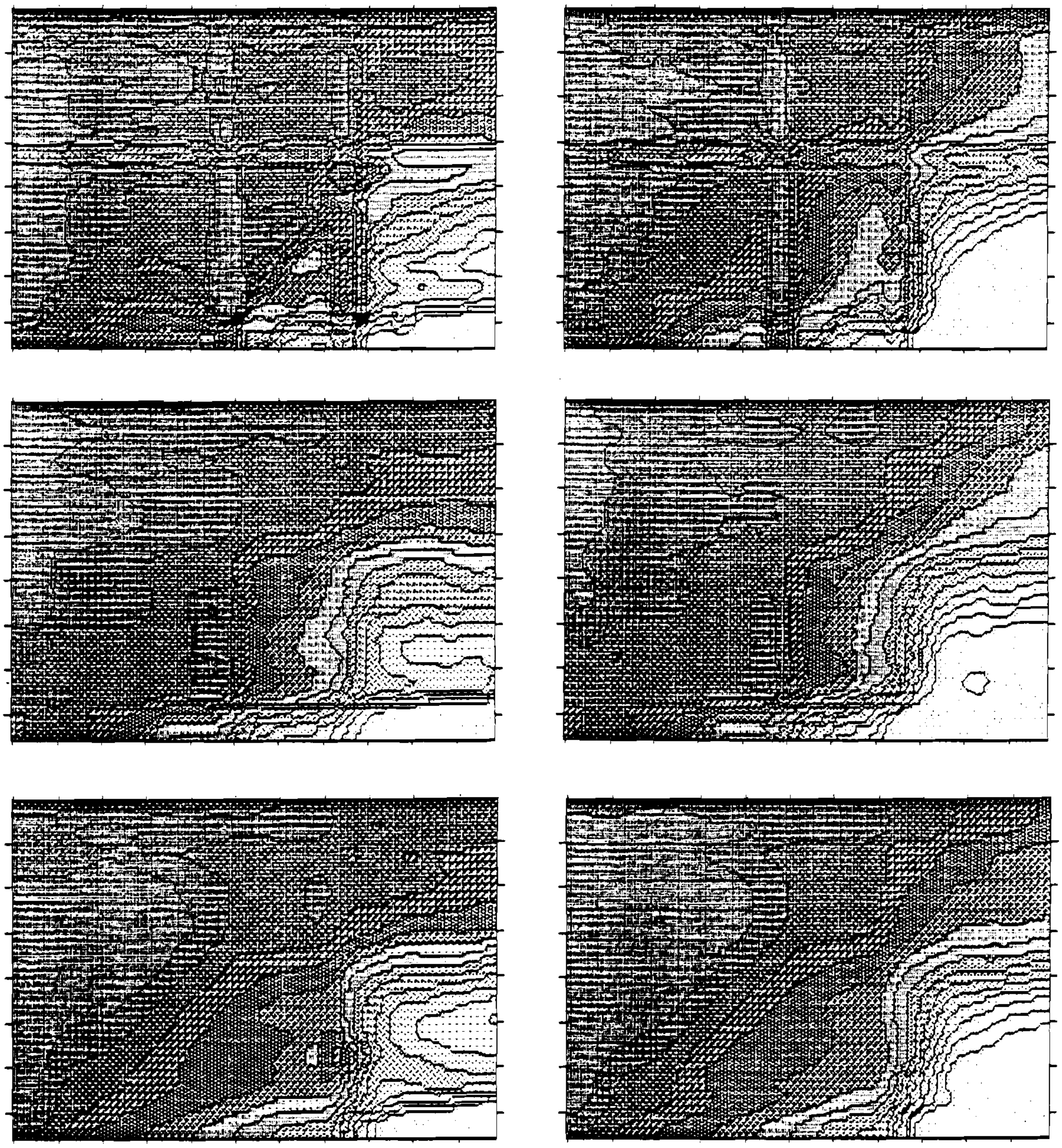
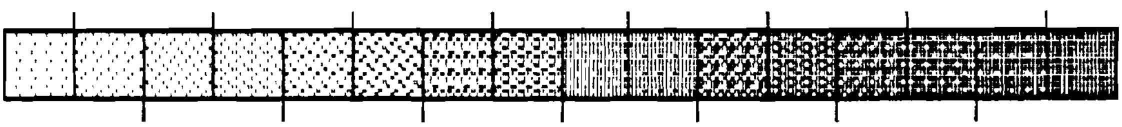


Figure 24: Mortality Rates Relative to 1870 Cohort Levels: Male Mortality in the first column, female mortality in the second column and Italian, Swedish, and England and Wales from top to bottom - with contours from .1 to 1.28 at multiples of 1.2 ,smoothed on a 5 by 5 square From Age 5 to 79 and Year 1870 to 1978



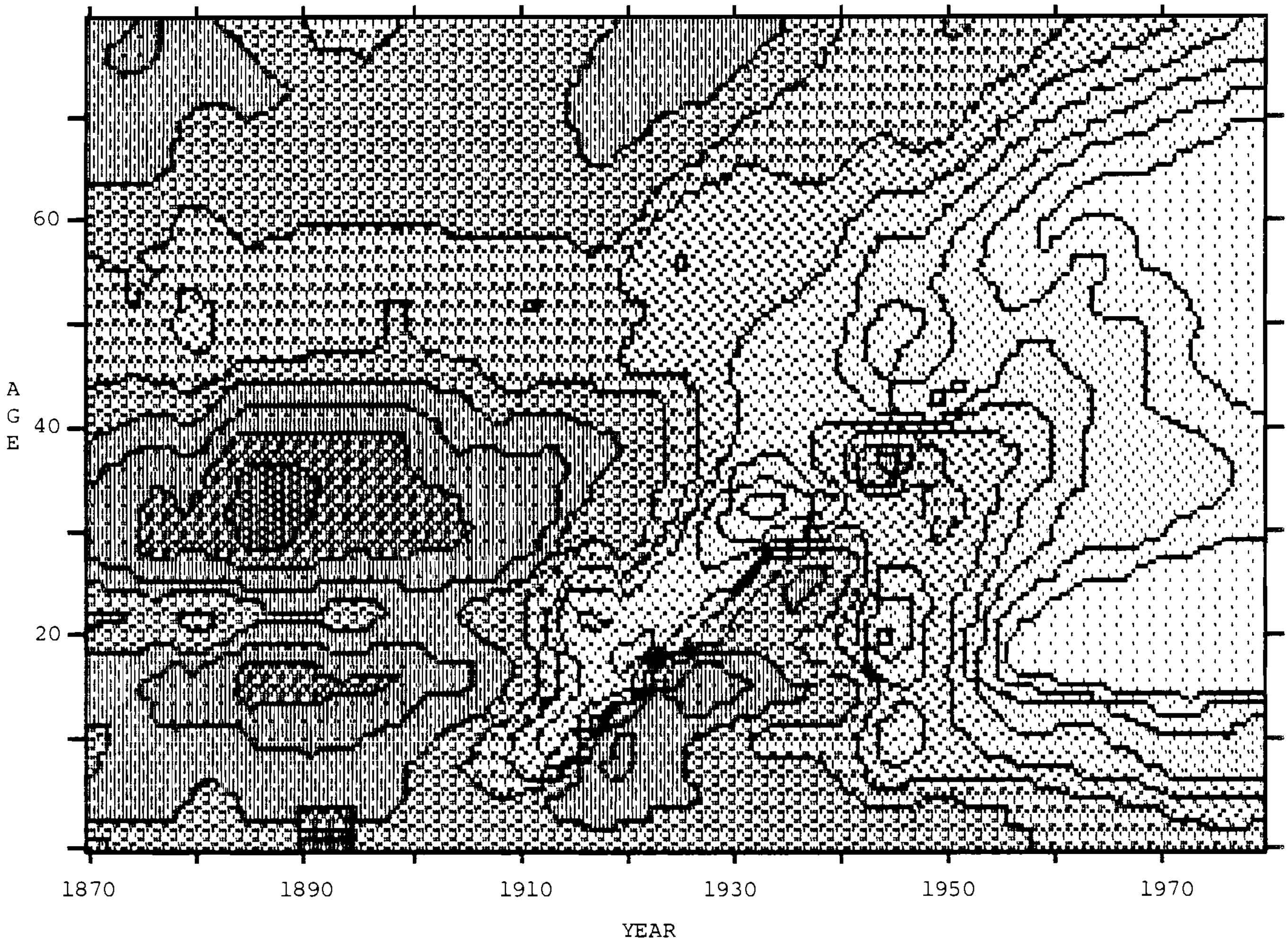
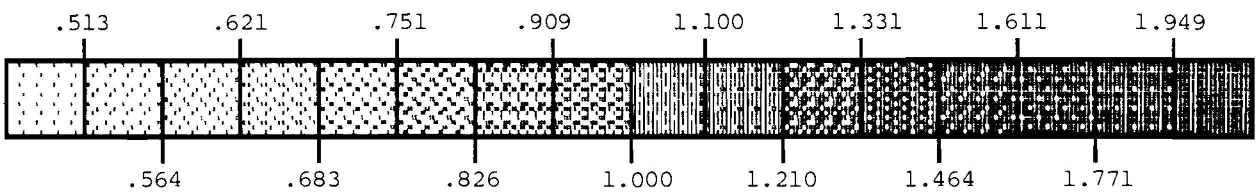


Figure 25a: Italian Female Mortality Rates Divided by Italian Male Mortality Rates - with contour lines from .51 to 1.95 at intervals of 10 percent change From Age 0 to 79 and Year 1870 to 1979, smoothed on a 5 by 5 square



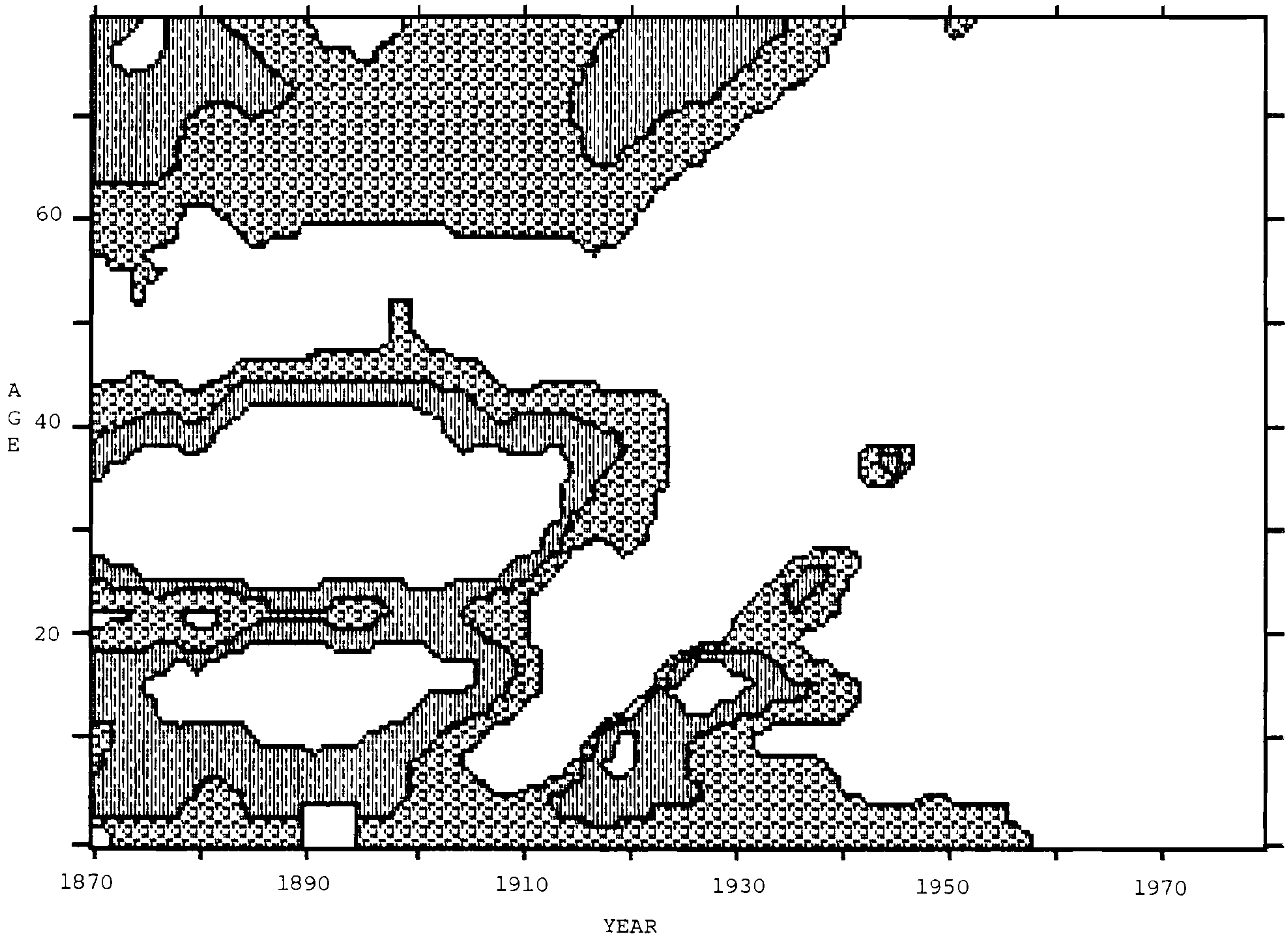
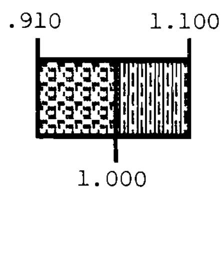


Figure 25b: Italian Female Mortality Divided by Italian Male Mortality - with selected contour lines, smoothed on a 5 by 5 square From Age 0 to 79 and Year 1870 to 1979



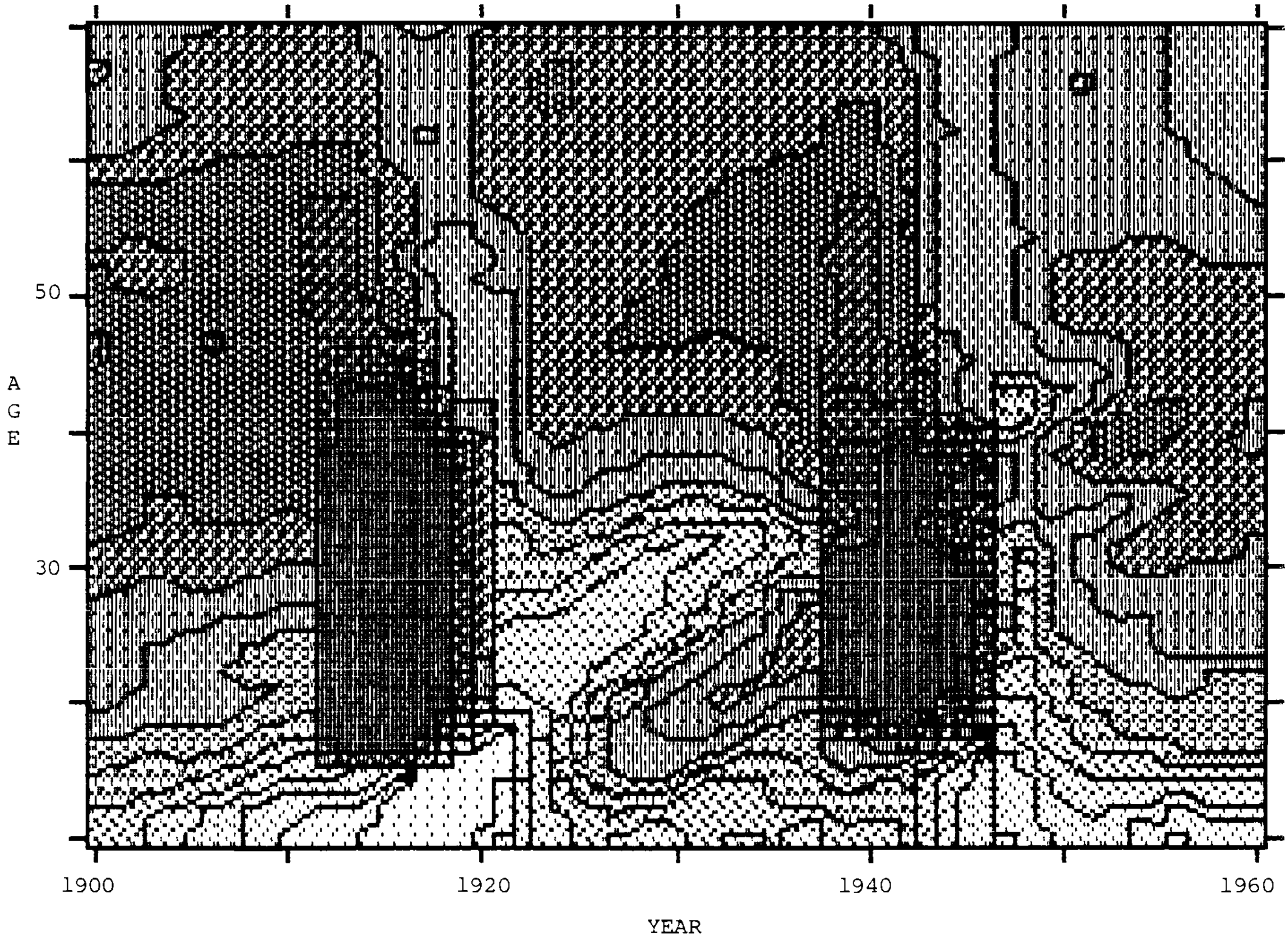
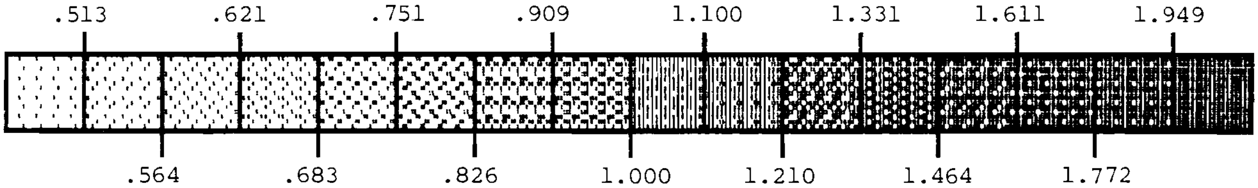


Figure 26: French Male Mortality Divided by Italian Male Mortality - with contour lines from .51 to 1.95 at multiples of 1.1 , smoothed on a 5 by 5 square From Age 10 to 70 and Year 1900 to 1960



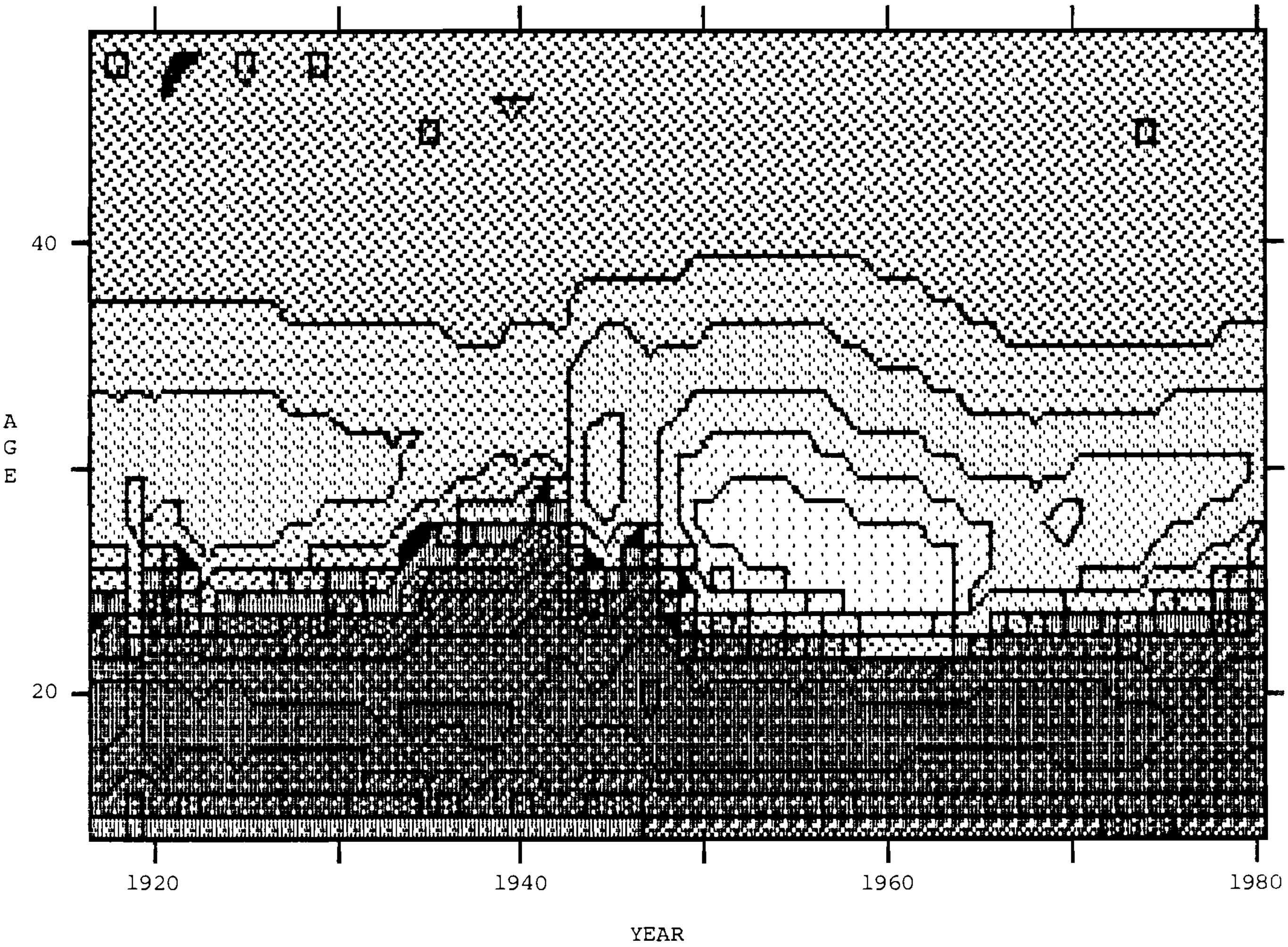
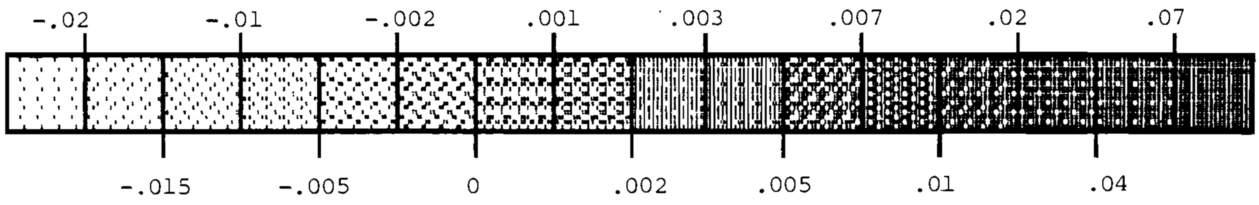


Figure 27: U.S. Fertility: Parity One Minus Parity Two - with contour lines selectively placed from -.02 to .07
From Age 14 to 49 and Year 1917 to 1980



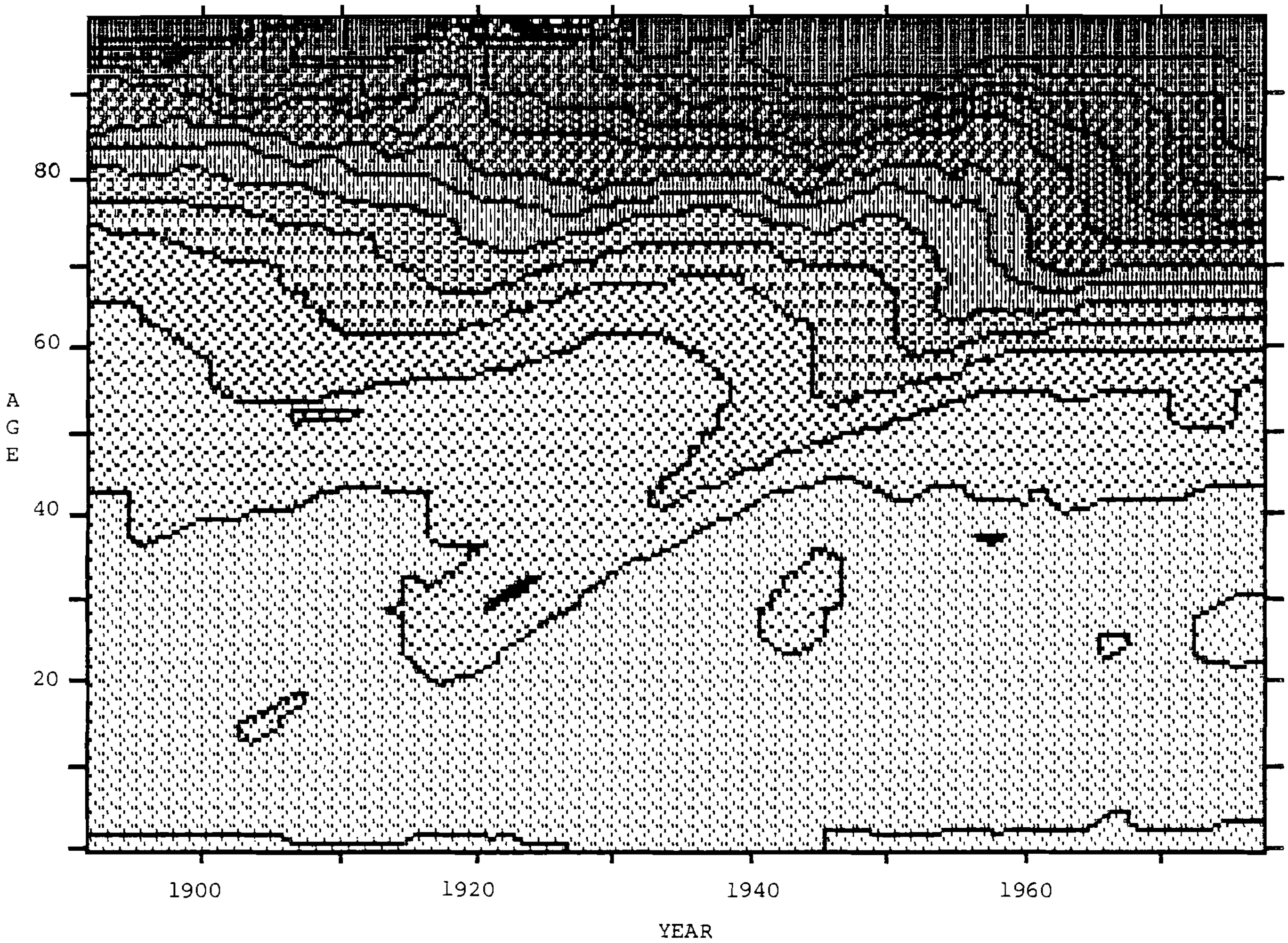
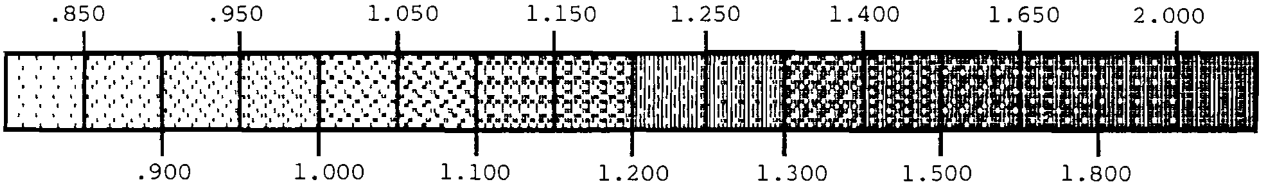


Figure 28: Belgian Female Population Divided by Belgian Male Population - with contour lines selectively placed from .85 to 2.0, 5 by 5 smoothing From Age 0 to 99 and Year 1892 to 1977



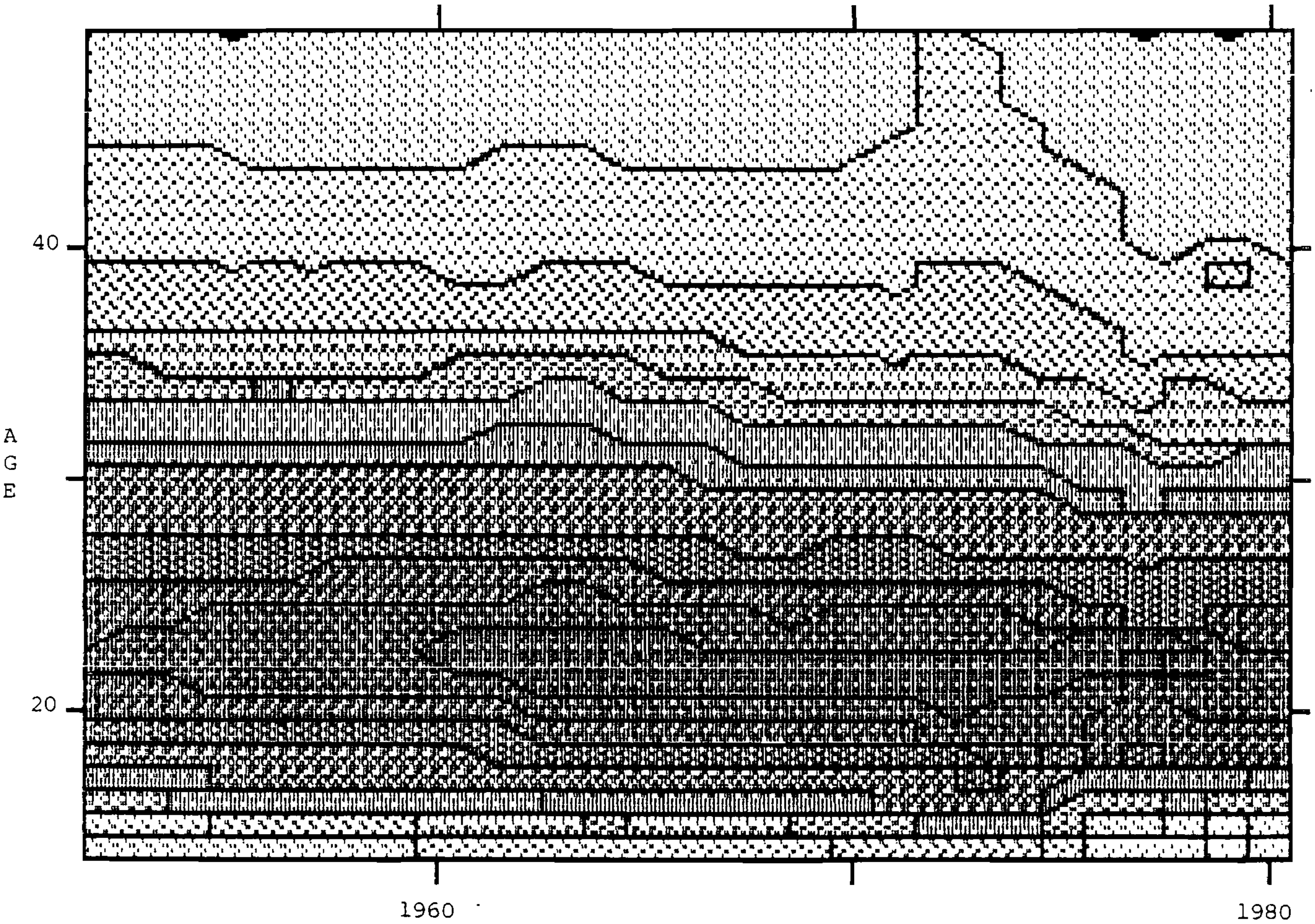
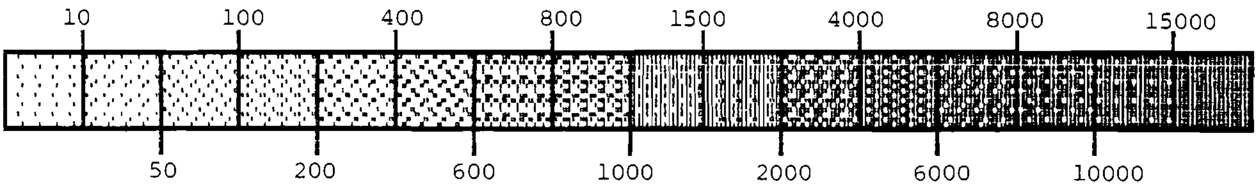


Figure 29: Age Specific Marriage Data for Italian Females - with contour lines from 10 to 15,000 at selected intervals
From Age 0 to 49 and Year 1952 to 1981



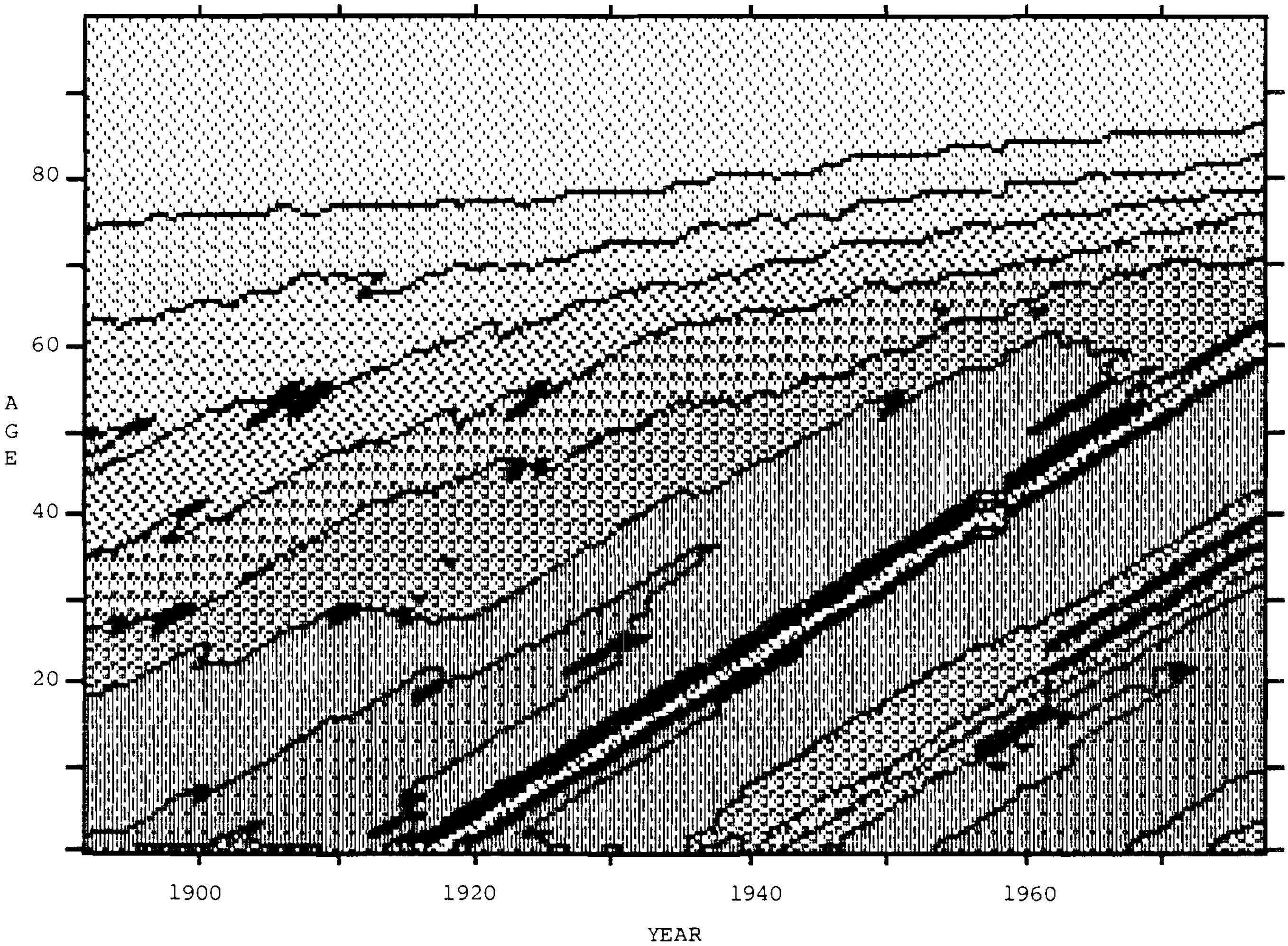
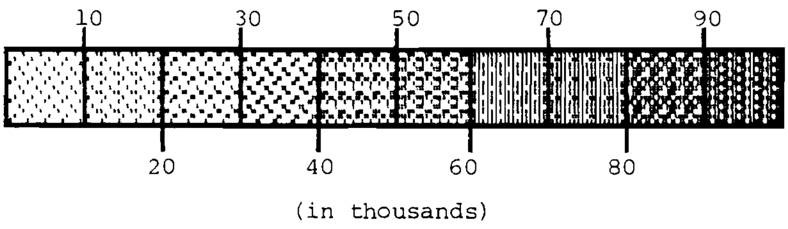


Figure 30: Belgian Female Population - with contour lines from 10,000 to 90,000 at selected intervals
From Age 0 to 99 and Year 1892 to 1977



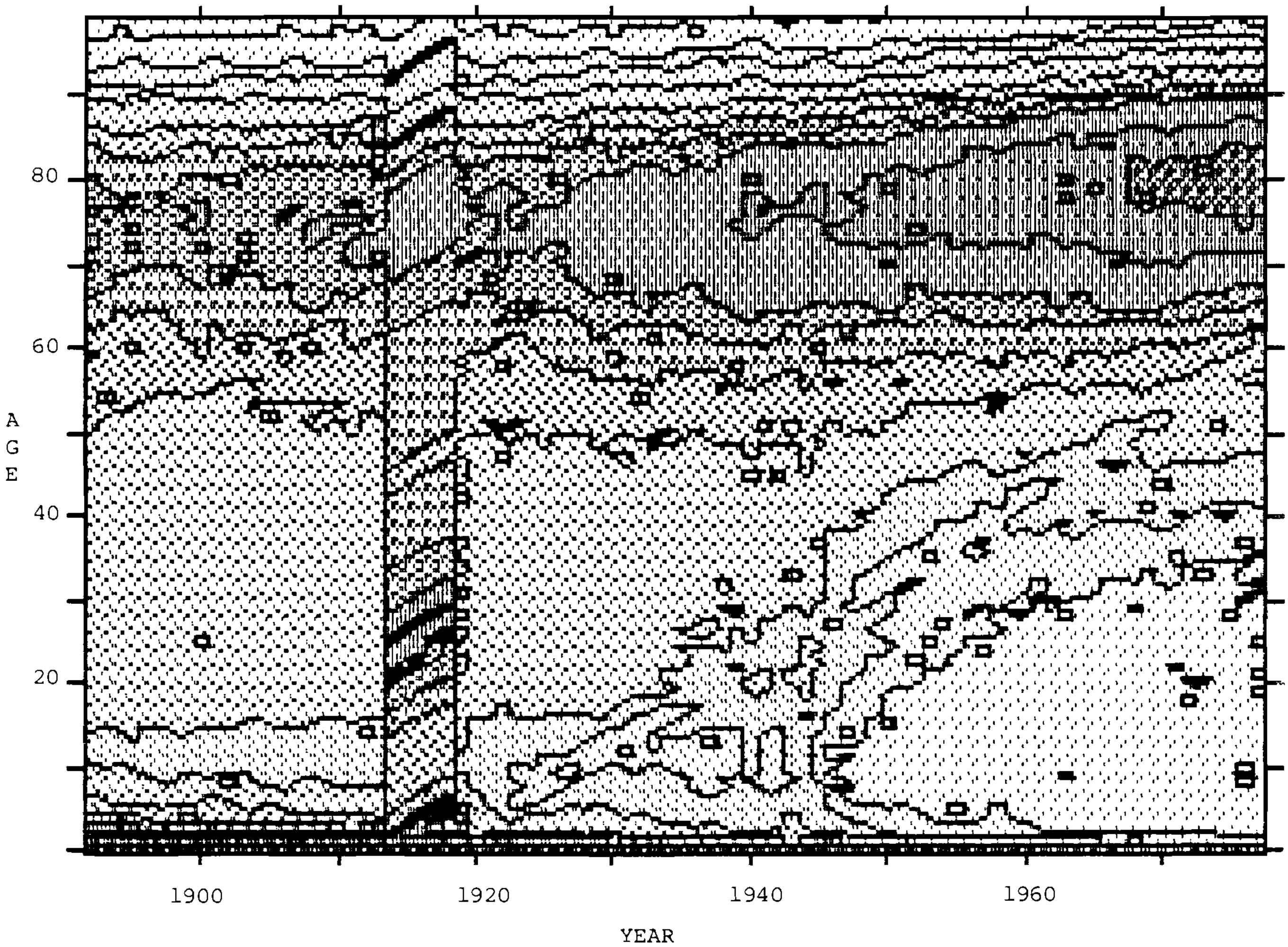
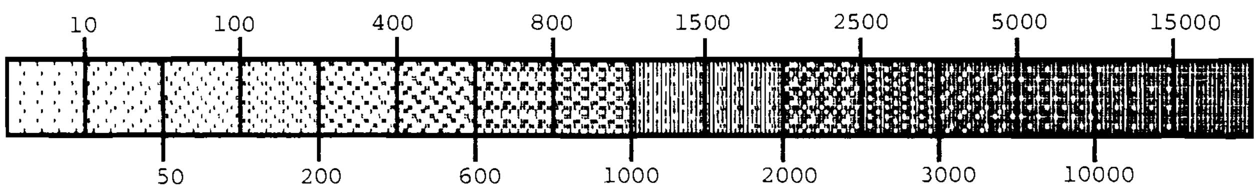


Figure 31: Belgian Female Deaths - with contour lines from 10 to 15,000
From Age 0 to 99 and Year 1892 to 1977



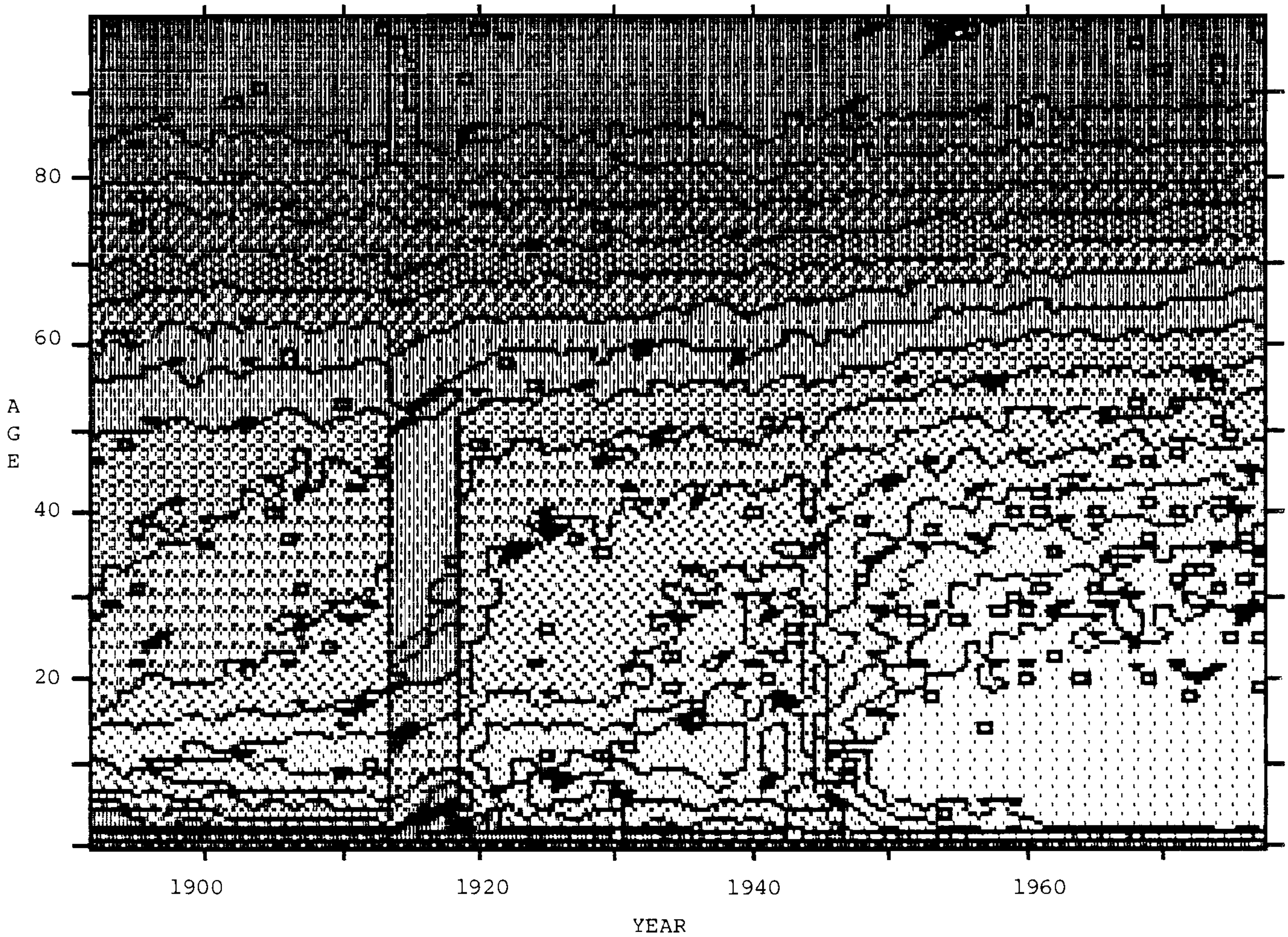
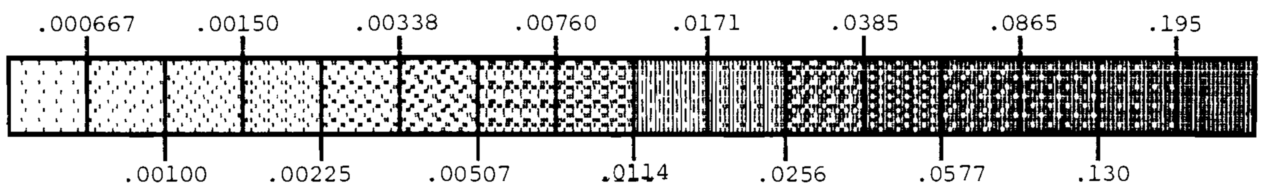


Figure 32: Belgian Female Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From Age 0 to 99 and Year 1892 to 1977



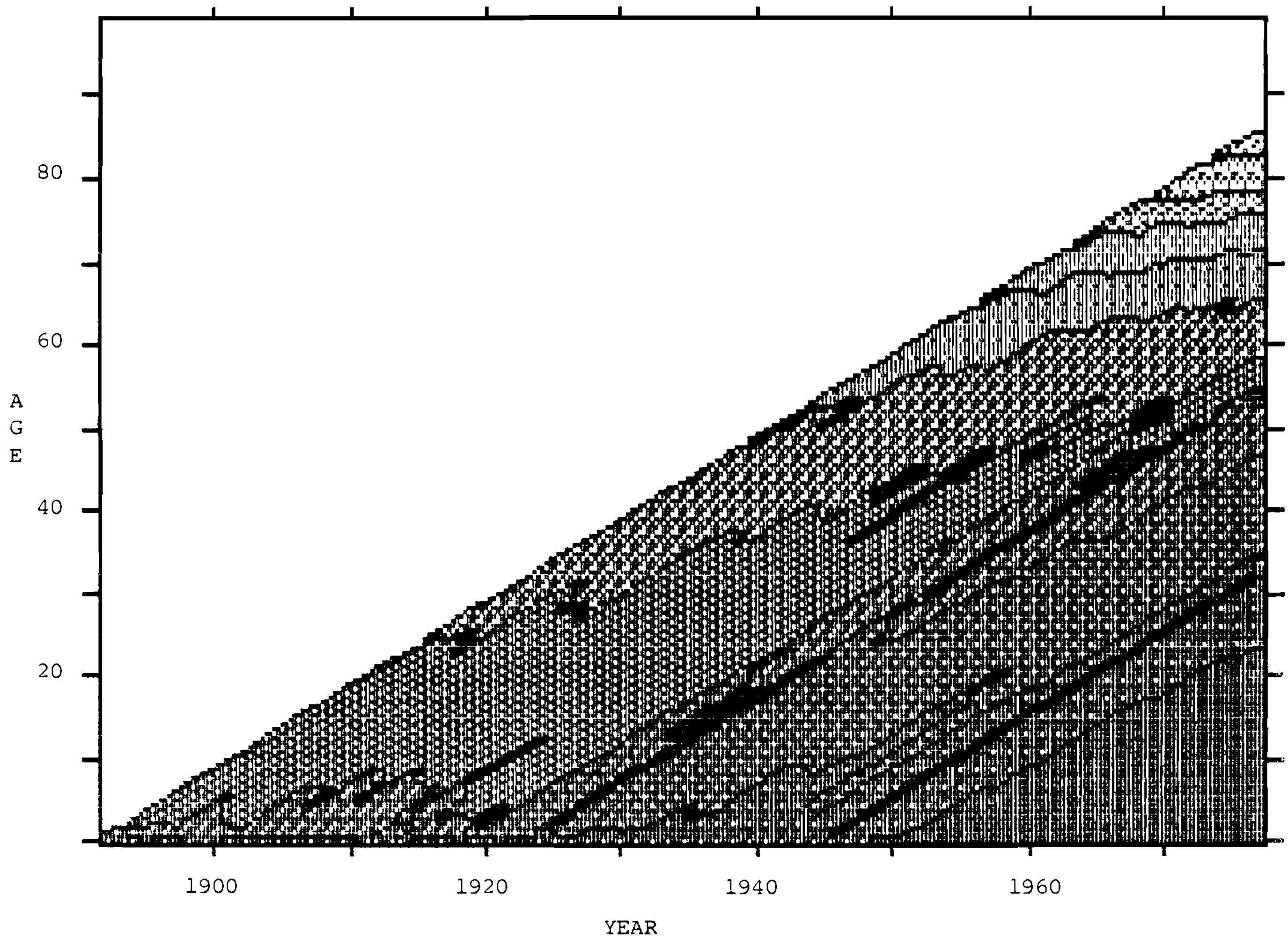
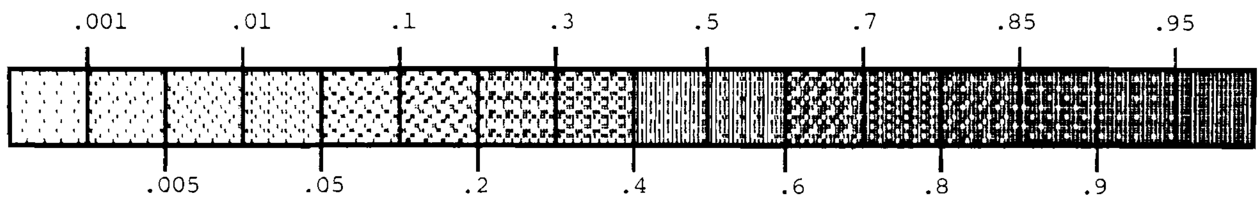


Figure 33: Belgian Female Period Survivorship - with contour lines selectively placed from .001 to .95
From Age 0 to 99 and Year 1892 to 1977



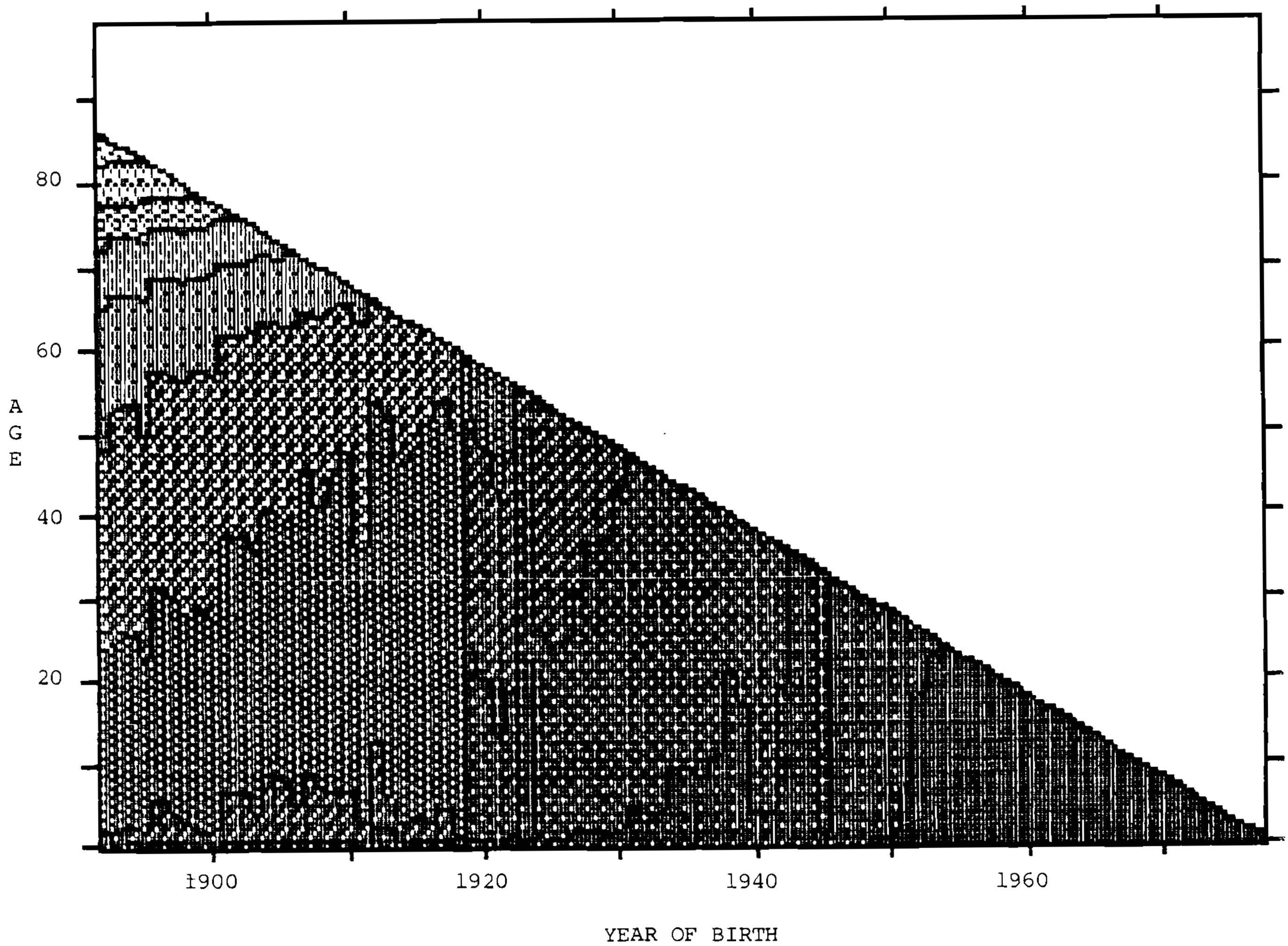
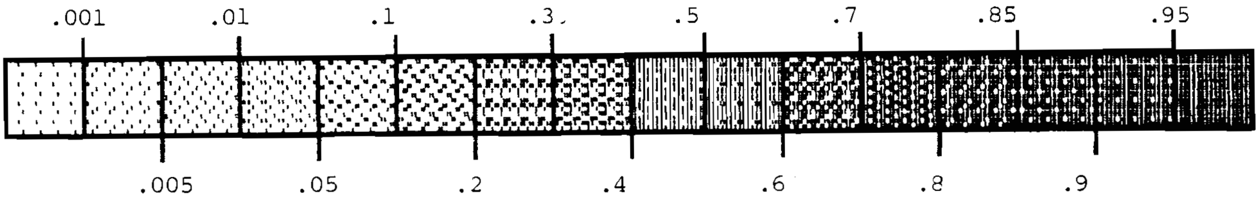


Figure 34: Belgian Female Cohort Survivorship - with contours selectively placed from .001 to .95
From Age 0 to 99 and Year 1892 to 1977



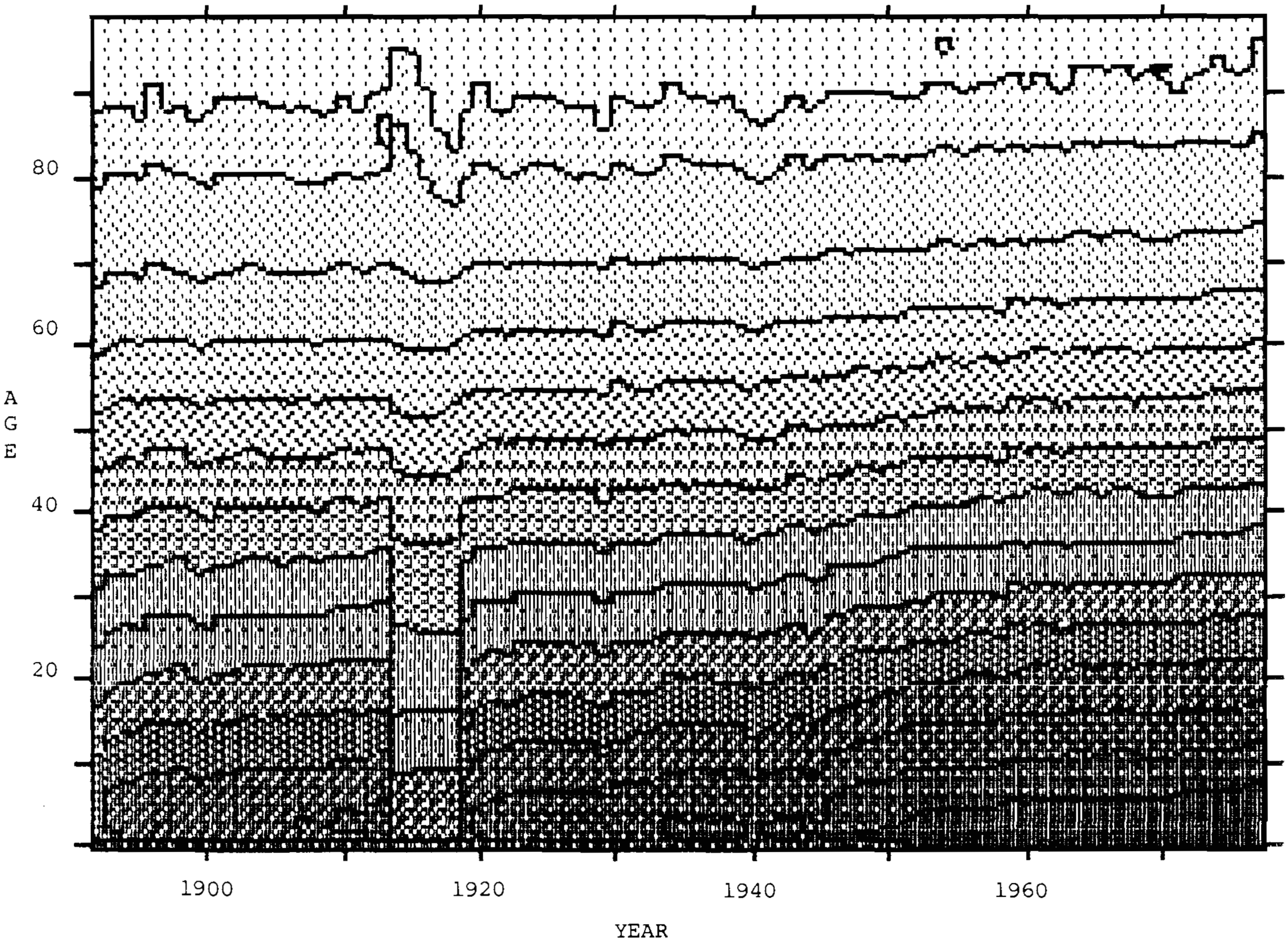
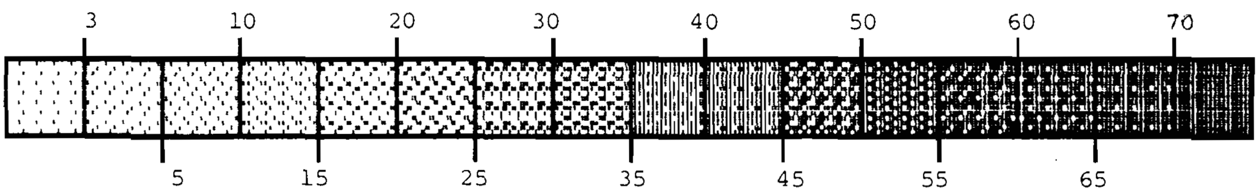


Figure 35: Belgian Female Period Life Expectancy - with contours selectively placed from .3 to 70
From Age 0 to 99 and Year 1892 to 1977



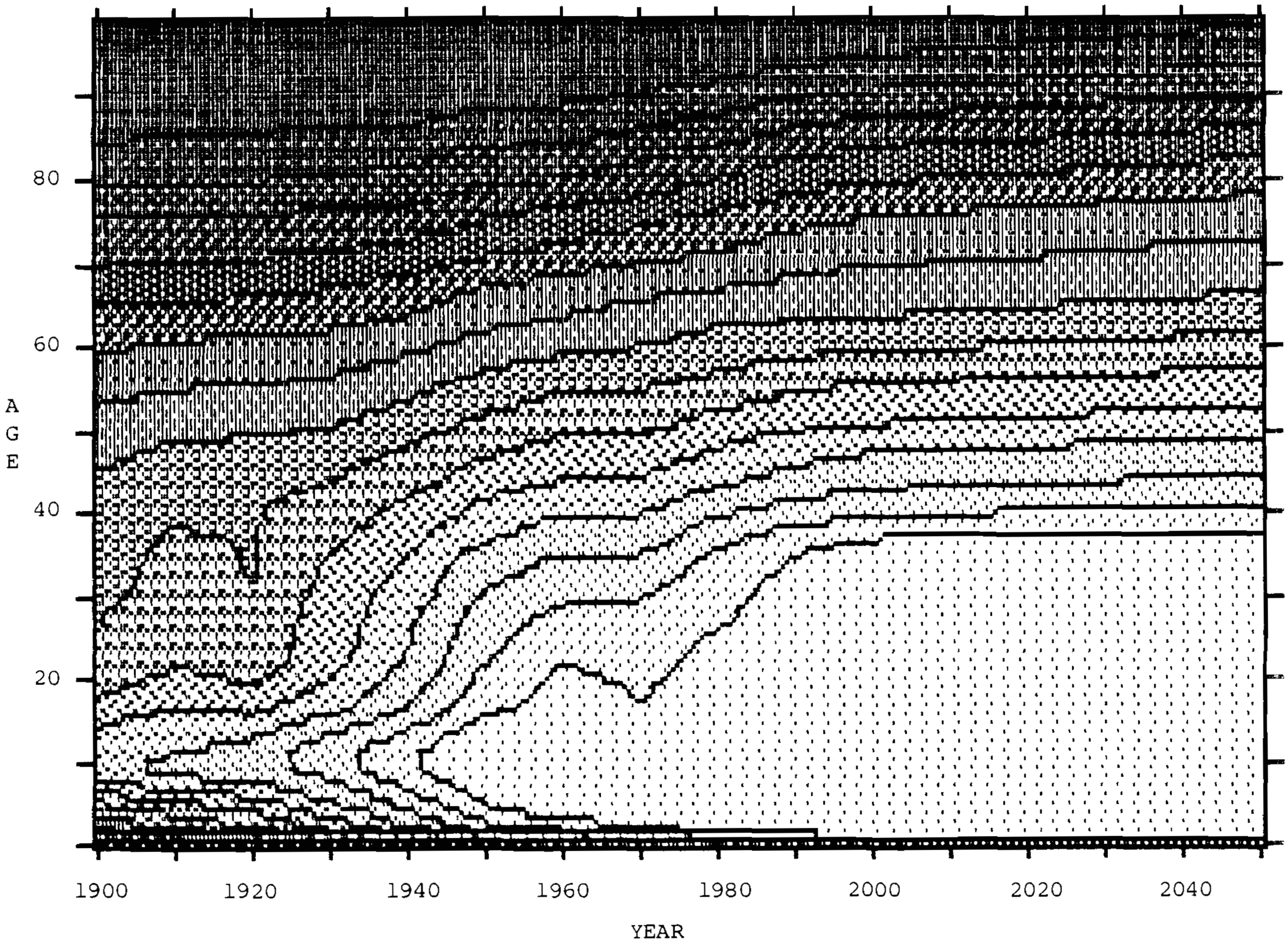
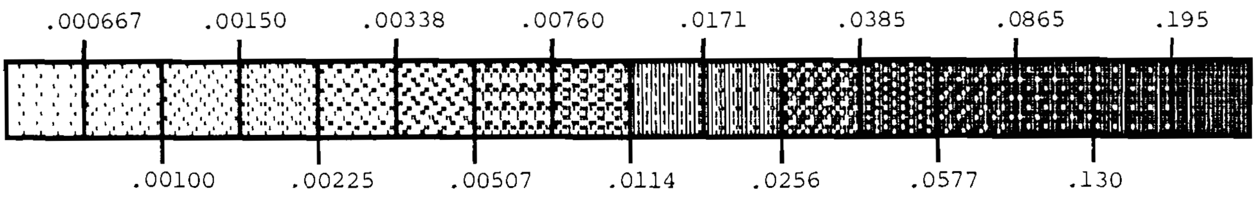


Figure 36: Projected U.S. Female Mortality Rates Based on Faber Life Tables - with contour lines from .000667 to .195 at multiples of 1.5
From Age 0 to 99 and Year 1900 to 2050



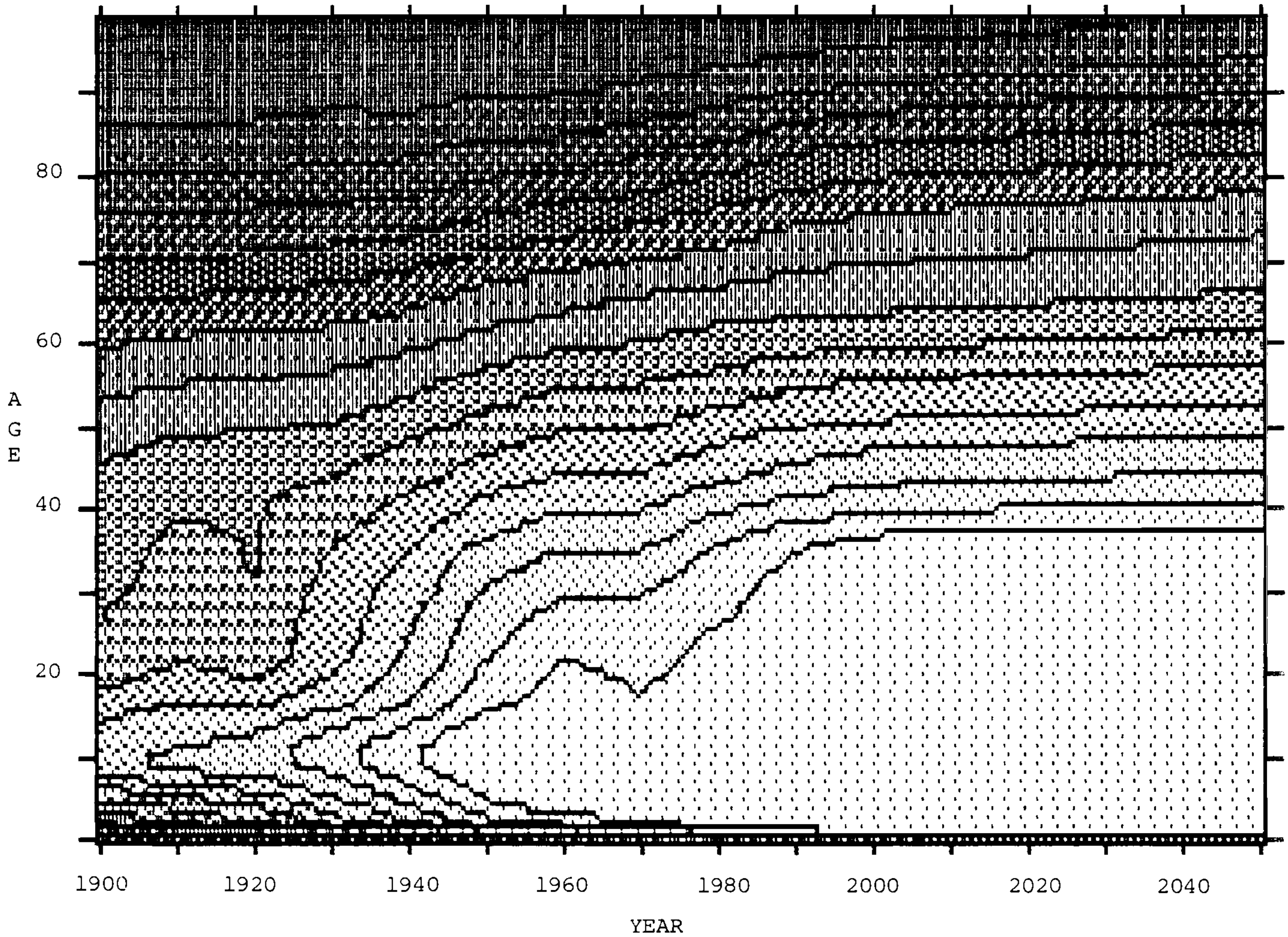
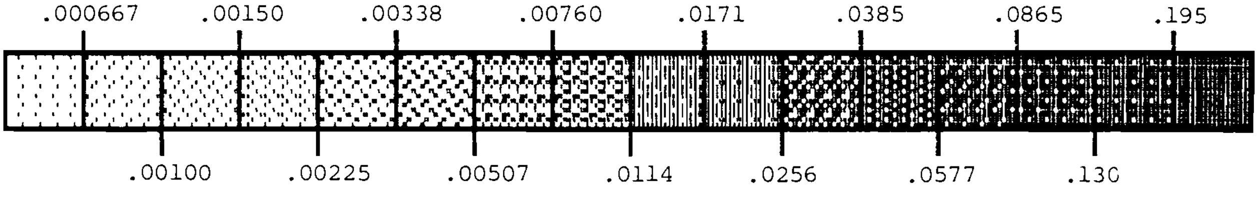


Figure 37: Projected U.S. Force of Mortality Based on Faber Life Tables - with contour lines from .000667 to .195 at multiples of 1.5 From Age 0 to 99 and Year 1900 to 2050



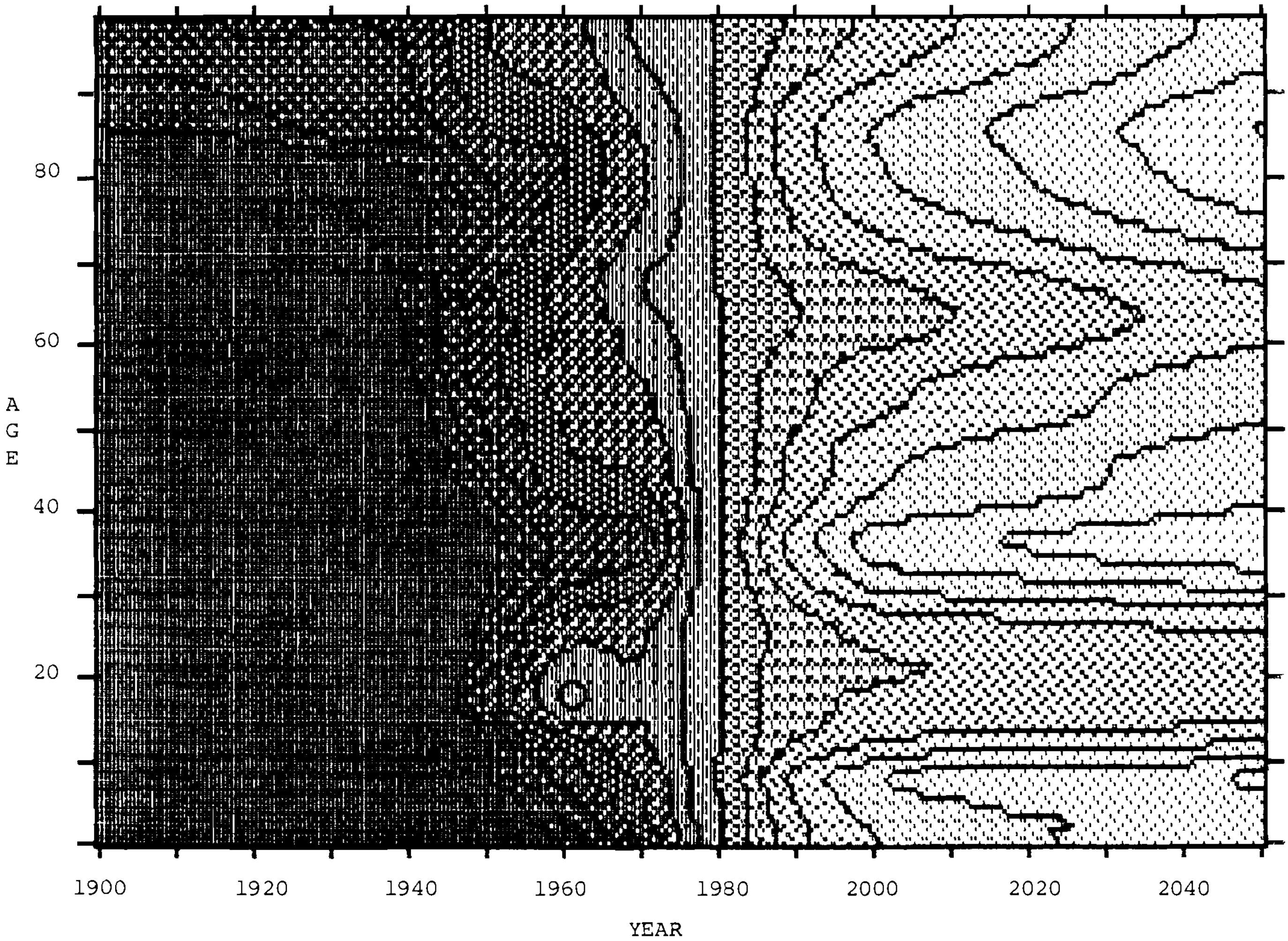
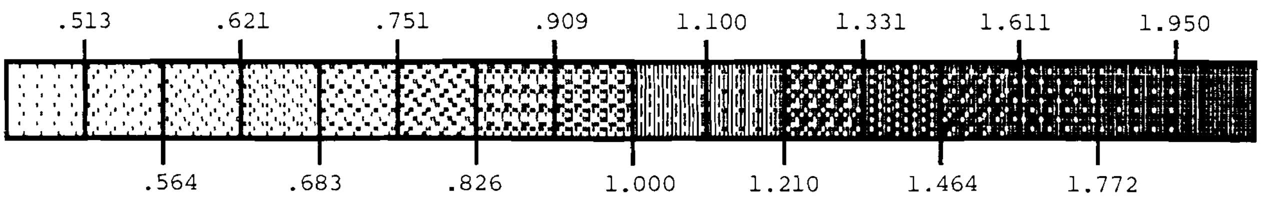


Figure 38: Projected Force of Mortality Rates Relative to 1980 Age Specific Levels - with contour lines from .51 to 1.95 at multiples of 1.1, 5 by 5 smoothing From Age 0 to 99 and Year 1900 to 2050



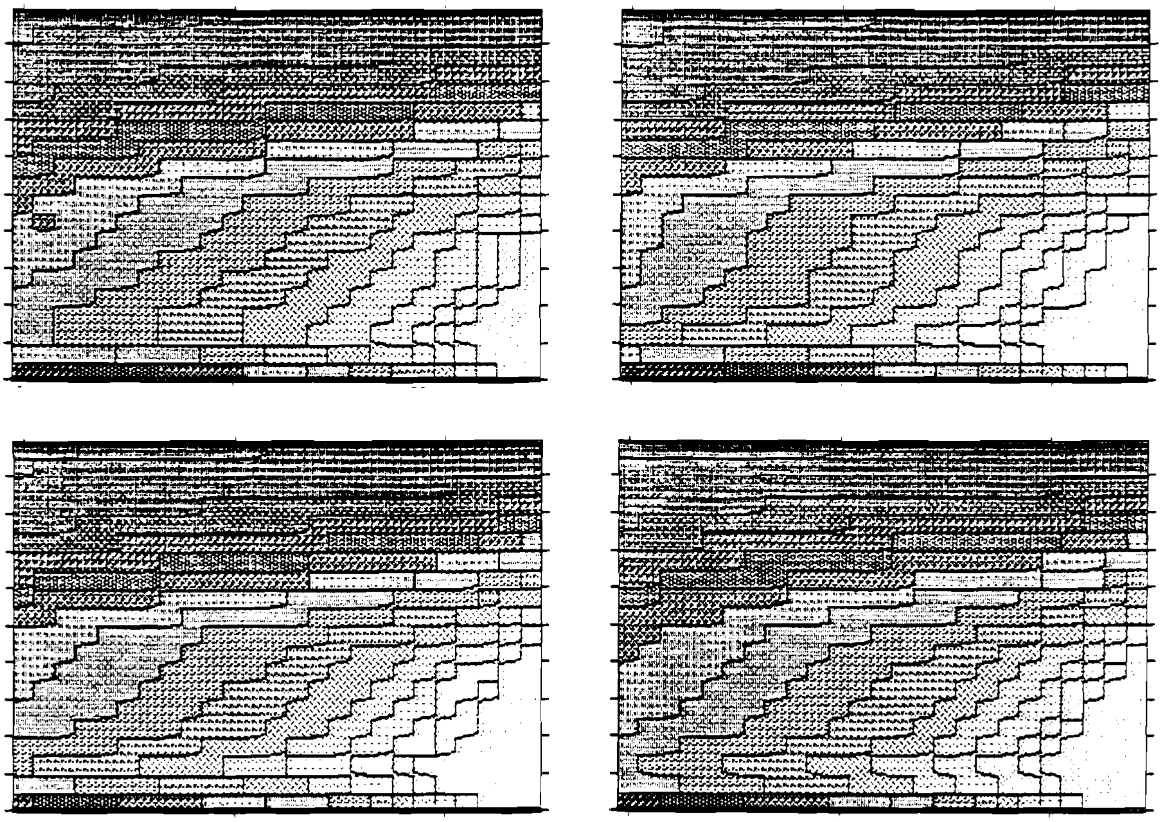
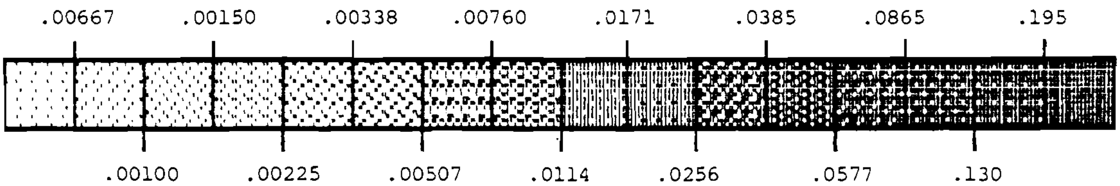


Figure 39: Coale and Demeny Model Data, with North, South, East, and West from Left to Right and Top to Bottom and contours from .000667 to .195
From Age 0 to 99 on the Verticle Axis and Life Expectancy 20 to 80 at intervals of 2.5 on the Horizontal Axis



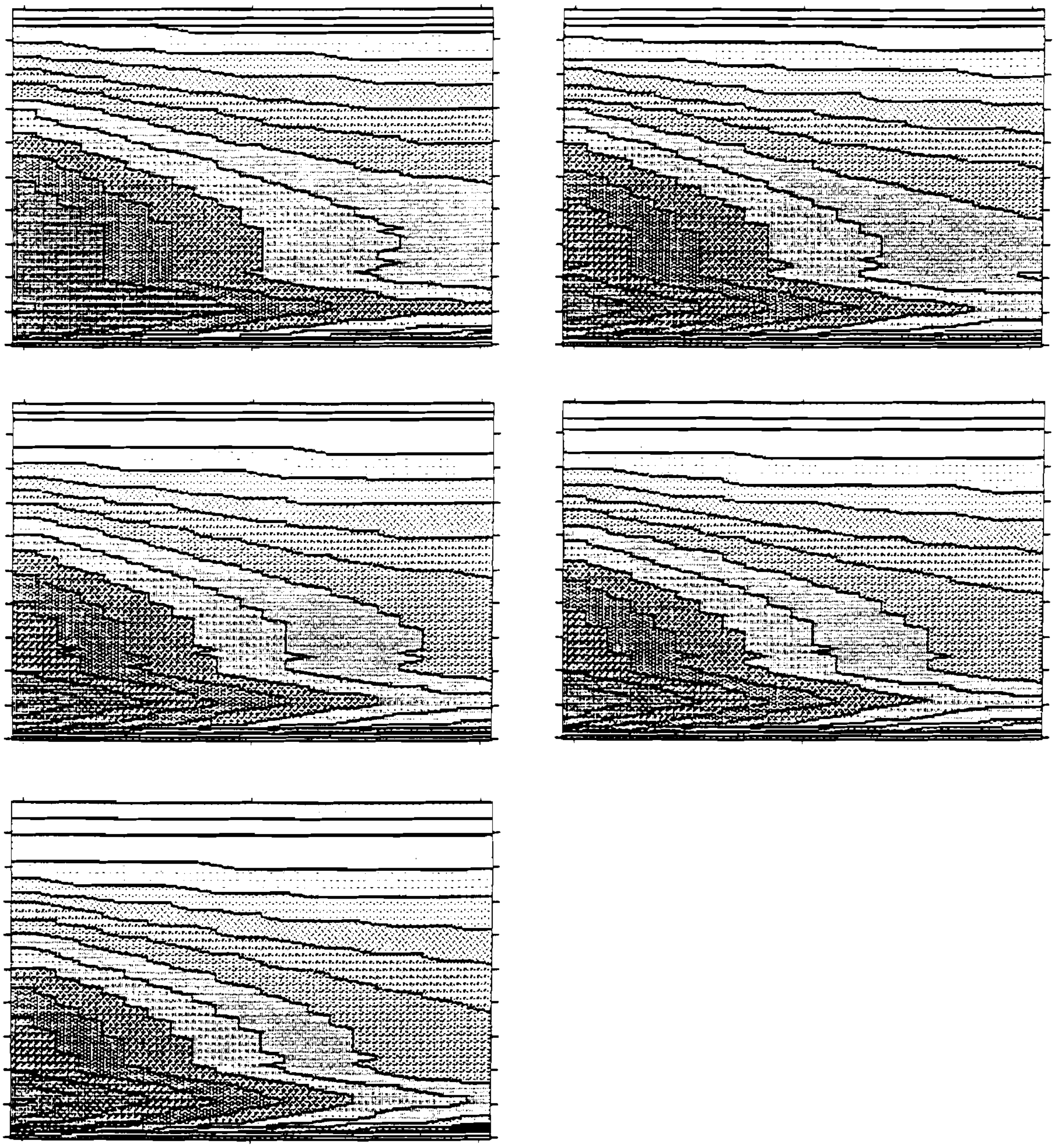
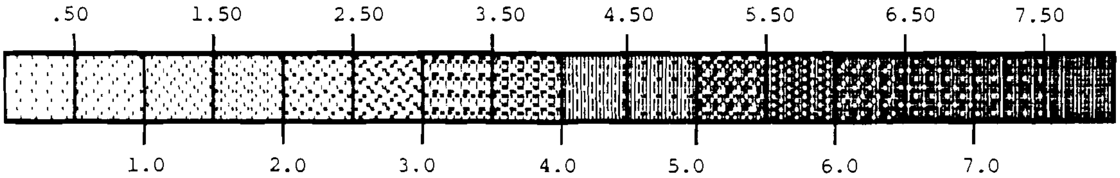


Figure 40: Brass's Model With Maps From Top to Bottom at Values of b=.6, .8, 1, 1.2, 1.4 - and contour lines evenly spaced from .5 to 7.5 From Age 0 to 99 on the Verticle Axis and Values of a From -1 to 1 at intervals of .1 on the Horizontal Axis



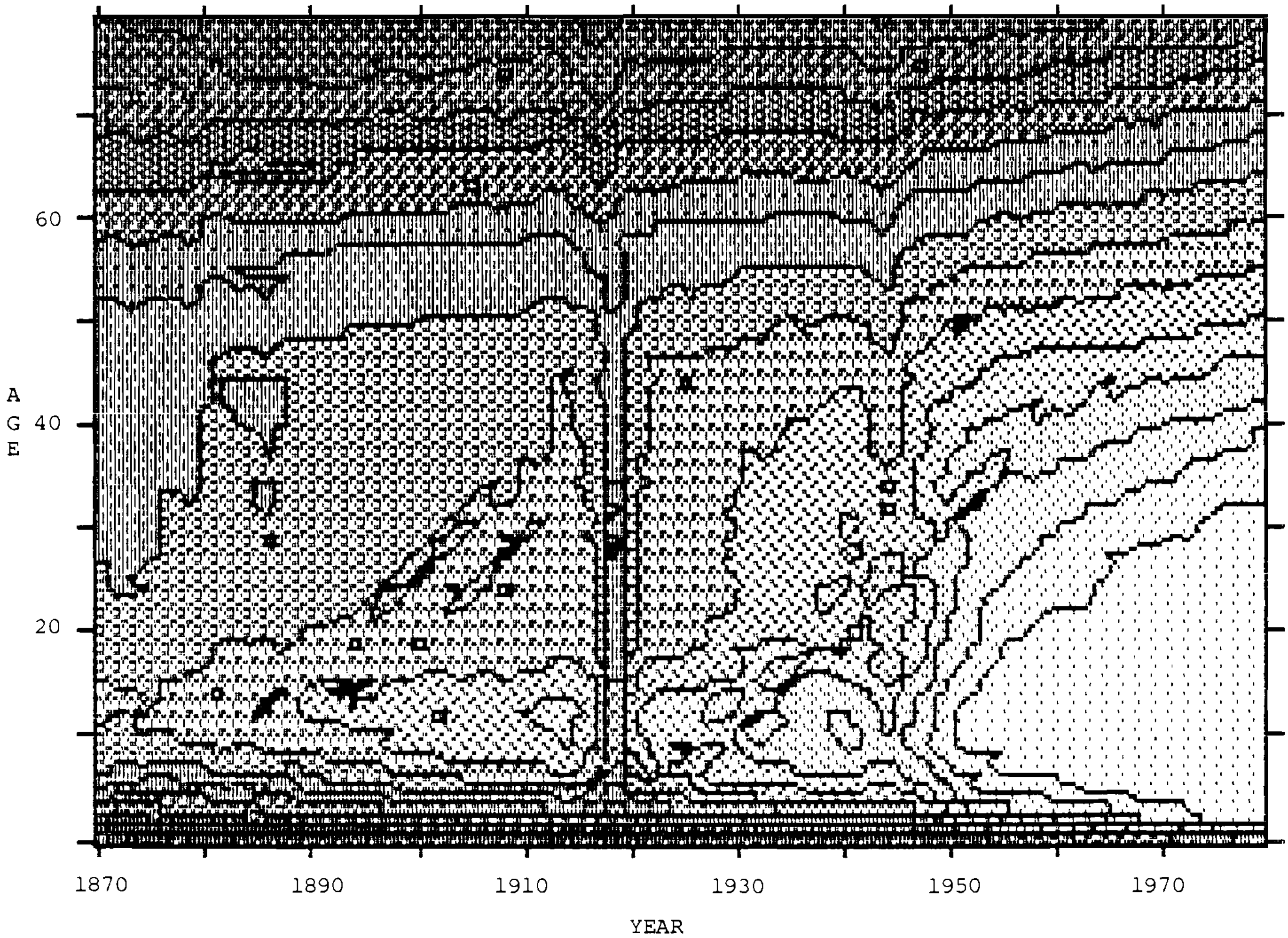
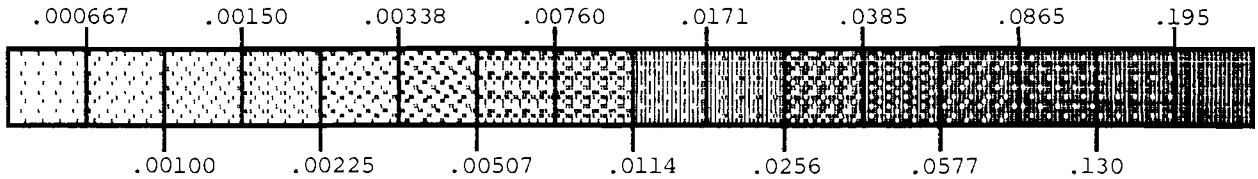


Figure 41a: Italian Female Mortality Rates - with contour lines from .000667 to .195 at multiples of 1.5
From Age 0 to 79 and Year 1870 to 1979



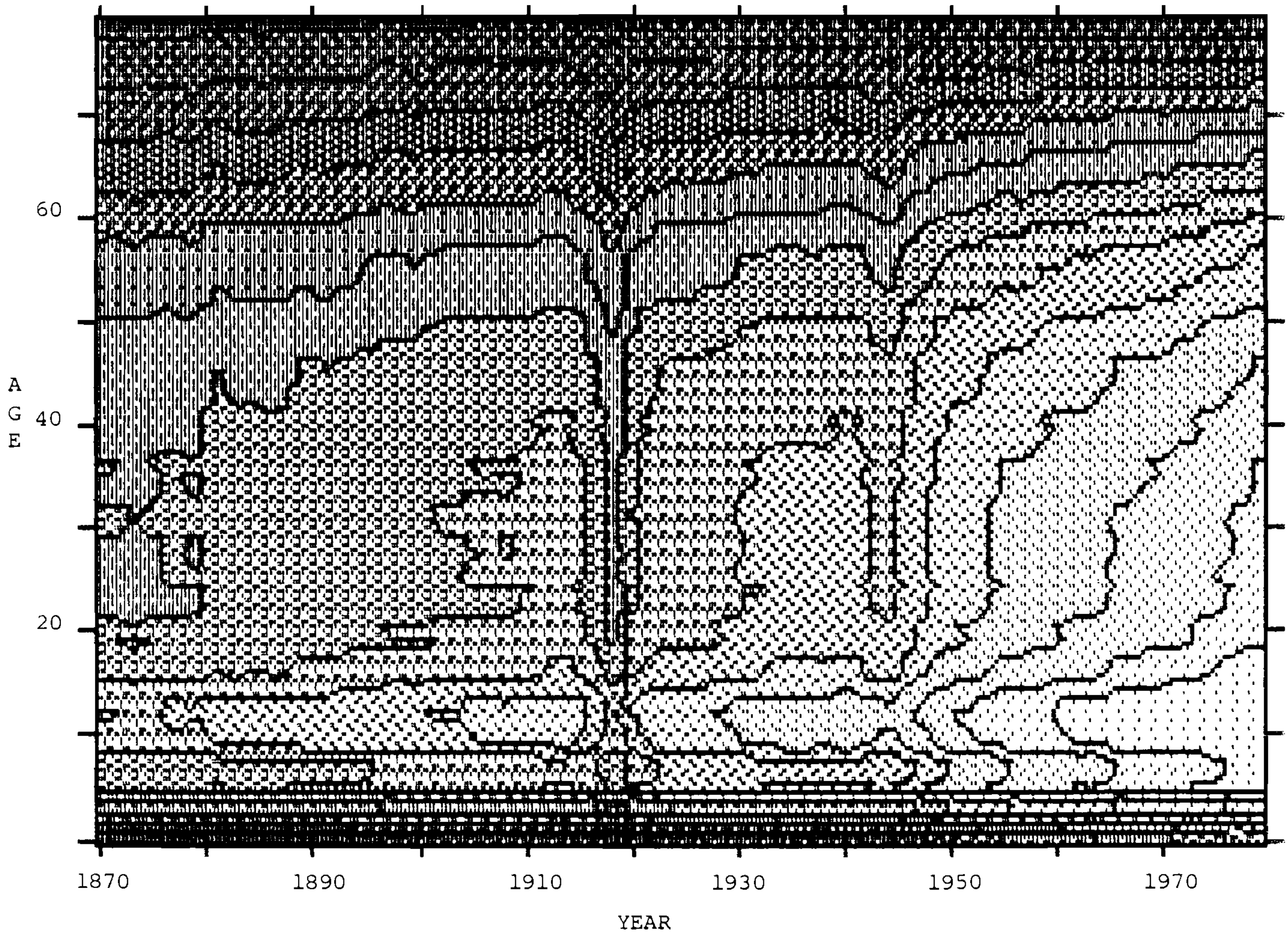
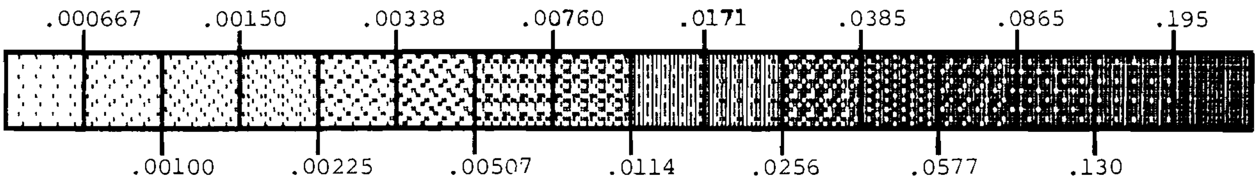


Figure 41b: Italian Female Mortality Rates Given by Brass's Model - with contour lines from .000667 to .195 at multiples of 1.5
From Age 0 to 79 and Year 1870 to 1979



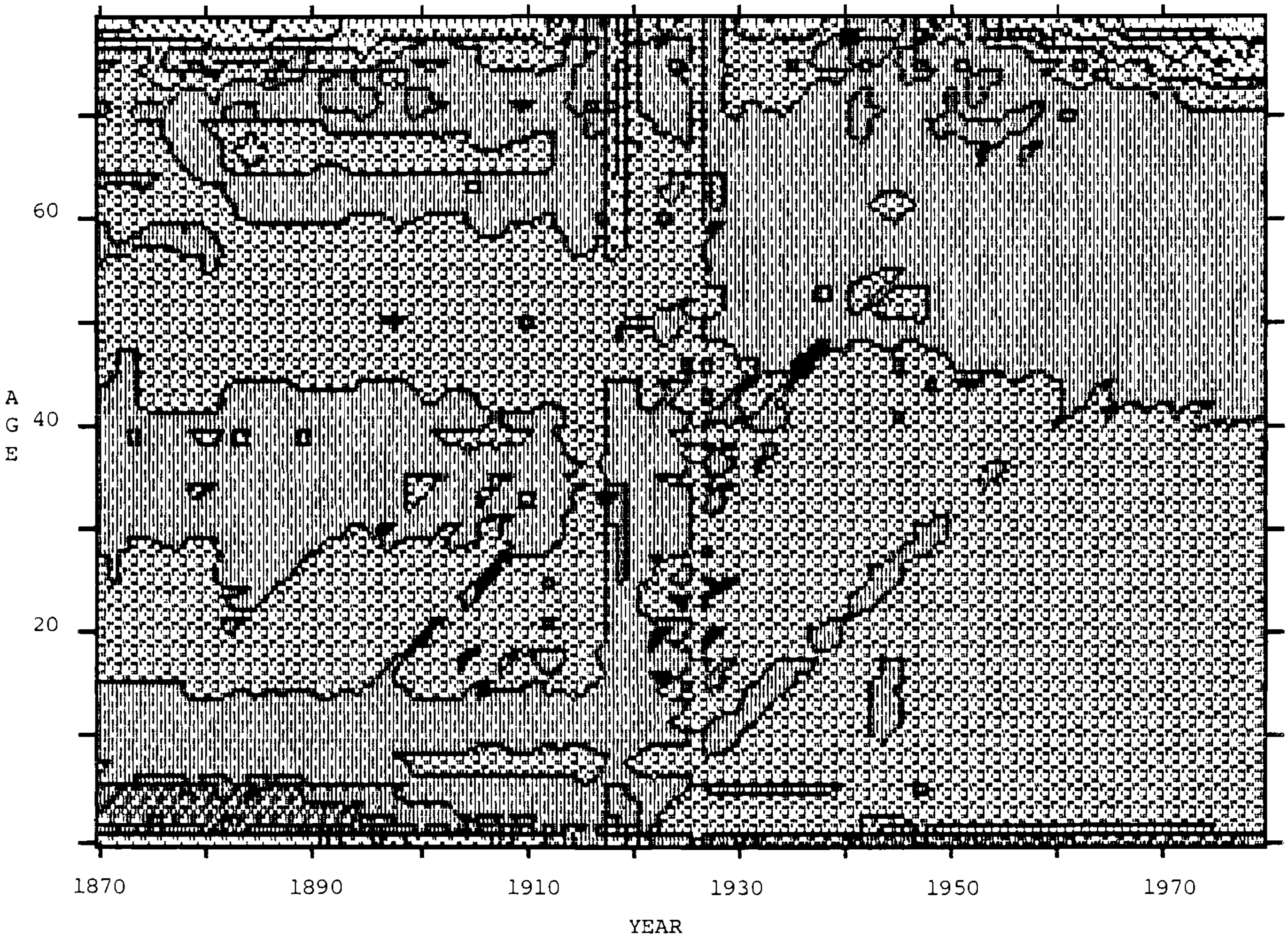
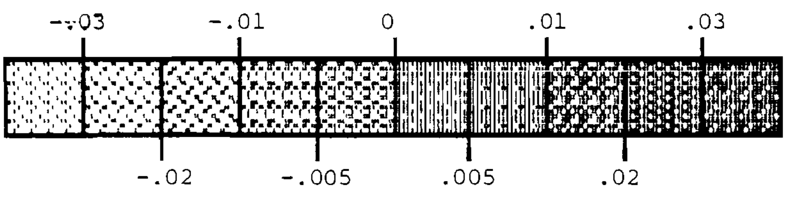


Figure 41c: Residuals From Brass's Model - with contour lines selectively placed from -.03 to .03
From Age 0 to 79 and Year 1870 to 1980



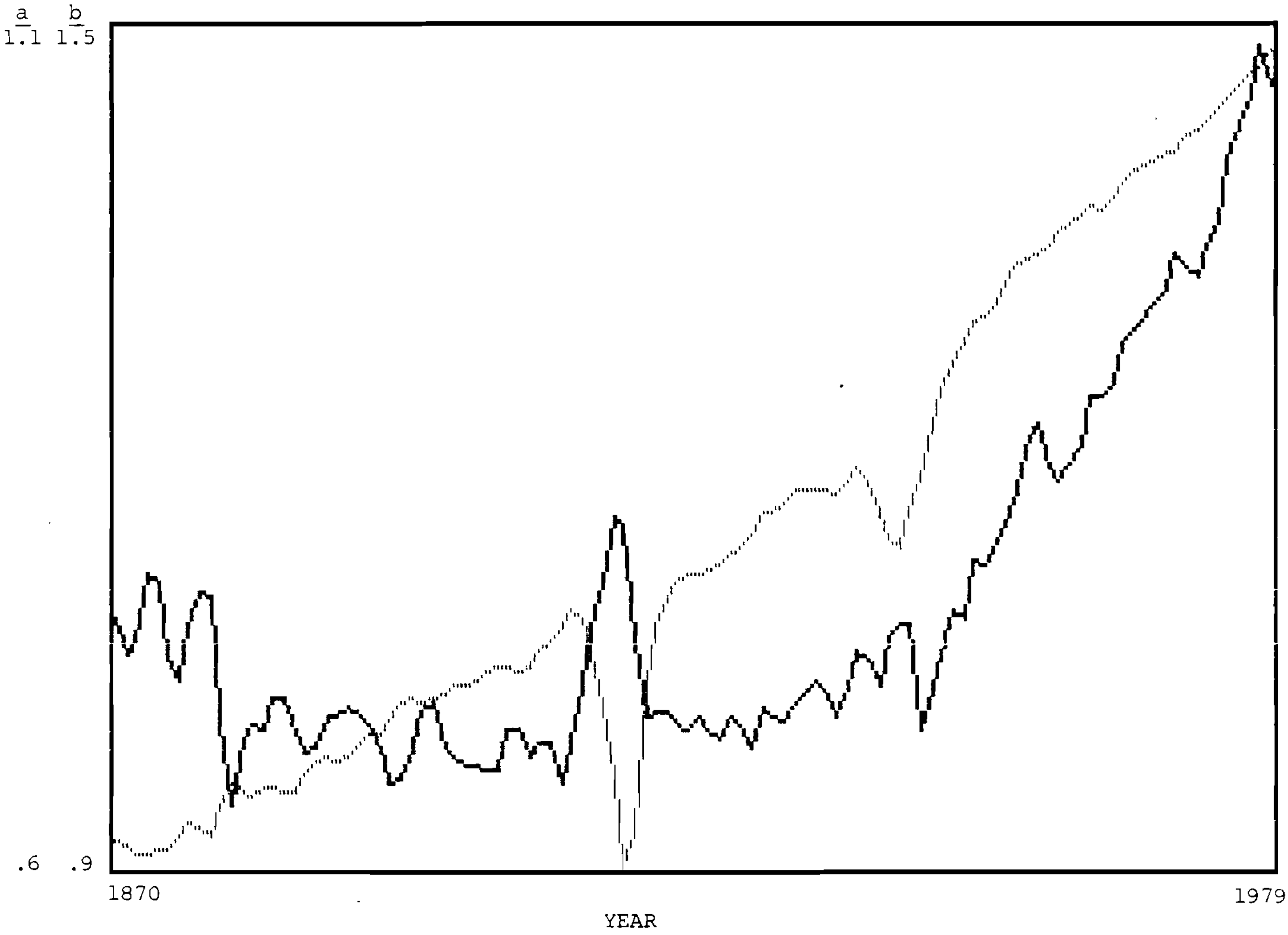


Figure 41d: Plot of a's and b's Used in Producing Italian Female Mortality Rates from Brass's Model - curve b is the darker curve

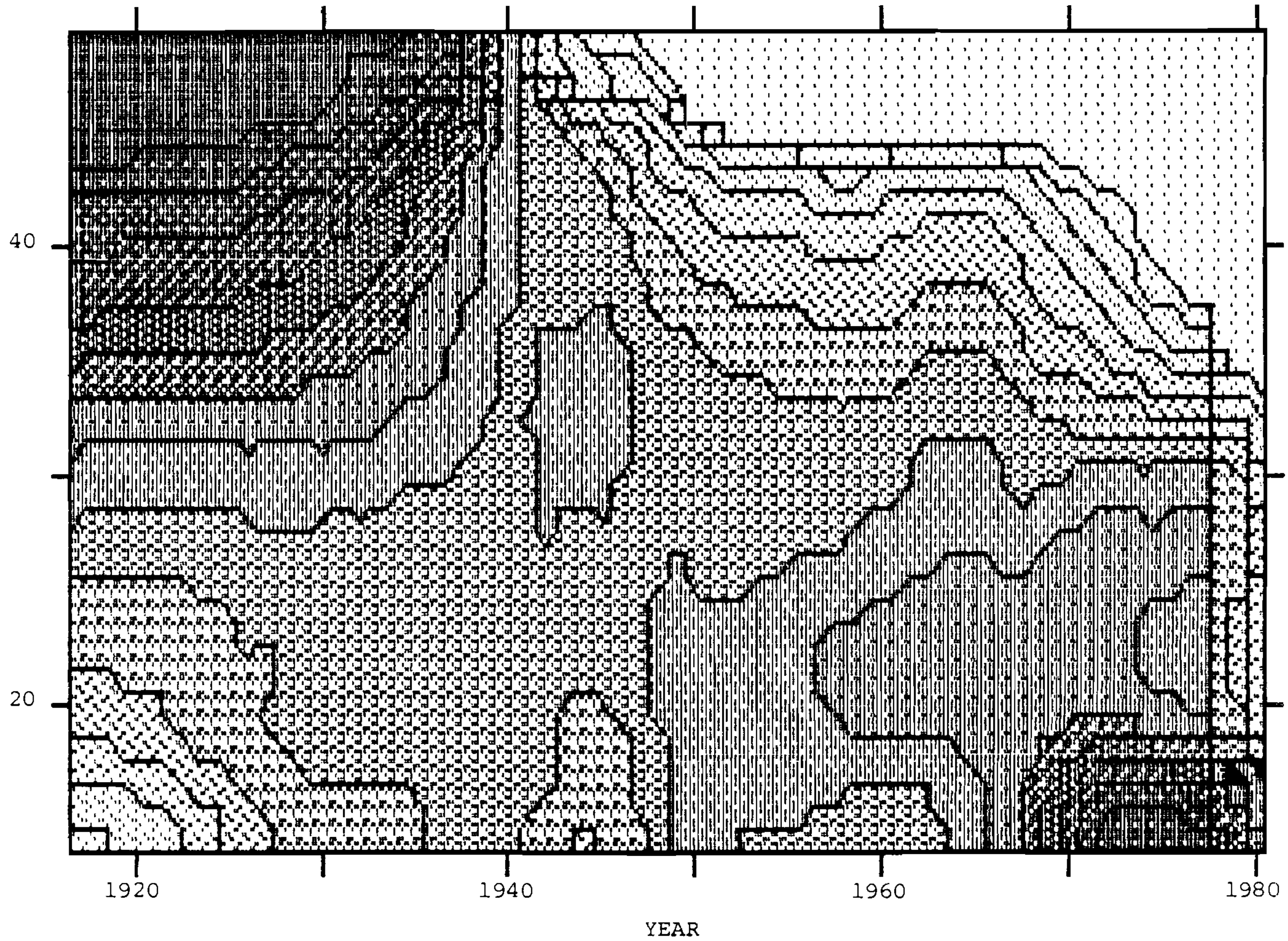
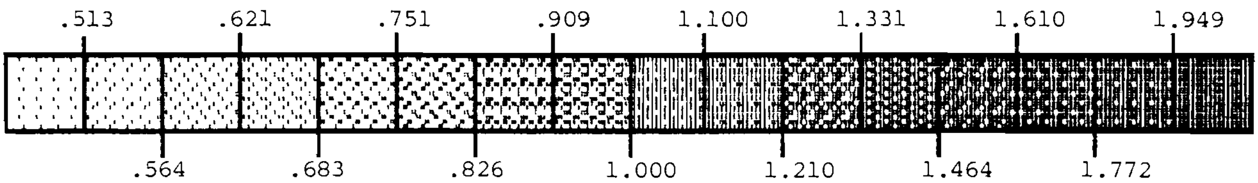


Figure 42a: U.S. Fertility Rates with Age and Period Effects Removed - with contour lines from .51 to 1.95
From Age 14 to 49 and Year 1917 to 1980



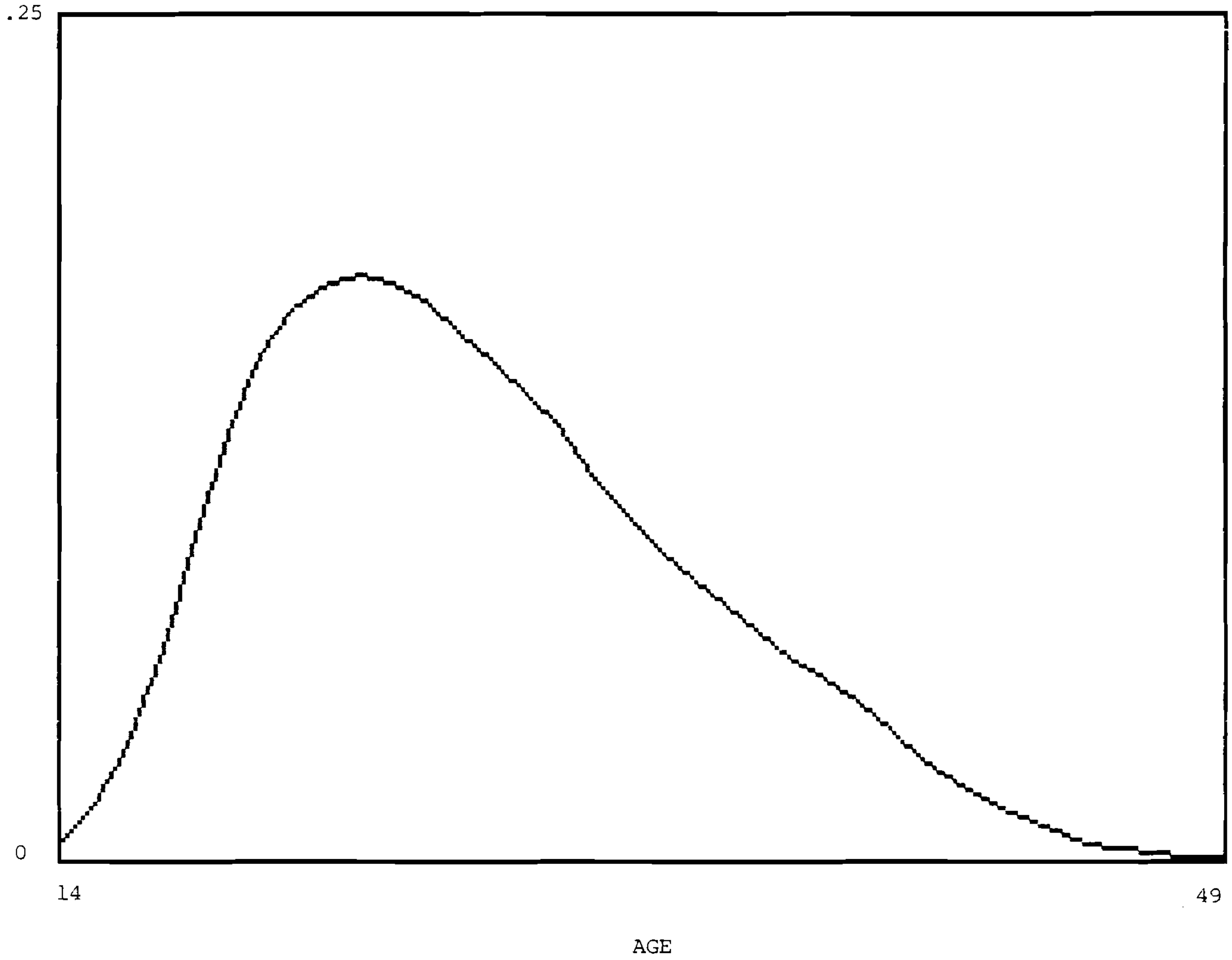


Figure 42b: Average U.S. Fertility From 1917 to 1980 Over Age 14 to 49

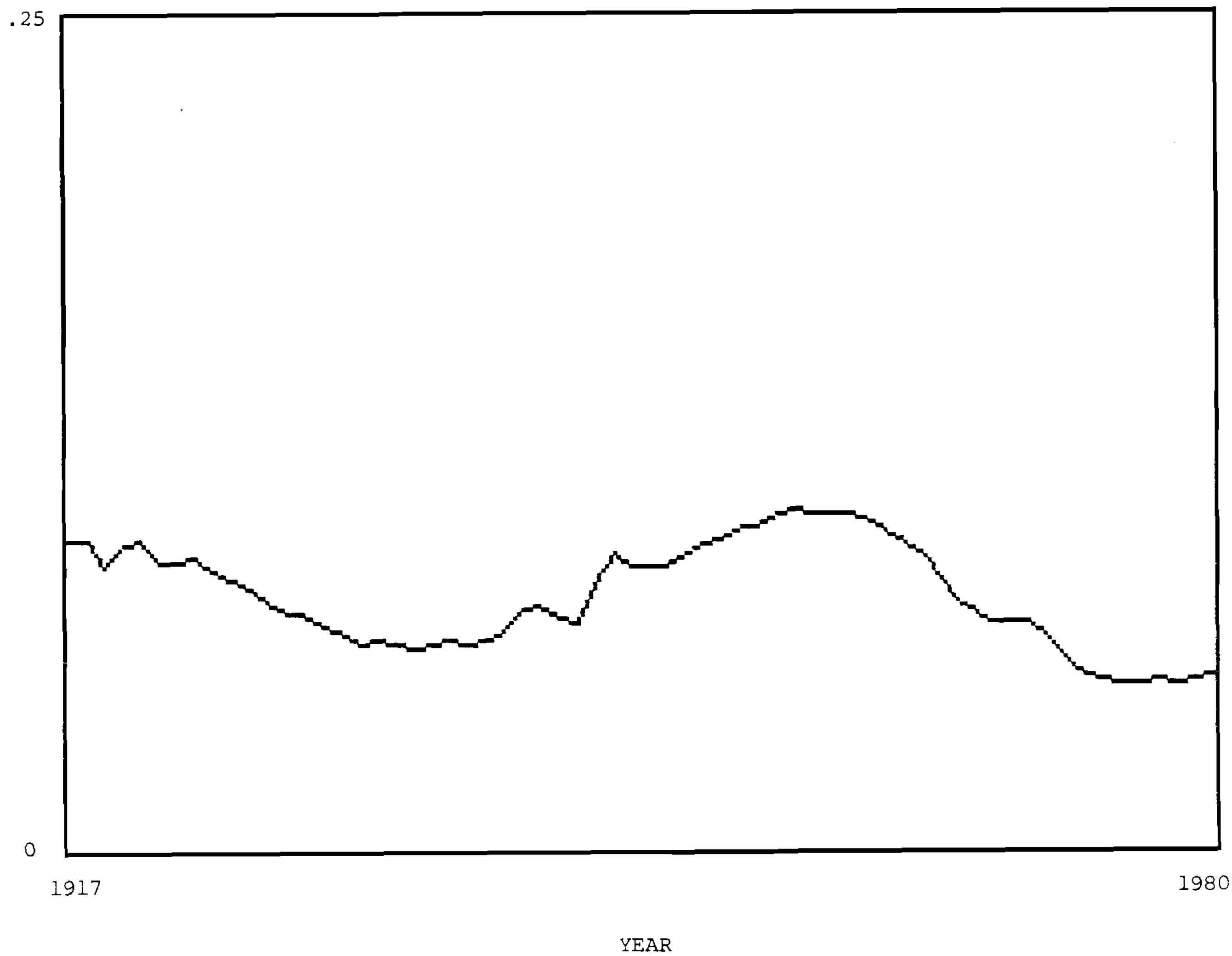


Figure 42c: Average Fertility Rates For Each Year from 1917 to 1980

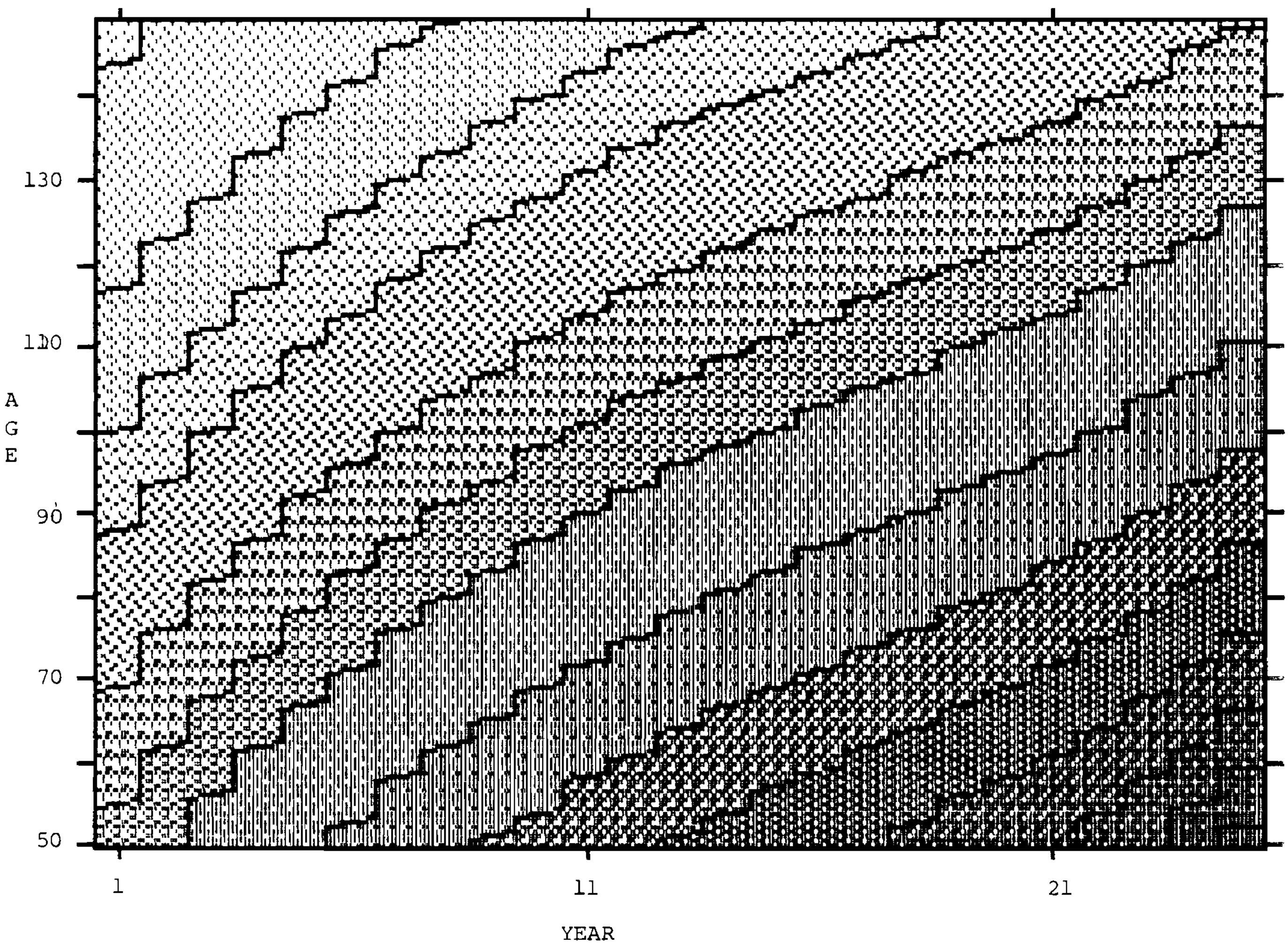
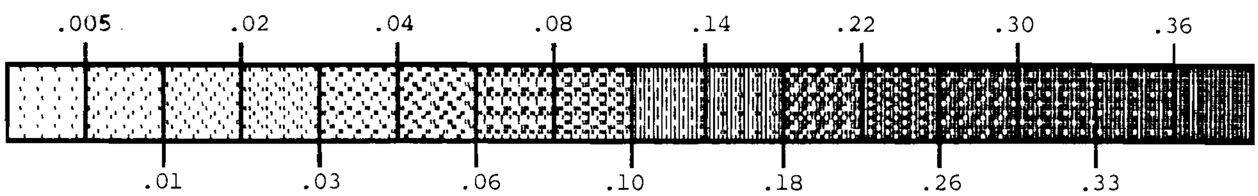


Figure 43: Proportion of People Above Age 65 - with contour lines selectively placed from .005 to .36
From Age 50 to 149 and Year 1 to 25



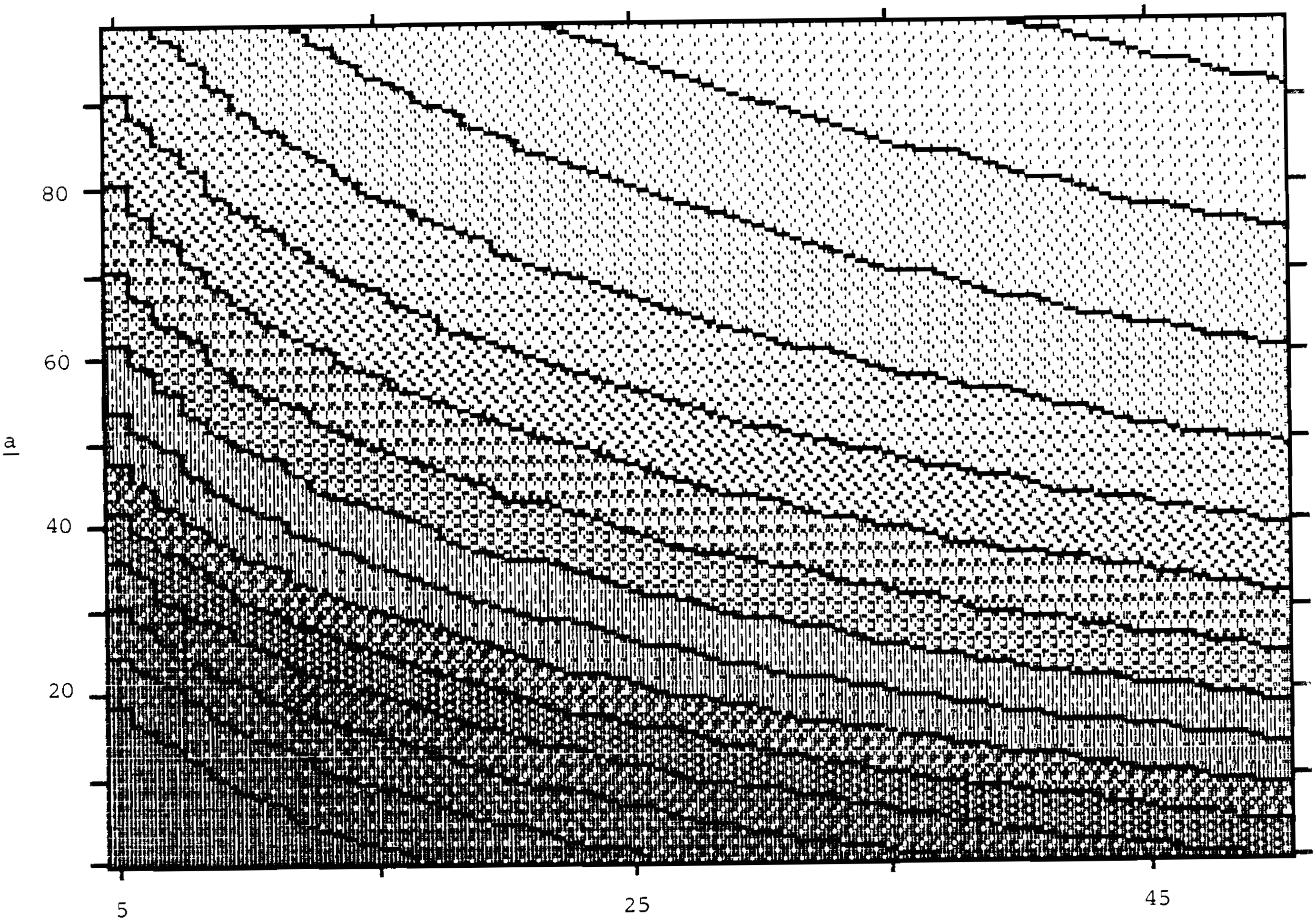
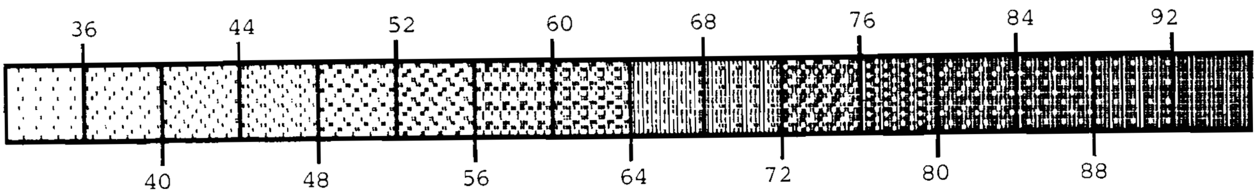


Figure 44: Changes in Life Expectancy at Birth Given by Gompertz Curve as a and b vary - with contour lines from 36 to 92 at selected intervals From a 0 to 99 and b 5 to 50



APPENDIX BY BRADLEY GAMBILL

The computer program used to create the contour maps featured in this paper was the product resulting from frequent interaction between a demographer with some knowledge of computers and a computer programmer with keen interest in demography. Through experimentation and frequent error, Vaupel and I developed what we hope will be a widely used tool for demographers and other analysts. Contours maps offer a simple, effective method for perceiving global patterns from large arrays of data. This appendix gives a brief description of the algorithm used in writing the program and describes a few of the program's capabilities.

The first, and perhaps most important, step in creating demographic contour maps is the selection of line levels. The program allows the user to choose between eight menu options that include many of the lines drawn in this paper as well as an option for entering user-defined lines. Much can be learned by varied placement of the isograms, making the user-entered lines a powerful option.

After line levels are selected, the extraordinary computational capabilities of the personal computer are put to use. As those who have had computer experience know, the choice constantly faced by programmers is memory vs. run-time. Reading in and evaluating data is the only point in the program where I have deliberately sacrificed run time for memory. This allows the program to run on the most inexpensive and widely available computer equipment. Because of this a maximum of 3 time years of data is contained in computer memory at one time (unless the smoothing option is used, in which case there may be up to 10 years). This seems to give the optimal run-time/memory usage combination.

Once the data is contained in memory, desired smoothing or relative rate operations are calculated for the data. After these user-entered transformations are executed, specific data points are replaced with integers representing the contour defined regions, and the original data are discarded.

The graphical representations are created using two separate programs—one does the calculations and creates digital specification files, the other turns the digitalized data into a visual representation of the surface. The separate drawing program allow the surface to be recreated much more quickly from saved specification files than if all calculation and comparison (in some cases as many as 200,000 for a surface) had to be redone.

After converting all specific data to region defined data, the digitalized surface is evaluated in terms of the eight points immediately surrounding it, with unique (or at least exclusive) combinations determining the direction and exact location of the line segment to be drawn. With eight points and sixteen possible regions, we have 8^{16} possible combinations (2.8×10^{14}), making it impractical to check all possible combinations. Instead, the program distinguishes which of the eight points are unequal to the first point, and then which of the eight points are unequal to each other. Then, in order to avoid complete checking of all the thousands of combinations that still exist even under these less restrictive assumptions, general rules and priorities are set. The program avoids redundancy by only drawing straight lines between the current point and points already checked. Also, diagonals drawn from left to right have priority over those drawn from right to left. Note that the surface is not evaluated along contour lines, but simply bottom to top and left to right on the screen, making it especially important to set strict guidelines for drawing the line segments so that all will meet to create the completed level lines.

Some problems result from the assumption that a simple matrix of data forms a continuous surface. We encountered great drops and peaks in sloping data. To alleviate this problem and make the surface maps more informative, the program has been designed to draw horizontal and vertical lines, but not diagonal lines, between the point being tested and the points around it if the surrounding points are not equal. Another problem was drops greater than one contour level from one year to the next. Because the shading differentiates between the various surface levels, drawing more than one line is unnecessarily time-consuming. Also, in some cases, because of low screen resolution, it is impossible to draw all of the lines that would be required of a normal, non-shaded contour map. Thus, the program draws, at most, one line between points in the surface. A third difficulty was deciding the direction of surface flow when the corners of two regions meet in the same place. To handle this, I simply assume that the surface flows from left to right in all such situations.

The actual drawing and displaying of maps is done in two stages. First, the contour lines are drawn from the digitalized data contained in specification files. Then, the surface is painted using either screen colors or printer colors, depending on the final destination of the newly created map. A distinction between colors is necessary because the colors that look the best on the screen do not look the best on the printer, since the lowest priced IBM printers paint only in shades of black and white. The printer colors used in this paper were designed so that as the contour levels increase, there is a proportionate rise in the number of dots per unit.

A few final things should be pointed out about the appearance of the contour maps. Maps that cover a shorter period of time either in the vertical or horizontal direction look somewhat rougher than those covering long periods of time. This is because the program does no interpolating of the given data, but leaves this to the user to do beforehand: this avoids redundant, very time-consuming calculations. Also, it may appear odd that the year and age marks that correspond to the first and last years on the axes do not line up directly with the axes themselves. The reason for this is that each data point occupies not just a point on the surface, but an entire square, and the tick marks along the axes match the very center point of the boxes which the corresponding data points represent.

We hope in the near future to make the contour program available to interested researchers at a nominal charge to cover production costs. Information on receiving a copy of the program for an IBM P.C. (or IBM compatible) can be obtained by writing to: Data Services, Population Program, IIASA, A-2361 Laxenburg, Austria.

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