WORKING PAPER

WHEN IS THE OPTIMAL ECONOMIC ROTATION LONGER THAN THE ROTATION OF MAXIMUM SUSTAINED YIELD?

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ABSTRACT

Contrary to the assertions of many, the rotation which maximizes the net present value of timber receipts may be ***longer*** than the rotation which maximizes average annual physical yield. This circumstance may arise even in a very simple economic model once regeneration costs are recognized. Along with a theoretical comparison of the two rotation criteria, an example using *Pinus patula* plantations in Tanzania demonstrates the potential practical importance of this conclusion.

FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forest policy, forest industrial strategy, and related trade policies.

The key elements of structural change in the forest industry are related to a variety of issues concerning demand, supply, and international trade in wood products. Such issues include the growth of the global economy and population, development of new wood products and of substitute for wood products, future supply of roundwood and alternative fiber sources, development of new technologies for forestry and industry, pollution regulations, cost competitiveness, tariffs and non-tariff trade barriers, etc. The aim of the Project is to analyze the consequence of future expectations and assumptions concerning such substantive issues.

This article represents a background study on timber supply economics. The optimal rotation has been studied in the presence of costs which are nonlinear relative to timber removals. In particular, the effect of fixed regeneration costs for short rotation forestry plantations is analyzed and illustrated with numerical examples.

Markku Kallio
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INTRODUCTION

Can the optimal economic rotation ever exceed the rotation which maximizes the average annual yield of the forest? This question has been discussed for years. Many (e.g. Samuelson, 1976; Clark, 1976; Bentley and Teeguarden, 1964; Hyde, 1980; Chang, 1983 and Andersson and Lesse, 1984) have argued that the optimal economic rotation (the rotation which maximizes the net present value of timber receipts over an infinite planning horizon) approaches the culmination of mean annual increment (or the point of maximum sustained physical yield) only as the discount rate approaches zero. This paper shows that this conclusion is valid only if regeneration and management costs are ignored. Once even a very simple cost formulation is introduced into the problem, economic rotations may equal or exceed the maximum sustained yield rotation.

This result is important from several perspectives. On purely theoretical grounds it is interesting to note that no firm conclusions can be drawn about the best rotation without careful consideration of precise costs and returns arising in a specific situation. From a somewhat more pragmatic point of view, that the economic rotation may be longer than the maximum sustained yield (MSY) rotation gives rise to the "backward bending" long run timber supply curve – the supply curve has negative rather than positive slope.

This paper first summarizes the economic and sustained yield optimization problems. Comparing the solutions to these problems shows how the economic rotation may be longer than the MSY rotation. The most interesting situations arise with fast growing, short rotation species. Consequently we consider an empirical example based on *Pinus patula* plantations in Tanzania. The paper concludes with some comments on the significance of the theoretical and empirical results.

1. TWO ROTATION MODELS

Consider two simple rotation problems. The rotation problems are particularly simple because we take the perspective of stand level optimization. Hence no forest-level constraints or stand interactions will obscure the main point of the analysis.

Economic Rotations

The first model is similar in structure to those used by Chang (1983), Hyde (1980), and Jackson (1980) if management intensity is taken as fixed, and is identical, in terms of the rotation age decision, to the models of

Samuelson (1976) and Clark (1976:257-263).

The economic optimization problem requires five important assumptions.

i. Capital markets are perfect so the forest owner can lend and borrow at a known interest rate i which is constant through time.

ii. Demand for timber is certain and constant so prices equal p/unit for all periods.

iii. Timber yield $v(t)$ per unit area is a known function of stand age t which does not change over time.

iv. The cost per unit area of regenerating the stand is c which does not change over time, and

v. The even-aged forest is regenerated promptly after clearcutting if it is profitable to do so.

Other more complex (and perhaps more realistic) assumptions can produce results similar to those derived below. For example, Dykstra (1985) showed that when unit logging costs decline with tree size, the optimal economic rotation can exceed the MSY rotation. The presence of valuable nontimber forest products can lengthen (Hartman, 1976) or shorten (Bowes et al., 1984) the optimal economic rotation compared to that when net timber receipts alone are considered. A very simple model was adopted here to show that, even for very simple cases, the relationship between the economic and MSY rotation age is ambiguous.

Our assumptions imply that the problem is stationary in the sense that the solution for the first rotation will be identical to the solution for the second and subsequent periods. The economic rotation problem is then

$$\max_t \pi(t) = -c + pv(t)e^{-it} + \pi(t)e^{-it} \qquad 1.1$$

Rearranging terms gives a continuous version of the familiar Faustmann or land expectation model.

$$\max_t \pi(t) = \frac{-c + pv(t)e^{-it}}{1 - e^{-it}} \qquad 1.2$$

The first order optimality condition for t^*, the optimal economic rotation, can be easily found by solving $\frac{d\pi}{dt} = 0$ (see Jackson, 1980; Hyde, 1980, or Chang, 1983)

$$\frac{\dot{v}}{v - \frac{c}{p}} = \frac{i}{1 - e^{-it}} \qquad 1.3$$

where

$$\dot{v} \equiv \frac{dv}{dt} .$$

Maximum Sustained Yield

Consider a forest of unit area. If all age classes are equally represented in the forest, each will occupy an area of $1/t$ where t is the oldest age class or the rotation age. The oldest age class contains a volume of $v(t)$/unit area, and each year this entire age class is harvested. Consequently the annual yield of this unit forest is $v(t)/t$. The maximum sustained yield rotation can then be found by solving the problem

$$\max_t \frac{v(t)}{t} \qquad 1.4$$

The first order condition for t_{MSY} is

$$\frac{\dot{v}}{v} = \frac{1}{t} \qquad 1.5$$

2. COMPARISON OF MSY AND ECONOMIC ROTATIONS

To see the relationship between the economic and MSY rotations, compare the first order conditions for the two cases, equations 1.3 and 1.5, respectively. Figure 1 shows these graphically. To see that the various curves are drawn in proper relationship, first observe that

$$\frac{i}{1-e^{-it}} \geq \frac{1}{t} \quad \text{for all } t \qquad 2.1$$

This can be seen by considering the series expansion of e^{-it}

$$e^{-it} = 1 - it + \frac{(it)^2}{2!} - \frac{(it)^3}{3!} + \cdots \qquad 2.2$$

$$= 1 - it + o(it) \ , \ o(it) \geq 0$$

Substituting into the left hand side of 2.1 gives

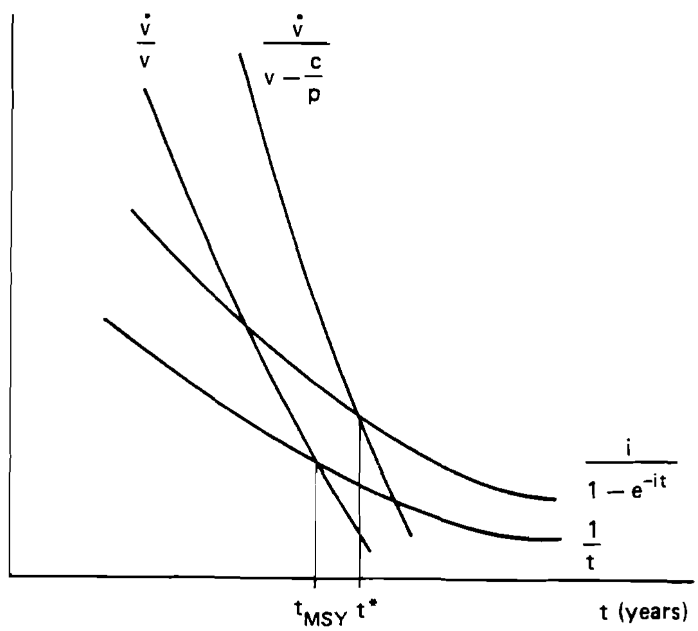

FIGURE 1. Optimal economic and MSY rotations.

$$\frac{i}{1 - 1 + it - o(it)} = \frac{1}{t - \frac{o(it)}{i}} \geq \frac{1}{t} \qquad 2.3$$

From 2.3, we can also see that

$$\lim_{i \to 0} \frac{i}{1 - e^{-it}} = \frac{1}{t} \qquad 2.4$$

Second, if both c and p are positive

$$\frac{\dot{v}}{v - \frac{c}{p}} > \frac{\dot{v}}{v} \qquad 2.5$$

From these arguments, we see that Figure 1 depicts a plausible set of relationships among the first order conditions 1.3 and 1.5. As drawn, c/p is large enough so the optimal economic rotation is greater than the MSY rotation.

Finally, note that

$$\lim_{p \to \infty} \frac{\dot{v}}{v - \frac{c}{p}} = \frac{\dot{v}}{v}, \text{ and} \qquad 2.6a$$

$$\left. \frac{\dot{v}}{v - \frac{c}{p}} \right|_{c=0} = \frac{\dot{v}}{v} \qquad 2.6b$$

This latter case, 2.6b, combined with the limiting behavior noted in 2.4, gives rise to the contention that the optimal economic rotation approaches the MSY rotation as the discount rate approaches zero. However, this assertion is true only if production costs are zero, are in the limit as stumpage prices grow very large.

Having established graphically that the MSY rotation may be shorter than the economic rotation, it is interesting to examine the point at which they coincide. To do so, it is useful to rewrite 1.3 as

$$\frac{\dot{v}}{v} = \frac{i\left[1 - \dfrac{c}{pv}\right]}{1 - e^{-it}} \qquad 2.7$$

Equating the right hand sides of 1.5 and 2.7 gives the relationship among c, p, i and v which must hold in order for the economic and MSY rotation to be identical

$$\frac{c}{p} = v\left[1 - \frac{1 - e^{it}}{it}\right] \qquad 2.8$$

In the case of a competitive economy, we can add the proviso that $\pi \geq 0$ (otherwise the optimal plan is to cut whatever trees are standing and abandon the land), which from 1.1 implies that

$$\frac{c}{p} < ve^{-it} \qquad 2.9$$

The term c/p can be eliminated from the problem by equating the right hand sides of 2.8 and 2.9 . This gives the limiting values of i and t where the economic rotation exceeds the MSY rotation but $\pi > 0$. Rearranging the resulting equation gives

$$(1 - e^{-it})(it - 1) = 0 \qquad 2.10$$

Which implies either

$$1 - e^{-it} = 0 \qquad 2.11a$$

or

$$it - 1 = 0 \qquad 2.11b$$

In the first case, either $t = 0$ or $i = 0$. In the second, more interesting case, $i = 1/t$. Thus for economic rotations to be greater than the MSY rotation, the interest rate must be less than the inverse of the MSY rotation. For comparatively slow growing species such as Douglas-fir, the white pines or many temperate hardwoods, the MSY rotation will be on the order of 100 years. In this case, 2.11b implies a maximum discount rate of 1/100 or $i = 0.01$, and the situation described here is of little practical importance. On the other hand, fast growing species such as the southern pines (particularly on high sites managed for cubic rather than board foot production) or tropical forest plantations of species such as *Pinus patula*, *Pinus caribaea* or *Gmelina arborea* may reach the culmination of mean annual increment in less than 20 years. In such situations the optimal economic rotation might be greater than MSY under a wide range of cost and price parameters.

3. AN EXAMPLE: *PINUS PATULA* PLANTATIONS IN TANZANIA

To give a better sense of the practical importance of this situation, equations 2.8 and 2.9 were computed for high site (33 m at age 20) *Pinus patula* plantations at Sao Hill in southern Tanzania. This fast growing pine, native to Mexico, has been widely planted throughout East Africa. Plantation yields at Sao Hill are similar though not identical to the yields of P. patula plantations elsewhere in Africa, including Kenya, Malawi and Uganda (Adegbehin, 1982). For analytical convenience, Adegbehin's (1982) yield estimates were fitted to a two-parameter yield function:

$$\begin{array}{llll} \ln[v(t)] = & 7.42 & - \ 15.5/t & R^2 = 0.989 \\ & (73.4) & (-32.7) & n = 17 \end{array} \qquad 3.1$$

where $v(t)$ refers to the total stand volume, outside bark, in m^3/ha. The numbers in parentheses are t-statistics for the null hypothesis that the coefficient equals zero.

The Durbin-Watson statistic for the initial ordinary least squares estimate of this model suggested the presence of positive serial correlation among the residuals, a condition which renders the estimates inefficient but unbiased. Consequently, the estimates in 3.1 were obtained by an iterative generalized least squares procedure where in each step the estimated lag-one serial correlation coefficient is used to correct the variance/covariance matrix of the GLS estimates until satisfactory convergence of the estimated coefficients is obtained.

Figure 2 graphs 2.8 and 2.9 is a function of the discount rate i for this yield curve. The two equations divide the graph into four areas. In regions I and II, the c/p ratio is high enough so $t^* > t_{MSY}$. In region II c/p is so high that timber production is unprofitable. Unless subsidized, we would expect no production at all for these combinations of c/p and i. In regions III and IV, the c/p ratio is low enough so that $t^* < t_{MSY}$. The two curves cross at $i = 1/t$, or at $i = 0.065$. That is, at interest rates greater than $i = 0.065$ the optimal economic rotation will always be less than the MSY rotation.

At a discount rate of $i = 0.04$, c/p ratios between 157 and 332 lead to $t^* > t_{MSY}$. As a point of reference, in 1980 $c = 3171$ shs/ha (Kowero, 1984), which gives a possible range in prices of 9.6-20.2 shs/m^3. In 1980, the royalty for P. patula stumpage in this size range was 20 shs/m^3 (Dykstra, 1985). Consequently, this simple economic model suggests that, at a discount rate of $i = 0.04$, the optimal economic rotation for these plantations would be greater than the age of maximum sustained yield.

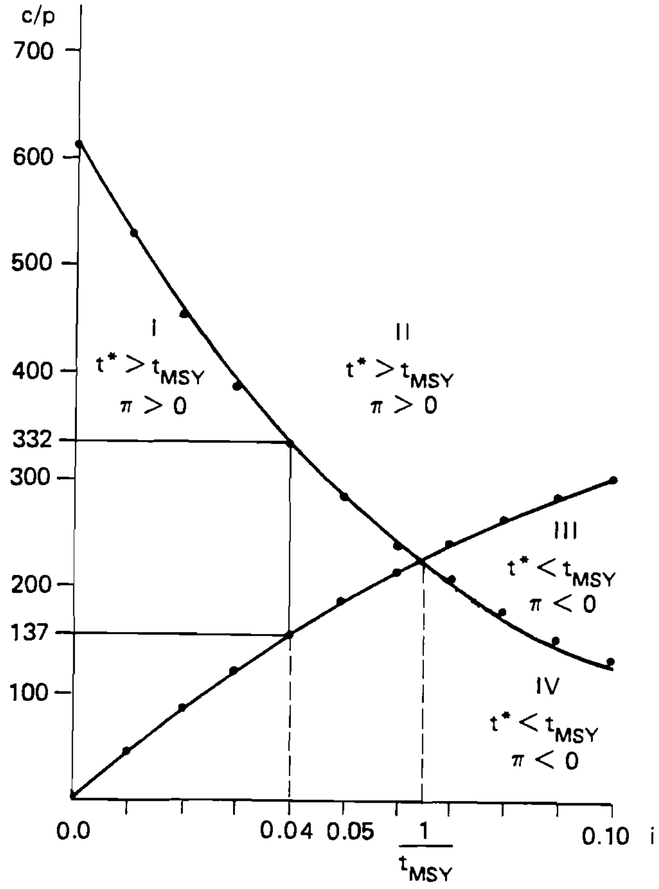

FIGURE 2. $t^* > t_{MSY}$, *Pinus patula*.

4. CONCLUSIONS

In general, economic rotations may be greater than, equal to, or less than the rotation which maximizes the sustained physical output of the forest. The relationship between the MSY and economic rotations depends on the yield function for the species in question, the management costs, the stumpage price and the interest rate. In practice, economic rotations exceeding MSY rotations are more likely to arise with fast than with slow growing species.

The relationship between the economic and MSY rotation is important for understanding long run timber supply. It is well-known that an increase in stumpage price will lead to a decrease in the optimal rotation (differentiate 1.3 with respect to t to see this, or see Chang, 1983:271). If prices are high enough so the economic rotation is shorter that MSY, then the reduction in rotation attending a price increase will lead to a ***lower*** level of average output. The long run timber supply curve will thus bend backward. If high production costs, high interest rates and low growth conspire to make profits negative at any rotation longer than MSY, no part of the long run supply curve will have a positive slope. Binkley (1985) gives a more complete account of the long run timber supply model.

A more realistic model of forest management would relax the assumptions of the present analysis. For example, prices would respond to changing supply/demand balances. Management costs would change in response to changes in labor and other factor markets. Yields might change over time as biological knowledge accumulates or as environmental degradation takes its toll. The nontimber products of the forest would be recognized in the economic optimization. These complications will surely upset the simple comparisons between economic and MSY rotations presented here but are not likely to alter the ambiguous relationship between the two approaches.

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