ESTIMATION OF INPUT-OUTPUT COEFFICIENTS USING NEOCLASSICAL PRODUCTION THEORY

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PREFACE

Many of today's most significant socioeconomic problems, such as slower economic growth, the decline of some established industries, and shifts in patterns of foreign trade, are inter- or transnational in nature. But these problems manifest themselves in a variety of ways; both the intensities and the perceptions of the problems differ from one country to another, so that intercountry comparative analyses of recent historical developments are necessary. Through these analyses we attempt to identify the underlying processes of economic structural change and formulate useful hypotheses concerning future developments. Our research concentrates primarily on the empiricial analysis of interregional and intertemporal economic structural change, on the sources of and constraints on economic growth, on problems arising from changing patterns of international trade, resource availability, and technology.

The aim of this paper, which was presented at the last Input-Output Modeling Task Force Meeting and is therefore limited to 11 pages, was to combine well-known theoretical approaches from the theory of production and to apply them to a data base drawn up within the framework of modern input-output statistics. The changes in I/O coefficients observed for the Canadian basic metal industry are attributed to changes in microtechnologies brought about by shifts in the relative prices and the output structure of this industry.

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1. INTRODUCTION

Over the years there has been much research and investigation into the question of change in input–output (IO) coefficients, which lie at the heart of any IO model. This research has taken many productive directions. Besides technical progress, two main reasons for changes in IO coefficients have been identified:

- Input factor substitution (including substitution of domestic products by imported commodities) caused by changes in the input price system (price effects), and
- Changing output structures of the industries concerned (product-mix effects).

An extensive literature exists on price effects: Tilanus (1966) concluded that the classical assumption of IO analysis, namely that value coefficients are constants, is less workable than the hypothesis that value coefficients (cost shares) are stable. Klein (1952) proved that this hypothesis requires a multiproduct Cobb–Douglas function. Using recent production theory, much more flexible assumptions were used by Frenger (1978), Bonnici (1983), Nakamura (1984), and Andersson ***et al.*** (1984), by applying Diewert (generalized Leontief) production or cost functions to IO data. Frenger (1978) analyzed the price-responsiveness of IO coefficients for textiles, construction, and metals and concluded that "there would seem to be little doubt that the Leontief assumption would have to be rejected ... relative prices have a significant effect on the viability of IO coefficients". Bonnici (1983) estimated a complete set of price-dependent IO coefficients derived from corresponding Diewert cost functions for all 17 sectors covered by a time series of annual IO tables. A comparison of the traditional method (forecasting on the basis of the coefficients from the most recent year available) with the generalized Leontief model showed that "... the forecasts of the generalized Leontief model outperform those of the (common) IO model in two out of every three cases". Contrary to Tilanus, Bonnici concluded that, whenever a time series of IO tables is available, there is considerable scope for relaxing the somewhat rigid assumption of fixed IO coefficients.

Another body of literature is devoted to product-mix effects. Here, the idea is that changes in the input coefficients of aggregate industries are attributable to changes in the industries' internal output profiles rather than to shifts caused by changes in the production processes.

Sevaldson (1960) wrote in the introduction to the 1954 Norwegian IO tables: "Lack of sector homogeneity makes product mix the dominant source of changes in the coefficients". A cross-sectional analysis on an establishment level by Forssell (1969), for six fairly homogeneous industry groups, showed that two-thirds of the explained dispersion of input coefficients among establishments could be attributed to heterogeneity in commodity mix while just one-third was found to be due to replacement of particular inputs by other commodities. Lager (1983) analyzed the changes in the energy coefficients of five of the most energy-intensive sectors in Austria and found that explicit consideration of product-mix effects produced a significant decline in the price elasticities. This result might encourage the assumption that changes in the input price system lead not only to changes in the (micro)technologies involved but also to remarkable effects on the output structure, and therefore that they contribute in two ways to changes in the technical coefficients of industry groups. However, it is generally agreed that changes in technology as well as shifts in production structure have explanatory power for estimating changes in the input coefficients. Consequently, emphasis on product-mix effects leads to rather large IO tables and disaggregated, but simple, models. On the other hand, the introduction of factor substitution implies flexible production functions and more or less aggregated, but complicated, modeling.

2. GENERAL CONSIDERATIONS

The aim of this study is to contribute to this "trade off" in such a way that both product-mix effects and factor substitution caused by changes in prices can play a role in explaining shifts in IO coefficients. This approach has been supported and stimulated by recent developments in the availability and structure of IO statistics: more and more IO tables are now compiled according to the concepts of the System of National Accounts (SNA) 1968 (UN 1968). Industrial interactions are described by two matrices: the ***make matrix*** shows the production of commodities by industries while the ***use matrix*** shows the demand of industries by commodities. The demand of an industry for a certain commodity (x_i) can be specified as

$$x_i = \sum_k a_{ik} \cdot Q_k$$

where Q_k is the volume (value at constant prices) of commodity k produced in that industry and a_{ik} is the input coefficient, which specifies the requirement of input i for output k.

If we assume that the input coefficients a_{ik} are functions of the input price indices $p = (p_1, p_2, \ldots, p_n)$, we can relate the changes in the industrial input requirements to changes in the price system ***and*** to changes in the production structure:

$$x_i = \sum_k a_{ik}(p) \cdot Q_k$$

If we choose a flexible functional form for the input coefficients $Q_{ik}(p)$ that allows for changes in the substitution elasticities, we would soon have problems associated with the estimation of too many parameters. A typical problem with the estimation of a sophisticated production function is that the observations are frequently not well distributed over the complete possibility set, but are grouped in clumps close together. This makes it very difficult to distinguish between different functional forms. Statistically speaking, one must also be very careful with the number of degrees of freedom assigned to a given problem, and it should be remembered that it is hard to separate very similar effects by using statistical analysis.

Therefore, following recent production theory, we will define a multi-input/multi-output technology for a whole industry and then try to derive micro demand functions for single commodities.

Suppose that an industry faces a series of competitive input markets with given input prices $[p = (p_1, p_2, \ldots, p_n)]$. Suppose further that there exists a technologically determined input requirement set that determines inputs for each exogenously determined (e.g. by demand, capacity) set of producible outputs $[Q = (Q_1, Q_2, \ldots, Q_m)]$. The cost function[1] for the industry is then defined by

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[1]Instead of using cost functions we could also use a profit function that relates profits to input as well as output prices.

$$C = C(p,Q)$$

and specifies the least cost of producing the output bundle Q at given input prices p. (For the sake of simplicity, technical progress is ignored here.)

Further, we assume that the technology used for a single product is in no sense related to the production processes for other commodities produced in the same industry. For example, the input requirements for steel products do not depend on the quantity of aluminum produced in the same industry. Therefore, for any individual output Q_k, a separable, non-joint, single-output cost function can be specified:

$$C_k = C_k\ (p, Q_k)$$

The total cost of production is then simply

$$C = \sum_k C_k\ (p, Q_k)$$

In addition, we assume *linear homogeneity* for all commodity cost functions and therefore

$$C = \sum_k g_k\ (p) \cdot Q_k$$

Using Shephard's Lemma, $x_i = \partial C / \partial p_i$, we obtain again

$$x_i = \sum_k a_{ik}\ (p) \cdot Q_k$$

where

$$a_{ik}\ (p) = \frac{\partial g_k\ (p)}{\partial p_i}.$$

Therefore, the input coefficients for a multi-product technology can be derived from a linear-homogeneous, non-joint cost function:

$$a_{ik}\ (p) = \frac{\partial^2 C(p,Q)}{\partial q_k\ \partial p_i}.$$

3. THE TRANSLOG COST FUNCTION WITH LINEAR HOMOGENEITY IN THE INPUT PRICES AND CONSTANT RETURNS TO SCALE

To test the restrictions described in Section 2 we start with a more general approach. Thus, we define a production possibility frontier that does not imply non-jointness or constant returns to scale *a priori*, but that does enable us to apply statistical tests to these restrictions. For this purpose we choose the translog function introduced by Christenson *et al.* (1973), which is a second-order approximation of any function.

We approximate the cost function at $p_i = 1$, $Q_k = 1$ by:

$$\ln (C) = \alpha_0 + \sum_{i=1}^{i=n} \alpha_i \ln(p_i) + \sum_{k=1}^{k=m} \beta_k \ln(Q_k) + \frac{1}{2} \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} \gamma_{ij} \ln(p_i) \ln(p_j)$$

$$+ \frac{1}{2} \sum_{k=1}^{k=m} \sum_{l=1}^{l=m} \vartheta_k \ln(Q_k) \ln(Q_l) + \sum_{i=1}^{i=n} \sum_{k=1}^{k=m} \delta_{ik} \ln(p_i) \ln(Q_k)$$

The parameters of the translog function equal the first- and second-order derivatives at the point of expansion:

$$\alpha_0 = \ln C_0 \ , \ \alpha_i = \frac{\partial \ln C}{\partial \ln p_i} \ , \ \beta_k = \frac{\partial \ln C}{\partial \ln Q_k}$$

$$\gamma_{ij} = \frac{\partial^2 \ln C}{\partial \ln p_i \partial \ln p_j} \ , \ \vartheta_{kl} = \frac{\partial^2 \ln C}{\partial \ln Q_k \partial \ln Q_l} \ , \ \delta_{ik} = \frac{\partial^2 \ln C}{\partial \ln p_i \partial \ln Q_k}$$

Symmetry of the second-order derivatives requires that $\gamma_{ij} = \gamma_{ji}$ and $\vartheta_{kl} = \vartheta_{lk}$.

One usual condition for a cost function is linear homogeneity in input prices. It is easy to prove that this requires that

$$\sum_{i=1}^{i=n} \alpha_i = 1, \ \sum_{i=1}^{i=n} \gamma_{ij} = 0, \ \sum_{k=1}^{k=m} \delta_{ik} = 0.$$

Constant returns to scale requires linear homogeneity in the outputs. Thus, taking the symmetry restriction and linear homogeneity into account, we obtain an additional set of restrictions:

$$\sum_{k=1}^{k=m} \beta_k = 1, \ \sum_{k=1}^{k=m} \vartheta_{kl} = 0, \ \sum_{i=1}^{i=n} \delta_{ik} = 0,$$

and

$$\sum_{j=1}^{j=n} \gamma_{ij} = 0, \ \sum_{l=1}^{l=m} \vartheta_{kl} = 0.$$

4. NON-JOINTNESS RESTRICTION ON THE TRANSLOG COST FUNCTION

As described in Section 2, non-joint production requires a cost function of the type:

$$C = \sum_k C_k(p, Q_k).$$

Consider the first- and second-order derivatives of this general non-joint cost function:

$$\frac{\partial \ln C}{\partial \ln Q_k} = \frac{1}{C} \frac{\partial C_k}{\partial \ln Q_k}, \quad k = 1,2,\dots,m,$$

$$\frac{\partial^2 \ln C}{\partial \ln Q_k \cdot \partial \ln Q_l} = -\frac{1}{C^2} \frac{\partial C_k}{\partial \ln Q_k} \frac{\partial C_l}{\partial \ln Q_l}, \quad k \neq l, \ k,l = 1,2,\dots,m.$$

The first- and second-order derivatives of $\ln C$ equal the parameters β_k and ϑ_{kl} at the point of expansion. Consequently, non-jointness requires

$$\vartheta_{kl} = -\beta_k \cdot \beta_l \quad \text{for } all \ k,l, \ k \neq l.$$

As described above, the translog function is a second-order approximation of the cost function at a point of expansion. Consequently, this restriction defines non-jointness only around that point of expansion.

5. HOW TO ESTIMATE THE TRANSLOG COST FUNCTION

Using Shephard's Lemma we obtain a system of n cost-share equations

$$s_i = \frac{\partial \ln C}{\partial \ln p_i} = \alpha_i + \sum_{j=1}^{j=n} \gamma_{ij} \ln(p_j) + \sum_{k=1}^{k=m} \delta_{ik} \ln(Q_k)$$

where

$$s_i = P_i \frac{X_i}{C}$$

The use of the share equations makes it possible to justify the parameter restrictions that arise from the imposition of linear homogeneity. Since the sum of all the shares must be one, and the linear homogeneity and symmetry constraints are used, only $n-1$ equations remain to be estimated. The last equation depends on the others, and must be calculated from them.

The share equations described above do not permit the estimation of the complete cost function. In order to estimate the parameters ϑ_{kl} and β_k, we need to define additional equations. The cost function itself can be used to get the missing parameters. The other way out is to specify an output price rule.

If we assume that the manufacture of each product breaks even, we can relate total costs to total outputs, $C_k = p_k Q_k$. Therefore the nominal product-mix coefficient is defined as $v_k = C_k / C$.

Non-jointness requires that

$$\frac{\partial \ln C}{\partial \ln Q_k} = \frac{1}{C} \frac{\partial C_k}{\partial \ln Q_k} = \frac{C_k}{C} \frac{\partial \ln C_k}{\partial \ln Q_k}$$

Constant returns to scale in the micro cost function C_k yields

$$\frac{\partial \ln C_k}{\partial \ln Q_k} = 1$$

Consequently,

$$\frac{\partial \ln C}{\partial \ln Q_k} = \frac{C_k}{C} \equiv v_k$$

This enables us to define an additional, estimatable set of m nominal product-mix equations, which now include the parameters ϑ_{kl} and β_k :

$$v_k \equiv \frac{C_k}{C} = \beta_k + \sum_{l=1}^{l=m} \vartheta_{kl} \ln q_l + \sum_{i=1}^{i=n} \delta_{ik} \ln p_i$$

From the nominal product-mix equations v_k we obtain micro cost functions C_k. Applying Shephard's Lemma, we can devise demand equations for each single output k:

$$x_{ik} = \frac{C}{p_i} \left[\delta_{ik} + v_k \cdot s_i \right] \ .$$

Dividing x_{ik} by Q_k, we can obtain commodity-by-commodity IO coefficients.

6. PRICE AND SUBSTITUTION ELASTICITIES

Here we begin by defining the price elasticity of input demand as the percentage change in input x_i when the input price p_j changes by one percent

$$\varepsilon_{ij} = \frac{\partial \ln(x_i)}{\partial \ln(p_j)},$$

$$Q = \text{constant}, \ p_i = \text{constant, for } i \neq j.$$

Next, the Allen elasticities of substitution are defined as follows:

$$\sigma_{ij}^A = \frac{C \cdot C_{ij}}{C_i \cdot C_j}, \quad C_i = \frac{\partial C}{\partial p_i}, \ C_{ij} = \frac{\partial^2 C}{\partial p_i \partial p_j}$$

The Allen elasticities are symmetric, $\sigma_{ij}^A = \sigma_{ji}^A$.

Using Shephard's Lemma, a relation can be obtained between the Allen elasticities and the price elasticities:

$$s_i \cdot \sigma_{ij}^A \ = \ s_i \cdot \frac{C(\partial x_i / \partial p_j)}{x_i x_j} = \frac{(\partial x_i / \partial p_j)}{(x_i / p_j)} = \varepsilon_{ij}$$

One of the major advantages of the translog function is that the elasticities ε_{ij} and σ_{ij} are not, *a priori*, constant but depend on the cost shares. To obtain the explicit derivation, it is best to compute the Allen elasticities first. The use of the translog cost function yields:

$$\sigma_{ij}^A \ = \ \begin{cases} \dfrac{\gamma_{ij} + s_i s_j}{s_i s_j} & \text{for } i \neq j \\ \dfrac{\gamma_{ii} + s_i^2 - s_i}{s_i^2} & \text{for } i = j \end{cases}$$

Having computed the Allen elasticities, the fundamental relation shown above can be used to obtain the price and substitution elasticities:

$$\varepsilon_{ij} = s_i \sigma_{ij}^A \ = \ \begin{cases} \dfrac{\gamma_{ij} + s_i s_j}{s_j} & \text{for } i \neq j \\ \dfrac{\gamma_{ii} + s_i^2 - s_i}{s_i} & \text{for } i = j \end{cases}$$

Assuming a multi-product industry sector, we can also explain the effects of a change in the product mix. For this purpose we define an input/output elasticity Θ_{ik}, which tells us what happens to the input x_i if the output Q_k changes:

$$\Theta_{ik} = \frac{\partial \ln x_i}{\partial \ln Q_k}$$

$$p = \text{constant}, Q_l = \text{constant, for } l \neq k.$$

We will calculate the input/output elasticities Θ_{ik} from the ith cost share:

$$s_i = \alpha_i + \sum_{j=1}^{j=n} \gamma_{ij} \ln p_j + \sum_{k=1}^{k=m} \delta_{ik} \ln Q_k$$

Use of the cost shares then yields:

$$\frac{\partial x_i}{\partial \ln Q_k} = \frac{\delta_{ik}}{s_i} + \frac{\partial \ln C}{\partial \ln Q_k} = \frac{\delta_{ik}}{s_i} + \beta_k + \sum_{j=1}^{j=n} \delta_{jk} \ln p_j + \sum_{l=1}^{l=m} \vartheta_{lk} \ln Q_l \ .$$

7. APPLICATION TO A REAL DATA SET: PRELIMINARY RESULTS

The approach described in the preceding sections has been applied to a series of make and use tables for the Canadian economy covering the period 1961–1978. The data are expressed in terms of both actual and constant 1971 dollar producer prices and were supplied by Statistics Canada. We utilized the M (medium) aggregation level, in which these rectangular tables are classified into 43 industries and 92 commodities. The approach described below was applied to the "primary metal" industry. The outputs were aggregated into three commodities, as shown in Table 1, while the six inputs shown in Table 2 were distinguished.

TABLE 1. Outputs of the Canadian basic metal industries in 1971.

Output	10^6 Dollars	% of total
Iron and steel	2019.4	39.5
Nonferrous metals	2587.7	50.5
Other	512.0	10.0
Total	5119.1	100.0

TABLE 2. Inputs into the Canadian basic metal industries in 1971.

Input	10^6 Dollars	% of total
Iron ores and concentrates	151.1	3.0
Other metal ores and concentrates	1284.5	25.1
Energy	265.9	5.2
Basic metal products	879.2	17.2
Other inputs (including margins, indirect taxes)	918.2	17.9
GDP at factor costs	1620.2	31.6
Total	5119.1	100.0

For each of these six inputs a producer price index[2] was calculated.

The results of the analysis presented in this section are rather preliminary in nature: the significance of the elasticities has not yet been tested and therefore caution should be exercized with any interpretation of the results.

Table 3 presents a second-order approximation of the commodity-by-commodity IO coefficients for the base year (1971).

We restricted nonferrous ore input to basic ferrous products and ferrous ore input to basic nonferrous products. The input coefficients for "other inputs" are calculated as a residual. With relatively few exceptions, the estimates for the commodity-by-commodity coefficients seem to be reasonable: all negative coefficients are insignificant, and steel production requires much more energy per

[2] For this preliminary report no attempt was made to calculate margins or indirect taxes on the commodity inputs so that purchasers' price indexes could be derived.

TABLE 3. Approximation of commodity-by-commodity IO coefficients for the Canadian basic metal industries in 1971 (t-values in parentheses).

Input	Output		
	Iron and steel products	Nonferrous metal products	Other products
Iron ores	0.056	0	0.067
	(7.6)		(2.6)
Nonferrous ores	0	0.540	-0.179
		(13.9)	(1.0)
Energy	0.126	0.023	-0.029
	(6.2)	(1.1)	(0.4)
Metal products	0.274	-0.058	0.541
	(7.1)	(1.6)	(5.1)
GDP at factor costs	0.315	0.343	0.103
	(14.1)	(13.0)	(1.0)
Other inputs	0.231	0.153	0.497

unit of output (value) than does the production of nonferrous metals. Statistics Canada (1978) reported that, in Canada in 1971, 209 GJ was required per 1000 $ worth of output of the iron and steel industries, while for the aluminum or copper industries the corresponding values were 163 GJ and 8 GJ, respectively.

As nonferrous ores are much more expensive than iron ores, the high nonferrous-ore input coefficient and the correspondingly small iron-ore coefficients seem reasonable. On the other hand, it is not reasonable that the production of "other products" should require more ferrous ores than does basic ferrous metal production. No attempt has been made to estimate a time series of commodity-by-commodity IO coefficients.

The influence of prices and changing output structures on the input requirements of the basic metal industries is demonstrated by a set of the relevant elasticities. To begin with, the symmetric Allen elasticities of substitution are presented in Table 4.

TABLE 4. Allen elasticities of substitution for the Canadian basic metal industries in 1977.

	Nonferrous ores	Energy	Metal products	Other inputs	Value added
Iron ores	0.042	0.009	-0.003	-0.078	0.226
Nonferrous ores		0.001	-0.170	-0.104	-0.044
Energy			0.002	-0.012	0.138
Metal products					0.051
Other inputs					0.087

No large elasticities of substitution were found, thus indicating that relative prices have only a small impact on input relations. GDP is found to be a partial substitute for ores and for energy. As might be expected, metal products are not substitutes for energy or ores, but are complementary to nonferrous ores. It seems reasonable that all inputs (trade and transport margins, taxes, overheads) are complementary to most of the inputs.

Own-price elasticities were calculated for all of the inputs. For energy, GDP, ferrous ores, and other inputs, negative elasticities were found. Table 5 presents a time series of own-price elasticities and Allen elasticities of substitution for energy and GDP expressed in terms of value added.

TABLE 5. Own-price elasticities ($\varepsilon_E, \varepsilon_{VA}$) and Allen elasticities of substitution ($\sigma^A_{E,VA}$) for energy and GDP (VA) for the Canadian basic metal industries, 1961–1977.

Year	ε_E	ε_{VA}	$\sigma^A_{E,VA}$
1961	-0.326	-0.730	0.154
1962	-0.271	-0.726	0.161
1963	-0.273	-0.723	0.164
1964	-0.212	-0.725	0.168
1965	-0.190	-0.714	0.178
1966	-0.185	-0.718	0.175
1967	-0.167	-0.740	0.161
1968	-0.156	-0.743	0.160
1969	-0.124	-0.736	0.168
1970	-0.212	-0.759	0.144
1971	-0.273	-0.732	0.157
1972	-0.300	-0.733	0.154
1973	-0.260	-0.749	0.147
1974	-0.378	-0.776	0.119
1975	-0.475	-0.760	0.120
1976	-0.480	-0.775	0.111
1977	-0.468	-0.733	0.138

A relatively large and constant own-price elasticity, varying smoothly in the region of -0.75, was found for GDP, indicating that there has been a significant and constant impetus to increase the productivity of primary inputs.

Comparatively smaller own-price elasticities were found for energy, and these varied over time with a characteristic pattern. In the course of the sixties, when real energy prices went down, elasticities moved from -0.32% to -0.12%. In the seventies, when energy became more expensive, the sensitivity of energy use to price grew again noticeably. This is reflected in the growth of the own-price elasticity, which jumped from -0.26 in 1973 to -0.48 in 1975.

The Allen elasticities of substitution for GDP and energy are rather small. The most surprising result is that, especially from 1971 to 1976, substitution elasticities fell. In 1977 the Allen elasticities started to increase again. To summarize: price-sensitive changes in the own-price elasticities for energy indicate that rising energy prices are likely to improve energy efficiency, while the relatively small and price-insensitive elasticities of substitution show us that rising energy prices do not stimulate substitution between energy and value added in the short term. The increase in the Allen elasticity noted for 1977 may indicate that there exists a time lag of about three years between a change in energy price and a response in terms of substitution behavior.

Finally, the changes in the inputs resulting from changes in the outputs were analyzed using the IO elasticities shown in Table 6.

Output elasticities for nonferrous ores varied around 1, indicating that the corresponding IO coefficients are rather stable, while the elasticities for iron ores

TABLE 6. Selected IO elasticities for the Canadian basic metal sector in 1965, 1970, and 1975.

Outputs	Year	Iron ores	NF ores	Energy	Metals	GDP
Ferrous	1965	0.767	0	0.965	0.781	0.412
metal	1970	0.758	0	0.919	0.772	0.387
	1975	0.756	0	0.766	0.788	0.452
Non-	1965	0	1.105	0.153	-0.206	0.566
ferrous	1970	0	1.055	0.187	-0.219	0.597
metals	1975	0	1.109	0.245	-0.204	0.517

were somewhat lower at around 0.75, showing that the iron ores coefficients are not only affected by the output of ferrous metal but also by other factors.

Both the energy and the metal elasticities are different for the two groups of output. Therefore it seems that shifts in product mix influence both the energy and the metal input coefficients for the industry as a whole. The energy elasticities of around 0.9 noted in the sixties and early seventies indicated that the energy/output ratio for ferrous metal was relatively constant, while the declining elasticities since 1975 show again that there have been some attempts to save energy since the first oil shock. The negative elasticities calculated for metals transformed into nonferrous metals output are not significant.

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