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# DEVELOPMENT OF SIMPLIFIED MODELS OF REGIONAL GROUNDWATER AND SURFACE WATER FLOW PROCESSES BASED ON COMPUTATIONAL EXPERIMENTS WITH COMPREHENSIVE MODELS

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## PREFACE

The *Regional Water Policies* project of IIASA focuses on intensively developed regions where both groundwater and surface water are integrating elements of the environment. Our research is directed towards the development of methods and models to support the resolution of conflicts within such socio-economic environmental systems. For that reason complex decision support model systems are under development for important test areas. One of these test areas is an open-pit lignite mining area in the GDR. A fundamental presumption for the development of such systems are appropriate submodels of the basic environmental processes to be considered. These submodels have to reflect the processes sufficiently accurately but should be on the other hand simple enough for their integration in complex model systems.

The paper deals with groundwater and surface water flow processes. It presents a methodology for the development of simplified reduced submodels based on computations with comprehensive flow models. The research has been done within the framework of a collaborative agreement between IIASA and the Institute for Water Management in Berlin. Besides the Institute for Water Management, the Institute for Lignite Mining, Grossräschen, GDR, took part in this research. This paper is the final report for the second stage of collaboration.

Although the methodology has been developed with special regard to open-pit lignite mining areas the given approaches and models are intended to be more generally applicable.

Sergei Orlovski
Project Leader
Regional Water Policies Project

**ABSTRACT**

The development of complex decision support model systems for the analysis of regional water policies for regions with intense socio-economic deveiopment affecting and being affected by the water resources system is of increasing importance. One of the most illustrative examples are regions with open-pit lignite mining.

Such model systems have to be based on appropriate submodels, e.g. for water quantity processes. The paper describes submodels for groundwater and surface water flow with special regard to open-pit lignite mining regions. Starting with a problem definition in Section 2 the methodological background is given. The state-of-the-art of comprehensive models of regional water flow processes based on groundwater flow models and of stochastic long-term management modeling are described in details.

Section 3 gives the methodological approach for model reduction. The application of this approach is illustrated in Section 4 for the modeling of mine drainage and groundwate tables, for the modeling of remaining pit management and of groundwater-surface water interactions.

In the appendix computer programs of some submodels are given being suitable for a more general application.

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- Department for Surface Water Management
- Department for Groundwater Management

and in the joint Research Group for Open-Pit Lignite Mining and the Dresden University of Technology.

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CONTENTS

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## 1. Problem Definition

Due to the complex interrelations between society and water resources, and the rapidly increasing intensity of water resources use in many countries a higher level in water resources systems planning and control has to be achieved. This necessitates both, advanced techniques (methods and models), and "advanced" trained decision makers. Consequently, a more comprehensive and detailed analysis has to be done of the availability of water resources in their spatial and time variability, as well as of water demands. Such an analysis has to investigate all conditions and effects of appropriate measures initiated for a rational use and a sufficient protection of water resources considering all demands of the society and also their expected developments.

Figure 1 demonstrates the typical decision process in water resources planning and control. The decision making procedure in its full complexity has to take into account the following elements:

- the **objectives** of water management with regard to its main tasks
  - water supply for the society and national economy
  - protection of the society against damages caused by water
  - protection of the water resources against depletion and pollution,

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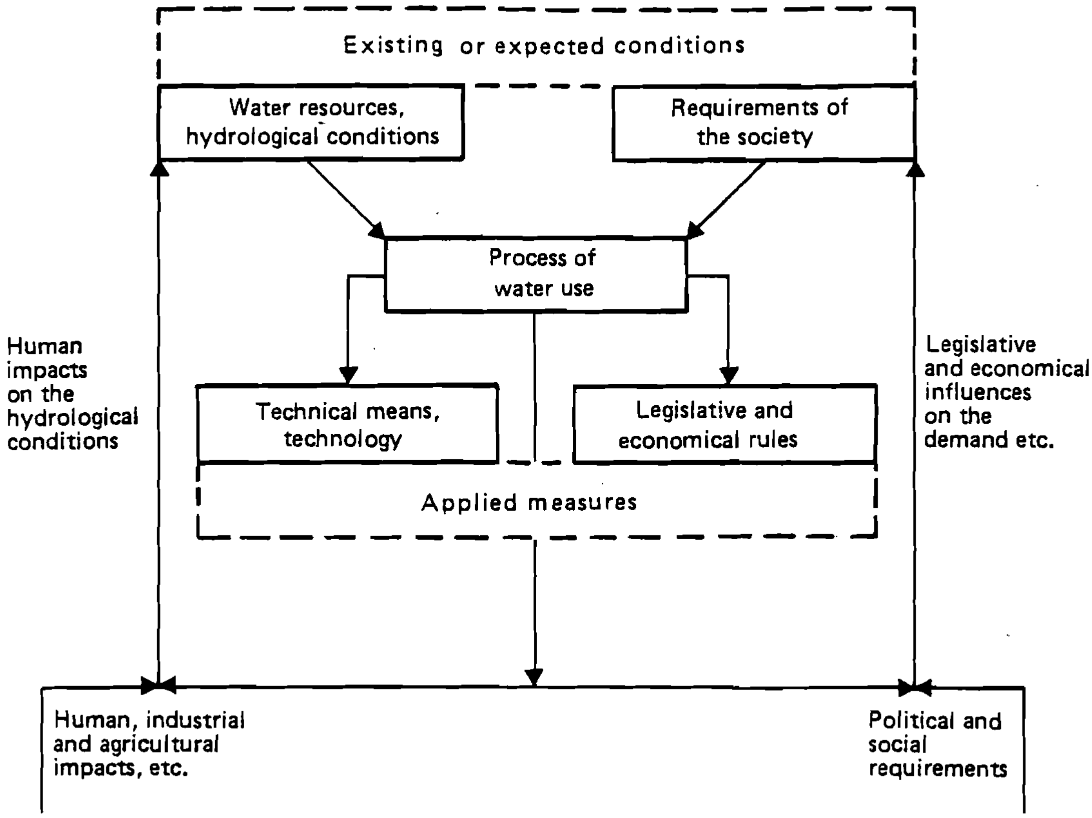



Figure 1: Schematized water management process

- the **time horizon** of the decision making directed towards
  - conceptional long-term planning
  - aspects of management and operational control,
- the existing and expected **conditions** for water management as
  - the hydrological regime determining the available water resources
  - the effects (in space and time, water quantity and quality) of applied measures for water use, management, protection, etc.
  - the consequences of other influences and impacts (human, industrial, agricultural etc.) on the water resources and demands
  - the requirements of the society upon water resources, depending on political, economical and social conditions, and objectives,
- possible **measures** to be applied in water management, e.g.
  - legislative means as laws, standards
  - economical rules, for example prices, fines
  - technical means (e.g. reservoirs, treatment plants) and techniques (e.g. reservoir operation rules).

Obviously a complex decision problem is given if the water management system under consideration comprises the entity of the water resources system and of the control and utilization facilities in the whole basin. Thus the task of water management includes the solution of the following three main problems:

- to elaborate development plans for the water resources and management system in river basins,
- to derive rational long-term management strategies for existing and/or planned water management systems,
- to provide a basis for the integration of short-term (real-time) operation policies into those systems.

In general because of this complexity a universal solution with a single method or model is impossible. Rather, separated investigations of the three main problems by means of adequate methods becomes more and more the general practice:

- application of *planning models* , being based on a high abstraction of the water management system, putting particular emphasis on socio-economic aspects,
- application of *management models* with detailed consideration of the optimum long-term water supply problem,
- application of *operational models* reflecting in detail the real-time behaviour of the water management system and considering extreme hydrologic or water quality situations.

There is, of course, a hierarchy

- planning
  - management
    - real-time operation.

For a number of years it had been assumed that for long-term planning and management modern mathematical programming techniques could provide an unique optimal solution for a given water resources system. It was neglected that the decision making process, generally being characterized by multiple criteria and frequently by multiple decision makers, cannot be totally formalized and modeled, in opposition to many technical and technological processes.

Nowadays it is evident that methods are required which reflect the complex, interactive, and subjective character of the decision making process. They should support an objective decision making by mathematical means, but without neglecting the subjective and more qualitative experiences of the decision makers.

Regarding to these requirements an advanced system of decision aids is needed which allows

- to consider the controversy among different water users and interest groups,
- to include multiple criteria some of which cannot be evaluated quantitatively,
- to take into account the uncertainty and the stochastic character of the system inputs, as well as the limited possibilities to analyze all the decisive natural and socio-economic processes and impacts,
- to offer a set of decision alternatives, demonstrating the necessary trade-offs between different water users and interest groups.

The main objective therefore is to develop relatively simple policy-oriented and computerized procedures that can assist in addressing the above mentioned generic issues, or in other words to develop a policy-oriented **Decision Support Model System (DSMS)**. The research within the Regional Water Policies Project at IIASA is directed towards this aim, see Orlovski and van Walsum 1984, Kaden et al. 1985a, b.

To be generaly applicable such a DSMS has to integrate submodels of all subsystems and sub-processes which are important for the analysis of regional water policies. Undoubtless this results in new requirements to the submodels. Whereas in earlier water resources modeling periods the primary interest had been directed towards the development of rather detailed submodels, considered as being appropriate for an adequate process simulation, it is now more important to develop submodels as simple as possible with regard to the complex problems and solution concepts.

A typical example for a complicated decision making process in water management represents open-pit mining areas. On the one hand the dewatering of lignite mines is an unavoidable part of the mining technology. On the other hand this dewatering results in a regional cone of groundwater depression and consequently in extensive changes of the hydrological regime and of the conditions for water resources use and mangement, also in downstream river basins.

A detailed analysis of the water management problems related to open-pit lignite mining is given by Kaden 1983, Kaden et al. 1985b. The following informations shall illustrate the integrated effects and influences of various hydrological processes in lignite mining areas:

(1) Infiltration losses of surface water caused by mine dewatering reduce the water supply for downstream water users and increase the groundwater pumpage necessary for dewatering the lignite mines.

(2) Essential changes in groundwater recharge are caused by the extensive changes of geographical and ecological conditions in open-pit mining areas, e.g.

- changes in land use, in soil-geological and biological conditions due to devastation and recultivation,
- changes of the groundwater level in connection with the disappearance of surface water or, in some cases, rising of surface water levels.

For example, the natural groundwater recharge of a wooded area with sandy soils is changing under the climatic conditions of the GDR from about 100 $mm \cdot a^{-1}$ up to 350−400 $mm \cdot a^{-1}$ after devastation (see Figure 2).

(3) The rate of water pumped from the mining area into the surface water system amounts up to 30 - 50 % of the total river discharge (70 % under low flow conditions).

Obviously all these interacting processes have to be taken into account for water management and planning in lignite mining areas. In collaboration between IIASA and research institutes in the GDR and Poland a **DSMS** for the analysis of regional water policies in open-pit lignite mining areas is under development, Kaden et al. 1985a,b. The scientific work being documented in our collaborative paper is a part of this study. To specify the problem a short summary of the methodological approach for the **DSMS** shall be given.

In general, dynamic problems of long-term regional water management are approached by time-discrete dynamic systems models. The step size depends on the variability in time of the processes to be considered, on the required criteria and their reliability, and on the frequency of decisions (control actions) effecting the systems development. Taking into account the policy-making reality related to long-term regional water management and planning two different step-sizes discretizing the *planning horizon* T (of about 50 years) are of major interest:

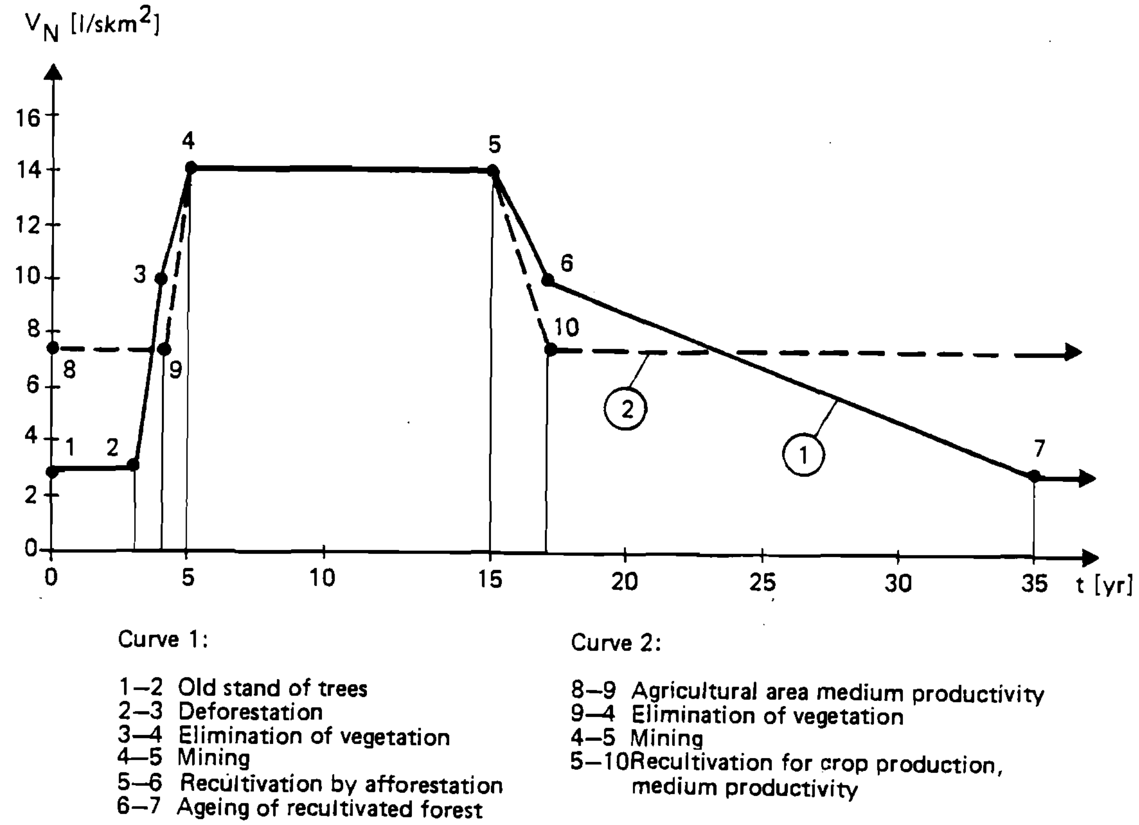



Figure 2: Variation of groundwater recharge in time in lignite mining areas

- the *planning periods* $\Delta T_j, j = 1, \ldots, J$ $(T = \sum_{j=1}^{J} \Delta T_j)$ as the time step for principal management/technological decisions, (e.g. water allocation from mines, water treatment, drainage technology)
- the *management periods* of one month for management decisions within the year related to short-term criteria as the satisfaction of monthly water demand (the classical criteria for long-term water resources planning).

The discretization of the planning horizon into a restricted number of planning periods enables principally to apply optimization techniques for multi-criteria analysis. Small time steps (for instance, $\Delta T_j = 1$ year) for the planning periods are favorable from the point of view of the evidence and accuracy of model results. Otherwise the number of planning periods should be minimized with respect to the available methods for multi-criteria analysis, computational facilities, and budget as well as time for analysis. As a compromise variable planning periods should be used, starting with one year and increasing with time. Taking into the account the uncertainties of long-term predictions of model inputs as the water demand and the required accuracy, decreasing with time, this approach is quite reasonable.

For monthly time steps (600 for a planning horizon of 50 years) the application of any optimization technique becomes unrealistic. To study monthly systems behaviour systems simulation is the only applicable tool. Furthermore this simulation opens an easy way to consider stochastic inputs (hydrological data, water demand etc.) applying the Monte Carlo method for stochastic simulation (see below).

Based on these assumptions a heuristic two-level model system is proposed, consisting of

- *planning model* for dynamic multi-criteria analysis for all planning periods in the planning horizon,
- *management model* for the stochastic simulation of monthly systems behaviour in the planning horizon.

Appropriate submodels have to be developed for both levels of the DSMS - the *planning model* and the *management model*. The requirement for the compatibility of the submodels for investigations at both levels can be expressed as follows: The results received from a computation with monthly time steps (management model) should be approximately the same as received from a computation with time steps of 1 year or more (according to the planning periods of the planning model).

The submodels for a DSMS should simulate the natural processes adequately, but they should be at the same time as simple as possible. This paper deals with **water quantity models of surface water and groundwater systems** and of their interrelations. It gives a methodology to develop **simplified flow models** based on **comprehensive models** for lowland regions with interrelated groundwater and surface water resources. Special concern is given to open-pit lignite mining areas, characterized by significant variations in the groundwater regime.

## 2. Methodological Background

### 2.1. Model Philosophy

Mathematical models found wide propagation in the GDR for the simulation of practical groundwater and surface water flow processes since the end of the sixtys. They replaced step by step the empirical methods used before, which considered the effects of selected input variables and boundary conditions on the investigated phenomenon, system outputs etc., analogously as later the black-box-models (see Section 3.).

In *surface water hydrology* the investigations were oriented from the beginning in two directions, that is the application of

- deterministic modeling techniques (with relatively short time increments of one day or less) for short term control problems,
- stochastic simulation techniques (usually with time increments of 1 month or more, according to the objectives) for long-term management and planning.

In *groundwater hydrology* only deterministic techniques were applied, in general with larger time increments, e.g. 1 month, taking into the account the dampened groundwater flow processes. Thus the application of stochastic simulation techniques was limited to surface water hydrology.

The analysis of long-term regional water policies, especially in lignite mining areas, requires an integration of the traditional modeling methods. In both fields, groundwater resources management and surface water management, there are a profound scientific background and manifold practical experiences in the GDR.

The title of our paper includes the terms *comprehensive model* and *simplified model* . It seems to be necessary to explain in what sense these terms are used in the following.

Generally, models of natural processes give a more or less simplified image of the reality. Abstraction and schematization are typical steps in the development of mathematical models. In a physical strong way, mathematical modeling of groundwater and surface water flow processes is based on the fundamental differential equations of fluid mechanics. Of course, this point of view is a highly theoretical one. In engineering practice these fundamental equations are used only in an integrated form. The principle of continuity is referred to volumes of practically relevant dimensions and the dynamic flow equations are substituted by integrated and/or simplified relations founded on empirical knowledge. In the field of hydrology the equations of DARCY and SAINT-VENANT give typical examples of this type of basic relations.

In lowland catchment areas with sandy aquifers the groundwater system is the integrating element in the complex hydrological system. Starting from the fundamental role of groundwater in the water balance, a comprehensive model of regional groundwater flow coupled with flow models of the main surface water bodies forms the prototype of a comprehensive model of the regional water flow processes. Regarding to the level of abstraction, both types of coupled models are adequate: groundwater flow is simulated as a horizontal-plane (two-dimensional) process, surface water flow is characterized by the surface water level and the discharge, summarized over cross-sections (one-dimensional). The comprehensive models constructed in such a way guarantee the complete utilization of all available data in practice. The abstractions inherent to the mathematical model are adequate to the quality of the data base acquired by the help of hydrological observations, hydrogeological research and exploration, etc.. From this point of view, the simplifications of the mathematical model are reasonable, at the same time they are necessary due to practical requirements. Applied to engineering problems, such comprehensive models give a sufficiently accurate image of the natural processes and represent the maximum justifiable effort for model development and application in practice.

*Comprehensive regional groundwater flow models* have been developed in the GDR in the last 10 years for essential objects of water supply and mining regions. For an overview see Kaden 1984. Details about the methodological level of these models and their application are given in Section 2.2. Generally, the mathematical model of horizontal-plane groundwater flow is used. In this case, the DUPUIT equation and the GIRINSKIJ potential are basic elements of the model development (Luckner and Schestakov 1975). The aquifer is assumed to be horizontally layered, hydrogeological parameters may be taken into account as variables in space and time. All types of boundary conditions (first, second and third kind) occur in the subterranean flow process and are included in the model concept. In principle, the model is deterministic, stochastic elements may be involved by stochastic boundary conditions. The effort for data acquisition, model calibration and application is high in consequence of the necessary discretization according to the applied finite difference or finite elements methods.

Due to the high computational effort such comprehensive models are generally applicable for a small number of simulations only. They are not usable both, for stochastic simulation, and for mathematical programming as it is required for the analysis of regional water policies. Methods for the integrated long-term analysis of regional water policies have to be based on *simplified* models.

*Long-term management models* to optimize surface water management are available for the important rivers in the GDR, see Schramm 1981. The theoretical base of the approach is given by the Monte-Carlo method. Using this method it is possible to create a realistic image of the management variants relevant for the water policies under consideration. As it has been discussed above, the concept of these models forms the basis for the management model of the DSMS.

The utilization of the Monte-Carlo method requires strong restrictions concerning the modeling of the water flow processes. Because of the high number of computational variants, the model concept has to be extremely simple to guarantee acceptable computer times. The same requirement holds for submodels to be included in mathematical programming problems. In difference to the groundwater flow models described above in the case of long-term management models the model of the surface water flow process is already reduced to the simplest form, that means the continuity equation. The high methodological level of these models, characterized in the following sections of the paper, gives an excellent basis for the development of the required DSMS submodels.

Frequently the term *simplified model* is used in the context of the so-called *model reduction*. A decisive *model reduction* is necessary for the submodels of the groundwater flow and the groundwater/surface water interactions, as it will be discussed in Sections 3 and 4. Concerning the mathematical apparatus, similar methods are used in model reduction as they are usual in the field of real-time processes surface water hydrology for modeling runoff formation in catchment areas.

From the methodological point of view of water management sciences, the incorporation of these reduced models into the management model forms an essential step forward. The model concept becomes more complicated, but principle mathematical problems do not arise. More important are the questions resulting from the interrelations between the planning model and the management model (see Section 1.). The results of the multi-criteria analysis using the planning model provide long-term oriented goal functions for the operational measures (operation rules) to be simulated in the management model.

## 2.2. Comprehensive Models of Regional Water Flow Processes

### 2.2.1. Simulation Models for Regional Groundwater Management

In the GDR in the last decade a system of highly sophisticated conceptual models for groundwater management has been elaborated, see Kaden 1984. For modeling of groundwater quantity a set of models is available, based on finite difference and finite elements methods. These models are formulated for the system descriptive mathematical model of saturated flow in porous media with distributed parameters.

Especially for the solution of mine dewatering problems by consideration of mining specific boundary conditions multilayer horizontal-plane flow models have been designed. They consider steady and nonsteady groundwater flow problems in confined and unconfined aquifers. Such models are available both, for orthogonal and for triangular finite elements grids. The following models found the widest application. Although all models are highly universal designed, nevertheless everyone of these models has a special sphere of application.

The HOREGO-model (Gutt 1984) is based on a discretization of the flow field in orthogonal elements. At maximum two hydrogeological coupled aquifers may be simulated. The consideration of an inhomogeneity of the underground (and under special conditions of anisotrophy too) is possible. To realize practical inner and outer boundary conditions time- or potential-dependent boundary conditions first, second and third kind are applicable.

The model HY 75 (Hennig et al. 1979) has been developed for triangular finite elements and considers up to 5 aquifers. Generally all practical mining specific boundary conditions and hydrogeological situations may be simulated.

The choice of the model depends on the aim of model application, but also on subjective positions of the model user, and above all on the robustness, universality, and reliability of the model.

Although partly subjective is the selection of a type of discretization. Generally, triangular grids permit a better adaptation to complicated inner and outer boundaries. But this advantage gets partly lost, if the location of boundaries changes in time, e.g. open-pit mine edges due to mining operation. Especially for regional groundwater flow models in areas with numerous mines the application of orthogonal grids seams to be advantageously, because later grid focusing and grid corrections are easier to realize in most cases than by the triangular grids. In this case detailed solutions concerning operations of single dewatering elements are not of interest.

The triangular grid discretization is mainly applied for open-pit mine models (see below), because more detailed informations about the operations of dewatering elements are necessary. The problem of modeling the advance of open-pit slopes can be solved by consideration of a corresponding operation time of dewatering wells.

Especially for the design and the control of mine drainage systems with a favored flow direction the multilayer one-dimensional horizontal-plane flow model TAFEGA (Kaden et al. 1976) has been elaborated. It is a stream-pipe model for solving of all groundwater flow simulation problems in open-pit mines and their immediate surrounding with a sufficient accuracy and justifiable computational effort. All boundary conditions (first, second and third kind) can be realized time-dependent and space-variable.

Finally a powerful three-dimensional model exists. The model AQUA 78 (Sames et al., see n.n. 1980) is based on an orthogonal finite elements grid. It can be used for the simulation of complicated geohydraulic conditions, as dewatering problems in spoils and flow through and under sheet-piling walls etc. Boundary conditions first and second kind can be realized, and the consideration of time-dependent changes of the geometry of percolated bodies is possible.

For regional flow problems the last mentioned models are of minor importance. All models, characterized before, are in principal suitable to be coupled with surface water models (see below) and groundwater quality models. They have been developed for the computers ES 1040/1055 (similar to IBM/360, IBM/370) and BESM 6. Detailed application informations are available. Generally, the programs are written in FORTRAN in order to realize a sufficient portability.

### 2.2.2. The Methodology of Continuously Working Models

Simulation models of environmental processes as groundwater flow form only an image of the reality with a certain accuracy. The accuracy of simulation results depends on the abstractions needed for model structuring and on the completeness and quality of input data. The lack of knowledge of processes and data in preliminary steps of analysis or in general requires the step by step improvement of the models and their data. For groundwater systems with their strong dampening and phase shifting between impact on the system and observable system response this can well be done by comparing simulation results with real systems responses. Based on that in the GDR the methodology of Continuously Working Models has been developed organizing information processing according to cybernetical principles (see Luckner 1973, Peukert 1979, Peukert et al. 1982).

As a so-called **Continuously Working Model (CWM)** we understand a methodology and a model system for monitoring and controlling of long-term man-made impacts on groundwater resources. Generally the model system consists of two main-parts:

- a specific data bank for information storing, and
- a simulation model for information processing.

By means of simulation the influence of feasible control measures can be simulated before their implementation in practice. The sequence of data acquisition, data processing and simulation includes the feedback of information being active either already during the simulation run or in the phase of implementation. In such a way, the simulation is guaranteed to be based on the latest available data situation.

The model system is operating parallel in time, but discontinuously with the running original mining process. The steps of its operation depend on size and importance of the model.

Usually a hierarchical system of models is needed to meet different requirements in the spatial and temporal resolution. It is not possible to simulate both the regional groundwater flow process and the operation of single dewatering wells inclusive the frequency of submersible motor pumps during the whole dewatering time.

For groundwater management in lignite mining areas a 3-level hierarchy has proven to be suitable, as shown in Figure 3. Generally, this hierarchical system is structured in

- regional models (3rd level),
- open-pit mine models (2nd level) and
- operational models (1st level).

*Regional models* are mainly used for medium- and long-term predictions for regional planning, water management and mine drainage. In principle regional models enclose the influence areas of a number of different impacts on the groundwater system as open-pit mines, water works, etc. For the simulation in general two-dimensional horizontal-plan flow models are used, e.g. HOREGO based on an orthogonal grid. Regional models should be actualized and applied in time intervals between 2 and 5 years. As simulation results detailed forecasts are obtained of the prognostic systems behavior of the whole regional groundwater flow process (groundwater lowering or/and rebound). Besides the mentioned application for medium- and long-term planning these results are used to estimate boundary conditions for lower level models in the hierarchy of continuously working models as open-pit mine models. Furthermore the regional models help to improve systematical information acquisition in the considered region, e.g. hydrogeological exploration.

*Open-pit* mine models in the second level are mainly used for medium-term predictions for planning and design of mine drainage systems and for water management measures in some cases. A second level model encloses the area influenced by the dewatering system of one mine or the catchment area of a groundwater work. For the simulation in most cases two-dimensional models as HY75 are used based on triangular grids. The results of the simulation, e.g. the local development of groundwater tables, are the basis for the project of detailed dewatering measures of one mine including the design of the monitoring system (monitoring wells, gauges, etc).

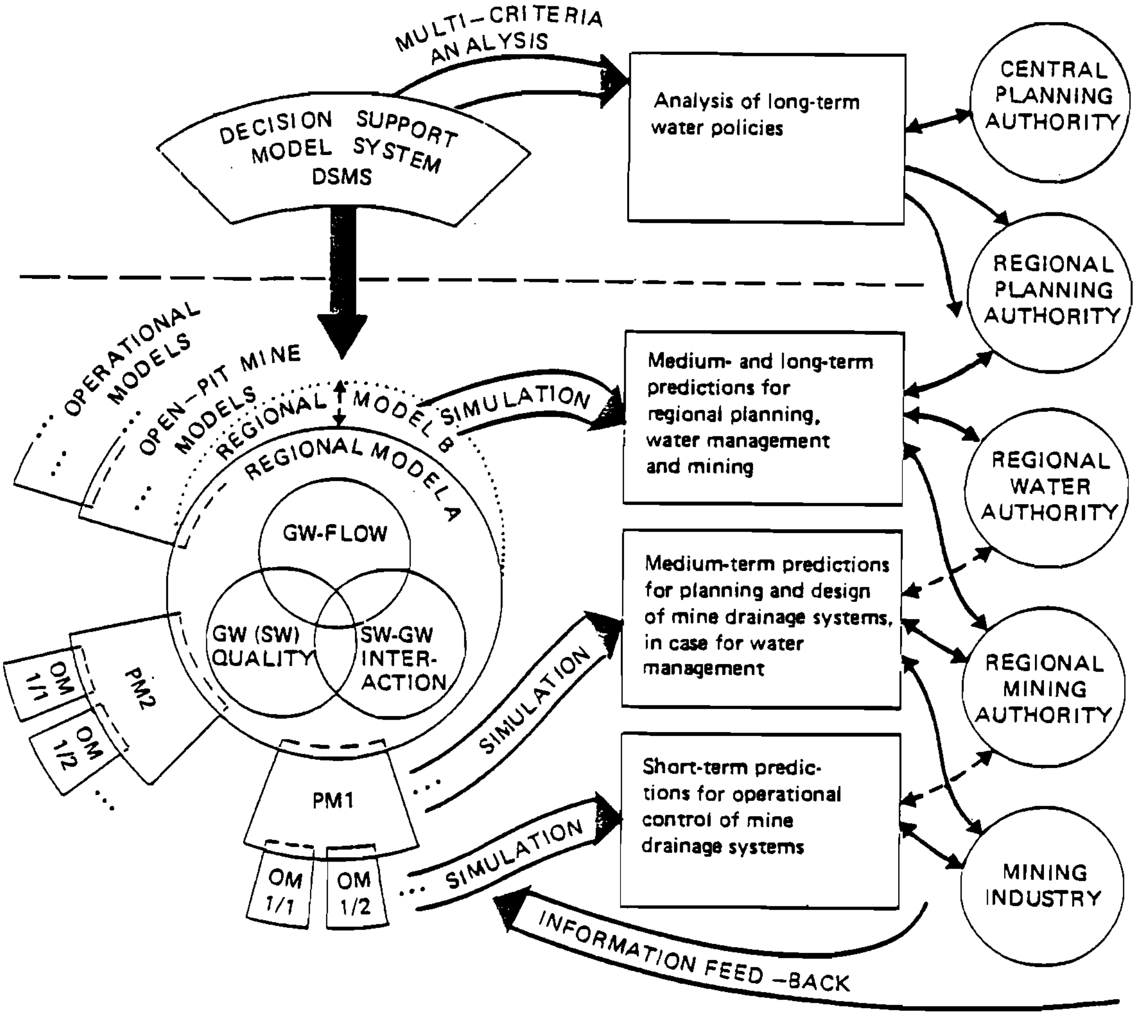



Figure 3: Hierarchy of Continuously Working Models

By means of the information feedback based on measurements of groundwater levels and water pumpage as well as by information acquisition from higher or lower level models a permanent actualization of open-pit mine models is done. The actualized results are useful to optimize the drainage and monitoring system during their operation, and to improve the forecast for future systems design. For that reason the interval between model applications should not be longer than 1 to 2 years.

*Operational models* are used for short-term predictions for operational control of especially important parts of mine drainage systems. In most cases a one-dimensional model (TAFEGA) is used for simulation, but for special conditions it is necessary to apply two-dimensional models. The aim of investigations is, to get exact predictions for the operation and control of dewatering systems during the operation time and to estimate the groundwater table at the slopes of the open-pit mine, as well as the outflow from these slopes in order to satisfy their geomechanical stability. These models are applied in monthly time intervals, yearly at maximum.

The models of all three levels are embedded in the hierarchical process of policy-making. In analyzing critically the features and application of continuously working models it has to be emphasized that the continuously working models are a helpful tool in medium-term planning and design as well as short-term control, but their applicability for long-term planning as an integrated part of the regional policy-making process is restricted.

The main obstacles to their use in the long-term policy-making process are as follow, Kaden and Luckner 1984.

(1) The continuously working models are tools to analyze an environmental system without considering socio-economic aspects. Economic modeling of groundwater management in mining areas is not done or economic models are not coupled with continuously working models. The key role of the interactions between the socio-economic development and the environmental processes is not considered implicitly.

(2) There is a gap between the complicated sophisticated models and the existence and quality of the relevant data, the latter being uncertain. Unreliable results may discredit models with the policy makers.

(3) Frequently the available time and budget for model structuring is incompatible to the necessary effort of collecting and verifying of the required data. This incompatibility is especially apparent when one considers groundwater quality and groundwater-surface water interactions.

(4) Mangement/technological alternatives have to be fixed exogenously for the simulation models. In the case of multiple objectives and decision makers that we have to deal with concerning the mining regions, the manual selection of efficient scenarios is very difficult and time as well as money consuming, or even impossible. Continuously working models are purely environmental models, not directly answering all questions asked in the reality by the decision makers. Systems analysts are needed to mediate.

Summarizing, there is an apparent need for the analysis of long-term, regional water policies in mining areas, to reconcile the conflicting interests within such socio-economic environmental systems. Scientifically sound but practically simple policy-oriented methods and computerized procedures have to be developed which can assist in addressing the above mentioned topics in water resources policy design. Such a system could be interpreted as a fourth level above the model -hierarchy of continuously working models (see Figure 3). It will be applied to decision making of central and regional planning authorities. The lower level models are used for more detailed and specific investigations.

### 2.2.3. Specific Boundary Conditions for Regional Flow Modeling

As stated in Section 2.1 regional groundwater flow models form the basis for a comprehensive regional flow model. This requires the consideration of boundary conditions in the groundwater model being specific for *regional* flow problems. Generally these problems include:

- natural groundwater recharge,
- infiltration/exfiltration from/in surface water systems, that means groundwater-surface water interactions.

Additionally in areas with open-pit mining activities we have to deal with substantial boundary conditions for groundwater:

- moving open-pit mine slopes during operation of mines,
- remaining pits after abandoning of mines being filled by natural groundwater flow, in case with artificial surface water inflow.

The character of the boundary condition ***groundwater recharge*** strongly depends on the distance between earth surface and groundwater table, with other words on the depth of the unsaturated zone. If the unsaturated zone is larger than $2\ m$, the natural groundwater recharge practically does not depend on the groundwater table.

For the determination of long-term mean values of groundwater recharge (precipitation minus real evapotranspiration) the program RASTER has been developed (Glugla et al. 1976, Enderlein et al. 1980). Based on long-term mean values of precipitation and potential evapotranspiration the real evapotranspiration is estimated using the BAGROV-approximation.

Above others the following input data for the RASTER-model are needed: precipitation, main form of land use (agricultural areas, forestry, water bodies ...), agricultural yield, soil type, groundwater level below earth's surface.

Natural groundwater recharge in mining areas is subject to strong temporal alterations, caused by the necessary process of devastation and reclaiming/recultiviation of large areas. Changes in soil exploitation and morphological and soil geolocical conditions are caused by the process of mining and overburden disposal coupled with the following recultivation. Furthermore, changes of groundwater and surface water tables due to mine drainage have to be considered. Size and duration of these alterations depend on the intensity of the changes due to the mining process in comparison with the original existing conditions and on its lapse of time.

The program RASTER is used to consider all these aspects, neglecting seasonal variations of groundwater recharge. The simulated values of groundwater recharge for the mostly uncovered glacial aquifers, we find in the test region, coincide in general with the long-term mean values of the natural groundwater recharge, used for simulation of regional groundwater flow processes.

In Figure 2 (see Section 1.) an example is depicted for the variation of local natural groundwater recharge caused by mining activities for typical conditions in the test area. Generally groundwater recharge increases in mining areas up to about $10\ l/s\ km^2$ on former forestral areas and about $5\ l/s\ km^2$ on agricultural areas.

In general, *surface water/groundwater interactions* can be found in all natural catchment areas. However, in mining areas the extensive cone-shaped groundwater depressions influence these interactions especially strong resulting in runoff balance variations of whole regions.

A common method for modeling regional water resources systems is to model the essential subsystem (groundwater or surface water) and to consider the other system by means of boundary conditions. This method is based on the assumption that the flow process of the subsystem being considered as boundary condition does not depend on the flow process of the modeled subsystems. In this case for the groundwater flow model the surface water systems are modeled via inner boundary conditions third kind. For details see Section 4.5.2. In systems with strong interactions between the subsystems as in mining areas this assumption does not hold for short-term variations. For that reason sometimes in mining regions surface water and groundwater interactions have to be considered explicitely in coupling the models of the subsystems.

As it has been mentioned in Section 2.1 two different model concepts are to consider. In surface water management hydrological one-dimensional flow models are used. In groundwater management system-descriptive deterministic models of quasi horizontal-plane flow processes are typically.

The essential systems variables for coupling are the water tables (groundwater and surface water) and the flux between surface- and groundwater. Principally the models may be coupled over the grid nodes (groundwater flow model) and balance segments (surface water flow model). Therefore a large number of grid nodes is required, resulting in relatively high expenses for data aqquisition and computation. It is reasonably to substitute each balance segment of the surface water flow system by a fictive well (point-source, -sink, respectively) situated in the centre of this segment. In doing this, the surface water system is transformed in a well contour system (or well plane systems), Luckner 1978. The parameters of the interactions between both systems depend on the geometric parameters of the space-discretized segment of the surface water model, on the distance between the centre of the surface water segment and the neighbored nodal point element of the groundwater flow model, on the transmissivity of the aquifer and on existing parameters of colmation of the surface water beds (see Rechenberger 1984). The coupled model system is solved either solving the entire system of equations or by iteration.

The main problem of modeling in mining areas is to realize the hydraulic effect of the mining position, varying in time, caused by the ***movement of the open-pit mine slopes***. In this case the groundwater flow process is characterized by lowering the groundwater table down to the bottom of the lignite seam, to be exploited, at the date of mining of this seam. This can be realized by the help of boundary conditions of first kind. This hydraulic boundary condition becomes complicated due to the movement of the mine and consequently the movement of the boundary condition in time. Related to a nodal point of an element in the finite elements grid this means the following: If the moving slope reaches the node, the dewatering aim (given groundwater table) has to be satisfied there. Therefore the boundary condition at this node has to be considered as many years earlier as the dewatering requires and it has to operate till overburden disposal begins on this location.

In Figure 4 it is shown how the process of the passage of a moving slope over a node can be modeled by means of an additional hydraulic resistance.
As the simulation result we obtain the amount of drainage water at the node as function in time.

Generally, this approach is sufficiently accurately for regional models. But by investigations with lower level models as strip mine models more detailed informations concerning the amount of water pumpage of all dewatering wells are required for the proper design of the dewatering system.

Appropriate solutions for the consideration of *dewatering wells* as inner boundary conditions are e.g given by Knapik et al. 1977 for an orthogonal element grid.

*Water bodies* as lakes or remaining pits being in direct interaction with the groundwater system can be considered as boundary conditions 3rd kind. The water body and the aquifer are coupled by the help of a hydraulic resistance, taking into the account the area of the water body belonging to the finite element under consideration as well as an additional hydraulic resistance for the transformation of vertical flow into horizontal ones, see Figure 5. The water body is modeled as a nonlinear function between water table and storage volume.
The solution is done iteratively. First, the water table of the water body is estimated taking into the account the previous water table and the inflow/outflow

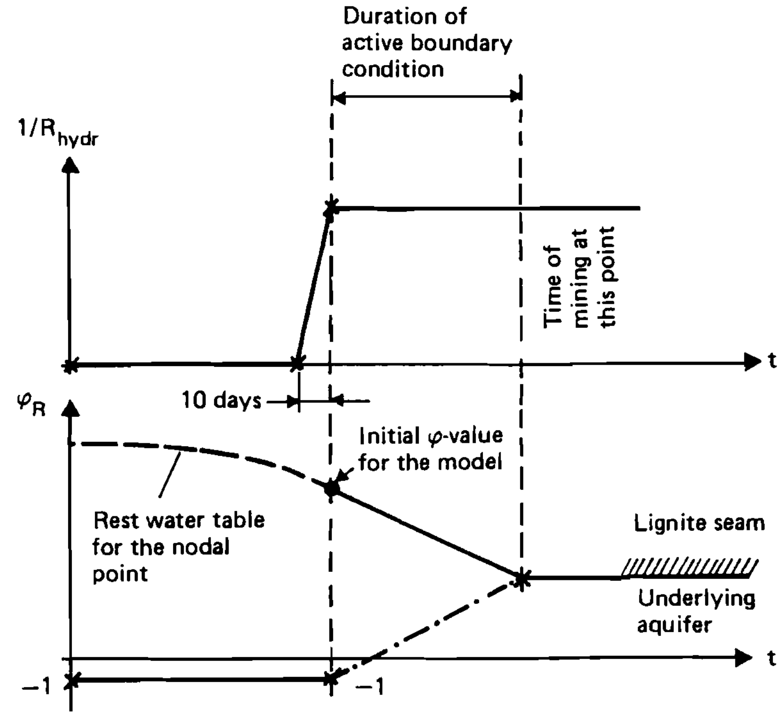



Figure 4: Simulation of a moving slope by an additional hydraulic resistance

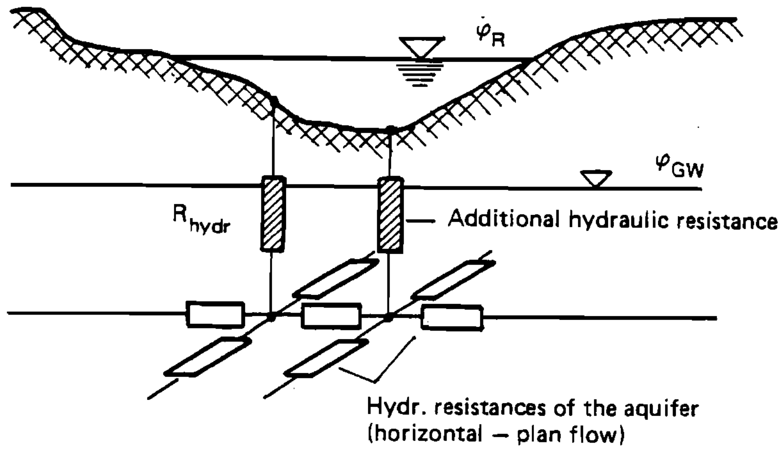



Figure 5: Interaction between water bodies and aquifer

during the studied time step. Second, the groundwater model is applied to estimate the flow between water body and aquifer via the hydraulic resistance. This flow is used to correct the water table of the water body according the first step.

This approach is especially helpful for the modeling of remaining pits in mining areas.

## 2.3. Stochastic Long-term Management Modeling

### 2.3.1. Place and Task of Stochastic Management Modeling

As it has been discussed in Section 1. for water management modeling a hierarchical model system for

- planning
- management
- real-time operation

has to be used. The goal figures for water management given by the respective management level of higher priority are relatively rough. Their assessment by more detailed investigations is done in the respective lower level of the above model hierarchy. Some aspects of this hierarchy are illustrated in Figure 6.

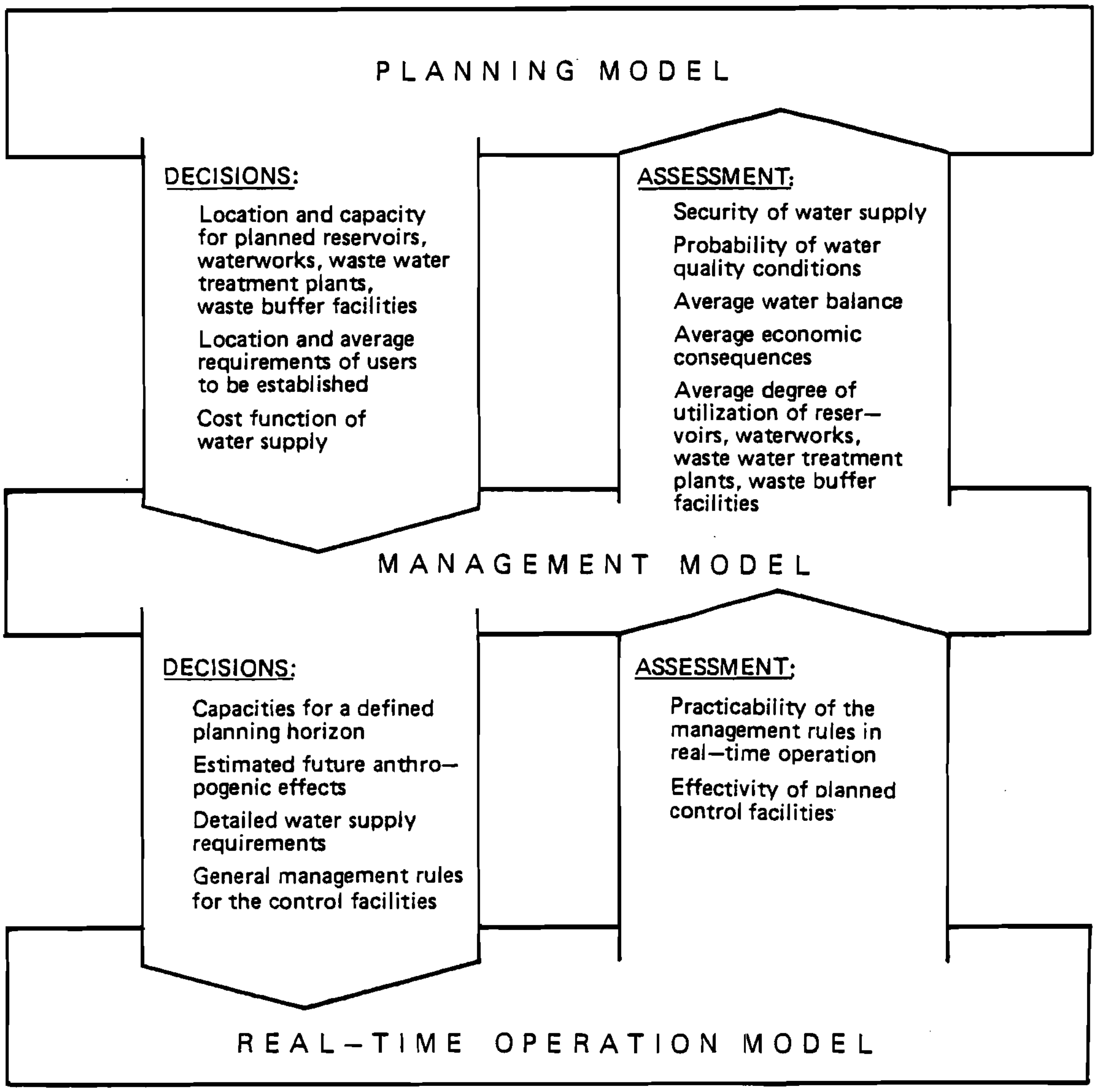



Figure 6: Hierarchy in water management

Obviously the management model has a key position herein. Its two highly interconnected main tasks to derive an effective long-term management strategy from the goal figures given by the planning model and to assess that strategy can reliably be solved only, if the following preconditions are fulfilled:

(1) The real water management system in a river basin and its essential processes must be reflected by the management model without systematic errors.

(2) The stochastic character of the natural processes taken as input variables of the model must adequately be reflected.

It turned out, that even in relatively small river basins analytical methods failed to meet these requirements. Therefore in the G.D.R. in the last fifteen years stochastic management models on the basis of the Monte Carlo method have been developed and applied (see Schramm 1981). Originally they have been designed to simulate the surface water management processes in river basins with respect to water quantity only. However during the last years first investigations have been carried out to integrate groundwater flow processes and water quality factors. In these investigations the basic principles of the management model, i. e. the Monte Carlo simulation of the surface water processes, remained unchanged.

### 2.3.2. The Basic Model for Stochastic Water Management Simulation

#### 2.3.2.1. Modeling principle

According to the probability theory main meteorological factors such as precipitation, gross radiation, air temperature etc. can be considered to be stochastic processes. Hence, both groundwater- and surface water runoff processes governed by the above mentioned factors must inevitably more or less strong reflect that stochastic behaviour. Furthermore, the demand of some of the water users within a river basin - e.g. municipal water supply or irrigation - highly depend on the actual, partly stochastic meteorological situation. So the problem to find a rational long-term water management strategy (as outlined in the previous Section) may be defined as a stochastic optimization problem. The state-of-the-art in this field does not allow for a comprehensive analytical solution of those kinds of problems. However, by applying the Monte Carlo method an "experimental" approach to the problem is provided. In Figure 7 the main steps of this approach are depicted. The model time step is selected with regard to the length of the available observation series for the input variables, to the diversity of water demands and their combined impacts. Usually a monthly discretization is considered to be adequate.

Thus, the basic model for stochastic water management simulation - which covers surface runoff balancing only - can be characterized as follows:

- by using stochastically generated runoff series the stochastic management model can be reduced to a deterministic balance (or allocation) scheme;
- owing to the rough monthly time step the runoff processes in the river network can be described by simple continuity equations rather than by complex hydrodynamic models;
- thus, analoguously to the time series approach, the deterministic balance model is able to reflect the manifold water allocation and utilization processes in a illustrative way;
- the statistical registration of simulated state variables and events combined with final frequency representation provides for a high versatility in assessing a given management strategy;

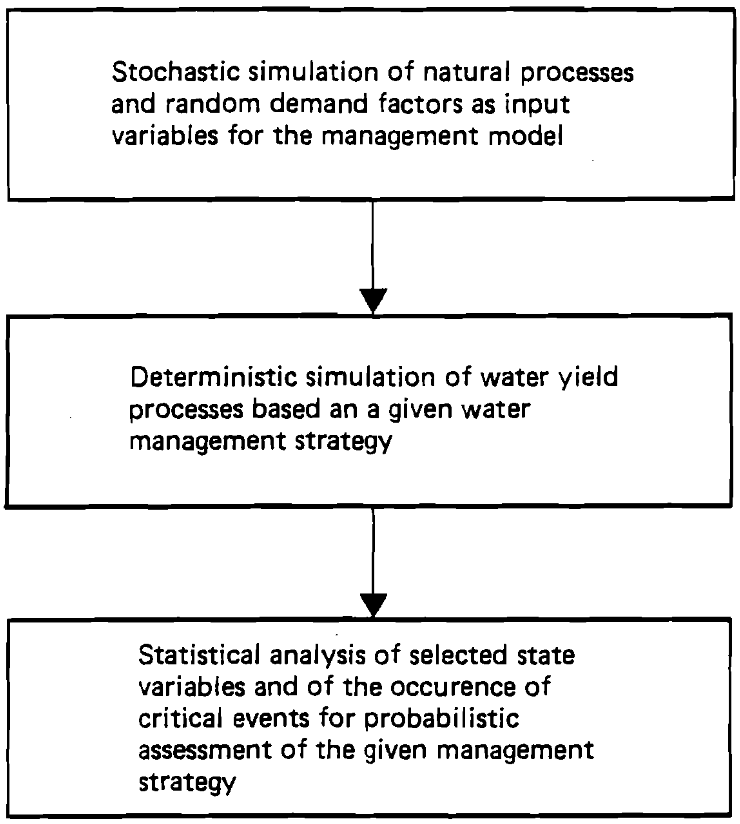



Figure 7: Monte Carlo method for water management

- a rational (approximate optimum) management strategy can be found by successive simulation runs with varying parameters.

### 2.3.2.2. Subdivision of the river basin area

As in most cases large scale river basins are to be investigated, the area to be considered is usually subdivided. At first, the description of the natural runoff process calls for a subdivision into so-called "simulation subareas" (German abbreviation STG). That partition is mainly determined by the location of long-term observed river gages, of tributary and water transfer junctions as well as of reservoirs and important water users (see Figure 8).
The subdivision of the basin into STG is generally rather rough. For a more detailed specification of user locations a further subdivision by additional balance profiles (BP) is required. Then, if for each of the balance profiles the next downstream one is specified, obviously the configuration of the entire river network is uniquely defined.

### 2.3.2.3. Stochastic simulation of the natural runoff process

A comprehensive analysis of several long duration time series showed for the climatic conditions of the GDR and with the monthly time step, that the runoff in rivers approximately posses the following properties, see Schramm 1975, Dyck 1980:

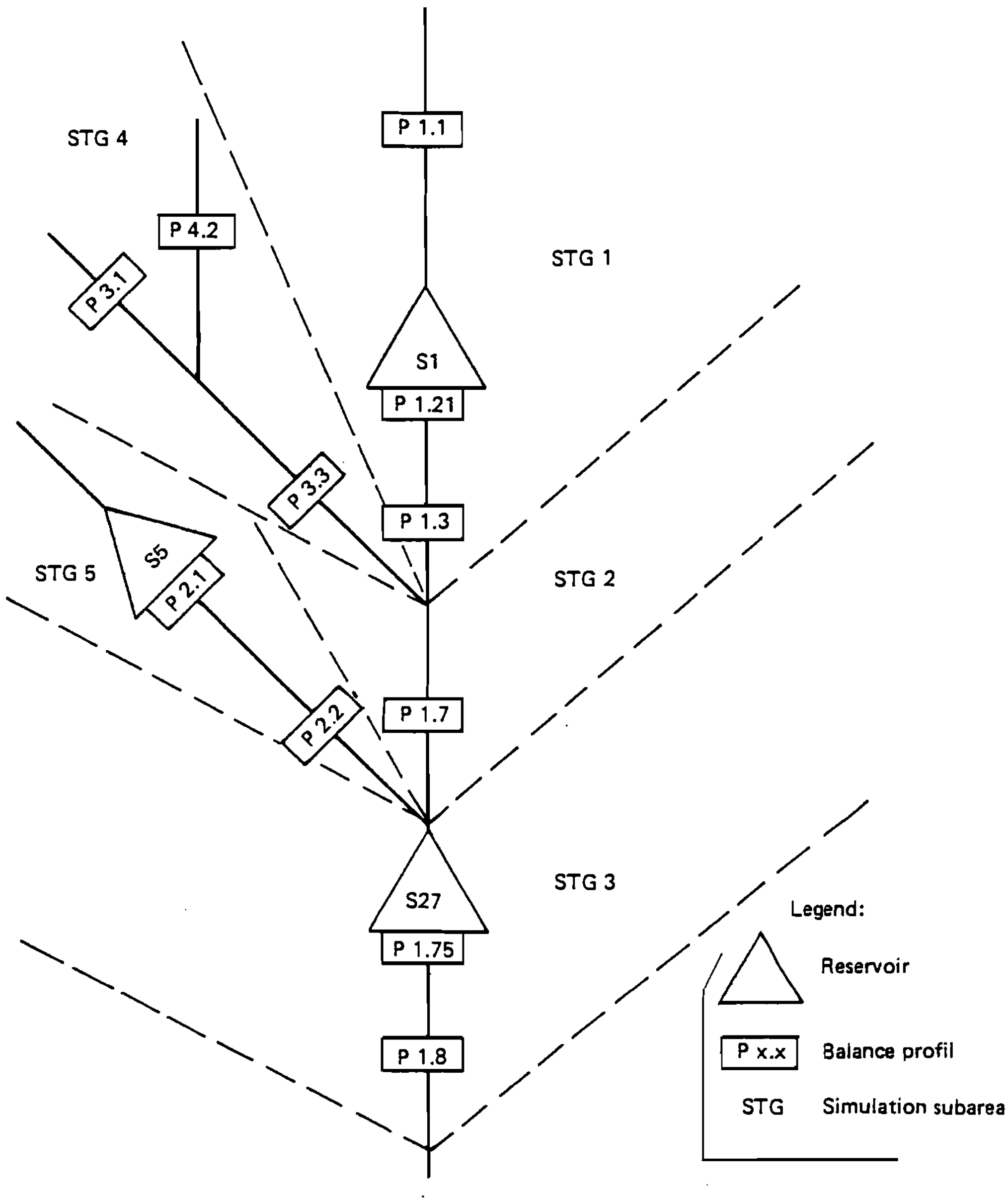



Figure 8: Subdivision of a river basin for water management modeling

- it is nonstationary and cyclic with the period being one year;
- its monthly one-dimensional distribution function can sufficiently well be approximated by a transformed normal distribution, e.g. by
  - three-parametric log-normal distribution (LN3-distribution) with the transformation

$$X = F(Q) = \frac{\ln(Q - q_0) - \bar{q}}{\sigma} \tag{2.1}$$

or four-parametric JOHNSON distribution with the transformation

$$X = F(Q) = \frac{\ln\frac{Q-a}{b-Q} - \bar{q}}{\sigma} \tag{2.2}$$

with

| | | |
|---|---|---|
| $Q$ | - | mean monthly runoff |
| $X$ | - | transformed, $N(0,1)$-distributed runoff |
| $a, b, \bar{q}, q_0, \sigma$ | - | parameters of the distribution function; |

- its sharp decreasing autocorrelation function for small time lags indicates its Markovian character.

Starting from the special transformation $F$ of the runoff and the estimates of the distribution and correlation functions, a multidimensional runoff process is appropriately simulated in the following two steps:

(1) Simulation of the transformed process in the defined month $k$ ($k = 1, 2, \cdots, 12$) by means of an autoregressive scheme

$$x_k = \sum_{l=1}^{m} A_{k-l} \cdot x_{k-l} + B_k \cdot x_k + C_k \cdot \varepsilon_k \tag{2.3}$$

where the coefficients contained in the matrices are least-square calculated from the auto- and cross correlation estimates of the transformed process. The symbols denote:

| | | |
|---|---|---|
| $A_k, B_k$ | - | regression matrices |
| $C_k$ | - | diagonal matrix of the residual standard deviations |
| $\varepsilon_k$ | - | $N(0,1)$ distributed random vector |
| $m$ | - | order of the model. |

(2) Simulation of the runoff process by inversion of the transformation:

$$q_k = F^{[-1]}(x_k). \tag{2.4}$$

The time series analysis including the estimation of parameters and matrix coefficients is done by means of the computer program SIKO. For the runoff generation itself and the transfer of the generated data to external storage units the program SIMO is used. The two programs are described in the Appendices A1 and A2.

### 2.3.2.4. Modeling of water users and flow control facilities

For the purpose of management modeling in a river basin all of the water supply requirements are represented by "users". Each user is specified as follows (see Figure 9):

- a decimal user identification code,
- assignment of water diversion and intake points of the river to balance profiles,
- the seasonal (monthly) water demand in dependency of the planning horizon, of the runoff or other model variables,
- the amount of target return flow,
- a rank number, which determines the priority of water supply among all users.

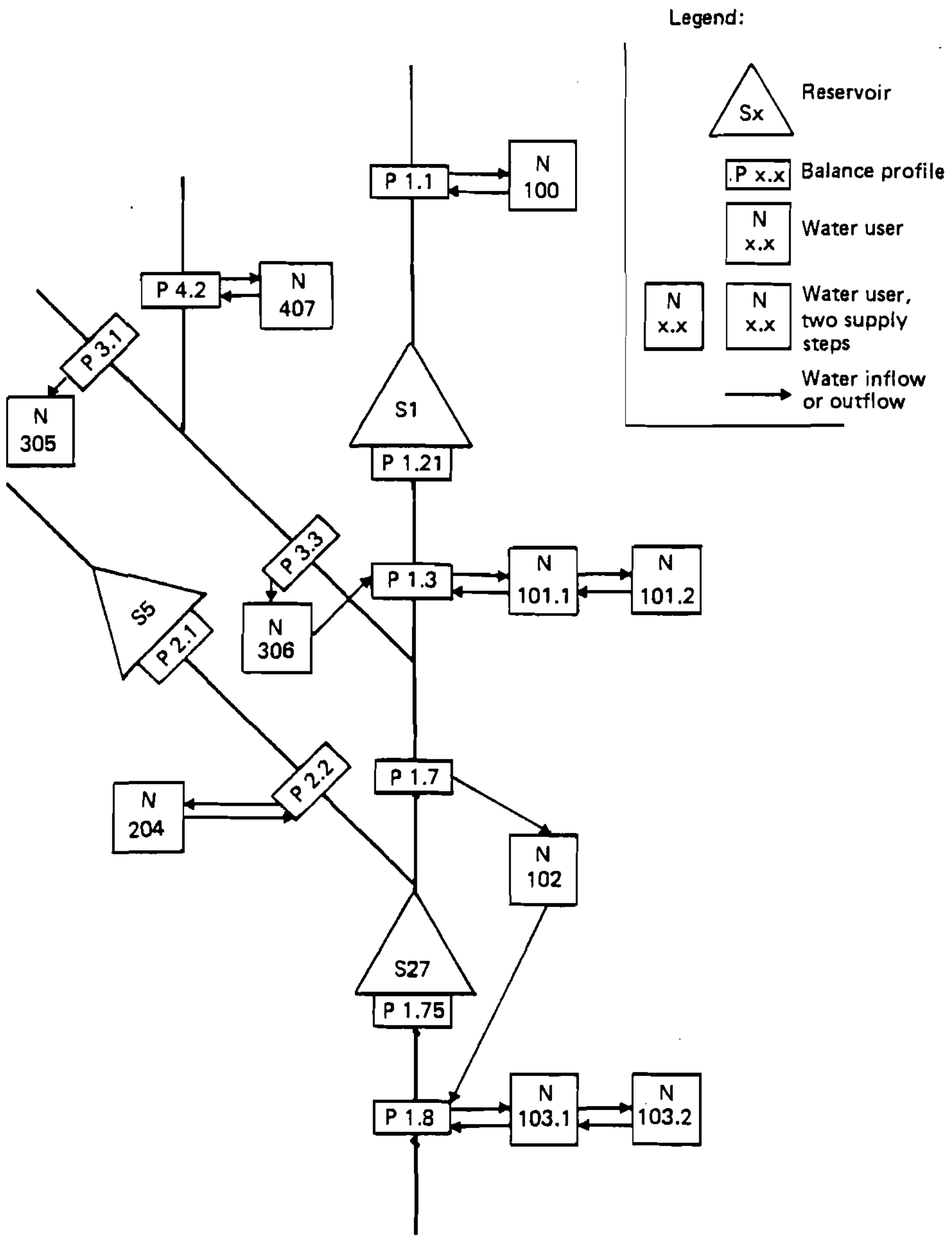



Figure 9: Location of water users with respect to the balance profiles

The model allows for more than one user to be defined at one balance point. It is as well possible to split up a single water supply requirement into a number of "user elements" with different priority (multistage supply scheme).

To define a reservoir in the model a decimal identifier, its location, capacity and seasonal varying usable storage volume must be specified. The simulation of reservoir operation is based on the principle "water release according to user requirements" with due consideration to given upper and lower storage limits. Furthermore, the model enables to reserve particular storage zones and to assign them primarily to certain users or user groups. User elements can also be applied to describe the transfer of water from one river section to other parts of the basin.

In general by the above specifications the long term strategy of a river basin management system may be sufficiently described.

### 2.3.2.5. Deterministic simulation of the water management processes

Based on the above mentioned runoff simulation and description of the management strategy a deterministic simulation of the monthly water allocation and reservoir operation activities can be performed, i.e. water management balance and reservoir release computations. Within the model the water allocation procedures (allocation of the available water resources either in space or in time) are represented by the following state variables:

$D(l)$ - actual runoff at balance point $l$
$DS(l)$ - saved portion of the actual runoff $D(l)$ at balance point $l$, that reflects the amount of water already assigned to users of higher priority
$SI(s)$ - actual storage in reservoir $s$
$AE(n)$ - actual water supply to user $n$.

In opposition to the real water utilization processes which simultaneously occur at the respective user sites, in the model the users are balanced successively according to the given ranking sequence. Based on the reservoir operation procedures the reservoir release is inserted into the sequence of user balancing. As a result of each of the users balancing and supply or reservoir operation activities the values of the respective state variables are immediately estimated. Thus the entity of the state variable values reflects at any intermediate simulation step all water consumptions, minimum flow requirements and reservoir operation steps already being considered. This principle allows a simple and illustrative formulation of the actual constraints for the balancing of a distinct user.

Rather simplified, the overall simulation procedure within a certain month can be described as follows:

(1) Define the initial systems state.
- Read in the generated runoff values for all simulation subareas (STG) for the current month.
- Allocate the respective runoff portions to the balance profiles (according to the area percentage).
- Cumulate the runoff contributions downstream to establish the initial runoff state.
- Establish the initial reservoir state by setting the according variables to the resulting values of the previous month.

(2) Compute the balance of all users according to their ranking with consideration of reservoir releases at the proper place of the rank list.
- Compute the actual water demand for the user to be balanced.
- Set up the users balance for the respective balance profiles considering of the saved runoff portions $DS$ at all downstream balance profiles,
  - in case of positive balance: full supply,
  - in case of negative balance: reduced supply (potential reservoir releases are already contained in runoff variables $D$).
- Compute actual consumption and return flow values and carry out the resulting runoff state corrections (decreasing D).

(3) Compute the final reservoir state. According to the reservoir operation principle a final check has to be made: What portion of the (maximum) release was actually necessary to meet the users' requirements? Then the free runoff (i.e. the amount of water not required) is taken from the river and (with due regard to the upper storage limits) "filled back" to the respective reservoirs.

#### 2.3.2.6. Registration and analysis of the systems state and critical events

In order to complete the monthly balance calculations a statistic counting procedure is called for registration of:

- state variables, e.g. runoff at selected balance profiles, actual users water supply and related quantities (such as extra costs, losses etc.), actual reservoir release and storage values,
- selected events, e.g. beginning and duration of user supply deficit periods.

Besides this, informations are stored to provide for final computations of water supply reliablities. There are three different types of reliability values:

1. Reliability of frequency $P_H$

$$P_H = \frac{number\ of\ nonfailure\ years}{total\ number\ of\ simulated\ years} \tag{2.5}$$

2. Reliability of duration $P_D$

$$P_D = \frac{number\ of\ nonfailure\ months}{total\ number\ of\ months} \tag{2.6}$$

3. Reliability of amount $P_M$ for a fixed time period

$$P_M = \frac{accumulated\ actual\ water\ supply}{accumulated\ water\ demand} \tag{2.7}$$

The frequency distribution tables or the single frequency figures obtained from the monthly registrations converge to probability distribution tables or reliability values respectively, provided the runoff simulation model is fairly reliable. In practise, a reasonable accuracy is achieved with simulation runs over 1000 to 2000 years.

#### 2.3.2.7. The program system GRM

The current level of stochastic management modeling is represented by the program system GRM implemented on a BESM-6 computer, see Kozerski 1981. The GRM model is a generalized program system which has entirely been developed according to the principle "automatic model setting by input data instead of writing an individual program". The user-oriented data representation scheme allows a direct model application by the local water management authorities.

All specifications needed to adapt the generalized GRM model to the basin to be investigated are classified into seven input data groups (DIC = data group identification code), see Table 1.

Data groups 1 - 4 and 9 have already been described in previous Sections. Group 8 is a numerical data modification option facilitating the formulation and running of series of typical alternatives for the estimation of rational management strategies. The so-called "dynamic elements" can optionally be applied, when special management activities are to be inserted into the simulation procedure, which cannot be described by standard model elements (standard user balancing and reservoir operation algorithms). Dynamic elements are free programmable sections, that consists of single statements and/or calls of external subroutines or

Table 1: Input data groups for the model system GRM

| DIC | **specified model object** |
|---|---|
| 1 | balance profiles (river network configuration) |
| 2 | basic runoff values (parameters for setting up the initial state) |
| 3 | user elements |
| 4 | reservoirs |
| 7 | dynamic elements |
| 8 | data modification option |
| 9 | registration |

procedures. By an internal generating module they are inserted into the standard computation sequence. They can be applied for a large variety of model functions, e.g. tests and/or settings of state variables or state-dependent parameter manipulations of standard elements. That option of the model essentially contributes to the model's flexibility and its capability to cover even unusual strategies as well as to be coupled with complete external models (e.g. for economic assessment etc.).

With the configuration and the management strategy of the system under investigation being specified by input data according to the above mentioned data groups and, if the standard functions of the program need to be altered, the selection of the respective model options, all further steps of the computer run are automatically controlled. That includes: setting up the internal configuration, definition of the required arrays and their dimensions, input of the numerical parameters, generating the individual sequence of management simulation algorithms, start of the simulation run and final output of the results.

### 2.3.3. Extensions of the Basic Management Model

#### 2.3.3.1. Disadvantages of the basic stochastic management model

Since the program system GRM has been put into regular practical application, the basic stochastic management model has become a powerful and versatile tool in long-term management of surface water resources in river basins. Obviously this is a consequence of its simple and illustrative input data representation and the capability to reflect the stochastic character of the runoff process. The latter aspect is of increasing importance in groundwater management and water quality investigations. These advantages of the basic model could be achieved owing to some heavy simplifications, which however cannot be accepted generally:

(1) The basic model assumes a fixed system configuration and defined management strategy for one simulation run, that reflects either the present conditions or a future balance horizon. A model extension is possible covering balance periods (planning periods) with varying system configuration and strategy instead of fixed time horizons. This kind of extension is necessary, if there are trends in the runoff formation processes due to men's activity (e.g. in large scale open-cast mining areas) or if the filling process of very large reservoirs has to be considered. Those variations affect data input and registration only while the modeling principle is left unchanged.

(2) With the time step of one month the actual runoff variations are smoothed out. It is particularly impossible therefore to integrate flood management directly into the model. This disadvantage can be remedied: If high values of monthly runoff occur, stochastically generated flood hydrographs are used to replace the constant monthly mean value, and the management algorithm is switched over to a flood procedure with a suitable time step (e.g. 1 day), see Thiele 1981.

(3) For the basic model it is assumed, that there is no coupling between groundwater and surface water management. However this is valid for hilly regions only, while in plain river basins this cannot be assumed because of the intensive water exchange between the components of the water resources system. The following two chapters deal with possibilities to remedy this disadvantage.

(4) The basic model does not allow for water quality management problems to be investigated. Only a few water quality parameters (conservative tracers, e.g. salt) can easily be integrated into the model. Such parameters depend directly on the runoff and can be controlled by means of reservoir releases or buffer pools. However most of the water quality processes - due to their complexity and internal interdependences as well as the dependence on the water quantity process - are often causing serious modeling problems, even if a relatively detailed model is applied. Therefore the issue of an adequate representation of water quality aspects within the stochastic management model is far from being solved in the next future.

#### 2.3.3.2. Application of reduced groundwater flow models

A possible approach to the integration of groundwater management into complex system models is shown in Section 4. Based on computer runs of different alternatives with the comprehensive groundwater flow model reduced submodels can be derived. In principle these models fulfill most of the requirements for their integration in the GRM model. For the GDR test area e.g. the following series of annual mean values have been estimated:

- inflow of mine drainage water into the river system,
- water level rise in the remaining pits due to refilling from the aquifer,
- water exchange rate between groundwater and surface water in the river system (infiltration/exfiltration).

The essential precondition for the calculations with the "comprehensive groundwater flow model" is - besides the heavy simplifications of the river system in the model - the assumption that the natural groundwater recharge is constant over the entire planning period. The spatial variability of the groundwater recharge conditions is considered in the individual elements and is updated for each planning period. This procedure is based on the assumption that the time-dependency of the groundwater recharge is almost entirely smoothed out by the subsurface flow process and that the significant factors are the local conditions governed by lignite mining activities. A higher temporal resolution in the above mentioned submodels is necessary to simulate the effects of certain management activities:

- The utilization of the remaining pits as water reservoirs results in variations of the natural refilling process.
- Significant runoff changes in the river system caused by management activities (water diversion or water intake) lead to water level changes which in turn influence the exchange rates between groundwater and surface water.

For the purpose of regional modeling these processes are considered to be local as a preposition for the desired simplification of the model. The requirement put on thereby are fulfilled by the methodology of model reduction developed in Sections 3. and 4.

#### 2.3.3.3. Application of deterministic catchment models

In opposition to the direct runoff simulation scheme described above a second approach to the integration of groundwater management issues is the indirect runoff generation based on meteorological factors. For this purposes advantage may be taken of the deterministic catchment models being well-known in surface water hydrology, which are mainly applied for flood forecasting, in an increasing degree also for continuous flow simulation.

Based on the long-term experience in the application of the conceptual catchment model EGMO (Becker 1975) some advanced versions of this model have been developed recently, above others:

- the model EGMOF for continuous flow simulation and forecasting on a daily basis (Becker 1983) and
- the model EGMOD for long-term flow simulation with time increments of 10 days or 1 month.

The model EGMOD is a reduced modified version of EGMOF. It is designed to transform the input variables precipitation (monthly sum), potential evaporation (monthly mean) and air temperature (monthly mean) into the mean monthly runoff of the catchment under consideration. Hence for the indirect runoff generation a stochastic simulation of the above three meteorological processes is required. It turned out, that the simulation technique described in Section 2.3.2.3. can be applied to these processes, too.

For the purpose of the model it is assumed, that the basin is horizontally classified into three hydrographic types (deep groundwater level, shallow groundwater level and water tables, see Figure 11) with the first two types having an additional vertical subdivision into three or two layers respectively (see Figure 10).
The model output consists of three runoff components:

- overland flow (surface runoff),
- interflow (hypodermic runoff),
- base flow (groundwater runoff).

By modifying the respective portions of the above classified areas (see Figure 11) subsurface water exchange with neighboring basins can be simulated.

Thus in a simple way time-dependent groundwater depression cones can be included into the model. In the EGMOD-model these depression areas do not provide a significant natural contribution to the river runoff, rather their runoff is considered as a portion of the water extraction from the cone of depression.

The EGMOD-model is adapted to an individual basin by specifying 6 hydrographic parameters which are obtained from special hydrographic maps as well as 9 system model parameters estimated from runoff series of at least five year length. In applying the EGMOD-model in extended groundwater depression areas for the runoff simulation some additional data are necessary, which can only be obtained by means of comprehensive or reduced groundwater flow models. Those data are (with respect to a defined balance horizon):

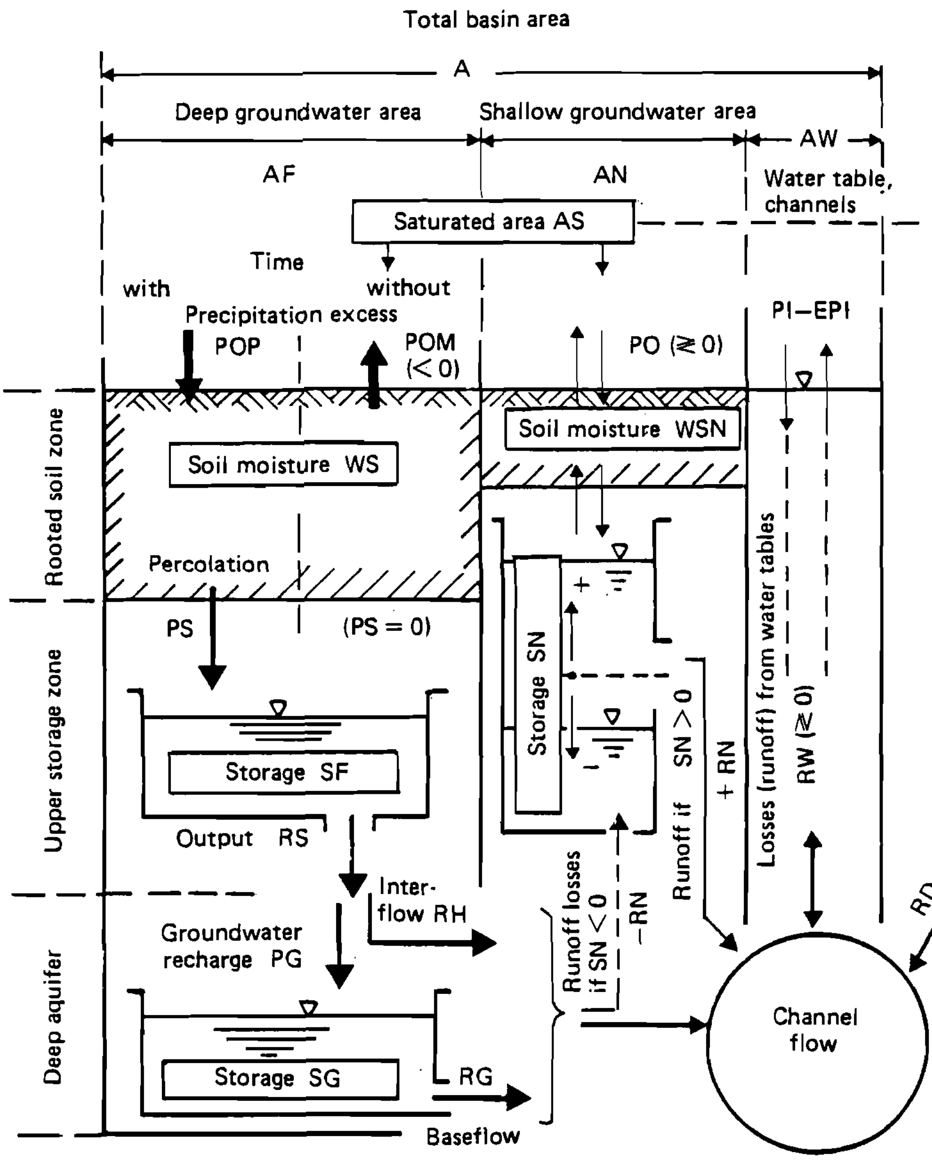



Figure 10: Schematic representation of the EGMOD catchment model

- local extension of the depression cones,
- infiltration rate from the river system to the cone,
- inflow of mine drainage water into the river system.

## 3. Methodology of Model Reduction

### 3.1. Principle Working Steps

Discussions in the previous sections demonstrate an apparent need for the development of reduced models of regional flow processes. The demand for such models results from the concept of complex Decision Support Model Systems (DSMS). Furthermore, the need for reduced models may be interpreted as a modified form of the requirement to apply *problem-adequate* models. The integration of a comprehensive regional model as an element into a DSMS is not only a computer problem, but primary a question of reasonable means. The effort for the

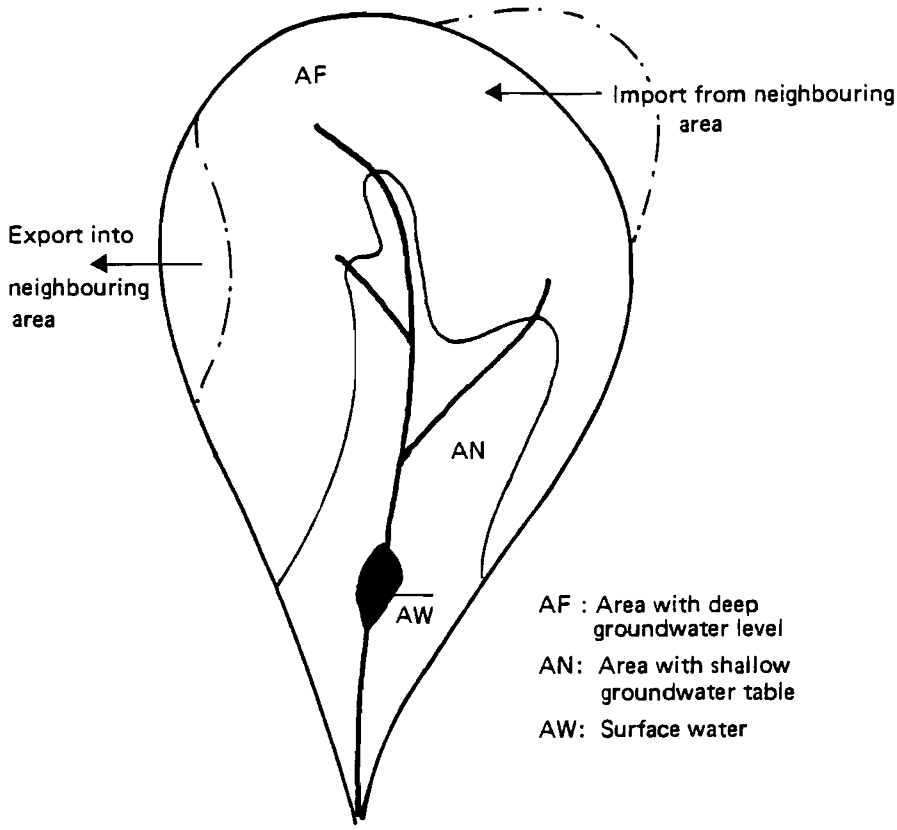



Figure 11: Hydrographic classification of a river basin

development of a submodel has to be in accordance to the importance of the process under consideration.

Generally, there are two different approaches for model reduction:

(1) Simplification of the mathematical model of the flow process, which is supposed to be a comprehensive model in the sense of Section 2.1, in such a manner that the simplified model can be used as an element of the DSMS.

(2) Utilization of the comprehensive model as an analogue of the natural processes for synthetic data generation. Reduced models are derived by fitting to these synthetic data.

Both procedures have advantages and disadvantages. Obviously, the type of the required submodel and the way for model reduction depends on the model level under consideration.

The *first approach* results in general applicable solutions supposed it does not include empirical object-specific data. The latter is the case for typical hydrological applications. For the model development especially in the operational hydrology the completeness and reliability of observed time series become fundamental. An adequate approach has been discussed in Section 2.3.

The *second approach* is directed towards a rational storage and processing of computational results of the comprehensive regional model. It is aspired to realize a similar accuracy as with the comprehensive model. This is possible if the results of the model can be processed as a time series, see Sections 3.2, 4.2. Such an approach is reasonable above all for the planning model based on the planning periods as time steps $\Delta t \geq 1$ *year*. The state of the hydrological system is described by discrete time functions (state descriptive functions), in many cases depending on only a few parameters. Typical examples of such submodels are given in Section 4.2. Fluctuations of infiltration/exfiltration during the year and even between years depending on the river flow are negligible. The natural groundwater recharge is considered to be constant in time over the planning period, but variable in space due to the recharge conditions effected by open-pit mining.

The management model is based on monthly time steps. With respect to the interannual water balance situation it is necessary to consider all groundwater/surface water interaction processes as systems variables. Therefore, adequate submodels involving the history of the process are needed. The reduced model has to be not only an effective means for storage of computed data, but it has to simulate the functional relationships for defined parts of the comprehensive model. In most cases for that a linearization becomes necessary.

Only box-models may be used as reduced models in this sense, conceptual or black-box type. With regard to the transition function box-models may be deterministic or stochastic. In the field of groundwater management deterministic box-models are dominant.

Another point of view is the way for obtaining the transition function. In the case of conceptual box-models the transition function is derived from special analytical solutions of the system descriptive model. Therefore, the parameters of this type of models allow for a clear physical interpretation. Such models have the advantage that they may be derived for regions even if no comprehensive model is available.

A physical interpretation is not possible and not necessary for black-box models. The parameters of transition functions are obtained by fitting empirical or theoretical formulas to observation data or calculations using the comprehensive model. With respect to the compatibility of the planning model and the management model, however, it is helpful, if the coefficients of the black-box model are given as explicite functions of the time step under consideration.

Grey-box models are a compromise between conceptual box-models and black-box models, see Section 3.2.3.

In Figure 12 an overview on the possibilities of model reduction is given.

Examples for the different ways of model reduction are described in Section 4. In the following, some mathematical approaches for model reduction are discussed.

## 3.2. Types of Reduced Models

### 3.2.1. Time Series of Systems Descriptive Values

It is out of question that a comprehensive regional flow model is not suitable as a submodel for a complex DSMS. But, in many cases computational results of regional models, prepared as time series, are an adequate form for the quantitative description of the relevant subprocesses. Such types of reduced models may be called *parameter-free models*.

This approach is above all well suited for the systems descriptive values within a planning model. The small number of time steps (about 10 planning periods) enables to process the results of the comprehensive groundwater flow model even in off-line mode. Variable influence values (decision variables) can be considered by interpolation between the computational results for different variants. This interpolation is done purly formal without analyizing the physical contents of the flow model.

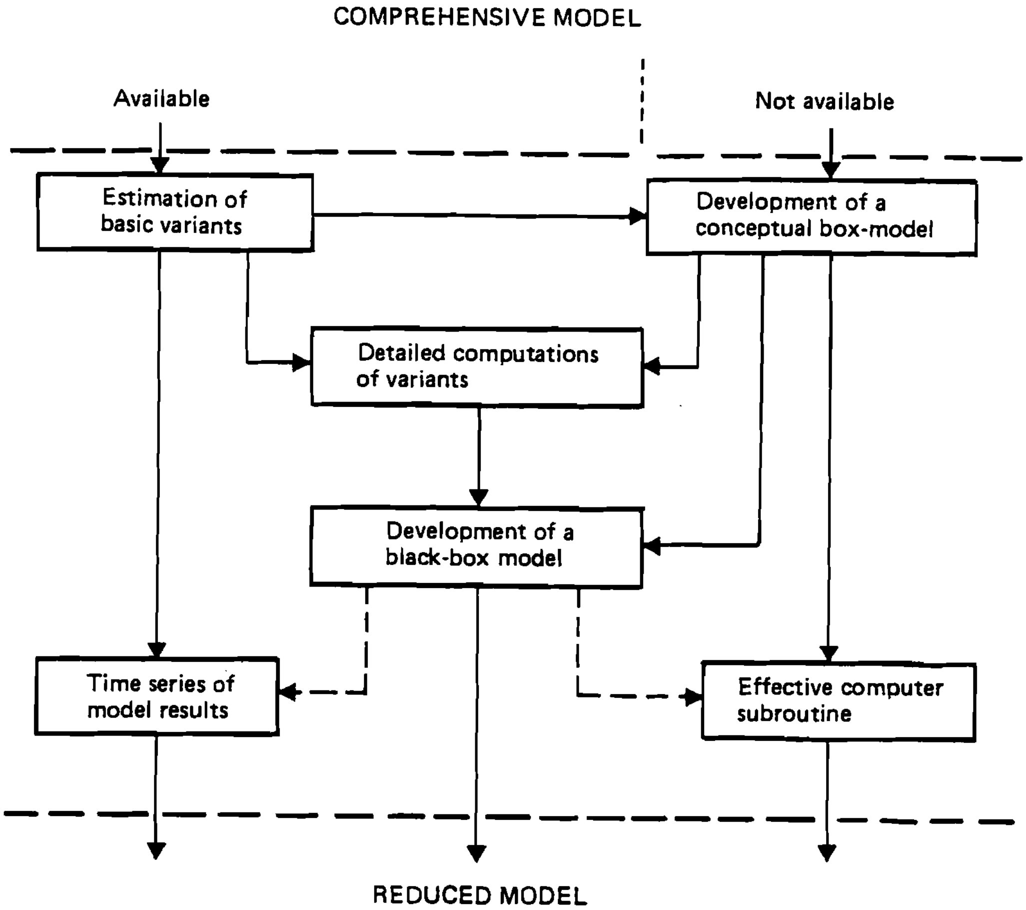



Figure 12: Ways for model reduction

The following approach is proposed (Kaden et.al. 1985a):

In a first step the comprehensive regional flow model is used for the simulation of an average systems behavior $\bar{S}$ in each planning period $j$, caused by mean expected inputs $\bar{I}$ and decisions $\bar{D}$, considering the nonlinearity of the groundwater flow in the entire region. As a result we get expectation values for groundwater tables $\bar{h}(t)$, groundwater pumpage $\bar{c}(t)$ etc. as functions in time. It is assumed that the actual inputs $I$ and decisions D are close to the mean expected values $\bar{I}$ and $\bar{D}$.

In a second step the comprehensive regional groundwater flow model is used for the estimation of the consequences on the systems development $\Delta S$, caused by changes of inputs $\Delta I$ or decisions $\Delta D$. E.g. for the GDR Test Area we studied separately the consequences of the changes in the filling process of the remaining pit as well as changes of the predewatering process in one of the mines on the development of the groundwater tables in an agricultural area $\Delta h_{ag}$.

By means of superposition of the average systems behavior $\bar{S}$ and the separately studied consequences $\Delta S$ in a third step a usable model of the systems behavior S is obtained, assuming that the error due to the nonlinearity would be small. This can be checked by the comprehensive regional flow model.

This type of model reduction fits well to the mode of work and output of complex regional models. In Section 4.2 examples for the GDR Test Area are given. Its bounds of possibilities are exceeded if systems descriptive values have to be modeled. In such case functional relationships have to be derived. Then the computational results of the regional flow model form the data base for the development of reduced models.

### 3.2.2. Recursive Difference Equations

From the computational point of view *recursive difference equations* are a particular advantageous form of reduced models. As a simple example let us examine the case of one *systems descriptive value* $y(t)$ depending on one *influence value* (decision variable) $w(t)$. The time dependency of $w(t)$ is approximated as a step function

$$w(t) = w_i \quad \text{for} \quad (i-1)\cdot\Delta t \leq t < i\cdot\Delta t \quad . \tag{3.1}$$

For the function values $y_k = y(k\cdot\Delta t)$ the following statement is given:

$$y_k = \sum_{i=1}^{m} a_i\cdot y_{k-i} + \sum_{i=0}^{n} b_i\cdot w_{k-i} \quad . \tag{3.2}$$

The parameters $m$ and $n$, which characterize the "memory" of the investigated process, have to be estimated in the framework of model adaption. Our investigations did result in the conclusion that the parameters $m = 2$ and $n = 1$ are practically most important. It can be proven analytically that the recursion equation

$$y_k = a_1\cdot y_{k-1} + a_2\cdot y_{k-2} + b_0\cdot w_k + b_1\cdot w_{k-1} \tag{3.3}$$

may be interpreted as an analogue of a linear differential equation second order with constant coefficients. The coefficients $a_1$, $a_2$, $b_0$, $b_1$ have to satisfy certain consistency conditions (see Section 4.3.3).

This interpretation is methodological important because it enables us to comprehend the dependency between the coefficients and the time steps $\Delta t$ in a mathematical expression. And this is the solution for the problem of compatibility between the planning model and the management model formulated in Section 1. In both model levels submodels of the form (3.3) are used. Their coefficients result from different $\Delta t$ values.

The estimation of the coefficients $a_i$, $b_i$ based on simulation results is a typical approximation problem to be solved e.g. by the *last square method.* Some additional possibilities result if the function $y(t)$ is described by an analytical transition function. Then a discrete weighting function can be derived:

$$y_k = y_0 + \sum_{i=0}^{k-1} g_{i+1}\cdot w_{k-i} \quad . \tag{3.4}$$

Inserting Eq. (3.4) into (3.2) results in conditional equations for the coefficients $b_i$ of the influence values $w_{k-i}$ and $a_i$ of the systems descriptive values $y_k$. These equations form an overdetermined system of equations. It may be solved approximately with common methods. Another approach is to satisfy the equations for the actual influence values $(w_k, w_{k-1}, w_{k-2}, \ldots)$ and to realize the exact stationary value of the transition process ($w = const.$).

In Section 4.5 an example is explained.

### 3.2.3. Grey-Box Models

Another powerful approach for the development of reduced models is the application of simplified mathematical models (differential equations) of the investigated processes. In difference to the black-box models the model parameters may be interpreted physically, but their optimal values are generally adapted to given simulated or observed data.

This intuitive way of model reduction plays the dominant role in modeling water quality subprocesses, see Luckner et.al. 1985.

Here it is applied for modeling the interdependencies remaining pit - groundwater, see Sections 4.3 and 4.4. Formal the same results are obtained as interpreting the recursive Eq. (3.3) as a differential equation. The submodels are well suited both for the planning model and for the management model.

## 4. Examples for the Development of Reduced Models

In the following several types of submodels are discussed which are needed both, for a planning model with time steps of one year and larger, and/or for a management model for monthly simulation of the essential hydrological processes. All examples refer to the GDR test area being considered in the IIASA case-study, see Kaden et al. 1985a,b.

A short description of the test area is given in the following Section 4.1. In Section 4.2 the estimation of submodels for the planning model is described, based on the regional groundwater flow model developed by Peukert 1979. In Sections 4.3.-4.5. typical variants of the proposed methodology of model reduction are presented especially for the management model.

### 4.1. The GDR Test Area

The selected test area is located in the south-eastern part of the German Democratic Republic in the so called Lusatian Lignite District, the most important and eldest lignite mining centre of our country. Since the middle of the 19-th century open-pit lignite mines are in operation there. The test region depicted in Figure 13 is an about 500 $km^2$ large part of the mining area.

The aquifer system of the test area can be schematized in tree aquifers (the first being unconfined), separated by aquitards (lignite). The boundary of the test area is not identically with the subsurface catchment area. Groundwater inflow, outflow respectively have to be considered. The region is crossed by a stream and some tributaries.

Due to mine drainage the groundwater flow is strong influenced. The cone-shaped groundwater depressions caused by the dewatering meassures of these mines superimpose each other. This superposition-state is permanently changing in space and time due to the progress of mining. Consequently municipal and industrial water supply plants as well as agricultural and environmental valuable areas are influenced. The groundwater and surface water resources are closely interrelated (baseflow into surface waters under natural conditions, infiltration (percolation) of surface water into the aquifer in the course of groundwater lowering due to mine drainage).

After closing lignite mining (in the test area mine A), resulting from the abandoning of dewatering measures the regional groundwater lowering process is additionally superimposed by the groundwater rebound. This groundwater rebound process is forced by a newly formed remaining pit, used as a water reservoir and for flood regulation by the water management authority. The rise of the water table in the remaining pit up to the planned final water levels increases the amount of water pumpage of drainage wells operating in the vicinity of these remaining pits.

The inflows into the region from the stream and the tributaries are natural ones depending on the hydro-meteorological situation in the upstream catchment areas.

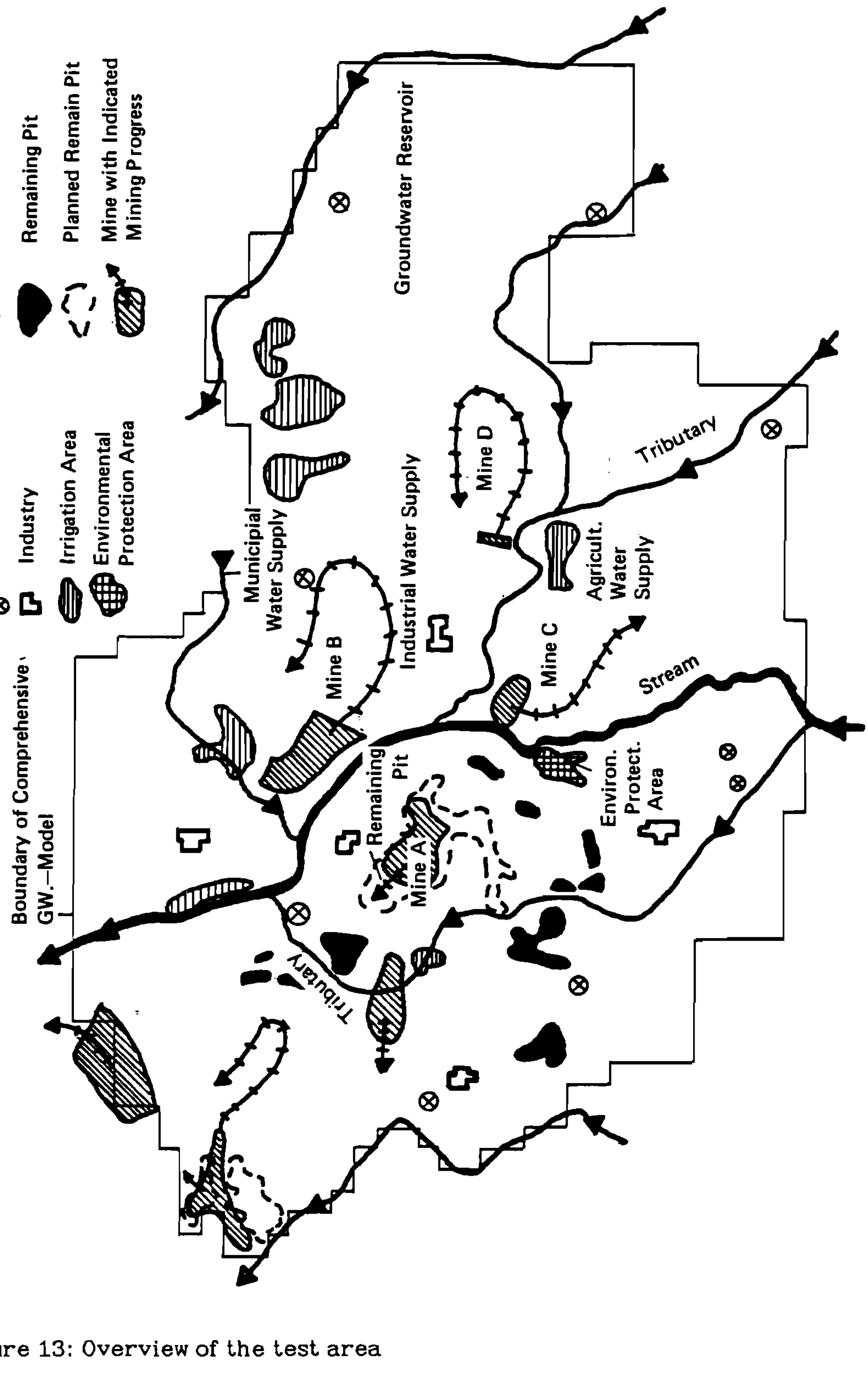



Figure 13: Overview of the test area

In the test regions all important impacts on the water resources system being typical for mining areas are considered.

### 4.2. Modeling Mine Drainage and Groundwater Tables in Selected Areas

In order to analyse the impacts of lignite mining on groundwater flow it was necessary to use a comprehensive regional groundwater flow model.

For the whole area a regional continuously working model is already existent. This model is in operation in its third stage of improvement and complementation. For the simulation the two-dimensional non-steady model HOREGO was chosen, see Section 2.2. The finite elements grid consists of about 1000 elements with an area between 1 and 4 $km^2$ (see Peukert 1979 and Peukert et al. 1982).

The model was calibrated for a period of 8 years. The groundwater flow process in the test region was well known for this period due to sufficient measurements of the groundwater table and of water pumpage in the individual open-pit mines. The calibration of the groundwater flow model was done by trial-and-error. Especially the transmissivity of the aquifer system and boundary conditions of the model have been varied.

In the following some results of the simulation for the development of submodels and for the estimation of the consequences of varying inputs and decisions (control variables) by use of the existing comprehensive regional groundwater flow model are demonstrated, see also Kaden et al. 1985a,b, especially Figure 1 in Kaden et al. 1985b.

In the upper part of Figure 14 the development of the groundwater table in an agricultural area is shown. We see that in general the groundwater table is lowering with the movement of mines closer to the agricultural area. The main influence results from mine D in this case. The process of groundwater rebound in this area is postponed due to the drainage measures of mine B. This can be recognized in the curve after the 25th year. At this time mine D is already far away from the agricultural area, but the dewatering systems of mine B are still operating and the groundwater rebound process is delayed.

Curve 1 shows the development of the groundwater table at normal predewatering conditions in mine D, that means 3 years in advance. Curve 2 holds if the start of the dewatering measures is two years earlier and curve 3 if this start is two years later. The coincidence of all three lines after the simulation year 25 elucidates the decreasing influence of mine D at the groundwater table in this area.

In the lower part of this figure the development of the groundwater table in an environmental protection area is shown influenced by the filling process in the neighboring remaining pit. At the beginning the groundwater table development is influenced by the dewatering measures of the mines C and A. In the year 10 of simulation mine C is located in the closest distance from the environmental protection area. At the same time the drainage of mine A is getting out of operation. The groundwater rebound in this region begins. In the 17th year of investigation starts the filling process in the newly formed remaining pit situated in the area of the former mine A. Curve 4 shows the development of the groundwater table if the remaining pit will be filled with water from the river (100 $Mill.\ m^3/annum$) and curve 5 shows this development if the remaining pit will be filled only by inflow of natural groundwater. The impact of the filling process of the remaining pit on the development of the groundwater table in the environmental protection area is strongly dampened because the distance is about 6 $km$.

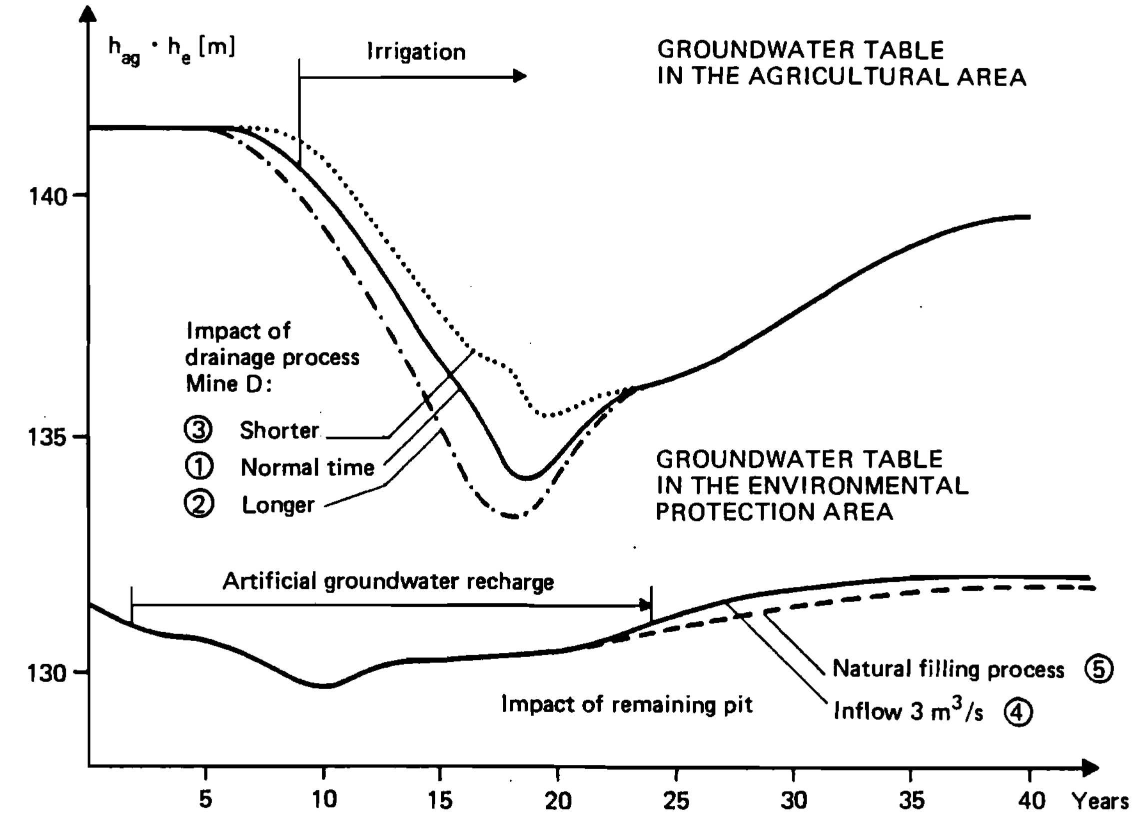



**Figure 14:** Influence functions for groundwater tables

In the lower part of Figure 14 the time period is marked for which an artificial groundwater recharge is necessary in the influenced area in order to prevent changes in the environmental protection area.

In the upper part of the Figure 15, the development of the exfiltration/infiltration behaviour of a river section is shown influenced by the filling process of the remaining pit. The exfiltration in this section is decreasing to about the 13th simulation year due to the drainage of the neighbouring mines A and B. Because of closed dewatering of mine A near the year 10 a short-term increasing of the exfiltration rate can be recognized. In the year 15, the influence of mine B becomes significant resulting in infiltration in this river section. From the 20th year the influence of the filling process in the remaining pit is obviously superimposed by the slowly reduced influence of mine B.

In the lower part of this figure the development of the exfiltration/infiltration behaviour of another river section is shown influenced by the changing dewatering process of mine D.

In Figure 16 the development of a simplified model for the groundwater pumpage in mine D is demonstrated, according to the approach in Section 3.2.1.

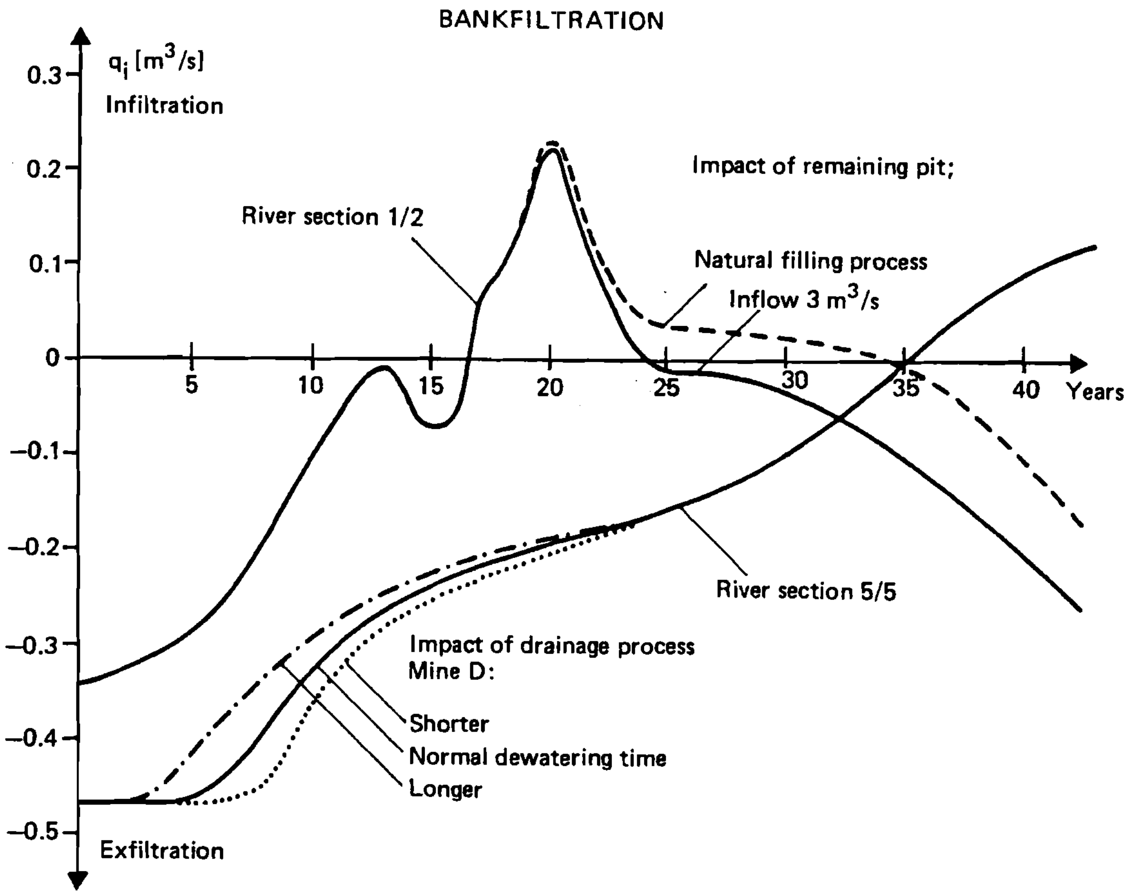


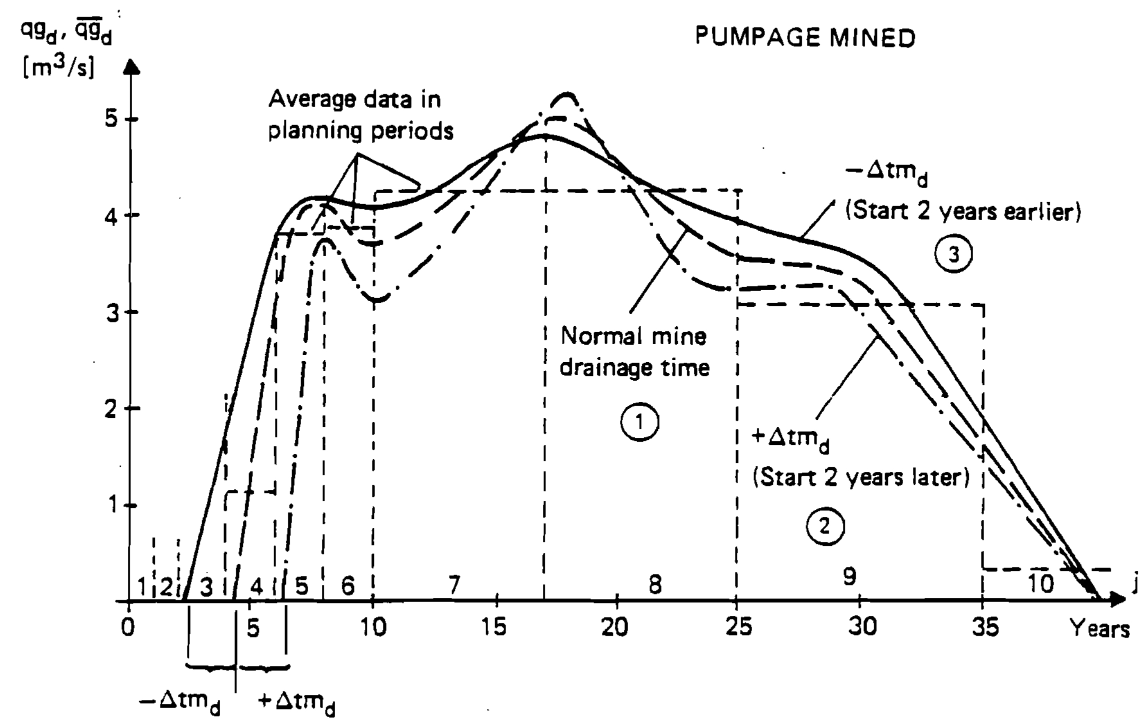

**Figure 15:** Influence functions for bank filtration

As mentioned above, three functions are shown, curve 1 for common dewatering time, curve 2 for dewatering two years later ($\Delta tm_d$ = +2) and curve 3 for two years earlier ($\Delta tm_d$ = −2). We recognize that the consequences of maximum values of water pumpage, caused by drastic changes of dewatering measures are dampened more or less depending on the size of the cone-shaped groundwater depression. It is evident that a longer dewatering time causes a larger cone-shaped groundwater depression than a shorter dewatering time and therefore the dampening is more or less strong.

In the lower part of this figure an analytic function is given, characterizing the development of water pumpage in mine D sufficiently accurately. The coefficients $a_1$ to $a_3$ can be taken from the table for each planning period $j$.

Altogether for the GDR test area the following submodels (system descriptive functions) were derived from the comprehensive groundwater flow model, using the same methodology:

- development of the groundwater level in an agricultural area,
- development of the groundwater level in an environmental protection area,




$$qg_d(j) = a_1(j) + a_2(j) \cdot \Delta tm_d + a_3(j) \cdot \Delta tm_d^2$$

| j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_1$ | – | – | – | 1.13 | 3.80 | 3.85 | 4.25 | 4.25 | 3.09 | 0.29 |
| $a_2$ | – | – | –.183 | –.700 | –.412 | –.175 | –.225 | –.063 | –.128 | –.023 |
| $a_3$ | – | – | .092 | .068 | –.131 | –.013 | –.013 | –.019 | –.011 | –.003 |

BOUNDS: $-2$ YEARS $\leq \Delta tm_d \leq +2$ YEARS

**Figure 16:** Submodel for groundwater pumpage

- development of the groundwater level in the vicinity of wells for municipal water use,
- groundwater pumpage of all mines,
- raising process of the water level in the remaining pit (see Section 4.3.),
- increasing of pumpage of mine B due to the filling process in the remaining pit (see Section 4.4.),
- development of infiltration/exfiltration behavior for all river sections (see Section 4.5).

The proposed methodology should be generally applicable. Nevertheless, results of modeling using such strongly simplified models, should be verified using comprehensive flow models.

## 4.3. Remaining Pit Management

### 4.3.1. Analysis of the Problem

In previous sections it has already been explained which impacts on the process of runoff formation are caused by the large-area drawdown of groundwater during the drainage of open-pit mines. These problems are not immediatly eliminated at the end of the mine drainage after closing mines. On the contrary, after abandoning of mine-drainage significant water balance deficits in the streams will occur temporary because the aquifer has to be recharged by precipitation and in the streams a balance compensation is no longer ensured by the inflow of mine drainage water.

An effective compensation of deficits can only be realized by the help of water reservoirs being included into the management of the surface water system. The hydrological utilization of the remaining pits in mining areas is the most preferable solution for a reasonable recultivation of the mining areas, to avoid water deficits, and to satisfy flood protection.

The use of remaining pits as reservoirs requires a considerable investment. Therefor its effectivity and its management strategy has to be estimated in advance. Two major stages have to be distinguished, the stage of recharging the remaining pit, and the management stage for its utilization in water management.

*Recharge stage*

The management of remaining pits takes place within a "usable storage layer". In order to get the water table of the remaining pit within this layer it is necessary to recharge the remaining pit after abandoning the drainage wells around the open-pit mine. This can either be done by natural groundwater inflow or additionally by artificial surface water or mine water inflow. The latter results in water losses by infiltration from the remaining pit into the aquifer.

Altogether the following problems arise in this stage:

- to determine the usable storage layer for management and for flood protection,
- to determine the water table of the remaining pit in time for the case of the natural recharge process up to the usable storage layer,
- to estimate a rational strategy for artificial recharge (refill).

*Management stage*

After reaching the usable storage layer the remaining pit can be used as a water reservoir. In such a case, the management is analogously to reservoir management including flood protection. One difference is that the storage basin is located in the by-pass of the stream. Due to this, a pumping station can be included in the management to transfer water between the remaining pit and the stream especially for flow augmentation.

Within the framework of a time-variable management (on monthly basis) the following problems have to be solved:

- to estimate the response of the remaining pit on variable monthly discharge and intake;
- to determine the water intake needed for compensation of infiltration losses;
- to investigate an effective management strategy;

- to estimate the consequences of the management on the surroundings (e.g. increased mine drainage for neighbored mines).

### 4.3.2. Approach for Model Reduction

For model reduction the comprehensive groundwater flow model described in Section 4.2 has been used, especially the facilities for modeling water bodies. Detailed computations of variants of the recharge and management stage has been done with the comprehensive flow model in order to get comprehensive synthetical data about the nonlinear process of remaining pit management as a base for model reduction.

In Figures 19 and 22 computational results for management variants with the comprehensive groundwater flow (and reduced models) are depicted. As the dominant input, the difference between the inflow into and the discharge from the remaining pit was varied over an interval being realistic from the hydrological point of view. These model results serve further on to quantify and/or to evaluate the model precision of the reduced models.

In Figure 17 the principle way of model reduction is depicted.

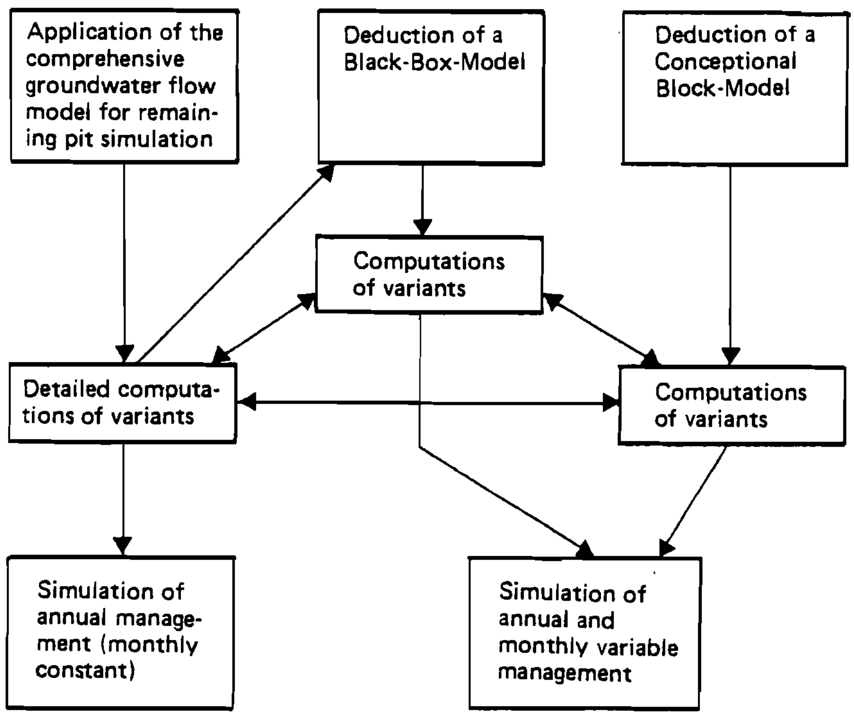



Figure 17: Steps for model reduction of remaining pit management

As a first step of model reduction, a black-box model in terms of a difference equation considering a history of 2 years was found to be the best suited model. According to Section 3.2.2., this type of model is considered as analogue of a differential equation second order. In the following Section 4.3.3. a physical interpretation of this model is given. Simultaneously a conceptual box-model of the remaining pit management has been developed, see Section 4.3.4.

All models are designed to describe the dynamic behavior of the water table in the remaining pit in dependence on its management.

### 4.3.3. Grey-Box Model

The model structure is derived by the help of a block concept subdividing the area under consideration into three blocks:

- the remaining pit,
- the aquifer around the remaining pit (GWL1) which is directly influenced by the remaining pit,
- the neighbored part of the aquifer (GWL2) which is undisturbed by the other blocks (see Figure 18).

The water table in the remaining pit and the groundwater table are the state variables, assumed to be constant within their blocks. The blocks are connected by the continuity equation and a kinetic equation. In order to get a simple model structure two assumptions have been formulated which enable an approximate linearization of the problem:

(1) The horizontal area of the remaining pit at any water level is proportional to the corresponding area of exchange between the remaining pit and the GWL1. Such an assumption permits the linearization of the dynamic behavior if the reaction of the storage GWL1 on the remaining pit is negligible.

(2) The influence of the remaining pit management on the dynamic systems behavior is small and is assumed to be approximately linear. For example this holds true if the variation of the exchange area of the remaining pit is small in relation to its water table.

Especially within the usable storage layer (from $108\,m < h_p < 118\,m$ ) both assumptions are justified. Possible influences of external boundary conditions on the dynamic systems behavior are separated by subtraction of two different management variants with the same external boundary conditions.

Based on the assumptions above we obtain for the dynamic behaviour of the water table for two interacting storages (Kindler 1972):

$$D_1 \cdot D_2 \cdot \frac{d^2 \tilde{h}_p}{dt^2} + (D_1 + D_2) \cdot \frac{d\tilde{h}_p}{dt} + \tilde{h}_p = K \cdot q_p \tag{4.1}$$

with

| | | |
|---|---|---|
| $D_1, D_2$ | - | time constants |
| $K$ | - | proportionality constant |
| $q_p$ | - | constant inflow into ( > 0 ) or discharge from ( < 0 ) the remaining pit |
| $\tilde{h}_p$ | - | difference between the actual water table and that for natural recharge. |

The homogeneous solution of the differential equation (4.1) can be given as a homogeneous recurrence equation of second-order:

$$\tilde{h}_p(t_j) = (P_1 + P_2) \cdot \tilde{h}_p(t_{j-1}) - P_1 \cdot P_2 \cdot \tilde{h}_p(t_{j-2}) \tag{4.2}$$

$$\text{with:} \quad P_1 = e^{(-\frac{\Delta t}{D_1})}, \; P_2 = e^{(-\frac{\Delta t}{D_2})}, \; \Delta t = t_j - t_{j-1}$$

After integration of the differential equation (4.1) with the initial conditions

$$\tilde{h}_p(0) = 0 \; , \; \frac{d\tilde{h}_p(0)}{dt} = \frac{\Delta t}{D_1 \cdot D_2} \cdot K \cdot q_p \tag{4.3}$$

we get the transition function $S(\Delta t)$ in the following form:

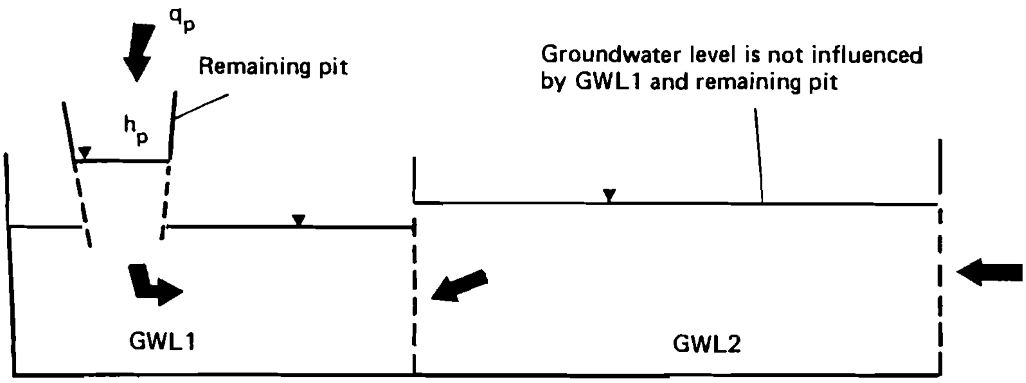



Figure 18: Block structure of the grey-box model

$$S(\Delta t) = 1 - c_1 \cdot P_1 - c_2 \cdot P_2 \text{ , with } c_1 = \frac{\Delta t - D_1}{D_2 - D_1} \text{ , } c_2 = 1 - c_1 \quad . \qquad (4.4)$$

Through superposition of the discrete time-dependent inflow into or discharge from the remaining pit we obtain based on the homogeneous solution:

$$h_p(i) = h_p^0(i) + (P_1 + P_2) \cdot \tilde{h}_p(i-1) - P_1 \cdot P_2 \cdot \tilde{h}_p(i-2) + \qquad (4.5)$$
$$+ (1 - c_1 \cdot P_1 - c_2 \cdot P_2) \cdot K \cdot q_p(i) + (P_1 \cdot P_2 - c_2 \cdot P_1 - c_1 \cdot P_2) \cdot K \cdot q_p(i-1)$$

with

| | | |
|---|---|---|
| $i$ | - | time step in years |
| $h_p^0(i)$ | - | water table in the remaining pit for natural recharge at the end of the year $i$ (see Appendix B) |
| $h_p(i)$ | - | water table in the remaining pit at the end of the year $i$ |
| $q_p(i)$ | - | inflow into or discharge from the remaining pit within the year $i$ in $\frac{Mill.\ m^3}{year}$. |

This inhomogeneous recurrence equation of second-order has three parameters - the two time constants $D_1$, $D_2$ and the proportionality constant $K$. These parameters are quantified by adaptation of Eq. (4.5) to discrete annual values of the water level in the remaining pit (calculated by means of the comprehensive grundwater flow model, see Section 4.2) for different management variants. The following parameters have been estimated for water tables within the usable storage layer:

$$D_2 = 4.21400\ [years] \text{ , } D_1 = 0.66234\ [years] \text{ , } K = 0.31276\ [\frac{years}{km^2}] \text{ .}$$

To apply the Eq. (4.5) also for water tables below the usable storage layer it was empirically modified with the arbitrary function $\gamma(i)$ .

$$\gamma(i) = \begin{cases} 2.22 - 0.004 \cdot q_p(i) & \text{for } i \le ip + 1 \\ 1.45 - 0.003 \cdot q_p(i) & \text{for } i = ip + 2 \\ 1 & \text{for } i > ip + 2 \end{cases} \qquad (4.6)$$

with $ip$ - year of opening the remaining pit.

Based on that the *grey-box model* for yearly time steps gets the following form:

*Annual model*

$$h_p(i) = h_p^0(i) + a_1 \cdot \tilde{h}_p(i-1) + a_2 \cdot \tilde{h}_p(i-2) + \\ + b_0 \cdot \gamma(i) \cdot q_p(i) + b_1 \cdot \gamma(i-1) \cdot q_p(i-1) \qquad (4.7)$$

with

$$\tilde{h}_p(i) = h_p(i) - h_p^0(i)$$

$$a_1 = 1.00971 \ , \ a_2 = -0.17428 \ , \ b_0 = 0.08295 \ , \ b_1 = -0.03150$$

Through modification of the time interval in the annual model (modification of parameters $P_1$ and $P_2$ in Eq. (4.2) we get the following monthly model:

*Monthly model*

$$h_p(i,k) = h_p^0(i,k) + a_1 \cdot \tilde{h}_p(i,k-1) + a_2 \cdot \tilde{h}_p(i,k-2) + \\ + b_0 \cdot \gamma(i) \cdot q_p(i,k) + b_1 \cdot \gamma(i-1) \cdot q_p(i,k-1) \qquad (4.8)$$

$$\tilde{h}_p(i,k) = h_p(i,k) - h_p^0(i,k)$$

$$h_p^0(i,k) = h_p^0(i-1\ ,\ 12) + (h_p^0(i\ ,\ 12) - h_p^0(i-1\ ,\ 12)) \cdot \frac{k}{12} \qquad (4.9)$$

with

$$a_1 = 1.86219 \ , \ a_2 = -0.86451 \ , \ b_0 = 0.00906 \ , \ b_1 = -0.00833$$

| | | |
|---|---|---|
| $k$ | - | number of month, $k = 1,...,12$ |
| $h_p^0(i\ ,\ 12)$ | - | water level in the remaining pit for natural recharge at the end of the year $i$ in meters |
| $h_p(i,k)$ | - | water level in the remaining pit at the end of month $k$ in the year $i$ in meters |
| $q_p(i,k)$ | - | constant inflow into or discharge from the remaining pit within the month $k$ in the year $i$ in $\frac{Mill.\ m^3}{year}$. |

In Figure 19 the water table in the remaining pit is depicted for different management variants comparing the results of the reduced model and of the comprehensive groundwater flow model. The standard deviation between the results of both models is 0.56 $m$ for a range of 36.95 $m$.

### 4.3.4. Conceptual Block-Model

#### 4.3.4.1. Derivation of the fundamental solution

In simplifying its geometry, a remaining pit can be considered as a well with a large diameter. Consequently it is possible to use analytical solutions of the well hydraulics as a transition function. The inner boundary condition of the classical THEISS-solution ($r \rightarrow 0$) has to be replaced by an adequately modified one since the storage effect of the "well" (remaining pit) is not negligible (see Figure 20).
According to Cooper et al. 1967 we get the following approach applying the **Laplace**-transformation:

*Differential equation:*

$$\frac{\delta^2 h}{\delta r^2} + \frac{1}{r} \cdot \frac{\delta h}{\delta r} = \frac{S}{T} \cdot \frac{\delta h}{\delta t} \qquad (4.10)$$

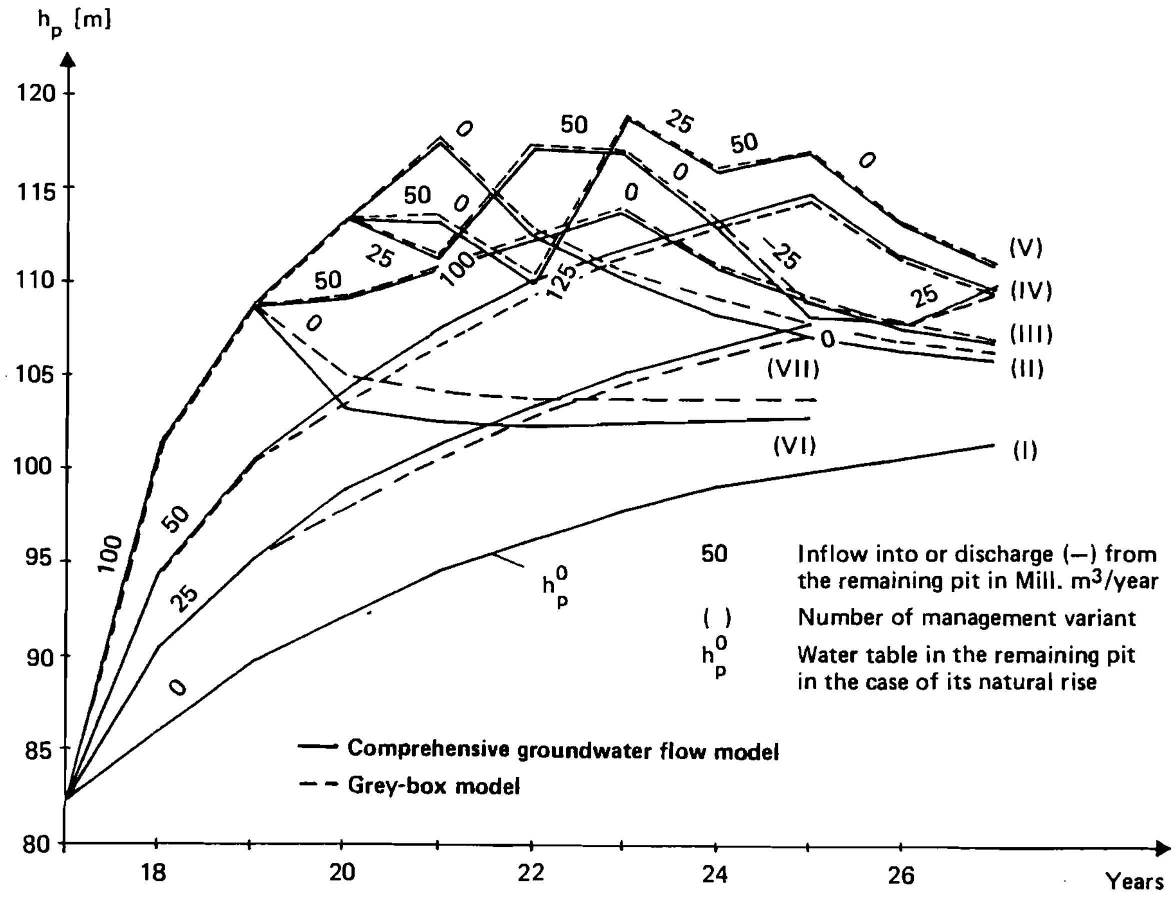



Figure 19: Water table in the remaining pit for different management variants - grey-box model

*Boundary and inital conditions*:

$$h(r_s,t)=H(t) \quad , \quad h(\infty,t)=0 \quad , \quad h(r,0)=0 \tag{4.11}$$

$$H(0)=\frac{\Delta V}{\pi \cdot r_c^2} \tag{4.12}$$

$$2\pi r_s T \cdot \frac{\delta h(r_s,t)}{\delta r}=\pi r_c^2 \cdot \frac{\delta H(t)}{\delta t} \tag{4.13}$$

*Solution of* **Laplace**-*transformed differential equation*:

$$\bar{h}(r,p)=\frac{S \cdot r_s \cdot H(0) \cdot K_0(\sigma \cdot r)}{\sigma T \cdot [\sigma r_s \cdot K_0(\sigma \cdot r_s)+2\alpha \cdot K_1(\sigma \cdot r_s)]} \tag{4.14}$$

with

| | | | |
|---|---|---|---|
| $K_0$, $K_1$ | - | **Bessel**-functions | |
| $r$ | - | space coordinate | [m] |
| $t$ | - | time | [sec.] |
| $r_c$, $r_s$ | - | see Figure 20 | |
| $\alpha$ | - | geohydraulic time constant | [sec/m$^2$] |

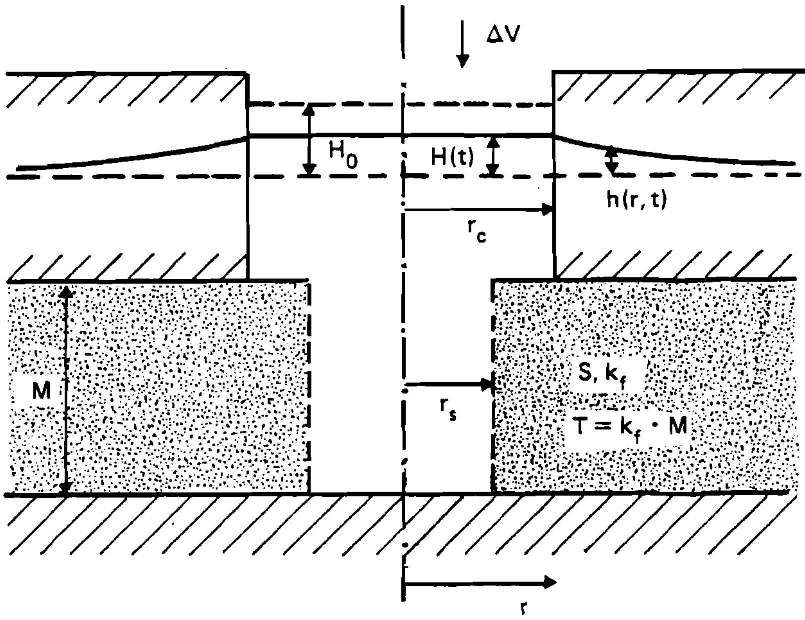



Figure 20: Idealized representation of a finite-diameter well

| | | | |
|---|---|---|---|
| $S$ | - | storage coefficient | [-] |

$$\alpha = \frac{r_s^2}{r_c^2} \cdot S \qquad \sigma = \sqrt{p \cdot \frac{S}{T}} = \sqrt{p \cdot \alpha} \ . \qquad (4.15)$$

*Solution of the problem by inverse* **Laplace**-*transformation:*

$$H(t) = h(r_s,t) = F \cdot H(0) = L^{-1}\left\{\bar{h}(r_s,p)\right\} \cdot H(0) \qquad (4.16)$$

The factor $F$ results from the inverse **Laplace**-transformation. From the analytical inverse transformation we get according to Carslaw, Jaeger 1959:

$$F = 8\,\alpha \,/\, \pi^2 \int_0^{\infty} \frac{e^{-\beta u^2/\alpha}}{u \cdot \Delta(u)} \cdot du \qquad (4.17)$$

with

$$\beta = \frac{T \cdot t}{r_c^2} \ , \quad \alpha = \frac{r_s^2}{r_c^2} \cdot S \qquad (4.18)$$

$$\Delta(u) = [u \cdot J_0(u) - 2\,\alpha \cdot J_1(u)]^2 + [u \cdot Y_0(u) - 2\,\alpha \cdot Y_1(u)]^2 \qquad (4.19)$$

and $J_0, J_1, Y_0, Y_1$ - generalized **Bessel**-functions.

### 4.3.4.2. Modification of the fundamental solution

For management modeling of the remaining pit the solution (Eq. 4.16) is in the given form not yet applicable because a few typical conditions have not been considered:

(1) Variations of groundwater dynamics due to external boundary conditions;

The influence of external boundary conditions is eliminated by the help of separation calculations. The actual variation of the storage volume $v_p$ of the remaining pit results on the one hand from the inflow/outflow due to external boundary conditions (natural recharge $q_p^0$ ) and on the other hand from intakes/discharges $\Delta q_p$ and exfiltrations/infiltrations $qi_p$ resulting therefrom (see Figure 21). The following balance equation holds for a planning horizon from time $t_B$ to $t_E$:

$$v_p(t_E) = \int_{t_B}^{t_E} (q_p^0 + \Delta q_p - qi_p)\, dt + v_p(t_B) \qquad (4.20)$$

with

$$h_p(t_p) = fh_p(v_p(t_p)) \,. \qquad (4.21)$$

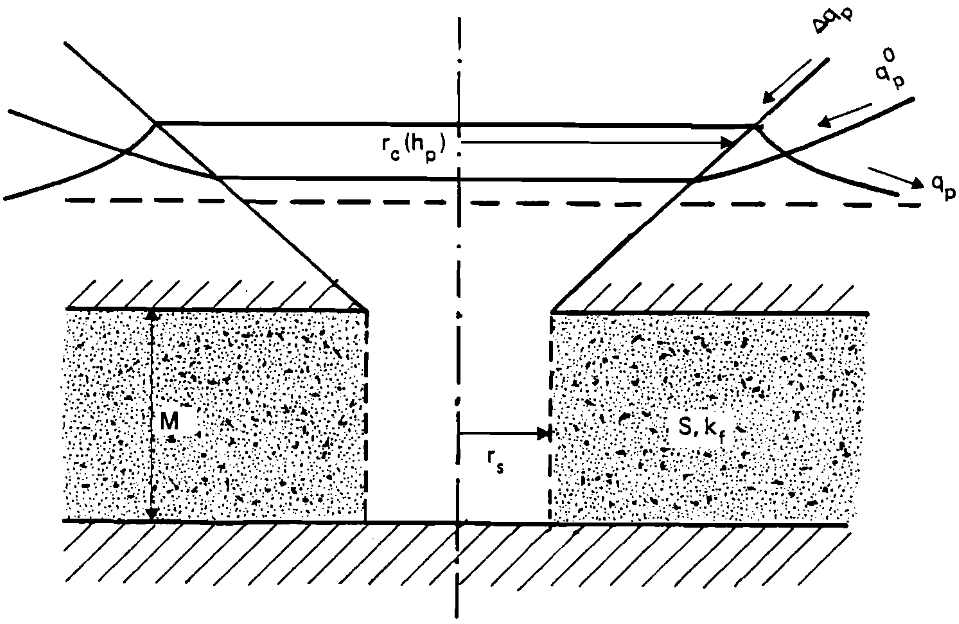



Figure 21: Separation of balance components

(2) Differing geometry of the remaining pit from the cylindrical well form (non-linear dependency between storage water table and volume);

The geometrical deviation of the remaining pit from a cylindrical form is characterized by the relationship $r_c = f(h_p)$. This nonlinearity is eliminated updating the radius $r_c$ by step (step by step linearization).

(3) Unconfined flow conditions;

The unconfined flow condition ( transmissivity $T = f(h_p)$ ) is simplified introducing a mean constant transmissivity.

(4) Time-variable management (artificial inflow);

The consideration of the time-variable management is possible on the basis of the superposition principle by the use of the convolution operation.

(5) Consideration of an additional hydraulic resistance reflecting the transformation of flow from vertical to horizontal direction.

Within the comprehensive groundwater flow model the remaining pit is modeled in the form of an inner boundary condition of third kind. In order to consider this in the block-model, the relevant radius for the exchange area, $r_{s,old}$ , is reduced at the recommendation in Busch, Luckner 1972:

$$r_{s,new} = \frac{r_{s,old}}{e^{2\pi T R_{hydr}}} \tag{4.22}$$

The reciprocal value of $R_{hydr}$ results as a sum of the reciprocal additional hydraulic resistances (parallel circuit) used in the boundary condition in the comprehensive flow model, see Section 2.2.3.

Based on all mentioned modifications we obtain the following time-discrete algorithm with $t_k = k \cdot \Delta t$, $k_B \leq k \leq k_E$:

$$v_p(t_E) = v_p(t_B) + \sum_{k=k_B}^{k_E-1} \left[ v_p^0(t_k) + \Delta v_p(t_k) - vi_p(t_k) \right] \tag{4.23}$$

The individual components are determined as follows:

$$v_p^0(t_k) = fv_p(h_p^0(t_{k+1}) - fv_p(h_p^0(t_k)) \tag{4.24}$$

$$\Delta v_p(t_k) = \Delta q_p(t_{\frac{k+1}{2}}) \cdot \Delta t \tag{4.25}$$

$$vi_p(t_k) = qi(t_k) \cdot \Delta t \tag{4.26}$$

with

$$qi(t_k) = \sum_{l=k_B}^{k} fv_p(h_p(t_l) + \Delta h_p(t_l)) - fv_p(h_p(t_l) + F(t_{k+1} - t_l) \cdot \Delta h_p(t_l)) \tag{4.27}$$

$$F(t_k - t_l) = L^{-1} \left\{ \frac{S \cdot r_s \cdot K_0(\sigma \cdot r_s)}{\sigma \cdot T \cdot [\sigma \cdot r_s \cdot K_0(\sigma \cdot r_s) + 2 \frac{r_s^2}{r_{c,l}^2} K_1(\sigma \cdot r_s)]} \right\} \tag{4.28}$$

$$\sigma = \sqrt{p \cdot \frac{S}{T}} = \sqrt{p \cdot a} \tag{4.29}$$

$$r_{c,l} = \sqrt{\frac{\Delta v_p(t_l)}{\pi \cdot \Delta h_p(t_l)}} \tag{4.30}$$

$$\Delta h_p(t_l) = fh_p(v_p(t_l) + \Delta v_p(t_l)) - h_p(t_l) \ . \tag{4.31}$$

$h_p^0$ is the water level in the remaining pit for natural recharge, see Appendix B for the Test Area.

The program for this algorithm is given in Appendix A3. The inverse **Laplace**-transformation is done numerically. On the basis of the numerical integration the computing time is reduced by a factor of about 15 as compared with the computation of $F$ in Eq. (4.17).

#### 4.3.4.3. Simulation of management variants

The developed program has been tested for the remaining pit management in the GDR test area. For the system-descriptive parameters the set of parameters of the comprehensive groundwater flow model has been used. The discrete geohydraulic parameters of that model (permeability coefficient, storage coefficient) have been transformed by arithmetic averaging into the integrated form needed.

The mean geohydraulic time constant has been estimated by $\bar{\alpha} = \alpha(r_s, t_E)$ see Busch, Luckner 1972. The undisturbed mean groundwater level for the area is about 118 meters. For the computations monthly time steps have been used in accordance with the requirements of the management model.

By computations with the comprehensive groundwater flow model the results obtained for natural recharge ($h_p^0 = f(t)$) have been used to separate influences due to external boundary conditions.

In Figure 22 the computational results for different management variants are compared with those of the comprehensive model.

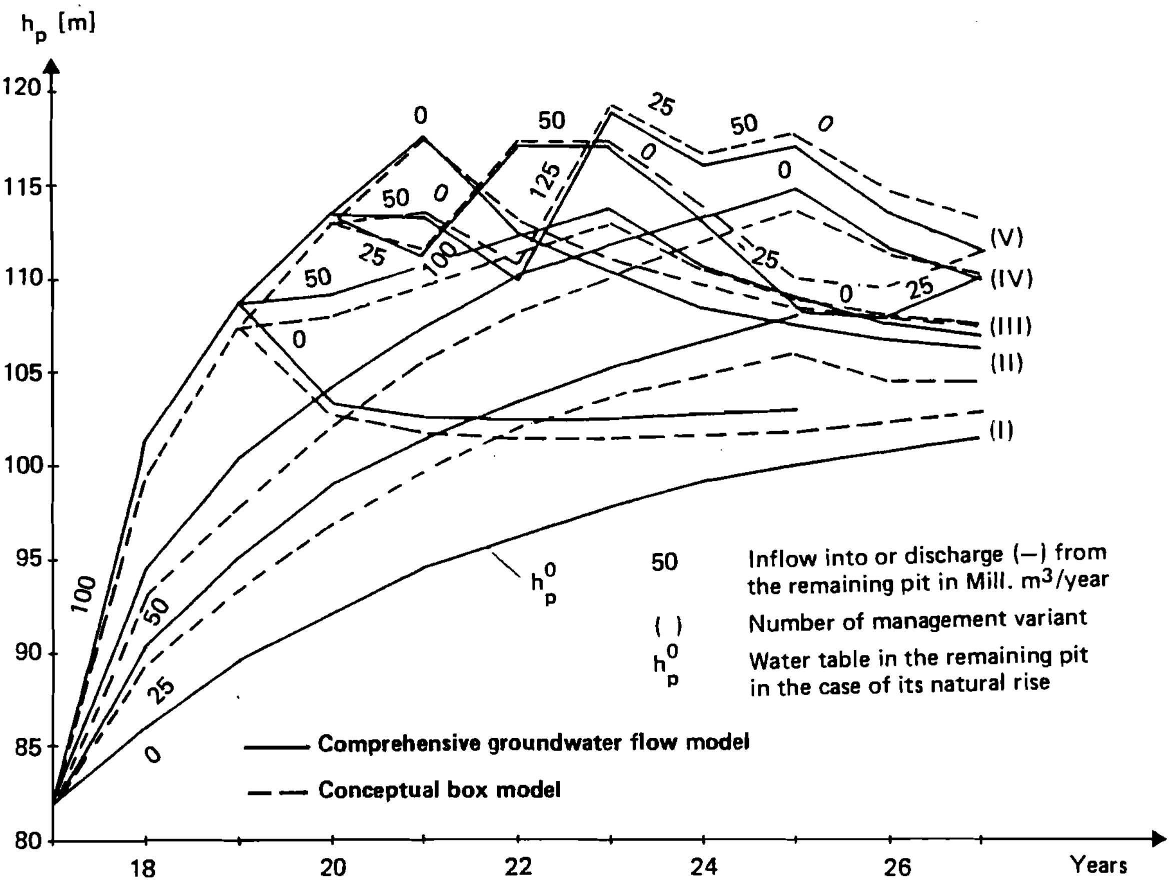



Figure 22: Water table in the remaining pit for different management variants - conceptional block-model

The modeling of the remaining pit management with the comprehensive flow model practically does not permit the simulation of monthly responses on corresponding management variants (monthly constant or variable). The reason for that is above all a significantly increased computing time. With the given model concept it is possible, for instance, to provide computational results for monthly time steps without any growth of computing time or to enable a simulation of the monthly varying management with small additional effort.

Because the response of the conceptional block-model agrees well with that of the comprehensive flow model for monthly constant management, we conclude from these results at the reliability of the conceptional model in the case of monthly varying management. Figure 23 shows the comparison between computing results executed in different ways for monthly managements.

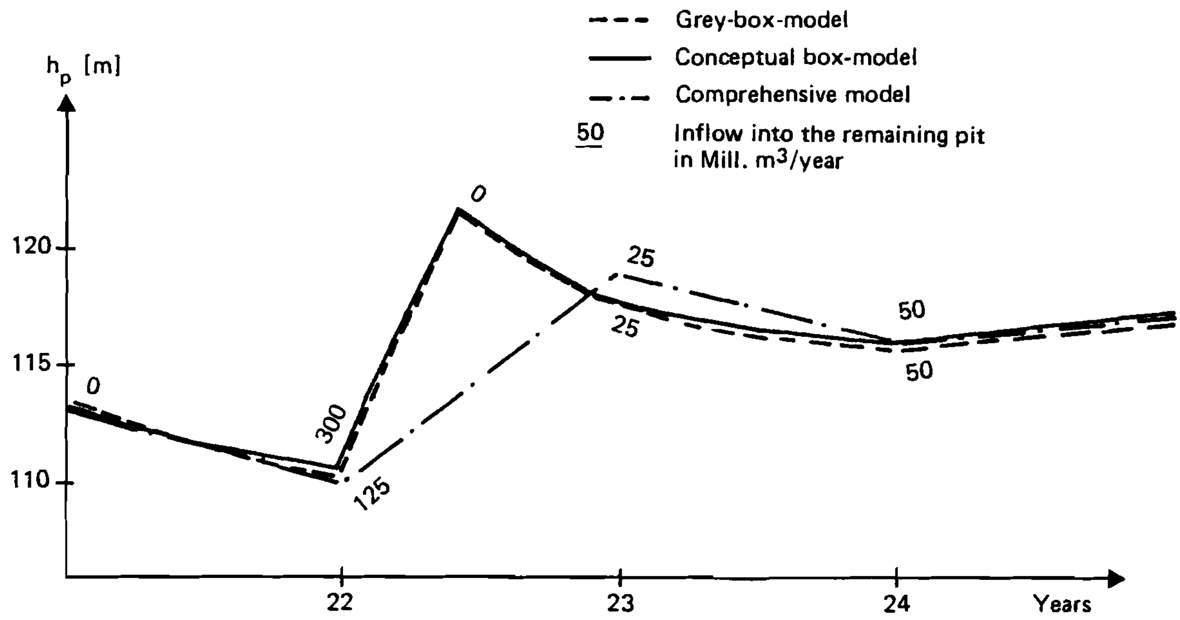



Figure 23: Computational results for monthly management

With the computational results it is possible both, to detect that different management on a monthly base has a strong influence on the annual values, and to test the monthly values of other more simplified models as the black-box-model being described in Kaden et al. 1985 and used for the planning model of the DSMS for the GDR test area.

### 4.4. Impact of Remaining Pit Management on Mine Drainage

If a remaining pit is located close to an operating open-pit mine, the drainage of this mine may be affected. This is the case for the mine B in the GDR test region, see Kaden et.al. 1985a.

Basis of the reduced model are computations with the comprehensive groundwater flow model. Analogously to the submodel "remaining pit management" (Section 4.3.3) for the model reduction the system is simplified into a few blocks. Combining the mathematical models of the blocks results in a grey-box model. The following blocks are considered:

- open-pit mine,
- remaining pit,
- aquifer being directly affected by the mine and the remaining pit,

- aquifer not directly affected.

The objective of the modeling is to estimate the increased mine drainage due to the management of the remaining pit. As the state variable the water level, groundwater table respectively is considered. Impacts of external boundary conditions on the mine drainage are separated by subtraction of the results for differing management alternatives with the same external boundary conditions.

The dynamics of increased mine drainage is for the given block concept described as a linear reservoir (Kindler 1972):

$$D \cdot \frac{d\tilde{qg}_b(t)}{dt} + \tilde{qg}_b(t) = -K \cdot \tilde{qg}_p(t) \qquad (4.32)$$

with

$$\tilde{qg}_b(t) = qg_b(t) - qg_b^0(t) \quad , \quad \tilde{qg}_b(0) = 0 \qquad (4.33)$$

| | | |
|---|---|---|
| $t$ | - | time |
| $\tilde{qg}_b(t)$ | - | actual amount of mine drainage |
| $qg_b^0(t)$ | - | amount of mine drainage in the case of natural recharge of the remaining pit (see Appendix B1) |
| $D$ | - | time constant |
| $K$ | - | proportionality constant |
| $\tilde{qg}_p(t)$ | - | difference of groundwater inflow into the managed remaining pit to the inflow for natural recharge. |

The value $\tilde{qg}_p(t)$ is for annual mean values approximated by the following function:

$$\tilde{qg}_p(i) = A \cdot (\tilde{h}_p(i) - \tilde{h}_p(i-1)) - q_p(i) \qquad (4.34)$$

with

| | | |
|---|---|---|
| $i$ | - | time in years |
| $q_p(i)$ | - | constant inflow/outflow of the remaining pit for the year $i$ |
| $\tilde{h}_p(i)$ | - | difference of the water table in the managed remaining pit to the water table for natural recharge at the end of the year $i$ |
| $A$ | - | average horizontal area of the remaining pit. |

Assuming linearity we obtain based on the superposition principle the following *grey-box model* for annual mean values of the increased mine drainage depending on the management of the remaining pit:

$$\tilde{qg}_b(i) = a_1 \cdot \tilde{qg}_b(i-1) + b_0 \cdot q_p(i) + b_1 \cdot (\tilde{h}_p(i) - \tilde{h}_p(i-1)) \qquad (4.35)$$

with

$$\tilde{qg}_b(i) = 0 \text{ for } i \le ip + 2 \quad , \quad \tilde{h}_p(i) = h_p(i) - h_p^0(i)$$

$$a_1 = 0.45929 \quad , \quad b_0 = 0.02245 \quad , \quad b_1 = -0.31948$$

| | | |
|---|---|---|
| $ip$ | - | year of opening the remaining pit |
| $h_p(i)$ | - | water table in the remaining pit at the end of the year $i$ |
| $\tilde{qg}_b(i)$ | - | increased mine drainage due to management of the remaining pit for the year $i$ in $\frac{Mill.\ m^3}{year}$. |

The model parameter are estimated adapting Eq. (4.35) to results of the comprehensive groundwater flow model. Only such data of the comprehensive model have been selected which are characterized by water levels in the remaining pit within the usable storage layer.

In Figure 24 the results of the reduced model and the comprehensive model are depicted for two management variants. The water level in the remaining pit has been estimated with the submodel "remaining pit", see Section 4.3.3.

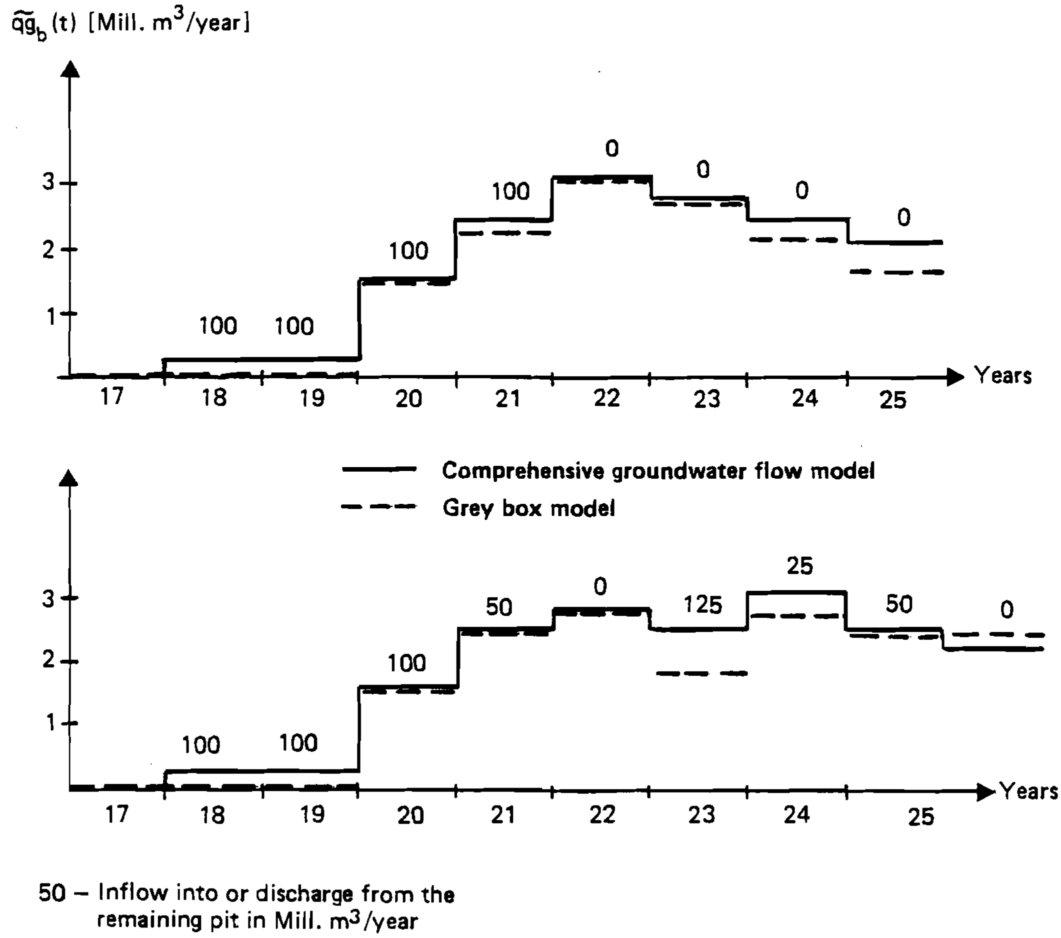



Figure 24: Increased mine drainage due to remaining pit management

## 4.5. Simulation of Exchange Processes between a Stream and Groundwater

### 4.5.1. Analysis of the Problem

In the water balance of lowland areas, the balance component *infiltration/exfiltration* is of similar magnitude as the other balance components (discharge, inflow, natural groundwater recharge etc.). Especially in areas with significant man-made impacts on the water resources system the importance of this balance element is severe increasing. A characteristic example in this connection is the infiltration from the stream into the aquifer in large-area groundwater depression zones as they are typically for open-pit mining regions.

For the estimation of *long-term mean values* of the infiltration/exfiltration (yearly and larger time intervals) the spatial and temporal changes of groundwater tables are the primary independent influence parameter. Changes of the water level in the stream in this case can be neglected because they are assessed to be minimal in comparison with the changes of ground water tables and are not subject to trends. In this case computations with the comprehensive groundwater flow model give detailed informations about the development of infiltration/exfiltration behavior within the test area, see Section 4.2., Figure 16. Such results are well suited for the planning model.

For the management model shorter time intervals have to be considered, i.e. *monthly mean values*. In this case the exfiltration/infiltration conditions are different. Processes with shorter time constants become essential, such as the variation of exfiltration/infiltration due to changes of the water level in the stream. In the areas of groundwater depression this process practically represents the only natural and rapid component which influences the runoff in the stream. Hypodermic and groundwater runoffs do not occur and surface runoff is in lowland areas almost negligible.

Consequently, only two subprocesses have to be simulated in order to model the exfiltration/infiltration processes in mining regions sufficiently accuratly. Due to the fact that the time constants differ between each subprocess by magnitudes it is possible to consider them separately in mathematical submodels. The results of both submodels have to be superimposed to get the appropriate balance values, see Figure 25 and Eq. (4.36).

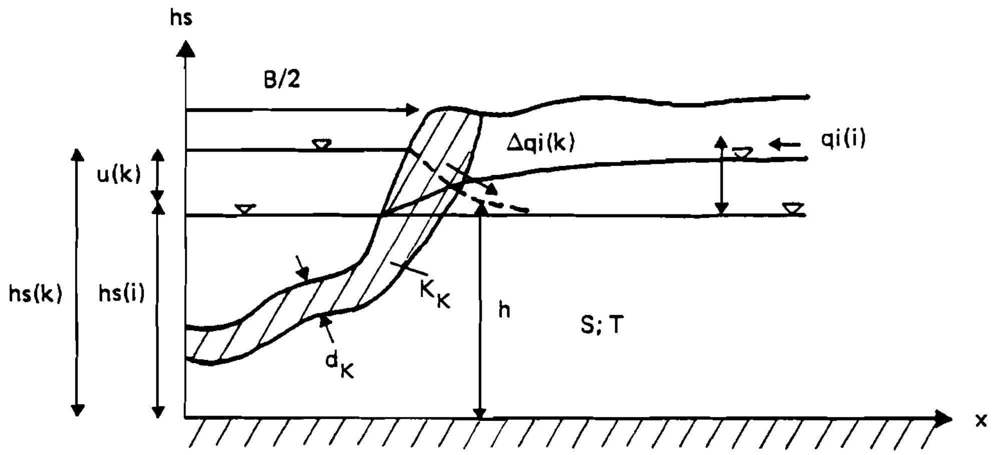



Figure 25: Exfiltration/infiltration process between stream and groundwater

$$qi(k) = \Delta qi(k) + qi(i) \tag{4.36}$$

with

$\Delta qi(k)$ - exfiltration/infiltration due to change of water level
$qi(i)$ - exfiltration/infiltration for mean water level in the stream

The component $qi(i)$ can be estimated as time series directly by computations with the comprehensive groundwater flow model (see Section 4.2). This is sufficiently to describe the exchange process in the planning model. A different situation is given for the component $\Delta qi(k)$ which cannot directly be obtained from that model (work effort, computing time). For this reason, the comprehensive groundwater flow model can only be used as an aid for the estimation of reduced models. The following approach of model reduction has been applied:

(1) Execution of test calculations with the comprehensive groundwater flow model;

(2) Derivation of a deterministic *conceptional block model*;

(3) Calculations of step-response functions with the reduced model and actual geohydraulic data;

(4) Derivation of a deterministic *black-box model*.

### 4.5.2. Test Calculations with the Comprehensive Groundwater Flow Model

The test calculations executed on a small range have been focused on two points:

- to support the selection of a model concept for a conceptional block-model;
- to use the computational results for a qualitative evaluation of the conceptional block-model.

In the comprehensive groundwater flow model (see Sections 2.2.3 and 4.2) the stream is considered via inner boundary conditions of third kind. The water level in the stream is in this case the independent variable. In addition to the geohydraulic variables of the corresponding finite element, a hydraulic resistance is considered. It represents on the one hand the colmation resistance of the bottom of the stream and on the other hand the transformation of the line-shaped boundary condition into the nodal points of the finite element model.

The corresponding equations are:

$$R_{hydr} = R_{hydr,GV} + R_{hydr,K} \quad , \quad R_{hydr,GV} = \frac{\Delta L}{T \cdot L} \quad , \quad R_{hydr,K} = \frac{d_K}{K_K \cdot L \cdot B} \qquad (4.37)$$

with

| | | | |
|---|---|---|---|
| $R_{hydr}$ | - | additional hydraulic resistance | $[sec. / m^2]$ |
| $K_K$ | - | hydraulic conductivity of the colmated layer | $[m / sec.]$ |
| $d_K$ | - | depth of the layer | $[m]$ |
| $L$ | - | length of the stream in the finite element | $[m]$ |
| $B$ | - | width of the stream in the finite element | $[m]$ |
| $\Delta L$ | - | mean distance between stream and nodal point | $[m]$. |

Special finite elements of the comprehensive groundwater flow model have been selected. First, the groundwater tables and the exfiltration/infiltration have been estimated monthly for a mean stream water table for a selected year. Second, the same computations have been carried out for monthly variable water levels. The resulting differences represent the effect of fluctuating water levels in the stream.

Figure 26 shows the effects on the groundwater table as a function of the distance to the stream, and Figure 28 represents the derivations of the exfiltration/infiltration from those for mean stream water level.

The results of the test computations can be summerized by the following statements:

- The variation of exfiltration/infiltration due to the change of the water level in the stream is a relevant balance component for the stream segments under consideration;
- The effects of variations of the water level in the stream on the groundwater tables are locally limited and do not effect the external boundary conditions of the flow field.

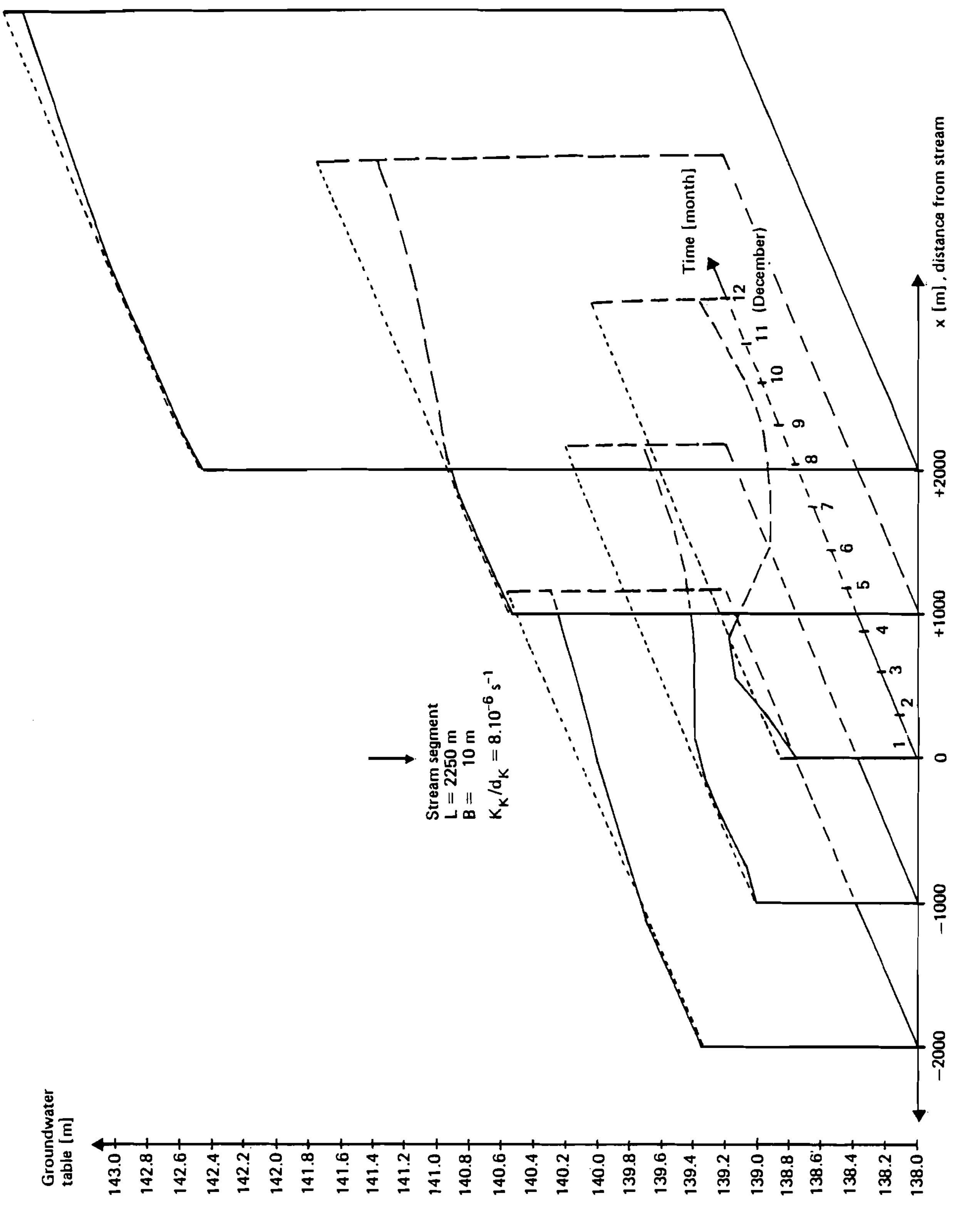



Figure 26: Groundwater table as a function of the stream distance

For the model concept of the deterministic conceptional block-model we obtain the following requirements or simplifications respectively:

- Groundwater process modeling as one-dimensional horizontal-plan groundwater flow;
- Formulation of the external boundary condition at infinity;
- Consideration of a potential-dependent resistance formulating the inner boundary condition (colmation).

### 4.5.3. Deterministic Conceptual Block-Model

Based on the above mentioned assumptions the process of exfiltration/infiltration is described by the linear differential equation of one-dimensional horizontal-plan groundwater flow to a channel:

*Differential equation*

$$\frac{\delta^2 h}{\delta x^2} = a \cdot \frac{\delta h}{\delta t} \quad , \quad a = \frac{S}{T} \tag{4.38}$$

*Boundary and initial conditions*

$$h(\infty,t) = 0 \quad , \quad h(x,0) = 0 \tag{4.39}$$

$$h(0,t) = A \frac{dh(0,t)}{dx} = H(0) \quad , \quad A = \frac{K}{K_K} \cdot d_K \tag{4.40}$$

with (see also above Eq.(4.37))

| | | | |
|---|---|---|---|
| $h$ | - | variation of groundwater table | $[m]$ |
| $x$ | - | space coordinate | $[m]$ |
| $t$ | - | time | $[sec]$ |
| $K$ | - | hydraulic conductivity of the aquifer | $[m/sec.]$ |
| $\Delta H$ | - | variation of the water level in the stream (step) | $[m]$ |
| $S$ | - | storage coefficient | $[-]$ |
| $T$ | - | transmissivity | $[m^2/sec.]$. |

The solution of the differential equation is obtained by means of **Laplace-**transformation:

*Differential equation*

$$\frac{\delta^2 \bar{h}}{\delta x^2} = a \cdot p \cdot \bar{h} - h(x,0) \tag{4.41}$$

*Boundary conditions*

$$\bar{h}(\infty,p) = 0 \quad , \quad \bar{h}(0,p) - A \frac{d\bar{h}(0,p)}{dx} = \frac{\Delta H(0)}{p} \tag{4.42}$$

with $p$ -time coordinate $[1/sec.]$.

The solution is:

$$\bar{h}(x,p) = \frac{\Delta H(0)}{p \cdot (1 + A\sqrt{a \cdot p})} e^{-\sqrt{a \cdot p}\, x} \tag{4.43}$$

For the actual volumetric flux we obtain with

$$q(x,t) = -2T \cdot L \frac{dh(x,t)}{dx} \quad : \tag{4.44}$$

$$\bar{q}(x,p) = \frac{2T \cdot L \cdot \Delta H(0) \cdot \sqrt{a \cdot p}}{p \cdot (1 + A\sqrt{a \cdot p})} \cdot e^{-\sqrt{a \cdot p}\, x} . \tag{4.45}$$

The total exfiltration/infiltration for $x=0$ is:

$$Q(t) = \int_0^t q(0,t)\,dt = L^{-1}\left\{\frac{1}{p}\cdot\bar{q}(0,p)\right\} \tag{4.46}$$

$$Q(t) = L^{-1}\left\{\bar{Q}(p)\right\} = L^{-1}\left\{\frac{2T\cdot L\cdot\Delta H(0)\cdot\sqrt{a\cdot p}}{p^2\cdot(1+A\,\sqrt{a\cdot p})}\right\} \tag{4.47}$$

The inverse **Laplace**-transformation is performed by an efficient numerical algorithm applying the residue theorem.

For the monthly mean values of exfiltration/infiltration needed for the management model we get with $\Delta t = 2.625\cdot10^5$ *sec.* $\approx 1$ *month*:

$$qs(t_k) = \frac{1}{\Delta t}\cdot\left[L^{-1}\left\{\bar{Q}(\frac{1}{t_k})\right\} - L^{-1}\left\{\bar{Q}(\frac{1}{t_{k-1}})\right\}\right] \tag{4.48}$$

To consider a time-variable step function the superposition principl of the individual step is used by the helps of the convolution operation ($k = t_k$).

$$\Delta qi(k) = \frac{u(m)}{\tilde{R}_{hydr}} + \sum_{l=1}^{k} w(k+1-l)\cdot S(l) \tag{4.49}$$

$$u(k) = hs(k) - hs(i) \quad , \quad w(k) = u(k) - u(k-1)$$

with

| | | | |
|---|---|---|---|
| $k$ | - | actual month | |
| $i$ | - | actual year | |
| $S(l)$ | - | step response function | $[m^2/sec.]$ |
| $hs$ | - | surface water level above its bottom | $[m]$ |
| $\tilde{R}_{hydr}$ | - | hydraulic resistance for free percolation | $[sec./m^2]$ |

For the hydraulic resistance holds:

$$\tilde{R}_{hydr} = 1/\sum_{l=1}^{M}\frac{1}{R_{hydr,l}} \tag{50}$$

with $M$ - number of finite elements of the comprehensive groundwater flow model per balance segment with free percolation.

The step-response function of a balance segment in the management model results from:

$$S(k) = \sum_{l=1}^{N} qs(k)_l \tag{4.51}$$

with $N$ - number of finite elements of the comprehensive groundwater flow model per balance segment (without free percolation).

The $qs(k)_l$-values have been estimated with the program CHANGE (see Appendix A4) for each element with a step $w(0) = 1\ m$ and superimposed. In Figure 27 the results are depicted for three balance segments.

Figure 28 compares for a finite element the computational results from the conceptional block-model and those of the comprehensive groundwater flow model.

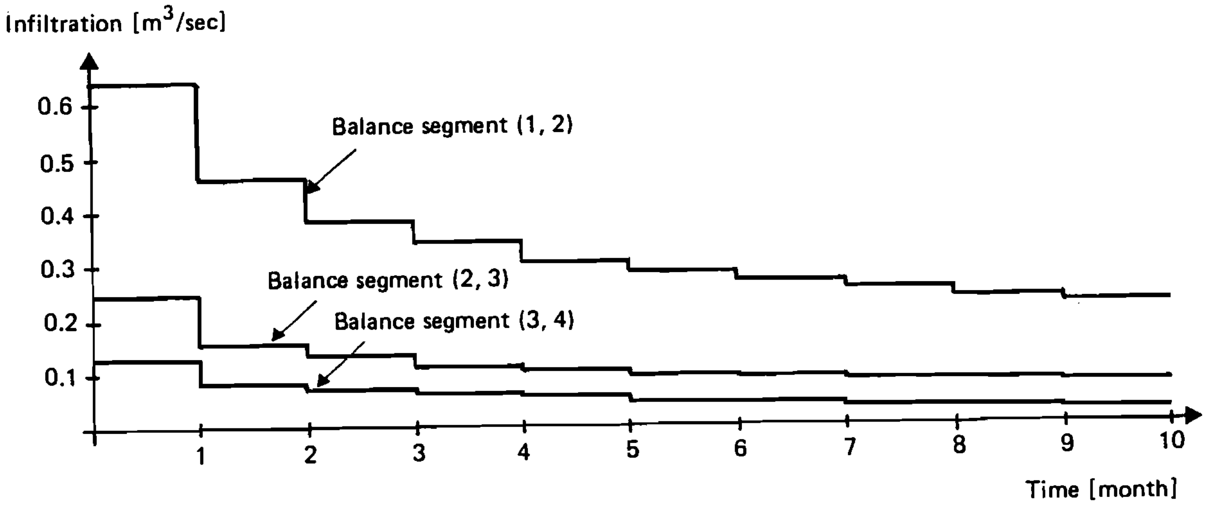



Figure 27: Step-response functions for $w(0) = 1\ m$

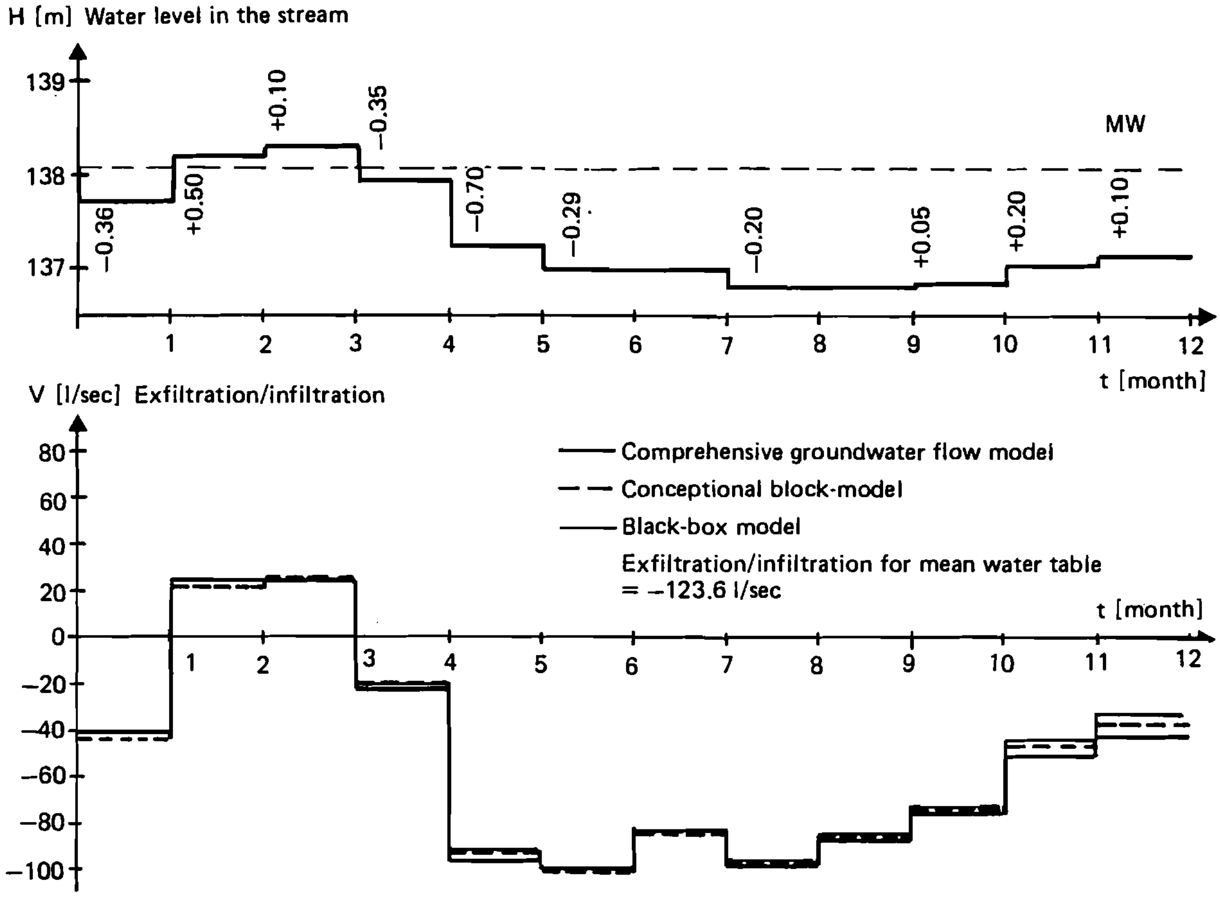



Figure 28: Comparison of model results

### 4.5.4. Black-Box Model

To compute the actual variation of exfiltration/infiltration per balance segment the computation with CHANGE has been proved to be too expensive as a submodel for the management model. That is why the step-response function is approximated by a pulse-response function based on a difference equation with a history of two time steps (see Kaden et.al., 1985):

$$\Delta qi(k) = a_1 \cdot \Delta qi(k-1) + a_2 \cdot \Delta qi(k-2) + (b_0 + c_0) \cdot u(k) + \\ + b_1 \cdot u(k-1) + b_2 \cdot u(k-2) \qquad (4.52)$$

The conditions for the approximation have been selected in such a way that the first four values and the stationary value of the step-response function are reflected exactly. The results of the black-box model (recursive equation) are also depicted in Figure 28.

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# APPENDIX A

## Computer Programs of Submodels

### Appendix A1

SIKO - Mathematical and statistical analysis of hydrologic and meteorologic time series

```
c*************************************************************************
c
c *** Mathematical and statistical analysis of hydrologic and
c *** meteorologic monthly time series;
c *** computation of regression coefficients for the stochastic simulation
c
c*************************************************************************
      program siko
c
c *** ng        - number of time series ( <=25 )
c *** n         - number of observation years ( <=100 )
c *** ive       - maximum shift for estimation of cross-correlation
c ***             ( 0<=ive<=12 )
c *** is        - number of simulation proposals ( <=25 )
c *** id        - logical number of unit for input of time series
c *** ver       - type of distributation function
c ***             =1      : normal disribution
c ***             =2      : logarithmic normal distribution
c ***             =3      : power-normal distribution
c ***             =4      : johnson-distribution
c *** pas(l)    - logical control parameter for series l
c ***             =.true. : parameters of all month' will be estimated
c ***             =.false.: not all parameters will be estimated
c *** stapa(l)  - logical control parameter for series l
c ***             =.true. : output of estimated parameters
c ***             =.false.: no output
c *** e         - time series (chronological), gaps filled with -900.0
c *** par(j)    - logical control parameter (for pas(l)=..false.)
c ***             =.true. : input of parameters of distrib. function
c ***                       for month j of series l
c ***             =.false.: estimation of parameters
c *** ew        - mean
c *** saw       - standard deviation
c *** ugo       - auxiliary parameter for estimation of lower limit
c ***             of logarithmic normal distribution
c *** ugw       - lower limit
c *** ugb/ogb   - raster width for estimation of lower and upper
c ***             limits for power-normal and johnson distribution
c *** exb       - raster width for estimation of exponent of power
c ***             normal distribution
c *** ex        - exponent
c *** ogw       - upper limit
c *** io(l)     - length of l-th simulation proposal
c *** stako(l)  - logical control parameter for output of coefficients
c ***             of l-th simulaion proposal
c ***             =.true. : output
c ***             =.false.: no output
c *** ig(m)     - ordinal number of m-th series of k-th proposal
c ***             m=1,...,io(k)
c *** iv(m)     - appertaining shifts

      dimension e(100,12,25),ver(25),pas(25),b(100),xb(100),par(12),
     *          sb(100,12),kk(12,12),c(23),mc(23)
      dimension io(25),ig(25),iv(25),stako(25),stapa(25)
      dimension sk(25,12),d(25),a(25,25)
      real      kk,no2w
      integer   ver
      logical   par,pas,stako,stapa
      data c /-2.3264,-2.0538,-1.6449,-1.2816,-1.0364,-0.8416,-0.6745,
     1        -0.5234,-0.3853,-0.2534,-0.1257, 0.0000, 0.1257, 0.2534,
     2         0.3853, 0.5234, 0.6745, 0.8416, 1.0364, 1.2816, 1.6449,
     3         2.0538, 2.3264/
      data mc /1,2,5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,
     1         90,95,98,99/
      data iin,iou/5,6/

c *** IIASA-routine for assignment of input/output files
      call usearg

      je=12
      write (iou,2000)
      write (iou,2100)

c *** input of control data
      read  (iin,1000) ng,n,ive,is,id
      write (iou,1000) ng,n,ive,is,id
      read  (iin,1000) (ver(l),l=1,ng)
      write (iou,1000) (ver(l),l=1,ng)
      read  (iin,1200) (pas(l),l=1,ng)
      write (iou,1200) (pas(l),l=1,ng)
      read  (iin,1200) (stapa(l),l=1,ng)
      write (iou,1200) (stapa(l),l=1,ng)
      do 490 l=1,ng
        write (iou,2200) l,(j,j=1,je)
c ***   input of the time series
        read  (id,1100)  ((e(i,j,l),j=1,je),i=1,n)
        write (iou,1300) ((e(i,j,l),j=1,je),i=1,n)
        do 10 j=1,je
   10   par(j)=.false.
        lver=ver(l)
        goto (20,100,200,300), lver
c ***   normal distributed series
   20   write (iou,2310)
        if (pas(l)) goto 25
        read  (iin,1200) (par(j),j=1,je)
        write (iou,1200) (par(j),j=1,je)
   25   do 90 j=1,je
                do 30 i=1,n
   30           b(i)=e(i,j,l)
                if (par(j)) goto 40
                call nv(b,n,ew,saw)
                if (stapa(l)) write (7,1400) ew,saw
                goto 50
   40           read  (iin,1400) ew,saw
   50           call tnv(b,xb,n,ew,saw)
                do 60 i=1,n
   60           e(i,j,l)=xb(i)
                call no2(xb,n,no2w)
                if (.not.par(j)) write (iou,1500) j,ew,saw,no2w
                if (par(j))      write (iou,1510) j,ew,saw,no2w
                do 65 k=1,23
   65           sk(k,j)=c(k)*saw+ew
                call ord(b,n)
                do 70 i=1,n
   70           sb(i,j)=b(i)
   90   continue
        goto 400
c ***   logarithmic normal distributed series
  100   write (iou,2320)
        if (pas(l)) goto 120
```

```
      read  (iin,1200) (par(j),j=1,je)
      k=0
      do 110 j=1,12
            if (.not.par(j)) k=1
110   continue
      if (k.eq.1) goto 120
      goto 130
120   read  (iin,1100) ug0
      write (iou,2321) ug0
130   write (iou,2322)
      do 190 j=1,je
            do 140 i=1,n
140         b(i)=e(i,j,l)
            if (par(j)) goto 150
            call ln3v(b,n,ug0,ew,saw,ugw)
            if (stapa(l)) write (7,1400) ew,saw,ugw
            goto 160
150         read (iin,1400) ew,saw,ugw
160         call tln3v(b,xb,n,ew,saw,ugw)
            do 170 i=1,n
170         e(i,j,l)=xb(i)
            call no2(xb,n,no2w)
            if (.not.par(j)) write (iou,1500) j,ew,saw,ugw,no2w
            if (par(j))      write (iou,1520) j,ew,saw,ugw,no2w
            do 175 k=1,23
175         sk(k,j)=ugw+exp(saw*c(k)+ew)
            call ord(b,n)
            do 180 i=1,n
180         sb(i,j)=b(i)
190   continue
      goto 400
c *** power normal distributed series
200   write (iou,2330)
      if (pas(l)) goto 220
      read  (iin,1200) (par(j),j=1,12)
      k=0
      do 210 j=1,12
            if (.not.par(j)) k=1
210   continue
      if (k.eq.1) goto 220
      goto 230
220   read  (iin,1100) ugb,exb
      write (iou,2331) ugb,exb
230   write (iou,2332)
      do 290 j=1,je
            do 240 i=1,n
240         b(i)=e(i,j,l)
            if (par(j)) goto 250
            call pnv(b,n,ugb,exb,ew,saw,ugw,ex)
            if (stapa(l)) write (7,1400) ew,saw,ugw,ex
            goto 260
250         read  (iin,1400) ew,saw,ugw,ex
260         call tpnv(b,xb,n,ew,saw,ugw,ex)
            do 270 i=1,n
270         e(i,j,l)=xb(i)
            call no2(xb,n,no2w)
            if (.not.par(j)) write (iou,1500) j,ew,saw,ugw,ex,no2w
            if (par(j))      write (iou,1530) j,ew,saw,ugw,ex,no2w
            do 275 k=1,23
                  h=c(k)*saw+ew
                  if (h.gt.0.) goto 274
                  sk(k,j)=-ugw
                  goto 275
274               sk(k,j)=h**(1./ex)-ugw
275         continue
            call ord(b,n)
            do 280 i=1,n
280         sb(i,j)=b(i)
290   continue
      goto 400
c *** johnson distributed series
300   write (iou,2340)
      if (pas(l)) goto 320
      read  (iin,1200) (par(j),j=1,12)
      k=0
      do 310 j=1,12
            if (.not.par(j)) k=1
310   continue
      if (k.eq.1) goto 320
      goto 330
320   read  (iin,1100) ugb,ogb
      write (iou,2341) ugb,ogb
330   write (iou,2342)
      do 390 j=1,je
            do 340 i=1,n
340         b(i)=e(i,j,l)
            if (par(j)) goto 350
            call jov(b,n,ugb,ogb,ew,saw,ugw,ogw)
            if (stapa(l)) write (7,1400) ew,saw,ugw,ogw
            goto 360
350         read  (iin,1400) ew,swa,ugw,ogw
360         call tjov(b,xb,n,ew,saw,ugw,ogw)
            do 370 i=1,n
370         e(i,j,l)=xb(i)
            call no2(xb,n,no2w)
            if (.not.par(j)) write (iou,1500) j,ew,saw,ugw,ogw,no2w
            if (par(j))      write (iou,1530) j,ew,saw,ugw,ogw,no2w
            do 375 k=1,23
                  eh=exp(c(k)*saw+ew)
375         sk(k,j)=(ugw+ogw*eh)/(eh+1.)
            call ord(b,n)
            do 380 i=1,n
380         sb(i,j)=b(i)
390   continue
400   continue
      write (iou,2400) (j,j=1,je)
      write (iou,1300) ((sb(i,j),j=1,je),i=1,n)
      write (iou,2500) (j,j=1,je)
      do 410 k=1,23
410   write (iou,1600) w(k),(sk(k,j),j=1,je)
      write (iou,2600) (j,j=1,je)
      write (iou,1700) ((e(i,j,l),j=1,je),i=1,n)
      if (ive.eq.0) goto 490
c *** computation of auto-correlation coefficients
      do 470 j=1,je
            na=0
            do 470 jv=1,ive
                  jjv=j-jv
                  k=0
                  if (jjv.gt.1) goto 450
```

```
                  na=n-1
                  jjv=jjv+12
                  k=1
  450             do 460 i=1,na
                          b(i)=e(i+k,j,l)
  460             xb(i)=e(i,jjv,l)
  470    call koko(b,xb,kk(jv,j),na)
         write (iou,2700) (j,j=1,je)
         do 480 jv=1,ive
                mjv=0-jv
  480    write (iou,1600) mjv,(kk(jv,j),j=1,je)
  490 continue
      if (ng.eq.1) goto 610
c *** computation of cross-correlation coefficients
      ivel=ivel1
      ng1=ng-1
      do 600 l=1,ng1
        l1=l+1
        do 599 ll=l1,ng
              write (iou,2750) l,ll,(j,j=1,je)
              do 530 j=1,je
                  na=n
                  do 530 jv=1,ivel
                          jv1=jv-1
                          jjv=j-jv1
                          k=0
                          if (jjv.ge.1) goto 510
                          na=na-1
                          k=n-1
                          k=1
                          jjv=jjv+12
  510                     do 520 i=1,na
                                  b(i)=e(i+k,j,l)
  520                     xb(i)=e(i,jjv,ll)
  530         call koko(b,xb,kk(jv,j),na)
              do 540 jv=1,ivel
                  mjv=1-jv
  540         write (iou,1600) mjv,(kk(jv,j),j=1,je)
              if (ive.eq.0) goto 599
              write (iou,2750) ll,l,(j,j=1,je)
              do 580 j=1,je
                  na=n
                  do 580 jv=1,ive
                          jjv=j-jv
                          k=0
                          if (jjv.ge.1) goto 560
                          na=n-1
                          k=1
                          jjv=jjv+je
  560                     do 570 i=1,na
                                  b(i)=e(i+k,j,ll)
  570                     xb(i)=e(i,jjv,l)
  580         call koko(b,xb,kk(jv,j),na)
              do 590 jv=1,ive
                  mjv=0-jv
  590         write (iou,1400) mjv,(kl(jv,j),j=1,je)
c 600 continue
  599   continue
  600 continue
  610 if (is.eq.0) goto 775
```

```
c *** input of control dates for the simulation variants
      write (iou,2100)
      read  (iin,1000) (io(l),l=1,is)
      write (iou,1000) (io(l),l=1,is)
      read  (iin,1200) (stako(l),l=1,is)
      write (iou,1200) (stako(l),l=1,is)
      do 990 il=1,is
        ib=io(il)
        write (iou,2100)
        read  (iin,1000) (ig(i),i=1,ib)
        write (iou,1000) (ig(i),i=1,ib)
        read  (iin,1000) (iv(i),i=1,ib)
        write (iou,1000) (iv(i),i=1,ib)
        do 830 j=1,je
c ***           construction of the regression matrix
                ibs=ib-1
                na=n
                do 620 i=1,ib
                        mm=j+iv(i)
                        if (mm.lt.1) goto 630
  620           continue
                goto 635
  630           na=n-1
  635           do 800 i=1,ib
                do 799 ij=1,ib
                        if (i.gt.ij) goto 696
                        if (i.eq.ij) goto 694
                        ki=i
  640                   k=0
                        mm=j+iv(ki)
                        if (na.eq.n) goto 660
                        if (mm.lt.1) goto 650
                        k=1
                        goto 660
  650                   mm=mm+je
  660                   if (kl.eq.ij) goto 680
                        igi=ig(l)
                        do 670 ka=1,na
  670                   b(ka)=e(ka+k,mm,igi)
                        ki=ij
                        goto 640
  680                   igij=ig(ij)
                        do 690 ka=1,na
  690                   xb(ka)=e(ka+k,mm,igij)
                        call koko(b,xb,kk(i,ij),na)
                        goto 799
  694                   kk(i,ij)=1.
                        goto 799
  696                   kk(i,ij)=kk(ij,i)
  799           continue
  800           continue
c ***           computation of simulation coefficients
c ***           by solution of a set of equations
                do 810 i=1,ibs
                        d(i)=kk(1,i+1)
                        do 810 ii=1,ibs
  810           a(i,ii)=kk(i+1,ii+1)
                call gauss(a,d,ibs)
                do 820 i=1,ibs
  820           si(i,j)=d(i)
```

```
               rs=0.
               do 830 i=1,ibs
 830           rs=rstd(i)*kk(1,i+1)
               rs=1.-rs
               if (rs.lt.0.) rs=100000000.
               sk(ib,j)=sqrt(rs)
 850     continue
         write (iou,2800) il,(j,j=1,je)
         write (iou,1700) ig(1),iv(1)
         do 870 i=1,ibs
               ii=i+1
               write (iou,1700) ig(ii),iv(ii),(sk(i,j),j=1,je)
 870     continue
         write (iou,2900)
         write (iou,1800) (sk(ib,j),j=1,je)
         if (.not.state(i)) goto 990
         do 900 j=1,je
 900     write (7,1900) (sk(i,j),i=1,ib)
 990 continue
 995 continue
1000 format (16i5)
1100 format (12f6.2)
1200 format (16i5)
1300 format (12f10.4)
1400 format (8f10.4)
1500 format (5x,i5,5(4x,f10.4))
1510 format (5x,i5,3(4x,f10.4),'     input-parameters')
1520 format (5x,i5,4(4x,f10.4),'     input-parameters')
1530 format (5x,i5,5(4x,f10.4),'     input-parameters')
1600 format (i5,2x,12f7.4)
1700 format (i5,i4,12f9.4)
1800 format (9x,12f9.4)
1900 format (8f10.4)
2000 format (41x,'*    p r o g r a m    ** s i k o **    *'/,
    1        41x,'****************************************'//,15x,
    2        'mathematical and statistical analysis of hydrological',
    3        'and meteorological monthly time series',/)
2100 format (/,'  control data :',/)
2200 format (/,55x,'series',i4,//,4x,i2,11(8x,i2),/)
2310 format (//,5x,'parameters of the normal-distribution',//,
    1        6x,'month',6x,'mean        stand.-dev.    no2-fitness',/)
2320 format (//,5x,'parameters of the logarithmic normal distribution')
2321 format (/,5x,'control parameter for the estimation of the lower ',
    1        'limit    ug0=',f6.2)
2322 format (/,6x,'month',6x,'mean          stand.dev.    lower limit    ',
    1        'no2-fitness',/)
2330 format (//,5x,'parameters of the power normal distribution')
2331 format (/,5x,'step control      ugb=',f6.2,'      a*b=',f6.2)
2332 format (/,6x,'month',6x,'mean          stand.dev.    lower limit',
    1        '          exponent     no2-fitness',/)
2340 format (//,5x,'parameters of the johnson distribution')
2341 format (/,5x,'step control      ugb=',f6.2,'      ogb=',f6.2)
2342 format (/,6x,'month',6x,'mean          stand.dev.    lower limit    ',
    1        'upper limit    no2-fitness',/)
2400 format (//,50x,'graded series by size',//,4x,i2,11(8x,i2),/)
2500 format (//,51x,'special quantiles',//,' p.c.       ',i2,11(7x,i2),/)
2600 format (//,51x,'transformed series',//,4x,i2,11(8x,i2),/)
2700 format (//,45x,'auto-correlation coefficients',//,' order',4x,i2,
    1        11(7x,i2),/)
2750 format (//,35x,'cross-correlation coefficients    series  ',i2,
    1        ' and ',i2,//,' order',4x,i2,11(7x,i2),/)
2800 format (//,31x,'coefficients for the stochastic simulation - ',
    1        'variant  ',i2,//,5x,12(7x,i2),/)
2900 format (/,44x,'residual standard deviation',/)

      end
c
c************************************************************************
c
      subroutine nv(b,n,ew,saw)
c *** parameter estimation of the normal distribution
c *** b         - series
c *** n         - length of b
c *** ew        - mean
c *** saw       - standard deviation

      dimension b(n)
      k=0
      ew=0.
      saw=0.
      do 10 i=1,n
        if (b(i).lt.-500.) goto 10
        k=k+1
        ew=ew+b(i)
        saw=saw+b(i)*b(i)
   10 continue
      ew=ew/k
      saw=(saw-k*ew*ew)/(k-1)
      saw=sqrt(saw)
      return
      end
c
c************************************************************************
c
      subroutine tnv(b,xb,n,ew,saw)
c *** transformation from normal into normed normal distribution

      dimension b(n),xb(n)
      do 10 i=1,n
        if (b(i).gt.-500.) goto 5
        xb(i)=b(i)
        goto 10
    5   xb(i)=(b(i)-ew)/saw
   10 continue
      return
      end
c
c************************************************************************
c
      subroutine ln3v(e,k1,pug,xm,xs,xop)
c**** parameter estimation of the logarithmic normal distribution
c     e         - time series
c     k1        - length of e
c     pug       - control parameter for the lower limit
c     xm        - mean
c     xs        - standard deviation
c     xop       - lower limit
      dimension e(k1),xn(3),t1(3),t2(3),t3(3)

      i1=0
```

```
      pin=1000000.
      l=0
    2 if (l.eq.k1) goto 5
      l=l+1
      if (e(l).le.-500.) goto 2
      if (e(l).lt.pin) pin=e(l)
      goto 2
    5 if(pug.le.0) goto 6
      j=1
      m=2
      xo(1)=pin-pug
      xo(2)=pin-0.01
      goto 50
    6 if(pin.gt.0.01) goto 7
      j=1
      m=1
      xo(j)=0
      goto 50
    7 j=1
      m=2
      xo(1)=0.
      xo(2)=pin-0.01
      goto 50
   10 if(t3(1).ge.0.) goto 11
      if(t3(2)) 40,12,12
   11 if(t3(2).ge.0.) goto 43
   12 dx=(xo(2)-xo(1))/10.
      xo(1)=xo(2)-dx
      j=1
      m=3
      goto 50
   20 if(t3(1).ge.0.) goto 21
      if(t3(2)) 22,23,23
   21 if(t3(2).lt.0.)goto 23
   22 xo(2)=xo(1)
      t3(2)=t3(1)
      xo(1)=xo(1)-dx
      goto 50
   23 x=(xo(1)+xo(2))/2.
      j=3
      xo(j)=x
      goto 50
   30 if(abs(t3(3)).lt.0.01) goto 41
      if(t3(1).ge.0.) goto 31
      if(t3(3)) 32,33,33
   31 if(t3(3).lt.0.) goto 33
   32 t3(1)=t3(3)
      xo(1)=xo(3)
      goto 23
   33 t3(2)=t3(3)
      xo(2)=xo(3)
      goto 23
   40 xm=t1(1)
      xop=xo(1)
      xs=sqrt(t2(1))
      goto 60
   41 xm=t1(3)
      xop=xo(3)
      xs=sqrt(t2(3))
      goto 60
   43 xm=t1(2)
      xop=xo(2)
      xs=sqrt(t2(2))
      goto 60
   50 u=0.
      v=0.
      w=0.
      x=0.
      k11=0
      do 51 l=1,k1
        if (e(l).le.-500.) goto 51
        a=e(l)-xo(j)
        k11=k11+1
        b=alog(a)
        u=u+1./a
        v=v+b
        w=w+b*b
        x=x+b/a
   51 continue
      t1(j)=v/k11
      t2(j)=w/k11-t1(j)*t1(j)
      t3(j)=u*(t2(j)-t1(j))+x
      if(m-2) 40,52,53
   52 if(j-1) 10,54,10
   53 if(j-2) 20,20,30
   54 j=2
      goto 50
   60 return
      end
c
c***********************************************************************
c
      subroutine tln3v(b,xb,n,ew,saw,ugw)
c *** transformation from logarithmic normal into normed normal distr.

      dimension b(n),xb(n)
      do 10 i=1,n
        if (b(i).gt.-500.) goto 5
        xb(i)=b(i)
        goto 10
    5   xb(i)=(alog(b(i)-ugw)-ew)/saw
   10 continue
      return
      end
c
c***********************************************************************
c
      subroutine pnv(b,n,ugb,exb,ew,saw,ugw,ex)
c *** parameter estimation of the power-normal distribution
c     b          - series
c     n          - length of b
c     ugb        - step control
c     exb        - step control
c     ew         - mean
c     saw        - standard deviation
c     ugw        - lower limit
c     ex         - exponent

      dimension b(n),d(3,3)
      integer [illegible]
```

```
      iend=500
      k=0
   10 k=k+1
      if (b(k).le.-500.) goto 10
      ugg=b(k)
      do 20 i=k,n
        if (b(i).le.-500.) goto 20
        if (b(i).lt.ugg) ugg=b(i)
   20 continue
      ugg=-(ugg-0.1*ugb)
      i=1
      ugm=ugg+50*ugb
      exm=1.
      do 30 ie=1,3
      do 30 iu=1,3
        ua=ugmt(iu-2)*ugb
        ea=exmt(ie-2)*exb
        call lifup(b,n,ugg,fl,ew,saw,ua,ea)
   30 h(ie,iu)=fl
   40 fmax=h(1,1)
      iex=1
      iux=1
      do 50 ie=1,3
        do 47 iu=1,3
                if (fmax.ge.h(ie,iu)) goto 49
                fmax=h(ie,iu)
                iex=ie
                iux=iu
   49   continue
   50 continue
      vu=iux-2
      ve=iex-2
      ugm=ugm+vu*ugb
      exm=exm+ve*exb
      if (i.ge.iend) goto 200
      if (iex.eq.2.and.iux.eq.2) goto 200
      if (vu.eq.0) goto 70
      if (vu.eq.1) lun=0
      if (vu.eq.-1) iun=4
      do 60 in=1,2
        iun=iun+vu
        iu=iun+vu
        do 60 ie=1,3
   60 h(ie,iun)=h(ie,iu)
   70 if (ve.eq.0) goto 90
      if (ve.eq.1) ien=0
      if (ve.eq.-1) ien=4
      do 80 in=1,2
        ien=ien+ve
        ie=ien+ve
        do 80 iu=1,3
   80 h(ien,iu)=h(ie,iu)
   90 if (vu.eq.0) goto 110
      nu=2+vu
      ua=ugmt(nu-2)*ugb
      do 100 ie=1,3
        ea=exmt(ie-2)*exb
        call lifup(b,n,ugg,fl,ew,saw,ua,ea)
  100 h(ie,nu)=fl
      if (ve.eq.0) goto 130
  110 ne=2+ve
      ea=exmt(ne-2)*exb
      do 120 iu=1,3
        if (vu.eq.0) goto 115
        if (iu.eq.nu) goto 120
  115   ua=ugmt(iu-2)*ugb
        call lifup(b,n,ugg,fl,ew,saw,ua,ea)
        h(ne,iu)=fl
  120 continue
  130 i=i+1
      goto 40
  200 call lifup(b,n,ugg,fl,ew,saw,ugm,exm)
      ugw=ugm
      ex=exm
      return
      end
c
c**********************************************************************
c
      subroutine lifup(b,n,ugg,fl,e,s,ug,ex)
c *** likelihood-function of the power-normal-distribution

      dimension b(n)
      if (ug.le.ugg.or.ex.lt.0.01.or.ex.gt.2.0) goto 20
      con=1.837877
      e=0.
      s=0.
      hh=0.
      k=0
      do 10 i=1,n
        if (b(i).le.-500.) goto 10
        k=k+1
        h=(b(i)+ug)**ex
        hh=hh+alog(b(i)+ug)
        e=e+h
        s=s+h*h
   10 continue
      e=e/k
      s=(s-k*e*e)/k
      s=sqrt(s)
      fl=k*alog(ex/s/con)+(ex-1.)*hh-k/2.
      return
   20 fl=-1000000.
      return
      end
c
c**********************************************************************
c
      subroutine tpnv(b,xb,n,ew,saw,ugw,ex)
c *** transformation from power normal into normed normal distribution

      dimension b(n),xb(n)
      do 10 i=1,n
        if (b(i).gt.-500.) goto 5
        xb(i)=b(i)
        goto 10
    5   xb(i)=((b(i)+ugw)**ex-ew)/saw
   10 continue
      return
      end
```

```
c
c*************************************************************************
c
      subroutine jov(b,n,ba,bb,e,s,ama,amb)
c *** parameter estimation of the johnson-distribution
c     b          - time series
c     n          - length of b
c     ba         - step control
c     bb         - step control
c     e          - mean
c     s          - standard deviation
c     ama        - lower limit
c     amb        - upper limit

      dimension b(n),h(3,3)
      integer va,vb
      iend=500
      k=0
   10 k=k+1
      if (b(k).le.-500.) goto 10
      bma=b(k)
      bmi=b(k)
      do 20 i=k,n
        if (b(i).le.-500.) goto 20
        if (bmi.gt.b(i)) goto 17
        if (bma.ge.b(i)) goto 20
        bma=b(i)
        goto 20
   17   bmi=b(i)
   20 continue
      bmi=bmi-0.1*ba
      bma=bma+0.1*bb
      i=1
      ma=0
      mb=50
      ama=bmi-ma*ba
      amb=bma+mb*bb
      do 30 iz=1,3
      do 30 is=1,3
        aka=ama+(2-is)*ba
        akb=amb+(iz-2)*bb
        call lifuj(b,n,bmi,bma,f,e,s,aka,akb)
   30 h(iz,is)=f
   40 fmax=h(1,1)
      izx=1
      isx=1
      do 50 iz=1,3
        do 49 is=1,3
                if (fmax.ge.h(iz,is)) goto 49
                fmax=h(iz,is)
                izx=iz
                isx=is
   49   continue
   50 continue
      va=2-isx
      vb=izx-2
      ma=ma-va
      mb=mb+vb
      ama=bmi-ma*ba
      amb=bma+mb*bb
      if(i.ge.iend) goto 200
      if (izx.eq.2.and.isx.eq.2) goto 210
      if (va.eq.0) goto 70
      if (va.eq.1) isn=4
      if (va.eq.-1) isn=0
      do 60 in=1,2
        isn=isn-va
        is=isn-va
        do 60 iz=1,3
   60 h(iz,isn)=h(iz,is)
   70 if (vb.eq.0) goto 90
      if (vb.eq.1) izn=0
      if (vb.eq.-1) izn=4
      do 80 in=1,2
        izn=izn+vb
        iz=izn+vb
        do 80 is=1,3
   80 h(izn,is)=h(iz,is)
   90 if (va.eq.0) goto 110
      ns=2-va
      aka=ama+(2-ns)*ba
      do 100 iz=1,3
        akb=amb+(iz-2)*bb
        call lifuj(b,n,bmi,bma,f,e,s,aka,akb)
  100 h(iz,ns)=f
      if (vb.eq.0) goto 130
  110 nz=2+vb
      akb=amb+(nz-2)*bb
      do 120 is=1,3
        if (va.eq.0) goto 115
        if (is.eq.ns) goto 120
  115   aka=ama+(2-is)*ba
        call lifuj(b,n,bmi,bma,f,e,s,aka,akb)
        h(nz,is)=f
  120 continue
  130 i=i+1
      goto 40
  200 continue
      goto 230
  210 continue
  230 call lifuj(b,n,bmi,bma,fmax,e,s,ama,amb)
      return
      end
c
c*************************************************************************
c
      subroutine lifuj(b,n,bmi,bma,ll,e,s,u,o)
c *** likelihood-function of the johnson-distribution

      dimension b(n),ha(100),hb(100)
      if (u.gt.bmi.or.o.lt.bma) goto 30
      con=1.837877
      e=0.
      s=0.
      l=0.
      do 10 i=1,n
        if (b(i).le.-500.) goto 10
        k=k+1
        ha(i)=alog(b(i)-u)
        hb(i)=alog(o-b(i))
```

```
      hc=ha(i)-hb(i)
      e=e+hc
      s=s+hc*hc
   10 continue
      e=e/k
      s=(s-k*e*e)/k
      sh=1./s
      s=sqrt(s)
      ff=k*alog(o-u)-k*alog(s)-0.5*k*con
      do 20 i=1,n
        if (b(i).le.-500.) goto 20
        ff=ff-(ha(i)+hb(i)+0.5*sh*(ha(i)-hb(i)-e)**2)
   20 continue
      return
   30 ff=-1000000.
      return
      end
c
c**********************************************************************
c
      subroutine tjov(b,xb,n,ew,saw,ugw,ogw)
c *** transformation from Johnson into normed normal distribution

      dimension b(n),xb(n)
      do 10 i=1,n
        if (b(i).gt.-500.) goto 5
        xb(i)=b(i)
        goto 10
    5   xb(i)=(alog((b(i)-ugw)/(ogw-b(i)))-ew)/saw
   10 continue
      return
      end
c
c**********************************************************************
c
      subroutine ord(x,n)
c *** size-grading
c                   - series
c     n             - length of x

      dimension x(n)
      i=0
   10 i=i+1
      s=x(i)
      k=i
   20 k=k+1
      if (k.gt.n) goto 30
      if (s.le.x(k)) goto 20
      x(i)=x(k)
      x(k)=s
      s=x(i)
      goto 20
   30 if (i.lt.n-1) goto 10
      return
      end
c
c**********************************************************************
c
      subroutine no2(t,n,tno)
c *** computation of the n-omega-square value for the fit ...
c     t             - series
c     n             - length of t
c     tno           - n-omega-square value
c     erf           - error function

      dimension t(n)
      call ord(t,n)
      k=0
      do 5 i=1,n
        if (t(i).lt.-500.) goto 5
        k=k+1
    5 continue
      sum=0.
      do 10 i=1,n
        if (t(i).lt.-500.) goto 10
        x=t(i)/1.4142135
        c=erf(x)/2.+0.5
        a=c-(i-0.5)/k
        sum=sum+a*a
   10 continue
      tno=sum+0.083333/k
      return
      end
c
c**********************************************************************
c
      subroutine koko(a,b,r,nn)
c *** computation of the correlation coefficient
c     a,b           - series
c     r             - correlation coefficient
c     nn            - length of a and b

      dimension a(nn),b(nn)
      s1=0.
      s2=0.
      s3=0.
      s4=0.
      s5=0.
      k=0
      do 10 m=1,nn
        if (a(m).le.-500..or.b(m).le.-500.) goto 10
        k=k+1
        s1=s1+a(m)
        s2=s2+b(m)
        s3=s3+a(m)*a(m)
        s4=s4+b(m)*b(m)
        s5=s5+a(m)*b(m)
   10 continue
      h1=s5-s1*s2/k
      h2=s3-s1*s1/k
      h2=h2*(s4-s2*s2/k)
      h2=sqrt(h2)
      r=h1/h2
      return
      end
```

```
c
c****************************************************************************
c
      subroutine gausse(a,b,n)
c *** solution of a linear quadratic set of equations
c     a          - matrix of coefficients
c     b          - absolute vector and solution
c     n          - size of a and b

      dimension a(25,25),b(25),ha(25)
      in=0
   10 in=in+1
      in1=in+1
      amax=abs(a(in,in))
      ie=in
      do 20 il=in1,n
        if (amax.ge.abs(a(il,in))) goto 20
        amax=abs(a(il,in))
        ie=il
   20 continue
      if (ie.eq.in) goto 40
      do 30 il=in,n
        ha(il)=a(in,il)
        a(in,il)=a(ie,il)
   30 a(ie,il)=ha(il)
      hb=b(in)
      b(in)=b(ie)
      b(ie)=hb
   40 do 60 iz=in1,n
        if (abs(a(in,in)).lt.1.e-6) goto 100
        p=-a(iz,in)/a(in,in)
        do 50 is=in1,n
   50   a(iz,is)=a(iz,is)+p*a(in,is)
   60 b(iz)=b(iz)+p*b(in)
      if (in1.lt.n) goto 10
      in=n
   70 in1=in+1
      s=0.
   80 if (in1.gt.n) goto 90
      s=s+b(in1)*a(in,in1)
      in1=in1+1
      goto 80
   90 if (abs(a(in,in)).lt.1.e-6) goto 100
      b(in)=(b(in)-s)/a(in,in)
      in=in-1
      if (in.ge.1) goto 70
      return
  100 do 110 i=1,n
  110 b(i)=0.0
      return
      end
```

* p r o g r a m ** s i k o ** *
******************************************

mathematical and statistical analysis of hydrologicaland meteorological monthly time series

control data :

2 10 3 2 5
2 2
t t
t t

series 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1300 | 0.8600 | 0.3400 | 0.5200 | 0.4700 | 0.6100 | 0.1100 | 0.1700 | 0.1200 | 0.0700 | 0.1900 | 0.4400 |
| 0.2600 | 0.2400 | 0.2600 | 0.9100 | 0.1400 | 0.0800 | 0.1800 | 0.1000 | 0.0500 | 0.0500 | 0.0500 | 0.0800 |
| 0.0500 | 0.0500 | 0.3400 | 0.4900 | 0.1900 | 0.2200 | 0.1400 | 0.3400 | 0.2400 | 0.3500 | 0.4800 | 0.1800 |
| 0.1100 | 0.1500 | 0.1700 | 0.3900 | 0.4700 | 0.1500 | 0.0700 | 0.0600 | 0.0400 | 0.0800 | 0.3600 | 0.3000 |
| 0.3400 | 0.1800 | 0.6200 | 0.8800 | 0.6300 | 0.3000 | 0.2100 | 0.2700 | 0.1300 | 0.0800 | 0.3000 | 1.1000 |
| 0.4500 | 0.7300 | 0.3500 | 0.7100 | 0.2900 | 0.3400 | 0.5400 | 0.2800 | 0.2200 | 0.2300 | 0.2900 | 0.8100 |
| 0.4300 | 0.5800 | 0.5200 | 0.4600 | 0.1500 | 0.2500 | 0.1000 | 0.0600 | 0.0900 | 0.1200 | 0.1600 | 0.6400 |
| 0.4300 | 0.1900 | 0.5000 | 0.6000 | 0.3000 | 0.2700 | 0.1400 | 0.1400 | 0.3000 | 0.4600 | 0.1300 | 0.1300 |
| 0.0800 | 0.1400 | 0.4000 | 1.3200 | 0.7200 | 0.6800 | 0.1600 | 0.1100 | 0.1100 | 0.0700 | 0.1700 | 0.1100 |
| 0.0900 | 0.3200 | 0.4800 | 1.5600 | 0.9100 | 0.1600 | 0.2000 | 0.3000 | 0.1500 | 0.3000 | 0.5000 | 0.4200 |

parameters of the logarithmic normal distribution

control parameter for the estimation of the lower limit ug0= 0.

| month | mean | stand.dev. | lower limit | no2-fitness |
|---|---|---|---|---|
| 1 | -1.9986 | 1.0234 | 0.0308 | 0.0770 |
| 2 | -1.3811 | 0.8256 | 0. | 0.0424 |
| 3 | -0.9802 | 0.3594 | 0. | 0.0468 |
| 4 | -1.2425 | 0.9817 | 0.3465 | 0.0227 |
| 5 | -1.6877 | 1.1810 | 0.1182 | 0.0689 |
| 6 | -1.3649 | 0.6095 | 0.0004 | 0.0279 |
| 7 | -2.2149 | 0.7166 | 0.0441 | 0.0476 |
| 8 | -2.0470 | 0.7357 | 0.0194 | 0.0567 |
| 9 | -2.1033 | 0.6168 | 0. | 0.0340 |
| 10 | -2.5603 | 1.1797 | 0.0400 | 0.0723 |
| 11 | -1.5192 | 0.6602 | 0. | 0.0383 |
| 12 | -1.6384 | 1.2839 | 0.0643 | 0.0487 |

graded series by size

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0500 | 0.0500 | 0.1700 | 0.3900 | 0.1400 | 0.0800 | 0.0700 | 0.0600 | 0.0400 | 0.0500 | 0.0500 | 0.0800 |
| 0.0800 | 0.1400 | 0.2600 | 0.4600 | 0.1500 | 0.1500 | 0.1000 | 0.0600 | 0.0500 | 0.0700 | 0.1300 | 0.1100 |
| 0.0900 | 0.1500 | 0.3400 | 0.4900 | 0.1900 | 0.1600 | 0.1100 | 0.1000 | 0.0900 | 0.0700 | 0.1600 | 0.1300 |
| 0.1100 | 0.1800 | 0.3400 | 0.5200 | 0.2900 | 0.2200 | 0.1400 | 0.1100 | 0.1100 | 0.0800 | 0.1700 | 0.1800 |
| 0.1300 | 0.1900 | 0.3500 | 0.6000 | 0.3000 | 0.2500 | 0.1400 | 0.1400 | 0.1200 | 0.0800 | 0.1900 | 0.3000 |
| 0.2600 | 0.2400 | 0.4000 | 0.7100 | 0.4700 | 0.2700 | 0.1600 | 0.1700 | 0.1300 | 0.1200 | 0.2900 | 0.4200 |
| 0.3400 | 0.3200 | 0.4800 | 0.8800 | 0.4700 | 0.3000 | 0.1800 | 0.2700 | 0.1500 | 0.2300 | 0.3000 | 0.4400 |
| 0.4300 | 0.5800 | 0.5000 | 0.9100 | 0.6300 | 0.3400 | 0.2000 | 0.2800 | 0.2200 | 0.3000 | 0.3600 | 0.6400 |
| 0.4300 | 0.7300 | 0.5200 | 1.3200 | 0.7200 | 0.6100 | 0.2100 | 0.3000 | 0.2400 | 0.3500 | 0.4800 | 0.8100 |
| 0.4500 | 0.8600 | 0.6200 | 1.5600 | 0.9100 | 0.6800 | 0.5400 | 0.3400 | 0.3000 | 0.4600 | 0.5000 | 1.1000 |

special quantiles

| p.c. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0433 | 0.0368 | 0.1626 | 0.3759 | 0.1301 | 0.0633 | 0.0547 | 0.0427 | 0.0291 | 0.0450 | 0.0471 | 0.0741 |
| 2 | 0.0473 | 0.0461 | 0.1794 | 0.3850 | 0.1346 | 0.0775 | 0.0692 | 0.0479 | 0.0344 | 0.0468 | 0.0564 | 0.0782 |
| 5 | 0.0559 | 0.0646 | 0.2078 | 0.4039 | 0.1447 | 0.0942 | 0.0777 | 0.0579 | 0.0443 | 0.0511 | 0.0739 | 0.0878 |
| 10 | 0.0673 | 0.0872 | 0.2367 | 0.4286 | 0.1589 | 0.1174 | 0.0877 | 0.0697 | 0.0554 | 0.0570 | 0.0939 | 0.1018 |
| 15 | 0.0777 | 0.1068 | 0.2586 | 0.4509 | 0.1726 | 0.1362 | 0.0961 | 0.0796 | 0.0644 | 0.0627 | 0.1104 | 0.1156 |
| 20 | 0.0880 | 0.1254 | 0.2773 | 0.4729 | 0.1867 | 0.1534 | 0.1038 | 0.0889 | 0.0726 | 0.0686 | 0.1256 | 0.1302 |
| 25 | 0.0987 | 0.1440 | 0.2945 | 0.4954 | 0.2016 | 0.1679 | 0.1114 | 0.0980 | 0.0805 | 0.0748 | 0.1402 | 0.1460 |
| 30 | 0.1101 | 0.1631 | 0.3109 | 0.5192 | 0.2179 | 0.1861 | 0.1191 | 0.1072 | 0.0884 | 0.0816 | 0.1549 | 0.1635 |
| 35 | 0.1221 | 0.1828 | 0.3267 | 0.5443 | 0.2356 | 0.2024 | 0.1269 | 0.1166 | 0.0962 | 0.0889 | 0.1697 | 0.1827 |
| 40 | 0.1357 | 0.2029 | 0.3426 | 0.5716 | 0.2553 | 0.2193 | 0.1351 | 0.1265 | 0.1044 | 0.0972 | 0.1852 | 0.2046 |
| 45 | 0.1499 | 0.2265 | 0.3587 | 0.6017 | 0.2776 | 0.2378 | 0.1439 | 0.1371 | 0.1130 | 0.1065 | 0.2015 | 0.2296 |
| 50 | 0.1663 | 0.2513 | 0.3752 | 0.6352 | 0.3032 | 0.2559 | 0.1533 | 0.1485 | 0.1221 | 0.1171 | 0.2189 | 0.2586 |
| 55 | 0.1849 | 0.2788 | 0.3926 | 0.6731 | 0.3328 | 0.2762 | 0.1636 | 0.1610 | 0.1319 | 0.1274 | 0.2378 | 0.2926 |
| 60 | 0.2064 | 0.3098 | 0.4110 | 0.7167 | 0.3677 | 0.2985 | 0.1750 | 0.1750 | 0.1427 | 0.1439 | 0.2588 | 0.3333 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | 0.2318 | 0.3454 | 0.4310 | 0.7679 | 0.4097 | 0.3235 | 0.1880 | 0.1708 | 0.1543 | 0.1614 | 0.2823 | 0.3829 |
| 70 | 0.2623 | 0.3871 | 0.4529 | 0.8291 | 0.4614 | 0.3518 | 0.2030 | 0.2091 | 0.1686 | 0.1829 | 0.3092 | 0.4447 |
| 75 | 0.3010 | 0.4386 | 0.4782 | 0.9062 | 0.5284 | 0.3857 | 0.2211 | 0.2315 | 0.1850 | 0.2108 | 0.3417 | 0.5262 |
| 80 | 0.3514 | 0.5034 | 0.5078 | 1.0060 | 0.6179 | 0.4270 | 0.2436 | 0.2592 | 0.2051 | 0.2480 | 0.3815 | 0.6367 |
| 85 | 0.4222 | 0.5913 | 0.5446 | 1.1450 | 0.7472 | 0.4808 | 0.2735 | 0.2962 | 0.2313 | 0.3018 | 0.4339 | 0.7993 |
| 90 | 0.5338 | 0.7239 | 0.5948 | 1.3623 | 0.9584 | 0.5583 | 0.3176 | 0.3509 | 0.2691 | 0.3896 | 0.5101 | 1.0713 |
| 95 | 0.7604 | 0.9771 | 0.6777 | 1.7977 | 1.4087 | 0.6965 | 0.3990 | 0.4524 | 0.3367 | 0.5767 | 0.6484 | 1.6698 |
| 98 | 1.1376 | 1.3675 | 0.7850 | 2.5145 | 2.2098 | 0.8776 | 0.5198 | 0.6044 | 0.4332 | 0.9094 | 0.8494 | 2.7782 |
| 99 | 1.4964 | 1.7152 | 0.8658 | 3.1797 | 3.0041 | 1.0550 | 0.6224 | 0.7344 | 0.5126 | 1.2392 | 1.0169 | 3.9155 |

transformed series

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -0.3045 | 1.4902 | -0.2744 | -0.5187 | 0.5444 | 1.4271 | -0.7045 | 0.2094 | -0.0276 | -0.8000 | -0.2144 | 0.5137 |
| 0.5136 | -0.0557 | -1.0209 | 0.6813 | -1.8112 | -1.9136 | 0.3055 | -0.6401 | -1.4469 | -1.7312 | -2.2365 | -1.7580 |
| -1.9077 | -1.9557 | -0.2744 | -0.7121 | -0.8014 | -0.2482 | -0.1810 | 1.2363 | 1.0962 | 1.1796 | 1.1893 | -0.4036 |
| -0.5244 | -0.6250 | -2.2032 | -1.9283 | 0.5444 | -0.8780 | -2.0080 | -1.5717 | -1.8087 | -0.5561 | 0.7536 | 0.1506 |
| 0.8061 | -0.4042 | 1.3973 | 0.6256 | 0.8618 | 0.2616 | 0.5839 | 0.9015 | 0.1022 | -0.5561 | 0.4774 | 1.3035 |
| 1.1035 | 1.2917 | -0.1938 | 0.2347 | -0.0625 | 0.4672 | 2.1119 | 0.9547 | 0.9551 | 0.7647 | 0.4261 | 1.0476 |
| 1.0557 | 1.0131 | 0.9078 | -0.9511 | -1.4912 | -0.0380 | -0.5074 | -1.5717 | -0.4940 | 0.0314 | -0.4747 | 0.8461 |
| 1.0557 | -0.3387 | 0.7987 | -0.1324 | -0.0146 | 0.0884 | -0.1810 | -0.0925 | 1.4580 | 1.4371 | -0.7892 | -0.8442 |
| -0.9893 | -0.7086 | 0.1778 | 1.2382 | 0.9990 | 1.6054 | 0.0834 | -0.4811 | -0.1686 | -0.8000 | -0.3829 | -1.1268 |
| -0.8097 | 0.2728 | 0.6851 | 1.4627 | 1.2313 | -0.7719 | 0.4971 | 1.0552 | 0.3342 | 1.0375 | 1.[illegible] | 0.4711 |

autocorrelation coefficients

| order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -1 | 0.9702 | 0.5852 | 0.1470 | 0.4972 | 0.2787 | 0.4849 | 0.1111 | 0.6430 | 0.7198 | 0.8058 | 0.5574 | 0.6057 |
| -2 | 0.6358 | 0.7743 | 0.3222 | 0.3470 | 0.1592 | [illegible] | -0.0957 | [illegible] | [illegible] | [illegible] | 0.7005 | 0.2696 |
| -3 | 0.1404 | 0.6047 | 0.1606 | 0.0089 | -0.0143 | 0.3632 | [illegible] | [illegible] | 0.4178 | 0.2398 | 0.4833 | 0.1937 |

series

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1600 | 0.7200 | 0.4700 | 0.5900 | 0.4100 | 0.5500 | 0.2800 | 0.3800 | 0.2800 | 0.1800 | 0.2800 | 0.4600 |
| 0.2800 | 0.2000 | 0.1600 | 0.8500 | 0.4000 | 0.2100 | 0.2400 | 0.1100 | 0.1300 | 0.1300 | 0.1600 | 0.1700 |
| 0.1300 | 0.0800 | 0.3600 | 0.6200 | 0.2200 | 0.2300 | 0.1800 | 0.4000 | 0.2100 | 0.3300 | 0.4100 | 0.1600 |
| 0.0800 | 0.0700 | 0.1800 | 0.3800 | 0.3300 | 0.1100 | 0.0700 | 0.0700 | 0.0500 | 0.1800 | 0.4300 | 0.3300 |
| 0.3000 | 0.1200 | 0.4700 | 1.0400 | 1.1200 | 0.3200 | 0.1500 | 0.2300 | 0.1400 | 0.1400 | 0.2800 | 0.7600 |
| 0.4000 | 0.9000 | 0.3700 | 0.8200 | 0.2700 | 0.5700 | 0.5700 | 0.2700 | 0.2600 | 0.3700 | 0.3400 | 0.7900 |
| 0.4700 | 0.8600 | 0.7400 | 0.6400 | 0.2400 | 0.3100 | 0.1300 | 0.0900 | 0.1800 | 0.2500 | 0.2300 | 0.6500 |
| 0.4000 | 0.2000 | 0.5400 | 1.2100 | 0.4300 | 0.1900 | 0.1600 | 0.1300 | 0.6500 | 0.6400 | 0.1400 | 0.1400 |
| 0.1200 | 0.1700 | 0.2700 | 1.0700 | 0.4500 | 0.4300 | 0.1300 | 0.1500 | 0.1300 | 0.0700 | 0.1400 | 0.0800 |
| 0.0700 | 0.2100 | 0.3300 | 1.4300 | 1.0500 | 0.1500 | 0.3200 | 0.3800 | 0.1400 | 0.6300 | 0.9500 | 0.6800 |

parameters of the logarithmic normal distribution

control parameter for the estimation of the lower limit us0= 0.01

| month | mean | stand.dev. | lower limit | no2-fitness |
|---|---|---|---|---|
| 1 | -2.2283 | 1.2083 | 0.0600 | 0.0715 |
| 2 | -2.0715 | 1.4263 | 0.0700 | 0.0533 |
| 3 | -1.3119 | 1.1943 | 0.1500 | 0.1759 |
| 4 | -1.1221 | 1.2666 | 0.3700 | 0.1533 |
| 5 | -1.9525 | 1.3231 | 0.2100 | 0.0557 |
| 6 | -1.9973 | 1.1048 | 0.1000 | 0.0486 |
| 7 | -2.2004 | 1.0015 | 0.0600 | 0.0543 |
| 8 | -2.2426 | 1.1017 | 0.0600 | 0.0607 |
| 9 | -2.1200 | 1.0022 | 0.0400 | 0.1014 |
| 10 | -1.9417 | 1.1290 | 0.0600 | 0.0457 |
| 11 | -2.3050 | 1.4010 | 0.1300 | 0.0829 |
| 12 | -1.5748 | 1.3396 | 0.0700 | 0.0746 |

graded series by size

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0700 | 0.0800 | 0.1600 | 0.3800 | 0.2200 | 0.1100 | 0.0700 | 0.0700 | 0.0500 | 0.0700 | 0.1400 | 0.0800 |
| 0.0900 | 0.0700 | 0.1800 | 0.5700 | 0.2400 | 0.1500 | 0.1300 | 0.0900 | 0.1300 | 0.1300 | 0.1400 | 0.1400 |
| 0.1200 | 0.1200 | 0.2700 | 0.6200 | 0.2700 | 0.1900 | 0.1300 | 0.1100 | 0.1300 | 0.1400 | 0.1600 | 0.1600 |
| 0.1300 | 0.1700 | 0.3600 | 0.6400 | 0.3300 | 0.2100 | 0.1500 | 0.1500 | 0.1400 | 0.1800 | 0.2300 | 0.1700 |
| 0.1600 | 0.2000 | 0.3700 | 0.8200 | 0.4000 | 0.2300 | 0.1600 | 0.1900 | 0.1400 | 0.1800 | 0.2800 | 0.3300 |
| 0.2800 | 0.2000 | 0.3900 | 0.8500 | 0.4100 | 0.3100 | 0.1800 | 0.2300 | 0.1800 | 0.2500 | 0.2800 | 0.4600 |
| 0.3000 | 0.2100 | 0.4700 | 1.0400 | 0.4300 | 0.3200 | 0.2400 | 0.2700 | 0.2100 | 0.3700 | 0.3400 | 0.6500 |
| 0.4000 | 0.7200 | 0.4700 | 1.0700 | 0.4500 | 0.4300 | 0.2800 | 0.3800 | 0.2600 | 0.3700 | 0.4100 | 0.6800 |
| 0.4000 | 0.8600 | 0.5400 | 1.2100 | 1.0500 | 0.5500 | 0.3200 | 0.3800 | 0.2800 | 0.6300 | 0.4300 | 0.7900 |
| 0.4700 | 0.9000 | 0.7400 | 1.4300 | 1.1200 | 0.5700 | 0.5700 | 0.4000 | 0.6500 | 0.6400 | 0.9500 | 0.7600 |

special quantiles

| p.c. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0665 | 0.0744 | 0.1601 | [illegible] | 0.2165 | 0.1124 | 0.0708 | 0.0687 | 0.0517 | 0.0704 | 0.1338 | 0.0792 |
| 2 | 0.0690 | 0.0767 | 0.1641 | 0.3741 | 0.2174 | [illegible] | [illegible] | 0.0711 | [illegible] | 0.0741 | 0.1354 | 0.0832 |
| 5 | 0.0748 | 0.0821 | 0.1729 | 0.4105 | 0.2261 | [illegible] | 0.0813 | 0.0773 | 0.0631 | [illegible] | 0.1400 | 0.0929 |
| 10 | 0.0827 | 0.0903 | 0.1853 | 0.4342 | 0.2360 | 0.1329 | [illegible] | [illegible] | [illegible] | [illegible] | 0.1444 | 0.1072 |
| 15 | 0.0908 | 0.0987 | 0.1974 | 0.4576 | 0.2440 | 0.1471 | 0.0992 | 0.0937 | [illegible] | 0.1045 | 0.1574 | 0.1217 |
| 20 | 0.0990 | 0.1077 | 0.2078 | 0.4821 | [illegible] | 0.1574 | 0.1077 | 0.1020 | 0.0916 | 0.1155 | 0.1607 | 0.1371 |
| 25 | 0.1077 | 0.1181 | 0.2230 | 0.5086 | 0.2681 | 0.1643 | 0.1164 | 0.1105 | 0.1011 | 0.1270 | 0.1608 | 0.1539 |
| 30 | 0.1172 | 0.1297 | 0.2374 | 0.5378 | 0.2810 | 0.1760 | 0.1256 | 0.1197 | 0.1110 | 0.1394 | 0.1779 | 0.1727 |
| 35 | 0.1276 | 0.1427 | 0.2531 | 0.5699 | 0.2952 | 0.1905 | 0.1353 | 0.1295 | 0.1216 | 0.1529 | 0.1881 | 0.1936 |
| 40 | 0.1393 | 0.1578 | 0.2707 | 0.6062 | 0.3115 | 0.2024 | 0.1459 | 0.1403 | 0.1331 | 0.1678 | 0.1999 | 0.2175 |
| 45 | 0.1525 | 0.1753 | 0.2906 | 0.6477 | 0.3302 | 0.2179 | 0.1577 | 0.1525 | 0.1458 | 0.1845 | 0.2137 | 0.2450 |
| 50 | 0.1677 | 0.1960 | 0.3133 | 0.6956 | 0.3519 | 0.2354 | 0.1708 | 0.1642 | 0.1600 | 0.2035 | 0.2278 | 0.2771 |
| 55 | 0.1854 | 0.2207 | 0.3398 | 0.7518 | 0.3776 | 0.2556 | 0.1856 | 0.1820 | 0.1761 | 0.2253 | 0.2490 | 0.3150 |
| 60 | 0.2063 | 0.2509 | 0.3711 | 0.8198 | 0.4084 | 0.2792 | 0.2029 | 0.2004 | 0.1947 | 0.2510 | 0.2723 | 0.3607 |
| 65 | 0.2316 | 0.2883 | 0.4088 | 0.9085 | 0.4463 | 0.3073 | 0.2229 | 0.2223 | 0.2166 | 0.2816 | 0.3012 | 0.4169 |
| 70 | 0.2627 | 0.3353 | 0.4552 | 1.0018 | 0.4936 | 0.3415 | 0.2471 | 0.2490 | 0.2428 | 0.3170 | 0.3377 | 0.4874 |
| 75 | 0.3033 | 0.3997 | 0.5156 | 1.1351 | 0.5564 | 0.3853 | 0.2777 | 0.2832 | 0.2740 | 0.3472 | 0.3867 | 0.5811 |
| 80 | 0.3578 | 0.4885 | 0.5963 | 1.3155 | 0.6421 | 0.4432 | 0.3173 | 0.3284 | 0.3170 | 0.4310 | 0.4544 | 0.7093 |
| 85 | 0.4368 | 0.6225 | 0.7132 | 1.5801 | 0.7692 | 0.5756 | 0.3727 | 0.3726 | 0.3791 | 0.5223 | 0.5561 | 0.8999 |
| 90 | 0.5667 | 0.8539 | 0.9048 | 2.0208 | 0.9835 | 0.6580 | 0.4598 | 0.4758 | 0.4736 | 0.6677 | 0.7308 | 1.2226 |
| 95 | 0.8460 | 1.3861 | 1.3149 | 2.9854 | 1.4609 | 0.9336 | 0.6352 | 0.7102 | 0.6641 | 0.9789 | 1.1295 | 1.9451 |
| 98 | 1.3482 | 2.4231 | 2.0483 | 4.7601 | 2.3588 | 1.4097 | 0.9264 | 1.0803 | 0.9801 | 1.5181 | 1.9024 | 3.3127 |
| 99 | 1.9509 | 3.5488 | 2.7788 | 6.5706 | 3.2920 | 1.8679 | 1.1984 | 1.4376 | 1.2755 | 2.0436 | 2.7267 | 4.7419 |

transformed series

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -0.0615 | 1.1503 | 0.5631 | -0.3095 | 0.2593 | 1.0869 | 0.6852 | 1.0013 | 0.6914 | -0.1582 | 0.2911 | 0.4727 |
| 0.5911 | 0.0219 | -2.3388 | 0.3064 | 0.2206 | -0.1882 | 0.4848 | -0.6836 | -0.2873 | -0.6356 | -0.3577 | -0.5433 |
| -0.3567 | -1.7764 | 0.2104 | -0.2086 | -2.0048 | -0.0370 | 0.0800 | 1.0564 | 0.3473 | 0.5601 | 0.7366 | -0.6220 |
| -1.3935 | -1.2904 | -1.4189 | -2.7499 | -0.1268 | -2.3586 | -2.4011 | -2.1445 | -2.4798 | -0.1582 | 0.7859 | 0.1700 |
| 0.6631 | -0.6480 | 0.5631 | 0.5697 | 1.4044 | 0.4372 | -0.2072 | 0.4272 | -0.1822 | -0.5173 | 0.2911 | 1.0886 |
| 0.9513 | 1.3217 | 0.2493 | 0.2554 | -0.6506 | 1.1262 | 1.5247 | 0.5282 | 0.6045 | 0.5601 | 0.5313 | 0.9304 |
| 1.1063 | 1.2871 | 1.0753 | -0.1478 | -1.1745 | 0.3971 | -0.4582 | -1.1473 | 0.1535 | 0.2489 | 0.0017 | 0.7689 |
| 0.9513 | 0.0219 | 0.7287 | 0.7482 | 0.3314 | -0.3698 | -0.1020 | 0.1110 | 1.6222 | 1.2373 | -1.6419 | -0.8096 |
| -0.4843 | -0.1620 | 0.2866 | 0.6043 | 0.3971 | 0.8062 | -0.4532 | -0.1501 | -0.2873 | -2.3591 | -1.6419 | -2.2622 |
| -1.9672 | 0.0739 | 0.0813 | 0.9319 | 1.3439 | -0.9019 | 0.8520 | 1.0013 | -0.1822 | 1.2219 | 1.5036 | 0.8066 |

auto-correlation coefficients

| order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -1 | 0.9446 | 0.4580 | 0.3378 | 0.3787 | 0.3302 | -0.0783 | 0.6658 | 0.7741 | 0.6630 | 0.3593 | 0.4614 | 0.7348 |
| -2 | 0.5634 | 0.4701 | 0.2680 | 0.3335 | -0.0782 | 0.5949 | 0.0613 | 0.5498 | 0.7016 | 0.2711 | -0.3442 | 0.5365 |
| -3 | 0.4010 | 0.1366 | 0.1188 | 0.3150 | 0.0789 | 0.5234 | 0.7302 | 0.1190 | 0.6636 | 0.2051 | 0.1743 | -0.0483 |

cross-correlation coefficients series 1 and 2

| order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.7560 | 0.9396 | 0.7705 | 0.8390 | 0.6328 | 0.6575 | 0.9208 | 0.9135 | 0.8098 | 0.7624 | 0.7494 | 0.9240 |
| -1 | 0.7718 | 0.2971 | 0.3039 | 0.0466 | 0.0382 | 0.0274 | 0.6039 | 0.6824 | 0.7635 | 0.4675 | 0.3331 | 0.6532 |
| -2 | 0.6677 | 0.7855 | 0.3654 | 0.1757 | 0.2547 | [illegible] | 0.1185 | 0.3551 | 0.5585 | 0.4743 | [illegible] | 0.3226 |
| -3 | 0.3119 | 0.4932 | 0.0899 | -0.0641 | -0.1324 | 0.7157 | 0.7177 | [illegible] | 0.4416 | [illegible] | 0.3151 | [illegible] |

cross-correlation coefficients series 2 and 1

| order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -1 | [illegible] | 0.6145 | 0.2203 | 0.7844 | 0.5944 | [illegible] | 0.1412 | 0.5617 | 0.4473 | [illegible] | [illegible] | 0.3548 |
| -2 | 0.1045 | 0.2570 | 0.1054 | 0.1189 | 0.2811 | 0.3772 | -0.0874 | 0.7754 | 0.5036 | [illegible] | [illegible] | [illegible] |
| -3 | -0.0678 | 0.3638 | 0.0772 | 0.2115 | 0.1776 | 0.9086 | 0.5704 | 0.2897 | 0.3679 | 0.1075 | 0.3581 | -0.0125 |

control data :

| | |
|---|---|
| 3 | 4 |
| t | t |

control data :

| | | |
|---|---|---|
| 1 | 1 | 2 |
| 2 | -1 | -1 |

coefficients for the stochastic simulation - variant 1

| 1 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | 1.0350 | 0.8417 | -1.1814 | 1.1353 | 0.9004 | 0.7810 | -0.5037 | 0.2343 | 0.1748 | 1.2414 | 0.7150 | 0.3106 |
| 2 | -1 | -0.0788 | -0.3392 | 1.4139 | -0.8282 | -0.7172 | -0.4648 | 0.9351 | 0.4737 | 0.6403 | -0.5378 | -0.2120 | 0.4234 |

residual standard deviation

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2382 | 0.7799 | 0.8626 | 0.6885 | 0.8709 | 0.7959 | 0.7009 | 0.7164 | 0.6435 | 0.5010 | 0.8215 | 0.7237 |

control data :

| | | | |
|---|---|---|---|
| 2 | 1 | 2 | 1 |
| 0 | -1 | -1 | 0 |

coefficients for the stochastic simulation - variant 2

| 2 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | -0.9036 | -0.1733 | 0.0496 | 0.5071 | 0.6748 | -0.8888 | -0.2463 | -0.4272 | -0.9972 | -1.2360 | -0.9726 | -0.6788 |
| 2 | -1 | 1.0528 | 0.3068 | 0.0654 | -0.0195 | -0.2528 | 0.4324 | 0.4967 | 0.5376 | 0.9393 | 0.6898 | 0.8705 | 0.7463 |
| 1 | 0 | 0.8221 | 0.9498 | 0.7434 | 0.5877 | 0.4409 | 1.0775 | 0.5482 | 0.7873 | 0.7723 | 1.4364 | 0.9777 | 0.7612 |

residual standard deviation

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2615 | 0.2712 | 0.6279 | 0.3419 | 0.6351 | 0.5036 | 0.3041 | 0.2565 | 0.4181 | 0.4999 | 0.2752 | 0.2580 |

## Appendix A2

SIMO - Stochastic simulation of monthly time series

```
c********************************************************************************
c
c ***   stochastic simulation of monthly time series
c
c********************************************************************************
        program simo
c
c ***   ng      - number of time series ( <=25 )
c ***   n       - number of simulation years
c ***   ver     - type of distribution function, see program SIKO
c ***   par     - parameters of distribution finction, see SIKO
c ***   io/ig   - parameters for simulation, see SIKO
c ***   sk      - simulation coefficients
c
c ***   The output of SIKO is used as input for SIMO
c
        dimension x(2,12,25),b(12,25),io(25),ig(25,25),iv(25,25),
     *            ver(25),sk(25,12,25),par(4,12,25)
        integer   ver
        data iin,iou,ires/5,6,7/

c ***   IIASA-routine for assignment of input/outpit units
        call usearg

        write (iou,4000)
        read  (iin,1000) ng,n
        write (iou,1000) ng,n
        write (iou,3000)
        read  (iin,1000) (ver(l),l=1,ng)
        write (iou,1000) (ver(l),l=1,ng)
        do 10 l=1,ng
              write (iou,3000)
              lver=ver(l)
              goto (1,3,5,5), lver
    1         do 2 j=1,12
                    read  (iin,2000) (par(i,j,l),i=1,2)
                    write (iou,2000) (par(i,j,l),i=1,2)
    2         continue
              goto 10
    3         do 4 j=1,12
                    read  (iin,2000) (par(i,j,l),i=1,3)
                    write (iou,2000) (par(i,j,l),i=1,3)
    4         continue
              goto 10
    5         do 6 j=1,12
                    read  (iin,2000) (par(i,j,l),i=1,4)
                    write (iou,2000) (par(i,j,l),i=1,4)
    6         continue
   10   continue
        write (iou,3000)
        read  (iin,1000) (io(l),l=1,ng)
        write (iou,1000) (io(l),l=1,ng)
        do 20 l=1,ng
              write (iou,3000)
              iol=io(l)
              read  (iin,1000) (ig(l,i),i=1,iol)
              write (iou,1000) (ig(l,i),i=1,iol)
              read  (iin,1000) (iv(l,i),i=1,iol)
              write (iou,1000) (iv(l,i),i=1,iol)
              write (iou,3000)
              do 20 j=1,12
                    read  (iin,2000) (sk(l,j,i),i=1,iol)
                    write (iou,2000) (sk(l,j,l),i=1,iol)
   20   continue
        write (iou,5000) n
        ix=72351
        do 30 l=1,ng
              do 30 j=1,12
                    x(1,j,l)=0.0
   30   continue
        do 130 i=1,n
              do 120 j=1,12
              do 120 l=1,ng
                    xsim=0.0
                    iol=io(l)-1
                    do 60 ioz=1,iol
                          igi=ig(l,ioz+1)
                          ivi=iv(l,ioz+1)
                          lq=j+ivi
                          if(lq.ge.1) goto 40
                          iq=iq+12
                          goto 50
   40                     xa=x(2,lq,igi)*sk(l,j,ioz)
                          goto 60
   50                     xa=x(1,lq,igi)*sk(l,j,ioz)
   60               xsim=xsim+xa
                    call nozu(ix,zu1,zu2)
                    x(2,j,l)=xsim+sk(l,j,iol+1)*zu1
                    lver=ver(l)
                    goto (80,90,100,110), lver
   80               b(j,l)=x(2,j,l)*par(2,j,l)+par(1,j,l)
                    goto 120
   90               b(j,l)=exp(x(2,j,l)*par(2,j,l)+par(1,j,l))
     *                                           +par(3,j,l)
                    goto 120
  100               h=x(2,j,l)*par(2,j,l)+par(1,j,l)
                    if(h.gt.0.0) goto 105
                    b(j,l)=-par(3,j,l)
                    goto 120
  105               b(j,l)=h**(1./par(4,j,l))-par(3,j,l)
                    goto 120
  110               h=exp(x(2,j,l)*par(2,j,l)+par(1,j,l))
                    b(j,l)=(par(3,j,l)+par(4,j,l)*h)/(h+1.)
  120         continue
              do 130 j=1,12
              do 125 l=1,ng
                    x(1,j,l)=x(2,j,l)
  125         continue
              write (ires,2000) (b(j,l),l=1,ng)
              write (iou,2000) (b(j,l),l=1,ng)
  130   continue
 1000   format (16i5)
 2000   format (8f10.4)
 3000   format (/)
 4000   format (/10x,'***** p r o g r a m   s i m o ****',//,6x,
     *         'stochastic simulation of monthly time series',
     *         //,'  control dates ',/)
 5000   format (//,5x,i5,'  y e a r s   g e n e r a t e d',
     *         //'  series 1    series 2'/)
        end
```

```
c
c*******************************************************************************
c
      subroutine nozu(ix,zu1,zu2)
c ***  generation of N(0,1)-distributed random numbers zu1,zu2

   10 ik=ix*1156+152897
      ix=mod(ik,1157625)
      z1=ix/1157625.
      ik=ix*1156+152897
      ix=mod(ik,1157625)
      z2=ix/1157625.
      if (z1.lt.1.e-6) goto 10
      a=(-alog(z1))*2.
      b=z2*6.2831853072
      zu1=sqrt(a)*cos(b)
      zu2=sqrt(a)*sin(b)
      return
      end
```

```
***** p r o g r a m   s i m o ****

  stochastic simulation of monthly time series

control dates

 2   10


 2    2


-1.9785     1.0274     0.0308
-1.7811     0.8256     0.
-0.9802     0.3594     0.
-1.2425     0.9817     0.3465
-1.6877     1.1810     0.1182
-1.3649     0.6095     0.0004
-2.2149     0.7166     0.0441
-2.0470     0.7357     0.0194
-2.1033     0.6168     0.
-2.5428     1.1797     0.0400
-1.5192     0.6602     0.
-1.6384     1.2839     0.0643


-2.2283     1.2093     0.0600
-2.0715     1.4263     0.0700
-1.8119     1.1943     0.1500
-1.1721     1.2666     0.3700
-1.9525     1.3231     0.2100
-1.9993     1.1048     0.1000
-2.2004     1.0015     0.0600
-2.2426     1.1017     0.0600
-2.1200     1.0022     0.0400
-1.9417     1.1290     0.0600
-2.3050     1.4010     0.1300
-1.5740     1.3396     0.0700


 3    4


 1    1    2
 0   -1   -1


 1.0350    -0.0788     0.2382
 0.0417    -0.3392     0.7799
-1.1814     1.4139     0.8676
 1.1353    -0.8282     0.6995
 0.9004    -0.7172     0.8709
 0.7810    -0.4648     0.7759
-0.5037     0.9351     0.7009
 0.7543     0.4737     0.7164
-0.1748     0.6403     0.6435
 1.2414    -0.5378     0.5010
 0.7150    -0.2120     0.6215
 0.3106     0.4204     0.7287
```

```
 2    1    2    1
 0   -1   -1    0

-0.9036     1.0528     0.8721     0.2615
-0.1733     0.3068     0.9498     0.2712
 0.0496     0.0654     0.7434     0.6277
 0.5071    -0.0195     0.5877     0.3419
 0.6748    -0.2528     0.4409     0.6351
-0.8988     0.4324     1.0775     0.5036
-0.2463     0.4967     0.5482     0.5041
-0.4272     0.5876     0.7873     0.2565
-0.9972     0.9893     0.7723     0.4181
-1.2360     0.6888     1.4364     0.4999
-0.9726     0.8705     0.9977     0.2952
-0.6788     0.7463     0.7612     0.2580


       10  y e a r s   g e n e r a t e d

series 1   series 2

 0.1776     0.1907
 0.1899     0.1569
 0.3817     0.2765
 0.8905     0.8373
 0.4347     0.5052
 0.5129     0.2496
 0.1782     0.1602
 0.3134     0.4806
 0.3548     0.2717
 0.4821     0.1696
 0.1588     0.1399
 0.1048     0.0854
 0.0782     0.0785
 0.2308     0.1811
 0.1788     0.1654
 0.3873     0.3908
 0.2118     0.2655
 0.1394     0.1542
 0.1076     0.1304
 0.1528     0.1973
 0.0555     0.0941
 0.0517     0.1079
 0.4580     1.7378
 0.7830     0.9165
 0.3028     0.4177
 0.2625     0.2394
 0.5807     0.7112
 2.2132     5.3082
 0.4477     0.8201
 0.5115     0.6607
 0.1086     0.1203
 0.1737     0.1836
 0.1363     0.1264
 0.2576     0.2774
 0.1735     0.1440
 0.1682     0.1607
```

```
 0.0858     0.1103
 0.0891     0.0935
 0.2240     0.3111
 0.3770     0.3817
 0.2815     0.2603
 0.5382     0.3082
 0.1911     0.1629
 0.1284     0.1153
 0.1769     0.2050
 0.2298     0.3233
 0.1878     0.1959
 0.2906     0.3384
 0.1690     0.2173
 0.4658     0.2450
 0.4118     0.5489
 0.7180     1.0150
 0.1802     0.2673
 0.1417     0.1541
 0.1121     0.1139
 0.2360     0.2794
 0.1943     0.2319
 0.1517     0.1564
 0.2038     0.1705
 0.1611     0.1380
 0.1053     0.0849
 0.3760     0.2066
 0.2266     0.1726
 0.6835     0.5490
 0.5690     0.4106
 0.1206     0.1079
 0.0743     0.0686
 0.0364     0.0640
 0.0236     0.0558
 0.0454     0.0781
 0.0794     0.1437
 0.1069     0.1792
 0.0643     0.1171
 0.0502     0.0852
 0.8712     2.7756
 1.7550     4.6209
 0.3038     0.6740
 0.2884     0.3349
 0.2801     0.3699
 0.1807     0.2290
 0.1258     0.1054
 0.1448     0.2614
 0.1599     0.1645
 0.1547     0.1875
 0.0767     0.0949
 0.1977     0.1645
 0.3636     0.2347
 0.7125     0.9246
 0.3403     0.4019
 0.1608     0.1468
 0.0765     0.0060
 0.1046     0.1073
 0.1306     0.1735
 0.1252     0.1543
 0.3948     0.3495
 0.3318     0.1562
```

```
 0.3020     0.0788
 0.4601     0.2343
 0.1712     0.1547
 1.1459     0.6746
 1.1232     1.3065
 0.2442     0.1200
 0.0740     0.0797
 0.0876     0.1267
 0.1530     0.3884
 0.1357     0.3986
 0.4481     0.9246
 0.5588     1.0784
 0.1986     0.3424
 0.2575     0.3121
 0.3726     0.3492
 0.9474     0.7052
 0.2932     0.5401
 0.4176     0.8167
 0.5818     0.6193
 0.1070     0.1329
 0.1146     0.2013
 0.1163     0.6680
 0.0944     0.2045
 0.1359     0.2975
```

**Appendix A3**

REMPIT- Remaining Pit Management

```
c*************************************************************************

c ***    Remaining Pit Management

c*************************************************************************
       program rempit

       common      /bl1/ alpha,rs,s,t
       dimension   v(20),h(20),bt(50),q(90),tq(90),hoehe(50),tg(20),
     *             hg(20),h0(500),r(500),t1(500),rss(500),hanf(500),
     *             btj(50)
       real*8      alpha,zeit,time,htime,htim,f
       real        kw,mgw,me
       character*72 text1,text2
       character*4  cch,ccvh,cct,ccht,ccqt,ccbt
       equivalence(btj(1),hoehe(1))
       data     bta,gdvs,ggwin,dvs,rh,qi0,qrbs         /7*0./
     *          time                                   /0.d00/
     *          qa,ze,me                               /3*1./
     *          isteu1,isteu2,isteu3,ih,iz             /5*0/
     *          i1                                     /1/
     *          pi                                     /3.1415926/
       data cch,ccvh,cct,ccht,ccqt,ccbt
     *          /'   h','v(h)','   t','h(t)','q(t)','  bt'/
       data iin,iou/5,6/

c ***  IIASA-routine for assignment of input/output files
       call usearg

c ***  input and print of data
       write(iou,2000)
2000   format (//53x,'program rempit'/,53x,14('*')/)

       read(iin,'(a72/a72)') text1,text2
c ***
c      text1   - comment1
c      text2   - comment2
c ***
       write(iou,2010) text1,text2
2010   format(2(1x,a72//))

       read(iin,*) isteu1,isteu2,isteu3
c ***
c              *** code:  no -> 0      yes -> 1     ***
c
c      isteu1  - exchange through bottom of remaining pit ?
c      isteu2  - hydraulic resistance (e.g. clogging) to consider ?
c      isteu3  - consideration of nonlinearity by
c                successive linearization of differential equation ?
c ***
       if (isteu1.ne.0.or.isteu2.ne.0.or.isteu3.ne.0) write(iou,2020)
2020   format(' conditions for program control:'/1x,30(1h*))
       if (isteu1.ne.0) write(iou,2030)
2030   format(39x,'*   exchange through bottom of remaining pit')
       if (isteu2.ne.0) write(iou,2040)
2040   format(39x,'**  hydraulic resistance is considered')
       if (isteu3.ne.0) write(iou,2050)
2050   format(39x,'*** nonlinearity is considered')

       read(iin,*) s,kw,sohle,mgw
c ***
c      s       - storage coefficient
c      kw      - hydraulic conductivity
c      sohle   - bottom altitude of the remaining pit above sea-level
c      mgw     - average groundwater table in the anthropogenic
c                undisturbed area
c ***
       write(iou,2060) s,kw,sohle,mgw
2060   format(//49x,'input data control print'/49x,25(1h*)//
     *        ' 1.geohydraulic values'/1x,22(1h-)/10x,
     *        'storage coefficient',10x,':     S  = ',e10.3/10x,
     *        'hydraulic conductivity       :    k  = ',e10.3,' m/s'/
     *        10x,'bottom altitude',14x,':    hs = ',f10.3,
     *        ' m above nn'/,10x,'mean gw-table',16x,
     *        ':    mw = ',f10.3,' m above nn')

       if (isteu1.ne.0) read(iin,*) sfl
c ***
c      sfl     - exchange area
c ***
       if (isteu1.ne.0) write(iou,2070) sfl
2070   format(7x,'*  exchange area',16x,':    as = ',f10.3,' km**2')

       if (isteu2.ne.0) read(iin,*) rh
c ***
c      rh      - hydraulic resistance
c ***
       if (isteu2.ne.0) write(iou,2080) rh
2080   format(7x,'** hydraulic resistance          :    rh = ',e10.3,
     *        ' s**2/m')

       read(iin,*) dt,ifk,ig,ie,ib,ze,me
c ***
c      dt      - time step
c      ifk     - number of interpolation nodes of the
c                water table / storage volume relationship)
c      ig      - number of interpolation nodes of the water table curve
c                for natural recharge of the remaining pit
c      ie      - number of interpolation nodes of the recharge function
c                of the remaining pit (artificial recharge)
c      ib      - number of computing times
c      ze      - time unit
c      me      - quantity unit
c ***
       dt=dt*ze
       dtj=dt/86400.
       write(iou, 2090) dt,dtj,ifk,ig ,ie,ib
2090   format(/' 2.dimensioning and control of computations'/
     *        1x,42(1h-)/10x,'time step',20x,':     dt = ',e10.4,' s'/
     *        43x,'dt = ',f10.1,' days'/10x,'number of nodes'/
     *        11x,'twater table/storage volume :    n1 = ',i10/
     *        11x,'tnatural recharge (w.table) :    n2 = ',i10/
     *        11x,'tartificial recharge (flux) :    n3 = ',i10/
     *        11x,'tcomputing times            :    n4 = ',i10)

       write(iou,2100) me
2100   format(/' 3.storage volume (in  ',e10.4,' * m**3) as ',
     *        'function of water table (in m above nn)')
       write(iou,2101) (cch,ccvh,i=1,3)
2101   format(1x,99('-')/1x,1hl,6x,a4,9x,1hl,8x,a4,7x,1hl,6x,a4,9x,
```

```
     *           1h1,8x,a4,7x,1h1,6x,a4,9x,1h1,7x,a4,7x,1h1)
        read(iin,*) (h(i),i=1,ifk),(v(i),i=1,ifk)
c ***
c       h(i)     - interpolation nodes - water table
c       v(i)     - interpolation nodes - storage volume
c ***
        write(iou,2110) (h(i),v(i),i=1,ifk)
2110    format(3(6x,f10.2,10x,e10.4,4x))
        do 10 i=1,ifk
                v(i)=v(i)*me
10      continue

        write(iou,2120)ze
2120    format(/' 4.influence of outer boundary conditions (in m',
     *          ' above nn) as function in time (in  ',e10.4,
     *          ' * sec.)')
        write(iou,2101) (cct,ccht,i=1,3)
        read(iin,*) (tg(i),i=1,ig),(hg(i),i=1,ig)
c ***
c       tg(i)    - interpolation nodes - time
c       hg(i)    - interpolation nodes - water table
c ***
        write(iou,2130) (tg(i),hg(i),i=1,ig)
2130    format(3(6x,e10.4,10x,f10.2,4x))
        do 20 i=1,ig
                tg(i)=tg(i)*ze
20      continue

        write(iou,2140)me,ze,ze
2140    format(/' 5.inflow   (in  ',e10.4,' * m**3/ ',e10.4,' * sec.'
     *          ,') as function in time (in  ',e10.4,' * sec.)')
        write(iou,2101) (cct,ccqt,i=1,3)
        read(iin,*) (tq(i),i=1,ie),(q(i),i=1,ie)
c ***
c       tq(i)    - interpolation nodes - time
c       q(i)     - interpolation nodes - inflow
c ***
        write(iou,2150) (tq(i),q(i),i=1,ie)
2150    format(3(6x,e10.4,10x,f10.5,4x))
        do 30 i=1,ie
                tq(i)=tq(i)*ze
                q(i)=q(i)/ze *me
30      continue

        write(iou,2160) ze
2160    format(/' 6.computing times (in  ',e10.4,' * sec. and',
     *          ' in sec.)')
        write(iou,2101) (ccbt,i=1,6)
        read(iin,*) (bt(i),i=1,ib)
c ***
c       bt(i)    - computing times
c ***
        do 40 i=1,ib
                btj(i)=bt(i)*ze
40      continue
        write(iou,2170) (bt(i),btj(i),i=1,ib)
2170    format(3(6x,e10.4,10x,e10.4,4x))

        ib1=ib-1
        bt(ib)=btj(ib)
        do 55 i=1,ib1
                if (bt(i+1).gt.bt(i)) goto 50
                write(iou,2180)
2180            format(' wrong order of computing times !')
                stop
50              bt(i)=btj(i)
55      continue
        if (dt.le.bt(1)) goto 60
        write(iou,2190)
2190    format(' time step greater than first computing time')
        stop
60      write(iou,2200)
2200    format(//44x,'output of computational results',/44x,
     *          31(1h+)/1x,120(1h-)/1x,1h1,15x,1h1,
     *          '   remaing pit     ',1h1,7x,
     *          'computation for actual time interval',9x,1h1,
     *          'computation for whole interval!'/
     *          ' Icomputing time !',102(1h-),1h1/1x,1h1,15x,
     *          1h1,'w-tablel  volume  IInterval !      inflow     !',
     *          '    infiltration        I  inflow    !   infiltration',
     *          '   !'/1x,1h1,118(1h-),1h1)
        write(iou,2210)
2210    format(' ! a !',5x,1hs,5x,'Im u. nnl    m**3   !    d    !',
     *          'm**3/sl    m**3    !    m**3   Im**3/sl    %  !   m**3',
     *          '     !    m**3    !    %    !'/' h',118(1h-),1h1)

c ***   definition of initial values

        t=kw*(mgw-sohle)
        alpha=s

        call vlapla

        h2=exp(2.*pi*rh*t)
        if (isteu1.ne.0) rs=sqrt(sf1*1.e+06/pi)
        if (isteu1.ne.0.and.isteu2.ne.0) rs=rs/h2

c ***   determination of the initial values for water table and volume
c ***   in the remaining pit

        hr=pol(ig,tg,hg,time)
        v1=pol(ifk,h,v,hr)
        v1a=v1
        v2s=v1
        hg1=hr

c ***   determination of the current variation of the water level in the
c ***   remaining pit (jump)

70      qis=qi0
        iz=iz+1
        if (iz.gt.500) goto 210
        hanf(iz)=hr
        iza=iz
        if (isteu3.ne.0) t1(iz)=(hr-sohle+(mgw-hr)*0.5)*kw
        if (ih.eq.0) goto 75
        time=htim
        ih=0
        iz=iz-1
        iza=iz
```

```
      dvs=dvs-dv
      v2s=v2s-dv
      v1=v1b
      hr=hra
      htime=time
      if (time.gt.tq(ie).and.qa.eq.0.) goto 90
      if (time+dt.lt.bt(i1)) goto 80
75    htime=time
      if (time.le.bt(i1).and.(time+dt).gt.bt(i1)) goto 90
      if (time.gt.tq(ie).and.qa.eq.0.) goto 90
80    time1=time+dt/2.
      time=time+dt
      goto 100
90    time1=time+(bt(i1)-htime)/2.
      htim=time
      time=bt(i1)
      v1b=v1
      hra=hr
      ih=1
100   qa=pol(ie,tq,q,time1)
      if (abs(qa).gt.1.e-05) goto 110
      qa=0.
      h0(iza)=0.
      dv=0.
      goto 160
110   dv=qa*(time-htime)
      v2=v1+dv
      dvs=dvs+dv
      if (v2.gt.v(ifk)) goto 200
      h1=pol(ifk,v,h,v2)
      h0(iz)=h1-hr

c *** determination of the parameter alpha

      r(iz)=dv/(pi*h0(iz))
      if (isteu1.eq.0.and.isteu2.ne.0) rss(iz)=sqrt(r(iz))/h2
120   if (h0(iza).eq.0.) goto 160
      if (isteu3.ne.0) t=t1(iza)
      if (isteu1.ne.0) goto 140
      if (isteu2.ne.0) goto 130
      rs=sqrt(r(iza))
      goto 150
130   rs=rss(iza)
140   alpha=rs*rs*s/r(iza)

c *** determination of the time-related infiltration according
c *** to management

150   zeit=time-htime

      call lapla(f,zeit)

      ha=f*h0(iza)
      ws1=hanf(iza)+h0(iza)
      ws2=hanf(iza)+ha
      qi=pol(ifk,h,v,ws1)-pol(ifk,h,v,ws2)
      qis=qis+qi
      if (f.gt.0.01) goto 160
      qi=pol(ifk,h,v,ws1)-pol(ifk,h,v,hanf(iza))
      h0(iza)=0.
      qi0=qi0+qi
160   iza=iza-1
      htime=(iza-1)*dt
      if (iza.ge.1) goto 120

c *** determination of the current values for water level and volume
c *** in the remaining pit

      hg2=pol(ig,tg,hg,time)
      qrb=pol(ifk,h,v,hg2)-pol(ifk,h,v,hg1)
      hg1=hg2
      qrbs=qrbs+qrb
      v2s=v2s+dv
      v1=v2s+qrbs-qis
      hr=pol(ifk,v,h,v1)
      test=abs(bt(i1)-time)
      if (test.lt.bt(i1)*1.e-04) goto 170
      goto 70

c *** computation of balance components

170   time=bt(i1)
      hoehe(i1)=hr
      xbt=time/3.15e+07
      ter=(time-bta)/86400.
      dvq=dvs/(time-bta)
      gwin=dvs-(v1-v1a)
      gwin1=gwin/(time-bta)
      if (dvs.lt.1.e-05) gwin2=0.
      if (dvs.lt.1.e-05) goto 180
      gwin2=gwin/dvs*100.
180   gdvs=dvs+gdvs
      ggwin=ggwin+gwin
      if (gdvs.lt.1.e-05) pgwin=0.
      if (gdvs.lt.1.e-05) goto 190
      pgwin=ggwin/gdvs*100.
190   bta=time
      v1a=v1

c *** print of the computational results

      write(iou,2220) xbt,time,hr,v1,ter,dvq,dvs,gwin,gwin1,gwin2,
     *                gdvs,ggwin,pgwin
2220  format(' ',f3.0,1h|,e11.5,1h|,f7.2,1h|,e10.4,1h|,f9.2,1h|,
     *       f6.3,1h|,e10.3,1h|,e10.3,1h|,f6.3,1h|,f6.1,1h|,e11.4,
     *       1h|,e10.3,1h|,f7.2,1h|)
      dvs=0.
      i1=i1+1
      if (i1.gt.ib) goto 220
      goto 70
200   write(iou,2230) v2
2230  format(' max. storage volume exceeded (',e10.3,')')
      stop
210   write(iou,2240)
2240  format(' array-dimension to small, increase dt ')
      stop
220   write(iou,2250)
2250  format(1x,120(1h-))

      stop
      end
```

```
      function pol(k,x,y,xs)

c ***  function for linear interpolation between interpolation nodes
c ***  of a polygon

      dimension x(k),y(k)

      if(k.eq.1) goto 2
      x0=xs
      if (x0-x(1).lt.0.) x0=x(1)
      do 1 i=1,k
              xt=abs(x(i)-x0)
              if (xt.le.1.e-9) goto 3
              if (x(i).gt.x0) goto 4
    1 continue
    2 pol=y(k)
      return
    3 pol=y(i)
      return
    4 pol=(y(i)-y(i-1))*(x0-x(i-1))/(x(i)-x(i-1))+y(i-1)

      return
      end
c
c*******************************************************************************
c
      complex*16 function fs(s)

      common /bl1/ alpha,rs,sp,tp
      complex*16   s,q,hk0,hk1,k0,k1
      real*8       alpha

      q= zsqrt(s*sp/tp)
      hk0=k0(rs*q)
      hk1=k1(rs*q)
      fs=rs*sp* hk0/(tp*q*(rs*q*hk0+2.*alpha*hk1))

      return
      end
c
c*******************************************************************************
c
      complex*16 function i0 (x)

      complex*16 x

      if (zabs(x).gt.3.75d0) goto 1
      x=(x/3.75d0)
      x=x*x
      i0=(((((.0045813d0*x+.0360768d0)*x+.2659732d0)*x+1.2067492d0)
     *                    *x+3.0899424d0)*x+3.5156229d0)*x+1.d0
      return
    1 x=3.75d0/x
      i0=((((((((.00392377d0*x-.01647633d0)*x+.02635537d0)*x-
     *        .02057706d0)*x+.00916281d0)*x-.00157565d0)*x+
     *         .00225319d0)*x+.01328592d0)*x+.39894228d0))*
     *                    zexp(3.75d0/x)/ zsqrt(3.75d0/x)
      return
      end

      complex*16 function i1 (x)

      complex*16 x,t

      if( zabs(x).gt.3.75d0) goto 1
      t=(x/3.75d0)
      t=t*t
      i1=((((((.00032411d0*t+.00301532d0)*t+.02658733d0)*t+
     *       .15084934d0)*t+.51498869d0)*t+.87890594d0)*t+.5d0)*x
      return
    1 t=3.75d0/x
      i1=((((((((-.00420059d0*t+.01787654d0)*t-.02895312d0)*t
     *         +.02282967d0)*t-.01031555d0)*t+.00163801d0)*t-
     *            .00362018d0)*t-.03988024)*t+.39894228d0)*
     *                                       zexp(x)/ zsqrt(x)
      return
      end
c
c*******************************************************************************
c
      complex*16 function k0(x)

      complex*16 x,t,i0

      if( zabs(x).gt.2.d0) goto 1
      t=(.5d0*x)
      t=t*t
      k0=(((((.00000740d0*t+.00010750d0)*t+.00262698d0)*t+
     *       .03488590d0)*t+.23069756d0)*t+.42278420d0)*t-
     *       .57721566d0-zlog(.5d0*x)*i0(x)
      return
    1 t=2.d0/x
      k0=((((((.00053208d0*t-.00251540d0)*t+.00587872d0)*t-
     *       .01062446d0)*t+.02189568d0)*t-.07832358d0)*t+
     *       1.253331414)/ zexp(x)/ zsqrt(x)
      return
      end
c
c*******************************************************************************
c
      complex*16 function k1(x)

      complex*16 x,t,i1

      if (zabs(x).gt.2.d0) goto 1
      t=.5d0*x
      t=t*t
      k1=((((((-.00004686d0*t-.00110404d0)*t-.01919402d0)*t-
     *       .18156897d0)*t-.67278579d0)*t+.15443144d0)*t+
     *       1.d0+x* zlog(.5d0*x)*i1(x))/x
      return
    1 t=2.d0/x
      k1=((((((-.00068245d0*t+.00325614d0)*t-.00780353d0)*t+
     *       .01504268d0)*t-.03655620d0)*t+.23498619d0)*t+
     *       1.25331414d0)* zexp(-x)/zsqrt(x)
      return
      end
c*******************************************************************************
c     The subroutines/functions lapla, vlapla, dreal have to be
c     taken from the program change, appendix 4.
```

## Appendix A4

CHANGE- Infiltration/exfiltration in a river caused by water table variations

program remeit
**************

testexample for collaborative paper

***********************************

conditions for program control:
*******************************

* exchange through bottom of remaining pit
** hydraulic resistance is considered

input data control print
*************************

1.geohydraulic values

| | | | | |
|---|---|---|---|---|
| storage coefficient | : | s = | 0.250e+00 | |
| hydraulic conductivity | : | k = | 0.600e-03 | m/s |
| bottom altitude | : | hs = | 68.000 | m above nn |
| mean gw-table | : | mw = | 118.000 | m above nn |
| * exchange area | : | as = | 12.325 | km**2 |
| ** hydraulic resistance | : | rh = | 0.203e+01 | s**2/m |

2.dimensioning and control of computations

| | | | |
|---|---|---|---|
| time step | : | dt = | 0.2625e+07 s |
| | | dt = | 30.4 days |
| number of nodes | | | |
| +water table/storage volume | : | n1 = | 8 |
| +natural recharge (w.table) | : | n2 = | 11 |
| +artificial recharge (flux) | : | n3 = | 3 |
| +computing times | : | n4 = | 10 |

3.storage volume (in 0.1000e+07 * m**3) as function of water table (in m above nn)

| h | v(h) | h | v(h) | h | v(h) |
|---|---|---|---|---|---|
| 68.00 | 0. e+00 | 70.00 | 0.1400e+01 | 80.00 | 0.3700e+01 |
| 90.00 | 0.1000e+02 | 100.00 | 0.2800e+02 | 110.00 | 0.7000e+02 |
| 118.00 | 0.1295e+03 | 127.50 | 0.2000e+03 | | |

4.influence of outer boundary conditions (in m above nn) as function in time (in 0.2625e+07 * sec.)

| t | h(t) | t | h(t) | t | h(t) |
|---|---|---|---|---|---|
| 0. e+00 | 82.00 | 0.1200e+02 | 86.05 | 0.2400e+02 | 89.67 |
| 0.3600e+02 | 92.15 | 0.4800e+02 | 94.42 | 0.6000e+02 | 96.31 |
| 0.7200e+02 | 97.84 | 0.8400e+02 | 99.13 | 0.9600e+02 | 100.08 |
| 0.1080e+03 | 100.77 | 0.1200e+03 | 101.50 | | |

5.inflow (in 0.1000e+07 * m**3/ 0.2625e+07 * sec.) as function in time (in 0.2625e+07 * sec.)

| t | q(t) | t | q(t) | t | q(t) |
|---|---|---|---|---|---|
| 0. e+00 | 8.33300 | 0.4800e+02 | 8.33300 | 0.4800e+02 | 0. |

6.computing times (in 0.2625e+07 * sec. and in sec.)

| dt | dt | dt | dt | dt | dt |
|---|---|---|---|---|---|
| 0.1200e+02 | 0.3150e+08 | 0.2400e+02 | 0.6300e+08 | 0.3600e+02 | 0.9450e+08 |
| 0.4800e+02 | 0.1260e+09 | 0.6000e+02 | 0.1575e+09 | 0.7200e+02 | 0.1890e+09 |
| 0.8400e+02 | 0.2205e+09 | 0.9600e+02 | 0.2520e+09 | 0.1080e+03 | 0.2835e+09 |
| 0.1200e+03 | 0.3150e+09 | | | | |

output of computational results
+++++++++++++++++++++++++++++++

| computing time | | remaing pit | | computation for actual time interval | | | | | | computation for whole interval | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | w-table | volume | interval | inflow | | infiltration | | | inflow | infiltration | |
| a | s | m u. nn | m**3 | d | m**3/s | m**3 | m**3 | m**3/s | % | m**3 | m**3 | % |
| 1. | 0.31500d+08 | 99.29 | 0.2672e+08 | 364.58 | 3.174 | 0.100e+09 | 0.782e+08 | 2.484 | 78.2 | 0.1000e+09 | 0.782e+08 | 78.23 |
| 2. | 0.63000d+08 | 107.47 | 0.5738e+08 | 364.58 | 3.174 | 0.100e+09 | 0.673e+08 | 2.138 | 67.3 | 0.2000e+09 | 0.146e+09 | 72.79 |
| 3. | 0.94500d+08 | 112.83 | 0.9104e+08 | 364.58 | 3.174 | 0.100e+09 | 0.683e+08 | 2.169 | 68.3 | 0.3000e+09 | 0.214e+09 | 71.31 |
| 4. | 0.12600d+09 | 117.38 | 0.1249e+09 | 364.58 | 3.174 | 0.100e+09 | 0.662e+08 | 2.100 | 66.2 | 0.4000e+09 | 0.280e+09 | 70.02 |
| 5. | 0.15750d+09 | 113.04 | 0.9258e+08 | 364.58 | 0. | 0. e+00 | 0.323e+08 | 1.025 | 0. | 0.4000e+09 | 0.312e+09 | 78.09 |
| 6. | 0.18900d+09 | 111.02 | 0.7757e+08 | 364.58 | 0. | 0. e+00 | 0.150e+08 | 0.476 | 0. | 0.4000e+09 | 0.327e+09 | 81.85 |
| 7. | 0.22050d+09 | 109.73 | 0.6887e+08 | 364.58 | 0. | 0. e+00 | 0.870e+07 | 0.276 | 0. | 0.4000e+09 | 0.336e+09 | 84.02 |
| 8. | 0.25200d+09 | 108.39 | 0.6322e+08 | 364.58 | 0. | 0. e+00 | 0.565e+07 | 0.179 | 0. | 0.4000e+09 | 0.342e+09 | 85.43 |
| 9. | 0.28350d+09 | 107.80 | 0.6077e+08 | 364.58 | 0. | 0. e+00 | 0.245e+07 | 0.078 | 0. | 0.4000e+09 | 0.344e+09 | 86.05 |
| 10. | 0.31500d+09 | 107.57 | 0.5979e+08 | 364.58 | 0. | 0. e+00 | 0.985e+06 | 0.031 | 0. | 0.4000e+09 | 0.345e+09 | 86.29 |

```
c***********************************************************************************

c ***   Determination of Infiltration/Exfiltration in Consequence of
c ***   Water Table Variations in a River

c***********************************************************************************

        program change

        real*8     h6,hh6
        common     al(10),s(10),t(10),r(10),ie1
        dimension  ws(10,12),st(10,12),so(10,12),
     1             bt(20),bvol(10,20),wm(10),sum(20),
     2             hf(20),delt(10,20),dvk(10,20),gvk(10,20),zd1(20),
     3             zd2(20),bt1(300),h(300)
        character*72 text,textu
        data       iin,iou /5,6/

c ***   IIASA-routine for assignment of input/output files
        call userq

c ***   input and print of date

        write(iou,2000)
2000    format(//53x,'programm change'/
     *         53x,15(1h*)//49x,'(code : yes => 1',' no => 0)'/)

        read(iin,'(a72/a72)') text,textu
c ***
c       text  - comment 1
c       textu - comment 2
c ***
        write(iou,2010) text,textu
2010    format(2(1x,a72//))

        read(iin,*) isteul
c ***
c       isteul  - summation of results of all river segments?
c ***
        write(iou,2020) isteul
2020    format(' conditions for program control:',
     *         10x,'summation     : ',i2/1x,38(1h*)/)

        read(iin,*) if,is,ib
c***
c       if      - number of river segments
c       is      - number of steps of water table
c       ib      - number of computing times
c****
        write(iou,2030)
2030    format(//47x,'input data control print'/47x,26(1h*)/)
        write(iou,2040) if,is,ib
2040    format(' number of river segments:',i3,4x,'number of',
     *         ' steps of water table:',i3,4x,
     *         'number of computing times:',i3/)
        read(iin,*) ze
c ***
c       ze      - time unit
c ***
        write(iou,2050) ze
2050    format(' time unit',15x,':  ',e10.4,' sec.')

        read(iin,*) (bt(i),i=1,ib)
c ***
c       bt(i)   - computing times
c ***
        read(iin,*) (al(i),s(i),t(i),r(i),wm(i),i=1,if)
c ***
c       al(i)   - lengths of river segments
c       s(i)    - storage coefficients
c       t(i)    - transmissivities
c       r(i)    - hydraulic resistences
c       wm(i)   - long-term mean values of water table in river segm.
c ***
        do 1 i=1,ib
                bt(i)=bt(i)*ze
                hf(i)=bt(i)/86400.
1       continue
        write(iou,2060)
2060    format(/' computing times',9x,':')
        write(iou,2070) (bt(i),hf(i),i=1,ib)
2070    format(3(3x,e10.2,' sec. (',f7.2,' days)'))

        do 3 i=1,if
        do 3 j=1,is
                read(iin,*) ws(i,j),st(i,j)
3       continue
c ***
c       ws(i)   - interpolation nodes of the input function (surface
c                 water table for each section)
c       st(i)   - interpolations nodes of the input function (time)
c ***
        write(iou,2080)
2080    format(//45x,'characteristics of river segments'/
     *         45x,33(1h-))
        do 6 ie=1,if
                so(ie,1)=ws(ie,1)-wm(ie)
                st(ie,1)=st(ie,1)*ze
                if(is.eq.1)goto 5
                do 4 i=2,is
                        so(ie,i)=ws(ie,i)-ws(ie,i-1)
                        st(ie,i)=st(ie,i)*ze
4               continue
5               continue
                write(iou,2090) ie,al(ie),s(ie),t(ie),r(ie),wm(ie)
2090            format(///' segment',i3/1x,12(1h-)//' length:',f10.2,
     *                 ' m     S-value:',f10.2,'     T-value:',
     *                 e10.2,'  m**2/s      resistance(r):',f10.2,
     *                 ' m'/' inital water table (wm):',f10.2,' m',
     *                 //' water table ws at time st (in sec.):')
                write(iou,2120) (ws(ie,j),st(ie,j),j=1,is)
                write(iou,2110)
2110            format(/1x,'corresponding steps so at time',
     *                 ' st(in sec. ):')
                write(iou,2120) (so(ie,j),st(ie,j),j=1,is)
2120            format(/3(1x,f7.2,' m      (',e10.3,')      '))
6       continue

c ***   control of input data
```

```
      if (ib.eq.1) goto 8
      do 7 i=2,ib
              if (bt(i).gt.bt(i-1)) goto 7
              write(iou,2130)
2130          format(' computing times not in increasing order!')
              goto 1000
7     continue
8     do 9 i=1,if
      do 9 j=1,is
              if(j.eq.1) goto 9
              if(st(i,j).gt.st(i,j-1)) goto 9
              write(iou,2140)
2140          format(' interpolation nodes not in icreas. order!')
              goto 1000
9     continue
      do 10 ie=1,if
              if(bt(1).gt.st(ie,1)) goto 10
              write(iqu,2150)
2150          format(1x,'first coputation time to small!')
              goto 1000
10    continue

      write(iou,2200)
2200  format(//23x,'p r i n t   o f   c o m p u t a t i o n a l',
     *        '   r e s u l t s'/23x,57(1h*))
      write(iou,2210)
2210  format(//' tabl-key:'/1x,9(1h-)//11x,'column number',
     *        37x,'description',30x,'unit'/11x,13(1h-),37x,11(1h-),
     *        30x,4(1h-)/16x,1h1,10x,'computational time',57x,
     *        'sec.'/16x,1h2,10x,'computational time',57x,'days'
     *        /16x,1h3,10x,'time difference to last computational ',
     *        'time',33x,'sec.'/16x,1h4,10x,'time difference to '
     *        ,'last computational time',33x,'days'/
     *        16x,1h5,10x,'relative change of storage volume during'
     *        ,' time difference ',18x,'m**3'/16x,1h6,10x,
     *        'mean volumetric flux according to position 3 and 5',
     *        25x,'m**3/s'/16x,1h7,10x,'absolute change of storage'
     *        ,' volume refered to zero at inital time ',10x,
     *        'm**3'/16x,1h8,10x,'mean volumetric flux according to'
     *        ,' position 1 and 7',25x,'m**3/s')

c ***  setting of initial values
      call vlapla

c ***  starting of actual determinations of each river segment

      do 25 ie=1,if
              iel=ie
              l=0

c ***         determination and classification of time differences

              do 11 j=1,is
              do 11 i=1,ib
                      if(bt(i).le.st(ie,j)) goto 11
                      l=l+1
                      bt1(l)=bt(i)-st(ie,j)
11            continue
12            ic=0
13            h6=1.0d+30
      do 14 l1=1,l
              if(bt1(l1).lt.0.) goto 14
              if(bt1(l1).ge.h6) goto 14
              l4=l1
              h6=bt1(l4)
14    continue
      lc=ic+1

c ***  determination of the actual value of the transition
c ***  function

      call lapla(hh6,h6)
      hvk=hh6
      h(l4)=hvk

c ***  classification of solutions

      do 15 l3=1,l
              if((bt1(l4).ne.bt1(l3)).or.(l3.eq.l4)) goto 15
              h(l3)=hvk
              bt1(l3)=-1.
15    continue
      bt1(l4)=-1.
      do 16 l6=1,l
              if(bt1(l6).le.-1.) goto 16
              goto 13
16    continue
      lc=0
      la=1
      do 17 ih=1,ib
               bvol(ie,ih)=0.
17    continue
      do 20 j=1,is
18            if(st(ie,j).lt.bt(la)) goto 19
              la=la+1
              if(st(ie,j).ge.bt(ib)) goto 21
              goto 18
19            continue

c ***         determination of the actual value of
c ***         infiltration/exfiltration

      do 20 l=la,ib
              lc=lc+1
              bvol(ie,l)=h(lc)*so(ie,j)+bvol(ie,l)
20    continue

c ***  determination of balance components

21    delt(ie,1)=bvol(ie,1)
      zd1(1)=bt(1)
      zd2(1)=zd1(1)/86400.
      gvk(ie,1)=bvol(ie,1)/bt(1)
      dvk(ie,1)=gvk(ie,1)
      do 22 l=2,ib
              delt(ie,l)=bvol(ie,l)-bvol(ie,l-1)
              zd1(l)=bt(l)-bt(l-1)
              dvk(ie,l)=delt(ie,l)/zd1(l)
              zd2(l)=zd1(l)/86400.
              gvk(ie,l)=bvol(ie,l)/bt(l)
```

```
22          continue

c ***       print of computational results

            write(iou,2220) ie
2220        format(//1x,'river segment',i3/1x,17(1h*))
23          write(iou,2230)
2230        format(//19x,89(1h-)/19x,
     *             8(1h|,10x),1h|/19x,'|     1     |     2     |'
     *             ,'     3     |     4     |     5     |     6',
     *             '     |     7     |     8     |'/19x,8(1h|,10x),
     *             1h|/19x,89(1h-))
            do 24 i=1,ib
                   write(iou,2240) bt(i),h1(i),zd1(i),zd2(i),
     *                             delt(ie,i),dvk(ie,i),bvol(ie,i)
     *                             ,gvk(ie,i)
2240               format(19x,9(1h|,10x),/19x,1h|,e10.3,1h|,f10.2
     *                    ,1h|,e10.3,1h|,f10.2,1h|,e10.3,1h|,
     *                    e10.3,1h|,e10.3,1h|,e10.3,1h|)
24          continue
            write(iou,2250)
2250        format(19x,89(1h-)//)
25      continue

        if(isteul.eq.0) goto 1000
        do 26 i=1,ib
        do 26 ie=1,if
            sum(i)=sum(i)+dvk(ie,i)
26      continue
        write(iou,2260) (bt(i),sum(i),i=1,ib)
2260    format(///20x,'*****  summation of position 6 for all',
     *         ' river segments  *****',//34x,'computational times',
     *         20x,'sum'/40x,'sec.',27x,'m**3/s'/
     *         (/38x,e10.3,20x,e10.3))

1000    stop
        end
c
c*******************************************************************************
c
        complex*16 function fs(s)

c ***   function for the computation of solution in the laplace-space

        common  al(10),sp(10),t(10),r(10),i
        complex*16 s,u
        u=zsqrt(s*sp(i)/t(i))
        fs=2.*al(i)*t(i)*u/(s*s*(1.+r(i)*u))
        return
        end
c
c*******************************************************************************
c
        subroutine lapla (ft,t)

c ***   subroutine for numerical determination of laplace's
c ***   inverse transformation

        common/bl2/ s(20),ds(20),pibyn,n,nm1
        complex*16  fs,s,ds,zz,sum,b,b1,b2
        real*8      dreal,ft,t,psi,cp,sp,v2,lambda,tau,z,pibyn
        data z,tau /0.0d0,6.0d0/

        zz=cmplx(z,z)
        lambda=tau/t
        psi=tau*pibyn
        cp=2.0d0*dcos(psi)
        sp=dsin(psi)
        b=zz
        b1=b
        do 1 ka=1,nm1
               k=n-ka
               v2=dexp(dmax1(tau*dreal(s(k)),-1.5d2))
               b2=b1
               b1=b
               b=cp*b1-b2+v2*ds(k)*fs(lambda*s(k))
1       continue
        sum=dexp(tau)*fs(lambda+zz)*0.5d0+cp*b-2.0d0*(b1-b*cmplx(z,sp))
        ft=lambda*dreal(sum)/n

        return
        end
c
c*******************************************************************************
c
        subroutine vlapla

c ***   subroutine for computation of constant values for numerical
c ***   laplace's inverse transformation

        common/bl2/ s(20),ds(20),pibyn,n,nm1
        complex*16  s,ds
        real*8      pibyn,theta,alpha,nu,pi
        data pi,nu /3.14159265358932d0,1.0d0/

        n=8
        pibyn=pi/n
        nm1=n-1
        do 1 k=1,nm1
               theta=k*pibyn
               alpha=theta*dcos(theta)/dsin(theta)
               s(k)=cmplx(alpha,theta)
               ds(k)= cmplx(nu,theta+alpha*(alpha-1.0d0)/theta)*0.5d0
1       continue

        return
        end
c
c*******************************************************************************
c
        real*8 function dreal(x)
        complex*16 x
        dreal=x
        return
        end
```

programm change
**************

(code : yes => 1 no => 0)

testrechnung uebergabe programm change

**************************************

conditions for program control: summation : 0
**************************************

input data control print
**************************

number of river segments: 1 number of steps of water table: 12 number of computing times: 12

time unit : 0.2592e+07 sec.

computing times :

| | | |
|---|---|---|
| 0.26e+07 sec. ( 30.00 days) | 0.52e+07 sec. ( 60.00 days) | 0.78e+07 sec. ( 90.00 days) |
| 0.10e+08 sec. ( 120.00 days) | 0.13e+08 sec. ( 150.00 days) | 0.16e+08 sec. ( 180.00 days) |
| 0.18e+08 sec. ( 210.00 days) | 0.21e+08 sec. ( 240.00 days) | 0.23e+08 sec. ( 270.00 days) |
| 0.26e+08 sec. ( 300.00 days) | 0.29e+08 sec. ( 330.00 days) | 0.31e+08 sec. ( 360.00 days) |

characteristics of river segments

segment 1

length: 2250.00 m S-value: 0.25 T-value: 0.60e-01 m**2/s resistance(r): 1647.00 m
inital water table (wm): 138.10 m

water table ws at time st (in sec.):

| | | | | | |
|---|---|---|---|---|---|
| 137.71 m | ( 0. e+00) | 138.21 m | ( 0.259e+07) | 138.31 m | ( 0.518e+07) |
| 137.96 m | ( 0.778e+07) | 137.26 m | ( 0.104e+08) | 137.01 m | ( 0.130e+08) |
| 137.01 m | ( 0.156e+08) | 136.81 m | ( 0.181e+08) | 136.81 m | ( 0.207e+08) |
| 136.86 m | ( 0.233e+08) | 137.06 m | ( 0.259e+08) | 137.16 m | ( 0.285e+08) |

corresponding steps sd at time st(in sec. ):

| | | | | | |
|---|---|---|---|---|---|
| -0.39 m | ( 0. e+00) | 0.50 m | ( 0.259e+07) | 0.10 m | ( 0.518e+07) |
| -0.35 m | ( 0.778e+07) | -0.70 m | ( 0.104e+08) | -0.25 m | ( 0.130e+08) |
| 0. m | ( 0.156e+08) | -0.20 m | ( 0.181e+08) | 0. m | ( 0.207e+08) |
| 0.05 m | ( 0.233e+08) | 0.20 m | ( 0.259e+08) | 0.10 m | ( 0.285e+08) |

# print of computational results

tabl-key:

| column number | description | unit |
|---|---|---|
| 1 | computational time | sec. |
| 2 | computational time | days |
| 3 | time difference to last computational time | sec. |
| 4 | time difference to last computational time | days |
| 5 | relative change of storage volume during time difference | m**3 |
| 6 | mean volumetric flux according to position 3 and 5 | m**3/s |
| 7 | absolute change of storage volume refered to zero at inital time | m**3 |
| 8 | mean volumetric flux according to position 1 and 7 | m**3/s |

river segment 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 0.259e+07 | 30.00 | 0.259e+07 | 30.00 | -0.121e+06 | -0.466e-01 | -0.121e+06 | -0.466e-01 |
| 0.518e+07 | 60.00 | 0.259e+07 | 30.00 | 0.593e+05 | 0.229e-01 | -0.614e+05 | -0.119e-01 |
| 0.778e+07 | 90.00 | 0.259e+07 | 30.00 | 0.696e+05 | 0.269e-01 | 0.820e+04 | 0.105e-02 |
| 0.104e+08 | 120.00 | 0.259e+07 | 30.00 | -0.525e+05 | -0.202e-01 | -0.442e+05 | -0.427e-02 |
| 0.130e+08 | 150.00 | 0.259e+07 | 30.00 | -0.254e+06 | -0.979e-01 | -0.298e+06 | -0.230e-01 |
| 0.156e+08 | 180.00 | 0.259e+07 | 30.00 | -0.280e+06 | -0.108e+00 | -0.578e+06 | -0.372e-01 |
| 0.181e+08 | 210.00 | 0.259e+07 | 30.00 | -0.239e+06 | -0.924e-01 | -0.817e+06 | -0.451e-01 |
| 0.207e+08 | 240.00 | 0.259e+07 | 30.00 | -0.277e+06 | -0.107e+00 | -0.109e+07 | -0.528e-01 |
| 0.233e+08 | 270.00 | 0.259e+07 | 30.00 | -0.247e+06 | -0.955e-01 | -0.134e+07 | -0.575e-01 |
| 0.259e+08 | 300.00 | 0.259e+07 | 30.00 | -0.213e+06 | -0.821e-01 | -0.156e+07 | -0.600e-01 |
| 0.285e+08 | 330.00 | 0.259e+07 | 30.00 | -0.139e+06 | -0.538e-01 | -0.169e+07 | -0.594e-01 |
| 0.311e+08 | 360.00 | 0.259e+07 | 30.00 | -0.111e+06 | -0.428e-01 | -0.181e+07 | -0.580e-01 |

## APPENDIX B

Reference Values for the Natural Recharge of the Remaining Pit In the GDR Test Area

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| i | | | | | | | 17 | 18 | 19 | 20 |
| $qg_b^0(i)$ | [m³/ sec] | | | | | | | 7.07 | 7.10 | 6.85 |
| $h_p^0(i)$ | [m] | | | | | | 82.0 | 86.1 | 89.7 | 92.1 |
| $i$ | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| $qg_b^0(i)$ | 6.75 | 6.24 | 5.32 | 5.07 | 4.85 | 4.79 | 4.76 | 4.75 | 4.74 | 4.74 |
| $h_p^0(i)$ | 94.4 | 96.3 | 97.8 | 99.1 | 100.1 | 100.9 | 101.6 | 102.1 | 102.6 | 102.9 |
| $i$ | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| $qg_b^{(i)}$ | 4.7 | 4.7 | 4.7 | 4.6 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 |
| $h_p^0(i)$ | 103.2 | 103.5 | 103.7 | 103.8 | 104.0 | 104.1 | 104.2 | 104.2 | 104.3 | 104.3 |
| $i$ | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| $qg_b^0(i)$ | 3.0 | 2.7 | 2.4 | 2.2 | 2.2 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| $h_p^0(i)$ | 104.4 | 104.4 | 104.5 | 104.5 | 104.6 | 104.6 | 104.6 | 104.6 | 104.6 | 104.6 |