A SIMPLE MODEL OF THE ECONOMIC LONG WAVE

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FOREWORD

A summary of this paper was presented at the IIASA/IRPET Conference on Long-Waves in Siena/Florence in October 1983. It was not included in the published proceedings (CP-85-9) of that Conference as we decided to publish it separately.

There are several reasons for this. First the paper presents, in concise form, the essence of an important school of thought on long-waves which is based on the model described therein. Second, this model depicts the dynamics of many microeconomic factors which are important for practice and therefore it comes closest to the business community (recently, even in the form of business games).

From the classification suggested by the authors of this model it can be labelled as an endogenous, structural (not correlative), and disequilibrium/dynamic one.

We hope that the paper will meet with interest among not only the research community, but also policy makers in industry.

Boris Segerstahl
Deputy Director

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John D. Sterman

ABSTRACT

Recent economic events have revived interest in the economic long wave or Kondratiev cycle, a cycle of economic expansion and depression lasting about fifty years. Since 1975 the System Dynamics National Model has provided an increasingly rich theory of the long wave. The theory revolves around "self-ordering" of capital, the dependence of the capital-producing sectors of the economy, in the aggregate, on their own output. The long-wave theory growing out of the National Model relates capital investment, employment and workforce participation, monetary and fiscal policy, inflation, productivity and innovation, and even political values. The advantage of the National Model is the rich detail in which economic behavior is represented. However, the complexity of the model makes it difficult to explain the dynamic hypothesis underlying the long wave in a concise manner.

This paper presents a simple model of the economic long wave. The structure of the model is shown to be consistent with the principles of bounded rationality. The behavior of the model is analyzed, and the role of self-ordering in generating the long wave is determined. The model complements the National Model by providing a representation of the dynamic hypothesis that is amenable to formal analysis and is easily extended to include other important mechanisms that may influence the nature of the long wave.

INTRODUCTION

Recent events have revived interest in the economic long wave, sometimes known as the Kondratiev cycle, a cycle of economic expansion and depression of approximately fifty years' duration.[2] Most students of the subject date the troughs of the cycle as the 1830s, 1870s-1890s, 1930s and possibly the 1980s.[3] Originally proposed by Van Gelderen, De Wolff, and Kondratiev (Van Duijn 1983), early long-

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[1] This work is based on a model originally developed in 1979. I am indebted to Dana Meadows, Dennis Meadows, Jorgen Randers, Leif Ervik, and Elizabeth Hicks for assistance with the 1979 version, and to the Gruppen for Ressursstudier, Oslo, for its hospitality. This research was supported in part by the Sponsors of the System Dynamics National Model Project. All errors are mine.

[2] Kondratiev (1935) remains the classic of early long-wave research. Van Duijn (1983) provides a comprehensive survey and analysis of long-wave theories and empirical evidence. A good overview of early long-wave work and a sampling of recent work also provided by the August and October 1981 *Futures* 13(4,5), edited by Christopher Freeman; Freeman et al. (1982) focus on unemployment and innovation.

[3] Long-wave dating is necessarily imprecise due to the lack of reliable data. Van Duijn (1977) and (1981).

wave work was based primarily on the detection of long cycles in economic time series.

Early theories of long cycles stressed war and monetary factors such as gold discoveries as causal factors (Tinbergen 1981). Until modern times, Schumpeter's (1939) long-wave theory was the most complete and revolved around innovation.[4] After languishing in the postwar era, the late 1970s witnessed the emergence of long-wave theories based on innovation (Delbeke 1981; Mensch et al. 1981; Mensch 1979), labor dynamics (Freeman 1979; Freeman et al. 1982), resource scarcity (Rostow 1978, 1975), and capital accumulation and class struggle (Mandel 1981, 1980). As Ernest Mandel (1981, p. 332) notes,

> It is amusing that the long waves of capitalist development also produce long waves in the credibility of long-wave theories, as well as additional long waves of these theories themselves.

Yet despite the revival of interest, most economists reject the idea of the long wave. The existence of at most four cycles and the lack of reliable data for most of that period hamper empirical studies. Most important, neoclassical theory is unable to account for a disequilibrium mode of behavior with a period of half a century. In the absence of formal, testable theories of the long wave, economists have correctly remained skeptical.

Since 1975 the System Dynamics National Model has provided an increasingly rich theory of the long wave (Forrester 1981, 1979, 1977, 1976; Graham and Senge 1980; Senge 1982). As discussed below, the core of the theory is the "self-ordering" of capital by the capital sector of the economy: the dependence of capital-producing industries, in the aggregate, on their own output. But the long-wave theory growing out of the National Model is not monocausal: it relates capital investment, employment and work force participation, aggregate demand, monetary and fiscal policy, inflation, debt, innovation and productivity, and even political values. The advantages of the National Model are its wide boundary and the rich detail in which economic behavior is represented. However, the complexity of the model and the lack of published documentation make it difficult to explain the dynamic hypothesis underlying the long wave in a simple and convincing manner.

This paper presents a simple model of the economic long wave based on the self-ordering hypothesis. The model demonstrates that self-ordering can account for long waves, and isolates the minimum structure sufficient to generate a long wave. In addition, the paper stresses the role of bounded rationality in generating the long wave. It is shown that the decision rules represented in the model for managing production, investment, and so on are locally rational. However, when interacting in the context of the system as a whole, they produce "irrational" behavior: periodic over- and under-expansion of the economy.

THE DYNAMIC HYPOTHESIS: SELF-ORDERING

This section outlines the dynamic hypothesis of self-ordering and sketches a conceptual model illustrating the most important mechanisms that contribute to the long wave.[5] Consider the economy divided into two sectors: the capital sector and the goods sector. The capital-producing industries of the economy (the construction, heavy equipment, steel, mining, and other basic industries) supply each other with the capital plant, equipment, and materials each needs to operate. Viewed in

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[4] The renaissance of interest in Schumpeter's classic work (1939) is illustrated in, e.g., Van Duijn (1981), Mensch et al. (1981), and Kleinknecht (1981).

[5] The notion of a dynamic hypothesis is discussed by Randers (1980).

the aggregate, the capital sector of the economy orders and acquires capital from itself, hence "self-ordering".

If the demand for consumer goods and services increases, the consumer-goods industry must expand its capacity and so places orders for new factories, equipment, vehicles, etc. To supply the higher volume of orders, the capital-producing sector must also expand its capital stock and hence places orders for more buildings, machines, rolling stock, trucks, etc., causing the total demand for capital to rise still further, a self-reinforcing spiral of increasing orders, a greater need for expansion, and still more orders.[6]

Figure 1 shows the most basic positive feedback loop created by self-ordering. The strength of the self-ordering feedback depends on a number of factors, but chiefly on the capital intensity (capital/output ratio) of the capital-producing sector. A rough measure of the strength of self-ordering can be calculated by considering how much capital production expands in equilibrium in response to an increase in investment in the rest of the economy.

Production of capital equals the investment in plant and equipment of the goods sector plus the investment of the capital sector:

$$KPR = GINV + KINV \tag{1}$$

where

KPR = Capital sector, production (capital units/year)

$GINV$ = Goods sector, investment (capital units/year)

$KINV$ = Capital sector, investment (capital units/year)

In equilibrium, investment equals physical depreciation. If the average lifetime of capital (the aggregate of plant and equipment) were twenty years, one-twentieth of the capital stock would have to be replaced each year. Thus

$$KINV = KC / KALC \tag{2}$$

where

KC = Capital sector, capital stock (capital units)

$KALC$ = Capital sector, average life of capital (years)

The capital stock KC is related to capital production KPR by the capital/output ratio $KCOR$ (years):

$$KC = KPR * KCOR \tag{3}$$

Substituting for $KINV$ and KC yields

$$KPR = GINV^{*} \left(\frac{1}{1 - KCOR / KALC}\right) \tag{4}$$

Equation 4 indicates how much capital production must increase in the long run when the investment needs of the rest of the economy rise, taking into account the extra capital needed to maintain the capital sector's own stock at the higher level.

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[6] Self-ordering is closely related to the investment accelerator, which is commonly thought to be a factor in the 4- to 7-year business cycle. However, recent work as well as classics such as Metzler (1941) indicate the business cycle revolves around inventory management and suggest the accelerator is primarily involved in longer modes (Forrester 1982; Low 1980; Mass 1975).

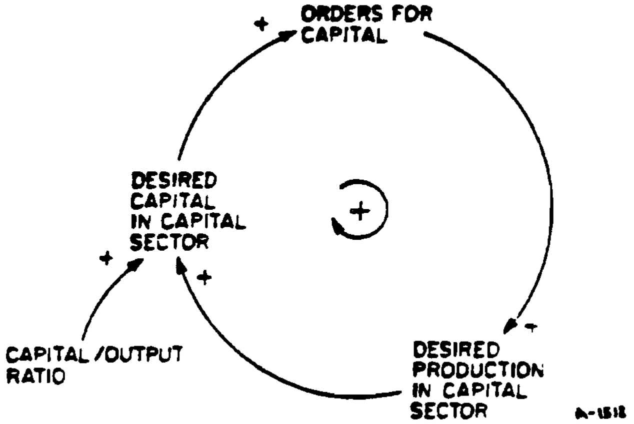

Figure 1. Basic self-ordering loop

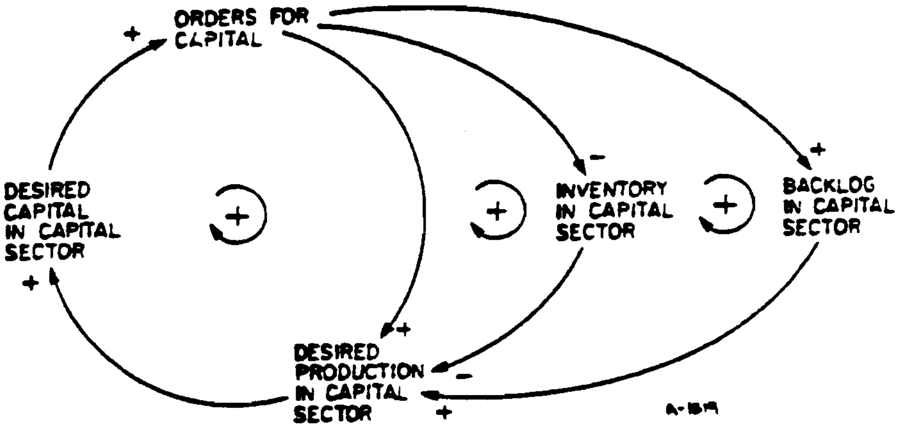

Figure 2. Amplification added by inventory and backlog adjustments

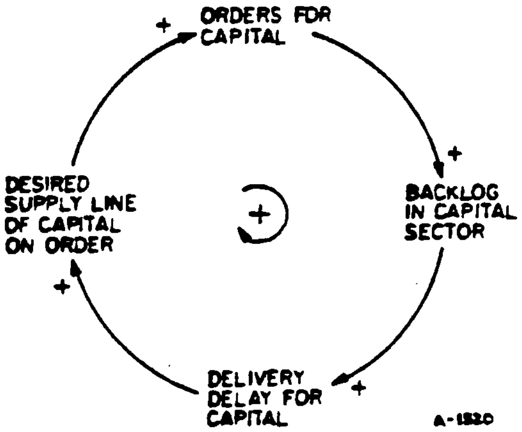

Figure 3. Rising delivery delays stimulate additional ordering

• Assuming an average life of capital of twenty years and an average capital/output ratio of three years (approximate values for the aggregate economy), the expression above yields a multiplier effect of 1.18: in the long run, an increase in investment in the rest of the economy yields an additional 18% increase in total investment through self-ordering.

The long wave is an inherently disequilibrium phenomenon, however, and during the transient adjustment to the long run the strength of self-ordering is greater than in equilibrium. As shown in Figure 2, an increase in orders for capital not only increases the steady-state rate of output required, but, because production of capital lags behind orders, depletes the inventories and swells the backlogs of the capital sector. To correct the imbalance, firms must expand output above the order rate, causing desired capital to expand further, and further swelling the total demand for capital. Production must remain above orders long enough to restore inventories and backlogs to normal levels.

Production lags behind orders for several reasons. It takes time for firms to recognize that an unanticipated change in demand is permanent enough to warrant a change in output. And once desired output rises, it takes time to increase employment and especially to increase capacity.

The disequilibrium pressures of low inventory and high backlog can significantly amplify the effect of an unanticipated change in demand, further strengthening the basic self-ordering loop.[7] Other mechanisms create additional amplification: when orders for capital exceed production, delivery times begin to rise. Faced with longer lead times and spot-shortages of specialized equipment, firms must hedge by ordering farther ahead and placing orders with more than one supplier, a process described by Thomas W. Mitchell in 1923 (p.645):

> Retailers find that there is a shortage of merchandise at their sources of supply. Manufacturers inform them that it is with great regret that they are able to fill their orders only to the extent of 80 per cent; there has been an unaccountable shortage of materials that has prevented them from producing to their full capacity. They hope to be able to give full service next season, by which time, no doubt, these unexplainable conditions will have been remedied. However, retailers, having been disappointed in deliveries and lost 20 per cent or more of their possible profits thereby, are not going to be caught that way again. If they want 90 units of an article, they order 100 so as to be sure, each, of getting the 90 in the pro rata share delivered. Probably they are disappointed a second time. Hence they increase the margins of their orders over what they desire, in order that their pro rata shares shall be for each the full 100 per cent that he really wants. Furthermore, to make doubly sure, each merchant spreads his orders over more sources of supply.

The hoarding phenomenon described by Mitchell is quite common, most recently contributing to the gasoline crisis of 1979 (Neff 1982).

For the aggregate capital sector, however, ordering farther ahead to compensate for a rising lead time adds to the total demand for capital, causing lead times to rise still further and creating still more pressure to order (Figure 3).

Other sources of amplification include growth expectations — the spread of optimism and pessimism — as described by Wesley Mitchell (1941, p.5):

> Virtually all business problems involve elements that are not precisely known, but must be approximately estimated even for the present, and

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[7] Mass (1980) discusses amplification created by stock-and-flow disequilibrium.

> forecast still more roughly for the future. Probabilities take the place of certainties, both among the data upon which reasoning proceeds and among the conclusions at which it arrives. This fact gives hopeful or despondent moods a large share in shaping business decisions... Most men find their spirits raised by being in optimistic company. Therefore, when the first beneficiaries of a trade revival develop a cheerful frame of mind about the business outlook, they become centers of infection, and start an epidemic of optimism.

To the extent expectations of future growth lead to expansion of investment, self-ordering ensures the demand for capital will in fact rise, validating and strengthening the forecast of continued growth (Figure 4).

Interactions with the labor market further strengthen self-ordering (Figure 5). To boost output, the capital sector expands employment as well as its capital stock. As the pool of unemployed is drawn down, the labor market tightens and wages rise. Scarcity of skilled workers and higher labor costs encourage the substitution of capital for labor throughout the economy, further augmenting the demand for capital. Thus one would expect the early phases of a long wave to involve expansion of labor and capital together, followed by a period of stagnant employment but continued growth in capital and output. Such patterns emerge from simulations of the National Model and have been documented for both the US, Europe, and Japan (Freeman 1979; Freeman et al. 1982; Graham and Senge 1980; Senge 1982).

Still more amplification is due to interactions with the financial markets (Figure 6). Rising capital demand boosts prices and profitability, leading to expansion of existing firms and the entry of new firms. In addition, the expansion of the asset and earnings base of the capital sector increases the external financing available for expansion. It is through these channels that monetary policy will influence the long wave, by providing (or withholding) credit sufficient to finance the demand for investment. Further amplification can be added if, as investment slows near the peak of a long wave, the monetary authority expands credit and lowers interest rates in an effort to buoy up the boom.[8]

Additional amplification arises from the familiar consumption multiplier: the expansion of the capital sector's output and employment boosts aggregate income, which feeds back to further stimulate investment demand by augmenting the demand for consumer goods and housing (Figure 7).

Interactions between self-ordering and innovation, international trade, and political values also exist and may further amplify the long wave.[9]

According to the theory derived from the National Model, the net effect of the positive feedback loops described above is to significantly amplify the basic self-ordering loop. Once a capital expansion gets under way, these loops sustain it until production catches up to orders, excess capital is built up, and orders begin to fall. At that point, the loops reverse: a reduction in orders further reduces investment demand, leading to a contraction in the capital sector's output and declining employment, wages, aggregate demand, and output. Capital production must remain below the level required for replacement and long-run growth until the

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[8] Monetary stimulus in the latter phases of the long-wave expansion may account in part for the historic movement of aggregate prices over the long wave.

[9] On innovation, see the work of Mensch and Freeman. Content analysis of political platforms has documented 50-year cycles in both American and British political values that correspond to the timing of the economic cycle (Namenwirth 1973; Weber 1981).

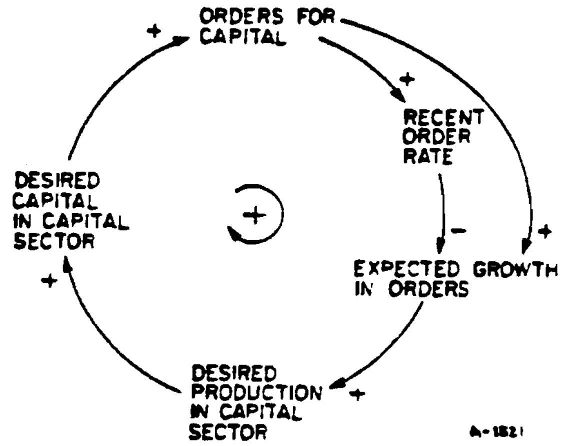

Figure 4. Amplification added by growth expectations

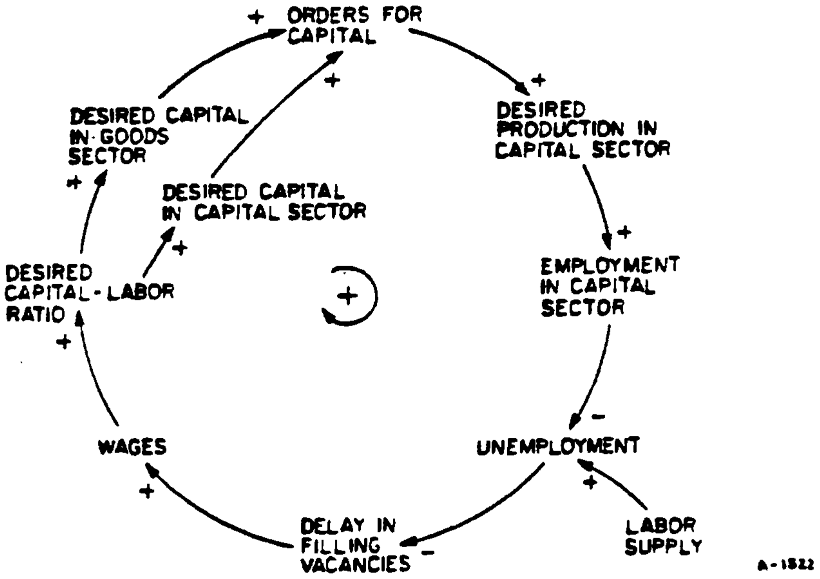

Figure 5. Rising wages encourage substitution of capital for labor

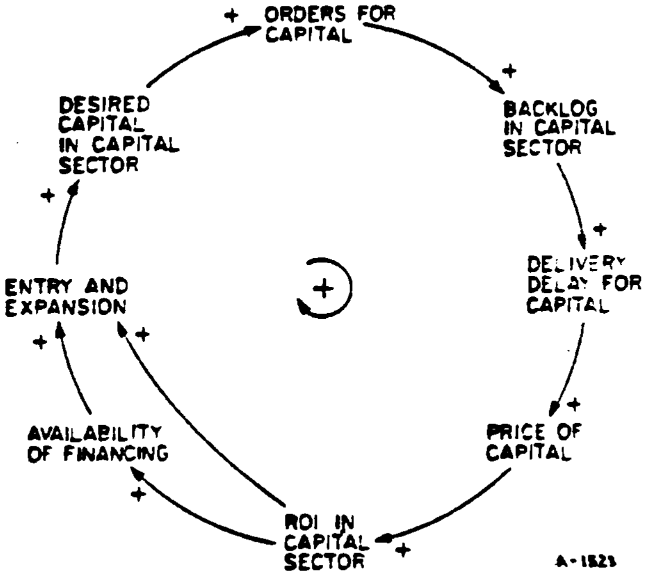

Figure 6. Rising return encourages entry and expansion

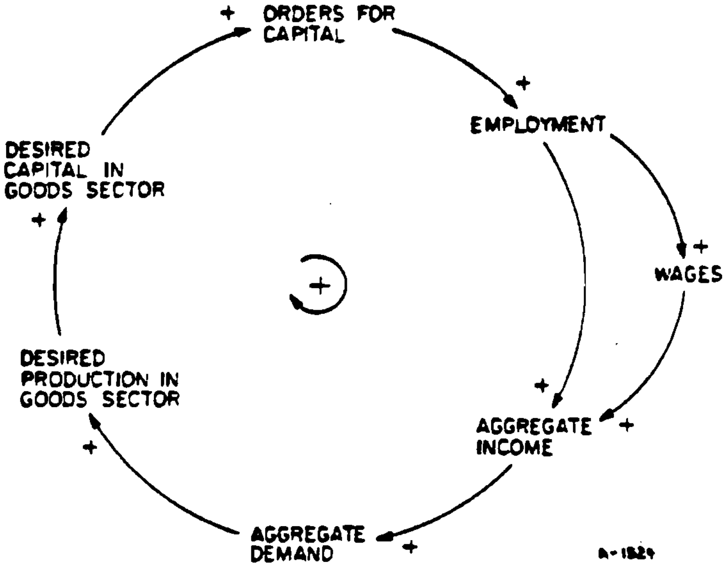

Figure 7. Amplification added by feedback through aggregate demand

excess physical and financial capital is depreciated – a process that may take a decade or more due to the long lifetimes of plant and equipment. Once the capital stock is worn out, investment rises, triggering the next upswing.

Figure 8 shows a typical series of long waves generated by the National model.[10] The simulation exhibits the short-term (4- to 7-year) business cycle as well as a 48- to 56-year long wave. Several features of the simulation bear comment:

1. The long wave is strongest in the capital sector, while the goods sector is relatively unaffected.
2. Capital stock in the capital sector peaks after production (due to construction delays) and declines slowly, depressing capital production.
3. The delivery delay for capital peaks before the peak of production.

The preceding discussion does not comprise a complete model of the long wave. Many important relationships have been omitted. Rather, the relationships above constitute a dynamic hypothesis — the essential feedback structure believed to be important in the genesis of the long wave. To be a useful hypothesis, the importance of self-ordering must be evaluated in a formal model that permits reproducible tests to be made. Further, the relative importance of the various self-ordering loops must be evaluated. The model developed below is used to address the following questions:

1. Is self-ordering sufficient to produce a long wave?
2. What factors control the period and amplitude of the long wave?
3. What nonlinearities are important in causing the long wave?
4. How might mechanisms excluded from the model alter its behavior?

BOUNDED RATIONALITY

Before proceeding to the model, this section reviews the behavioral underpinnings of the theory. The model presented below is based in part on the theory of bounded rationality (Cyert and March 1963; March 1978; Merton 1936; Nelson and Winter 1982; Simon 1947, 1957, 1978, 1979). The essence of the theory is summarized in the principle of bounded rationality, as formulated by Herbert Simon (1957, p.198):

> The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problem whose solution is required for objectively rational behavior in the real world or even for a reasonable approximation to such objective rationality.

The theory of bounded rationality is supported by an extremely large and diverse body of empirical research, which not only documents the limitations of human information processing, but highlights the systematic biases and errors deeply embedded in the heuristics people use to make decisions. While a complete catalogue of bounded rationality in its many guises is beyond the purpose of this paper, those aspects most important for theories of economic behavior in general and for this paper in particular can be stated quite simply.[11]

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[10] The behavior is triggered by exponentially autocorrelated noise in exogenous consumer demand with a time constant of 0.25 years and a standard deviation of 2.5% of the mean.

[11] Complete references cannot be given here. Excellent discussion and references to the literature can be found in Kahneman et al. (1982) and Hogarth (1980). Morecroft (1983) provides an excellent treatment of the relationships between bounded rationality and system dynamics. See also Dutton and Starbuck (1971).

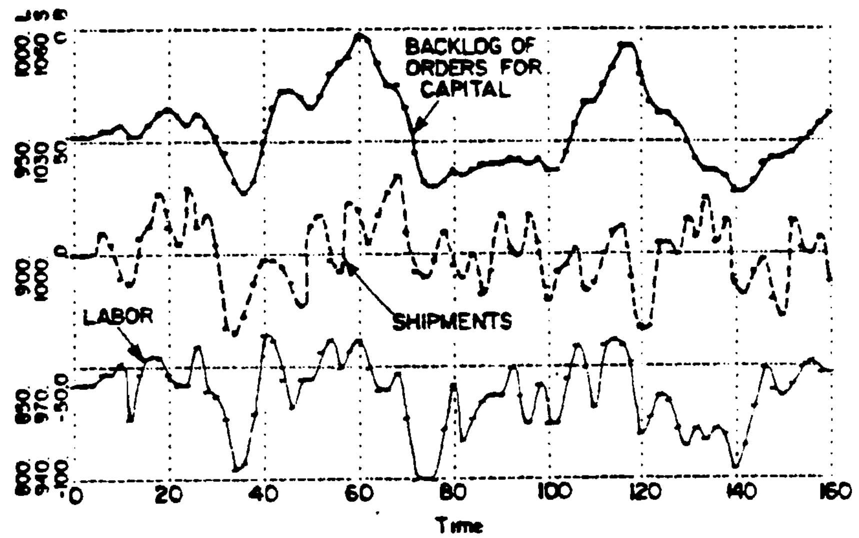

Figure 8a. Long wave generated by the National Model: goods sector

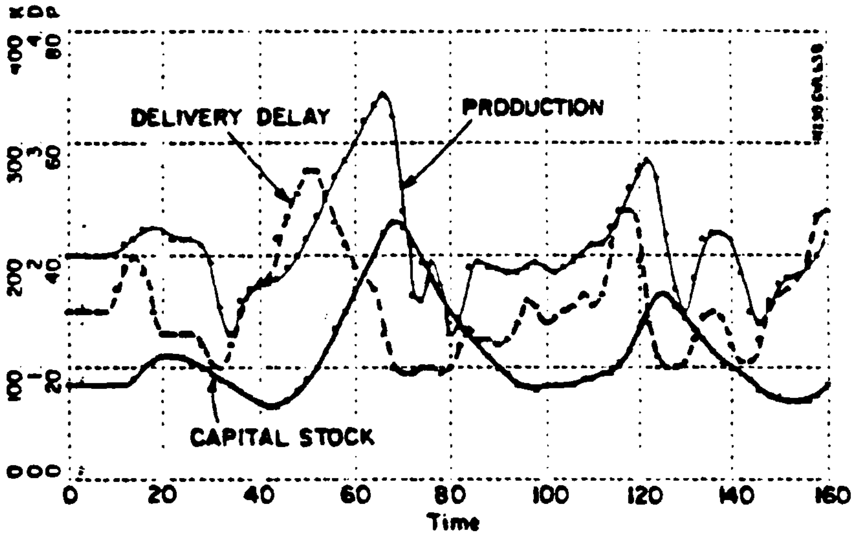

Figure 8b. Long wave generated by the National Model: capital sector

1. Limited Information-Processing Capability

Humans have a limited ability to process information. As a consequence, "perception of information is not comprehensive but *selective*" (Hogarth 1980, p.4; emphasis in original). For both physiological and psychological reasons, people take only a very few factors or cues into account when making decisions. Further, the cues that are taken into account are not those with the best predictive ability. Rather people focus on cues they judge to be relatively certain, systematically excluding uncertain or remote information regardless of its importance (Hogarth 1980, p. 36; Kahneman et al. 1982, esp. Ch. 4, 7-10). Additionally, since "people give more weight to data that they consider *causally* related to a target object...," they focus on cues they believe to be meaningful (Hogarth 1980, p. 42-43, emphasis in original). However, precisely because of limited information-processing capability and the aversion to uncertainty, people are notoriously poor judges of causality and correlation, and in controlled experiments systematically create mental models at variance with the known situation.[12] Ironically, "people tend to believe that they pay attention to many cues, although models based on only a few cues can reproduce their judgements to a high degree of accuracy" (Hogarth 1980, p. 48). Though sometimes aware of the pitfalls in judgement and inference, people, including many professionally trained in statistics, consistently assert that their own performances are immune, are reluctant to abandon their mental models and selectively use hindsight to "validate" their mental models.[13]

2. Decentralized Decision Making

As a consequence of limited information-processing ability, organizations (and the individuals within them) divide the total task of the organization into smaller units. By establishing subgoals assigned to subunits within the organization, the complexity of the total problem is vastly reduced. The subunits in the hierarchy ignore, or treat as constant or exogenous, those aspects of the total situation that are not directly related to their subgoal (Simon 1947, p.79):

> Individual choice takes place in an environment of "givens" – premises that are accepted by the subject as bases for his choice ...

Limited information-processing ability also forces people within organizational subunits to evolve simple heuristics or rules of thumb to make decisions. The rules of thumb rely on relatively certain information that is locally available to the subunit. Rules of thumb are also computationally simple (Morecroft 1983, p.133):

> In the short run, these procedures do not change, and represent the accumulated learning embodied in the factored decision making of the organization. Rules of thumb need employ only small amounts of information... Rules of thumb process information in a straightforward manner, recognizing the computational limits of normal human decision makers under pressure of time.

Such factoring is central to the management of all but the smallest enterprises. Indeed, organization, as Simon (1947, p.80) states, "permits the individual

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[12] Hogarth (1980) discusses numerous separate sources of bias in decision making. Among the common fallacies of causal attribution are the gambler's fallacy and the regression fallacy. (Tversky and Kahneman 1974).

[13] See Kahneman et al. (1982), especially Ch.2,9-12,20, and 23. Goffman's (1959) "dramaturgic" model of public behavior is relevant here: People constantly adjust their public performances so as to enhance their status and competence in the eyes of others.

to approach reasonably near to objective rationality." The implicit assumption (necessitated by the complexity of the total situation and the limited time available for decision making) is that the task is separable in the sense that achieving the subgoals ensures attainment of the larger goal.

THE MODEL

The model will be presented in several stages. First, a simplified, generic model of a firm or sector of the economy will be developed (the "production sector"). It will be shown, through partial model tests, that the decision rules for production and investment yield rational behavior in the simplified environment presumed by each subunit of the organization. The model will then be used to represent the aggregate capital-producing sector of the economy, including self-ordering. Finally, simulation experiments will be used to establish the relative contribution of the structural and parametric assumptions to the resulting long-wave behavior.[14]

$$PR_t = PC_t * CU_t \ . \tag{1}$$

$$CU_t = f_1(IP_t / PC_t) \quad f_1(0) = 0, \ f_1(1) = 1, \ f_1' > 0, \ f_1'' < 0 \tag{2}$$

$$IP_t = B_t / NDD \tag{3}$$

where

PR = Production rate (units/year)

PC = Production capacity (units/year)

CU = Capacity utilization (fraction)

IP = Indicated production (units/year)

B = Backlog of unfilled orders (units)

NDD = Normal delivery delay (years)

Equations 1 through 3 describe production and capacity utilization. Production rate PR is determined by production capacity PC and the rate of capacity utilization CU. Capacity utilization is determined by the ratio of indicated production to production capacity, a measure of demand relative to supply. Indicated production represents the rate of production that would be required to deliver an order with the normal delivery delay NDD. The normal delivery delay represents the time required, in equilibrium, to process, produce, and deliver an order.

As shown in Figure 9, capacity utilization varies nonlinearly with the ratio IP/PC. When $IP/PC > 1$, the rate of production required to meet the normal delivery delay exceeds capacity, which becomes a binding constraint on production. If indicated production drops below capacity, however, output is curtailed. (Since inventories are not represented, production and shipments are always

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[14] The model is formulated in continuous time as a set of integral equations, and was simulated using Euler integration (see Appendix).

equal, and if there are no orders to be filled, production must decline to zero unless one assumes firms simply throw the extra output away.) If firms wanted to maintain the normal delivery delay regardless of capacity, capacity utilization would fall in proportion to the decline in demand, production would equal indicated production, and *CU* would lie along line A. If firms wanted to continue to operate at full capacity at all times, even in the face of diminished demand, utilization would fall only when the sector was producing at the minimum delivery delay, defined by line B.[15] Capacity utilization is specified as a compromise between these two extremes: if indicated production drops below capacity, firms are assumed to reduce utilization only slightly, preferring to maintain relatively full utilization (and hence revenues) by drawing down their backlogs. Delivery delays would become shorter than normal. If backlog continued to fall, utilization would be cut back, but at less than proportional rates, until firms were producing at the minimum delivery delay. Further declines in backlog then force proportional reductions in output. The behavior described by the capacity utilization formulation is illustrated by the following description of the machine tool industry (*Business Week*, 14 March 1982, p.20):

> Bad as they are, shipments are outpacing orders by a very wide margin, forcing a continued rundown in the industry's order backlog... At the average shipment rate of the past three months, backlogs provide less than six months of production, in an industry that had a one-year backlog when the recession began... the low level of capacity utilization suggests that shipments will run ahead of orders well into summer.

$$PC_t = C_t \,/\, COR \tag{4}$$

where

C = Capital stock (capital units)

COR = Capital/output ratio (years)

Production capacity is determined by capital and the capital/output ratio. For simplicity, capital is the only explicit factor of production, and the capital/output ratio is assumed fixed, implicitly assuming other factors (particularly labor) are freely available.[16]

$$C_t = \int_{t_0}^{t} (CA_t - CD_t)\,dt + C_{t_0} \tag{5}$$

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[15] Line B determines the minimum delivery delay because the actual delivery delay or average residence time of an order in the backlog is given by $DD = B / PR = B / (PC{*}CU)$. When $CU = \delta * (IP / PC)$ for $\delta > 1$ and $\delta * (IP / PC) \leq 1$, i.e. when CU lies along line B $DD = B / (PC{*}\delta\,(IP / PC)) = B / (\delta{*}IP)$ but $IP = B / NDD$, so $DD = B / (\delta{*}B / NDD) = NDD / \delta = MDD$ where MDD = Minimum delivery delay (years).

[16] Though a more complete model would include a more sophisticated production function with both variable labor and a variable work week, the dynamics of labor acquisition are primarily associated with the short-term business cycle (see footnote 6). However, since rising wages contribute to the strength of self-ordering during a long-wave expansion (Figure 5), omission of labor as an explicit factor is likely to reduce the model's ability to generate a long wave. For a dynamic model with multiple factors of production that conforms to the principles of bounded rationality see Sterman (1981, 1982).

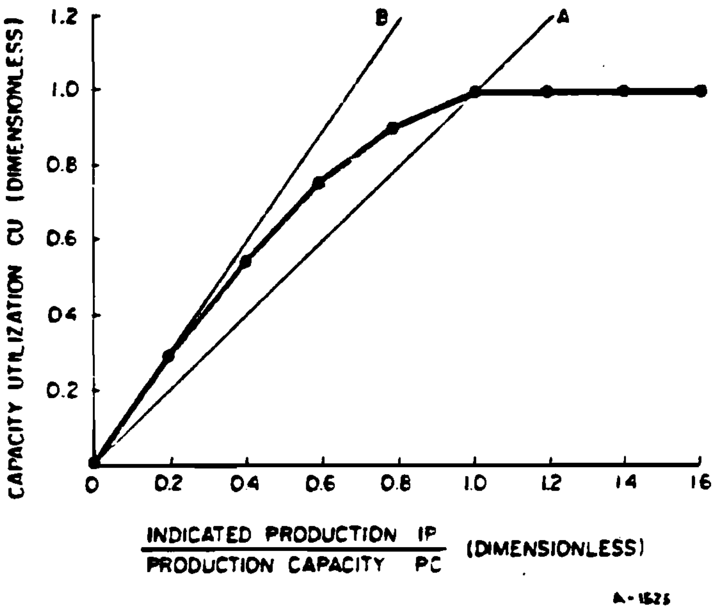

Figure 9. Capacity utilization

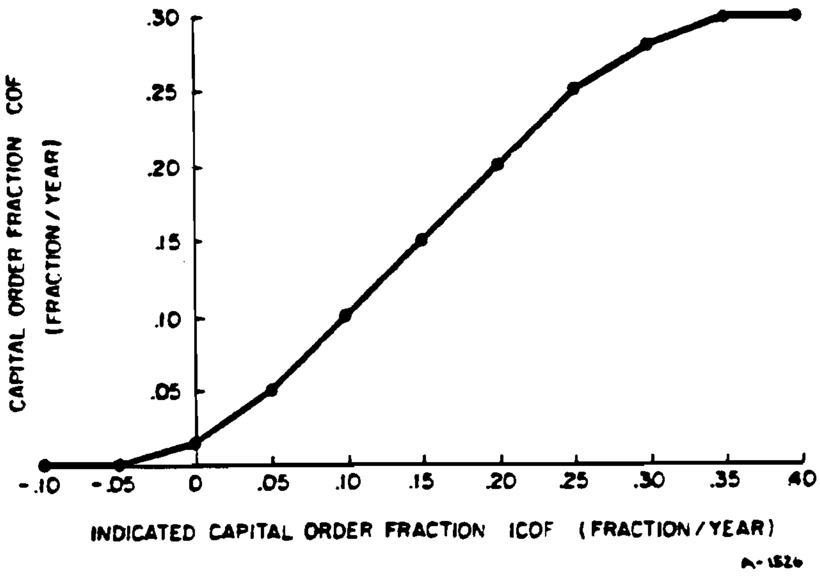

Figure 10. Capital order fraction

where

CA = Capital acquisitions (capital units/year)

CD = Capital discards (capital units/year)

Capital stock, representing both plant and equipment, is the accumulation of capital acquisitions CA less capital discards CD.

$$CD_t = C_t \,/\, ALC \tag{6}$$

The simplest formulation for capital discards is to assume all units have an equal probability of being discarded regardless of age, defining (in equilibrium) an exponential probability density for the age of individual units, with the mean physical life given by the average life of capital ALC. For simplicity, the average lifetime is assumed constant.[17]

$$CA_t = SL_t \,/\, DDC_t \tag{7}$$

where

SL = Supply line of unfilled orders for capital (capital units)

DDC = Delivery delay for capital (years)

Capital acquisition, or gross investment, is determined by the sector's supply line or backlog of unfilled orders for capital (including capital under construction) and the average delay in acquiring those units (including the time required for construction). In general, the delivery delay for capital will vary according to the capacity of the supplying industries relative to the demand.

$$SL_t = \int_{t_0}^{t} (CO_t - CA_t)\,dt + SL_{t_0} \tag{8}$$

where

CO=capital orders (capital units/year).

The sector's supply line is augmented as orders for capital are placed with suppliers, and is diminished when construction is completed and the capital enters the productive stock of the sector.

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[17] Sterman (1980) contrasts the lumped capital stock used here to a model with capital disaggregated by vintage. A more complete model would also include a variable average lifetime to represent variations in the discard rate.

$$CO_t = C_t * COF_t \quad (9)$$

$$COF_t = f_2(ICOF_t) \quad f_2' \geq 0 \quad (10)$$

$$ICOF_t = (CD_t + CC_t + CSL_t)/C_t \quad (11)$$

where

COF = Capital order fraction (fraction/year)

$ICOF$ = Indicated capital order fraction (fraction/year)

CC = Correction to orders from capital stock (capital units/year)

CSL = Correction to orders from supply line (capital units/year)

Though capital acquisition corresponds to investment, it is the order rate for capital that determines acquisitions. Three motivations for ordering capital are assumed: First, to replace discards; second, to correct any discrepancy between the desired and actual capital stock; and third, to correct any discrepancy between the desired and actual supply line.[18] The sum of these three pressures, as a fraction of the existing capital stock, defines the indicated capital order fraction $ICOF$. However, in extreme circumstances the indicated capital order fraction may take on unreasonable values. For example, an extreme excess of capacity could cause $ICOF$ to be negative. As shown in Figure 10, the actual order fraction COF is a nonlinear function of the indicated order fraction. Since gross investment must be positive, COF asymptotically approaches zero as $ICOF$ drops below 5% year.[19] Similarly, if demand far exceeds capacity, the indicated order fraction may take on unreasonably large values. It is assumed that the maximum capital order fraction is 30% of the capital stock per year. The limit reflects physical constraints to rapid expansion such as labor and materials bottlenecks, financial constraints, and organizational pressures.[20]

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[18] Investment resulting from growth expectations would have to be included in a more complete model. The investment function of the model is a simplified version of the System Dynamics National Model investment function. Senge (1978, 1980) shows the SDNM function reduces to the neoclassical investment function (e.g. Jorgenson 1963; Jorgenson *et al.* 1970) when a variety of equilibrium and perfect information assumptions are made. The SDNM function is shown to provide a better statistical fit of investment data and to behave more plausibly than the neoclassical function when faced with various test inputs.

[19] The formulation for COF excludes order cancellations. Disallowing cancellations is a simplifying assumption. A more complete model would disaggregate unfilled orders from units under construction and would represent cancellations explicitly (Sterman 1981). The formulation for COF smoothly approaches zero due to the aggregation of firms, some of which will be ordering nonzero amounts even when the average $ICOF < 0$. The values of COF for $ICOF < 0.05$ were estimated by assuming (1) the ordering function of a single firm is $COF = MAX(0, ICOF)$ and (2) $ICOF$ for the aggregate sector is distributed normally with a variance of 0.05/year.

[20] The values of COF for $ICOF \gg 0.05$ were derived by assuming the order function of an individual firm was $MIN(0.30, ICOF)$ and that $ICOF$ is distributed normally with a variance of 0.05/year.

$$CSL_t = (DSL_t - SL_t) / TASL \tag{12}$$

$$DSL_t = CD_t * PDDC_t \tag{13}$$

$$PDDC_t = DDC_t \tag{14}$$

where

DSL = Desired supply line (capital units)

$TASL$ = Time to adjust supply line (years)

$PDDC$ = Perceived delivery delay for capital (years)

DDC = Delivery delay for capital (years)

Equations 12 through 14 describe the management of the supply line. Firms strive to eliminate discrepancies between the desired and actual supply line within the time to adjust supply line $TASL$. To ensure an appropriate acquisition rate, firms must maintain a supply line proportional to the delivery delay they face in acquiring capital: as described by Mitchell (1923), if the delivery delay rises, firms must plan for and order new capital farther ahead, increasing the required supply line. The desired supply line is based on relatively certain information: the discard rate and the delivery delay for capital perceived by the firm. For simplicity, delays in perceiving the true lead time for capital are not represented, thus the perceived delivery delay for capital is assumed to equal the actual delivery delay. However, the relationship between delivery delay and the desired supply line is likely to be highly nonlinear: as Mitchell notes, initially a change in delivery delay may produce a more than proportional change in orders due to hoarding and panic. And rather than expand orders continually as lead times rise, chronically high delivery delays would eventually cause firms to seek substitutes, limiting the desired supply line. The sensitivity of the model to the decision rule for desired supply line is tested below.

$$CC_t = (DC_t - C_t)\, TAC \tag{15}$$

$$DC_t = RC * f_3(IC_t / RC) \quad f_3^{(0)} = 0, \quad f_3^{(1)} = 1, \quad f_3' \geq 0, \quad f_3'' \leq 0 \tag{16}$$

$$IC_t = IPC_t * COR \tag{17}$$

where

DC = Desired capital (capital units)

TAC = Time to adjust capital (years)

RC = Reference capital (capital units)

IC = Indicated capital (capital units)

IPC = Indicated production capacity (units/year)

Equations 15 to 17 describe the adjustment of capacity to desired levels. Like the supply line correction, firms attempt to correct discrepancies between desired and actual capital stock over a period of time given by the time to adjust capital. Desired capital is nonlinearly related to the indicated capital stock, which is the stock needed to provide the indicated production capacity IPC. (Indicated production capacity is the capacity judged necessary to meet expected demand.) As shown in Figure 11, diminishing returns to capital are assumed to set in when IC becomes large relative to a reference level of capital RC (set at the initial equilibrium of the system). Though labor is not explicitly represented, the linear range of the relationship between IC and DC implies employment can be expanded in proportion to capital. As the available labor supply is exhausted, however, further expansion of capital lowers the marginal productivity of capital and diminishes incentives for further expansion even if demand remains high.

$$IPC_t = EO_t + CB_t \tag{18}$$

$$CB_t = (B_t - IB_t)/TAB \tag{19}$$

$$IB_t = NDD * EO_t \tag{20}$$

where

EO = Expected orders (units/year)

CB = Correction from backlog (units/year)

IB = Indicated backlog (units)

TAB = Time to adjust backlog (years)

Equations 18 through 20 determine indicated production capacity. It reflects the capacity the sector judges necessary both to fill expected orders EO and adjust the backlog of unfilled orders to an appropriate level. The speed with which the sector strives to correct discrepancies between the actual and indicated backlog is determined by the time to adjust backlog, a reflection of the sector's sensitivity to abnormal delivery delays. Indicated backlog is the backlog that would be necessary to fill the expected order rate within the normal delivery delay.

$$EO_t = \int_{t_0}^{t} \frac{OR_t - EO_t}{TAO} d_t + EO_{t_0} \tag{21}$$

where

TAO = Time to average orders.

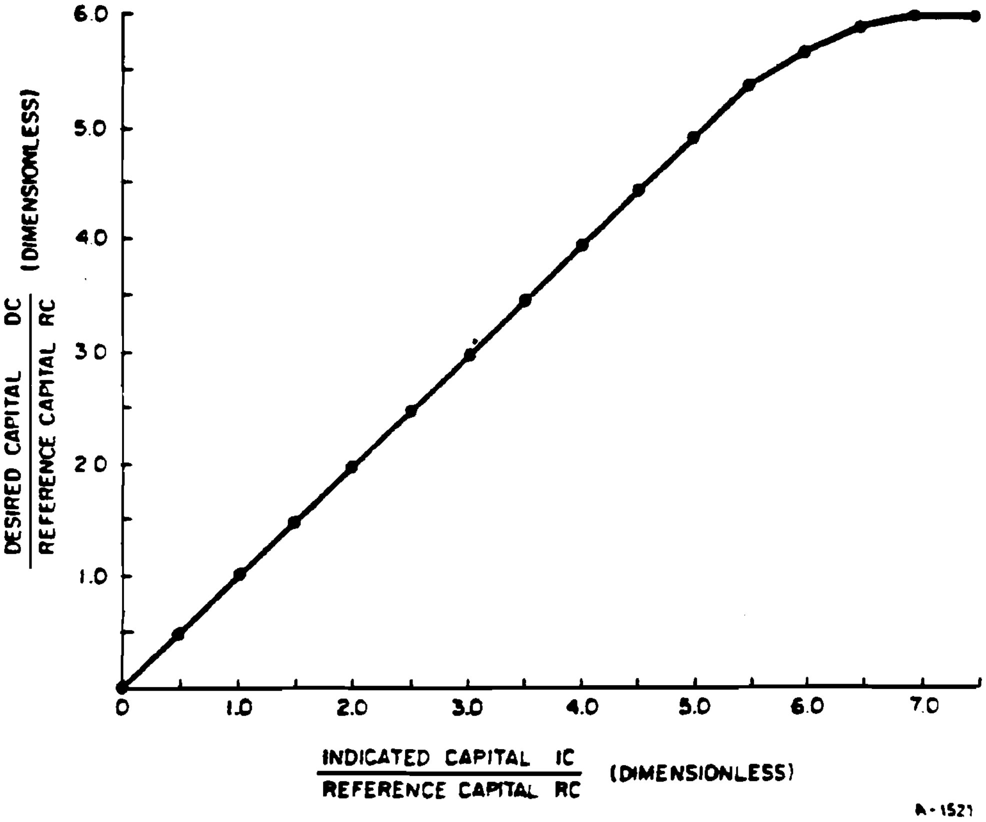

Figure 11. Desired capital stock

The expected order rate represents the sector's forecast of demand conditional on available information and the rules of thumb for forecasting used by the sector. The firm is assumed to forecast demand by averaging past orders. Orders are averaged because it takes time for firms to decide that an unanticipated change in demand is lasting enough to warrant capacity expansion. The averaging serves to filter out short-term noise in demand, providing a more certain measure of long-run demand than the raw order rate, and preventing wild swings in investment by allowing the backlog to buffer the system from the short-term variability of demand. First-order exponential smoothing is assumed for the averaging process. The smoothing time is given by the time to average orders *TAO*.[21]

$$DD_t = B_t / PR_t \tag{22}$$

$$B_t = \int_{t_0}^{t} (OR_t - PR_t)\,dt + B_{t_0} \tag{23}$$

$$OR_t = \text{exogenous} \tag{24}$$

$$DDC_t = NDD \tag{25}$$

Finally, the delivery delay for the sector's output, or average residence time of an order in the backlog, is given by the ratio of backlog to production. The backlog of unfilled orders accumulates orders less shipments (production). The order rate is assumed exogenous; delivery delay for capital is exogenous and assumed constant.

The parameter values assumed for the analysis are summarized in Table 1. The parameters were chosen to represent a producer of capital goods. The parameters are broadly consistent with survey and econometric evidence reported in various studies. But because the model excludes all but the most basic channels through which self-ordering operates, precise estimation is not warranted. The sensitivity of the model to the key parameters is analyzed below.

THE LOCAL RATIONALITY OF THE DECISION RULES

The behavioral formulations in the model conform to the principles of bounded rationality: management of the firm is broken down into several distinct decisions (production, investment, demand forecasting, etc.). The individual decision rules rely on locally available, relatively certain information. For example, desired production capacity relies on the backlog and average orders rather than the current and less certain order rate. Similarly, the desired supply line requires knowledge only of the replacement rate of investment and the delivery delay for capital experienced by the firm, and does not consider the condition of capital suppliers or the effect demand changes might have on availability. Simple rules of thumb are used to determine how much capital to keep on order, how fast to adjust production capacity, and how to manage backlogs. To test the local or intended rationality of the decision rules, this section describes partial model tests of the

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[21] Growth expectations would have to be included in a more complete model of demand forecasting, and would add amplification.

Table 1. Parameters

Symbol	Definition	Value (years)
ALC	Average Life of Capital	20
COR	Capital/Output Ratio	3
NDD	Normal Delivery Delay	1.5
DDC	Delivery Delay for Capital	1.5
TAB	Time to Adjust Backlog	1.5
TAO	Time to Average Orders	2
TAC	Time to Adjust Capital	3
TASL	Time to Adjust Supply Line	3

Sources/Comments:

ALC:	Coen [1975] found service lives ranging from 8 to 22 years for equipment and 20 to 50 years for structures. Sterman [1981] estimated a 20-year lifetime for the aggregate of plant and equipment.
COR:	The mean value of real private capital stock/real GNP (1958 $) from 1946 to 1970 = 2.9. [Historical Statistics of the U.S. Series F-470/F-32].
NDD & DDC:	Mayer [1960] found mean lead times for plant and equipment (planning to completion) of 22 months (5 months planning and 17 months ordering and construction delays). Since the sector represents a capital producer, NDD=DDC.
TAB:	TAB should be comparable to NDD: Firms would not want to try to adjust backlogs faster than products can be delivered; but TAB>>NDD implies a sluggish response to abnormal delivery delays. Senge [1978], using nondurable manufacturing data, found no statistically significant difference between NDD and TAB.
TAO:	TAO should be greater than TAB to reflect the low weight managers place on current and highly uncertain orders compared to the much more certain backlog. Senge [1978] found TAO>TAB (using shipments instead of orders as the measure of demand).
TAC & TASL:	Senge [1978] found TAC=12.1 quarters (est. std. dev. 2.2 quarters). TASL should be comparable to TAC so that orders in planning are weighted in the order decision as heavily as units in the productive stock. If TASL>TAC, overordering results as capital on order is partially ignored; if TASL<TAC, orders in the supply line are counted more heavily in the investment decision than capital itself.

production and investment decisions. A minimum requirement for intended rationality is that the individual decision rules respond well to shocks when the decision rules are tested in isolation.

1. Demand Forecasting and Backlog Management

Equations 18 through 21 describe the demand forecasting procedure and determination of desired capacity. To test the intended rationality of this decision rule, the sector was subjected to a sudden, unanticipated increase in orders of five percent at the start of year one. To isolate the decision rule, it was assumed that

$$PR_t = IPC_t \tag{1'}$$

Capacity then places no constraint on production, and the production scheduling equations become the only determinants of the sector's behavior.

The result (Figure 12) is a smooth and orderly response. Immediately after the shock, expected orders and production are unchanged and the backlog begins to rise. As backlog rises, however, firms recognize the growing discrepancy between the backlog and the backlog consistent with the normal delivery delay. Production is adjusted above expected orders by exactly enough to keep delivery delay constant. Simultaneously, as management comes to believe the new level of demand will persist, expected orders rise, gradually shifting the burden of adjustment from the correction from backlog to the demand forecast.[22] The response is extremely rational in the sense that: it is appropriate – in equilibrium, expected output, output, and backlog have all expanded by five percent. It is also orderly – expected orders, production, and backlog all smoothly approach their new equilibrium values. Even though expected orders lag behind actual orders, delivery delay remains constant at its normal value. The expected order rate covers 95% of the initial discrepancy in six years. Production covers 95% of the initial discrepancy within 4.5 years.

2. Investment and Capacity Acquisition

Equations 5 through 17 describe the determinants of investment and capacity acquisition. To test the local rationality of this decision rule, it is assumed that indicated production capacity is exogenous. The sector is subjected to a sudden, unanticipated increase in indicated production capacity of five percent in year one. It is assumed the sector faces a constant delivery delay for capital, eliminating the possibility of bottlenecks in the supplying industry.

Again, the response (Figure 13) is smooth and orderly. Immediately after the shock, there is a maximum discrepancy between desired and actual capital, and orders for capital rise to a peak. As the supply line fills, the order rate drops, for even though the capital stock does not increase immediately, the units ordered but not yet received are taken into account when placing future orders. Overordering, an obvious source of instability, is thus prevented. As the supply line rises, so too do acquisitions, which peak two years after the shock. As capital increases

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[22] The equations for indicated production capacity (18 through 20) reduce to:

$$\begin{aligned} IPC &= EO + CB = EO + (B - IB)/TAB \\ &= EO + (B - EO \cdot NDD)/TAB \\ &= EO(1 - NDD/TAB) + B/TAB \end{aligned}$$

The base case assumes $TAB = NDD$, so $IPC = B/TAB = B/NDD$, thus IPC always equals the production rate consistent with NDD, which is why DD remains constant in the test.

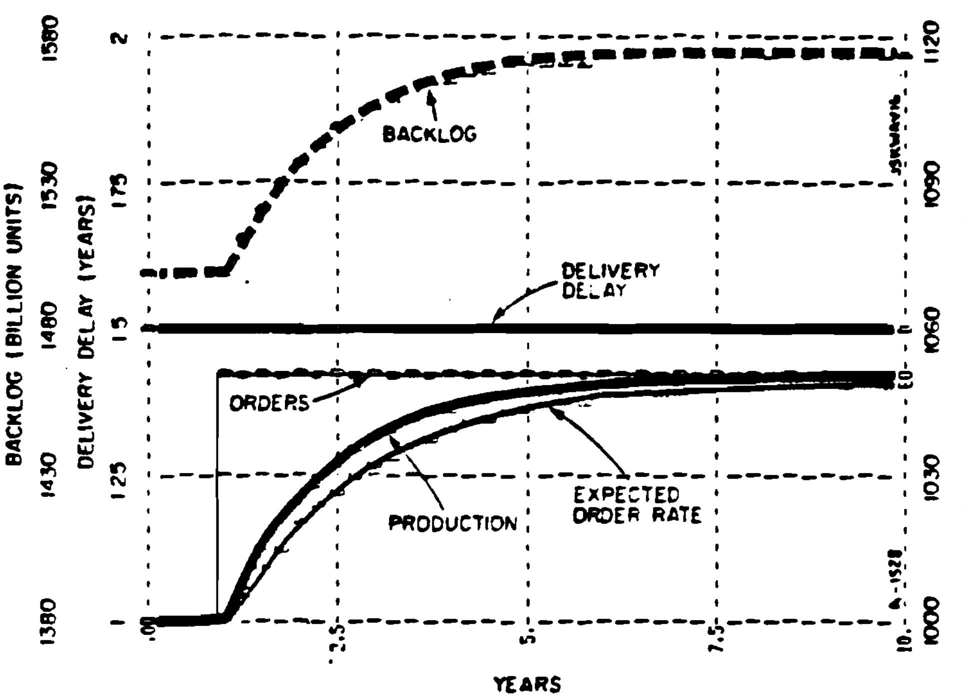

Figure 12. Response of production scheduling subsector to step in orders

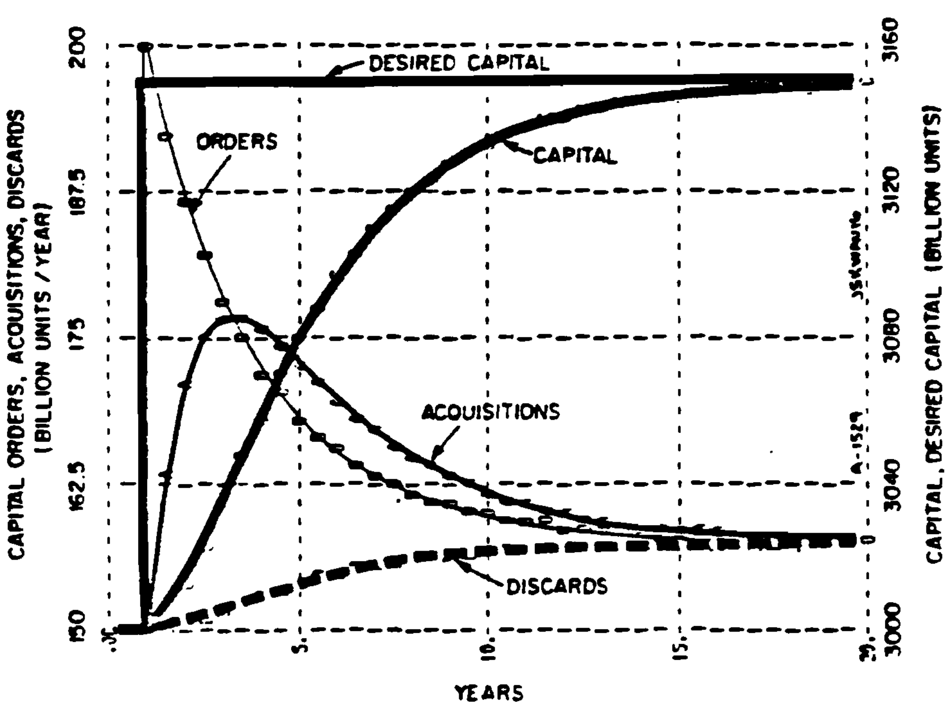

Figure 13. Response of investment subsector to step in desired capital

the burden of investment shifts back to replacements, and in equilibrium the desired and actual stock are again equal (likewise the desired and actual suply line). Like the production scheduling equations, the response is extremely rational: the adjustment is appropriate, orderly, and essentially completed (over 95%) within twelve years.

3. Testing the Complete Production Sector

The partial model tests show that the decision rule for investment can track changes in desired capacity without overshoot or instability. Similarly, the production scheduling decision can accommodate unanticipated changes in demand smoothly and without disruption. The next test examines the ability of the entire sector to respond to a change in demand. In the test, the sector faces a five percent unanticipated increase in orders at the start of year one. The delivery delay for capital is assumed constant.

The result (Figure 14) is a highly damped oscillation with a period of about twenty years. In contrast to the previous tests, production and capacity now overshoot orders, then undershoot slightly before reaching equilibrium. Because capacity (and production) lag behind orders, the backlog (and delivery delay) must rise. When production equals orders (in year six), backlog stops increasing and reaches its maximum. Delivery delay peaks slightly earlier. In order to reduce delivery delay to normal levels, production and capacity must continue to expand above orders. By year eight, delivery delay is once again normal, but production still rises due to growing capacity and industry reluctance to reduce utilization. By year ten, backlog has fallen enough to begin to force utilization down, but because firms prefer to maintain full utilization, output continues to exceed orders, and delivery delay falls below normal as firms draw down their backlogs to preserve profitability. Faced with excess capacity, investment is cut back, and by the twelfth year, capacity begins to decline. For delivery delay to return to normal, the backlog must rise, forcing output and capacity below orders. But when delivery delay has returned to normal, capacity is once again insufficient, triggering a second, though much smaller, overshoot.

The test shows that as the complexity of the system grows relative to the simplifying assumptions and decision rules used by the subsectors of the organization, the rationality of the organization's response to change is degraded. Yet despite the overshoot, the system's response is, on the whole, still rather rational. The majority of the behavior is a direct consequence of the physical constraints facing the firms in the sector. Since production must lag behind orders backlogs must initially rise. Therefore output and capacity must exceed orders to bring backlog back down. Overshoot is an inevitable consequence of the lags in expanding output. Oscillation, however, is not: the existence of oscillation is a consequence of decentralized decision making and the aggressiveness with which people attempt to correct perceived imbalances. Still, the system exhibits a high degree of damping (93% of the cycle is damped each period). And though output rises to a peak 65% greater than the change in orders, rising delivery delays are arrested within four years, production settles within 2% of its equilibrium value after fifteen years, and utilization never drops below 97%. The behavior represents a good compromise between a speedy response and stability.[23]

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[23] The 20-year cycle is consistent with earlier models of capital investment and empirical work on construction of Kuznets cycles. See Forrester (1982), Low (1980), and Mass (1975) for models of Kuznets-type cycles arising out of capital-investment policies. For empirical work on Kuznets cycles see, e.g., Hickman (1963) and Kuznets (1930).

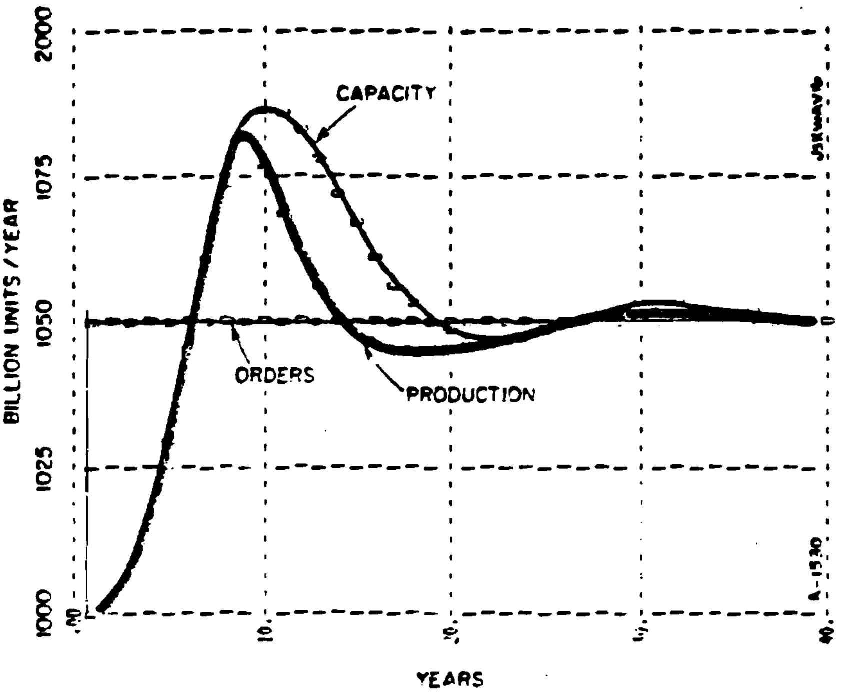

Figure 14a. Response of production sector to step in orders: output and capacity

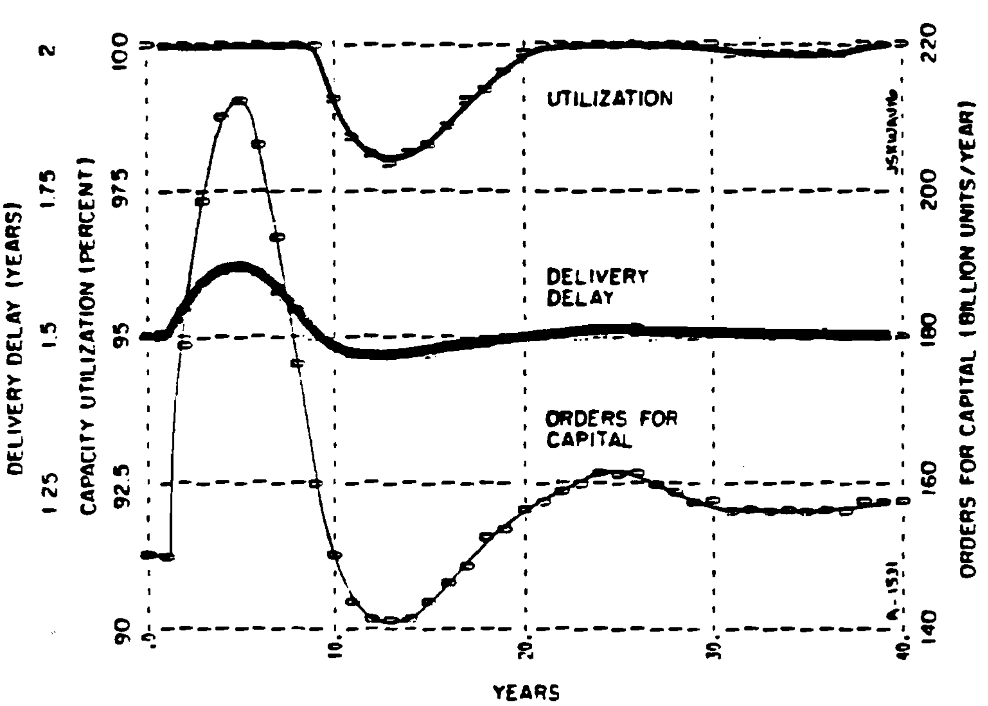

Figure 14b. Response of production sector to step in orders: utilization, delivery delay, and capital orders

TESTING THE DYNAMIC HYPOTHESIS

Having established the local rationality of the subsectors of the model, the production sector can now be used to test the dynamic hypothesis behind the long wave. To do so, the production sector is used to represent the capital-producing sector of the aggregate economy. The following equations are added or modified to implement the test:

$$B_t = GSL_t + SL_t \tag{23'}$$

$$OR_t = GCO_t + CO_t \tag{24'}$$

$$DDC_t = DD_t \tag{25'}$$

$$GSL_t = \int_{t_0}^{t} (GOR_t - GCA_t)\, dt + GSL_{t_0} \tag{26}$$

$$GCA_t = GSL_t \,/\, DDC_t \tag{27}$$

$$GCO_t = \text{exogenous} \tag{28}$$

where

GOR =Goods sector, capital order rate (units/year)

GSL =Goods sector, supply line of unfilled orders (units/year)

The total demand for capital (eq. 24') is now composed of two parts: an exogenous order rate for capital deriving from the goods sector of the economy (all noncapital industries) and the self-ordering component: the capital sector's own orders for capital. The backlog of the sector (eq. 23') becomes the sum of the supply lines of the goods and capital sectors. The supply line of the goods sector (eq. 26) accumulates the goods sector's orders for capital less acquisitions, which are determined by the delivery delay for capital (eq. 27). A direct consequence of self-ordering is that the delivery delay for capital faced by the capital sector is the time required to produce its own output (eq. 25'). In addition, it is assumed that each order in the backlog has an equal probability of being filled. As a consequence, the output of the capital sector is divided between the goods and capital sectors in proportion to their supply lines, implying the priority of the two sectors is equal.

Finally, note that the order rate for capital from the goods sector is exogenous. Thus only the most basic self-ordering mechanisms shown in Figures 1-3 are included. The self-ordering feedback loops that operate through growth expectations, labor markets, prices, financial markets, and aggregate demand (Figures 4-7) are not included.

The model was subjected to an unanticipated increase in orders for capital from the goods sector of one percent (a less than one-percent change in total capital demand). The response (Figure 15) is a large-amplitude limit cycle with a steady-state period of forty-nine years. Figure 16 shows one complete cycle drawn from the steady-state region of Figure 15. The gross qualitative features of the behavior correspond to the long wave produced by the full National Model (Figure 8):

1. The cycle has a period substantially longer than the business or Kuznets cycle and more than double the period of the production sector in isolation without self-ordering.
2. Output rises slowly as capital is accumulated but falls precipitously, followed by a long depression while the excess capital depreciates. Capital peaks after output.
3. The delivery delay for capital peaks before the peak of output.
4. The cycle is a limit cycle that persists without continuous exogenous triggering.

To clarify the sources of the behavior, consider the sequence of events shown in Figure 16. In the 110th year, the capital sector has excess capacity and is primarily producing for the goods sector. Net investment in the capital sector is negative, and capacity is falling. As a result, utilization is rising. In approximately the 118th year, capacity and orders become equal, but because backlog and delivery delay are below normal, output remains depressed. Capacity continues to fall until by year 120, capacity and output become equal, utilization reaches one hundred percent, and delivery delay becomes normal. However, the sector is not in equilibrium because capacity has fallen below orders, just as in the test of the sector without self-ordering.

However, unlike the response of the sector in isolation, capacity and output do not then rise smoothly to equilibrium, but continue to expand well beyond the equilibrium level of output. Self-ordering is directly responsible, through several channels.

Up until year 118, excess capacity meant the sector's gross investment was less than discards. As capacity falls towards orders, orders for capital rise to the replacement level. Acquisitions, however, lag behind by the delivery delay. As a result, capacity falls below orders, and delivery delay rises above normal. Additional orders are placed to correct this discrepancy, swelling the backlog of the sector, increasing desired output and causing still more orders for capital. This most basic of the self-ordering loops is the inevitable consequence of the fact that capital is an input to its own production. As orders for capital are placed in an attempt to reduce the discrepancy between demand and capacity, self-ordering acts to increase the discrepancy by expanding desired production with each new order. The sector chases its own shadow.

Second, because capacity is inadequate, delivery delay rises above normal. Thus as capital producers attempt to expand, they find capital acquisitions lagging further behind orders. As a result, capacity expands less rapidly than anticipated, widening the gap between desired and actual capital, causing still more orders to be placed, and further lengthening the delivery delay.

Third, faced with lengthening lead times, capital producers attempt to compensate by ordering further ahead, allowing orders to expand still further.

As a consequence, though output begins to grow rapidly, demand grows more rapidly, and the delivery delay rises. Within eight years capital acquisitions have expanded enough to allow capacity to gain ground on demand. By the 128th year, output is expanding as fast as orders, and delivery delay reaches its maximum

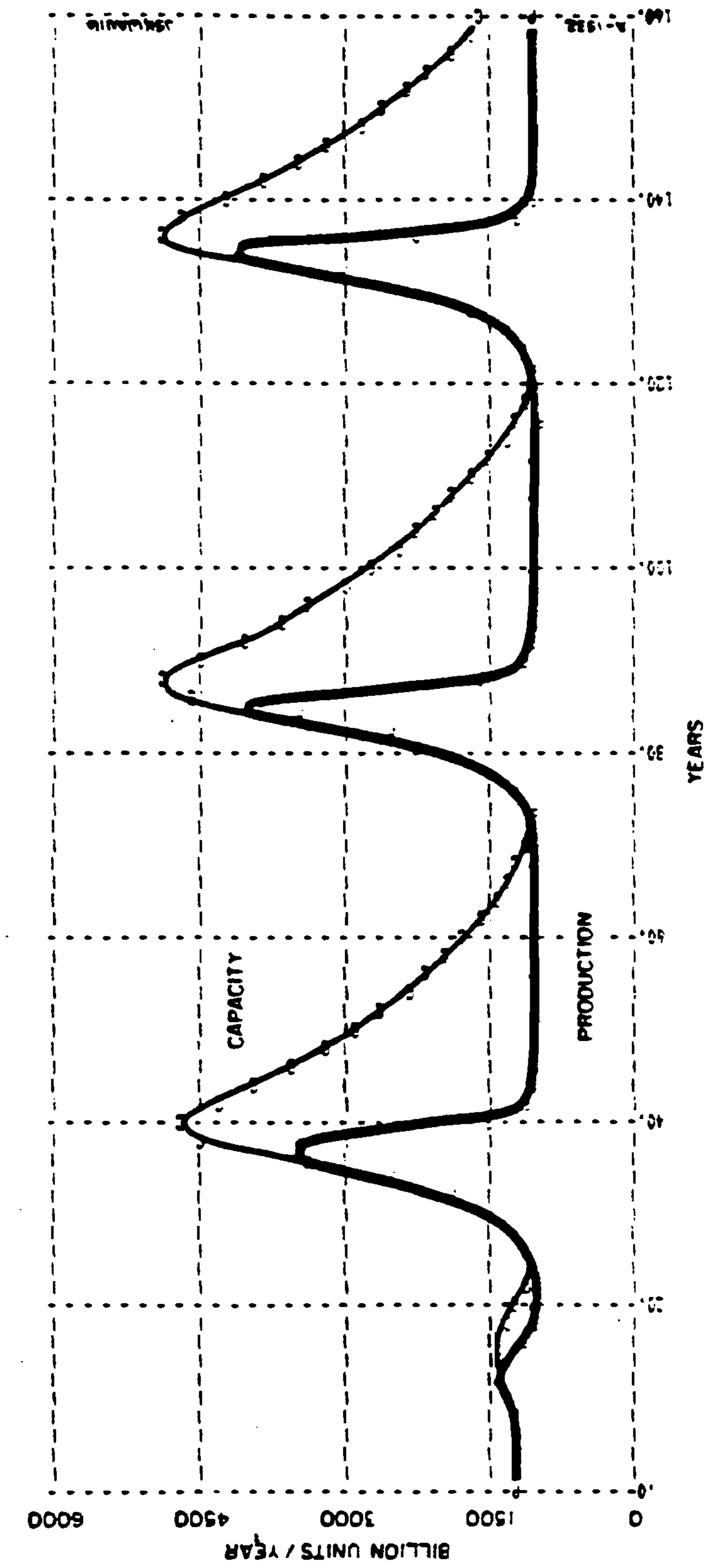

Figure 15. Long wave resulting from self-ordering

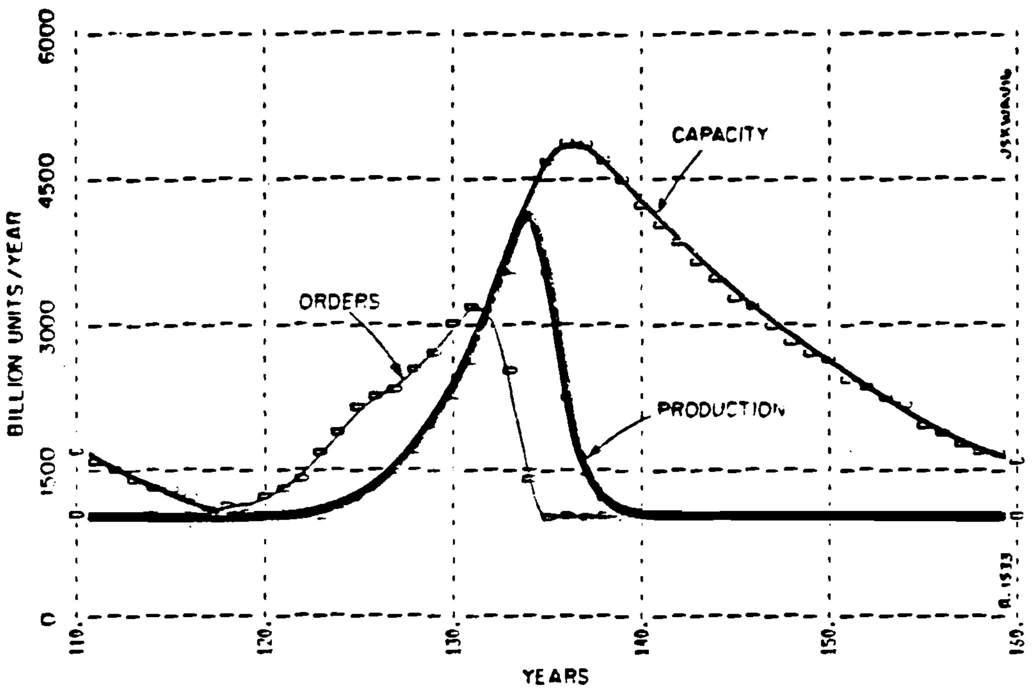

Figure 16a. Long wave: orders, production, and capacity

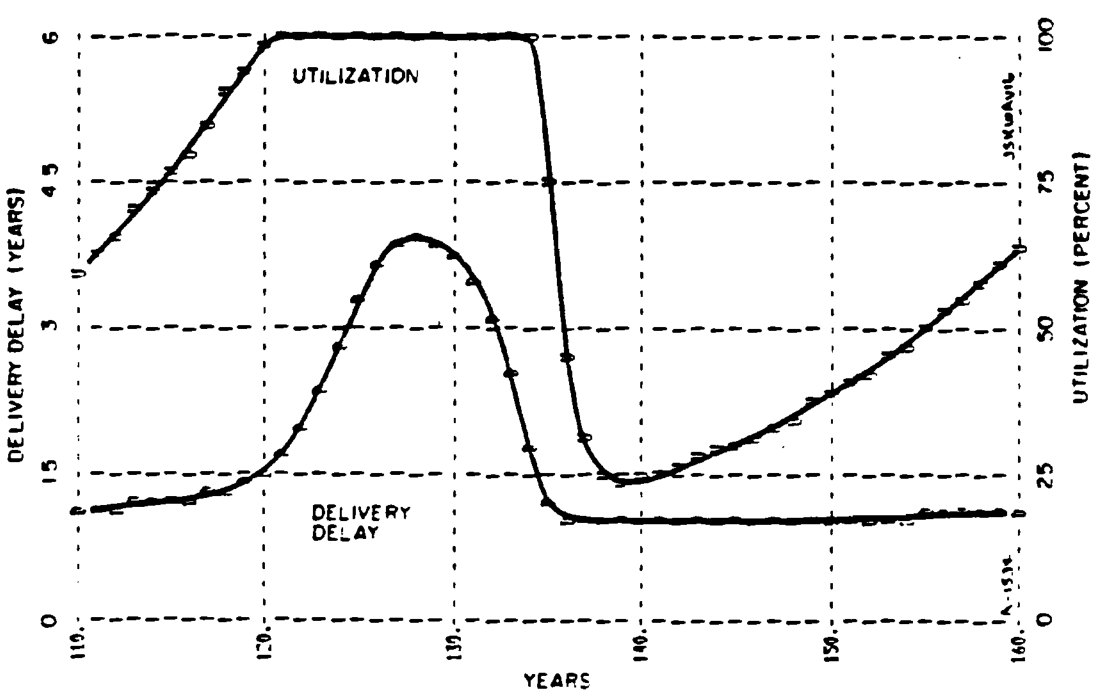

Figure 16b. Long wave: delivery delay and capacity utilization

value. The sector's output now rapidly begins to catch up to orders, though orders, through self-ordering, continue to rise even though they are now well above the equilibrium level. By the 132nd year, output overtakes orders and backlog reaches its peak. Delivery delay is now falling, reducing orders by accelerating acquisitions and reducing the required supply line. But though orders are now falling, backlog and delivery delay remain well above normal, forcing capacity to expand further. By the 134th year, delivery delay and backlog have return to normal, but capacity is much higher than its equilibrium level.

With output at record levels and orders plummeting, backlog and delivery delay reach and then drop below their normal values as firms attempt to maintain full utilization. The backlog is rapidly depleted, however, and utilization is forced down. Output drops precipitously, and the sector enters a period of depression with capacity far in excess of demand. Note that capacity continues to rise even after output has fallen. Though the sector's orders for capital peak in year 131 and then fall precipitously, capital already ordered continues to arrive, worsening overcapacity.[24] And since the lead time for capital drops below normal, capital on order is delivered faster than expected, expanding capacity beyond anticipated levels.

With its backlog depleted and capacity utilization at 25%, the sector has, by year 139, cut gross investment to zero. Output and gross investment remain depressed for the next two decades as capacity slowly depreciates, until capacity once again equals orders and the cycle begins again.

COMMENTS ON THE REALISM OF THE BEHAVIOR

The long cycle generated by the model with self-ordering closely resembles, in qualitative terms, the long wave generated by the National Model. But the magnitude of the fluctuation is extreme: delivery delay expands to over 250% of normal; capacity utilization falls to a minimum of under 25%; total gross investment falls by over 75% from the peak with investment in the capital sector collapsing to zero. In comparison, between 1929 and 1933, US real private investment fell 88%, real GNP fell by 30%, and unemployment reached 25%.

The extreme simplicity of the model is the cause of the extreme behavior. Since only the most basic channels for self-ordering are represented, the full burden of the disequilibrium pressures generated during the cycle must be borne by a few variables. One would expect that as additional structure and realism are added to the model, the burden borne by any individual channel would fall while the total amplification, to a first approximation, stayed the same. For example, the model excludes relative prices. In reality, as demand outstrips capacity, the price of capital would rise, easing some of the pressure on delivery delay by damping demand growth. At the same time, higher capital prices would encourage expansion and reinforce self-ordering through the mechanisms outlined in Figure 6. Given the extreme simplicity of the model, extensive comparison of the magnitudes of the variables to historical experience is not warranted.

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[24] A recent example is provided by the commercial construction industry (*Business Week*, 4 October 1982, pp.94-98).

THE ROLE OF SELF-ORDERING

The strength of self-ordering in equilibrium is governed primarily by the capital/output ratio. As calculated in equation (4), the equilibrium multiplier effect created by self-ordering is given by $1/(1-COR/ALC)$. Thus reducing the capital/output ratio should reduce both the amplitude and period of the cycle by reducing the magnitude of the capacity overshoot and hence the time required for capacity to depreciate. Table 2 shows period, amplitude, and damping as a function of *COR* given the other parameters of the model. When $COR \approx 0$, self-ordering is eliminated, and the behavior approaches that of the sector in isolation with a period of twenty years and a damping ratio of 93%. As *COR* rises, damping falls dramatically while the period remains relatively constant. At $COR \approx 1.6$, damping is eliminated and the oscillation reaches a fixed steady-state amplitude. Further increases in *COR* rapidly lengthen the period and boost the amplitude. The results verify the crucial role of self-ordering in lengthening the natural period of the accelerator mechanism portrayed in the production sector.

The other determinant of the strength of self-ordering is the average life of capital *ALC*. Altering *ALC* has two opposing effects. On the one hand, *ALC* controls the time required for excess capacity to depreciate during the depression phase, so shortening *ALC* should reduce the period. But shortening *ALC* also increases the strength of self-ordering, suggesting a larger amplitude. Table 3 shows that the amplitude is increased substantially as *ALC* falls. The period, however, is quite insensitive to *ALC*, and in fact tends to shrink as *ALC* gets shorter *or* longer. Though a shorter *ALC* implies faster decay of excess capacity, more rapid depreciation makes it more difficult for the capital sector to catch up to orders during the expansion phase. Output overtakes demand at a later and higher level, so even though excess capacity is eliminated more rapidly, more excess capacity is generated, resulting in a reduction in period of only six years when *ALC* is reduced from twenty to ten years. Similarly, a longer *ALC* extends the time required to eliminate excess capacity but reduces the strength of self-ordering so that output overtakes orders at a much lower level. The results, particularly the decrease in period with longer *ALC*, show the period of the cycle to be determined primarily by the strength of the self-ordering loop and not the life of capital. The insensitivity to *ALC* shows the cycle is not created by the echo effect that figures in some explanations of the long wave.[25]

Self-ordering also operates through other channels. During the upswing of the cycle, rising delivery delays slow capital acquisition, further augmenting the backlog and lengthening lead times. To test the importance of this channel, it was assumed that the capital sector has absolute priority over the goods sector when demand for capital exceeds capacity, and is always able to receive capital within the normal delivery delay:

$$CA_t = SL_t / NDD \qquad (7')$$

$$GCA_t = PR_t - CA_t \qquad (26')$$

The result is a 37-year cycle with an amplitude 70% as large as the base case. The qualitative features are largely unchanged. With priority over other sectors, the

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[25] Both Kondratiev and De Wolff invoked the echo effect to explain the period of the long wave (Van Duijn 1983, pp.62, 67).

Table 2. Sensitivity to Capital/Output Ratio

COR (years)	Period (years)	"Damping Ratio"[a]	Steady-State Amplitude[b] (% of Base)
0	20	.93	0
0.1	20	.88	0
0.5	20	.79	0
1.0	20	.59	0
1.6	20	$\simeq 0$	1
2.0	23	NA	20
2.5	34	NA	40
3.0 (base case)	49	NA	100
3.5	55	NA	140
4.0	60	NA	150

[a] "Damping Ratio" = 1 - Peak of cycle n/Peak of cycle n-1 (measured with respect to equilibrium values)

[b] Measured in Production Rate

Table 3. Sensitivity to Average Life of Capital

ALC (years)	Period (years)	Steady-state Amplitude[a] (% of Base)
10	43	170
15	45	120
20 (Base Case)	49	100
30	49	50
40	35	20

[a] measured in production rate PR

Table 4. Sensitivity to Aggressiveness of Backlog Adjustment

TAB (years)	Period (years)	Steady-State Amplitude[a] (% of Base)
0.5	55	130
1.0	53	120
1.5 (Base Case)	49	100
2.0	39	60
2.5	30	30

[a] measured in production rate PR

capital sector can catch up to orders sooner and at a lower level. However, it is unlikely that such allocation exists. All firms, to some extent, are involved in purchasing from each other. Capital producers do not know the extent to which their customers are coupled through self-ordering and certainly do not consult an input/output table to assign priorities on the basis of the technical coefficients of their customers. The assumption of equal priorities is probably roughly correct in the aggregate, at last for an approximately competitive economy.[26]

Self-ordering also operates through the backlog correction (Figure 2). The aggressiveness with which firms seek to maintain delivery delays at normal levels is controlled by the time to adjust backlog *TAB*. As shown in Table 4, the period and amplitude are inversely related to *TAB*. While the amplitude is quite sensitive to *TAB*, the period is relatively less sensitive.

Likewise, more aggressive adjustment of capital to desired levels (Table 5) lengthens the period and increases the amplitude by boosting orders even further above base-case levels for a given discrepancy between desired and actual capital. Again the variation in the period is less than the variation in the amplitude.

Speeding adjustment of the supply line, in contrast, is stabilizing (Table 6). Since the capital and supply line corrections oppose each other, more aggressive adjustment of the supply line relative to the capital stock effectively reduces the strength of self-ordering. Eliminating the supply line correction altogether means capital once ordered is forgotten until it arrives, destabilizing the system by causing overordering, as can be verified in simulations without self-ordering.

As described in Figure 3, the rising delivery delay boosts the desired supply line, adding still more to orders during the expansion phase. To test the importance of this channel, equation (14) was modified so that the desired supply line is always based on the normal delivery delay, effectively eliminating the hoarding phenomenon described by T.W. Mitchell:

$$PDDC_t = NDD$$

The result is a cycle with the same period and an amplitude (measured in output) 90% as large. The timing and character of the behavior are virtually unaffected. Therefore, the decision rule for the desired supply line, though contributing some amplification, does not appear to play a strong role in the long wave.

THE ROLE OF NONLINEARITY

The limit cycle behavior of the model implies one or more nonlinearities bound what would otherwise be an expanding oscillation. Two obvious nonlinearities are the limitation on orders as a fraction of capacity (eq.10), intended to capture bottlenecks and other constraints on the rate of expansion, and the diminishing returns to capital (eq.16). Eliminating both nonlinearities by setting

$$COF_t = ICOF_t \quad \text{for} \quad ICOF_t > 0.05 \tag{10'}$$

---

[26] The allocation issue raises the fascinating question of whether a centrally planned economy could minimize or eliminate the long wave through careful allocation of investment and output. Empirical evidence is inconclusive, and analysis is made difficult by entrainment of market and centrally planned economies through trade.

Table 5. Sensitivity to Aggressiveness of Capital Stock Adjustment

TAC (years)	Period (years)	Steady-State Amplitude[a] (% of Base)
1.5	56	150
2	54	120
3 (Base Case)	49	100
4	37	40
5	31	20

[a] measured in production rate

Table 6. Sensitivity to Aggressiveness of Supply-Line Correction

TASL (years)	Period (years)	Steady-State Amplitude[a] (% of Base)
1.5	34	40
2	42	70
3 (Base Case)	49	100
4	51	110
∞	57	140

[a] measured in production rate

$$DC_t = IC_t \tag{16'}$$

yields a period of 75 years and an amplitude nearly 3.5 times greater than the base case.[27] The test clearly shows constraints on either the level or rate of capital expansion to be important factors in bounding the period and amplitude of the cycle. However, without these nonlinearities the cycle still reaches a finite steady-state amplitude, suggesting another nonlinearity is primarily responsible for bounding the oscillation. That nonlinearity is the capacity utilization formulation (eq. 2).

In both Figures 14 and 16, output falls below capacity when the backlog drops below a level consistent with the normal delivery delay, restraining the overshoot of output and preventing the backlog from declining too far below its equilibrium value. If output were always equal to capacity, however, backlog would decline further below its equilibrium value, forcing larger cutbacks in investment and destabilizing the cycle. Setting

$$PR_t = PC_t \tag{1''}$$

without self-ordering leaves the period largely unaffected but reduces the damping ratio from 93% to 35%. When self-ordering is added, equation (1'') results in an expanding oscillation which soon drives backlog, delivery delay, and capital acquisitions below zero. As further confirmation of the importance of capacity utilization, the full model was simulated with

$$PR_t = IP_t \tag{1'''}$$

implying perfectly flexible capacity. The result is a highly damped response with a single overshoot of capacity above its equilibrium value.

Thus the crucial nonlinearity is the relationship between backlog and capacity. While constraints on the rate or level of capacity expansion may limit the period and amplitude of the cycle, fundamentally it is the fact that output can only rise as capacity grows that creates the disequilibrium, and the fact that output must fall as the backlog is depleted that limits it.[28]

---

[27] Eliminating only one of the nonlinearities simply allows the system to grow further until the remaining constraint becomes binding.

[28] Though the behavior of the model is dominated by nonlinearity, analysis of the eigenvalues of the linearized system verified the crucial role of capacity utilization in controlling damping. Linearizing around the initial equilibrium with eq.(1'') ($CU=1$)a yields dominant eigenvalues corresponding to expanding oscillation with a period of 25.5 years and a growth rate of the envelope of 29%/year. In contrast, linearization with eq. (1''') ($CU=IP/PC$) yielded eigenvalues corresponding to a highly damped oscillation with a period of 23.8 years and a decay rate of the envelope of 19%/year. Intuitively, the slope of CU determines the relative strengths of the oscillatory capital acquisition loop and the stable first-order production scheduling loop. During the expansion phase, utilization is at its maximum, and the unstable loop dominates. As excess capacity develops, CU falls, and dominance shifts to the stable loop, limiting the amplitude of the cycle.

CONCLUSIONS

The model presented here is not merely a set of equations which produce a long cycle. The decision rules portrayed in the model are consistent with the information-processing and decision making capabilities of economic agents. Further, the individual decision rules are locally or intendedly rational: they yield rapid, orderly, and appropriate adjustments to unanticipated shocks within the local environment of the organizational subunits responsible for the decision. Yet as the complexity of the environment grows, the overall rationality of the system's response is degraded. The results demonstrate what Simon (1947, p.81) calls

> "segments" of rationality...(the) behavior shows rational organization within each segment, but the segments themselves have no very strong interconnections.

The positive feedback loops created by self-ordering increase the amplitude and lengthen the period of oscillations created by the production and investment policies of the sector, policies which, from the vantage point of the firm, are quite rational. Indeed, an individual firm cannot distinguish orders that are part of the "true" long-run demand from the "false" orders generated by amplification and self-ordering. A firm or management team that attempted to turn away orders or expand less aggressively on the grounds that it would cause overexpansion in twenty years would not last long in the face of high delivery delays and rapid growth.

The results show that the dynamic hypothesis of self-ordering is sufficient to cause a long wave, given only the local rationality of the decision rules and the physical structure of capital accumulation. More precisely, the results show that self-ordering amplifies the disequilibrium pressures created by the interaction of locally rational decision rules and the lags in capital acquisition within a firm, verifying Forrester's statement (1977, p.534) that self-ordering "creates the 50-year cycle out of what would otherwise be a 20-year medium cycle in capital acquisition."

The model shows only the most fundamental feedback loops created by self-ordering, relationships which primarily involve the physical determination and allocation of output, are necessary to generate a robust long wave. But the sufficiency of the basic self-ordering channels does not mean other mechanisms are unimportant or irrelevant. Self-ordering also creates additional feedback channels through, for example, labor markets, growth expectations, prices, financial markets, and aggregate demand. These are portrayed in the full National Model. One would expect that adding these additional mechanisms would add to the net amplification created by self-ordering, strengthening the long wave while adding "fine structure" to the behavior and permitting realistic policy analysis.

The results should not be interpreted as excluding other mechanisms as amplifying or contributory factors in the long wave. However, those who would argue for the primacy of other mechanisms have yet to demonstrate the sufficiency of those mechanisms in a framework that permits reproducible testing. In particular, the model shows the long wave can arise with technology held completely constant (without even the technological changes implicit in varying the mix of capital and labor). The results suggest that the historical long-wave pattern in innovations is the result of entrainment by the physical process of self-ordering rather than vice-versa, as explained by Forrester (1977), and by Graham and Senge (1980). If the "long-wave theory of innovation" more nearly describes the situation than the "innovation theory of the long wave" favored by the neo-Schumpeterian school, policies directed at stimulating innovation may be insufficient to mitigate the effects of the current long-wave downturn.[29] These issues have important policy implications, and it is hoped that the methodological framework illustrated with the single model presented here can provide the common ground for systematic exploration of the forces behind the long wave, contributing toward an integrated theory of disequilibrium economic behavior.

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[29] Both Freeman *et al.* (1982) and Van Duijn (1983) argue for stimulus of innovation as prime components of an effective strategy to counter the long wave. While renewed commitment to R&D is needed, these results suggest dealing with excess physical capacity may be more important (Mass and Senge 1981; Sterman 1983).

REFERENCES

Coen, R. (1975) "Investment Behavior, The Measurement of Depreciation, and Tax Policy", *American Economic Review*, 65, 59-74.

Cyert, R., and J. March (1963) *A Behavioral Theory of the Firm* (Englewood Cliffs: Prentice-Hall).

Delbeke, J. (1981) "Recent Long-Wave Theories: A Critical Survey", *Futures*, 13, 246-57.

Dutton, J., and W. Starbuck (1971) *Computer Simulation of Human Behavior* (New York: Wiley).

Forrester, J. (1976) "Business Structure, Economic Cycles, and National Policy", *Futures*, 8, 195-214.

Forrester, J. (1977) "Growth Cycles", *De Economist*, 125(4), 525-43.

Forrester, J. (1979) "An Alternative Approach to Economic Policy: Macrobehavior from Microstructure" in *Economic Issues of the Eighties*, N. Kamrany and R. Day (eds.) (Baltimore: Johns Hopkins University Press).

Forrester, N. (1982) *A Dynamic Synthesis of Basic Macroeconomic Theory: Implications for Stabilization Policy Analysis*, Ph.D. Dissertation, MIT.

Freeman, C. (1979) "The Kondratiev Long Waves, Technical Change, and Unemployment", in *Structural Determinants of Employment and Unemployment*, Vol.2 (Paris: OECD), 181-96.

Freeman, C., J. Clark, and L. Soete (1982) *Unemployment and Technical Innovation: A Study of Long Waves and Economic Development* (Westport, Conn.: Greenwood Press).

Goffman, E. (1959) *The Presentation of Self in Everyday Life* (Garden City, NY: Doubleday).

Graham, A., and P. Senge (1980) "A Long-Wave Hypothesis of Innovation", *Technological Forecasting and Social Change*, 17, 283-311.

Hickman, B. (1963) "Postwar Growth in the United States in Light of the Long-Swing Hypothesis", *American Economic Review: Papers and Proceedings*, 53, 490-507.

Hogarth, R. (1980) *Judgement and Choice* (New York: Wiley).

Jorgenson, D. (1963) "Capital Theory and Investment Behavior", *American Economic Review*, 53, 247-59.

Jorgenson, D., J. Hunter, and M. Nadiri (1970) "A Comparison of Alternative Econometric Models of Quarterly Investment Behavior", *Econometrica*, 38, 187-212.

Kahneman, D., P. Slovic, and A. Tversky (1982) *Judgement Under Uncertainty: Heuristics and Biases* (Cambridge: Cambridge University Press).

Kleinknecht, A. (1981) "Observations on the Schumpeterian Swarming of Innovations", *Futures*, 13, 246-57.

Kondratiev, N. (1935) "The Long Waves in Economic Life", *Review of Economic Statistics*, 17, 105-15.

Kuznets, S. (1930) *Secular Movements in Production and Prices* (New York: Houghton Mifflin).

Low, G. (1980) "The Multiplier-Accelerator Model of Business Cycles Interpreted from a System Dynamics Perspective", in *Elements of the System Dynamics Method*, J. Randers (ed.) (Cambridge: MIT Press), 76-94.

Mandel, E. (1980) *Long Waves of Capitalist Development* (Cambridge: Cambridge

University Press).

Mandel, E. (1981) "Explaining Long Waves of Capitalist Development", *Futures*, 13, 332-38.

March, J. (1978) "Bounded Rationality, Ambiguity, and the Engineering of Choice", *Bell Journal of Economics*, 9, 587-608.

Mass, N. (1975) *Economic Cycles: An Analysis of Underlying Causes*, (Cambridge: MIT Press).

Mass, N. (1980) "Stock and Flow Variables and the Dynamics of Supply and Demand", in *Elements of the System Dynamics Method*, J. Randers (ed.) (Cambridge: MIT Press), 95-114.

Mass, N. and P. Senge (1981) "Reindustrialization: Aiming for the Right Targets", *Technology Review*, Aug./Sept., 56-65.

Mayer, T. (1960) "Plant and Equipment Lead Times", *Journal of Business*, 33, 127-32.

Mensch, G. (1979) *Stalemate in Technology*, (Cambridge: Ballinger).

Mensch, G., C. Coutinho, and K. Kaasch (1981) "Changing Capital Values and the Propensity to Innovate", *Futures*, 13, 276-92.

Merton, R. (1936) "The Unanticipated Consequences of Purposive Social Action", *Sociological Review*, 16, 894-904.

Metzler, L. (1941) "The Nature and Stability of Inventory Cycles", *Review of Economic Statistics*, 23, 113-29.

Mitchell, T. (1923) "Competitive Illusion as a Cause of Business Cycles", *Quarterly Journal of Economics*, 38, 631-52.

Mitchell, W. (1941) *Business Cycles and Their Causes* (Berkeley: University of California Press, Reprint Series edition, 1971).

Morecroft, J. (1983) "System Dynamics: Portraying Bounded Rationality", *Omega*, 11, 131-42.

Namenwirth, Z. (1973) "The Wheels of Time and the Interdependence of Value Change", *Journal of Interdisciplinary History*, III, 649-83.

Neff, T. (1982) "Short-Term Vulnerabilities: The World Oil Market in Transition", Working Paper MIT-EL82-001WP, Energy Laboratory, MIT, Cambridge, MA 02139.

Nelson, R. and S. Winter (1982) *An Evolutionary Theory of Economic Change* (Cambridge: Belknap Press of Harvard University Press).

Randers, J. (1980) "Guidelines for Model Conceptualization", in *Elements of the System Dynamics Method*, J. Randers (ed.) (Cambridge: MIT Press), 117-139.

Rostow, W. (1975) "Kondratiev, Schumpeter and Kuznets: Trend Periods Revisited" *Journal of Economic History*, 35, 719-53.

Rostow, W. (1978) *The World Economy: History and Prospect* (Austin: University of Texas Press).

Schumpeter, J. (1939) *Business Cycles* (New York: McGraw-Hill)

Senge, P. (1978) *The System Dynamics National Model Investment Function: A Comparison to the Neoclassical Investment Function*. Ph.D. Dissertation, MIT.

Senge, P. (1980) "A System Dynamics Approach to Investment Function Formulation and Testing", *Socio-Economic Planning Sciences*, 14, 269-80.

Senge, P. (1982) "The Economic Long Wave: A Survey of Evidence", Working Paper D-3262-1, System Dynamics Group, MIT, Cambridge, MA 02139.

Simon, H. (1947) *Administrative Behavior* (1st ed.; New York: MacMillan).

Simon, H. (1957) *Models of Man* (New York: Wiley).

Simon, H. (1978) "On How to Decide What to Do", *Bell Journal of Economics*, 9, 494-507.

Simon, H. (1979) "Rational Decision Making in Business Organizations", *American Economic Review*, 69, 493-513.

Sterman, J. (1980) "The Use of Aggregate Production Functions in Disequilibrium Models of Energy-Economy Interactions", Working Paper D-3234, System Dynamics Group, MIT, Cambridge, MA 02139.

Sterman, J. (1981) *The Energy Transition and the Economy: A System Dynamics Approach* (2 vols.), Ph.D. Dissertation, MIT.

Sterman, J. (1982) "Economic Vulnerability and the Energy Transition", *Energy Systems and Policy* (Forthcoming), Working Paper D-3356-1, System Dynamics Group, MIT, Cambridge, MA 02139.

Sterman, J. (1983) "The Long Wave" (letter), *Science*, 219 (18 March), 1276.

Tinbergen, J. (1981) "Kondratiev Cycles and So-Called Long Waves: The Early Research", *Futures*, 13, 258-63.

Tversky, A., and D. Kahneman (1974) "Judgement Under Uncertainty: Heuristics and Biases", *Science*, 185, 1124-31.

Van Duijn, J. (1977) "The Long Wave in Economic Life", *De Economist*, 125, 544-76.

Van Duijn, J. (1981) "Fluctuations in Innovation Over Time", *Futures*, 13, 264-75.

Van Duijn, J. (1983) *The Long Wave in Economic Life*. (London: Allen and Unwin).

Weber, R. (1980) "Society and Economy in the Western World System", *Social Forces*, 59(4), 1130-48.

APPENDIX

Equations for Simulation

```
00010 DOPT DEFFL,SDOCL,WUOPT=0
00020 * A SIMPLE MODEL OF THE ECONOMIC LONG WAVE
00030 NOTE
00040 NOTE  11 FEBRUARY 1983
00050 NOTE
00060 NOTE  JOHN D. STERMAN
00070 NOTE  ASSISTANT PROFESSOR
00080 NOTE  MASSACHUSETTS INSTITUTE OF TECHNOLOGY
00090 NOTE  ALFRED P. SLOAN SCHOOL OF MANAGEMENT
00100 NOTE  CAMBRIDGE MA 02139
00110 NOTE
00120 NOTE  CAPITAL SECTOR
00130 NOTE
00140 A     KPR.K=KSPR*KPC.K*KCU.K+(1-KSPR)*KIPC.K
00150 A     KCU.K=TABHL(KCUT,KIP.K/KPC.K,0,2.0,.2)
00160 T     KCUT=0/.3/.55/.75/.9/1/1/1/1/1/1
00170 A     KIP.K=KB.K/KNDD
00180 A     KPC.K=KC.K/KCOR
00190 L     KC.K=KC.J+(DT)(KCA.J-KCD.J)
00200 A     KCD.K=KC.K/KALC
00210 A     KCA.K=KSCA*(KSL.K/KDDC.K)+(1-KSCA)(KSL.K/KNDD)
00220 L     KSL.K=KSL.J+(DT)(KCO.J-KCA.J)
00230 A     KCO.K=KC.K*KCOF.K
00240 A     KCOF.K=TABXT(KCOFT,KICOF.K,-.1,.4,.05)
00250 T     KCOFT=0/0/.02/.05/.1/.15/.2/.25/.28/.3/.3
00260 A     KICOF.K=(KCD.K+KCC.K+KCSL.K)/KC.K
00270 A     KCSL.K=(KDSL.K-KSL.K)/KTASL
00280 A     KDSL.K=KPDDC.K*KCD.K
00290 A     KPDDC.K=KNDDC*KEDDSL.K
00300 A     KEDDSL.K=TABXT(KTPDDC,KDDC.K/KNDDC,0,3,.5)
00310 T     KTPDDC=0/.5/1/1.5/2/2.5/3
00320 A     KCC.K=(KDC.K-KC.K)/KTAC
00330 A     KDC.K=KRC*KRDRC.K
00340 A     KRDRC.K=TABXT(KTRDRC,KIC.K/KRC,-.5,7.5,.5)
00350 T     KTRDRC=0/0/.5/1/1.5/2/2.5/3/3.5/4/4.5/5/5.4/5.7/5.9/6/6
00360 A     KIC.K=KSDC*KIPC.K*KCOR+(1-KSDC)*KXDC.K
00370 A     KIPC.K=KEO.K+KCB.K
00380 A     KCB.K=(KB.K-KIB.K)/KTAB
00390 A     KIB.K=KNDD*KEO.K
00400 L     KEO.K=KEO.J+(DT/KTAO)(KOR.J-KEO.J)
```

```
00410 NOTE
00420 NOTE  PARAMETERS AND INITIAL VALUES
00430 NOTE
00440 C     KNDD=1.5
00450 C     KCOR=3
00460 C     KALC=20
00470 C     KTASL=3
00480 C     KTAC=3
00490 C     KTAB=1.5
00500 C     KTAO=2
00510 C     KSPR=1
00520 C     KSCA=1
00530 C     KSDC=1
00540 N     KNDDC=KNDD
00550 N     KDD=KNDD
00560 N     KRC=KC
00570 N     KPR=KPC
00580 N     KC=(1-KSSO)*GCO*KCOR+KSSO*GCO*KCOR*KALC/(KALC-KCOR)
00590 N     KSL=KDDC*KCD
00600 N     KEO=KPC
00610 NOTE
00620 NOTE  COUPLING EQUATIONS
00630 NOTE
00640 A     KOR.K=GCO.K+KSSO*KCO.K
00650 A     KB.K=GSL.K+KSSO*KSL.K
00660 A     KDDC.K=KSSO*KDD.K+(1-KSSO)*KNDD
00670 A     KDD.K=KB.K/KPR.K
00680 C     KSSO=1
00690 A     KXDC.K=KRC*(1+STEP(KFIDC,KTIDC))
00700 C     KFIDC=.05
00710 C     KTIDC=1
00720 NOTE
00730 NOTE  GOODS SECTOR
00740 NOTE
00750 L     GSL.K=GSL.J+(DT)(GCO.J-GCA.J)
00760 N     GSL=GDDC*GCO
00770 A     GCA.K=KSCA*(GSL.K/GDDC.K)+(1-KSCA)(KPR.K-KCA.K)
00780 A     GDDC.K=KDD.K
00790 A     GCO.K=GRCO*(1+STEP(GFICO,GTICO))
00800 C     GRCO=1E12
00810 C     GFICO=.05
00820 C     GTICO=1
00830 NOTE
00840 NOTE  SIMULATION CONTROL PARAMETERS
00850 NOTE
00860 SPEC  DT=.0625/LENGTH=0
00870 A     PLTPER.K=PLTP1+STEP(PLTP2-PLTP1,PLTIME)
00880 C     PLTP1=0
00890 C     PLTP2=2.5
00900 C     PLTIME=1000
00910 A     PRTPER.K=PRTP1+STEP(PRTP2-PRTP1,PRTIME)
00920 C     PRTP1=0
00930 C     PRTP2=0
00940 C     PRTIME=1000
```

LIST OF VARIABLES

SYMBOL	TYPE	DEFINITION
DT	S	SOLUTION INTERVAL (YEARS)
GCA	A	GOODS SECTOR, CAPITAL ACQUISITIONS (UNITS/YEAR)
GCO	A	GOODS SECTOR, CAPITAL ORDERS (UNITS/YEAR)
GDDC	A	GOODS SECTOR, DELIVERY DELAY FOR CAPITAL (YEARS)
GFICO	C	GOODS SECTOR, FRACTIONAL INCREASE IN CAPITAL ORDERS (FRACTION)
GRCO	C	GOODS SECTOR, REFERENCE CAPITAL ORDERS (UNITS/YEAR)
GSL	L N	GOODS SECTOR, SUPPLY LINE (UNITS)
GTICO	C	GOODS SECTOR, TIME TO INCREASE CAPITAL ORDERS (YEAR)
KALC	C	CAPITAL SECTOR, AVARAGE LIFE OF CAPITAL (YEARS)
KB	A	CAPITAL SECTOR, BACKLOG (UNITS)
KC	L N	CAPITAL SECTOR, CAPITAL STOCK (UNITS)
KCA	A	CAPITAL SECTOR, CAPITAL ACQUISITIONS (UNITS/YEAR)
KCB	A	CAPITAL SECTOR, CORRECTION FOR BACKLOG (UNITS/YEAR)
KCC	A	CAPITAL SECTOR, CORRECTION FOR CAPITAL (UNITS/YEAR)
KCD	A	CAPITAL SECTOR, CAPITAL DISCARDS (UNITS/YEAR)
KCO	A	CAPITAL SECTOR, CAPITAL ORDERS (UNITS/YEAR)
KCOF	A	CAPITAL SECTOR, CAPITAL ORDER FRACTION(FRACTION)
KCOFT	T	CAPITAL SECTOR, CAPITAL ORDER FRACTION TABLE
KCOR	C	CAPITAL SECTOR, CAPITAL OUTPUT RATIO (YEARS)
KCSL	A	CAPITAL SECTOR, CORRECTION FOR SUPPLY LINE (UNITS/YEAR)
KCU	A	CAPITAL SECTOR, CAPACITY UTILIZATION (FRACTION)
KCUT	T	CAPITAL SECTOR, CAPACITY UTILIZATION TABLE
KDC	A	CAPITAL SECTOR, DESIRED CAPITAL (UNITS)
KDD	N A	CAPITAL SECTOR, DELIVERY DELAY (YEARS)
KDDC	A	CAPITAL SECTOR, DELIVERY DELAY FOR CAPITAL (YEARS)
KDSL	A	CAPITAL SECTOR, DESIRED SUPPLY LINE (UNITS)
KEDDSL	A	CAPITAL SECTOR, EFFECT OF DELIVERY DELAY ON SUPPLY LINE (DIMENSIONLESS)
KEO	L N	CAPITAL SECTOR, EXPECTED ORDERS (UNITS/YEAR)
KFIDC	C	CAPITAL SECTOR, FRACTIONAL INCREASE IN DESIRED CAPITAL (FRACTION)
KIB	A	CAPITAL SECTOR, INDICATED BACKLOG (UNITS)
KIC	A	CAPITAL SECTOR, INDICATED CAPITAL (UNITS)
KICOF	A	CAPITAL SECTOR, INDICATED CAPITAL ORDER FRACTION (FRACTION)

KIP	A	CAPITAL SECTOR, INDICATED PRODUCTION (UNITS/YEAR)
KIPC	A	CAPITAL SECTOR, INDICATED PRODUCTION CAPACITY (UNITS/YEAR)
KNDD	C	CAPITAL SECTOR, NORMAL DELIVERY DELAY (YEARS)
KNDDC	N	CAPITAL SECTOR, NORMAL DELIVERY DELAY FOR CAPITAL (YEARS)
KOR	A	CAPITAL SECTOR, ORDER RATE (UNITS/YEAR)
KPC	A	CAPITAL SECTOR, PRODUCTION CAPACITY (UNITS/YEAR)
KPDDC	A	CAPITAL SECTOR, PERCEIVED DELIVERY DELAY FOR CAPITAL (YEARS)
KPR	A N	CAPITAL SECTOR, PRODUCTION RATE (UNITS/YEAR)
KRC	N	CAPITAL SECTOR, REFERENCE CAPITAL (UNITS)
KRDRC	A	CAPITAL SECTOR, RATIO OF DESIRED TO REFERENCE CAPITAL (DIMENSIONLESS)
KSCA	C	CAPITAL SECTOR, SWITCH FOR CAPITAL ACQUISITIONS (DIMENSIONLESS)
KSDC	C	CAPITAL SECTOR, SWITCH FOR DESIRED CAPITAL (DIMENSIONLESS)
KSL	L N	CAPITAL SECTOR, SUPPLY LINE (UNITS)
KSPR	C	CAPITAL SECTOR, SWITCH FOR PRODUCTION (DIMENSIONLESS)
KSSO	C	CAPITAL SECTOR, SWITCH FOR SELF ORDERING (DIMENSIONLESS)
KTAB	C	CAPITAL SECTOR, TIME TO ADJUST BACKLOG (YEARS)
KTAC	C	CAPITAL SECTOR, TIME TO ADJUST CAPITAL (YEARS)
KTAO	C	CAPITAL SECTOR, TIME TO AVERAGE ORDERS (YEARS)
KTASL	C	CAPITAL SECTOR, TIME TO ADJUST SUPPLY LINE (YEARS)
KTIDC	C	CAPITAL SECTOR, TIME TO INCREASE DESIRED CAPITAL (YEAR)
KTPDDC	T	CAPITAL SECTOR, TABLE FOR PERCEIVED DELIVERY DELAY FOR CAPITAL
KTRDRC	T	CAPITAL SECTOR, TABLE FOR RATIO OF DESIRED TO REFERENCE CAPITAL
KXDC	A	CAPITAL SECTOR, EXOGENOUS DESIRED CAPITAL (UNITS)
LENGTH	S	SIMULATION LENGTH (YEARS)
PLTIME	C	PLOT START TIME (YEAR)
PLTPER	A	PLOT PERIOD (YEARS)
PLTP1	C	PLOT PERIOD 1 (YEARS)
PLTP2	C	PLOT PERIOD 2 (YEARS)
PRTIME	C	PRINT START TIME (YEAR)
PRTP	A	PRINT PERIOD (YEARS)
PRTP1	C	PRINT PERIOD 1 (YEARS)
PRTP2	C	PRINT PERIOD 2 (YEARS)
STEP		STEP FUNCTION
TABHL		FUNCTION FOR NONLINEAR RELATIONSHIP
TABXT		FUNCTION FOR NONLINEAR RELATIONSHIP