

ECONOMIC STRUCTURAL CHANGES ANALYSIS
BY MEANS OF MATHEMATICAL FLOW MODELS

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Summary

Most of the formal methods using mathematical modelling to analyze socioeconomic phenomena are based on the assumption that the models describe these phenomena with sufficient accuracy and completeness. However, in many cases it is not possible to build mathematical models with the required properties and the user must spend a lot of effort verifying the practical applicability of the solutions obtained by standard schemes. This report describes an approach whereby it is possible to use incomplete mathematical models to produce logically correct results. But this is achieved at the expense of the insolubility of standard statements of the problems and the development of special software.

PART ONE : PROBLEM USER' MANUAL

Introduction

The chief measure of the quality of a mathematical model is its degree of correspondence to the modelled object, i.e. how accurately the model reflects all the features which essentially determine the behavior of the object. There are two reasons why a model may not be considered acceptable by the users. Firstly, the mathematical description may have been made in the absence of adequate information. Secondly, it might not be possible to formalize all of the essential features of the object by mathematical means, or these features may not be known at all. Therefore we may call a mathematical model containing a formal description (with an acceptable level of accuracy) *of not all* the essential features of the object under consideration an *incomplete mathematical model*.

It is clear that any developer of mathematical models wants to make them as complete as possible. And most of the mathematical tools developed to analyze these models are based on the assumption that they are complete.

Nevertheless, in practice this assumption of completeness is often invalid, which means that such models cannot be used *to generate a forecast or to find an optimal solution*. The user of an incomplete mathematical model should try to improve it by increasing the level of completeness; otherwise, he/she should restate the problem to be solved to avoid contradictions which arise from the incompleteness of the model.

This paper is concerned with the correct use of incomplete mathematical models.

Statement of Problems for Incomplete Mathematical Models

We define an incomplete mathematical model as a set of formalized descriptions which have been made with an acceptable level of accuracy, but which do not reflect all essential features (such as links, constraints, etc.) influencing the behavior of the modeled object.*

To obtain results of practical value it is necessary to take into consideration both formalized and nonformalized features of the object. The formalized features may be presented in the form of an incomplete mathematical model, but for the latter we must engage the *model* user in the process of decisionmaking. The main aim of this approach is to *combine the ability of the user to extract acceptable states of the model from the set of feasible solutions with the computer's ability to generate this set for a given incomplete model*.

* An incomplete model may be augmented by including new variables, constraints and so on, but not by changing the existing ones, otherwise it should be considered a different incomplete model.

Two definitions should be given here. A state of the model is *feasible* if it satisfies all formalized constraints included in the description of the incomplete model; and a state of the model is *acceptable* if the user has no objection to this state. It is obvious that the set of feasible states of the model includes the set of acceptable solutions, but not vice versa. As there is no formalized way to extract acceptable solutions from the set of feasible ones, the decisionmaker cannot use the computer to verify sufficient conditions of acceptability. He/she can only check (by means of formal tools) whether the necessary conditions of acceptability are valid, i.e. whether feasible solutions exist or not. This is why no optimization or forecasting problems can be solved using incomplete mathematical models. These models may help us to find out 'what will not happen', but not 'what will happen'.

The following scheme is suggested for seeking acceptable solutions, combining the abilities of human decisionmaking and formal computer analysis. As a first step the computer generates the set of feasible solutions for a given incomplete model, or determines that such solutions do not exist. Because it is practically impossible for the user to manipulate a whole set of solutions, the decisionmaker analyzes only one of them. If the solution is not acceptable, the user introduces additional constraints into the incomplete model, trying to eliminate unacceptable features of the solution. The computer corrects the feasible set of solutions in accordance with these new constraints and generates a new solution, the acceptability of which is to be tested by the user. The process is repeated until an acceptable solution is found.

This scheme is not concrete enough for one to make conclusions about its convergency from a purely formal viewpoint. In practice a decisionmaker will usually find a solution. The existence of the solution (or set of solutions) depends on the problem, but is not a property of the described scheme.

In spite of the theoretical simplicity of the approach, its practical use has been found to be difficult. In the next sections we will discuss in detail the problems that arise in the case of finite-dimensional mathematical models, describe the software for linear flow models, and give an example of the practical application of the approach.

The Case of Finite-Dimensional Models

Let a state of the mathematical model considered be described by an n -dimensional vector x , the components of which are x_1, x_2, \dots, x_n . We will assume that the relations

$$y_s(x) \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} 0, \quad s = [1, m], \quad (1)$$

are expressions of the only essential features of the modelled object which can be formalized at an acceptable level of accuracy. We will also assume that all $y_s(x)$ are convex functions of components of x defined for a nonempty domain $\Omega \subset E^n$.

Suppose now that the set of all x satisfying the system (1) is not empty, i.e. that there exists at least one x^* which is a feasible state of the model. The decisionmaker verifies whether x^* is an acceptable solution as well. If it is found to be acceptable, the procedure is finished. Otherwise, the user can insert additional constraints

$$g_t(x) \begin{bmatrix} \geq \\ = \\ \leq \end{bmatrix} 0, \quad t = [1, l]. \quad (2)$$

where functions $g_t(x)$ have the same properties as the functions $y_s(x)$.

The main purpose of these new constraints is to convert the feasible solution x^* to an acceptable solution. The difference between functions $g_t(x)$ and $y_s(x)$ is that the first ones may be unknown to the user before analysis of the feasible solution x^* , whereas $y_s(x)$ are known *a priori*. Together systems (1) and (2) are the *conditions of feasibility*.

This correcting procedure may be repeated several times until an acceptable solution is found. At each step new constraints are included in the system (1)-(2) which, generally speaking, make the domain Ω more narrow.

A difficulty which may arise at some step of the procedure is the infeasibility of the system (1)-(2). It is suggested that the following special procedure is used to avoid this situation. Let the set of constraints

$$g_{t^r}(x) \begin{cases} \geq \\ = \\ \leq \end{cases} 0, \quad t^r = [1, l^r] \quad (3)$$

cause the state of infeasibility. This means that the system (1)-(2)-(3) has an internal contradiction and all the conditions cannot be satisfied simultaneously. In this case it is possible to remove conditions (3) from the set of necessary conditions of the model and to start considering them only as 'desirable' conditions. But, on the other hand, this 'desirability' means that these constraints should be satisfied as exactly as possible. We can use the lack of uniqueness of the solution of the system (1)-(2) by choosing that solution which satisfies the new constraints (3) in the best way.

We may, for example, introduce a metric

$$\rho(x) = \max_{t^r \in [1, l^r]} \text{abs} \frac{g_{t^r} - g_{t^r}^*}{N_{t^r}^*}. \quad (4)$$

where $g_{i,r}^*$ are *reference values** for the 'desirable' constraints and $N_{i,r}^*$ are suitable normalizations.

The metric (4) has a disadvantage, namely that the minimization of $\rho(\mathbf{x})$ may not uniquely define all components of \mathbf{x} . To avoid this we may repeat the minimization several times, fixing all the components of \mathbf{x} which were defined uniquely during the previous steps. Technical details of this procedure, called *sequential fixation*, as well as choosing the reference values and normalization, will be discussed in the next section.

The last problem to be mentioned here is the possible infeasibility of the original system (1). If this is the case, parametric analysis is recommended to reconstruct the initial description of the incomplete model. A number of suitable algorithms and methods are known. One of them, called the *compact modelling approach*, was successfully tested in practice [Umnov, 1984].

Linear Flow Models

The ideas described in the previous sections are too general for a conclusion to be made about their practical effectiveness. Therefore it seems reasonable to move to a more concrete case: that of standard linear flow models.

Let us consider a mathematical model consisting of a network consisting of N nodes which may be linked by means of K component flows. Each of the nodes

*We use the term 'reference value' following Wierzbicki et al. [1984], because of the technical similarity, but the described approach is opposite to optimization in general (and to multiobjective optimization in particular) owing to the main assumption about the incompleteness of the considered mathematical model. The reference values are formal parameters of the procedure and have no practical interpretation.

may be a source, a sink, or both. Generally speaking, the graph of the network may not be connected.

Let the value of the flow from the i th node to the j th of the k th type be x_{ij}^k . A state of the model is described by the set of variables $\{x_{ij}^k, i, j = [1, N], k = [1, K]\}$.^{*} For the convenience of the decisionmaker additional variables are introduced which make it possible to operate with the sums of the original variables over different groups of indices. For example, the additional variable S_{i+}^k is defined as

$$S_{i+}^k = \sum_{j=1}^N p_{ij}^k x_{ij}^k,$$

where p_{ij}^k are coefficients permitting summation of the different kinds of flows in common units. Variables $S_{+j}^k, S_{ij}^+, S_{++}^k, S_{i+}^+, S_{+j}^+, S_{++}^+$ are defined in an analogous way.

The conditions of feasibility (1) are described in terms of a system of constraints, each of which is an equality or inequality imposed on both absolute and relative values of the variables. The decisionmaker may use the constraints

$$\begin{aligned} \underline{a}_{ij}^k &\leq x_{ij}^k \leq \bar{a}_{ij}^k \\ A_j^k &\geq S_{i+}^k \\ x_{ij}^k &= b_{ij}^k S_{+j}^k. \end{aligned} \tag{5}$$

and the like. The values of the parameters $\underline{a}_{ij}^k, \bar{a}_{ij}^k, A_j^k, b_{ij}^k, \dots$ are to be defined by the user.

To simplify the procedure of decisionmaking, a special subset of the 'soft' constraints (3) was used for the linear flow model. These constraints are to be

^{*} Here we give a short description of the 'fma. 12'-software system developed by the Regional Issues Group of IIASA in 1983, [Lenko, 1985].

equalities defining values of the primary variables x_{ij}^k . This means that the metric (4) should have the following form :

$$\rho(\mathbf{x}) = \max_{i,j,k} w_{ij}^k \text{ abs} \frac{x_{ij}^k - x_{ij}^{k*}}{x_{ij}^{k*}}, \quad (6)$$

where nonnegative numbers w_{ij}^k are weight coefficients and x_{ij}^{k*} are the components of the reference point expressed in terms of primary variables.

The procedure of sequential fixation is essential here because the metric (6) may not define uniquely all components of the vector $\tilde{\mathbf{x}}$, which is the minimum point for the function (6). For each step of the procedure all the components of \mathbf{x} which have nonzero dual values are fixed. The procedure is finished when all the components have been fixed or the minimum of (6) becomes zero. The obtained sequence of optimal values $\rho \{ \rho_1, \rho_2, \dots, \rho_P \}$ may be very useful for the decisionmaker because they rank the set of components of vector \mathbf{x} , measuring the minimal relative change necessary to transfer the reference point \mathbf{x}^* to a feasible solution.

The importance of sequential fixation is also demonstrated by the fact that in the case of a complex system (1) the maximum element from the set $\{ \rho_t, t = [1, P] \}$ may not give the correct description of model properties. For example, Table 1 and the corresponding Figure 1 present the dependences of the maximal ρ and an average $\bar{\rho}$ on the value of a parameter of the model described in Umnov [1984]. The average $\bar{\rho}$ was calculated using

$$\bar{\rho} = \sum_{t=1}^P \alpha_t \rho_t,$$

where α_t is the ratio of the sum of the flows fixed on the t th step of the sequential fixation to the sum of all the flows, and P is the number of steps of the procedure.

Finally, it should be noted that the weight coefficients w_{ij}^k may be used for the following purposes. Firstly, the decisionmaker can give zero weight to those flows which do not exist or are zero at the reference point. Sometimes this trick permits one to avoid an infeasibility *a priori*. Secondly, using very large weights, it is possible to find maximum or minimum values of the corresponding components of vector x . The decisionmaker should be careful to have maximum or minimum values for these components only at the reference point. If the decisionmaker introduces simultaneously a set of criteria and their trends are contradictory, then, as can be easily checked, a semi-effective equilibrium on the Pareto set is achieved.

NECESSARY STRUCTURAL CHANGES FOR 1990 (in %)

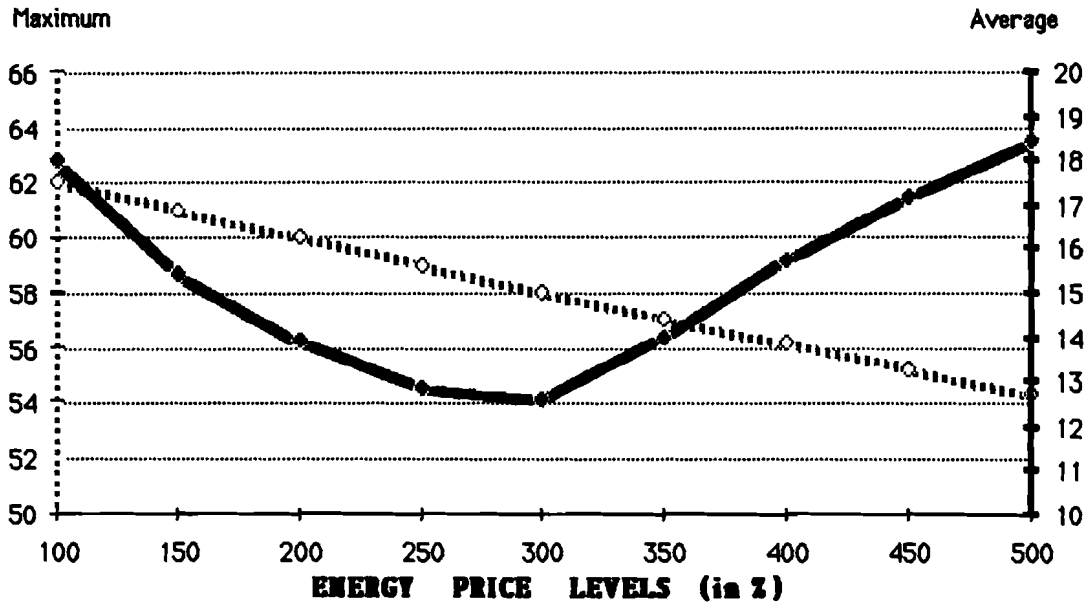


Figure 1.

Price level for energy products (in % of 1970)	Necessary structural change for balancing the state of the world trade market in 1990 (in %)	
	Maximal ρ	Average ρ
100	62.00	18.01
150	60.99	15.43
200	59.98	13.90
250	59.00	12.86
300	58.03	12.58
350	57.08	13.97
400	56.14	15.72
450	55.21	17.18
500	54.30	18.44

Table 1.

PART TWO : SYSTEM USER' MANUAL

GENERAL INFORMATION

Mathematical description of this problem can be found in the first part of the document. This part contains the formal description of the relevant software.

Source programs are in: /uc/lenko/FMA

Working version: FMA12 - with shortened MINOS, automatic fixation

MINOS subroutines are in /uc/lenko/short in compact form (shortened version) and in /tmp/lenko/short - object files after compilation

List of source files:

```
tma12.f    bdata12.f  routine12.f  vstup12.f  gener12.f
vypoc12.f  vystup12.f  min12.f     restr12.f  equat12.f
podprog12.f  m1w.f     m2w.f       m3w.f     m4w.f
ogrbas12.f
```

Link file is: link.fma

Executable task will be in: /tmp/lenko

If the task has once been solved, the user has a possibility to choose some other output tables according to specifications in the file for output description (des.out) without resolving the problem again. For this purpose use a program OMA12. All input and output files have the same format. To get this program use file link.oma to link all necessary subroutines together. Use file oma12.f instead of vypoc12.f from the list of source files and do not use files m1w.f m2w.f

m3w.f m4w.f min12.f ogrbas12.f and gener12.f.

FILES

All input information must be prepared on input files. Program reads this information, check data to some extent and prepares input file (mpsfile) for standard optimization process (where MINOS is used). After optimization output tables are prepared according to user's definitions. Process of optimization can be done as a simple process, or as a so called "automatic fixation process", where all the flows which are on the boarder are fixed and slightly modified process is solved until all flows are fixed, or the objective function is less then a given limit.

INPUT FILES:

spec file:

```
name of task      : FLOW MODEL ANALYSIS
inp.f. data base  : database
inp.f. nodes      : nodes
inp.f. flow.types : flow.types
inp.f. flow.equiv : flow.equiv
inp.f. restrictions : restr
inp.f. structure  : struct
inp.f. descr.output : des.out
out.f. data base  : newbase
out.f. graph      : graph
tollerance epstol : 1.00000e-4
tollerance epsil1 : 1.0000e-11
old mps file ?    : no
```

Input specification file has always name 'spec'. Here the user writes names of input and output files and other parameters of task (name of task etc). Filenames on lines 2,3,4,5 and 9 are necessary, all other are optional. If the user do not want to have some input or output files (concerning the lines 6,7,8,10) he writes the keyword 'no' instead of name of file. Filename can be 12 characters long. Name of task can be 40 characters long. All these lines are read with format: (22x,10a4) except the lines with tollerances, where the format is : (22x,e12.5). Here the tollerance epstol is used at preparing output graph table as a minimum relative distance to optimal value of each flow and also as a limit for objective function at automatic fixation. This parameter serves also as a criterium for the end of automatic fixation process. When the objective function is less then this parameter, the process is finished. Next parameter epsil1 serves as a criterium which flows are to be fixed. For the usage of old or new mpsfile keyword 'yes' or 'no' should be used. The keyword

'yes' can be used in the case when the user wants to restart the process and the mpsfile is ready.

nodes

There is an identifier (2 characters) and a name (20 chars) for each node here. The format is (a2,1x,5a4). The sequence of nodes defines the sequence of data in all output tables, indexes of flows etc.

flow.types

Each flow.type has its own name (20 characters), which is read with format (5a4). The sequence of flow.types defines their index.

database

Here the user gives the values of all flows in the system. Flows are separated in blocks where one block means all flows from one node to another. Each block has header, body and tail. The header has form: block OUIIN with format(6x,2a2) where OU is identifier of outgoing node and IN is identifier of incoming node. Body has a form : flow.type index, value of flow with format (2x,i3,g22.14). The flow should be greater than zero. The zero flow means that the flow does not exist now but should occur later. This flow must be declared in restriction file as 'free' (it does not take part in computing the objective function) The tail has form: 0 0. with format (i5,e12.5) It is not allowed to have 2 blocks with the same header in the database.

equivalent coefficients file

The user gives the values for equivalent coefficients in this file, which means the values with which you multiply the value of flow and so you get the flow in comparative equivalents. (You can make a sum of flows with different flow.types only in their equivalents.) This file can be one of three different

types, which is written as a number in first row of the file. The rest of the file has the same structure as database file. If type=1 then equivalent depends on flow types only. The content of file is one block with blanks as names for header block . Then for each flow type you can give the value of coefficient. Default value is 1. If type=2 then the equivalent depends on the outputting node and on the flow type, so the file can have maximal so many blocks how many nodes you have, one block for each node. In header you must give the identifier of the outputting node only. If type=3 the equivalent coefficients are for each flow. In header you must specify outputting and inputting node and in body you give values for corresponding flow types. You can give the values different from 1. which is the default value.

restr

The user can give restrictions to simple flows, to the sum of flows or to the difference between outcoming and incoming flow or sum of flows, which will be later marked as imbalance. Format of file is: (a2,1x,3a2,1x,a1,1x,e12.5). Sequence of items on each line is:

VAR,IND1,IND2,IND3,OP,VALUE where items VAR, IND1, IND2, IND3 form so called variable

VAR denotes type of restricted variable. This can be keyword:

VA - where variable is as value

EQ - where variable is as in a form of equivalent

IM - where variable is of type imbalance

IND1 denotes identifier of outputing node

IND2 denotes identifier of inputing node

IND3 denotes index of flow type

For all this three parameters we can use also keywords ++ or **

with the following meaning:

++ means the sum of all flows

** means that this item is substituted for all possible identifiers of nodes resp. indexes of flow types (in the place of IND3).

It means, that there are restrictions to the whole set of variables in one row.

OP can be:

> for bigger then

< for less then

= for equal to

W for weight for single flow

VALUE is restricted value.

In the case of weight it means the weight factor. If this parameter is missing the default value is 0. which means that variable is free (has no weight). In other cases if the value is missing it is substitutes by the value taken from source data. It means e.g. if the variable is simple flow, it will be value of the flow taken from the database. If the variable is sum of flows, the sum of corresponding

flows taken from the database will be computed and this will be the default value.

Some examples:

VA EUSU02 > 10.3

VA EUSU02 < 11.2 - simple flow will be in the range $10.3 < \text{flow} < 11.2$

EQ EUSU02 = 14. - simple flow will have the equivalent value of 14.

IM SU++++ > 140. - imbalance of total output from node SU to all nodes
and all flow types will be greater than 140.

EQ EU++02 = 4. - total output from EU to all nodes for flow type nro.2 = 4.

EQ ++US++ < 99. - full input to US will be < 99.

EQ SUUS** > - flows from SU to US of all flow types must be greater than
their given values

EQ ***** < - all flows in the system must get lower value than they have

EQ ++++++ > 1999. - total sum of all flow types in the market should be > 1999.

The user is responsible to give consistent restrictions.

struct

The user can define another constraints for the variables also. These constraints can be of following type:

$$\text{var1 } \$ \text{ coef1 } * \text{ var2 } + \text{ coef2}$$

where:

var1 or var2 - are variables used in system. All possible variables according to specification for restr file can be used, e.g. simple flows, total output, input imbalance, sum of outputing flow types etc.

coef1 or coef2 - are values for coefficients, coef1 should not be zero

\$ - is type of constraint, it can be < , > or =

Format of file is: (a2,1x,3a2,1x,a1,1x,e12.5,a2,1x,3a2,e12.5)

var1, var2 and coef1 cannot be missing. If coef2 is missing (blank)

its default value is 0.

des.out

This is a file with description of output tables. File consists of 3 parts, first part is for output table 1, second for output table 2 and third for output table 3. In first row of each part the user gives the name of output file. If there is a keyword 'no' here it means that no output file will be produced and user continues directly with next line. Do not put empty lines between these parts if keyword was 'no'. In next lines after the filename the user gives parameters about type of output tables. You have several possibilities to choose output. Your description consists of 4 parts.

NO ICR ACRW VE

where NO is identifier of node. The user can put in this place also identifiers **, which means all nodes, or ++ which means sum of all flows. For the second output table this parameter is index of flow type (here ++ ** can also be used). For the third output table it is also identifier of node. For all other option use letter Y (for yes) or N (for no) instead of letter above.

ICR option : here you can choose input(I=Y), correction (C=Y) or output table (O=Y).

ACRW option : here you can choose if data in the table will be as absolute value (A=Y), or they will be printed relatively to the sum of the column (C=Y), relatively to the sum of the row (R=Y), or relatively to the whole sum (W=Y).

VE option: here you can choose if the data will be printed as flow (values, V=Y), or as their equivalents (E=Y).

For each of this three options minimally one Y in each option must be chosed, but it is possible to use more Y in each option, then for each line with description you can get more output tables.

For the third output tables only ICR option is valid, no more options are possible.

specfile

This is an example of specfile:

```
begin specs for priklad
  minimize
  objective   obj
  rhs        rhs
  bounds     bnd
  rows       1000
  columns    1000
  elements   10000
  nonlinear variables  0
  mps file   2
  old basis file  0
  new basis file  4
  solution file  0
  solution    no
  cycle limit 1000
  cycle print 0
  crash option 1
  iterations  3000
  log frequency 100
  lu row tollerance 50.0
  factorize frequency 100
  partial price 1
  feasibility tol 1.0e-8
  problem number 0
end
```

NOTE: cycle limit - defines maximal number when minos is called. If we do not want to use automatic fixation, we must used value 1, if we want to use

automatic fixation, we must set the maximum possible number here (theoretically it is number of flows, when 1 flow will be fixed in one cycle). old basis file - if we want to use reastart from previous solution, we can use file with number 3 here. At first copy fort.4 to fort.3 and set old basis file 3.

NOTE: It is possible to use comment line in any place of files database, restr or struct. Comment lines must have == in first two columns, the rest of line is comment.

OUTPUT FILES:

outfile

This file is shortened version of standard outfile for MINOS. The necessary information about optimization process are written here. Here you can find standard output from MINOS if the solution was infeasible.

fort.9

There is an information about automatic fixation (value of objective function fixated flows, fixed values etc) stored here. All errors are printed here also.

output table 1

This is a table of flows according to flow types. The user can specify the outcoming flows, type of table (input, corrections, output, absolute, relative, values or equivalents) in file des.out.

output table 2

This file has the same structure as file output table 1 instead of that each table is for one particular flow type which is specified as item NO in file des.out.

output table 3

For each outputting node (NO item) the following table can be printed: (according to specifications of ICR options in file des.out) values for input flows, corrections, resulting flows in values and equivalences.

graph

Results are produced in a so called graphical form here. For each flow in the model the corresponding character will be printed. The meaning of characters is as following:

- + means that the flow is on upper border,
- means that the flow is on lower border,
- . means that the flow has zero value
- e means that the flow was fixed at the beginning of task,
- # means that the flow was marked as free,
- o means that the flow has nothing from upper given property (e.g. it is between max. and min. etc.)

NOTE: it is not interesting to prepare graph file after automatic fixation, because all flows will be fixed at the end of task. It is not possible to prepare the graph file if the number of flow types multiplied by number of nodes is greater than 128 because of the number of columns in one line of line printer.

WORKING FILES:

mpsfile

Standard mps file for MINOS solution is prepared within the program. The whole problem is transformed to the following linear programming problem:

minimize uu with respect to:

$$xxijjkk + AAijjkk * uu \geq 0. \quad (\text{rows rliijjkk})$$

$$-xxijjkk + AAijjkk * uu \geq 0. \quad (\text{rows ruiijjkk})$$

$$\text{sumjj sumkk } xxijjkk - \text{exii} = -\text{sumjj sumkk } AAijjkk \quad (\text{rows reii})$$

$$\text{sumii sumkk } xxijjkk - \text{imjj} = -\text{sumii sumkk } AAijjkk \quad (\text{rows rijj})$$

$$\text{exii} - \text{imii} - \text{saii} = 0. \quad (\text{rows rsii})$$

$$\text{sumii sumjj } xxijjkk - \text{xgkk} = -\text{sumii sumjj } AAijjkk \quad (\text{rows rgkk})$$

$$\text{sumjj } xxijjkk - \text{xoi99kk} = -\text{sumjj } AAijjkk \quad (\text{rows roii99kk})$$

$$\text{sumii } xxijjkk - \text{xd99jjkk} = -\text{sumii } AAijjkk \quad (\text{rows rd99jjkk})$$

$$\text{sumkk } xxijjkk - \text{xqii99} = -\text{sumkk } AAijjkk \quad (\text{rows rqii99})$$

$$\text{xoi99kk} - \text{xd99iikk} - \text{soii99kk} = 0. \quad (\text{rows rrii99kk})$$

$$\text{xqii99} - \text{xqjii99} - \text{sgii99} = 0. \quad (\text{rows rvii99})$$

$$\text{sumii exii} - \text{xt} = 0. \quad (\text{row rt})$$

$$\text{sumjj } xxijjkk / \text{piijjkk} - \text{xbii99kk} = -\text{sumjj } AAijjkk / \text{piijjkk} \quad (\text{rows rbii99kk})$$

$$\text{sumjj sumii } xxijjkk / \text{piijjkk} - \text{xc9999kk} = -\text{sumjj sumii } AAijjkk / \text{piijjkk} \\ (\text{rows rc9999kk})$$

$$-\text{xaiijjkk} + \text{xxijjkk} - \text{xxjjiikk} = \text{AAjjiikk} - \text{AAijjkk} \quad (\text{rows raiijjkk})$$

Here every row and column has its own meaning. Names consist mainly in a

form yyijjkk where yy are 2 characters denoting type of variable

ii is index of outputing node,

jj is index of inputing node,

kk is index of flow types.

When index is equal to 99 it means the sum. Here rows rliijjkk ruiijjkk reii rijj rsii are always generated, other rows are generated only when it is necessary (user gives request in restr or struct file). Besides that for each row from struct file corresponding eniiii row is generated with variables which names can be any of above. Here iiii is number of row in struct file (except ** option which makes the added generation of rows and so the shift with numbering rows in mpsfile).

basis files fort.3, fort.4

These are basis files which can be used for restart purposes.

scratch file nro 8

It is working file for minos which is scratched after solution.

ERROR MESSAGES:

The structure of each message is :

nro - subroutine - concerning file - description - how correct

- 1,line - vstuf - spec - file too short or error at reading file - correct data
- 2,line - vstuf - nodes - such a node already exists - correct data
- 3,line - vstuf - nodes - too many nodes - change size of arrays and parameter mxp
- 4,line - vstuf - nodes - no node - correct data
- 5,line - vstuf - flow types - too many flow types - change size of arrays and parameter mxg
- 6,line - vstuf - flow types - file too short or error at reading numeric value - correct data
- 7,line - vstuf - database - wrong identifier of node in the header - correct data
- 8,line - vstuf - database - wrong index of flow type - correct data
- 9,line - vstuf - database - value is less then 0. - correct data
- 10,line - vstuf - database - too many flows - change size of arrays and parameter mxv
- 11,line - vstuf - database - error at reading numeric value, block not closed or short file - correct data
- 12,line - vstuf - database - zero number of flows - correct data
- 13,line - restr - restr - wrong type of restricted variable (not VA,EQ,IM) - correct data
- 14,line - restr - restr - wrong type of constraint (not < > = W) - correct data
- 15,line - restr - restr - negative value of flow - correct data
- 16,line - getvar - restr/struct - type of restricted variable cannot be IM - correct data
- 17,line - getvar - restr/struct - wrong identifier of outputing node - correct data
- 18,line - getvar - restr/struct - wrong identifier of inputing node - correct data
- 19,line - getvar - restr/struct - wrong index of flow type - correct data
- 20,line - getvar - restr/struct - type of restricted variable cannot be FL - correct data
- 21,line - restr - restr - constraint for nonexisting flow - correct data

22,line - addr - restr/struct - too many constraints - change size of arrays
and parameter mxr

28,error - vypoc - 0 - error at minos solution - change definition of task

29,0 - vstap - 0 - flow(s) is zero and is not fixed or free

31,line - getvar - restr- wrong numeric value for index of flow type - correct data

32,line - restr - restr- wrong numeric value for flow - correct data

33,0 - open - 0 - error at opening file - consult with system programmer

34,line - restr - restr- error at reading file - correct data

38,index - outgr - outgraph - flow has zero value and is not free - declare
variable as free in database

39,0 - outgr - outgraph - it is not possible to make graph (too big) - put
'no' parameter as graphical output file

40,line - vstap - nodes - error at reading file - correct data

41,0 - matmod - 0 - cannot find flow (error in program) - consult with author
of program

42,0 - tma - 0 - small arrays for lngpg - change size of arrays and
parameter lngpg

43,line - equat - struct - bad type for variable 1 - correct data

44,line - equat - struct - bad type of structure - correct data

45,line - equat - struct - missing coefficient for structure - correct data

46,line - equat - struct - error at reading coef for structure - correct data

47,line - equat - struct - both types are simple flows for ** option - correct
data

48,line - addone - struct - too many structures (short arrays) - change size of
arrays and parameter mxr

49,line - equat - struct - ** does not match - correct data

51,line - equat - struct - bad data on struct file - correct data

52,line - equat - struct - bad type for variable 2 - correct data

53,line - equat - struct - not such a flow - correct data

54,line - addr - restr/struct - imbalance for the same node - correct data

55,line - matmod - 0 - nro of cycle is greater then size of arrays for storing
results - change size of arrays or reduce number of cycles

56,0 - tma - 0 - small arrays for output table 2 (ares,bres) - change size
of arrays and parameter mxp

57,line - restr - restr - weight cannod be for flows of type ++

59,line - restr - restr - weight can be for simple flow only

60,0 - restr - restr - lower > upper bound for total - correct data

61,index - restr - restr - lower > upper bound for imbalance - correct data

62,index - restr - restr - lower > upper bound for total output - correct data

63,index - restr - restr - lower > upper bound for total input - correct data

64,index - restr - restr - lower > upper bound for sum of all flow types - correct
data

70,0 - tma - 0 - more than 2000 flows or 25 nodes in common /trans/ -
change common in all subroutines

71,0 - vstup - spec - missing filename for database - correct data

72,0 - vstup - spec - missing filename for nodes - correct data

73,0 - vstup - spec - missing filename for flow types - correct data

74,0 - vstup - spec - missing filename for flow equivalences - correct data

75,0 - vstup - spec - missing filename for new database - correct data

76,line - vystup - des.out - wrong numeric value or file too short - correct data

77,line - vstup - flow.equiv - error at reading numeric value or file too short
or block not closed - correct data

78,lngg - vstup - flow types - no flow types (file empty) - correct data

79,0 - vystup - 0 - change dimension of pole()

81,line - vystup - des.out - wrong identifier for outputing node

82,line - vystup - des.out - wrong identifier for inputing node

85,line - oma - newbase - error at reading block, block too short or not closed

- correct data

86,line - oma - newbase - flow was not found - correct data

87,line - oma - newbase - wrong identifier for outputing or inputing node - correct data

88,line - oma - newbase - negative flow - correct data

90,jfound - matmod - 0 - wrong index for fixed variable (ask programmer)

91,itypp - vstup - flow.equiv - type of flow equivalent is not 1,2 or 3 -

correct data

92,line - vstup - flow.equiv - wrong identifier for outputing or inputing node - correct data

93,line - vstup - flow.equiv - wrong flow type index - correct data

94,line - vstup - flow.equiv - flow equivalent cannot be ≤ 0 . - correct data

NOTE : all error messages are displayed to file 'fort.9'. All errors cause finishing the program at the moment and place when they occur (generally when error is at reading input files or at minos solution, no output files will be prepared.) There is also one warning message which does not caused the finish of program and which occurs when the user gives inconsistent constraints for some variable. In this case the last constraint will be taken into consideration.

PART THREE : AN ILLUSTRATIVE EXAMPLE

Analysis of the Dynamics of the Energy Production-Consumption Structure for CMEA Countries

The approach described was applied to investigate trends of development of the energy production and consumption structures of the member countries of the Council for Mutual Economic Assistance (CMEA) up to the year 2000. The basic, incomplete model was developed by the Energy systems Group of IIASA in 1983/84; see, for example, Golovin [1985].

The main purpose of this investigation was to analyze the feasibility of different versions of consumption structures and evaluations of the potential growth of energy production. The following were taken into consideration:

- ranges of consumption levels consistent with the
planned rates of general economic growth;
- ranges of possible capacities for energy production;
- the requirement to achieve the target levels with
the minimal structural changes in energetics.

The first two conditions are the conditions of feasibility. The third condition is an informal definition of metric (6). The *reference state* of the model is the initial situation. Roughly speaking, we would like to change nothing to achieve the desired targets.

In terms of the linear flow model the problem may be formulated as follows. We have a system of eight nodes (Table 2) linked by a set of four component

flows (Table 3).

Identifier	Country
BG	Bulgaria
HU	Hungary
GR	GDR
PL	Poland
RO	Romania
SU	USSR
CS	Czechoslovakia
RW	Rest of the World (as a supplier-consumer for CMEA)

Table 2.

No.	Energy product	Unit of measurement	Coefficient of equivalence
1	Coal	mill. tce	1.000
2	Primary Electricity	bill. kWth	0.326
3	Crude Oil	mill. tons	1.454
4	Natural Gas	bill. cu. m.	1.188

Table 3.

The state of the production-consumption market for CMEA countries in 1980 was taken as the initial state. The description of the state is given in Tables 4, 5, 6, and 7 for coal, electricity, oil and gas, respectively. Each row of these tables describes the production and each column shows the consumption.

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	15.2	15.2
HU	.	10.0	0.1	10.1
GR	.	0.4	76.4	0.2	.	.	0.3	1.0	78.3
PL	0.1	0.7	2.3	143.5	0.3	5.4	1.6	13.6	167.5
RO	17.0	.	.	.	17.0
SU	5.6	1.1	4.4	0.6	1.2	487.1	2.9	2.6	505.5
CS	.	0.7	1.7	.	0.4	.	59.0	2.6	64.4
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	20.9	13.0	86.0	144.3	23.5	492.8	63.9	19.9	864.3

Table 4. Production-consumption of coal in 1980 (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	9.0	0.2	.	0.7	9.9
HU	.	0.13	0.13
GR	.	.	9.8	2.3	.	.	0.3	.	12.4
PL	.	.	0.5	1.0	.	.	1.0	.	2.5
RO	.	0.9	.	.	11.2	.	.	.	12.1
SU	4.6	8.3	.	1.5	0.5	220.8	1.2	3.1	240.0
CS	.	0.5	2.9	0.3	.	.	3.4	1.7	8.8
RW	0.1	0.3	0.3	.	0.7
Total consumption	13.7	10.13	13.2	5.1	11.7	221.0	6.2	5.5	286.53

Table 5. Production-consumption of electricity in 1980 (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.3	0.3
HU	.	1.0	1.0	2.0
GR
PL	.	.	.	0.3	0.3
RO	11.5	.	.	.	11.5
SU	12.0	8.0	19.0	13.1	0.4	481.2	18.3	51.5	603.5
CS
RW	1.0	0.3	2.9	3.2	15.5	7.0	1.0	.	30.9
Total consumption	13.3	9.3	21.9	16.6	27.4	488.2	19.3	52.5	648.5

Table 6. Production-consumption of oil in 1980 (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	6.1	6.1
GR	.	.	2.8	2.8
PL	.	.	.	6.3	6.3
RO	.	0.2	.	.	35.0	.	.	.	35.2
SU	4.6	3.8	6.8	5.3	1.3	375.5	8.7	29.0	435.0
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	4.8	10.1	9.6	11.6	36.3	378.0	9.2	29.0	488.6

Table 7. Production-consumption of gas in 1980 (bill. cu. m.)

The necessary conditions for feasibility were defined not only for the final point (year 2000) but also for intermediate points : 1985, 1990 and 1995. These conditions are inequalities for absolute and relative values of production-consumption volumes for different countries and different kinds of products (Tables 8, 9, 10 and 11). The hypothesis about the dynamics of the energy potential for CMEA countries predicts moderate growth of the coal industry, stabilization of crude oil production, and intensive development of both nuclear energy and natural gas production. The sources of information used to evaluate the potential volumes of energy production were the World Energy Conference [1983], Wilson [1983], the British Institutes Joint Energy Policy Programme [1983], Stern [1982], and the official statistical CMEA reports [1982, 1983]. Because of the essential differences between the forecast levels of energy consumption, two independent scenarios were considered. The first, called 'high consumption' scenario (Table 9), suggests that the planned 3% economic growth will be provided by an energy elasticity (relative to GNP) for the USSR ranging from 0.85 in 1985 to 0.65 in 2000, and for the other CMEA countries from 0.75 to 0.50, respectively. The 'low consumption' scenario (Table 10) is based on the assumption that the energy elasticity ranges from 0.50 to 0.25 for the USSR and from 0.30 to 0.10 for the other CMEA countries.

Table 11 contains the description of three possible structures of energy consumption. Structure A corresponds to the state just after 1980 and permits relatively narrow variations. Structure C differs essentially from A. The main differences are: a reduction in the share of crude oil and increases in the shares of primary electricity and natural gas. The coal dynamics depend on the policy of the individual country, but the average share is slightly decreased. Structure B is an intermediate variant between A and C.

Exporter	Energy Product	Reachable maximum levels of production			
		1985	1990	1995	2000
BG	Coal	17.2	18.0	19.0	20.0
	Electr.	13.8	19.4	35.0	54.0
	Oil	0.2	0.2	0.2	0.3
	Gas	0.2	0.2	0.2	0.2
HU	Coal	11.0	11.0	13.0	14.0
	Electr.	0.13	9.1	22.0	36.0
	Oil	2.0	1.8	1.6	1.5
	Gas	6.5	7.5	9.0	6.0
GR	Coal	80.0	80.0	80.0	80.0
	Electr.	14.6	20.6	39.0	58.0
	Oil
	Gas	2.8	2.8	2.9	3.0
PL	Coal	180.0	200.0	210.0	220.0
	Electr.	2.5	6.6	18.0	36.0
	Oil	0.3	0.2	0.1	0.3
	Gas	6.5	7.5	9.0	6.0
RO	Coal	22.0	30.0	40.0	55.0
	Electr.	12.5	16.6	23.0	33.0
	Oil	11.5	11.0	10.5	10.0
	Gas	30.0	30.0	30.0	33.0
SU	Coal	540.0	590.0	660.0	780.0
	Electr.	440.0	705.0	940.0	1200.0
	Oil	630.0	640.0	650.0	630.0
	Gas	630.0	780.0	880.0	1100.0
CS	Coal	65.0	65.0	65.0	65.0
	Electr.	18.9	23.6	31.0	48.0
	Oil
	Gas	0.5	0.5	0.5	0.5

Table 8. Reachable maximum levels of energy production

Importer	Necessary minimum levels of energy consumption			
	1985	1990	1995	2000
BG	57.5	66.0	74.0	80.9
HU	44.0	50.0	57.0	62.0
GR	138.0	145.0	148.0	152.0
PL	200.0	220.0	240.0	260.0
RO	125.0	143.0	161.0	176.0
SU	1985.0	2300.0	2600.0	2900.0
CS	115.5	132.0	149.0	162.0

Table 9. Necessary minimum levels of energy consumption : 'High' scenario
(mill. tce)

Importer	Necessary minimum levels of energy consumption			
	1985	1990	1995	2000
BG	57.5	60.0	62.0	63.9
HU	44.0	46.0	48.0	50.0
GR	138.0	144.0	148.0	150.0
PL	200.0	209.0	220.0	230.0
RO	125.0	133.0	137.0	140.0
SU	1985.8	2150.0	2300.0	2400.0
CS	115.5	122.0	127.0	130.0

Table 10. Necessary minimum levels of energy consumption: 'Low' scenario
(mill. tce)

Importer	Energy product	Possible structures of energy consumption (in % of total consumption)					
		Variant A		Variant B		Variant C	
		min	max	min	max	min	max
BG	Coal	39	41	36	40	34	38
	Electr.	8	12	10	16	20	24
	Oil	34	39	30	36	20	24
	Gas	10	14	13	19	18	22
HU	Coal	30	33	30	34	33	35
	Electr.	8	12	10	15	20	25
	Oil	30	33	26	30	20	25
	Gas	26	29	24	28	22	27
GR	Coal	62	65	58	62	54	56
	Electr.	3	5	5	10	11	14
	Oil	22	24	20	23	19	22
	Gas	8	10	8	12	9	14
PL	Coal	76	79	65	77	62	65
	Electr.	0.5	2	1	4	3	5
	Oil	11	14	12	15	14	17
	Gas	7	9	9	15	16	19
RO	Coal	21	26	25	30	36	38
	Electr.	3	4	4	8	5	7
	Oil	30	36	26	32	17	21
	Gas	37	40	36	40	35	39
SU	Coal	26	29	25	28	24	27
	Electr.	4	6	6	10	10	13
	Oil	37	40	32	38	25	30
	Gas	26	28	28	33	30	35
CS	Coal	57	60	50	55	46	48
	Electr.	2	5	4	9	10	14
	Oil	23	26	21	25	10	13
	Gas	11	16	16	24	30	36

Table 11. Possible structures of energy consumption for CMEA countries.

Analysis of Results

For the model described above two series of calculations have been performed. The first series was performed to investigate the feasibility of different combinations of consumption structures for the 'high' scenario and the second one for the 'low' scenario.

The calculations were made in the following way. As a first step a solution satisfying all necessary conditions of feasibility for 1985 was found, minimizing the 'distance' (6) between the states of 1980 and 1985. In the next step a solution was built which satisfied all constraints for 1990 and minimized the 'distance' between the states of 1985 and 1990, and so on, until the final point 2000 was reached or an infeasibility appeared.

Some additional constraints were introduced during the process. These are a constant or increasing the total consumption of primary electricity, maximization of crude oil exports, and so on. Sequential fixation was used during all calculations.

On the basis of the results obtained we may conclude that the up-to-date evaluations of the energy potential of the CMEA countries do not contradict the planned economic target up to the end of the century. There are enough energy resources not only to provide the 3% economic growth, but also to permit the sale of a considerable amount of energy outside the CMEA. But this can happen only if some changes are made in the structure of the energy consumption.

Structure A (Table 11) will be in contradiction with the plans for economic growth after 1990 for the 'high' scenario or after 1995 for the 'low' scenario. A condition for keeping structure A until the year 2000 is to increase imports of

oil after 1995 ('low' scenario) or to increase imports of oil after 1990 and coal after 1995 ('high' scenario). Evaluations of the relevant import levels are given in Tables 12 and 13.

On the other hand, the combination of structures A A B B (for the years 1985, 1990, 1995 and 2000, respectively) would avoid the contradiction and, hence, an increase in energy imports for the 'low' scenario. For the 'high' scenario the combination A B B C is found to be necessary. These results are presented in Table 14.

Year	Oil import from RW (mill. tons)	Coal import from RW (mill. tons)
1985	30.9	6.3
1990	101.0	15.2
1995	187.4	41.5
2000	274.7	58.1

Table 12. Dynamics of imports assuring feasibility for structure A : 'High' scenario

Year	Oil import from RW (mill. tons)
1985	30.9
1990	30.9
1995	34.7
2000	83.2

Table 13. Dynamics of imports assuring feasibility for structure A : 'Low' scenario

Variant	Combinations of the structures used				Solution found	Possible alternative
	1985	1990	1995	2000		
1	A	A	A	A	Infeasible after 1990	Increase of imports of oil and coal
2	A	B	B	B	Infeasible after 1995	Increase of imports of oil
3	A	B	B	C	Feasible state	-

Table 14. Results of the analysis: 'High' scenario

Tables 15 - 30 in Appendix A contain a description of a feasible state of the considered model for the structural set A B B C ('high' scenario). For comparison the analogous solution A A B B ('low' scenario) is given in Appendix B.

Finally, we would like to emphasize once again that these solutions may be unacceptable from the viewpoint of the decisionmaker, because the actual solving process has not been finished here. Our purpose here was only to demonstrate all the main principles of *incomplete modelling*, considering both the positive and the negative aspects of the approach.

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PART FOUR : APPENDIXES

Appendix A.

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	17.2	17.2
HU	.	10.95	0.05	11.0
GR	.	0.25	78.83	0.1	.	.	0.35	0.47	80.0
PL	0.15	1.05	3.44	157.6	0.93	8.08	2.4	6.35	180.0
RO	22.0	.	.	.	22.0
SU	5.08	0.7	3.63	0.3	3.73	522.75	2.61	1.21	540.0
CS	.	0.45	1.4	.	1.24	.	60.7	1.21	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	22.43	13.5	88.5	158.0	32.5	531.13	66.16	9.29	921.5

Table 15. Production-consumption of coal by CMEA countries in 1985 : 'High' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	18.0	18.0
HU	.	10.98	0.02	11.0
GR	.	0.41	78.73	0.06	.	.	0.52	0.28	80.0
PL	0.21	1.68	4.84	169.16	1.38	11.36	3.56	6.35	198.54
RO	30.0	.	.	.	30.0
SU	5.55	1.12	3.95	0.18	5.53	569.08	3.87	0.72	590.0
CS	.	0.71	0.72	.	1.39	.	61.55	0.63	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	23.76	15.0	89.44	169.4	42.9	580.74	69.61	7.99	998.84

Table 16. Production-consumption of coal by CMEA countries in 1990 : 'High' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	19.0	19.0
HU	.	12.99	0.01	13.0
GR	.	0.6	78.47	0.04	.	.	0.78	0.11	80.0
PL	0.31	2.49	6.4	175.64	0.62	16.85	5.27	2.43	210.0
RO	40.0	.	.	.	40.0
SU	7.33	0.99	2.04	0.09	2.47	641.73	5.07	0.28	660.0
CS	.	0.49	0.37	.	0.62	.	63.28	0.24	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	26.64	17.67	88.49	175.77	48.3	658.88	74.5	3.05	1093.3

Table 17. Production-consumption of coal by CMEA countries in 1995 : 'High' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	20.0	20.0
HU	.	14.0	14.0
GR	.	0.84	75.82	0.04	.	.	0.92	0.11	77.73
PL	0.37	3.46	6.18	168.89	0.83	20.03	6.27	2.43	208.45
RO	53.79	.	.	.	53.79
SU	8.67	1.38	1.66	0.08	3.32	758.82	5.86	0.22	780.0
CS	.	0.68	0.3	.	0.83	.	62.99	0.19	65.0
RW	.	0.1	1.16	.	4.6	0.3	0.1	.	6.26
Total consumption	29.04	20.46	85.12	169.0	63.36	779.15	76.14	2.95	1225.2

Table 18. Production-consumption of coal by CMEA countries in 2000 : 'High' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	13.18	0.29	.	0.33	13.8
HU	.	0.13	0.13
GR	.	.	11.54	2.71	.	.	0.35	.	14.6
PL	.	.	0.41	0.98	.	.	1.11	.	2.5
RO	12.5	.	.	.	12.5
SU	7.05	12.73	.	2.3	1.96	338.51	1.84	3.34	367.73
CS	.	0.77	4.45	0.46	.	.	5.21	1.83	12.72
RW	0.1	0.3	0.3	.	0.7
Total consumption	20.33	13.92	16.4	6.44	14.5	338.8	8.82	5.5	424.68

Table 19. Production-consumption of primary electricity by CMEA countries in 1985 : 'High' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	18.52	0.41	.	0.33	19.26
HU	.	0.18	0.18
GR	.	.	16.21	3.52	.	.	0.66	.	20.4
PL	.	.	0.58	1.27	.	.	2.07	.	3.92
RO	16.6	.	.	.	16.6
SU	9.91	17.88	.	2.99	2.97	475.67	3.43	3.34	516.2
CS	.	1.08	6.25	0.6	.	.	9.73	1.83	19.48
RW	0.1	0.3	0.3	.	0.7
Total consumption	28.53	19.44	23.04	8.38	19.57	476.08	16.2	5.5	596.75

Table 20. Production-consumption of primary electricity by CMEA countries in 1990 : 'High' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	22.77	0.67	.	0.33	23.76
HU	.	0.25	0.25
GR	.	.	26.23	5.7	.	.	0.98	.	32.91
PL	.	.	0.94	2.06	.	.	3.07	.	6.07
RO	23.0	.	.	.	23.0
SU	12.18	24.22	.	4.84	4.82	769.64	5.09	3.34	824.13
CS	.	1.46	10.11	0.97	.	.	14.41	1.83	28.78
RW	0.1	0.3	0.3	.	0.7
Total consumption	35.05	26.23	37.28	13.56	27.82	770.3	23.85	5.5	939.6

Table 21. Production-consumption of primary electricity by CMEA countries in 1995 : 'High' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	32.27	0.79	.	0.38	33.44
HU	.	0.28	0.28
GR	.	.	36.09	10.05	.	.	2.05	.	48.2
PL	.	.	1.29	3.63	.	.	6.44	.	11.36
RO	.	9.3	.	.	23.7	.	.	.	33.0
SU	17.27	27.3	.	8.54	5.44	914.89	10.67	3.92	988.02
CS	.	0.96	13.91	1.71	.	.	30.23	1.2	48.0
RW	0.1	0.3	0.3	.	0.7
Total consumption	49.63	38.14	51.29	23.93	29.14	915.68	49.69	5.5	1163.0

Table 22. Production-consumption of primary electricity by CMEA countries in 2000 : 'High' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	1.53	0.47	2.0
GR
PL	.	.	.	0.3	0.3
RO	11.5	.	.	.	11.5
SU	12.83	7.25	17.98	11.99	1.57	534.69	19.53	24.05	630.0
CS
RW	1.0	0.3	2.9	3.2	15.5	7.0	1.0	.	30.9
Total consumption	14.03	9.08	20.88	15.49	28.57	541.69	20.65	24.51	674.9

Table 23. Production-consumption of oil by CMEA countries in 1985 : 'High' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	1.52	0.28	1.80
GR
PL	.	.	.	0.2	0.2
RO	11.0	.	.	.	11.0
SU	13.74	8.26	18.18	14.76	2.38	546.69	21.7	14.3	640.0
CS
RW	1.0	0.3	2.9	3.2	15.5	7.0	1.0	.	30.9
Total consumption	14.94	10.08	21.08	18.16	28.88	553.69	22.7	14.58	684.1

Table 24. Production-consumption of oil by CMEA countries in 1990 : 'High' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	1.5	0.1	1.6
GR
PL	.	.	.	0.1	0.1
RO	10.5	.	.	.	10.5
SU	14.07	8.39	17.46	16.51	2.79	565.21	20.52	5.05	650.0
CS
RW	1.0	0.3	2.9	3.2	15.5	7.0	1.0	.	30.9
Total consumption	15.27	10.19	20.36	19.81	28.79	572.21	21.52	5.15	693.3

Table 25. Production-consumption of oil by CMEA countries in 1995 : 'High' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.24	0.24
HU	.	1.42	0.08	1.5
GR
PL	.	.	.	0.15	0.15
RO	9.35	.	.	.	9.35
SU	12.11	6.81	17.4	25.26	2.26	548.25	13.81	4.1	630.0
CS
RW	1.0	0.3	2.9	3.2	13.81	7.0	0.67	.	28.88
Total consumption	13.35	8.53	20.3	28.61	25.42	555.25	14.48	4.18	670.12

Table 26. Production-consumption of oil by CMEA countries in 2000 : 'High' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	6.5	6.5
GR	.	.	2.8	2.8
PL	.	.	.	6.5	6.5
RO	30.0	.	.	.	30.0
SU	6.58	4.24	8.82	8.13	8.93	465.35	13.34	29.0	544.37
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	6.77	10.74	11.62	14.63	38.93	467.85	13.84	29.0	593.37

Table 27. Production-consumption of gas by CMEA countries in 1985 : 'High' scenario (bill. cu. m)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	7.13	7.13
GR	.	.	2.8	2.8
PL	.	.	.	7.5	7.5
RO	30.0	.	.	.	30.0
SU	9.24	4.65	11.85	10.57	13.55	636.39	19.80	29.0	735.04
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	9.44	11.79	14.65	18.07	43.55	638.89	20.3	29.0	785.67

Table 28. Production-consumption of gas by CMEA countries in 1990 : 'High' scenario (bill. cu. m)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	8.13	8.13
GR	.	.	2.86	2.86
PL	.	.	.	9.0	9.0
RO	30.0	.	.	.	30.0
SU	11.36	5.3	12.09	17.1	21.99	719.72	29.33	29.0	845.9
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	11.56	13.43	14.95	26.1	51.99	722.22	29.83	29.0	899.09

Table 29. Production-consumption of gas by CMEA countries in 1995 : 'High' scenario (bill. cu. m)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	6.0	6.0
GR	.	.	3.0	3.0
PL	.	.	.	6.0	6.0
RO	.	2.09	.	.	30.91	.	.	.	33.0
SU	13.5	5.98	14.37	29.02	24.79	851.88	40.41	29.0	1009.0
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	13.7	14.06	17.37	35.02	55.71	854.38	40.91	29.0	1060.2

Table 30. Production-consumption of gas by CMEA countries in 2000 : 'High' scenario (bill. cu. m)

Appendix B.

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	17.2	17.2
HU	.	10.95	0.05	11.0
GR	.	0.25	78.83	0.1	.	.	0.35	0.47	80.0
PL	0.15	1.05	3.44	157.6	0.93	8.08	2.4	6.35	180.0
RO	22.0	.	.	.	22.0
SU	5.08	0.7	3.63	0.3	3.73	522.75	2.61	1.21	540.0
CS	.	0.45	1.4	.	1.24	.	60.7	1.21	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	22.43	13.5	88.5	158.0	32.5	531.13	66.16	9.29	921.5

Table 31. Production-consumption of coal by CMEA countries in 1985 : 'Low' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	18.0	18.0
HU	.	10.99	0.01	11.0
GR	.	0.17	79.29	0.03	.	.	0.39	0.13	80.0
PL	0.26	1.8	5.91	165.0	1.16	13.88	4.11	6.35	198.47
RO	27.41	.	.	.	27.41
SU	5.6	0.46	3.06	0.09	1.05	577.19	2.2	0.34	590.0
CS	.	0.29	1.26	.	0.35	.	62.75	0.34	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	23.86	13.8	90.72	165.11	34.58	591.37	69.56	7.18	996.18

Table 32. Production-consumption of coal by CMEA countries in 1990 : 'Low' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	19.0	19.0
HU	.	12.99	0.01	13.0
GR	.	0.08	79.38	0.02	.	.	0.46	0.06	80.0
PL	0.35	0.87	8.16	172.43	0.52	19.15	5.67	2.85	210.0
RO	29.87	.	.	.	29.87
SU	5.45	0.22	3.48	0.05	0.47	647.55	2.63	0.15	660.0
CS	.	0.14	1.02	.	0.16	.	63.53	0.15	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	24.8	14.4	93.24	172.5	35.62	667.0	72.39	3.22	1083.2

Table 33. Production-consumption of coal by CMEA countries in 1995 : 'Low' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	19.33	19.33
HU	.	13.65	0.01	13.66
GR	.	0.07	79.32	0.02	.	.	0.54	0.05	77.73
PL	0.34	0.83	9.2	181.63	0.49	18.18	6.62	2.7	220.0
RO	30.69	.	.	.	30.69
SU	5.54	0.23	3.93	0.06	0.49	677.52	3.06	0.15	690.97
CS	.	0.12	0.85	.	0.13	.	63.77	0.13	65.0
RW	.	0.1	1.2	.	4.6	0.3	0.1	.	6.3
Total consumption	25.2	15.0	94.5	181.7	36.4	696.0	74.1	3.04	1225.9

Table 34. Production-consumption of coal by CMEA countries in 2000 : 'Low' scenario (mill. tce)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	13.18	0.29	.	0.33	13.8
HU	.	0.13	0.13
GR	.	.	11.54	2.71	.	.	0.35	.	14.6
PL	.	.	0.41	0.98	.	.	1.11	.	2.5
RO	12.5	.	.	.	12.5
SU	7.05	12.73	.	2.3	1.96	338.51	1.84	3.34	367.73
CS	.	0.77	4.45	0.46	.	.	5.21	1.83	12.72
RW	0.1	0.3	0.3	.	0.7
Total consumption	20.33	13.92	16.4	6.44	14.5	338.8	8.82	5.5	424.68

Table 35. Production-consumption of primary electricity by CMEA countries in 1985 : 'Low' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	14.32	0.34	.	0.33	14.99
HU	.	0.13	0.13
GR	.	.	15.54	2.91	.	.	0.61	.	19.06
PL	.	.	0.55	1.05	.	.	1.91	.	3.51
RO	.	2.49	.	.	14.11	.	.	.	16.6
SU	7.66	12.94	.	2.47	2.21	395.36	3.16	3.34	427.14
CS	.	0.78	5.99	0.49	.	.	8.95	1.83	18.04
RW	0.1	0.3	0.3	.	0.7
Total consumption	22.09	16.64	22.09	6.92	16.32	395.71	14.92	5.5	500.18

Table 36. Production-consumption of primary electricity by CMEA countries in 1990 : 'Low' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	14.8	0.37	.	0.33	15.5
HU	.	0.13	0.13
GR	.	.	15.97	4.51	.	.	0.63	.	21.12
PL	.	.	0.57	1.63	.	.	1.99	.	4.19
RO	.	3.52	.	.	14.53	.	.	.	18.05
SU	7.92	12.94	.	3.83	2.28	422.95	3.3	3.34	456.56
CS	.	0.78	6.16	0.77	.	.	9.35	1.83	18.89
RW	0.1	0.3	0.3	.	0.7
Total consumption	22.82	17.67	22.7	10.73	16.81	423.31	15.58	5.5	535.13

Table 37. Production-consumption of primary electricity by CMEA countries in 1995 : 'Low' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	15.04	0.38	.	0.33	15.75
HU	.	0.13	0.13
GR	.	.	16.19	4.94	.	.	0.65	.	21.78
PL	.	.	0.58	1.78	.	.	2.04	.	4.4
RO	.	3.52	.	.	14.85	.	.	.	18.37
SU	8.05	12.94	.	4.19	2.33	441.34	3.38	3.34	475.57
CS	.	0.78	6.24	0.84	.	.	9.58	1.83	19.27
RW	0.1	0.3	0.3	.	0.7
Total consumption	23.19	17.67	23.0	11.75	17.18	441.72	15.95	5.5	555.96

Table 38. Production-consumption of primary electricity by CMEA countries in 2000 : 'Low' scenario (bill. kWth)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	1.53	0.47	2.0
GR
PL	.	.	.	0.3	0.3
RO	11.5	.	.	.	11.5
SU	12.83	7.25	17.98	11.99	1.57	534.89	19.65	24.05	630.0
CS
RW	1.0	0.3	2.9	3.2	15.5	7.0	1.0	.	30.9
Total consumption	14.03	9.08	20.88	15.49	28.57	541.69	20.65	24.51	674.9

Table 39. Production-consumption of oil by CMEA countries in 1985 : 'Low' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	1.67	0.13	1.80
GR
PL	.	.	.	0.2	0.2
RO	11.0	.	.	.	11.0
SU	12.93	7.52	18.89	12.41	0.94	562.21	18.3	6.81	640.0
CS
RW	1.0	0.3	2.9	3.2	15.5	7.0	1.0	.	30.9
Total consumption	14.13	9.49	21.79	15.81	27.44	569.21	19.3	6.94	684.1

Table 40. Production-consumption of oil by CMEA countries in 1990 : 'Low' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	1.6	1.6
GR
PL	.	.	.	0.1	0.1
RO	10.5	.	.	.	10.5
SU	12.34	6.85	18.49	12.09	7.76	574.38	18.09	.	650.0
CS
RW	1.95	1.45	3.9	4.45	10.0	10.9	2.	.	34.68
Total consumption	14.5	9.9	22.39	16.64	28.27	585.28	20.09	.	697.08

Table 41. Production-consumption of oil by CMEA countries in 1995 : 'Low' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.3	0.3
HU	.	1.5	1.5
GR
PL	.	.	.	0.3	0.3
RO	10.0	.	.	.	10.0
SU	4.3	3.0	17.2	11.11	6.89	571.43	16.07	.	630.0
CS
RW	10.13	5.82	5.5	6.0	11.99	39.3	4.5	.	83.23
Total consumption	14.73	10.32	22.7	17.4	28.89	610.73	20.56	.	725.32

Table 42. Production-consumption of oil by CMEA countries in 2000 : 'Low' scenario (mill. tons)

	BG	HU	GR	PL	RO	SU	CS	RW	Total product. on
BG	0.2	0.2
HU	.	6.5	6.5
GR	.	.	2.8	2.8
PL	.	.	.	6.5	6.5
RO	30.0	.	.	.	30.0
SU	6.58	4.24	8.82	8.13	8.93	465.35	13.34	29.0	544.37
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	6.77	10.74	11.62	14.63	38.93	467.85	13.84	29.0	593.37

Table 43. Production-consumption of gas by CMEA countries in 1985 : 'Low' scenario (bill. cu. m)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	6.61	6.61
GR	.	.	2.8	2.8
PL	.	.	.	6.98	6.98
RO	30.0	.	.	.	30.0
SU	6.87	4.31	9.32	8.72	14.78	504.23	15.93	29.0	593.17
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	7.07	10.92	12.12	15.7	44.78	506.73	16.43	29.0	642.75

Table 44. Production-consumption of gas by CMEA countries in 1990 : 'Low' scenario (bill. cu. m)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	6.85	6.85
GR	.	.	2.88	2.88
PL	.	.	.	7.41	7.41
RO	30.0	.	.	.	30.0
SU	7.11	4.47	9.58	9.26	16.13	539.59	16.6	29.0	631.73
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	7.31	11.31	12.46	16.67	46.13	542.09	17.1	29.0	682.06

Table 45. Production-consumption of gas by CMEA countries in 1995 : 'Low' scenario (bill. cu. m)

	BG	HU	GR	PL	RO	SU	CS	RW	Total production
BG	0.2	0.2
HU	.	6.0	6.0
GR	.	.	2.92	2.92
PL	.	.	.	6.0	6.0
RO	.	1.52	.	.	30.66	.	.	.	32.18
SU	7.22	4.47	9.71	10.14	16.48	563.16	17.01	29.0	657.18
CS	0.5	.	0.5
RW	2.5	.	.	2.5
Total consumption	7.42	11.99	12.63	16.14	47.14	565.66	17.51	29.0	707.48

Table 46. Production-consumption of gas by CMEA countries in 2000 : 'Low' scenario (bill. cu. m)