

WORKING PAPER

MAXIMIZING THE CARRYING CAPACITY OF FOREST
ECOSYSTEMS: MODELLING AND FORMATION OF THE
MOST PRODUCTIVE STANDS

L. Kairiukstis
A. Juodvalkis

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*L. Kairiukstis**
*A. Juodvalkis***

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** Deputy Leader, Environment Program
International Institute for
Applied Systems Analysis
A-2361 Laxenburg, Austria*

*** Lithuanian Research Institute of
Forestry, Kaunas, Girionys
Lithuanian SSR, USSR*

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

Foreword

The forest is one of the most important resources of the biosphere. Trees are necessary components of the ecological process that the earth a possible dwelling place for human beings. In the interaction between the sun's energy and the air's carbon dioxide, trees produce stored energy and oxygen, and the root systems bind the soil particles and raise the soil's productive capacity. From an economic point of view, trees are raw material for fuel, lumber, the chemical industry, modern medicines, etc. However, there are reports in the press almost daily about forest decline. Hence, the forest needs much more attention needs to be given to ways of improving forest management systems and increasing the carrying capacity of the forest itself.

Academician Leonardas Kairiukstis and his collaborator Dr. Antanas Juodvalkis, at IIASA and at the Lithuanian Research Institute of Forestry respectively, have both had long experience in research and modelling of forest ecosystems. In this paper they present interesting new approaches to modelling and the formation of a forest ecosystem of maximal productivity. Their approach is based on the effective utilization of solar energy and "*stress*" phenomenon which occurs during the creation of the forest ecosystem.

An optimization of crown parameters, their overlap and stand density is made for different phases of stand development. The model calculates the optimal number of trees in each time span (standard stands) which would enable an increase in carrying capacity of the forest ecosystem. An application of these standards in the thinning practice of the Lithuanian SSR has already resulted in a productivity increase of forest stands.

Professor R.E. Munn
Leader, Environment Program
International Institute for
Applied Systems Analysis
A-2361 Laxenburg, Austria

Summary

On the basis of long-term (over 30 years) and vast investigations (more than 400 permanent experimental plots), the theoretical basis for creating a maximally productive forest was developed and new methods to determine optimal density were suggested. Also, models to simulate stands of maximal productivity were constructed and standards to form maximally productive stands were worked out.

The elaborated methods of density optimization are based on a search for the optimal crown parameters and optimal growing space ensuring maximal increment of the whole stands. On the basis of a newly-revealed phenomenon, the so-called *stress effect* which occurs during the process of ecosystem (biocenoses) creation, it was found that the criteria of optimal stand density in different phases of stand development are different.

In young stands, (process of ecosystem creation) the optimal density is that density which eliminates the mutual influence (competition) of the trees and ensures maximum height increment for a possibly greater number of trees. Following ecosystem (biocenoses) creation, it was noted that in premature and middle-aged stands, the optimal density is such that it provides the maximum annual increment of the growing stock and maximal total stand productivity. This is achieved in the case when the crown closure is maximal with an optimal rate of mutual overlap and the stand is formed from maximally productive trees distributed at an optimal distance from one another.

Based on the above, the standards of maximally productive forest stands have been worked out. Such standards have been widely implemented in exploitive forests in the Lithuanian SSR as well as in western and north-western regions of the USSR. Specialized forest growth undertaken in accordance with set standards has already resulted in a productivity increase of up to 15–20%. The utilization of wood per unit area has been significantly augmented and the cutting cycle of stands has been noted to be much shorter.

MAXIMIZING THE CARRYING CAPACITY OF FOREST ECOSYSTEMS: MODELLING AND FORMATION OF THE MOST PRODUCTIVE STANDS†

L. Kairiukstis and A. Juodvalkis***

1. INTRODUCTION

One of the main tasks for forest management is to increase forest productivity. In tackling this problem, intermediate cutting is particularly important. It is one of the most effective means to ensure qualitative stands are obtained. However, it must be taken into account that positive results are achieved when efficient and qualitative thinning is applied and when the whole system of maximally productive stand formation is scientifically sound. Thus, a forester must know which, and how, stands must be formed according to species composition, structure and productivity, i.e., he must have a simulation model which provide an insight into the probable stand development. In practice, he must deal with prototypes or standards of a maximally productive forest and have concrete programs for forming such a forest. Such standards and programs must be differentiated according to species composition of stands, ecological and geographical regions as well as to site conditions.

We have, therefore, studied the biological grounds of forming a maximally productive forest in more detail. Our task was to:

- elucidate the regularities of natural and artificial stand formation;
- develop the principles and methods for constructing simulation models of maximally productive stands;
- elaborate standards for growing of stands of maximal productivity;
- elaborate purposeful programs for forming such stands in forestry practice particularly by means of thinning.

On the basis of the above, we constructed a simulation model, calculated the standards and developed the programs to form maximally productive stands.

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*Deputy Leader, Environment Program, International Institute for Applied Systems Analysis, A-2361 Laxenburg, Austria

**Lithuanian Research Institute of Forestry, Kaunas, Girionys, Lithuanian SSR, USSR

2. EXPERIMENTAL AND THEORETICAL BACKGROUND FOR THE MODEL CONSTRUCTION

Data from more than 400 permanent experimental plots of 5–30 years duration were used (Kairiukstis, 1977). The trees were mapped and measured. Intermediate cutting on permanent experimental plots was applied 2–6 times. Data from 80 temporary experimental plots were also used. The stands included different species composition, structure and age. The investigations resulted in establishing the following:

- major regularities of growth and formation of stands;
- impact of internal and external factors on the magnitude of the annual increment of a tree and stand; and
- the effect of intermediate cutting on tree growth and stand productivity.

To ascertain the correlation between separate assessment indices, more than 300 different equations were derived.

It was found that natural stands do not attain maximally possible productivity because 40–60% of the trees are of lower or low productivity. The processes of self-regulation in stands occur through intense competition between the individual trees. This results in wasted energy and, consequently, in retardation of forest growth. Thus, artificial regulation of stand density and structure at an optimal level is the essential factor ensuring increase in stand productivity. The investigations revealed that stand productivity on comparatively fertile soils is determined by the following conditions:

- **First:** quantity and quality of solar energy received in the stand. This depends very much upon the canopy surface and its depth being regulated through the thinning systems. The greater the dips in the surface of the canopy, the lesser the albedo, and the more solar energy is received in the stands (see Figure 1).
- **Second:** effectiveness of the solar energy utilization by trees and storeys and by the tree quality and productivity itself. The efficiency of the use of solar radiation obtained by the crowns of variously developed trees is not equal. The higher the tree productivity in annual increment per stand volume in m^3 and the more productive their needles or leaves in annual increment per leaves weight in tons, the higher the production of wood increment per unit of absorbed energy. The coefficient of profitable solar energy use for both the Physiologically Active Radiation (PhAR) and total Solar Radiation (SR) is assumed for class A* trees = 1.0, for trees of class B = 0.8–0.7 and for trees of class C = 0.7–0.5 respectively. The stand formation by thinning cuttings enable one to change the composition of tree classes and to increase the percentage of more productive trees. Class A trees are able, after thinning, with least expenditure of substance and energy, to produce a much more and better quality wood. This produces a positive influence on the overall productivity of stands (Figure 2).
- **Finally:** stand productivity is determined by the optimal number of trees per area unit. Due to the fact that great attention is given to the optimal stand density in forestry literature, we shall discuss this question in more detail.

*The classification of trees was established by L. Kairiukstis (1969).

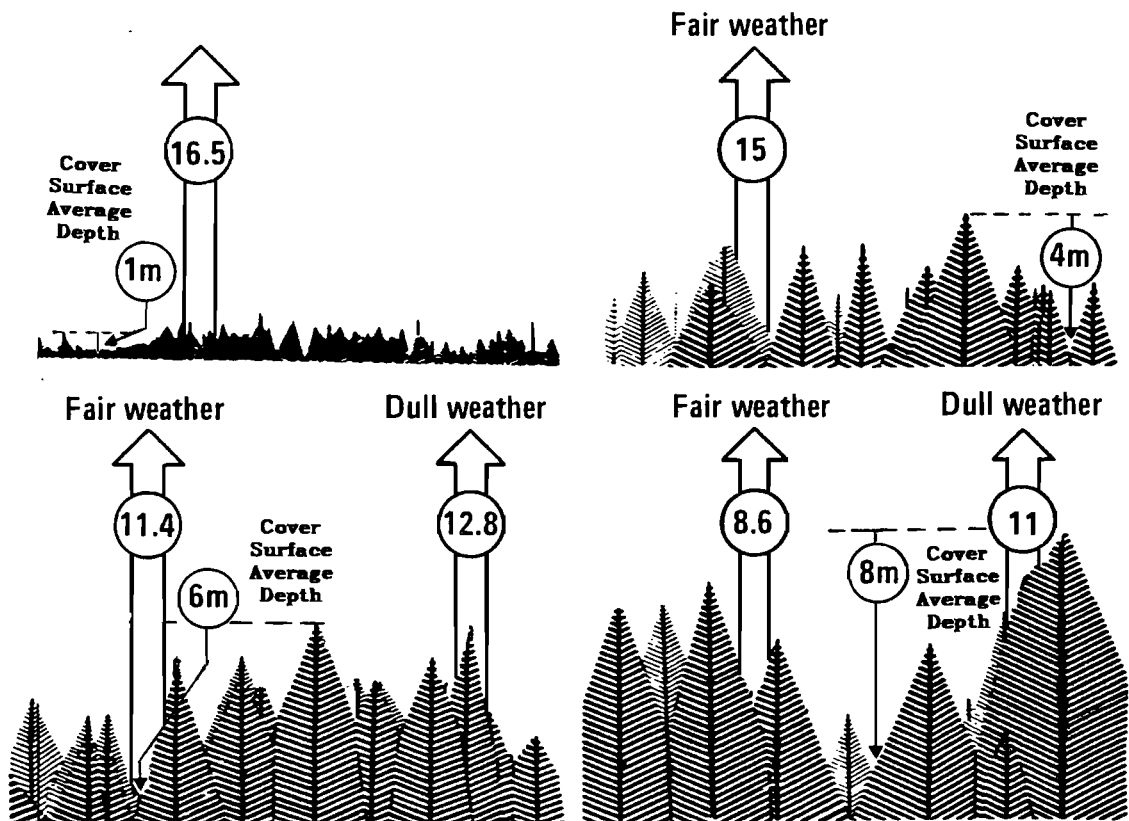


Figure 1. Albedo (in percent) from total solar radiation, depending upon the surface depth of spruce stand; in the total solar radiation case: at fair weather = 1.1–1.2 cal/cm² min; at dull weather = 0.4 cal/cm² min. Dubrava Experimental Forest, Kaunas Region, Lithuanian SSR, USSR.

Currently, many different methods are available to determine the optimal stand density, among which it is feasible to single out three major trends:

- a) the optimal stand density is established on the basis of investigations of growth processes of stand with different initial density (Timofejev, 1957; Kondratiev, 1959; Wiksten, 1965; Majorov, 1968; Kazimirov, 1972; and others);
- b) the optimal density is ascertained on the basis of the correlation between the indices characterizing the density of stands with different assessment indices of trees (Shustov, 1933; Reinecke, 1933; Wilson, 1946; Stahelin, 1949; Geworkiantz, 1947; Becking, 1954; Kramer, 1966; Thomasius, 1978; and others).
- c) the optimal stand density is determined on the basis of the dependence of the annual increment magnitude on stock density (Assman, 1961; Matuzanis et al., 1966; Zagrejev, 1962; Kozhevnikov, 1971, and others).

We have verified a vast number of methods and have noted that most of them are far from perfect for the construction of dynamic models of maximally productive stands. We carried out silvicultural/physiological investigations on the productivity of stands and class structure of trees in storeys (Kairiukstis, 1973). We find that the stand density reflects tree growth conditions better than the stock

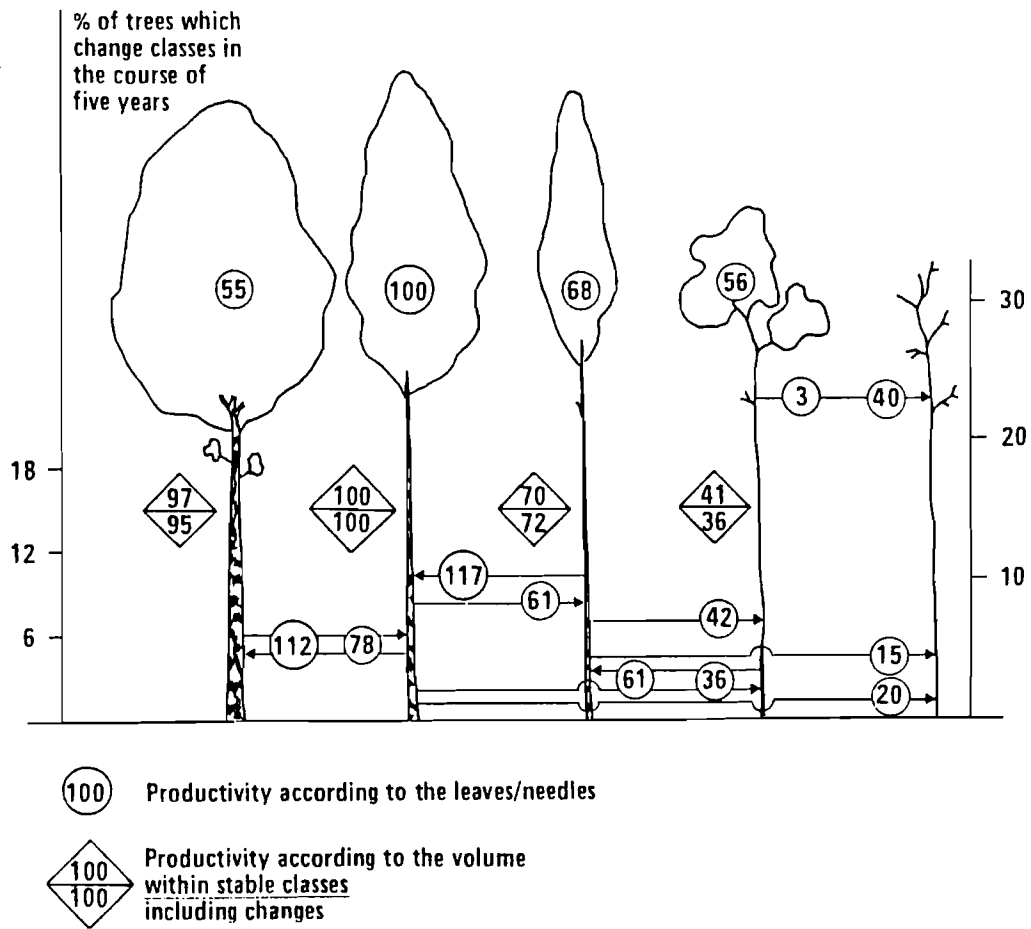


Figure 2. Productivity of trees of different development classes and its various stages during the transitions to other classes (in % from the productivity of class A).

density. We arrived at the conclusion that density optimization must be based on crown parameters of maximally productive trees (class A), on the optimum dynamic space for each tree in each time span. In such optimal space the highest annual increment of stock growth can be ensured. With regard to the crown, the following must be mentioned. First, the crown is a sensitive index simultaneously reflecting the development and productivity level of a tree and its state in space (canopy). Second, as previously determined, the correlation of the magnitude of annual tree increment with horizontal crown projection area is considerably closer ($r = 0.75$) than with growing space ($r = 0.47$). Therefore, the crown was selected as the main criterion of stand density optimization.

Thus, we can infer that the search of the optimal crown parameters and optimal space rates for each age class will enable the optimal stand density to be determined for the entire period of stand growth.

We have investigated growth characteristics and formation of stands in various phases of their development. We discovered a new phenomenon, the so-called *stress effect* which occurs during the process of creation of forest ecosystems (Kairiukstis and Juodvalkis, 1975). We also discovered fixed limits of the critical approach of crowns at which trees enter into an intra-specific competitive interrelationship, resulting in a high increment decrease irrespective of soil or climatic conditions. The limit of critical approaches of crown may be

expressed as follows:

$$y_{(cm)} = 73.2 - \frac{24.8}{x} \quad (\eta = 0.979) ,$$

where

x = height of tree, in meters (0.5–5.0).

Further, it was found that following the critical limit and ecosystem creation during the subsequent approach of crown, the mutual suppression of trees decreased while annual growth increment increased (Figure 3).

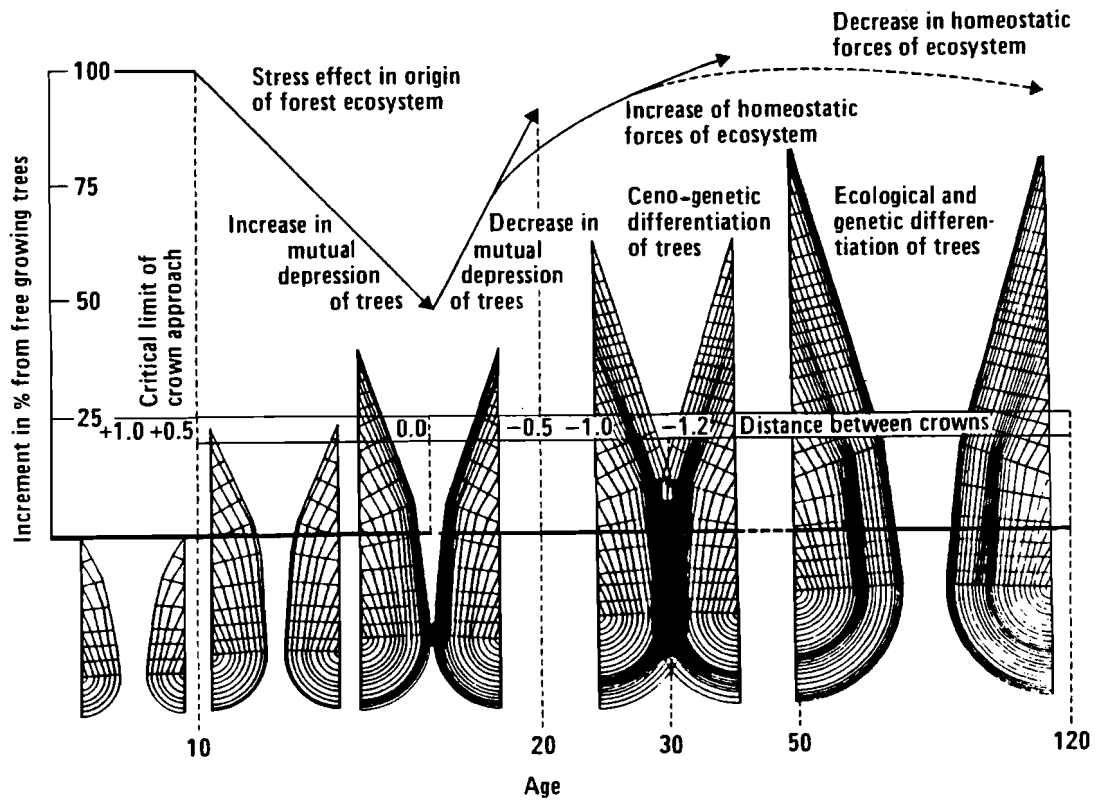


Figure 3. Generalized scheme of origin and formation of forest ecosystem.

Taking into consideration the above phenomenon, the optimal density criterion cannot be identical during the whole period of stand growth even when the aim is the same--to grow maximally productive stands.

Striving for significant growing stock increment in a young stand is senseless. In young stands, maximal growing stock involves considerable growth retardation of individual trees. Consequently, stand productivity and stability decrease and the stands mature later. Thus, in young stands, before the stress effect occurs, the optimal density must be considered such that it eliminates intra-specific competition, ensures maximal height and diameter increment for the greatest possible number of trees. Also, due to such an optimal density, the culmination of

maximal productivity occurs later and contributes to the total productivity during felling rotation and to speeding up the maturity process. During this period, the optimal stand density is determined by crown parameters of well-developed trees and the critical distance among crowns. It is established according to the following formula:

$$N_{opt} = \frac{Q}{S} ,$$

where

- N_{opt} = optimal number of trees/ha;
- Q = maximum possible crown cover area during crown closure, m²/ha;
- S = crown and root space for one tree (m²) at the critical distance between crowns.

Following the effect of stress and ecosystem creation, the optimal density in forest stands is that which provides maximal annual increment of the growing stock and maximum total stand productivity. Such density also provides the greatest timber volume by the age of final felling.

Our investigations revealed that stands meet such requirements when the following three conditions are combined in them:

- a) crown cover is maximized;
- b) the stand consists of maximally productive trees (class A according to the authors, or class I.8–II.2 according to G. Kraft);
- c) maximally productive trees are distributed at an optimal distance from one another.

Conforming with the **first condition**, *maximal stand productivity is obtained in the case where the ecosystem utilizes the space maximally, i.e., in the case where the area of the crown cover is maximal*. The investigations show that natural stands, regardless of their species composition and age, do not attain potentially possible crown closure and do not entirely utilize the whole complex of soil/light conditions. Therefore, they do not give their potentially possible productivity. The fact is, that in any stand there is always a certain area of "windows" or some small glades where trees might grow. Depending upon the species and age, the area of such windows in natural stands comprises 5–15% of the experimental plot area. These windows are artificially filled with trees while processing stand data in the laboratory. Then the area of maximal crown cover and that of the inevitable openings are ascertained. We have determined that the maximally possible crown cover area depending upon species composition and age comprises 7500–9000 m²/ha, while the area of inevitable openings is 10–25%.

Under the **second condition**, *maximal stand productivity is achieved only when possible crown cover area is chiefly covered by crowns of maximally productive trees, i.e., trees with the most productive crowns*—maximal annual increment of stem wood per 1 m² of crown area (Figure 4). Relative crown productivity of trees of various development classes displays great diversity. Within a stand there is a particular crown area for each individual tree in which the highest relative crown productivity is attained. An optimal area is established according to the extremum in the curve of the dependence of relative crown productivity on its horizontal projection area. We have ascertained that within a stand, the optimal area of horizontal crown projection is close to the mean crown area of well-developed trees (class A). Therefore, for practical purposes, in order to determine the optimal horizontal crown projection area, it is enough to establish the mean crown area of well-developed trees.

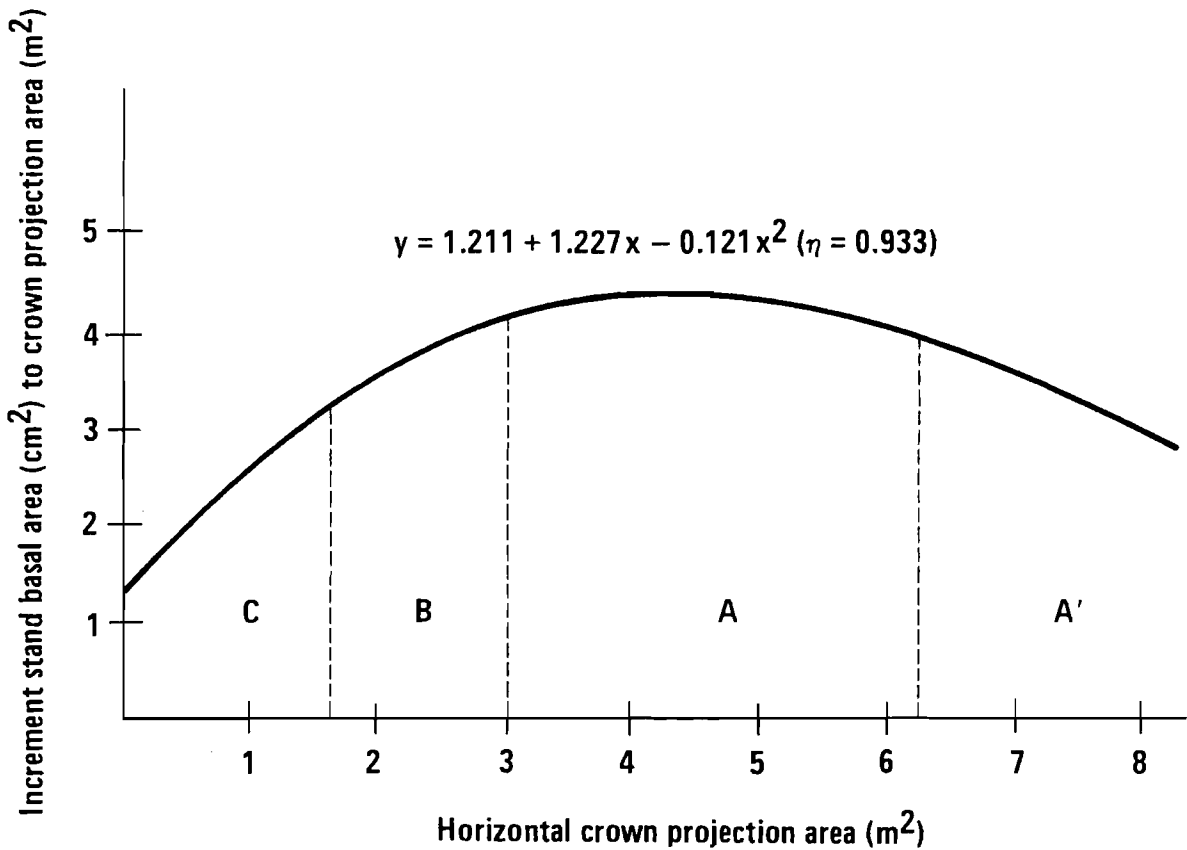


Figure 4. Dependence of relative crown productivity on its size (22 year-old oak stand; A¹, A, B and C classes of trees).

The investigations and theoretical calculations have indicated that when a stand consists of well-developed trees, and depending upon species and age, its productivity is by 5–25% higher than the annual increment of natural stands of the same density.

According to the **third condition**, *maximal stand productivity is achieved when the crowns are situated at an optimal distance from one another with optimal mutual overlap.*

A search for the best index reflecting the distance among trees showed that the correlation between the magnitude of the annual tree increment and the distance among trees is highest when the distance is expressed through crown parameters. The extent of crown overlap must be allowed for. Therefore, in the proposed method of stand density determination, the indices of the distance among trees may be replaced by those of crown diameter and the extent of the optimal crown overlap.

The optimal crown overlap is established according to the extremum of the curve of the dependence of the annual stand increment magnitude on the extent of crown overlap (Figure 5). It in turn, is determined from the data of the curve of the dependence in individual trees on the extent of their crown overlap. We have explained that within a stand the optimal crown overlap is similar to the mean overlap of crown of well-developed trees. Consequently, for practical purposes,

the indices of the optimal crown overlap may be replaced by those of the mean crown overlap of well-developed trees (class A).

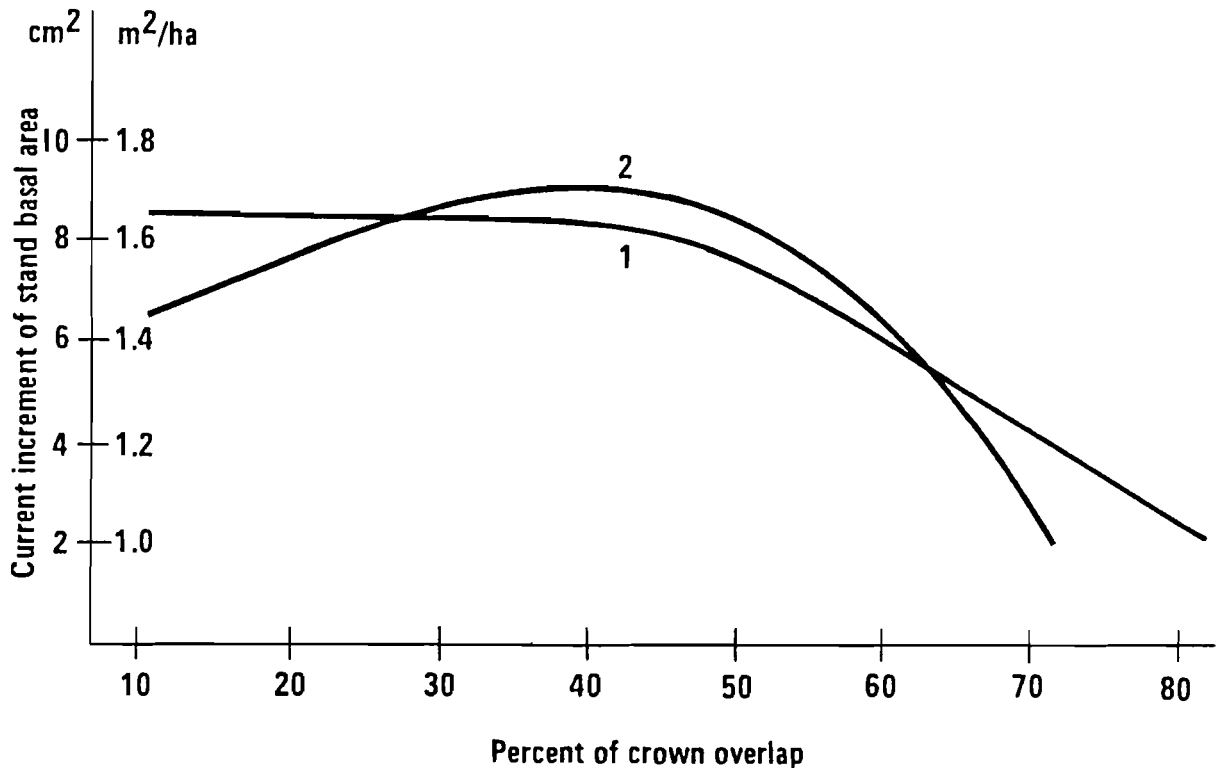


Figure 5. Dependency of annual increment of tree (1) and stand (2) basal area on the extent of crown overlap (36 year-old spruce stand).

Consequently, the optimal density of middle-age and those of maturing stands is ascertained with the help of the following formula:

$$N_{opt} = \frac{Q_{max}}{S_{opt} \left(1 - \frac{P_{opt}}{100}\right)}$$

where

- N_{opt} = optimal number of trees, trees/ha;
- Q_{max} = maximum possible crown cover area; m²/ha;
- S_{opt} = optimal area of horizontal crown projection of well-developed trees, m²;
- P_{opt} = extent of optimal crown overlap, %.

While establishing the optimal density and constructing the models of maximally productive one-storied stands, great attention was paid to the external and internal relations between single structural elements of a tree and a stand.

For two-storied stands, the optimal growth conditions for trees of the most valuable species was also ensured. The total productivity of mixed stands was considered to be negligible.

The stand growth and formation regularities and correlation between the single assessment indices of a tree and a stand as well as the method of stand density determination enable the construction of models of maximally productive stands. The models include pure and mixed stands formed from the main species of the Lithuanian SSR according to predominating forest types.

3. BLOCK SCHEME OF THE MODEL

Figure 6 presents a block diagram of the algorithm. The values of the parameters are:

- A = age, year;
- S = section area in ha;
- Q_{plot} = area of the layer projection in the section, in m^2 ;
- Q = area of the layer projection in m^2/ha ;
- q = area of crown projection of an individual tree without overlapping, in m^2 ;
- q_p = crown projection area of an individual tree in m^2 ;
- q_{opt} = crown area of the optimal projection in m^2 ;
- q_v = mean area of a crown projection in m^2 ;
- N = number of trees in the area unit;
- Z_g = increment of stand basal area of a separate tree in m^2 ;
- Z_G = increment of a stand basal area, in m^2/ha ;
- d = diameter of a tree in cm;
- d_{opt} = optimal diameter of a tree in cm;
- Z_r = radial increment during a one year period in cm;
- P = percent of the overlapping crown;
- T = index of a forest type;
- η = index of a tree species;
- h = height of a tree in m;
- h_{opt} = optimal height of a tree in m;
- G_A = optimal sum of stand basal area in m^2/ha ;
- N_{opt} = optimal number of trees in ha;
- M_A = optimal volume of a stand, in m^3/ha ;
- F = optimal form index; and
- a = equation parameters.

The model algorithm consists in determining the maximum density of the layer, calculating the area of the horizontal projection of the optimal crown, determining the optimal overlapping percent of the optimal crown, and calculating the number of trees of a certain age. To determine the maximum density of the layer (Block 2), one should use experimental plots in the most dense stands in the whole range indicated in the constraints to draw the plan of tree locations and crown projections. The whole area of crown projection (q_p) and area q , except when overlapping, are presented separately for each tree. These measurements are necessary to determine the area of inevitable glades in the stand and to calculate the optimal crown overlap. If the stand contains "windows" (or squares), i.e., areas which are greater than that of an average tree crown, a designated quantity of trees is artificially included in them and their crown are included in the initial

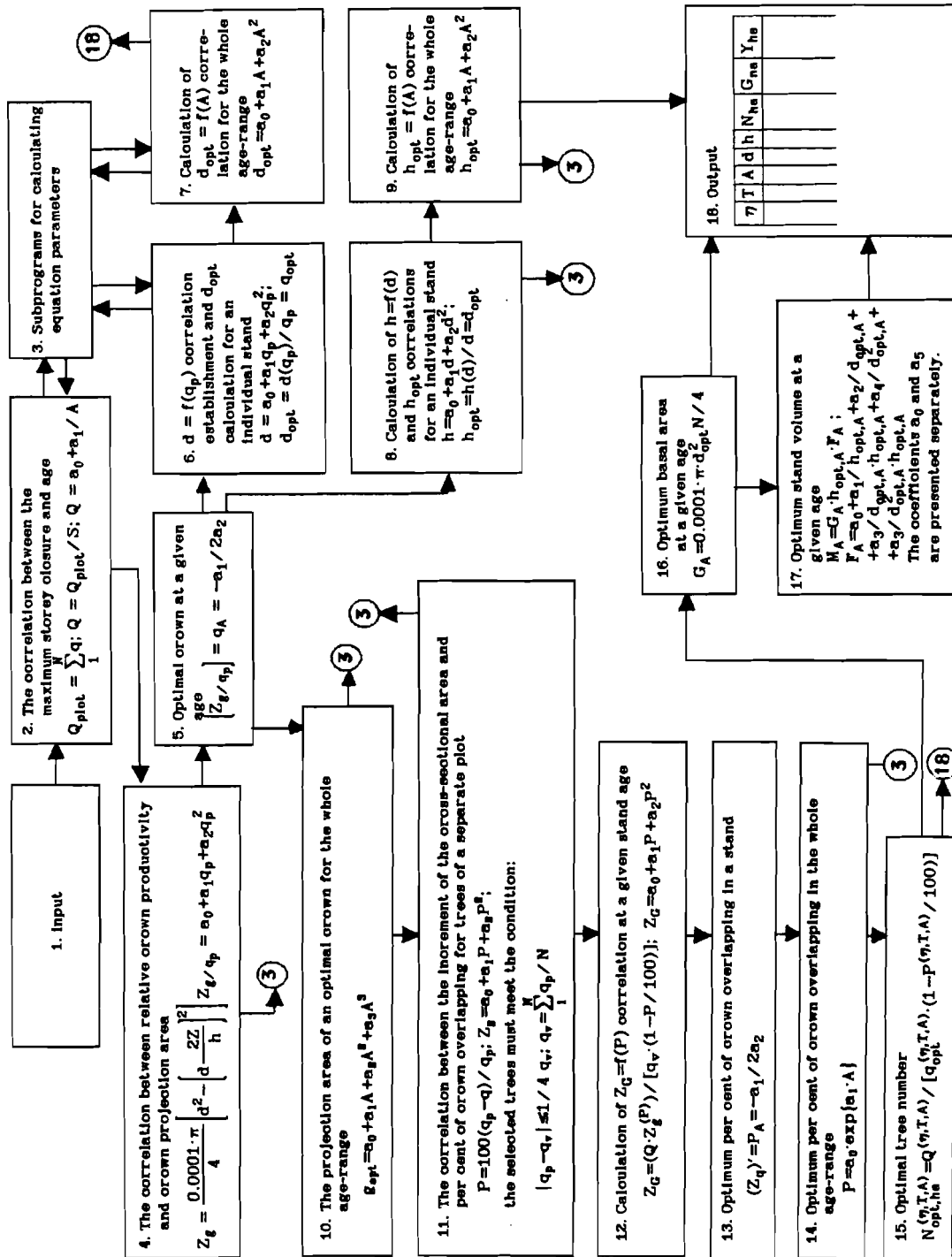


Figure 6. A block scheme of model for construction of standards of maximally productive stands.

information. The maximum density of the layer in 1 ha area is found by the computer. This is the sum of the crown projection areas without overlapping.

In order to determine the optimal crown parameters, the correlation between Z_g and g_p at a particular age is being sought. The crown producing the maximum wood increment in the area unit is considered optimal. Having the optimal crown parameters for a particular age interval, we find its relation with age in all the age ranges (Blocks 4, 5 and 10).

When the maximum layer density and the optimal crown area are known, the optimal number of trees in the area unit can be found. For this purpose, however, the optimal distance between the trees must be found. In the model, this distance is established in terms of the optimal crown parameters and the percent of optimal crown overlap. The calculation of optimal percentage of crown overlap is described in blocks 11-14. As a result of the calculations, the optimal number of trees, the optimal stand base area and volume affecting the maximum wood increment are obtained for the given type of forests and tree species (see Block 10).

4. STANDARDS OF MAXIMALLY PRODUCTIVE STANDS AND THEIR APPLICATION

Based on the above-mentioned model, the standards of the most productive stands for pure stands and main groups of species' composition and site conditions have been elaborated. These standards express optimal indices of stands in different periods of their growth. The standards reflect optimum stand density and structure at any age.

Determination of the optimal stand density and the model constructed solves only the first part of the problem, i.e. establishing standards of the most productive stands. The point is, that in nature there are practically no stands with optimal density and structure. The stands which most conform with the calculated standards must be formed artificially by intermediate cutting. Thus, a forester needs thinning programs with the aid of which he is able to create optimal stands. For this purpose it was necessary to know in detail all the regularities of change in stand growth and productivity occurring under the influence of intermediate cutting.

To tackle these problems, we used the data from more than 300 permanent experimental plots in which intermediate cutting of different density was applied 2-6 times. This allowed us to determine the following: the extent and duration of tree response to thinning, the dependence of the size of the annual timber volume increment on the extent of thinning, the optimal and critical extent of thinning (Figure 7). The optimal terms of re-thinning and the optimal regime of intermediate cutting in stands of different species composition, structure and age was established (Kairiukstis and Juodvalkis, 1985). An example for spruce (*Picea abies*, Karsten) is given in Figure 8.

Hence, programs to form maximally productive stands were elaborated and suggested for practical purposes. The essence of the programs lies in the fact that for a regular repetition of intermediate cutting, the standards discussed above indicate the number of well-developed (class A) trees, stand basal area, and timber volume that must be left after thinning. The number of trees is ascertained according to the conformity of the stand with the optimal density in the middle of the period between the two applications of intermediate cutting.

Table 1. Number of trees (trees/ha) that must be left after felling in pure stands of different species depending upon the mean height (m) of well-developed trees when opening of stands and clearings are repeated every five years while thinning and intermediate cutting every ten years.

Height of well-developed trees	Oak (<i>Quercus robur</i>) stands	Ash (<i>Fraxinus excelsior</i>) stands	Spruce (<i>Picea abies Kar</i>) stands	Aspen (<i>Populus tremula</i>) stands	Birch (<i>Betula verrucosa</i>) stands
3			2400		
4			2390		
5	5380	5360	2370	5910	5540
6	4250	4280	2320	5240	4900
7	3460	3530	2270	4640	4330
8	2890	2970	2210	4110	3830
9	2450	2540	2140	3650	3390
10	2100	2200	2070	3230	2990
11	1830	1920	1980	2860	2650
12	1600	1690	1890	2540	2340
13	1410	1500	1800	2250	2070
14	1250	1340	1700	1990	1830
15	1110	1200	1590	1770	1620
16	990	1070	1490	1560	1430
17	890	960	1380	1390	1270
18	800	870	1280	1230	1120
19	710	780	1180	1090	990
20	640	710	1080	960	880
21	580	640	980	860	770
22	520	580	890	760	680
23	470	520	800	670	600
24	420	470	720	600	540
25	370	420	650	530	470
26	330	380	590		420

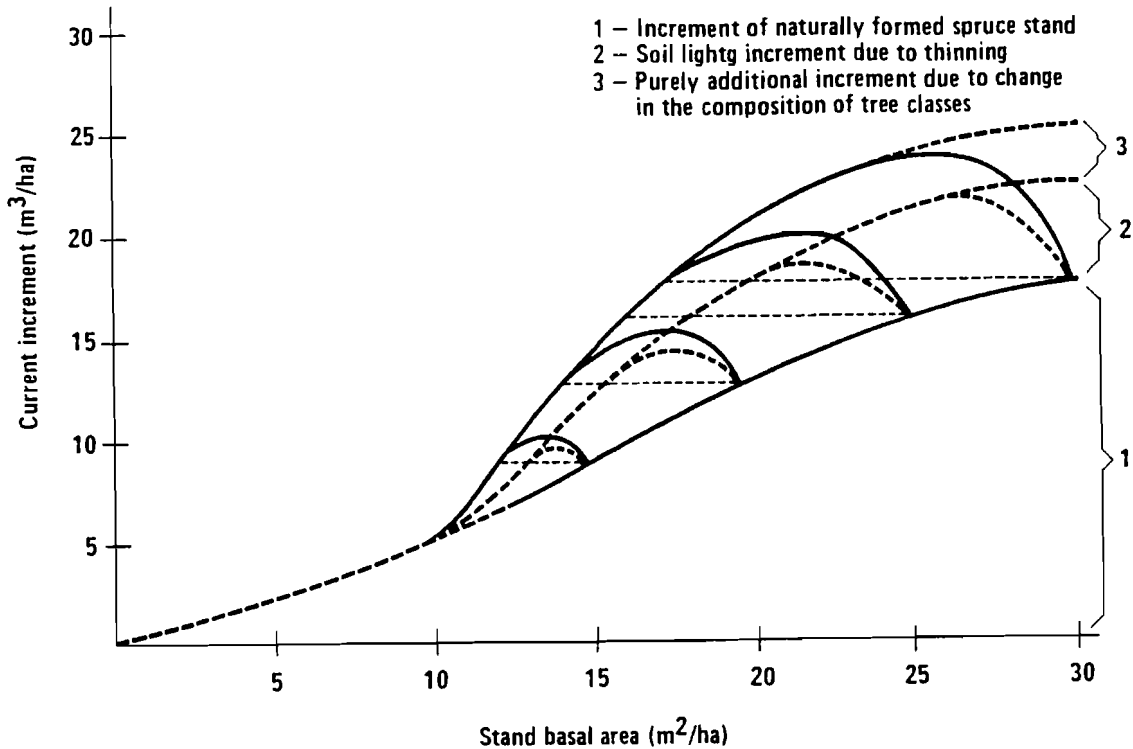


Figure 7. Homeostatic abilities and thinning effect (30 year-old spruce stand).

- horizontal dashed line - - - = reduction of stand basal area by thinning;
- convex curves ——— = extent and duration effects of thinning.

The standards are assigned not only for carrying out intermediate cutting but also in projecting forest management by computer. The main assessment index in carrying out intermediate cutting is the number of trees that must be left while projecting the intermediate cutting - the volume of cutting-timber that is to be left after felling.

In order to simplify the standards for practical use, the assessment indices of trees that must be kept are expressed depending upon the mean height of well-developed trees. A generalized standard for forest types of similar productivity can thus be applied.

In the process of standard forest formation in practice, the assessed indices of the specific stand at a certain height are compared with the corresponding indices given by the thinning program (Table 1). In the planning of intermediate cuttings, the volume or stand basal area in the forest is compared with the corresponding indices for such calculations.

In selecting trees for cutting in typical places where stands are being formed, areas of 10x10 m or 10x20 are allocated. Here the mean height of trees and their total number are compared with the standard.

Cutting is applied on the allocated areas in young stands whereas in older stands, the trees are selected and marked for felling. These areas are examples for carrying out cutting or for selecting trees for felling in the rest of the plot.

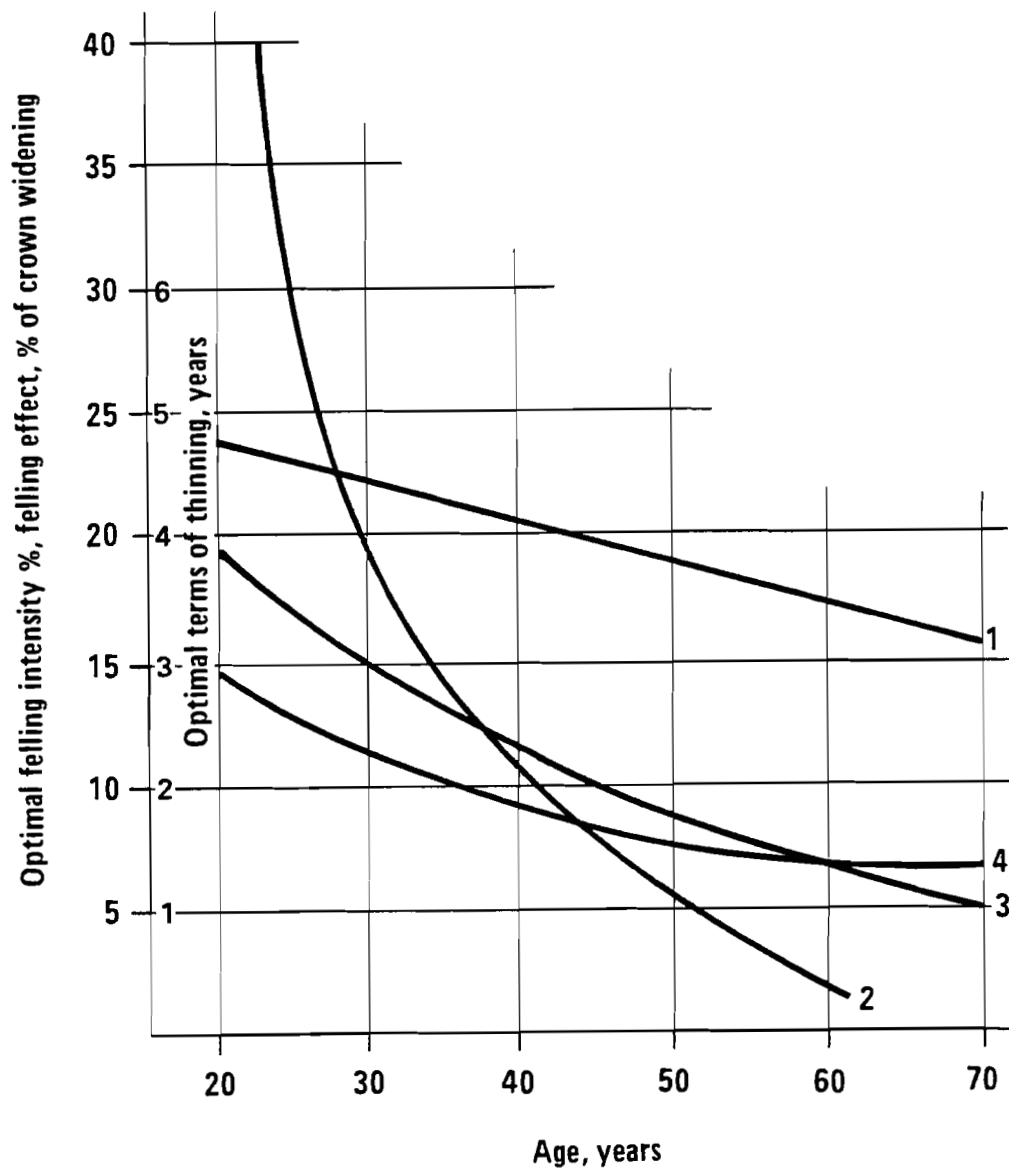


Figure 8. Optimum 6 time span of re-thinnings (1), maximum thinning effect (2) optimal intensity of thinning (3) and percent of crown widening (4) in *Oxadilosum* spruce stand depending upon age.

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