PULSE EXTRACTION UNDER RISK AND A NUMERICAL FORESTRY APPLICATION

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June 1987
WP-87-49

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FOREWORD

The presented study on discontinuous pulse extraction under specific assumptions about the stochastic price and growth processes is based on the results achieved during the Author's stay with the System and Decision Sciences YSSP Program, 1985. It was motivated by the forestry application described in the paper. The general method can be useful in the modeling of optimal use of other natural resources as well.

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ACKNOWLEDGMENTS

The author is grateful to Academician A. Kurzhanski, the leader of the System and Decision Sciences Program at IIASA, where this work began in 1985. Professor J. Dupačová at IIASA and Professor K.G. Löfgren at the Swedish University of Agricultural Sciences gave valuable comments on an earlier version of this paper.

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ABSTRACT

The problem of optimal pulse harvesting of a resource under risk is discussed. The fundamental importance of stationarity in the stochastic process is investigated and the limiting optimal stopping rule is derived. A numerical example from forestry is used to discuss the expected present value and the optimal stopping criterion as functions of time.

Finally, the probability distribution of different optimal harvesting ages is calculated.

In 1987, a publication will appear closely related to this one. It will contain a purely analytical derivation of the numerical results presented in this paper. Furthermore, it will contain a more general numerical optimization model, where any first order autoregressive price process and any price-age relationship can be used. Hence, this paper is a "popular" version of the final paper. Preliminary results show that the qualitative results discussed in this paper hold in very general cases.

CONTENTS

1 Introduction

1.1 General introduction

Future prices and future growth are generally not known with certainty. The question under consideration is whether or not this fact has any implications for the optimal resource management.

It is assumed that pulse extraction (harvest) of the resource is optimal. This means that extraction does not take place continuously (which is often the case in for instance oil extraction) but discontinuously (compare Lohmander (19)). Maybe the most typical case of pulse extraction can be found in forestry. In the first production stage, the forest is planted. Then, in some cases, thinning is undertaken. Finally, maybe one hundred years later, the clear cutting takes place.

The optimal rotation problem was originally studied under the assumptions of a constant price and a deterministic growth function by Faustmann (8). A modern treatment of the Faustmann problem can be found in Johansson and Löfgren (13) where many comparative static results are derived.

The effects of the introduction of risk on the optimal management (investment and extraction) have the unpleasant property that they depend on different things.

First of all, the question is <u>where</u> the risk occurs. Risk in the growth process may have other implications for the optimal behaviour than risk in the price process.

The second question is <u>when</u> the risk occurs. Since the management of resources is an intertemporal problem, this is important to know.

Thirdly, it is important to have some knowledge of the <u>stochastic properties</u> of the process where the risk is present. As will be proved, the consequences of risk are quite different under stationary and non-stationary (for instance Martingale) processes.

1.2 Tools

The original work on dynamic programming was made by Bellman (2). The main part of the analytical tools can be found in the publications by Fleming and Richel (9) and Malliaris and Brock (21).

The concept "risk" has been defined according to Rotschild and Stiglitz (26) and Sandmo (30). An introduction to the different risk concepts and their analytical treatment is given by Hey (11).

1.3 Optimal stopping in resource economics

Norstrøm (23) treats growth as a deterministic process but allows the timber price to be stochastic. He proposes that; - The expected present value in the stochastic model is at least as great as the present value in the corresponding deterministic model.

Norstrøm treats price as a stationary Markov process. The proposition is easy to believe since you have the option to wait for a price that is better than the expected price when the price is stochastic. (Of course the expected price in the stochastic model is equal to the price in the deterministic model.) The stationarity assumption is, however, crucial.

Risvand (25) has defined a stochastic dynamic programming model for the cutting decision in forestry. The model is used numerically to study how some variables are affected by changes in parameters such as the rate of interest and the development of prices. The variables under investigation are the harvest decision criterion and the probability of cutting.

Risvand makes the same general assumptions as Norstrøm; Growth is deterministic and price stochastic. He calculates the expected present value and reservation prices in the forest under the assumption that prices are distributed according to a Markov chain estimated from Norwegian data.

Kao (14) calculates the optimal stocking level under growth uncertainty numerically and claims that; - The effects of various degrees of indeterminateness, or risk in growth prediction, are that for larger variance of growth prediction, optimal regimes involve shorter rotation, lower stocking levels and lower mean annual increment. Kao uses the maximum mean annual increment criterion (which is generally not consistent with the expected profit criterion) and dynamic programming.

Brock, Rotschild and Stiglitz (4) (BRS) assume that the value of the stand follows a discrete time Markov process of the kind (1.3.1)

$$X_{t+1} = X_t + \tilde{\varepsilon}_t, \ E(\tilde{\varepsilon}_t) = \mu \tag{1.3.1}$$

where $\tilde{\varepsilon}_t$ are independent, identically distributed increments with expectation μ. They claim that the optimal strategy is a barrier strategy under their assumptions. There is a critical size $\hat{x}$ such that the tree should be harvested the first time it reaches $\hat{x}$. BRS show that the critical size is the same under risk and certainty if $P(\tilde{\varepsilon}_t > 0) = 1$. Löfgren and Ranneby (20) show that the critical size is the same under risk and certainty for $P(\tilde{\varepsilon}_t > 0) < 1$. Miller and Voltaire (22) extend the analysis a bit further of the same basic model (1.3.1) through some comparative dynamics.

However, let us consider the process (1.3.2).

$$X_{t+1} = X_{t+1}(X_t) + \varepsilon_t, \ E(\varepsilon_t) = 0 \tag{1.3.2}$$

The first order autoregressive process given in (1.3.3) is a special case of (1.3.2).

$$X_{t+1} = a + bX_t + \varepsilon_t, \ E(\varepsilon_t) = 0 \tag{1.3.3}$$

Obviously, if we restrict (1.3.3) further and assume that b = 1, we get (1.3.1).

(1.3.1) is a nonstationary process. If, on the other hand, $a > 0$, $0 < b < 1$, then we have a stationary process. A special case of a stationary process is (1.3.4) if $a_t = a$ = constant over time.

$$X_t = a_t + \varepsilon_t, \ E(\varepsilon_t) = 0 \tag{1.3.4}$$

In order to show that the results obtained through the analysis of the nonstationary process (1.3.1) do not generally hold for stationary processes, the simpliest possible stationary problem will be discussed below;

Assume that the expected present value of the profit should be maximized and that price is a constant (= 1) and the rate of interest is zero. The quantity of the resource is (1.3.4). The probability density function of X_t is $F_t'(X_t)$

$$F_t'(X_t) > 0 \qquad \text{for } X_t > 0 \tag{1.3.5}$$

When X_t is known, the expected present value is $\phi_t(X_t)$.

$$\phi_t(X_t) = \max\left[X_t, W_{t+1}\right] \tag{1.3.6}$$

W_{t+1} is the expected present value if harvest does not take place at t.

$$W_t = \int_0^\infty \phi_t(X_t)F_t'(X_t)dX_t \tag{1.3.7}$$

Assume that X_t^* exists such that

$$\begin{aligned}
&[X_t > X_t^*] \rightarrow [X_t > W_{t+1}]\\
&[X_t = X_t^*] \rightarrow [X_t = W_{t+1}]\\
&[X_t < X_t^*] \rightarrow [X_t < W_{t+1}]
\end{aligned} \tag{1.3.8}$$

then (1.3.7) implies (1.3.9).

$$W_t = \int_0^{X_t^*} W_{t+1}F_t'(X_t)dX_t + \int_{X_t^*}^\infty X_tF_t'(X_t)dX_t \tag{1.3.9}$$

X_t^* is called the optimal reservation value and maximizes W_t;

$$\frac{\partial W_t}{\partial X_t^*} = (\underbrace{W_{t+1} - X_t^*}_{=0})\underbrace{F_t'(X_t^*)}_{>0} = 0 \qquad (1.3.10)$$

The second order maximum condition is obviously satisfied;

$$\frac{\partial^2 W_t}{\partial X_t^{*2}} = - F_t'(X_t^*) < 0 \qquad (1.3.11)$$

Let us calculate another derivative!

$$\frac{\partial^2 W_t}{\partial X_t^* \partial W_{t+1}} = F_t'(X_t^*) > 0 \qquad (1.3.12)$$

Differentiation of (1.3.10) gives (1.3.13)

$$\frac{\partial^2 W_t}{\partial X_t^{*2}} dX_t^* + \frac{\partial^2 W_t}{\partial X_t^* \partial W_{t+1}} dW_{t+1} = 0 \qquad (1.3.13)$$

$$\frac{\partial X_t^*}{\partial W_{t+1}} = - \frac{\dfrac{\partial^2 W_t}{\partial X_t^* \partial W_{t+1}}}{\dfrac{\partial^2 W_t}{\partial X_t^{*2}}} > 0 \qquad (1.3.14)$$

Obviously, the reservation value in period t is strictly increasing in W_{t+1}! Denote total derivatives by § signs.

$$\frac{\S W_t}{\S W_{t+1}} = \frac{\partial W_t}{\partial W_{t+1}} + \underbrace{\frac{\partial W_t}{\partial X_t^*}}_{=0} \frac{\partial X_t^*}{\partial W_{t+1}} \qquad (1.3.15)$$

Obviously we get;

$$\frac{\S W_t}{\S W_{t+1}} = F_t(X_t^*) > 0 \qquad (1.3.16)$$

And from induction;

$$\frac{\S W_t}{\S W_{t+n}} > 0 \quad (n > 0) \tag{1.3.17}$$

Hence it is clear that the expected optimal present value (in this case identical to expected harvest volume) in period t is dependent on all future periods.

Assume for simplicity a two period world. If W_2 would be 0, then W_1 is given in (1.3.18)

$$W_1 = \int_0^{\infty} X_1 F_1'(X_1) dX_1 = E(X_1) \tag{1.3.18}$$

From (1.3.17) and (1.3.18) it is clear that;

$$[W_2 > 0] \rightarrow [W_1 > E(X_1)] \tag{1.3.19}$$

Assume that $E(X_1) > E(X_2)$, which means that X_1 is the optimal harvest volume under certainty. Then, we have shown that the expected optimal harvest volume under risk is strictly larger than under certainty under some assumptions including stationarity.

1.4 Stochastic properties of prices

Obviously, different authors make different assumptions concerning the stationary properies of the processes.

However, since the effects of increasing risk may be different under different assumptions, we must consider the problem through theoretical and empirical analysis. Qualitative results based on the use of reservation price models may be irrelevant if the martingale assumption (or (1.3.1)) is in reality a good approximation of price movements.

On the other hand, some qualitative results for the optimal management of natural resources are extracted from models that are based on the assumption of Martingale prices (or (1.3.1.)). If it can be demonstrated that prices are in reality unlikely to be Martingales, (or (1.3.1.)), then the relevance of these results should be questioned.

A Martingale is a stochastic process P_t where $E(P_{t+1}) = P_t$. This can be described through equation (1.4.1.).

$$P_{t+1} = P_t + u_t \tag{1.4.1}$$

u_t is a stochastic variable with expectation zero. It is obvious from (1.4.1.) that $P_{t+2} = (P_t + u_t) + u_{t+1}$.

$$\begin{aligned}(P_{t+1} - P_t) &= u_t \\ (P_{t+2} - P_{t+1}) &= u_{t+1}\end{aligned} \tag{1.4.2}$$

If we assume that the stochastic variable u_t is not serially correlated (u_{t+1} is not correlated with u_t) then the price differences $(P_{t+2} - P_{t+1})$ and $(P_{t+1} - P_t)$ are also uncorrelated.
A first order autoregressive process is described in (1.4.3.).

$$P_{t+1} = a + bP_t + u_t \tag{1.4.3}$$

It is obvious that (1.4.1) is a special case of (1.4.3). If (1.4.3) is a stationary series, then a > 0 and b < 1. The expected price in steady state is $\mu = a/(1-b)$. Let us investigate the differences of (1.4.3)

$$\begin{aligned}P_{t+2} &= a + b(a + bP_t + u_t) + u_{t+1} \\ (P_{t+2} - P_{t+1}) &= ba + b(b-1)P_t + (b-1)u_t + u_{t+1} \\ (P_{t+1} - P_t) &= a + (b-1)P_t + u_t\end{aligned} \tag{1.4.4}$$

Assume that $P_t = \mu$. Since $\mu = a/(1-b)$ it follows from (1.4.4.) that $E(P_{t+1} - P_t) = E(P_{t+2} - P_{t+1}) = 0$. Furthermore, $(P_{t+2} - P_{t+1})$ is negatively correlated with $(P_{t+1} - P_t)$ since $(P_{t+2} - P_{t+1})$ contains the term $(b-1)u_t$ and $b < 1$ by assumption.

The intuitive interpretation of this is that the expected price change after a positive change from equilibrium is negative. Hence, the equilibrium is "stable" (stationary).

Samuelson (28) shows that price differences are uncorrelated over time (the martingale property). He writes - "This means that there is no way of making an expected profit by extrapolating past changes in the futures price, by chart or any other esoteric devices of magic or mathematics".

In 1971, Samuelson (29) introduces a model for stochastic speculative prices. He now assumes that harvest in a natural resource is stochastic in every time period (for instance weat) and that an optimal inventory is held. Assume that growth is extremely high a certain year. It is then optimal to distribute this extra quantity between the harvest year and future years (inventory). Thus, if the demand equation is downward sloping, the price decreases during the high growth year and the following years (during which the inventory is consumed). Hence price is serially correlated. The series is stationary thanks to the assumption that harvest quantity in each time period is stochastic and independent of other periods. (The expected harvest is constant over time.)

Note that the stationarity in this model depends on the assumption of a stationary demand equation, a stationary growth equation and stationary harvesting costs. If one of these factors would not be stationary, then there would be no reason to believe that the weat price is stationary.

Alchian (1) discusses the same problem as Samuelson (28). From the litterature survey above it is clear that no theoretical answer can be given in the general case to the question whether prices are Martingales or stationary processes.

The roundwood market in Sweden has been analysed by Brännlund, Johansson and Löfgren (6) and by Brännlund (5). They have described the roundwood market as an equilibrium and a disequilibrium. However, they demonstrate that the disequilibrium model does not give a significantly better explanation of the market behaviour than the equilibrium model. Kuuluvainen (15) has made a similar econometric model for Finnish conditions. Kuuluvainen also includes optimal raw material stock as an explanatory variable in the demand equation. Solberg (31) introduces ARIMA and transfer function models in the analysis of sawn wood prices. This is not a roundwood market model but included here since the area is related. Note however, that the roundwood market models do not give any qualitative information about the stochastic properties of the roundwood price. They are possible to use only if the parameters of the demand and supply equations are known in advance.

How is the future affected by the present according to the different assumptions concerning the parameters in (1.4.3.)?

$$\frac{\partial P_{t+1}}{\partial P_t} = b \tag{1.4.5}$$

$$\frac{\partial P_{t+2}}{\partial P_t} = b^2 \tag{1.4.6}$$

$$\frac{\partial P_{t+n}}{\partial P_t} = b^n \tag{1.4.7}$$

Let us investigate the value of (1.4.7) for different assumptions about b!

$b = 0$	$0 < b < 1$	$b = 1$
$\frac{\partial P_{t+n}}{\partial P_t} = 0$	$\lim_{n \to \infty} \frac{\partial P_{t+n}}{\partial P_t} = 0$	$\frac{\partial P_{t+n}}{\partial P_t} = 1$

(1.4.8)

Hence we can make the following conclusions:

- If the price is a martingale, then a price increase today means that we should expect the future prices to increase equally much. Thus it is not clear that we should increase harvest today. Maybe it is equally good to wait longer?

- If the price is stationary, then a price increase today generally implies changes in the expected prices in future periods. However, as the length of the time interval approaches infinity, the influence of todays price on the expected price in the future approaches zero. The prices are practically independent over time when the periods are long. This is the assumption made in section 2.

2. A numerical forestry application

The general stochastic pulse harvesting problem under risk in the growth and the price processes has been analysed by Lohmander (18).
He found that the qualitative effects of increasing risk depended on the choice of risk definition and the number of stochastic processes in the problem. An updated version of the analytical paper will appear in 1987.

In this section we will investigate the phenomenon of stochastic prices and the effects on optimal harvesting. This is likely to be the most significant stochastic problem in forestry with respect to profitability.

Of course growth is also stochastic. However, the net prices may vary 100 % over a few years and the price observations are almost costless. The standing volume in the forest stand varies only slowly and is generally much easier to predict. Furthermore, the growth is not directly observable and growth information is not costless. To get reliable estimates, a large set of measurements must be made in the particular forest.

The stochastic properties of wood prices in the future are of course not known. Let us take a look at the past developments of wood prices in Norway and Finland!

In figure 1 and 2, the round wood prices are given in the two Nordic countries. According to a time series analysis in Lohmander (18), they can be described as stationary autoregressive processes. Hence, as the reader can find in the figures, the correlation between the prices in "neighbour time periods" decreases as the time interval between periods increases.

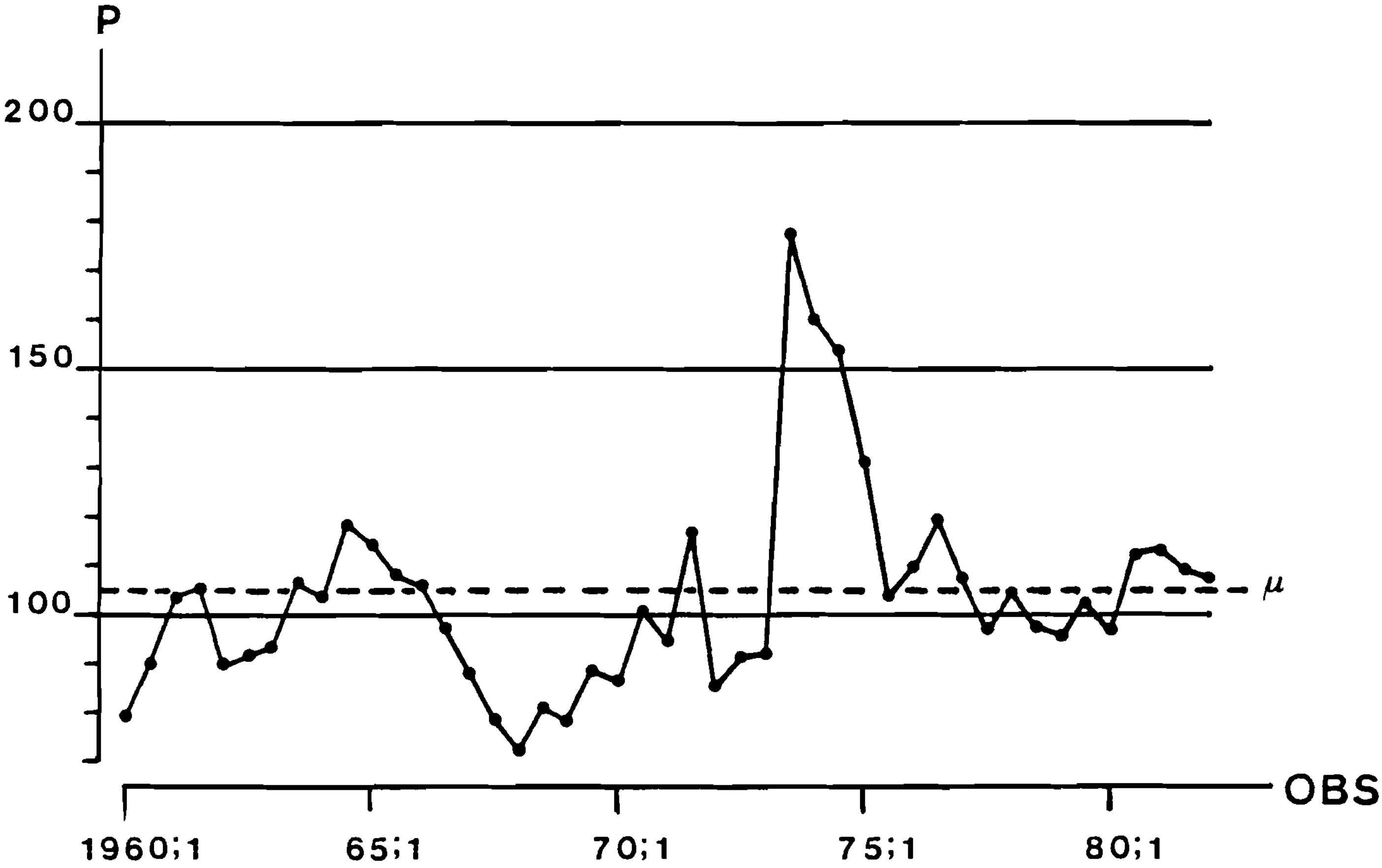


Figure 1. FINLAND, Real stumpage price (deflated by whole sale price index, 1977/2 = 100) Finnish mark/M^3. (Source; Kuuluvainen (15), Lohmander (18)).

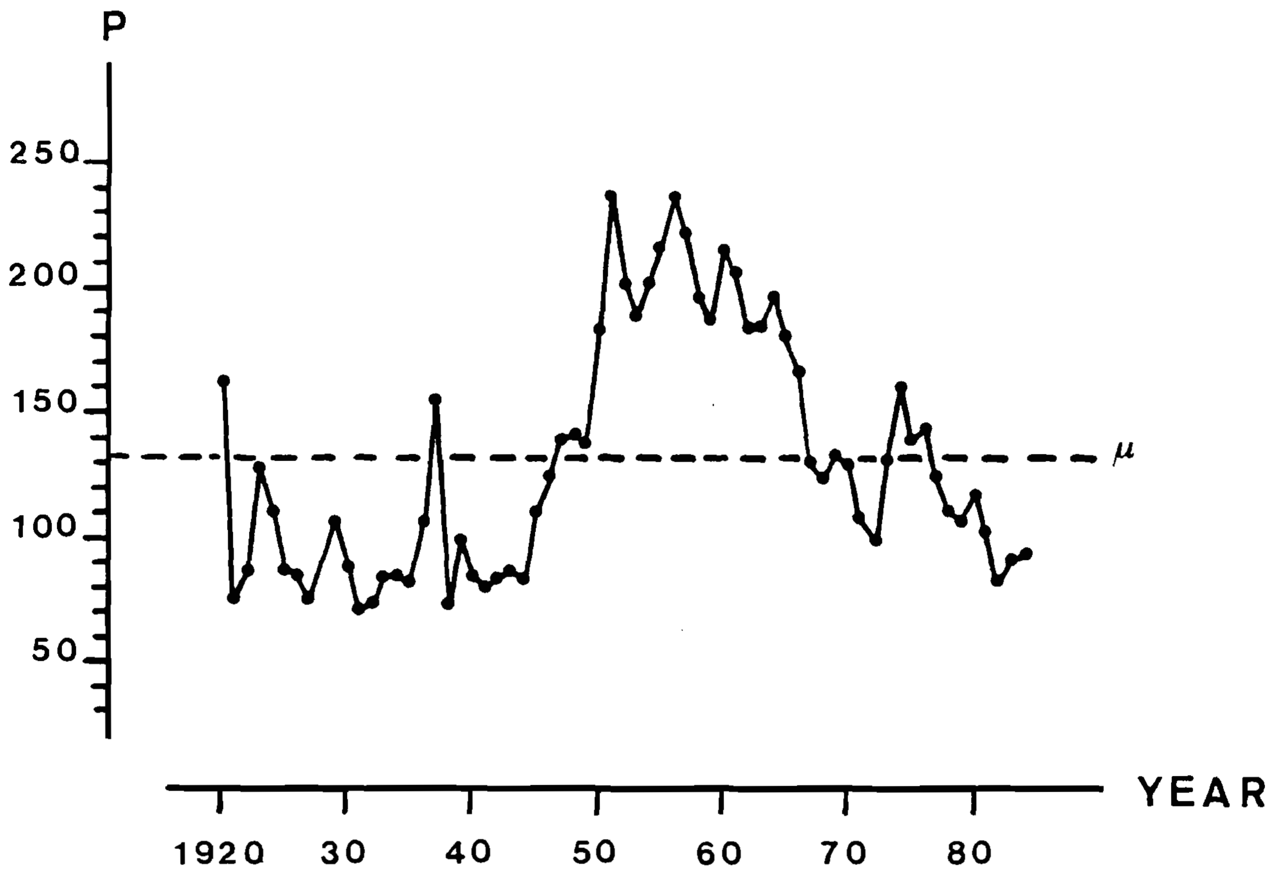


Figure 2. NORWAY, (mean price - variable costs) deflated by consumer price index (1979=100) Norwegian crowns/M^3 (Assumptions; 35 % high quality timber, 30 % low quality timber 35 % pulpwood) (source; Lohmander (18)).

2.1 General model assumptions

The object of the forest manager is to maximize the expected present value of the forest stand and the forest land.

In period t, the maximization problem can be stated as;

$$\max W(t) = W^{*}(t) = \int_{0}^{P^{*}(t)} W^{*}(t+1)F'(P)dP + \int_{P^{*}(t)}^{\infty} e^{-rt}[PV(t) + L]F'(P)dP \quad (2.1.1.)$$

where the variables and parameters are;

P	stochastic price process
W(t)	expected present value at time t just before P_t has been observed and the forest has not yet been harvested
$P^{*}(t)$	optimal reservation price at time t
L	expected present value of the land which is "released" at the time of harvest
F'(P)	Probability density function of P. The model can easily be generalized to the case of a deterministic trend in the price and $F'_t(P)$. $F(0) = 0,\ F(\infty) = 1$.
r	rate of return in the capital market
V(t)	Stand density as a function of time

2.2 General model results

Let us maximize the expected present value at time t. The problem is hence to choose $P^{*}(t)$ optimally. From (2.1.1) we derive (2.2.1), which is the first order optimum condition.

$$\frac{\partial W^{*}(t)}{\partial P^{*}(t)} = \{W^{*}(t+1) - e^{-rt}[P^{*}(t)V(t) + L]\}F'(P^{*}(t)) = 0 \quad (2.2.1)$$

If we assume that F'(P) is strictly positive everywhere, then (2.2.1) implies (2.2.2);

$$P^{*}(t) = \frac{e^{rt}W^{*}(t+1) - L}{V(t)} \quad (2.2.2)$$

(2.2.2) is the formula used to determine the optimal reservation prices through backward recursion. The second order maximum condition is easily investigated;

$$\frac{\partial^2 W^*(t)}{\partial P^*(t)^2} = -e^{-rt}V(t)F'(P^*(t)) < 0 \qquad (2.2.3)$$

Obviously we have an unique maximum!

(Clearly, if P_t is not stationary, and distant periods can not be regarded as independent, $W^*(t+1) = W^*(t+1, P_t)$. Then, an optimal reservation price may not exist. Furthermore, it may not be unique.)

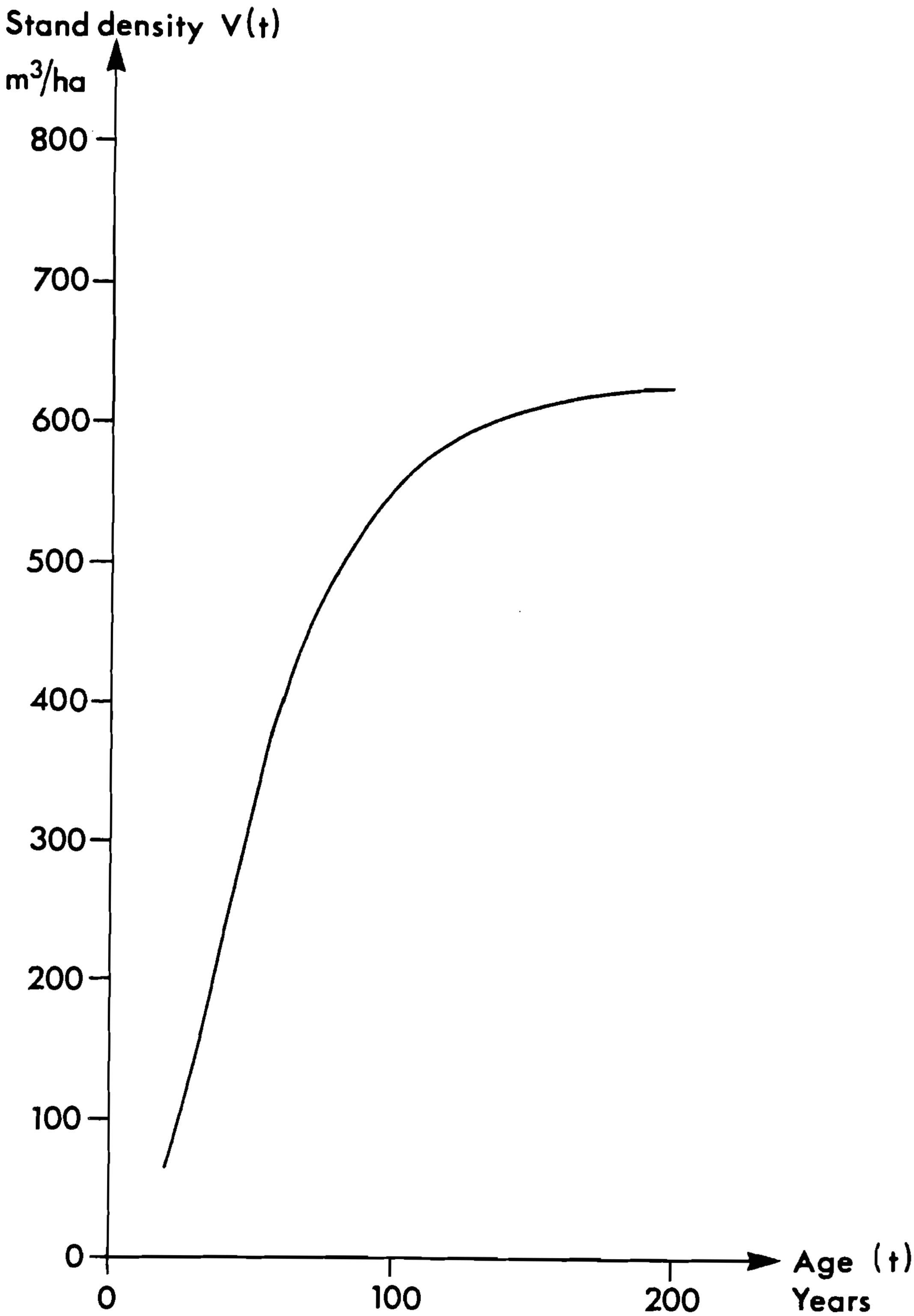
2.3 Numerical model assumptions

Since we need some numerical assumptions in order to get numerical results, we assume the following;

Volume function

Species; Pinus Contorta
Site index; H50 = 20 meters
1 500 stems/ha, no thinnings
MAI; 6.4 m^3/ha, year
Age at MAI; 60 years
$V(t) = 630.3744(1 - 6.3582^{(-t/60)})^{2.8967}$

(The empirical production data is presented in Hägglund (12) and the particular functional form of the volume function is suggested by Fridh and Nilsson (10).)



Stand density V(t)
m³/ha
800
700
600
500
400
300
200
100
0
0
100
200
Age (t)
Years

Figure 3. V(t)

Expected land value

L should in principle be determined endogenously as the present value of an infinite series of future forest generations. This would however complicate the analysis very much in the stochastic case since the rotation age in every generation is stochastic. The error made here treating L as a constant is likely to be small if L is small in relation to PV(t) for most P and t at the suggested rate of interest. Hence, L is given the value 1 000 crowns/ha.

Time periods

The last year under consideration (the time horizon) is year 200. The time scale consists of 40 periods, every period being 5 years. (It is assumed that there is one possibility to harvest every 5 year period.)

Rate of interest

r is given the value 3 %.

Net price distribution

The price is an independent identically distributed random variable (note that this is more or less consistent with stationary price processes as the length of the time periods increases according to (1.4.8)).

We assume that price has a uniform distribution with the probability density function F'(P)

$$F'(P) = \begin{cases} \frac{1}{b} & \text{for } a < P < a + b \\ 0 & \text{elsewhere} \end{cases}$$

	a	b
low risk	75	50
medium risk	50	100
high risk	0	200

(Note that E(P) = 100 crowns/m^3 in all cases.)

2.4 Numerical analysis

As a start, the deterministic optimum is calculated. We assume that the net price is 100 crowns/m^3. The present value as a function of rotation age is given in figure 4.

Obviously, the optimal rotation age is 45 years and the present value is 7 368 crowns/ha. (Note that the cost of the first regeneration is not included here!)

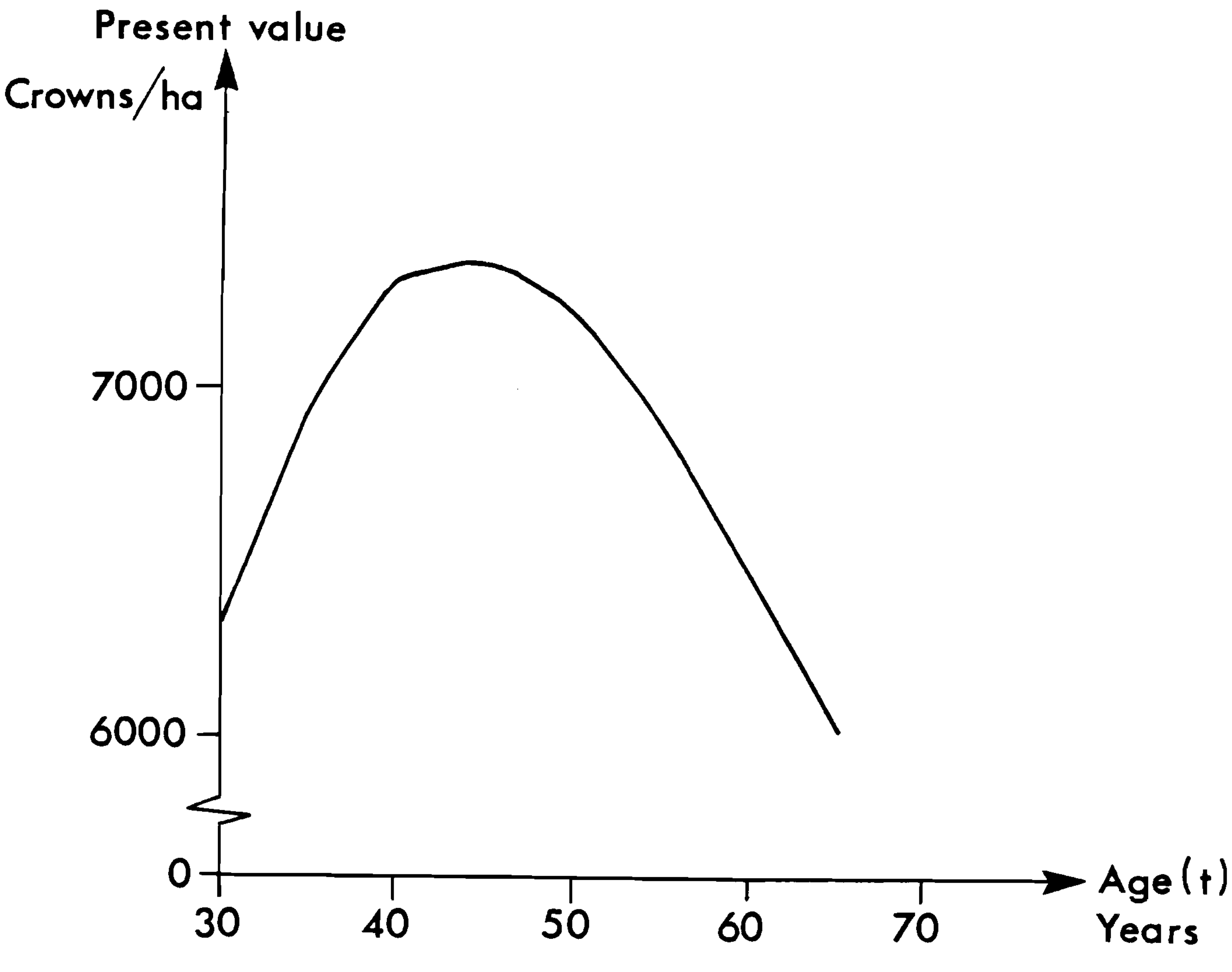


Figure 4. The deterministic case

Making use of the formulas (2.2.2) and (2.1.1), it is possible to calculate the optimal reservation prices and the expected present values recursively, starting from year 200 calculating backwards. We simply assume that the expected present value $W^*(T+1) = 0$, where T denotes the last period.

In figure 5, we find the optimal reservation prices as functions of time, one function for each risk assumption.

One should observe that the reservation prices increase with risk in the price distribution. Obviously one should require a high price in order to harvest during high risk conditions.

Another observation is that the reservation prices decrease as time goes. Of cource this depends on the particular growth function and the rate of discount. However, the Fridh and Nilsson (10) growth function has been fitted to a large set of different species and site indexes. The numerical phenomena discussed here hold for all cases.

The extreme value at year 200 is only an endpoint problem. However, it is very unlikely that we should wait so long anyway (compare figure 8.). In the next publication, the sensitivity of the expected present value to the last possible harvest year will be analysed.

A third observation is that the reservation prices are fairly constant within the time interval 80 - 195 years. Obviously, the optimizing forest owner should every year during that period have almost the same critical reservation criterion.

A final observation in figure 5 is that a person who owns a forest that is 180 years old (the optimal "deterministic rotation age" was 45 years) should not harvest his forest under all market conditions ! It is quite possible that it is optimal to wait longer!

Again, we should be aware that the growth function has been extrapolated in this particular case. However, the principles remain valid.

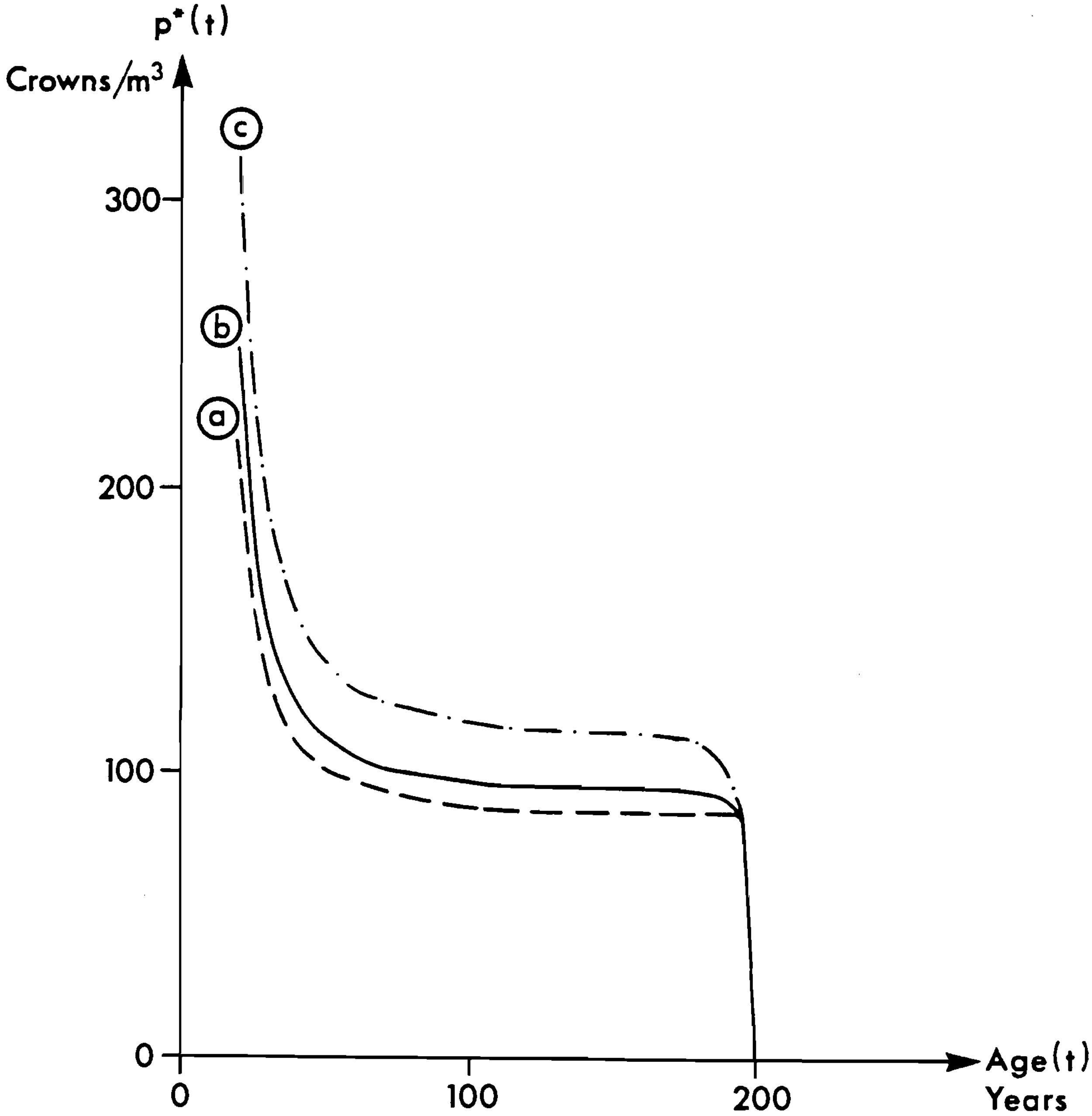


Figure 5. The optimal reservation price

a = low risk

b = medium risk

c = high risk

The graph of $W^*(t)$, the optimal expected present value, is given in figure 6. Again, there is an obvious difference between the different risk alternatives; when the risk increases, then the expected present value increases. It should also be noted that the expected present value decreases with time.

Assume that we select the reservation price $\underline{P}^*(t) = \infty$. (Note that $\underline{P}^*(t)$ is likely not to be the optimal reservation price!)

Then, from (2.1.1), (2.4.1) follows.

$$W(t) = W^*(t + 1) \tag{2.4.1}$$

However, since it is possible to select any reservation price and we should select the optimal one, it is clear that (2.4.2) follows;

$$W^*(t) > W^*(t + 1) \tag{2.4.2}$$

(2.4.2) is quite consistent with figure 6. The intuition behind (2.4.2) is that when the time is t, then we can always choose to wait without any harvest activity until time t + 1. It can not be worse to have the option to wait one period than to be in the next period directly. Hence, $W^*(t)$ must be at least as high as $W^*(t + 1)$.

Furthermore, when the time is t, there may be a positive probability to obtain a price in that period which makes it better to harvest than to wait until t + 1. Thus we get (2.4.2).

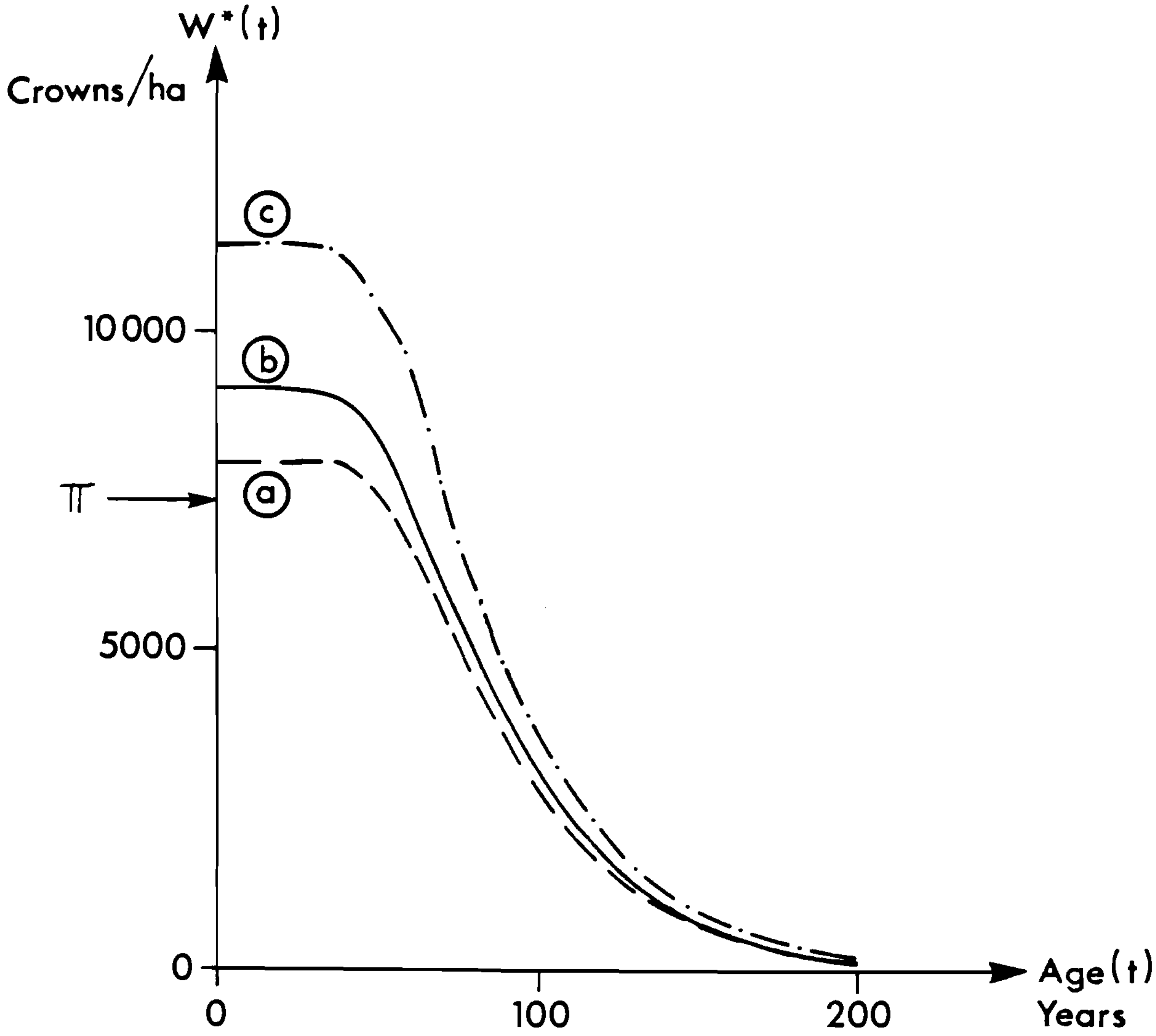


Figure 6. The optimal expected present value

a = low risk

b = medium risk

c = high risk

Π = maximum present value in the deterministic case.

Figure 7 shows the probabilities (at year zero) that different time periods are the optimal harvest periods under the different risk regimes. It it clear from the graphs that the optimal harvest age is not "deterministic". It is rather likely that we harvest ten years before or 15 years after the deterministic rotation age 45 years.

Of course, when the risk decreases, we approach the case of a constant price. Then the rotation age is 45 years with probability 1. The graph shows that the probability distribution function becomes more flat as the risk in the price distribution increases. In the case of high price risk, the probability that you should harvest at the age of 75 years is 2.2 %. In the low risk case, this probability is only 0.4 %. Again, similar qualitative numerical results have been obtained when other site indexes and species have been tested.

Figure 8 is closely related to figure 7. It shows the probabilites (at year zero) that it has not yet been optimal to harvest the stand. In other words, figure 8 also shows the probability (at year zero) that the stand is still "alive" at a given age.

Again, we find that the probabilities of early and late harvests are larger under the high risk regime than otherwise. Maybe figure 8 is the explanation why we today in some countries observe forest stands that are much older than what is recommended in forest acts? Maybe it simply is optimal to wait for better prices?

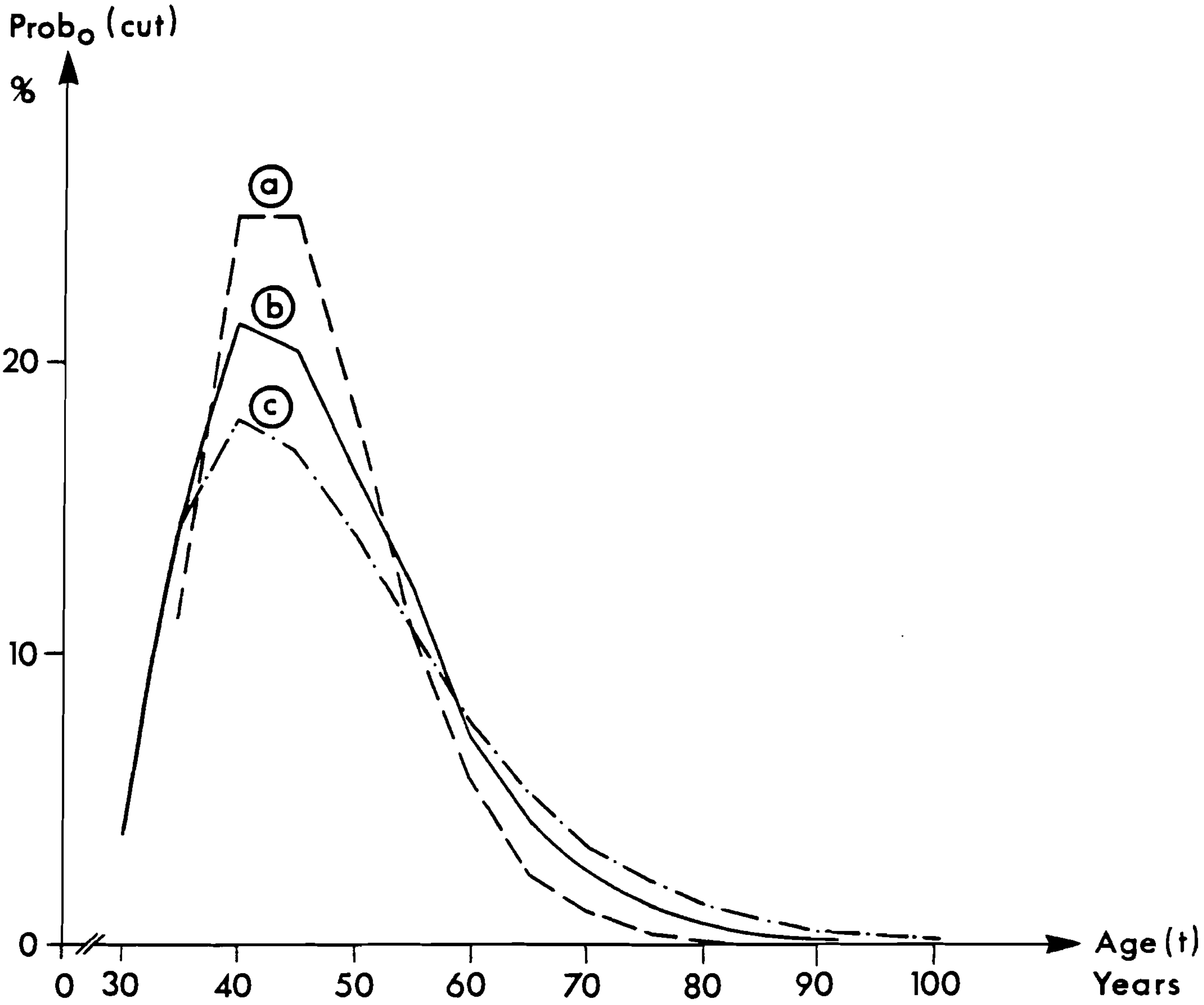


Figure 7. The probability (at year 0) that harvest is optimal.

a = low risk

b = medium risk

c = high risk

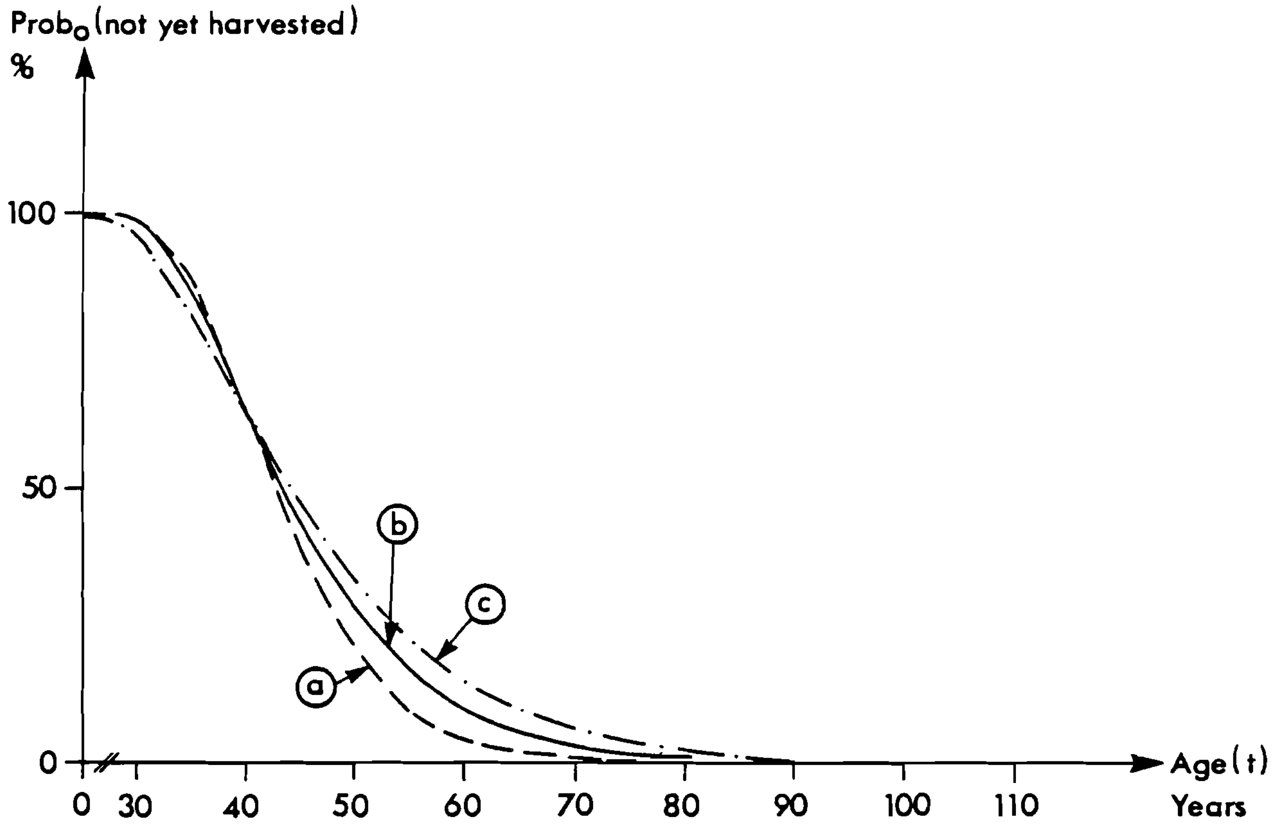


Figure 8. The probability (at year 0) that optimal harvest has not yet taken place.

a = low risk

b = medium risk

c = high risk

3. Discussion

It is always dangerous to draw conclusions based on results from particular numerical models. Furthermore, any model is just a model of reality. Important relationships in reality may be forgotten. The main limitations in this numerical analysis have been;

a. The growth function is very rough and based on a limited set of parameters. Furthermore, the production tables are based on a limited set of field experiments. Hence, the reliability is probably low, especially at high ages. Of course, in the numerical model, the growth function is extrapolated far outside the experimental material. This should be considered, but since the most probable harvest ages are rather low (compare figure 7), this is not critical to the derived results.

b. The price probability density function has no empirical support. However, compare figure 1 and figure 2, the price is (or has been) far from deterministic and constant, which is usually assumed in forest economics. Furthermore, three different risk cases are given which make it possible to look at the sensitivity to price risk assumptions. The author is convinced that future prices can not be perfectly predicted. The question is only the level of unpredictability.

c. It is quite possible that it would be optimal to do some thinning before the final felling. However, it is at the present difficult to make quantitative studies of the optimal thinning pattern over time. The reason is that existing production functions (at least in Sweden) are based on production experiments with "traditional" and fairly intensive thinnings (compare Eriksson (7)). When it comes to Pinus Contorta, some of the main Swedish producers advocate the method of no thinnings. Wieslander (32) and Bjurulf and Freij (3) have found that forestry without thinnings is more profitable than management according to traditional methods with thinnings in stands with Picea Abies in the south of Sweden.

d. It is quite possible that the net price is size dependent. This has not been taken into consideration in the present analysis. Probably the size dependence is not very high in the most probable harvest ages, particularily if the roundwood is used as input in the pulp industry. It is very easy to adjust the analysis through the introduction of a time dependent shift in the price distribution.

Obviously we live in a world which can be treated as stochastic (Compare figure 1 and 2) even if it is based on deterministic relationships that we can not observe or do not know perfectly. Obviously this has important implications for the optimal management of natural resources such as a forest (compare figure 5, 6, 7 and 8).

It is important that we use the correct decision criteria under these conditions. In the present example we were able to increase the expected present value with between 8 and 55 % at year zero!

A more general treatment of the problem under investigation will appear in 1987.

4. References

1. Alchian, A. A: Information, Martingales and Prices, The Swedish Journal of Economics, Vol. 76, No. 1, March 1974.

2. Bellman. R: Dynamic programming, Princeton University Press, Princeton, N. J. 1957.

3. Bjurulf, A., Freij, J: B-modellen, en alternativ skötsel-metod för gran i södra Sverige, Seminariearbete, Swedish University of Agricultural Sciences, Dept of Forest Management, 1986.

4. Brock, W., Rotschild, M., Stiglitz, J. E: Stochastic capital theory, University of Wisconsin, Madison, 1979 (unpublished mimeo).

5. Brännlund, R: Equilibrium versus disequilibrium; An econometric analysis of the Swedish sawtimber market, Swedish University of Agricultural Sciences, Dept of Forest Economics, No. 57, 1985.

6. Brännlund, R., Johansson, P-O., Löfgren., K-G: An econometric analysis of aggregate sawtimber and pulpwood supply in Sweden, Forest Science, Vol. 31, No. 3, sept. 1985.

7. Eriksson, H: Hög- eller låggallring, Sveriges Skogsvårdsförbunds Tidsskrift, Nr 2, 1986.

8. Faustmann, M: Berechnung des Wertes welchen Waldboden sowie noch nicht haubare Holzbestände für die Waldwirtschaft besitzen, Allgemeine Forst und Jagd-Zeitung, 25, 441-455, 1849.

9. Fleming, W. H., Rishel, R. W: Deterministic and Stochastic optimal control, Springer Verlag, 1975.

10. Fridh, M., Nilsson, N. E: Enkla avverkningsberäkningar baserade på en generell produktionsmall, Skogsstyrelsen, Sweden, stencil, dec. 1980.

11. Hey, J. D: Economics in disequilibrium, Martin Roberson & Co Ltd, Oxford, 1981.

12.Hägglund, B: Forecasting growth and yield in established forests, Swedish University of Agricultural Sciences, Dept. of Forest Survey, No 31, 1981.

13. Johansson, P-O., Löfgren, K-G: The economics of forestry and natural resources, Basil Blackwell, 1985.

14. Kao, C: Optimal stocking levels and rotation under risk, Forest Science, Vol. 28, No. 4, 1982.

15. Kuuluvainen, J: Short term demand for and supply of sawlogs in Finland, The Finnish Forest Research Institute, Dept of Forest Economics, Reports, No. 185, 1985.

16. Lohmander, P: On the optimal investment intensity under the influence of price risk and successive decisions and on the effects of price variations on the expected optimal harvest year, Swedish University of Agricultural Sciences, Dept. of Forest Economics, No. 29, 1984.

17. Lohmander, P: On the optimal choice of species under the influence of price risk, Scandinavian Forest Economics no. 28, Royal Veterinary and Agricultural University, Dept. of Forestry, Denmark, Helles editor, No 20, 1985.

18. Lohmander, P: Pulse extraction under risk, Swedish University of Agricultural Sciences, Dept of Forest Economics, WP- 47, 1985.

19. Lohmander, P: Continuous extraction under risk, International Institute for Applied Systems Analysis IIASA, Systems and Decisions Sciences, Mars 1986.

20. Löfgren, K-G., Ranneby, B: Two papers on some stochastic aspects of the supply and demand for timber, chapter 1, "On optimal stocking levels under uncertainty", Swedish University of Agricultural Sciences, Dept of Forest Economics and Dept of Biometrics and Forest Management, No. 46, 1983.

21. Malliaris, A. G., Brock, W. A: Stochastic methods in economics and finance, Advanced textbooks in economics, Bliss, C.J. & Intrilligator, M. D. (Editors), North-Holland, 1981.

22. Miller, R. A., Voltaire, K: A stochastic analysis of the tree paradigm, Journal of economic dynamics and control 6 (1983) 371-386 North-Holland.

23. Norstrøm, C. J: A stochastic model for the growth period decision in forestry, Swedish Journal of Economics, 1975.

24. Pindyck, R. S., Rubinfeld, D. L: Econometric models and economic forecasts, McGraw Hill, 2 ed. 1981.

25. Risvand, J: A stochastic model for the cutting policy decision in forestry, Agricultural University of Norway, Dept of Mathematics and Statistics, Vol. 55, 1976.

26. Rotschild, M., Stiglitz, J: Increasing risk: I. A definition, Journal of Economic Theory, 2, (225-243), 1970.

27. Rotschild, M., Stiglitz, J: Increasing risk: II. Its economic consequenses, Journal of Economic Theory, 3, (66-84). 1971.

28. Samuelson, P. A: Proof that properly anticipated prices fluctuate randomly, Industrial management review, Spring, 1965.

29. Samuelson, P. A: Stochastic Speculative Price, Proceedings from the National Academy of Sciences, February, 1971.

30. Sandmo, A: On the theory of the competitive firm under price uncertainty, The American Economic Review, March 1971.

31. Solberg, B: Univariate stochastic models and transfer function models for short term prognosis of sawnwood export prices to west Germany - some preliminary results, Symposium on forest products and roundwood markets, Scandinavian Society of Forest Economics, 1983.

32. Wieslander. N: Gallringsfritt granskogsbruk för större värdeproduktion, Sveriges Skogsvårdsförbunds Tidskrift, Nr 2, 1986.