WORKING PAPER

DISTRIBUTION MODELS FOR DIFFUSION OF
ADVANCED TECHNOLOGIES BY COMPANY SIZE

Akira Tani

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IIASA
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for Applied Systems Analysis

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FOREWORD

One of the most important tasks of the CIM Project is to prospect the diffusion of advanced manufacturing technologies such as CIM and its components.

The author formulated an integrated model of technology diffusion in his previous working paper, entitled "Penetration Mechanism of New Technologies: Integrated Formulation from the Company Level to the Macroeconomic Level".

The present paper shows the extensions and applications of the above model. In order to explain the actual data, some modifications are made to the proposed model.

The model extensions make it possible to take into account uncertainties through probabilistic modeling as well as more accurate modelling of learning curve effects. They reveal new hypotheses and new features of diffusion. The model extensions are also more realistic in the case of different benefits, but are still mainly restricted to technologies, the main effect of which is labor substitution.

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Contents

1. Introduction

Generally speaking, there are two different approaches to the diffusion problems. One is called the micro-level approach, which focuses on the decision-making problem in introducing the advanced technologies at the company (or factory) level. Cost/benefit analyses are carried out through a lot of case studies. The other approach is called the macro-level approach, which deals with the diffusion process of the technologies into the industries by means of statistical data and methods. Macroeconomic models are developed to forecast the future diffusion [Tani 1987a; Mori, 1987; Tchijov, 1987b]. However, there are many difficulties concerning the interrelations between these models.

In order to build a bridge between the two approaches, the author proposed in his previous paper, entitled "Penetration Mechanism of New Technologies: Integrated Formulation from the Company Level to the Macroeconomic Level" [Tani; 1988] a new approach to formulate the penetration or diffusion mechanism of advanced technological equipment into industry by introducing the company size factor.

In his earlier paper on the "Enterprise Size and its Impact on Penetration of Industrial Robots - Application of Econometric Analysis" [Tani , 1987b] the author shows that the distribution of company size is one of the most important factors affecting the penetration. Maly [1987a] has also pointed out the importance of company size.

A mathematical diffusion model, as formulated in the previous paper, is based upon several empirical laws given below (for more detail, see Appendix B).

a) Cost/benefit assessment at the company[1] level;

b) Economy of scale in user costs;

c) Wage gap between large and small companies;

d) Company size distribution;

e) Decreasing price of advanced technological equipment;

f) Wage increase.

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[1] In this paper "company" means establishment (or factory in most cases of manufacturers) rather than enterprise.

Although many parameters related to the above factors are introduced, a type of Gompertz[2] curves can be derived as a final mathematical form of our model by assuming an exponential distribution of company size.

However, in order to apply this model to the real-world cases, the following modifications are important, as already mentioned in the last chapter of the previous paper:

I) More adequate distribution function of company size;

II) Probabilistic function of decision-making;

III) Learning curve or economy of scale in production instead of simple trend function for prices.

Three types of models (Model I, II and III) are developed in this paper corresponding to the above three factors.

Model I is incorporated into Models II and III as a basic model.

2. Model Structure

This chapter reviews briefly the structure of our model proposed in the previous paper [Tani, 1988].

The diffusion rate R(t) can be expressed in the following equation.

$$R(t) = \int_{X(t)}^{\infty} f(x)dx \qquad (1)$$

where

x company size in terms of employees

f(x) labor distribution function by company size.

$$\int_{0}^{\infty} f(x)dx = 1 \qquad (2)$$

The following linear equation is assumed:

$$u(x) = u_0 \cdot x \qquad (3)$$

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[2]With regard to the Gompertz curve, see [Kotz & Johnson, 1983] and [Kurtz, 1984].

$u(x)$	number of units introduced at company size x	
$R(t)$	diffusion ration (= $U(t)/U_\infty$)	
	$U(t)$	number of units installed
	U_∞	ultimate number of units
$X(t)$	companies of larger number of employees than $X(t)$ introduce the advanced equipment	

$X(t)$ is calculated from the following condition on the cost/benefit assessment on the company level.

$$(C/B) = C(x,t)/B(x,t) \leq n \tag{4}$$

where

$$C(x,t) = p(x,t) \cdot u(x) \tag{5}$$

$$B(x,t) = w(x,t) \cdot l(x) \tag{6}$$

$C(x,t)$	cost for introducing advanced equipment at company of size x
$B(x,t)$	labor-saving benefit at company of size x
n	cost/benefit assessment criterion (years)
$p(x,t)$	unit cost of advanced equipment when introduced by companies of size x
$w(x,t)$	annual wage at company of size x
$l(x)$	labor saved by introducing advanced equipment at company of size x.

The following linear equation is assumed on $l(x)$

$$l(x) = l_0 \cdot x \tag{7}$$

The functional forms of $p(x,t)$ and $w(x,t)$ are assumed below:

$$p(x,t) = P_0 \cdot X^{-a} \cdot e^{-\alpha t} \tag{8}$$

$$w(x,t) = W_0 \cdot x^{b} \cdot e^{\beta t} \tag{9}$$

where x^{-a} and $e^{-\alpha t}$ denote the effects of "economy of scale" in user costs and the decreasing price by technological progress, respectively.

x^b and $e^{\beta t}$ denote the wage gap by company size and increasing wage, respectively.

Parameters P_o and W_o are constant coefficients.

By substituting equations (3), (5), (6), (7), (8) and (9) into (4), the cost/benefit criterion can be transformed in the following condition on company size x.

$$x \geq X(t) \tag{10}$$

where

$$X(t) = \left[\frac{P_o}{W_o \cdot l_o \cdot n} \right]^{\frac{1}{a+b}} e^{-\frac{(\alpha+\beta)}{(a+b)} t} \tag{11}$$

In the previous paper, assuming an exponential function $e^{-\lambda x}$ as f(x), we obtained R(t) as shown below.

$$R(t) = e^{-\lambda \cdot X(t)} \tag{12}$$

3. <u>Company Size Distribution in Japanese Manufacturing</u>

In our previous paper an exponential function was assumed as labor distribution function f(x). This assumption led us to a type of Gompertz curve as a final mathematical model. However, in order to express more precisely the company size distribution function in Japanese manufacturing, the other type of function is, according to the data analysis, better than the exponential one.

This chapter shows the result of regression analysis on the distribution of company size for the case of Japanese manufacturing.

Firstly, we introduce the cumulative labor distribution function by company size F(x) instead of f(x) as follows:

$$F(x) = \int_x^{\infty} f(x')dx' \tag{13}$$

F(x) denotes the share of labor who work at companies of a larger number of employees than x. The reason for using F(x) instead of f(x) lies in the format of available data as shown in Table 1.

According to the regression analysis on F(x), the best fitting function is a logarithmic normative function as shown below.[3]

$$F(x) = e^{-\lambda \cdot (\ln x)^2} \tag{14}$$

where

$$x \geq 1 \tag{15}$$

$$\lambda = 0.042246$$

The results of the regression analysis in Table 2 and Figure 1 show us a very good fitting with $R^2 = 0.99935$.

From F(x) in equation (14) f(x) can be derived through differentiation as follows:

$$f(x) = 2\lambda \cdot \left[\frac{\ln x}{x} \right] e^{-\lambda \cdot (\ln x)^2} \tag{16}$$

4. <u>Basic Model: Model I</u>

4.1 Model equations

By employing the new company size distribution function (equation (16)), our model is revised as follows:

$$R(t) = e^{-\lambda \cdot (\ln X(t))^2} \tag{17}$$

where

$$X(t) \geq 1 \tag{18}$$

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[3] If we assume $F(x) = e^{-\lambda \ln x}$, we can find the relationship between this model and the production function model [Tani, 1987a], see Appendix C.

Table 1. Format of data available

Company size (employees)	Number of workers	Cumulative share
1 - 4	L_1	$F(1) = 1.0$
5 - 9	L_2	$F(5) = F(10)+L_2/L_0$
10 - 29	L_3	$F(10) = F(30)+L_3/L_0$
30 - 49	L_4	$F(30) = F(50)+L_4/L_0$
50 - 99	L_5	$F(50) = F(100)+L_5/L_0$
100 -199	L_6	$F(100) = F(200)+L_6/L_0$
200 -299	L_7	$F(200) = F(300)+L_7/L_0$
300 -499	L_8	$F(300) = F(500)+L_8/L_0$
500 -999	L_9	$F(500) = F(1000)+L_9/L_0$
1000 -	L_{10}	$F(1000) = L_{10}/L_0$
Total	L_0	

Table 2. Results of regression analysis

Size x	Share* F(x)	X (ln(x))^2	Y ln(F(x))	Model Y
1	1.0000	0.0000	0.0000	0
5	0.9153	2.5903	-0.0886	-0.109429
10	0.8145	5.3019	-0.2052	-0.223984
30	0.6197	11.5681	-0.4786	-0.488709
50	0.5247	15.3039	-0.6449	-0.646532
100	0.4083	21.2076	-0.8957	-0.895939
200	0.3085	28.0722	-1.1760	-1.185941
300	0.2574	32.5331	-1.3570	-1.374400
500	0.1985	38.6214	-1.6170	-1.631603
1000	0.1285	47.7171	-2.0517	-2.015863

Regression Output:

Constant		0
Std Err of Y Est		0.0176093
R Squared		0.9993503
No. of Observations		10
Degrees of Freedem		9
X Coefficient(s)	-0.042246	
Std Err of Coef.	0.0002189	

*Source [MCA, 1983]

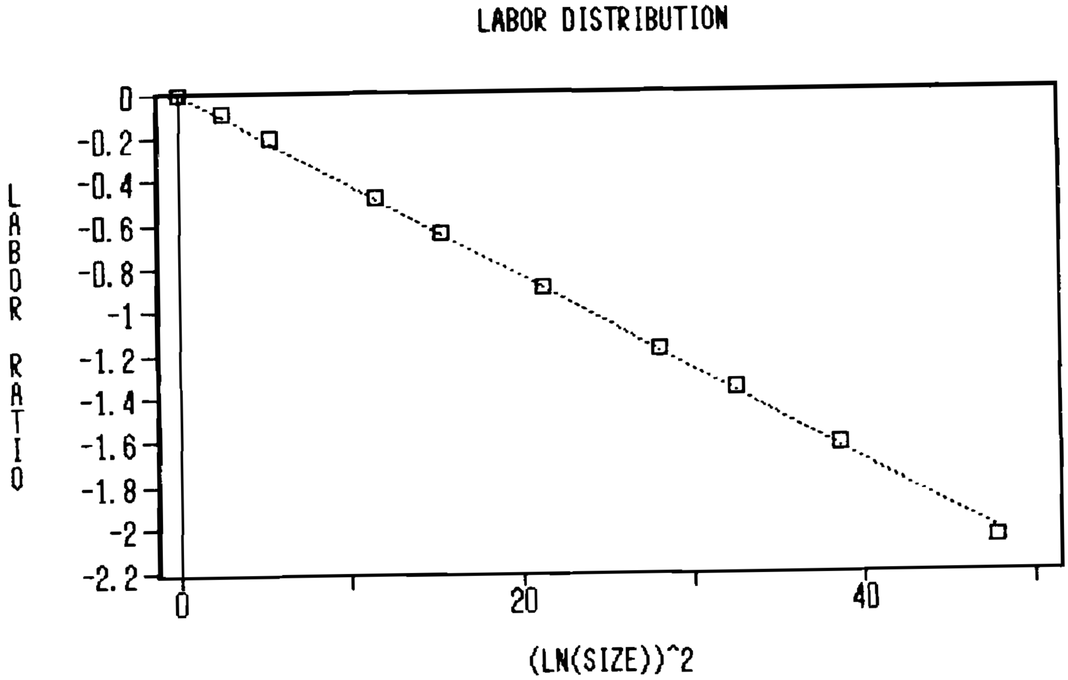

Figure 1. Fitting of regression curve.

By substituting X(t) of equation (11) to (17), the diffusion rate R(t) can be expressed explicitly in terms of time t.

$$R(t) = e^{-\lambda \cdot r^2 \cdot (t_c - t)^2} \tag{19}$$

where

$$t \le t_c \tag{20}$$

$$r = (\alpha+\beta)/(a+b) \tag{21}$$

$$t_c = \frac{1}{(\alpha+\beta)} \ln \left[\frac{P_c}{W_0 \cdot l_0 \cdot n} \right] \tag{22}$$

$$R(t) = 1, \quad \text{when } t \ge t_c \tag{23}$$

As shown in equation (19), our revised basic model (Model I) leads us to the left-hand side of the normal distribution function.

Parameter t_c denotes the time when the diffusion reaches the saturation level.

The structure of Model I is outlined in Figure 2.

4.2 Features of Model I

R(t) in equation (19) is also a kind of S-shaped curve[4] as shown below.

$$\frac{dR}{dt} \begin{cases} > 0 & \text{when } t < t_c \\ = 0 & \text{at } t = t_c \end{cases} \tag{24}$$

$$\frac{d^2R}{dt^2} \begin{cases} > 0 & \text{when } t_* > t \\ = 0 & \text{at } t = t_* \\ < 0 & \text{when } t_c > t > t_* \end{cases} \tag{25}$$

where t_* denotes a point of inflection defined below.

$$t_* = t_c - \frac{1}{r\sqrt{2\lambda}} \tag{26}$$

$$R(t_*) = e^{-0.5} \tag{27}$$

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[4]The meaning of S-shaped curves in diffusion of technologies is discussed in [Tchijov, 1987a].

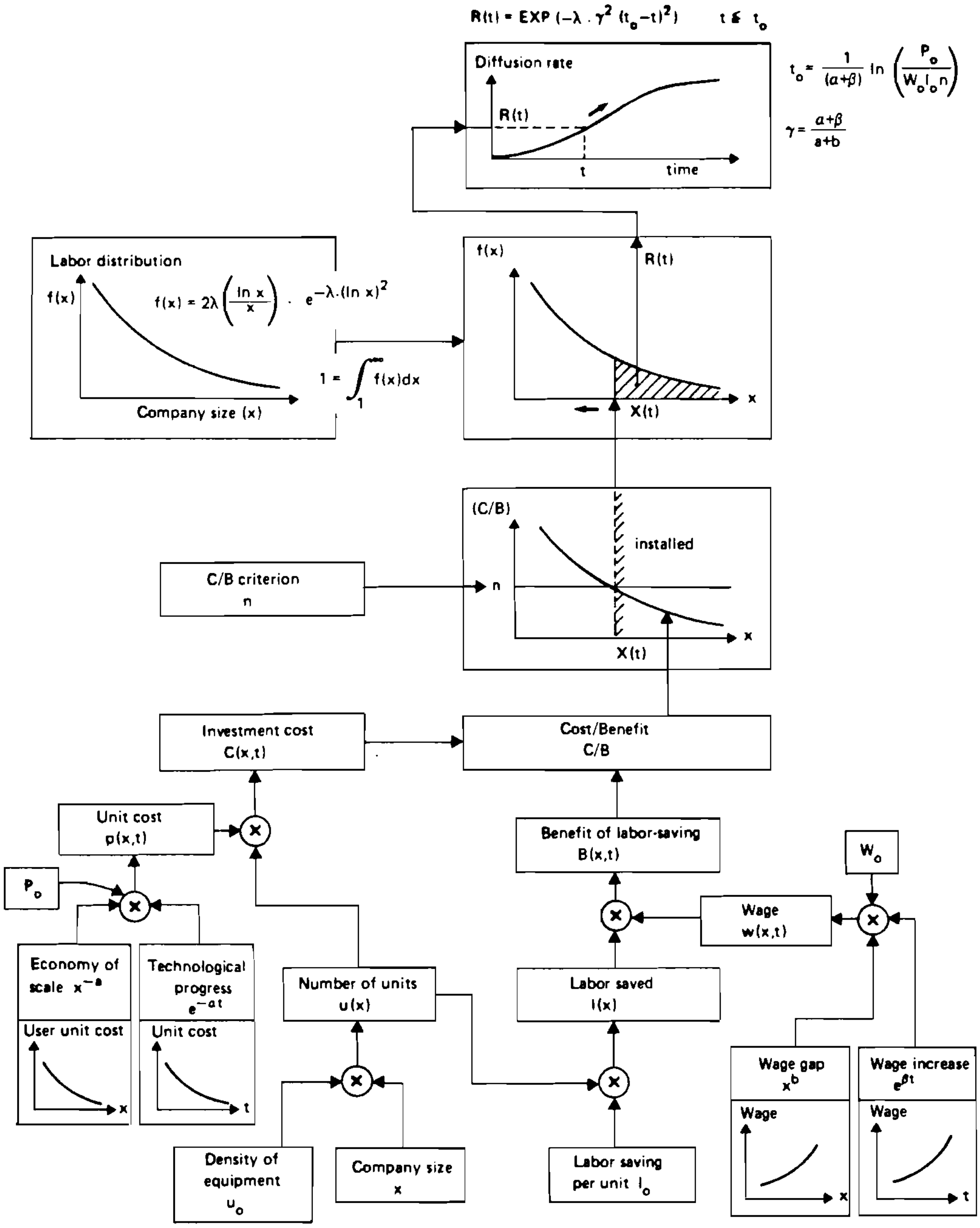

Figure 2. Structure of the Basic Model.

The impacts of the change of parameter values are summarized by the following partial derivatives.

$$\left.\begin{array}{l} \frac{\partial R}{\partial \lambda},\ \frac{\partial R}{\partial P_o} < 0 \\ \frac{\partial R}{\partial W_o},\ \frac{\partial R}{\partial l_o},\ \frac{\partial R}{\partial n},\ \frac{\partial R}{\partial a},\ \frac{\partial R}{\partial b} > 0 \\ \frac{\partial R}{\partial \alpha},\ \frac{\partial R}{\partial \beta} > 0 \text{ (when } t_o > t > 0) \end{array}\right\} \qquad (28)$$

The above impacts are almost similar to our previous model.

The major difference of the S-shape between the revised model and the previous model is considered to be the level of the inflection point as described below.

	diffusion level at inflection point
previous model (Gompertz curve)	36.8%
revised model (Model I)	60.7%

4.3 Application

In this chapter we apply our model, Model I, to forecasting the diffusion of advanced industrial robots in Japanese manufacturing. The advanced types of industrial robots comprise playback robots, NC robots, and intelligent robots, excluding manual manipulators, fixed sequence robots, and variable sequence robots [Tani, 1987a].

In order to apply our model, it is necessary to estimate the parameters of Model I.

Parameter λ, related to the company size distribution, was already estimated in Chapter 3.

$$\lambda = 0.042246 \qquad (29)$$

Parameter α, which denotes the decreasing rate of robot prices, can be obtained from the data, i.e., 7.685 million yen in 1985 and 12.616 million yen in 1980 [Tani, 1987a].

$$\alpha = 0.0944 \tag{30}$$

The wage increasing rate β is estimated in a similar way.

$$\beta = 0.0415 \tag{31}$$

The wage gap parameter b can be obtained from regression analysis on the wages by company size in 1984 [Tani, 1987b]. The results of the regression analysis are shown below.

$$\ln(W_{1984}) = \ln(1.6247) + 0.1337 \cdot \ln x \tag{32}$$

$$\bar{R}^2 = 0.983$$
$$b = 0.1337$$

By setting time to zero (t = 0) we obtain the year 1980, and parameter W_0 is obtained from the averaged wage data of 1980 and 1984 as shown below.

$$W_0 = 1.3593 \text{ million yen/person} \tag{33}$$

Parameter a, related to "economy of scale" in user costs, can be estimated as the relative coefficient to the wage gap effects [Tani, 1987b].

$$a = 0.375b = 0.05014 \tag{34}$$

In order to estimate parameter P_0, we assume that the averaged robot price in 1980 was at company size x = 1000. In addition, the ratio of system costs to robot price is set at 2.07 according to the data of JIRA [Tani, 1987a]. The estimated P_0 is as follows.

$$P_0 = 36.924 \text{ million yen/unit} \tag{35}$$

The other parameters, l_0 and n, are assumed below, according to the surveyed data of robot users by JIRA [JIRA, 1984; Mori, 1987].

$$l_o = 1.51 \text{ labor saved/unit} \quad (36)$$

$$n = 3.5 \text{ years} \quad (37)$$

By using the estimated parameters described above, our diffusion model can be expressed by the following equation.

$$R(t) = e^{-0.023085 \cdot (12.045 - t)^2} \quad (38)$$

where

$t \le t_o = 12.045$ (year of 1992).

The saturation year t_o is forecast to be 1992.

The resulting diffusion curve is shown in Figure 3.

The dotted line in Figure 3 shows the more realistic case with a gradual price saturation from 1985 to 1990. The resulting saturation year in such a case will be about ten years later than in the case without price saturation.

In order to estimate the population of advanced industrial robots by using R(t) described in equation (38), the following condition is introduced:

$$\sum_t (U_t - U_\infty \cdot R(t))^2 \to \min \quad (39)$$

where U_t denotes the observed population at time t, and U_∞ is a parameter to be estimated.

The results of the regression analysis are shown in Table 3 and in Figure 4. Figure 4 gives us a good fitting of Model I to the observed data. The saturation level is estimated to be 193 thousand units.

This saturation level, i.e. 193 thousand units, is much higher than that of 142 in the case of a simple logistic curve [Tani, 1987a].

The difference between these two levels might be explained by the meaning of saturation level in Model I, i.e. that even very small companies introduce the robots at that saturation stage. If the relative price of robots to wage rate, namely the cost/benefit ratio is saturated at some level, the diffusion of robots is limited for companies larger than some respective scale. In such

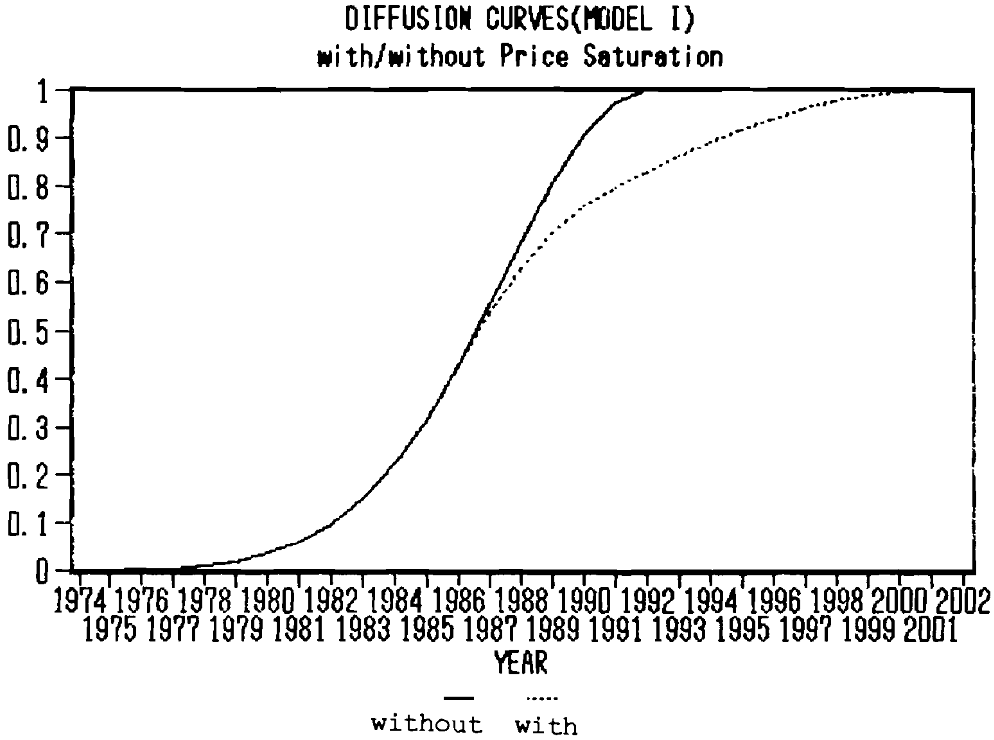

Figure 3. Diffusion curves (Model I)

Table 3. Results of Model I

Model 1: R(t) = EXP(-gamma*(TO-t)^2)

PARAMETERS			YEAR	R(t)	U(t) Est	U(t)
			1970	0.00001	0.00	0
LAMBDA	0.042246		1971	0.00004	0.01	0
ALPHA	0.0944		1972	0.00009	0.02	0
BETA	0.0415		1973	0.00023	0.04	0
a	0.05014		1974	0.00054	0.10	0.14
b	0.1337		1975	0.00122	0.24	0.26
PO	36.924		1976	0.00262	0.51	0.50
WO	1.3593		1977	0.00537	1.04	1.02
LO	1.51		1978	0.01052	2.03	1.68
n	3.5		1979	0.01966	3.79	2.85
			1980	0.03509	6.76	5.71
TO	12.05		1981	0.05980	11.52	10.51
gamma	0.0231		1982	0.09732	18.75	18.47
			1983	0.15122	29.14	28.57
t=0:	1980		1984	0.22437	43.24	44.18
			1985	0.31789	61.26	63.83
Regression Analysis			1986	0.43007	82.88	81.03
U(t) = A*R(t)			1987	0.55557	107.06	
			1988	0.68532	132.07	
Constant		0	1989	0.80722	155.56	
Std Err of U Est		1.0974	1990	0.90790	174.96	
R squared		0.9983	1991	0.97507	187.90	
No. of Observation		13	1992	0.99995	192.70	
Degrees of Freedom		12	1993	1.00000	192.71	
			1994	1.00000	192.71	
Coefficient (A) U_∞		192.71	1995	1.00000	192.71	
Std Err of Coef.		1.79	1996	1.00000	192.71	

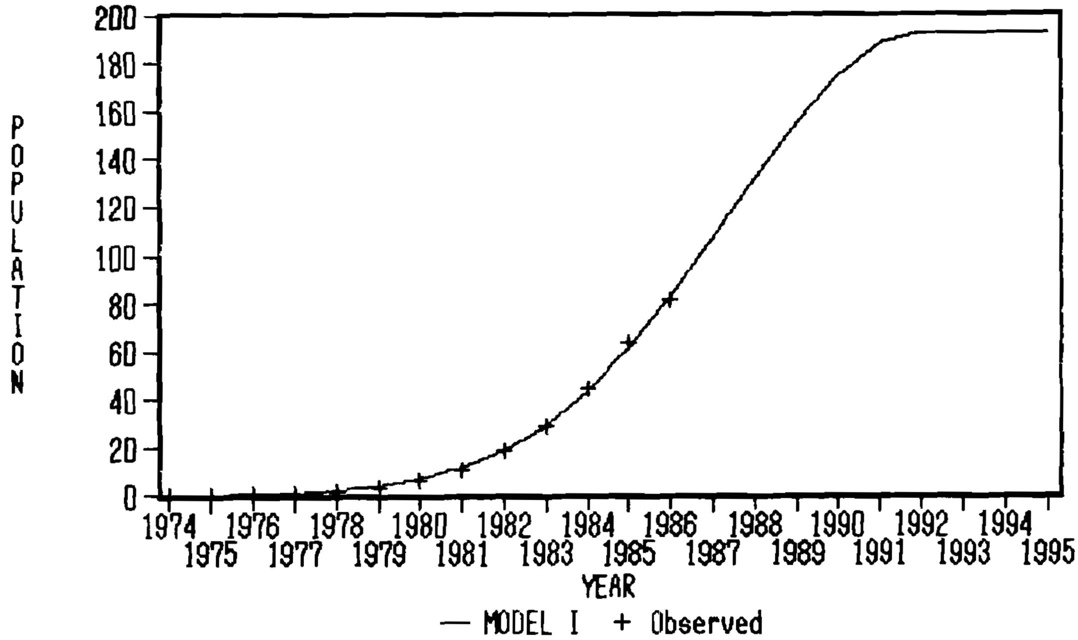

Figure 4. Diffusion curve of Model I.

a case the diffusion is saturated at the intermediate level of potential users.

Figure 5 shows the relationship between the saturated C/B ratio at a company size of one thousand employees and the saturation level of diffusion as a whole.

If the C/B ratio is saturated at 0.75 and 0.50 in the case of company size x = 1000, the saturation levels of diffusion are 30% and 66%, respectively, according to Figure 5.

This means that the saturation level depends greatly upon the future prospects of the saturated C/B ratio. In other words, in order to forecast the future saturation level of diffusion, it is necessary to set the future trend of the C/B ratio. The absolute saturation level, estimated by simple methods of growth curve fitting, such as a logistic curve model, might be considered unreliable, even though the statistical index apparently seems to be good. One of the reasons is that a small change of the C/B ratio trend in the future leads to a relatively large change of the saturation level if smaller companies have a large share in industry.

Another reason is related to the meaning of further diffusion. According to our model, the diffusion curve shows an extension of user companies into smaller sizes. This means that the structure of users in terms of industrial sectors and applications is also in the future assumed to remain unchanged. However, some new applications and diffusions into other industrial sectors usually appear in the real world, and they play an important role of promoting further diffusion in case of advanced technological goods.

5. Probabilistic Model: Model II

5.1 Model equations

It is assumed in Model I that the C/B ratio is determined -- without variances -- only by company size x and time t. The criterion of decision-making, namely the pay-back year (n) for investments, is also assumed to be deterministic. These assumptions lead to the deterministic company cut-off size X(t) in equation (11). Such a situation is not realistic. In the actual cases there are many differences in the C/B ratio and the

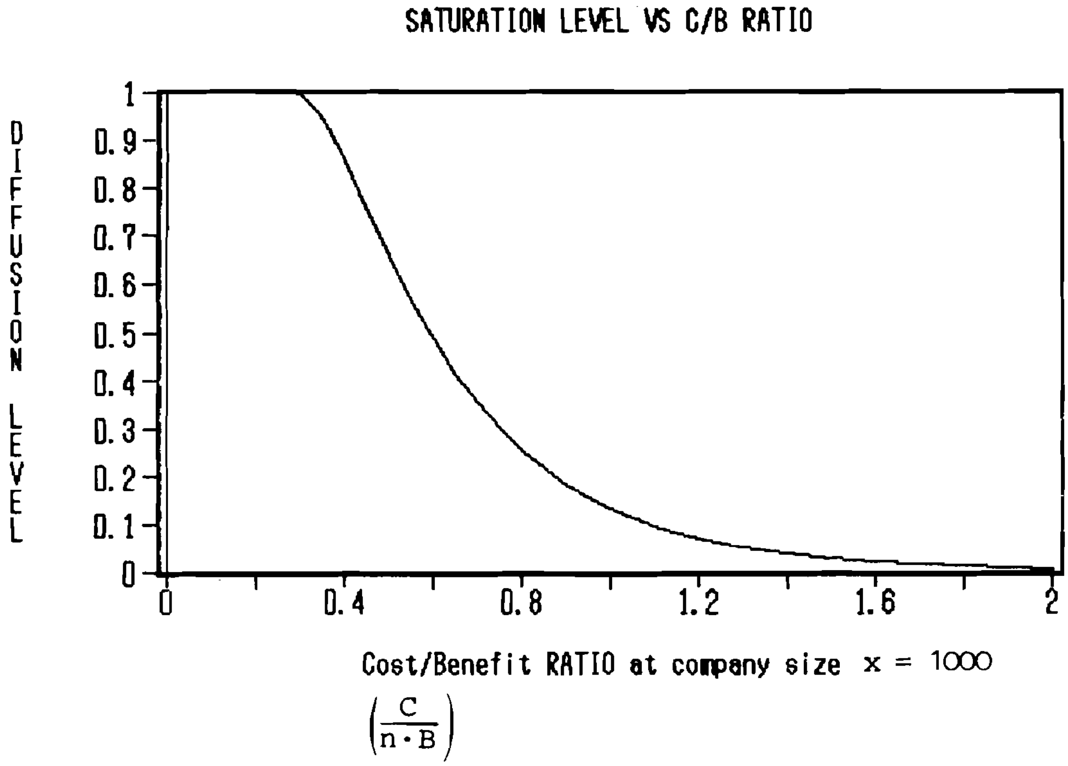

Figure 5. Saturation level vs. C/B ratio.

criterion n among companies of the same size. In other words, those variables should be considered as probabilistic variables.

Therefore we introduce a probabilistic variable to Model I in this chapter.

At first, the condition of decision-making in equation (4) is modified to the following equation by introducing a random variable (z) of normal distribution with a mean = 0 and a standard deviation = σ.

$$\ln \left[\frac{C}{n \cdot B} \right] + z \leq 0 \qquad (40)$$

The probabilistic distribution function of z, namely g(z), is expressed as follows:

$$g(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}} \qquad (41)$$

$$\int_{-\infty}^{+\infty} g(z)dz = 1 \qquad (42)$$

The condition (40) imposes the following condition on size x.

$$\ln x \geq \ln X \equiv \frac{1}{(a+b)} \left[\ln \left[\frac{P_o}{W_o 1_o n} \right] + z - (\alpha+\beta) \cdot t \right] \qquad (43)$$

In other words, the cut-off size X becomes probabilistic.

Based upon the above condition, the diffusion level R can be obtained as shown below.

$$R = \int_{-\infty}^{\infty} D(X) \cdot g(z)dz \qquad (44)$$

where

$$D(X) = \begin{cases} 1 & 0 < X < 1 \\ e^{-\lambda(\ln X)^2} & X \geq 1 \end{cases} \qquad (45)$$

By subtracting equation (43) on X into equation (44), R can be expressed as an explicit function of z.

$$R = \int_{-\infty}^{z_o} g(z)dz + \int_{z_o}^{\infty} e^{-\frac{(z-z_o)}{2\gamma^2}} g(z)dz \tag{46}$$

where

$$z_o = (\alpha + \beta) \cdot t - \ln(P_o/W_o l_o \cdot n) \tag{47}$$

$$\gamma = (a + b)/\sqrt{2\lambda} \tag{48}$$

The result of the integral in equation (46) can be expressed by using the cumulative normal distribution function[5] **N** as shown below.

$$R(t) = N\left[\frac{z_o}{\sigma}\right] + \left[\frac{Q \cdot \Sigma}{\sigma}\right] \cdot \left[1 - N\left[\frac{z_o - \mu}{\Sigma}\right]\right] \tag{49}$$

where

$$N(y) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{u^2}{2}} du \tag{50}$$

$$\frac{1}{\Sigma^2} \equiv \frac{1}{\gamma^2} + \frac{1}{\sigma^2} \tag{51}$$

$$\mu \equiv \left[\frac{\sigma^2}{\sigma^2 + \gamma^2}\right] z_o \tag{52}$$

$$Q \equiv \exp\left[-\frac{z_o^2}{2(\sigma^2 + \gamma^2)}\right] \tag{53}$$

As shown in Table 4 and Figure 6, the speed of diffusion becomes slower if the standard deviation σ is larger. In addition, Model II reduces to Model I when σ approaches zero.

5.2 Distribution of diffusion

Model II can give us the distribution of diffusion in terms of company size x as explained below.

Firstly, in order to obtain the distribution function of

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[5]With regard to the cumulative normal distribution function, see [Abramowitz & Stegun, 1970].

Table 4. Results of Impacts by σ

model II

parameters

a=	0.05014
b=	0.1337
lambda=	0.04224
alpha=	0.0944
beta=	0.0415
P0=	36.924
W0=	1.3593
L0=	1.51
n=	3.5
sigma=	
gamma=	0.63245
SIGMA=	0.39223
a+b=	0.18384
alpha+be	0.1359
lnPWLn=	1.63701

			CASES	
YEAR	sgm=1	sgm=0.5	sgm=0.1	0.01
1970	0.02208	0.00078	0.00001	0.00001
1971	0.02931	0.00145	0.00004	0.00003
1972	0.03843	0.00260	0.00011	0.00009
1973	0.04977	0.00453	0.00027	0.00023
1974	0.06365	0.00768	0.00064	0.00054
1975	0.08042	0.01265	0.00142	0.00122
1976	0.10040	0.02025	0.00299	0.00262
1977	0.12385	0.03150	0.00603	0.00538
1978	0.15100	0.04764	0.01161	0.01053
1979	0.18195	0.07002	0.02137	0.01967
1980	0.21670	0.10003	0.03761	0.03511
1981	0.25512	0.13888	0.06327	0.05983
1982	0.29692	0.18742	0.10174	0.09736
1983	0.34163	0.24584	0.15640	0.15127
1984	0.38870	0.31361	0.22984	0.22442
1985	0.43743	0.38929	0.32289	0.31794
1986	0.48711	0.47058	0.43362	0.43010
1987	0.53700	0.55437	0.55667	0.55558
1988	0.58639	0.63688	0.68317	0.68529
1989	0.63463	0.71423	0.80149	0.80716
1990	0.68107	0.78329	0.89891	0.90781
1991	0.72499	0.84234	0.96402	0.97495
1992	0.76550	0.89033	0.99323	0.99984
1993	0.80246	0.92716	0.99954	1
1994	0.83601	0.95390	0.99996	1
1995	0.86565	0.97205	0.99999	1
1996	0.89127	0.98397	0.99999	1
1997	0.91307	0.99148	0.99999	1
1998	0.93138	0.99585	0.99999	1
1999	0.94657	0.99808	0.99999	1
2000	0.95904	0.99906	0.99999	1

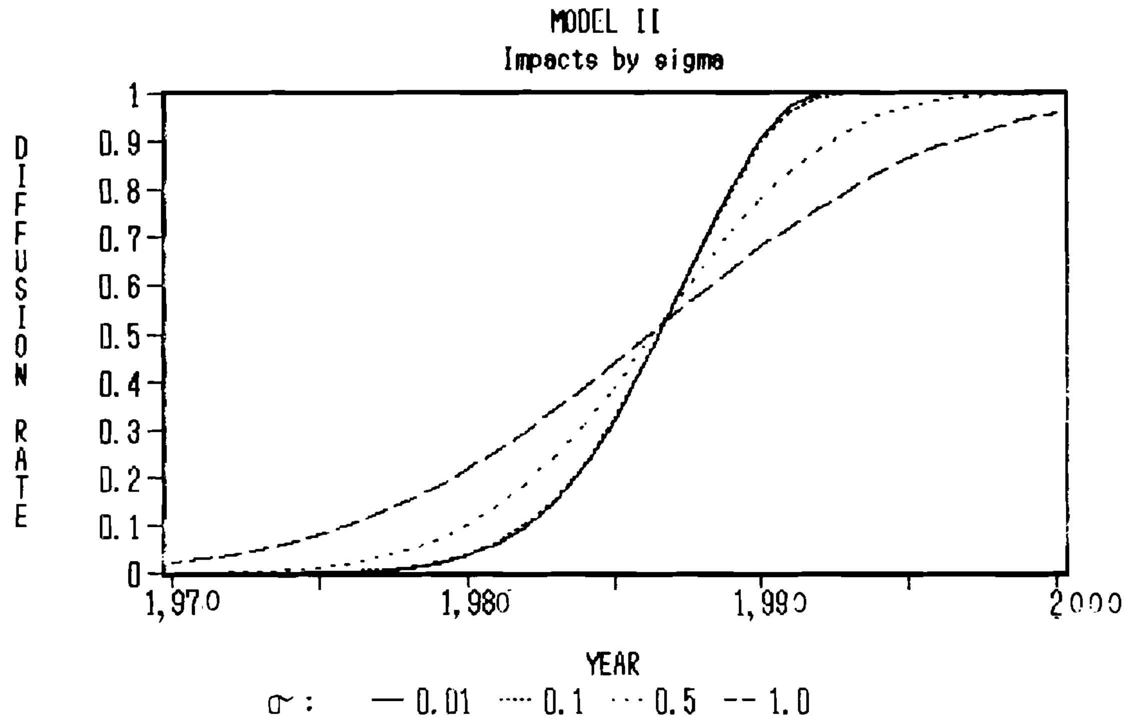


Figure 6. Diffusion curve of Model II

diffusion, we introduce the following function $h(x,t,z)$ (see Figure 7).

$$h(x,t,z) \equiv \begin{cases} e^{-\lambda(\ln x)^2} & \ldots \text{ if } x \geq X(z,t) \\ e^{-\lambda(\ln X)^2} & \ldots \text{ if } 1 \leq x < X(z,t) \end{cases} \qquad (54)$$

The function $h(x,t,z)$ denotes the cumulative diffusion level up to company size x from $+\infty$ in the case of random variable z.

An expectation of $h(x,t,z)$, namely the cumulative diffusion level up to x from $+\infty$ of company size $H(x,t)$ can be obtained by the integral using $g(z)$ as given below.

$$H(x,t) \equiv \int_{-\infty}^{+\infty} h(x,t,z)g(z)dz \qquad (55)$$

By substituting equations (54) and (43) into (55), $H(x,t)$ can be expressed as an explicit function of x and t.

$$H(x,t) = e^{-\lambda(\ln x)^2} \cdot N\left[\frac{\ln x - Y_0}{\Sigma_0}\right] + \left[\frac{v}{\Sigma_0}\right] \cdot e^{V_0} \cdot \left[1 - N\left[\frac{\ln x - \mu_0}{v}\right]\right] \qquad (56)$$

where

$$\Sigma_0 \equiv \sigma/(a+b) \qquad (57)$$

$$v \equiv \sigma/\sqrt{2\lambda\sigma^2 + (a+b)^2} \qquad (58)$$

$$Y_0 \equiv \frac{1}{(a+b)}\left[\ln\left[\frac{P_0}{W_0 1_0 n}\right] - (\alpha+\beta)\cdot t\right] \qquad (59)$$

$$\mu_0 \equiv \left[\frac{v}{\Sigma_0}\right]^2 \cdot Y_0 \qquad (60)$$

$$V_0 \equiv \frac{1}{2}\left[\left[\frac{v}{\Sigma_0}\right]^2 - 1\right]\left[\frac{Y_0}{\Sigma_0}\right]^2 \qquad (61)$$

Finally, we can obtain the distribution function of diffusion $F_0(x,t)$ as shown below.

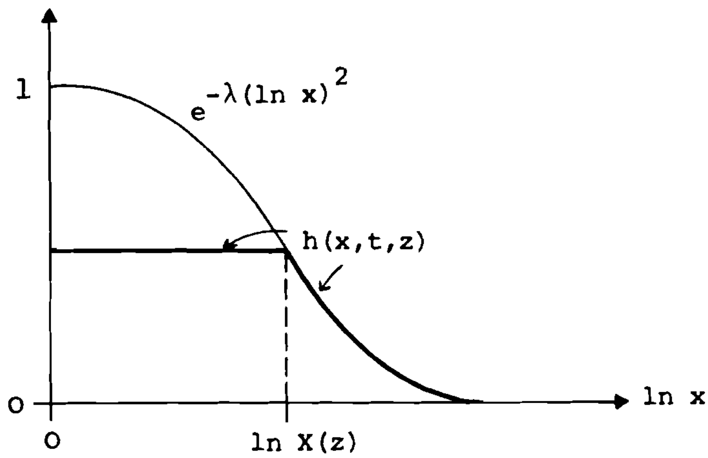


Figure 7. Function h(x,t,z).

$$F_c \equiv - \left[\frac{d\ H(x,t)}{d(\ln x)} \right] = 2\lambda \cdot \ln x \cdot e^{-\lambda(\ln x)^2}$$

$$\cdot N \left[\frac{\ln x - Y_c}{\Sigma_c} \right] \tag{62}$$

where $F_c(x,t)$ is also related to the total diffusion rate R(t) through the following equation.

$$R(t) = \int_{\ln x}^{\infty} F_c(u,t) d(\ln u) \tag{63}$$

From equation (61) $F_c(x,t)$ can be calculated as follows.

$$F_c(x,t) = 2\lambda \cdot (\ln x) \cdot e^{-\lambda(\ln x)^2} \cdot N \left[\frac{\ln x - Y_c(t)}{\Sigma_c} \right] \tag{64}$$

$F_c(x,t)$ approaches $f(x) \cdot x$ in equation (16) when $t \to t_\infty$. By using $F_c(x,t)$, we can see how the diffusion proceeds into smaller companies in the course of time. An example of $F_c(x,t)$ is shown in Figure 8.

If there are available data on the distribution of advanced industrial robots from the viewpoint of company size, we could estimate the parameter σ, namely the variance of the C/B ratio. However, such data are not available at present. The author estimated the distribution of total industrial robots including conventional robots [Tani, 1987b]. The estimated distribution of the year 1984 is shown in Table 4. The share of advanced robots was about 30% in 1984. Therefore the share of large companies would be considered higher than that in Table 4, if we could exclude the conventional type from the data.

In order to investigate the value of parameter σ, we tried to calculate the distribution for several values of σ by using $F_c(x,t)$. The results are also shown in Table 5. Compared with the data, σ seems to be less than 0.2. This means that more than 60% of the C/n·B value are within the range of ±22% from the mean value. In the case of σ = 0.2, the estimated trend of diffusion by size classes of companies is shown in Figure 9.

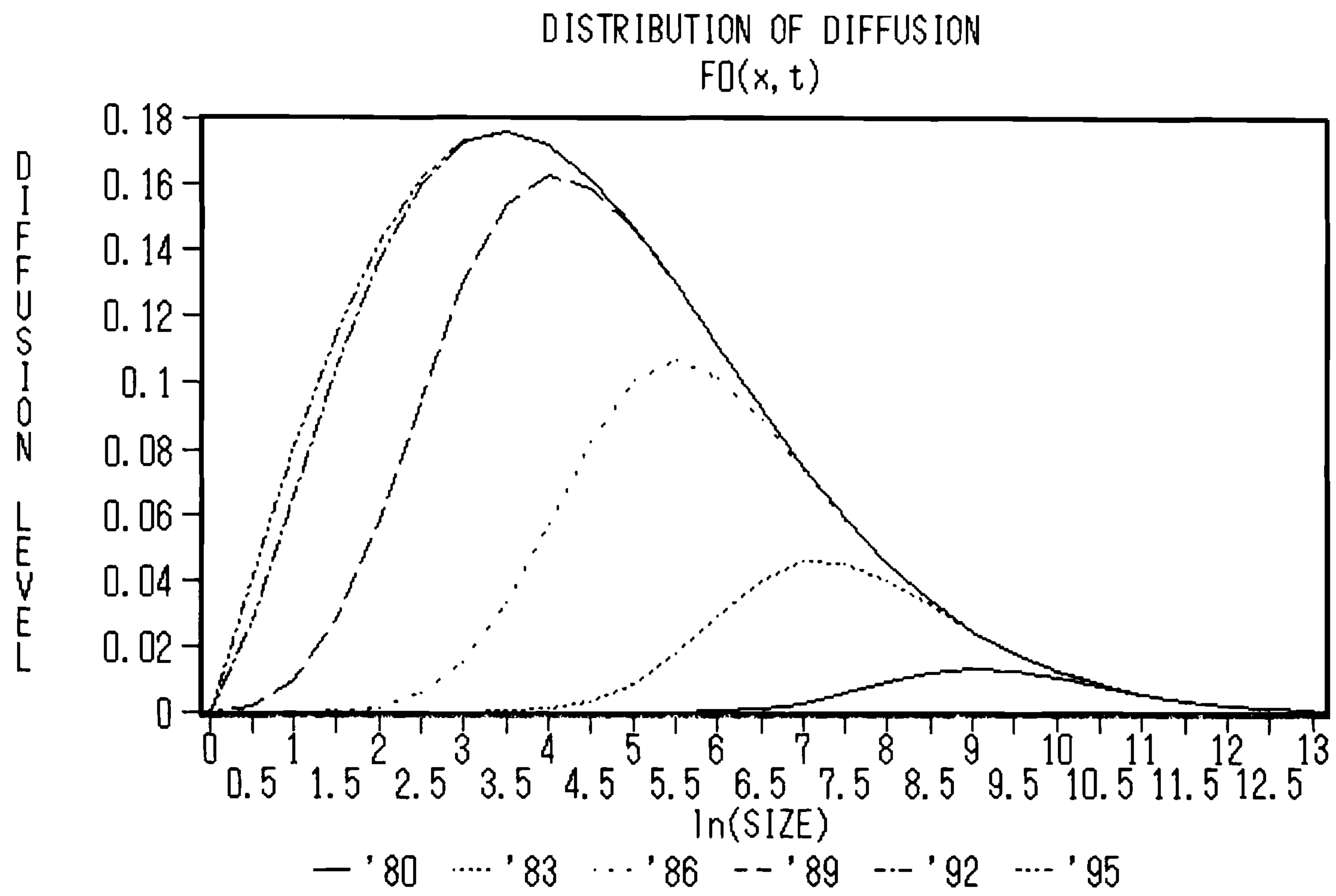


Figure 8. Distribution of diffusion.

Table 5. Advanced Type Robots

SIZE	ADVANCED TYPE ROBOTS (CASES ABOUT SEVERAL sigma) 1.0	0.5	0.4	0.3	0.2	0.1	TOTAL ROBOTS OBSERVED
4-	7.5%	2.6%	1.4%	0.4%	0.0%	0.0%	1.0
10-	8.4%	4.2%	2.5%	1.1%	0.0%	0.0%	2.0
20-	5.8%	3.6%	2.5%	1.2%	0.2%	0.0%	2.0
30-	19.6%	15.8%	13.4%	9.2%	3.6%	0.1%	6.0
100-	18.3%	19.0%	19.0%	18.1%	14.9%	6.7%	9.5
300-	16.8%	21.1%	23.0%	25.8%	29.7%	35.6%	22.1
1000-	23.7%	33.8%	38.3%	44.2%	51.5%	57.7%	57.3

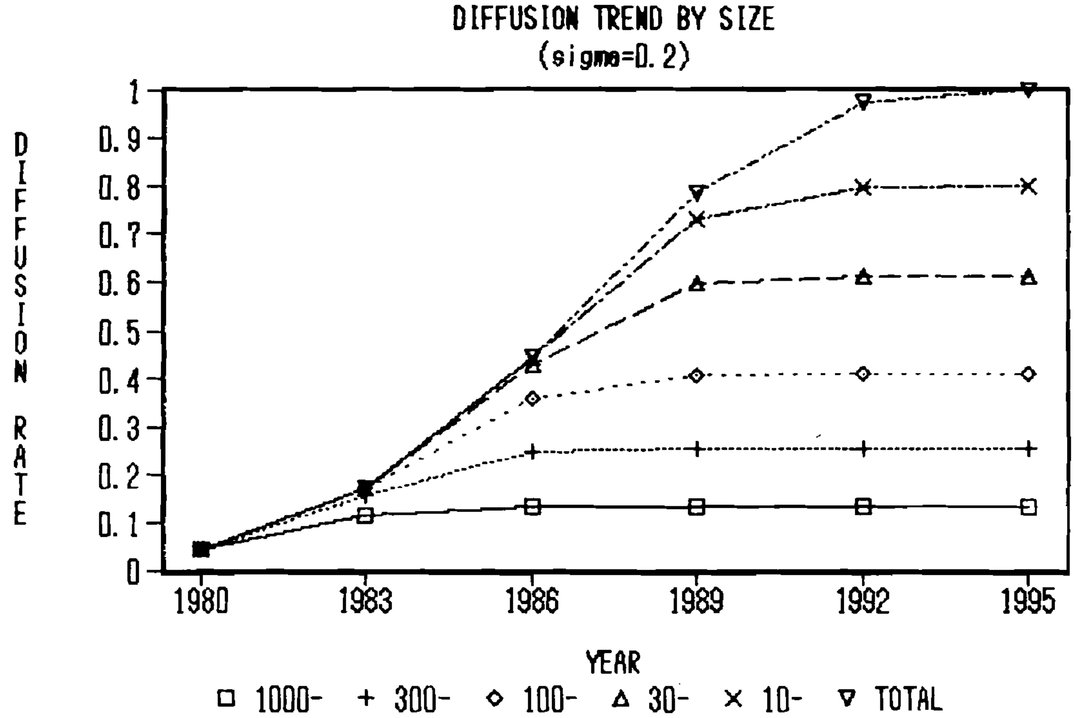


Figure 9. Diffusion trend by size.

6. Learning Curve Type Model (Model III)

6.1 Model equations

It is assumed in Model I that the price of advanced equipment decreases year by year due to technological progress. In other words, a time-trend function is used.

In order to clarify the diffusion mechanism, the learning curve effects or economy of scale effects in production should be incorporated into our model.

Therefore we introduce the following relationship between price and diffusion rate into Model I.

$$P_o = K_o \cdot R^{-c} \qquad (65)$$

where

$K_o > 0$, $c > 0$.

I.e., we use R in the above equation instead of the cumulative number of production (in case of the learning curve) or annual production (in case of economy of scale in production), because of the similar shapes of these three curves and for reasons of simplicity.

By substituting P_o of equation (65) into equation (19), we can obtain the following quadratic equation with respect to ln R.

$$Y = -\lambda \cdot r^2 \left[\frac{1}{(\alpha+\beta)} \left[\ln\left(\frac{K_o}{W_o l_o n}\right) - c \cdot Y \right] - t \right] \qquad (66)$$

where

$Y = \ln R$.

The solutions for Y can be derived from the above quadratic equation.

$$Y = (B - A/2) \pm \sqrt{A(A/4 - B)} \qquad (67)$$

where

$$A = \frac{(a+b)^2}{\lambda c^2} > 0 \qquad (68)$$

$$B = 1/c \left[\ln \left[\frac{K_o}{W_o l_o n} \right] - (\alpha+\beta) \cdot t \right] \qquad (69)$$

The condition for having real solutions in equation (67) is as follows:

$$A/4 \geq B \tag{70}$$

Condition (70) can be written as given below.

$$t \geq T_* \tag{71}$$

where

$$T_* = \frac{1}{\alpha+\beta}\left[\ln\left[\frac{K_o}{W_o l_o n}\right] - \frac{(a+b)^2}{4\lambda c}\right] \tag{72}$$

On the other hand, time (T_o) is, when saturation occurs, obtained by setting Y = 0 in equation (66).

$$T_o = \frac{1}{\alpha+\beta}\ln\left[\frac{K_o}{W_o l_o n}\right] \tag{73}$$

By introducing the following parameter Δ, equation (67) can be expressed in a simpler form.

$$Y = \left[\frac{\alpha+\beta}{c}\right]\left[(T_*-t-\Delta) \pm 2\sqrt{\Delta\cdot(t-T_*)}\right] \tag{74}$$

where

$$\Delta = \frac{(a+b)^2}{4\lambda c(\alpha+\beta)} = T_o - T_* \tag{75}$$

$$T_o \geq t \geq T_* \tag{76}$$

According to the condition that Y equals zero at time $t = T_o$, the following final solution is selected from the two solutions in equation (74).

$$R(t) = \mathrm{EXP}\left[\left[\frac{\alpha+\beta}{c}\right](T_* - t - \Delta + 2\sqrt{\Delta\cdot(t-T_*)})\right]$$

$$= \mathrm{EXP}\left[\left[\frac{\alpha+\beta}{c}\right](\sqrt{t - T_*} - \sqrt{\Delta})^2\right] \tag{77}$$

6.2 Features of Model III

At first, the diffusion curve R(t) has the following values at both boundaries of time t.

$$R(T_0) = 1 \text{ (saturation)} \qquad \text{when } t = T_0 \tag{78}$$

$$R(T_*) = EXP\left[- \frac{(a+b)^2}{4\lambda c^2} \right] < 1 \qquad \text{when } t = T_* \tag{79}$$

Secondly, the differentiation of R with respect to t shows us a monotonously increasing feature of R as given below.

$$\frac{dR}{dt} = R \cdot \left[\frac{\alpha+\beta}{c} \right] \cdot \left[-1 + \sqrt{\frac{\Delta}{(t-T_*)}} \right] \tag{80}$$

$$\left\{ \begin{array}{lll} \dfrac{dR}{dt} = 0 & & \text{when } t = T_0 \\ \dfrac{dR}{dt} > 0 & & \text{when } T_0 > t > T_* \\ \dfrac{dR}{dt} \to +\infty & & \text{when } t \to T_* \end{array} \right\} \tag{81}$$

Thirdly, the second derivative of R with respect to t is calculated as follows.

$$\frac{d^2R}{dt^2} = \left[\frac{\alpha+\beta}{c} \right] \cdot R \left[\left[\frac{\alpha+\beta}{c} \right] \left[\sqrt{\frac{\Delta}{t-T_*}} - 1 \right]^2 - \right.$$
$$\left. - \frac{\sqrt{\Delta}}{2} (t-T_*)^{-3/2} \right] \tag{82}$$

$$\left\{ \begin{array}{lll} \dfrac{d^2R}{dt^2} = - \left[\dfrac{\alpha+\beta}{2c\,\Delta} \right] < 0 & & \text{when } t = T_0 \\ \dfrac{d^2R}{dt^2} \to -\infty & & \text{when } t \to T_* \end{array} \right\} \tag{83}$$

In order to investigate the shape of R(t) in more detail, we check the points of inflection by setting

$$\frac{d^2R}{dt^2} = 0.$$

This leads us to the following equation.

$$q(y) = - y^3 + A(y-1)^2 = 0 \tag{84}$$

where

$$y = \sqrt{\frac{\Delta}{t-T_*}} \geq 1 \tag{85}$$

$$A = \frac{1}{2\lambda} \left[\frac{a+b}{c} \right]^2 > 0 \tag{86}$$

The function $q(y)$ has the same sign as

$$\left[\frac{d^2R}{dt^2} \right].$$

The features of $q(y)$ are shown below.

$$\left. \begin{array}{ll} q(1) = -1 & q(0) = A > 0 \\ q(+\infty) \to -\infty & q(-\infty) \to +\infty \end{array} \right\} \tag{87}$$

According to the values of parameter A, the function $q(y)$ has different shapes as shown in Figure 10. If A is less than 27/4, there is no point of inflection in $R(t)$ as shown in Figure 10a. On the other hand, if A is greater than 27/4, there are two points of inflection in $R(t)$ as shown in Figure 10b.

In usual cases, A is considered to be greater than 27/4 as explained in the following chapter. In other words, the diffusion curve of Model III is not a kind of simple S-shaped curve such as the logistic curve, the Gompertz curve and Model I in this paper. The curve obtained here is a more sophisticated growth curve.

Several interesting features, which our model shows, are summarized as follows.

The first one is the existence of a discontinuous starting point (T_*) in the diffusion curve. This means that there is a kind of "critical" mass ("volume") $R(T_*)$ for starting the diffusion as in nuclear fission reaction.

The second one is related to the number of inflection points. Our model has usually two points of inflection, T_1 and T_2, while an ordinary diffusion curve has only one point.

The first inflection point (T_1) might be considered to occur mainly because of the saturating trend in the price of advanced equipment as shown in equation (65). On the other hand, the

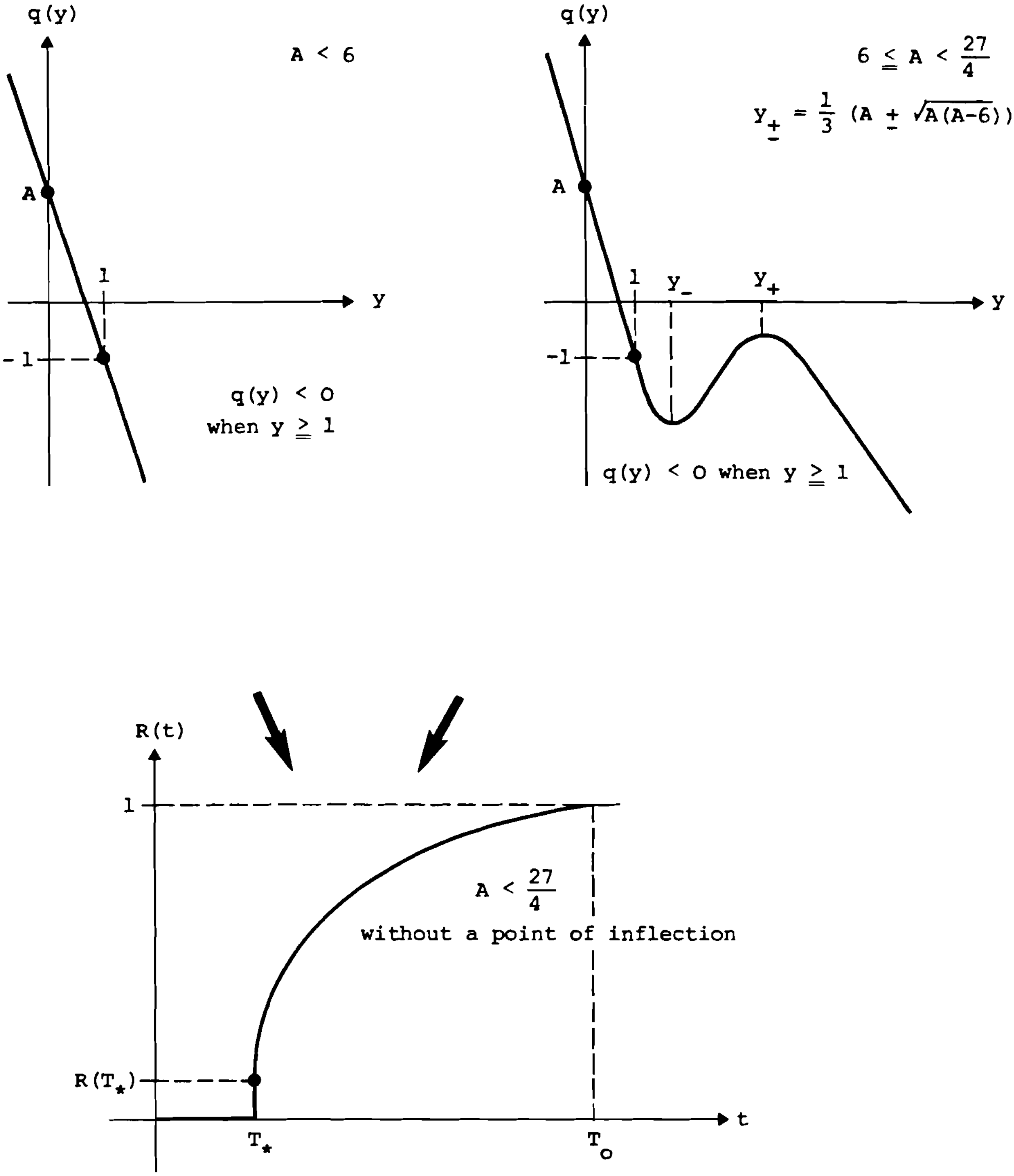


Figure 10a. Diffusion curve of Model III $(A < \frac{27}{4})$

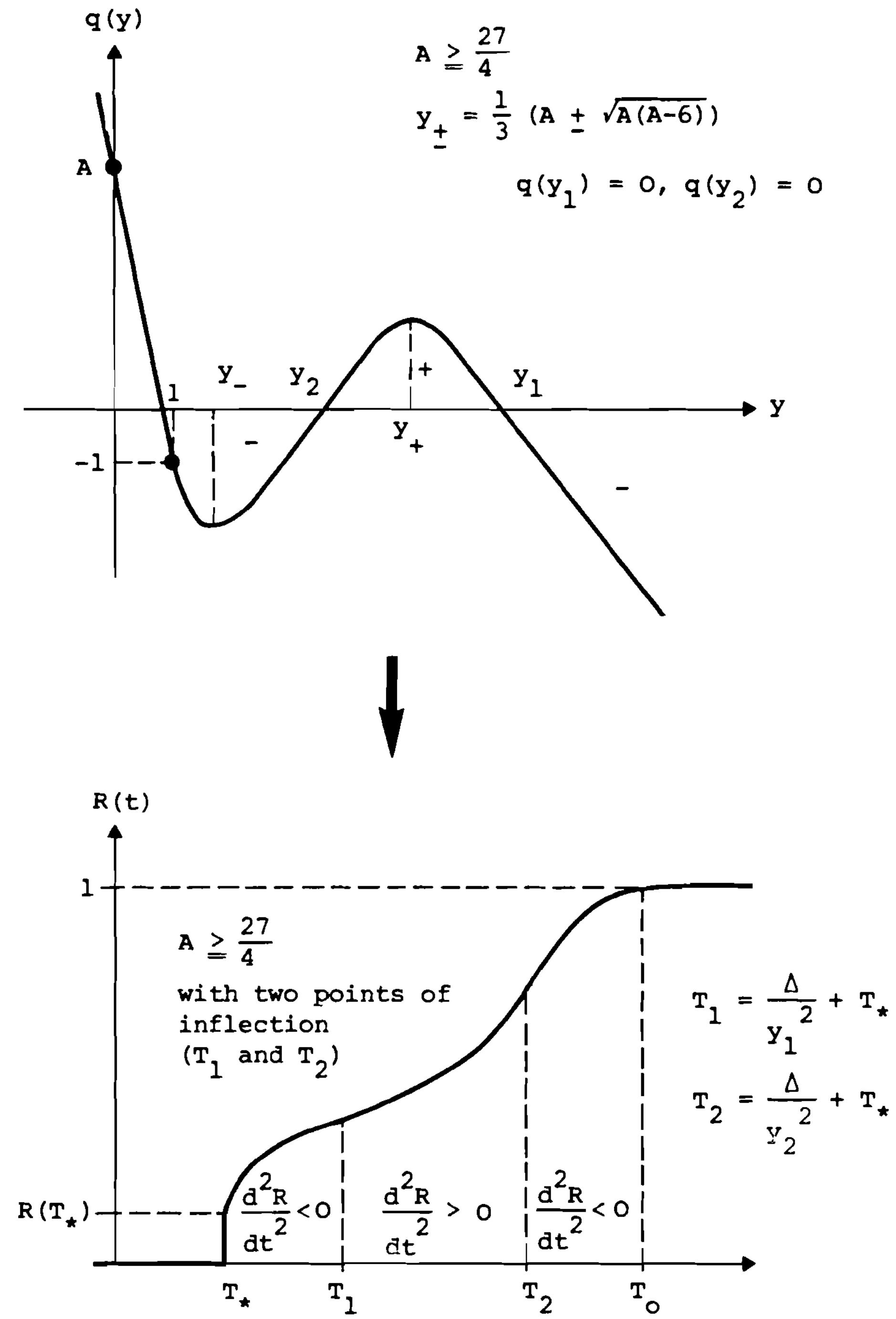


Figure 10b. Diffusion curve of Model III ($A \geqq \frac{27}{4}$).

second point (T_2) might be considered to occur mainly because of the saturating trend in cumulative company size distribution F(x) in equation (13).

The third feature is the period of diffusion from starting to saturation, namely Δ in equation (75). The impacts on the diffusion period Δ by the various parameters in our model are similar to Model I, as was expected.

6.3 Comparison with Model I

The parameters, except for α, c and K_0, are the same as set in Model I. Parameter α is set zero because this effect is considered by parameter c in Model III as follows.

$$\left.\begin{aligned} \ln P &= 2.8368 - \underset{(0.01607)}{0.19432} \ln U \\ R^2 &= 0.9734 \\ c &= 0.19432 \end{aligned}\right\} \qquad (88)$$

Parameter K_0 is set on the basis of the condition that the resulting P_0 is equal to that in Model I.

The results of Model III are summarized in Table 6. In addition, Figure 11 shows the differences of the diffusion curves between Model I and III.

By introducing a type of learning curve effects into Model I, the speed of diffusion in Model III becomes lower than that in Model I, as could be easily expected. Accordingly, the saturation year (T_0) is postponed from 1992 in Model I to 2004 in Model III.

The period of diffusion Δ is estimated to be about 25 years, i.e. from 1979 to 2004.

The starting point of diffusion is also estimated to be $R(T_*) = 0.005$ at $T_* = 1978.96$.

7. <u>Generalized Models</u>

The model equations in this paper are written with the assumption of constants α and β. These parameters are used in the form of

$$e^{-\alpha t} \text{ and } e^{\beta t} .$$

Table 6. Comparison of Model I and Model III

MODEL III

PARAMETERS	
alpha	0
lambda	0.042246
a+b	0.18384
beta	0.0415
VO	1.3593
LO	1.51
n	3.5
c	0.19432
KO	19.26
alpha+beta	0.0415
delta	24.80
T*	1978.96
TO	2003.76
R(T*)	0.0050084
t=0 (1980)	
A	10.593255 > 27/4

YEAR	R(t) MODEL I	R(t) MODEL III
1970	0.000	0.000
1971	0.000	0.000
1972	0.000	0.000
1973	0.000	0.000
1974	0.001	0.000
1975	0.001	0.000
1976	0.003	0.000
1977	0.005	0.000
1978	0.011	0.000
1979	0.020	0.007
1980	0.035	0.035
1981	0.060	0.068
1982	0.097	0.107
1983	0.151	0.152
1984	0.224	0.202
1985	0.318	0.257
1986	0.430	0.315
1987	0.556	0.374
1988	0.685	0.435
1989	0.807	0.496
1990	0.908	0.556
1991	0.975	0.614
1992	1.000	0.670
1993	1.000	0.723
1994	1.000	0.771
1995	1.000	0.816
1996	1.000	0.856
1997	1.000	0.892
1998	1.000	0.922
1999	1.000	0.947
2000	1.000	0.967
2001	1.000	0.983
2002	1.000	0.993
2003	1.000	0.999
2004	1.000	1.000
2005	1.000	1.000
2006	1.000	1.000
2007	1.000	1.000
2008	1.000	1.000
2009	1.000	1.000
2010	1.000	1.000

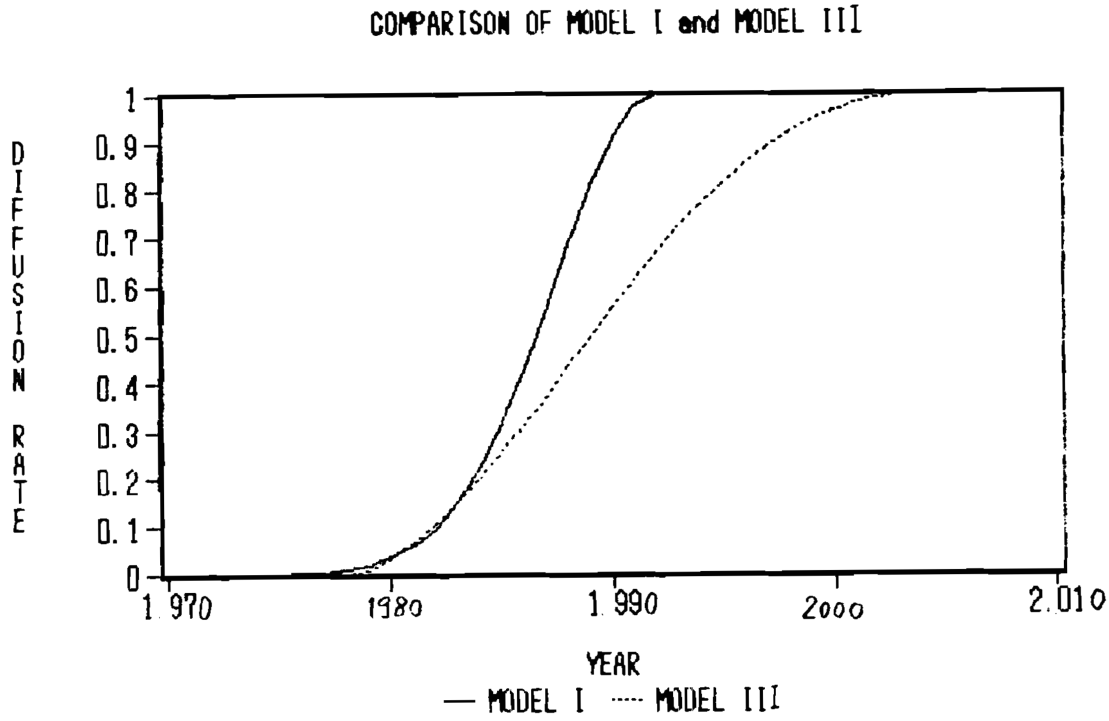


Figure 11. Comparison of Model I and Model III.

In order to apply our models to long-term forecasting for real cases, it is necessary to formulate the models in a more general form by using time-dependent general functions of prices and wages.

The results of these formulations are as follows.

We introduce the function $P_o(t)$ and $W_o(t)$ instead of

$$P_o \cdot e^{-\alpha t} \text{ and } W_o \cdot e^{\beta t}.$$

The resulting R(t) can be calculated from the following equations.

Model I

$$R(t) = EXP \left[- \frac{\lambda}{(a+b)^2} \left[\ln \left(\frac{P_o(t)}{W_o(t) \cdot l_o \cdot n} \right) \right]^2 \right] \tag{19'}$$

Model II

$$R(t) = N \left[\frac{z_o}{\sigma} \right] + \frac{Q \cdot \Sigma}{\sigma} \left[1 - N \left[\frac{z_o - \mu}{\Sigma} \right] \right] \tag{49'}$$

where

$$z_o = - \ln \left[\frac{P_o(t)}{W_o(t) \cdot l_o \cdot n} \right] \tag{47'}$$

In Model III we introduce only $W_o(t)$ instead of constant W_o, because the price P is, in this case, a function of R(t) as shown in equation (65).

$$R(t) = EXP \left[- \frac{(a+b)^2}{4\lambda c^2} \left[1 - \sqrt{1 - \frac{4\lambda c}{(a+b)^2} \ln \left(\frac{K_o}{W_o(t) \cdot l_o \cdot n} \right)} \right]^2 \right] \tag{77'}$$

$$T_o \geq t \geq T_* \tag{76'}$$

The starting point T_* and the saturation point T_o can be obtained by solving the following equations

$$W(T_*) = \frac{K_o}{l_o \cdot n} e^{-\frac{(a+b)^2}{4\lambda c}} \tag{72'}$$

$$W(T_o) = \frac{K_o}{l_o \cdot n} \tag{73'}$$

If we use the above generalized models, the resulting diffusion curves become more realistic, more sophisticated and more easily applicable than those of the simple functions of Models I, II and III.

8. Conclusions

Integrated models, namely Models I, II and III, have been developed in this paper for the diffusion of advanced technologies into industry. These models have also been applied to the diffusion curve of advanced industrial robots in Japanese manufacturing.

According to the results of this study, the following might be concluded with regard to our models:

The models developed here are applicable for forecasting the diffusion of advanced technological equipment into industry. By using these models, we can see the distribution of diffusion in industry from the viewpoint of the company size.

Our models can be applied to any diffusion problem if costs and benefits are expressed as a function of company size. In other words, the labor-saving benefit used in this paper is only an example of various benefits.

Finally, the following point should be kept in mind when we forecast the diffusion of advanced technologies:

As mentioned in the previous paper, advanced technology diffuses three-dimensionally (see Figure 12).

Our models should be applied to each application and sector. In other words, in order to apply our model, it is essential to extract the important applications and sectors which do not yet appear at present.

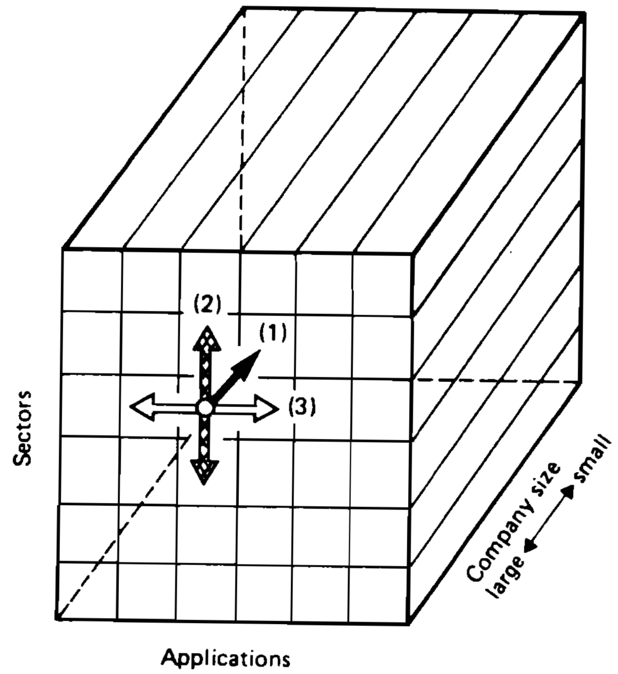


Figure 12. Three-dimensional diffusion of advanced technologies.

APPENDIX A: Notation of Variables and Parameters

x: company size (number of employees in company)

L: total number of employees in industry

$f(x)$: labor distribution density function with respect to company size x

$F(x)$: cumulative labor distribution function from $+\infty$ to x of company size

$$F(x) = \int_x^\infty f(x')dx'$$

λ: parameter of labor distribution

$$F(x) = EXP(-\lambda\cdot(\ln x)^2)$$

$u(x)$: number of units introduced in company of size x

u_0: density of advanced equipment (units per employee)

$l(x)$: labor saved by introducing advanced equipment in company of size x

l_0: labor saved per unit of equipment

$C(x,t)$: investment cost for introducing equipment in company of size x at time t

$p(x,t)$: unit cost of equipment in company of size x at time t

a: parameter showing the effect of "economy of scale" in user cost (cost $\propto x^{-a}$)

α: annual rate of price decrease (price $\propto e^{-\alpha t}$)

P_0: constant coefficient which denotes the unit cost of equipment in company size x=1 at time t=0

$B(x,t)$: benefit of labor saving in company of size x at time t

$w(x,t)$: annual wage in company size x at time t

b: parameter showing the effect of the wage gap between large and small companies (wage $\propto x^b$)

β: annual wage increase rate (wage $\propto e^{\beta t}$)

W_0: constant coefficient which denotes the annual wage in company size x=1 at time t=0

n: decision criterion for investment (years)

$X(t)$: minimum size of companies which decide to introduce advanced equipment at time t

$U(t)$: population of advanced equipment in industry at time to

U_∞: upper limit of the population

$R(t)$: diffusion rate of advanced equipment [$R(t) = U(t)/U_\infty$]

[Model I]

t_0: saturation time

$$t_0 \equiv \frac{1}{(\alpha+\beta)} \ln \left[\frac{P_0}{W_0 \cdot l_0 \cdot n} \right]$$

t_*: point of inflection

[Model II]

z: random variable which represents the variance of the C/B value

σ: standard deviation of the C/B value

g(z): normal distribution function with a mean = 0 and a standard deviation = σ

N(z): cumulative normal distribution function with a mean = 0 and a standard deviation = 1

h(x,t,z): cumulative diffusion level up to company size x from $+\infty$ in case of random variable z at time t

H(x,t): expected value of h(x,t,z)

$$H(x,t) \equiv \int_{-\infty}^{+\infty} h(x,t,z)g(z)dz$$

$F_0(x,t)$: distribution function of diffusion in terms of company size (ln x) at time t

$$F_0(x,t) \equiv - \left[\frac{dH(x,t)}{d(\ln x)} \right]$$

[Model III]

c: coefficient of learning curve effects [$P_0 = K_0 \cdot R^{-c}$]

K_0: constant of learning curve effects [$P_0 = K_0 \cdot R^{-c}$]

T_*: starting time of diffusion

T_0: saturation time of diffusion

Δ: period of diffusion [$\Delta \equiv T_0 - T_*$].

APPENDIX B: Empirical Laws in Technology Diffusion

When we review the past history of technology, we can observe some kinds of broad, general trends in the diffusion processes of new technological equipment into industries. These general trends might be called "empirical laws" in technology diffusion. The following "empirical laws" are used in our formulation of penetration mechanisms.

(1) Decreasing price of advanced equipment (see Figure B-1).
The price has a tendency to decrease year by year, although it is high at the initial stage of diffusion. The reasons generating this tendency could be classified into the following factors:
   a) The technological innovation effect [Ayres, 1987]
   b) The "Economy of scale" effect in production
   c) The "Experience curve" (or learning curve) effect in production [Tani, 1987b; Ayres & Funk, 1987].

   However, it is difficult to extract these three effects separately from the statistical data because they work simultaneously to reduce the costs of production in the real diffusion process.

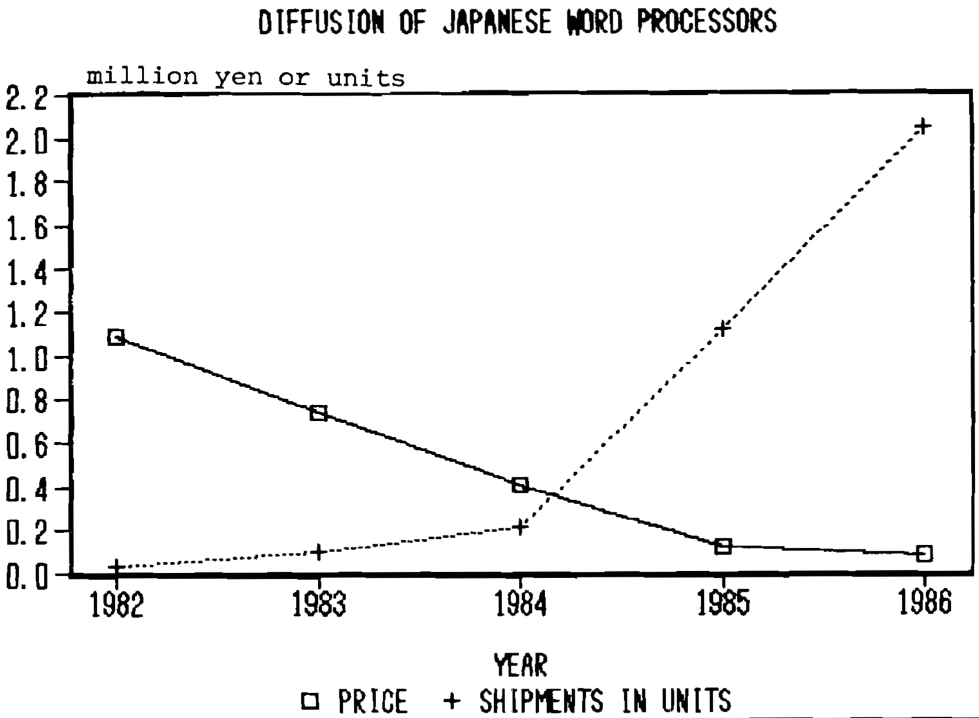
(2) Large companies (or factories) introduce the advanced equipment earlier than smaller companies in terms of the statistical (macro-level) diffusion rate [Tani, 1987b].

(3) Decision-making to introduce the advanced equipment at the company level depends mainly on the cost/benefit evaluation criterion [JIRA, 1984; Maly, 1987b; Sheinin & Tchijov, 1987; ECE 1986]. If the major benefit is a labor-saving effect, the relative cost of equipment to the wage rate becomes the most important factor in decision-making [Mori, 1987; Tani, 1987a; Ayres, Brautzsch & Mori, 1987].

(4) "Economy of scale" in user costs.
The cost-performance of advanced equipment has a tendency to be better in larger companies than in smaller companies [JIPDEC, 1987].

(5) The wage gap between large and small companies [MITI, 1986].

(6) Company size distribution [MCA, 1983].
Small and medium-size companies have a great share of labor in industry.



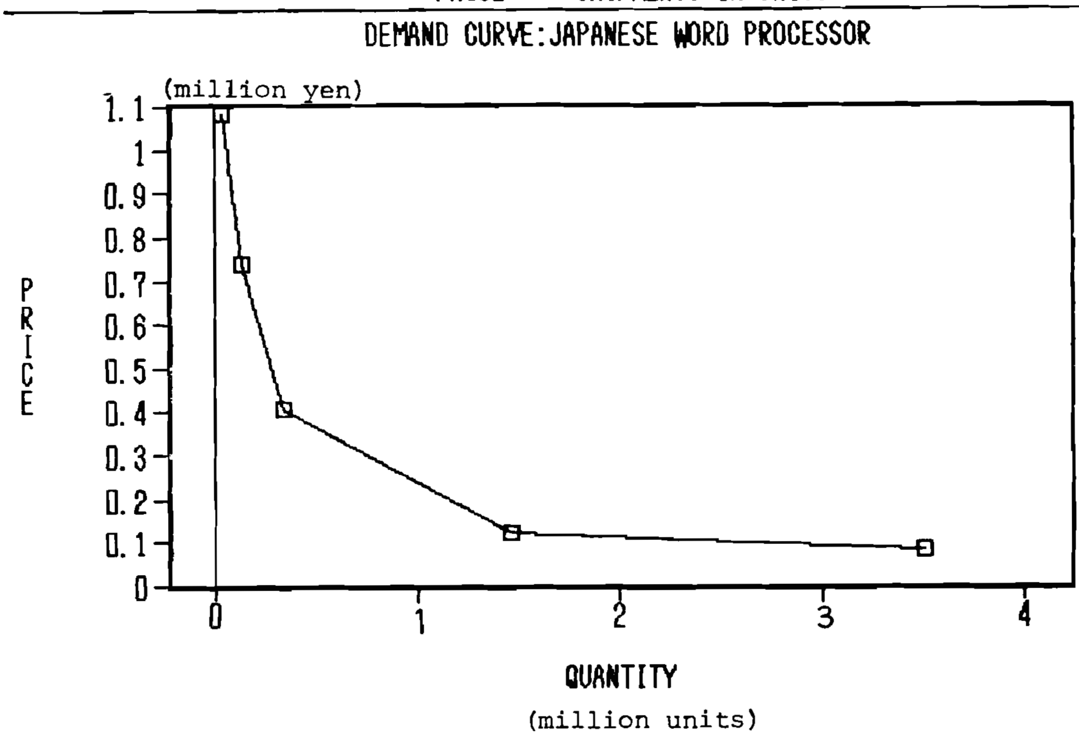


Figure B-1. An example of decreasing price.

The above two tendencies are clearly observed in Japan. The same patterns are also seen in the U.S.A., although the differences among company sizes are smaller than in Japan (see Figure B-2).

(7) As the diffusion proceeds with technological progress and price reduction, various applications of advanced equipment appear; the equipment is of higher quality and is used in the broader industrial sectors, which accelerates the further diffusion of technologies [JIRA, 1985; Tani, 1987c].

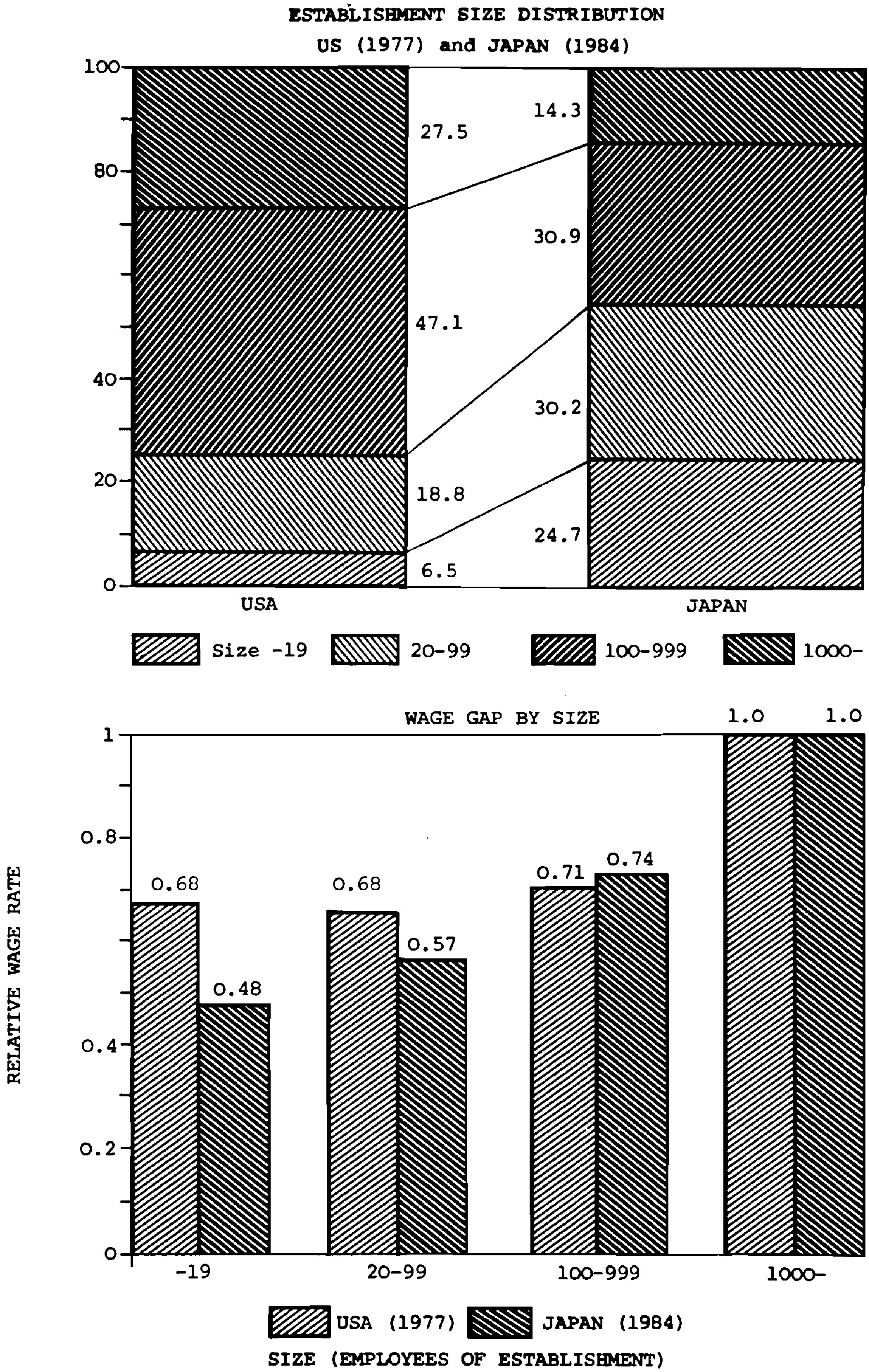


Figure B-2. Comparisons of company size distribution between the USA and Japan.

APPENDIX C: Comparison with the Production Function Model

The author applied the production function model proposed by S. Mori [Mori, 1987] to the diffusion of advanced industrial robots in Japanese manufacturing [Tani, 1987a].

According to the model, the robot density (U/L) is expressed as a function of the ratio of robot price to wage rate, (P/W).

$$(U/L) \propto (P/W)^{\frac{1}{a_0-1}}$$

$$a_0 = 0.7171$$

The parameter a_0 denotes an elasticity of substitution between robots and workers as shown in the following CES function.

$$\text{Equivalent Labor Force} = \left[L^{a_0} + A \cdot U^{a_0} \right]^{1/a_0}$$

On the other hand, if we assume

$e^{-\lambda \cdot \ln x}$ instead of $e^{-\lambda \cdot (\ln x)^2}$

in equation (14) as F(x), the diffusion rate R(t) has the following relation to (P_0/W_0) according to the model described in this paper.

$$R(t) \propto \left[P_0(t)/W_0(t) \right]^{-\frac{\lambda}{a+b}}$$

As L can be approximated as a constant in the above case, the parameter a_0 is considered to have the following relationship with a, b and λ.

$$\frac{1}{a_0-1} = -\frac{\lambda}{a+b} \quad \Rightarrow \quad a_0 = 1 - \left[\frac{a+b}{\lambda} \right]$$

By using the estimated λ, a_0 can also be estimated through the model in this paper.

The parameter λ takes various values for the range of sample data as shown in Table C-1, because

$e^{-\lambda \cdot (\ln x)^2}$ is more appropriate than $e^{-\lambda \cdot (\ln x)}$.

Table C-1. Comparison with production function type model [Tani, 1987a]

SIZE	F(x)	ln(x)	ln(F(x))
1	1	0	0
5	0.9153	1.609437	-0.08850
10	0.8145	2.302585	-0.20518
30	0.6197	3.401197	-0.47851
50	0.5247	3.912023	-0.64492
100	0.4083	4.605170	-0.89575
200	0.3085	5.298317	-2.17603
300	0.2574	5.703782	-1.35712
500	0.1985	6.214608	-1.61696
1000	0.1285	6.907755	-2.05182

a+b = 0.18384

Production Function Model

Equivalent labor force
= (L^a$_c$ + A*U^a$_c$)^(1/a$_c$)

a_c = 0.717
[Tani, 1987a]

$\ln F \propto e^{-\lambda \ln x}$

Estimated Parameters	Samples for Regression analysis (Size)						
	10-	30-	50-	100-	200-	300-	500-
λ =	0.396473	0.438491	0.462942	0.499933	0.545793	0.579913	0.627370
a_c =	0.536311	0.580744	0.602888	0.632271	0.663169	0.682987	0.706967

The estimated a_c has a tendency of being larger in the sample range of large company size as shown in Table C-1.

In case of sample range $(x \geq 500)$, a_c is estimated to be 0.7070, which is very near the value of 0.7171, estimated by the production function model.

This means that the parameter value, $a_c = 0.7171$ in the production function model is effective only in the early stage of diffusion.

At the later stage of diffusion, the lower value is considered more appropriate for a_c.

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