# WORKING PAPER

## ON FUTURE MORTALITY

*Nathan Keyfitz*

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IIASA
International Institute
for Applied Systems Analysis

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## Foreword

Population projection is the activity of demographers that is best known to the public. The record shows some successes, in which the projected population came close to the population that subsequently materialized, and some failures, in which the two were far apart. The accurate forecasting of population using nothing but demographic data is impossible, but two things can be done: marginal improvements in accuracy can be made, and the accuracy likely to be attained can be estimated in advance.

Since the future population of any area depends on the three components (future births, deaths, and migration) in a simple accounting identity, its forecasting comes down to forecasting these components. The present paper looks into the mortality component, examining past mortality on the basis of Canadian data for the period 1921 to 1981.

The examination shows that which past interval one takes as the indication of the pace of future population improvement is the most important element of the forecast of mortality. One reason that this finding is useful is that it enables the range of uncertainty in future mortality to be estimated from the range within which the future life table falls when we assume the pace of improvement of various past periods. Application of the same principle to fertility and migration will enable a calculation to be made of the uncertainty of population projections. This broader matter will be developed in a paper shortly to appear.

Nathan Keyfitz
Leader, Population Program

# ON FUTURE MORTALITY

**Nathan Keyfitz**

We can be reasonably certain that mortality will continue to fall; what we do not know is how fast. And the best way of describing how fast is in terms of past periods: will it be as fast as Canada showed in 1976-81, or only as fast as the average 1921-81, or as slow as 1926-31? This apparently simple question, asked in perfectly non-technical language, will be shown to embrace the question of future mortality. The whole matter of projecting mortality comes down to deciding what past period describes the future.

And what difference does the selected mortality schedule make to the projected population of Canada, say by the year 2021? Obviously this is not the whole range of ignorance of the future, but only that part that arises out of uncertainty on mortality. To the variation here described would have to be added (in a probability sense) the corresponding range for fertility and migration.

We could think of the mortality fall of the several five-year time intervals as a random variable, graduating to a normal curve, and then getting 95 percent limits, for instance. I have gone with this refinement only to the degree of taking 1976-81 (the fastest five-year improvement of the 60-year record) as an upper bound of prospective improvement; it is the upper extreme among 12 time intervals; similarly 1926-31 is the lower extreme of the 12. Between these extremes is presumably the range within which some large fraction of the probability for the future lies. More refined methods are given by Keilman and Kučera (1989).

This paper starts with geometric extrapolation using the minimum of data. First it finds the ratio of the $q_x$ of 1981 to that of 1976 at each separate age, and takes this as the ratio for all times in the future. The result is compared with the same geometric extrapolation, but on the complement of survivorship, $1 - l_x$, where again the ratio of improvement is taken from the last time interval, 1976-81 (Table 1).

Table 1. Life expectancy $\overset{o}{e}_x$ 1986-2021 on two methods, showing the effect on $\overset{o}{e}_x$ of geometric extrapolation on $q_x$ versus geometric extrapolation on the complement of survivorship, $1 - l_x$, where the ratio of improvement is taken from the last time interval, 1976-81.

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| $q_x$ | 76.805 | 77.960 | 78.999 | 79.937 | 80.786 | 81.556 | 82.256 | 82.894 |
| $1-l_x$ | 76.575 | 77.525 | 78.383 | 79.160 | 79.866 | 80.510 | 81.100 | 81.641 |

The geometric series based on $q_x$ gives higher survivorship, with 1.25 years more by 2021. Both of these are high compared with what we will see below, and that is due to the ratio used, 1981 to 1976, being the largest improvement of mortality in the 60 year record.

Table 2. Same as Table 1 giving $\overset{o}{e}_x$ but basing the ratio for the future on the average improvement of the last 6 intervals, i.e. on the average of 1951-1981.

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| $q_x$ | 76.287 | 77.002 | 77.667 | 78.289 | 78.873 | 79.423 | 79.941 | 80.432 |
| $1-l_x$ | 76.156 | 76.750 | 77.305 | 77.825 | 78.312 | 78.770 | 79.201 | 79.608 |

Table 3. Same as Table 1 giving $\overset{o}{e}_x$ but basing the improvement on the average improvement of all 12 intervals, i.e. on the average of 1921-1981, projecting with Brass, $q_x$ and $l_x$.

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| $q_x$ | 76.162 | 76.759 | 77.315 | 77.832 | 78.317 | 78.772 | 79.200 | 79.604 |
| $1-l_x$ | 76.048 | 76.541 | 77.002 | 77.433 | 77.838 | 78.218 | 78.577 | 78.915 |
| Brass | 76.019 | 76.925 | 77.775 | 78.573 | 79.322 | 80.025 | 80.686 | 81.306 |

Table 3 includes a third method due to Brass. It will be recalled that the Brass method consists in first transforming the $l_x$ to logits, say $Y_x$, then choosing one of the life tables (in our case the most recent) as the standard, then finding the simple regression of each of the other tables on the standard, so obtaining an $\alpha$ and a $\beta$ for each life table. Each of these forms a time series, and the two time series may be projected—in our case with a straight line fitted by least squares. The program provides for three different ways of projecting mortality: Brass, geometric projection of $q_x$, and geometric projection of the complement of $l_x$, $1 - l_x$. The Brass procedure gives higher expectancy than either of the other two.

The Brass method is applied only to the entire set of 13 life tables, but for each of the others the ratio for the geometric series projection is obtained in three ways: from the last pair of tables, those around the years 1981 and 1986; from the average improvement of mortality from 1950-1952 to 1980-1982, and the average improvement of the entire set of 13, extending from 1920-22 to 1980-82.

Table 4. Values of $l_1$ projecting with $q_x$, $l_x$, and Brass.

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| ON INCREASE OF LAST INTERVAL | | | | | | | | |
| $q_x$ | 0.9929 | 0.9947 | 0.9961 | 0.9971 | 0.9978 | 0.9984 | 0.9988 | 0.9991 |
| $1-l_x$ | 0.9929 | 0.9947 | 0.9961 | 0.9971 | 0.9978 | 0.9984 | 0.9988 | 0.9991 |
| ON AVERAGE INCREASE OF LAST 6 INTERVALS | | | | | | | | |
| $q_x$ | 0.9924 | 0.9940 | 0.9952 | 0.9962 | 0.9970 | 0.9976 | 0. 981 | 0.9985 |
| $1-l_x$ | 0.9924 | 0.9940 | 0.9952 | 0.9962 | 0.9969 | 0.9976 | 0.9981 | 0.9984 |
| ON AVERAGE INCREASE OF ALL 12 INTERVALS | | | | | | | | |
| $q_x$ | 0.9921 | 0.9934 | 0.9945 | 0.9954 | 0.9961 | 0.9968 | 0.9973 | 0.9977 |
| $1-l_x$ | 0.9921 | 0.9934 | 0.9946 | 0.9955 | 0.9962 | 0.9969 | 0.9974 | 0.9979 |
| Brass | 0.9917 | 0.9932 | 0.9943 | 0.9953 | 0.9961 | 0.9968 | 0.9973 | 0.9978 |

Table 4 shows all of the above for $l_1$, Table 5 for $l_{50}$, and Table 6 for $l_{85}$.

Table 5. Values of $l_{50}$ projecting with $q_x$, $l_x$, and Brass.

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| ON INCREASE OF LAST INTERVAL | | | | | | | | |
| $q_x$ | 0.9436 | 0.9527 | 0.9603 | 0.9665 | 0.9717 | 0.9760 | 0.9796 | 0.9827 |
| $1-l_x$ | 0.9425 | 0.9509 | 0.9581 | 0.9643 | 0.9695 | 0.9740 | 0.9778 | 0.9811 |
| ON AVERAGE INCREASE OF LAST 6 INTERVALS | | | | | | | | |
| $q_x$ | 0.9387 | 0.9442 | 0.9490 | 0.9533 | 0.9571 | 0.9605 | 0.9636 | 0.9664 |
| $1-l_x$ | 0.9386 | 0.9441 | 0.9491 | 0.9536 | 0.9578 | 0.9616 | 0.9650 | 0.9681 |
| ON AVERAGE INCREASE OF ALL 12 INTERVALS | | | | | | | | |
| $q_x$ | 0.9392 | 0.9451 | 0.9503 | 0.9550 | 0.9592 | 0.9629 | 0.9662 | 0.9692 |
| $1-l_x$ | 0.9387 | 0.9443 | 0.9494 | 0.9540 | 0.9582 | 0.9620 | 0.9655 | 0.9686 |
| Brass | 0.9385 | 0.9465 | 0.9536 | 0.9598 | 0.9652 | 0.9699 | 0.9739 | 0.9775 |

Table 6. Values of $l_{85}$ projecting with $q_x$, $l_x$, and Brass.

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| ON INCREASE OF LAST INTERVAL | | | | | | | | |
| $q_x$ | 0.3371 | 0.3728 | 0.4080 | 0.4423 | 0.4756 | 0.5077 | 0.5385 | 0.5680 |
| $1-l_x$ | 0.3239 | 0.3460 | 0.3673 | 0.3880 | 0.4080 | 0.4274 | 0.4461 | 0.4642 |
| ON AVERAGE INCREASE OF LAST 6 INTERVALS | | | | | | | | |
| $q_x$ | 0.3238 | 0.3464 | 0.3689 | 0.3912 | 0.4132 | 0.4348 | 0.4560 | 0.4769 |
| $1-l_x$ | 0.3130 | 0.3248 | 0.3365 | 0.3479 | 0.3591 | 0.3701 | 0.3809 | 0.3916 |
| ON AVERAGE INCREASE OF ALL 12 INTERVALS | | | | | | | | |
| $q_x$ | 0.3161 | 0.3311 | 0.3460 | 0.3607 | 0.3753 | 0.3898 | 0.4041 | 0.4182 |
| $1-l_x$ | 0.3082 | 0.3154 | 0.3224 | 0.3294 | 0.3363 | 0.3432 | 0.3500 | 0.3567 |
| Brass | 0.3055 | 0.3227 | 0.3405 | 0.3587 | 0.3774 | 0.3964 | 0.4157 | 0.4353 |

Table 7. Summary for the year 2021 of Tables 1-6 projecting with $q_x$, $l_x$, and Brass, and using 3 past periods for data.

| | $\overset{o}{e}_0$ | $l_1$ | $l_{50}$ | $l_{85}$ |
|---|---|---|---|---|
| ON INCREASE OF LAST INTERVAL | | | | |
| $q_x$ | 82.894 | 0.9991 | 0.9827 | 0.5680 |
| $1-l_x$ | 81.641 | 0.9991 | 0.9811 | 0.4642 |
| ON AVERAGE INCREASE OF LAST 6 INTERVALS--1951-1981 | | | | |
| $q_x$ | 80.432 | 0.9985 | 0.9664 | 0.4769 |
| $1-l_x$ | 79.608 | 0.9984 | 0.9681 | 0.3916 |
| ON AVERAGE INCREASE OF ALL 12 INTERVALS--1921-1981 | | | | |
| $q_x$ | 79.604 | 0.9977 | 0.9692 | 0.4182 |
| $1-l_x$ | 78.915 | 0.9979 | 0.9686 | 0.3567 |
| Brass | 81.306 | 0.9978 | 0.9775 | 0.4353 |

The summary in Table 7 demonstrates that which past set of data is used matters anything up to 3 times as much as the method of extrapolation from that data. Taking just the last interval and the whole 12 intervals, for $\overset{o}{e}_0$ we have

| | last interval | all 12 intervals | difference |
|---|---|---|---|
| $q_x$ | 82.894 | 79.604 | 3.290 |
| $1-l_x$ | 81.641 | 78.915 | 2.726 |
| difference | 1.253 | 0.689 | |

Note again that the Brass method shows higher survivorship, and accordingly higher life expectancies and higher projected population, than either of the other two projections, when all three are applied to the same set of data, i.e. the 13 Canadian life tables. But this is less important than the difference among past periods chosen as describing the future. We would have obtained even larger differences among periods by taking 1926-31, when if anything there was a fall in $\overset{o}{e}_x$, but have rather chosen to confine the comparisons to the three periods that could reasonably be chosen—the latest, the last 30 years, and the entire 60 years.

In earlier work I have elaborated a few of the innumerable possible methods of projecting mortality. Most obvious is extrapolating age by age, of which the admissible ways—that at least met the minimum requirement of producing probabilities between 0 and 1—included dividing the probability of not surviving $(1 - l_x)$ by 20, say, and adding one of the 1/20ths each five years; converting the $l_x$ to logits and then extrapolating, etc. For any of these we can choose the data base out of the existing record in many ways: the last 10 years, the whole interval of the record 1921-1981, etc. It appeared throughout that the choice of data base was more important than the choice of method.

## COMPARISON WITH THE UN PROCEDURE

Let us compare all these with the United Nations projections (Table 8). Apparently the UN $\overset{o}{e}_x$ is lower than the result of projecting with the mortality of the late 1970s; higher than the use of all 13 life tables by geometric series, mixed with the Brass method. It most nearly coincides with what we obtained by the use of the average ratio of improvement of the last 7 life tables, that is of the interval 1951-81.

Aside from checking our method against that of the United Nations, the comparison in Table 8 serves to evaluate the United Nations method, to see what implicit assumption underlies it. We are not informed what method the UN actually used, but its outcome is almost exactly equivalent to projecting $q_x$ in geometric progression, using the average ratio of 1951 to 1981. Since it seems likely that the future can show more progress than the average of 1921-1981, but probably not as much as 1976-81, this intermediate result seems about as good as anything we can do.

Table 8. Comparison of the three time periods of Tables 1-4, using $q_x$ in geometric progression, and the Brass method.

| | 1976-81 1) | 1951-81 2) | 1921-81 3) | Brass 4) | UN 5) | Departures from UN 1)-5) | 2)-5) | 3)-5) | 4)-5) |
|---|---|---|---|---|---|---|---|---|---|
| 1981 | 75.51 | 75.51 | 75.51 | 75.51 | 75.92 | -0.41 | -0.41 | -0.41 | -0.41 |
| 1986 | 76.80 | 76.28 | 76.16 | 76.02 | 76.69 | 0.11 | -0.41 | -0.54 | -0.68 |
| 1991 | 77.96 | 77.00 | 76.76 | 76.92 | 77.29 | 0.67 | -0.29 | -0.54 | -0.37 |
| 1996 | 79.00 | 77.67 | 77.31 | 77.77 | 77.97 | 1.03 | -0.30 | -0.65 | -0.19 |
| 2001 | 79.94 | 78.29 | 77.83 | 78.57 | 78.51 | 1.42 | -0.23 | -0.68 | 0.06 |
| 2006 | 80.79 | 78.87 | 78.32 | 79.32 | 79.03 | 1.76 | -0.16 | -0.71 | 0.29 |
| 2011 | 81.56 | 79.42 | 78.77 | 80.02 | 79.48 | 2.07 | -0.06 | -0.71 | 0.54 |
| 2016 | 82.26 | 79.94 | 79.20 | 80.69 | 79.95 | 2.30 | -0.01 | -0.75 | 0.73 |
| 2021 | 82.89 | 80.43 | 79.60 | 81.31 | 80.41 | 2.48 | 0.02 | -0.81 | 0.89 |

## EFFECT ON THE PROJECTED POPULATION

With each one of the mortality extrapolations considered we can make a full population projection in order to see what is the corresponding future population, using some standard set of the fertility and migration components.

Table 9 compares the consequences for the output population when we use a given set of data, and try different methods. In all cases, as Table 9 shows, the use of $q_x$ gives a larger population by about 200,000 to 300,000 than does the projections of $1 - l_x$, and again the Brass method is higher than either one. But the choice of the period from which the ratio is selected is again on the whole more important that the choice of method. For example, for the year 2021 we have from Table 9, in thousands of persons,

| | last interval | all 12 intervals | difference |
|---|---|---|---|
| $q_x$ | 32639 | 31874 | 765 |
| $1-l_x$ | 32350 | 31723 | 627 |
| difference | 289 | 151 | |

Again similar to the effect on $\overset{o}{e}_0$, where the base time period matters some 3 times as much as the method.

Table 9. Projected population 1986-2021 on three sets of data, showing the effect of different methods for any one set (thousands of persons).

| | 1986 | 1991 | 1996 | 2001 | 2006 | 2011 | 2016 | 2021 |
|---|---|---|---|---|---|---|---|---|
| ON INCREASE OF LAST INTERVAL | | | | | | | | |
| $q_x$ | 24089 | 25593 | 27101 | 28433 | 29619 | 30657 | 31644 | 32639 |
| $1-l_x$ | 24089 | 25579 | 27059 | 28352 | 29492 | 30477 | 31410 | 32350 |
| ON AVERAGE INCREASE OF LAST 6 INTERVALS: | | | | | | | | |
| $q_x$ | 24089 | 25566 | 27023 | 28282 | 29378 | 30310 | 31181 | 32048 |
| $1-l_x$ | 24089 | 25557 | 26996 | 28229 | 29295 | 30194 | 31031 | 31865 |
| ON AVERAGE INCREASE OF ALL 12 INTERVALS: | | | | | | | | |
| $q_x$ | 24089 | 25559 | 27000 | 28238 | 29308 | 30210 | 31046 | 31874 |
| $1-l_x$ | 24089 | 25550 | 26976 | 28192 | 29236 | 30110 | 30920 | 31723 |
| Brass | 24089 | 25550 | 27001 | 28269 | 29390 | 30364 | 31295 | 32242 |

## CURVE FITTING AND EXTRAPOLATION OF PARAMETERS

Regarded as especially promising is the parametrization of the life table $l_x$ by some function, algebraic or transcendental. Numerous analytical forms appear in the literature, starting more than 150 years ago with Gompertz. Four that have been referred to a good deal are due to Makeham, Perks, unnamed British actuaries, and Pollard-Heligman. These have respectively 3, 4, 5, and 8 parameters, with formulas as follows:

Makeham $\quad u_x = A + Bc^x$

Perks $\quad p_x = \dfrac{1}{1 + A - Hx + Bc^x}$

Actuaries $\quad u_x = \dfrac{A + bc^x}{kc^{-x} + 1 + Dc^x}$

Pollard–Heligman $\quad q_x = A^{(x+B)^c} + De^{-E(\ln x - \ln F)} + \dfrac{GH^x}{1 + GH^x}$

Other curves are also promising, especially that due to Petrioli and Berti (1979), but I have not carried the experimenting past the four mentioned. Stoto (1979) modifies the Brass method to use 4 constants rather than 2 in the regression, so improving the fit at the youngest and oldest ages. Again I have not applied this to projection.

Fitting the four curves to the life tables by least squares is not straightforward, and turned out to exceed my programming ability. I had to call on help from Professor A. Lewandowski of IIASA, and he produced the fits, which I have exhibited in another paper (Keyfitz, 1988). Extracted from that paper is the set of departures of the fitted from the observed for the 13 life tables (Table 10).

The way to use the parameters for purposes of projection would be to fit each of the 13 sets, say of $l_x$. The parameters would then be treated as time series and extrapolated, then the $l_x$ reconstructed from the extrapolated values of the parameters. The goodness of fit to the past is no guarantee that the future $l_x$ will accord with what comes to pass; it is a necessary condition but hardly a sufficient one.

What we find in Table 10 is that the Makeham and Actuaries curves are out of the running—for all of the tables their errors are far greater than for the other two, Perks and Pollard. The Makeham formula does especially badly, and the Actuaries does only slightly better. Perks is an order of magnitude better than either, and the Pollard-Heligman is on average the best of all.

As between the two closest, Pollard is considerably better up to 1946, after which Perks is somewhat better. On the average of the 13 life tables Pollard wins; on the last 7 Perks is better, though only slightly.

Table 10. Root mean square error of fit to $l_x$ for four functions at 13 dates.

| Year | Root mean square errors | | | |
|---|---|---|---|---|
| | Makeham | Perks | Pollard | Actuaries |
| 1921 | 0.03345 | 0.005108 | 0.000885 | 0.02482 |
| 1926 | 0.03542 | 0.005769 | 0.001261 | 0.02600 |
| 1931 | 0.03292 | 0.005045 | 0.001019 | 0.02494 |
| 1936 | 0.02921 | 0.003843 | 0.001583 | 0.02213 |
| 1941 | 0.02330 | 0.002911 | 0.001080 | 0.01777 |
| 1946 | 0.01960 | 0.001724 | 0.001206 | 0.01471 |
| 1951 | 0.01640 | 0.001370 | 0.002454 | 0.01265 |
| 1956 | 0.01355 | 0.001804 | 0.002412 | 0.01005 |
| 1961 | 0.01208 | 0.002323 | 0.003046 | 0.00884 |
| 1966 | 0.00996 | 0.001796 | 0.002743 | 0.00767 |
| 1971 | 0.00780 | 0.001548 | 0.001941 | 0.00565 |
| 1976 | 0.00574 | 0.001649 | 0.002207 | 0.00437 |
| 1981 | 0.00461 | 0.001611 | 0.001790 | 0.00307 |
| Mean | 0.018777 | 0.002807 | 0.001817 | 0.014053 |

## BEYOND FITTING: HOW TO EXTRAPOLATE THE PARAMETERS

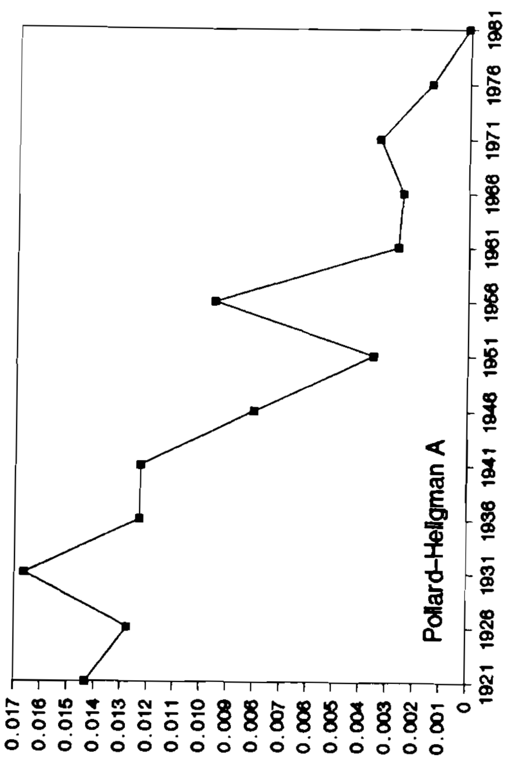
But closeness of fit by itself does not solve the projection problem as we see in the charts of the time series of the several parameters, shown here for the Pollard fit. For rather few of the curves would a straight line do for the projection, and in some there is no obvious trend that can be discerned. We illustrate this with the 8 Pollard constants, whose fitted values are shown in Table 11.

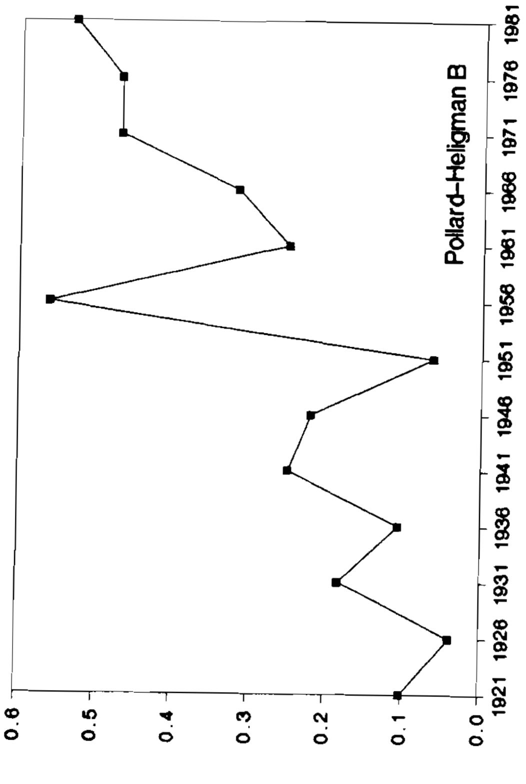
Table 11. The eight parameters of the Pollard-Heligman parameterization of the mortality curve. Fitting is by least-squares to the Canadian life tables from 1920-22 to 1980-82.

| Year | A | B | C | D |
|---|---|---|---|---|
| 1921 | 0.01431 | 0.1013 | 0.2202 | 0.002907 |
| 1926 | 0.01274 | 0.0388 | 0.1717 | 0.002984 |
| 1931 | 0.01664 | 0.1826 | 0.2764 | 0.002955 |
| 1936 | 0.01230 | 0.1055 | 0.2171 | 0.002371 |
| 1941 | 0.01229 | 0.2492 | 0.2963 | 0.001955 |
| 1946 | 0.00801 | 0.2197 | 0.2944 | 0.001309 |
| 1951 | 0.00353 | 0.0622 | 0.1987 | 0.000928 |
| 1956 | 0.00952 | 0.5614 | 0.4928 | 0.000536 |
| 1961 | 0.00264 | 0.2505 | 0.3556 | 0.000420 |
| 1966 | 0.00247 | 0.3167 | 0.3963 | 0.000466 |
| 1971 | 0.00334 | 0.4702 | 0.4625 | 0.000440 |
| 1976 | 0.00141 | 0.4702 | 0.5362 | 0.000447 |
| 1981 | 0.00006 | 0.5310 | 1.1650 | 0.000343 |

| Year | E | F | G | H |
|---|---|---|---|---|
| 1921 | 1.1730 | 29.98 | 0.000035 | 1.106 |
| 1926 | 1.5150 | 30.67 | 0.000046 | 1.103 |
| 1931 | 0.8464 | 30.83 | 0.000041 | 1.105 |
| 1936 | 1.0000 | 33.43 | 0.000044 | 1.104 |
| 1941 | 0.3672 | 40.12 | 0.000041 | 1.105 |
| 1946 | 0.1679 | 47.67 | 0.000047 | 1.102 |
| 1951 | 0.2907 | 48.07 | 0.000047 | 1.101 |
| 1956 | 0.0186 | 48.00 | 0.000047 | 1.101 |
| 1961 | 0.0001 | 30.00 | 0.000050 | 1.099 |
| 1966 | 0.0000 | 29.97 | 0.000048 | 1.099 |
| 1971 | 0.0303 | 30.04 | 0.000052 | 1.097 |
| 1976 | 0.0035 | 26.11 | 0.000050 | 1.097 |
| 1981 | 0.1055 | 8.81 | 0.000046 | 1.097 |

Though overall the most hopeful of the several parameterizations proposed for mortality is that of Pollard and Heligman, it offers difficulties for projection. The matter is investigated in more detail for all four curves in another paper (Keyfitz, 1988), but Table 11 gives an indication of the sensitivity of the projection of the parameters to the period that is chosen. Parameter D for example, if projected from the 1921-1956 would show a sharp downward tendency; if projected from 1956-1981 would be very nearly level. Similar remarks could be made about their parameter F, while on the other hand G and H show a trend that would vary less with respect to the past interval from which one extrapolates. The figures make these points clear.


Pollard–Heligman A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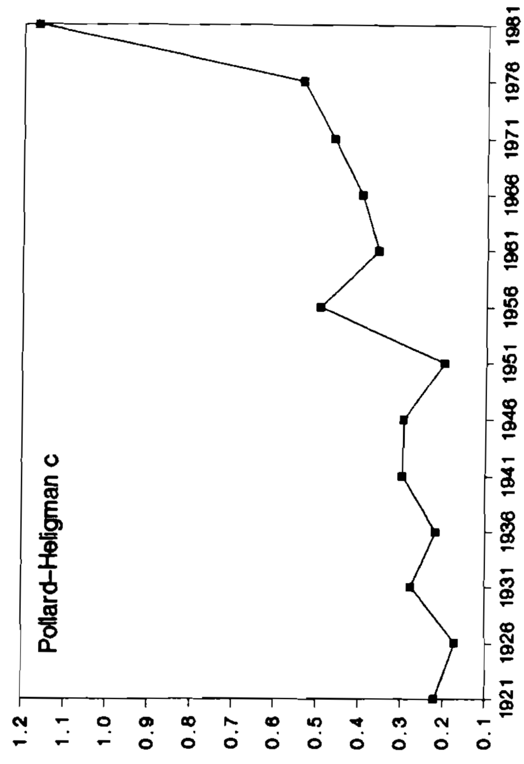
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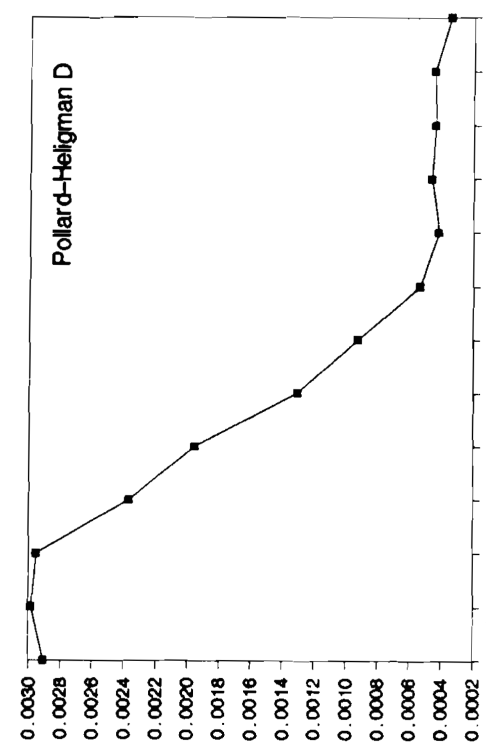
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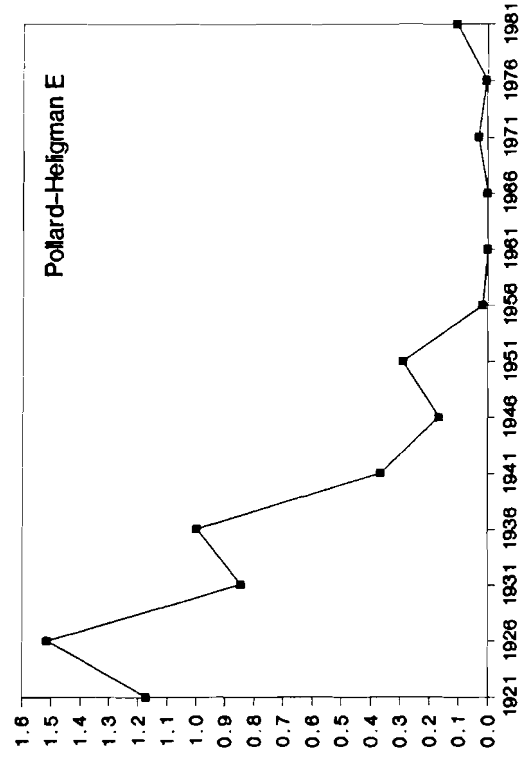
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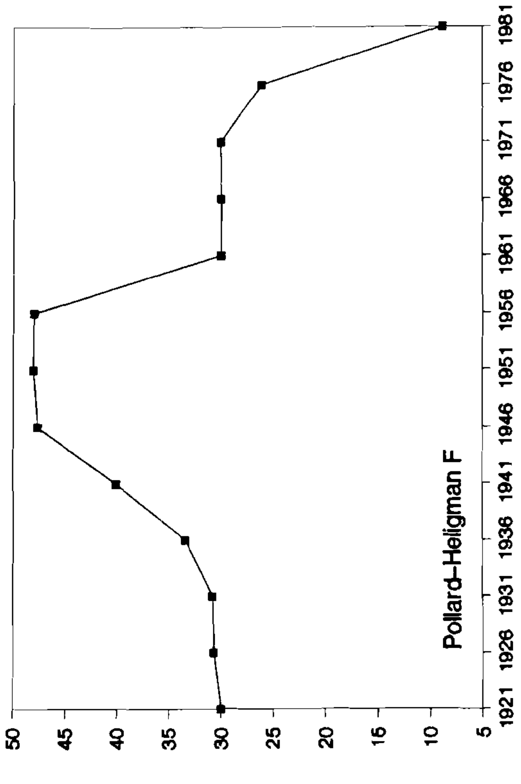
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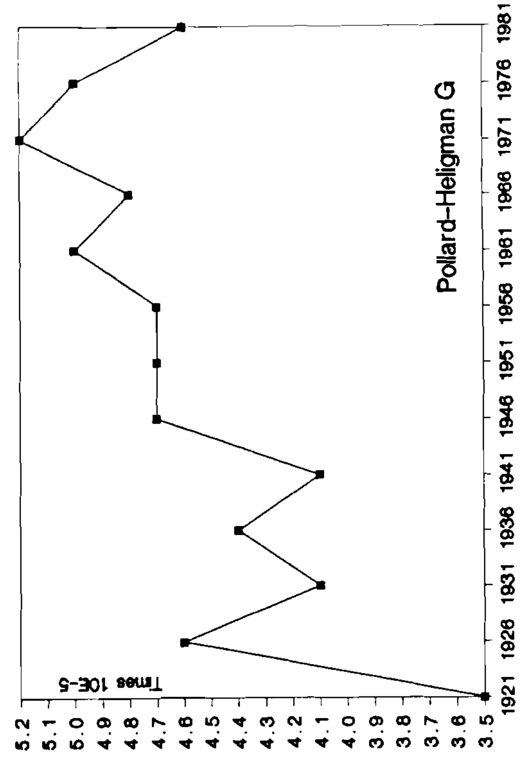
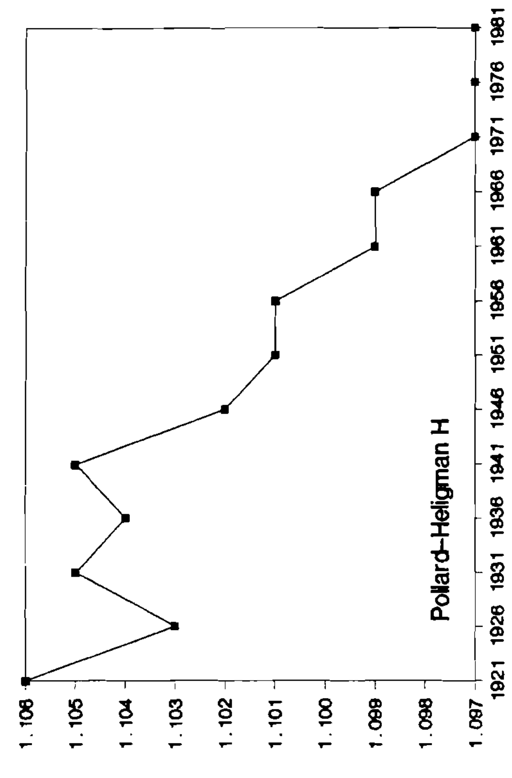
10
5
1921 1926 1931 1936 1941 1946 1951 1956 1961 1966 1971 1976 1981


Pollard-Heligman G
Times 10E-5
5.2
5.1
5.0
4.9
4.8
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4.0
3.9
3.8
3.7
3.6
3.5
1921 1926 1931 1936 1941 1946 1951 1956 1961 1966 1971 1976 1981


Pollard-Heligman H
1.106
1.105
1.104
1.103
1.102
1.101
1.100
1.099
1.098
1.097
1921 1926 1931 1936 1941 1946 1951 1956 1961 1966 1971 1976 1981

Thus the problem of selection of the base period reappears in this quite different projection method from that of Tables 1-8. Parameterization cannot avoid the decision on whether 1986-2021 will be like 1976-81, or like 1951-81, or like 1921-81.

## PROJECTION OF MORTALITY TREND BY REGRESSION

Can we bypass such fitting and extrapolation, and simply project the trend of the 60 years in the rate of improvement of life expectancy? The regression of the improvement in life expectancy against time in calendar years is given by

| | |
|---|---|
| Constant | 7.396 |
| Coefficient of X | -0.003 |
| Std Err of Coefficient of X | 0.012 |
| Std Err of Y Est | 0.750 |
| r Squared | 0.006 |

The coefficient of X is negative, -0.00313, but the amount is so small that we hardly need calculation to assure ourselves that it is not significant—in fact it is only about one quarter of its standard error.

We could elaborate this in various ways. One would be by using GNP as an independent variable. That would be unlikely to secure significance, and even if it did it would place on us the burden of estimating the GNP for the next 40 years. Another elaboration would be to take the trend age by age, but further pursuit of the regression option does not seem worthwhile.

## TAKE ADVANTAGE OF SERIAL CORRELATION?

Another possibility is to make use of serial correlation. A considerably armory of techniques exists that would enable us to go from the last mortality table to the one beyond, and then to one more, and so on. Of course the error would increase as we went far into the future, but that has still permitted useful applications in other fields. Let us test this out by finding what is the regression of each item on the preceding item in the five-year improvement in the $e_0$. For the years 1921 to 1981 we can construct 11 pairs of neighbors. The resulting correlations and regressions are as follows:

| | |
|---|---|
| Constant | 1.039 |
| Coefficient of X | 0.184 |
| Std Err of Coefficient of X | 0.331 |
| Std Err of Y Est | 0.776 |
| r Squared | 0.033 |
| r | 0.18 |
| Mean increase over 12 intervals is | 1.292 |

The standard error of the x-coefficient is nearly double the coefficient. The coefficient of correlation is the square root of 0.0336 or 0.18. The regression tells us that we can take the increase in $e_0$ for 1981-86 to be equal to the increase of 1976-81 times 0.184982 + 1.039076, but the standard error of this would be 0.776. This looks better than the simple trend of improvement, but still not good enough to use.

With no trend in the rate of improvement and no appreciable serial correlation our best bet on the future is the average of the past. Thus the forecasting problem comes down to "Which past?"

## LACKING SIGNIFICANT TREND ONE CAN ONLY USE THE HISTORIC AVERAGE OF AMOUNT OF IMPROVEMENT

No method of projecting mortality can escape the question: what will future decades be like? Will they be like the 1970s, with an improvement of 2.5 years? Or will they rather resemble the 1920s or the 1960s, with about 1.4 years of improvement? No trend calculation, or regression on economic variables will answer this, for an element enters that is more or less independent of prosperity or depression: technical advance—antibiotics for the 1940s and 1950s, new ways of handling heart disease in the 1970s.

In the face of inevitable ignorance of future technical development, how then should one make the projection? The answer to that is clear: simply suppose that the average improvement of the past 60 or the past 30 years will apply to the succeeding 40. For both sexes together one would have the results shown in Tables 1-7. Without much to back the choice I favor supposing future improvement to be the same as that of 1951-81. That also, we infer, was the choice of the UN.

A useful compilation of what it is that national offices do to produce official forecasts is provided by Cruijsen and Keilman (1989). They mostly avoid such sophistication as fitting parameters.

As for the sexes, can we suppose that the present differential will continue, it being a permanent biological fact? Or will it rather diminish as women tend to have careers similar to those of men? It might be wise to suppose that it goes down by 2021 to about half of what it is in 1981, a trend that would be consistent with the increasing similarity in the life styles of the two sexes. But there is no science to back such a decision.

## RECOGNIZING CAUSE OF DEATH

Whatever the method and period chosen, performing the projection by individual causes has been strongly recommended. In a time when infectious disease was large, and showed a different trend and different age incidence from chronic disease, the case for the recognition of causes was indeed strong. But now that infectious diseases have smaller effect on mortality, and the age impact of the chronic diseases does not differ much from one to another, the usefulness of breaking down the calculation by cause is considerably diminished.

## CONCLUSION

Study and comparison has been carried out on the various ways of projecting mortality. The choices seem to be as follows:

1) Extrapolating each age separately. This can be done equivalently on any of the life table functions, $l_x$, $q_x$, $M_x$, etc. If on the $l_x$ or $q_x$ transforming by the logit function, extrapolating, and then transforming back will ensure that the results come out between zero and unity. So will taking the future $q_x$ or $1-l_x$ in a geometric ratio obtained from some past period.

2) Fitting each past point of time with a suitable function, projecting each of the parameters treated as a time series, then reconstructing the future curves.

3) The Brass method: transforming to logits, regressing on a standard population, then extrapolating the time series of regression coefficients.

4) Regression over time on other series such as GNP per capita.

5) Auto-regression of the series on itself.

6) Performing any of the above by individual causes, then assembling the causes.

Of the above, 4) and 5) are dismissed by the low correlations that were found between mortality and income or other series; 6) would have been appropriate in an age when infectious disease was common, since it is distributed over different ages from chronic disease but is less appropriate now. In Canada, as in other advanced countries, infectious disease has diminished greatly, so most of the important causes of death now are chronic, and these have similar distributions. The usefulness of differentiating causes for projection purposes is much diminished. by the similarity of those age distributions.

There is no obvious trend in the rate of improvement of mortality—we cannot say that mortality is tending to improve faster as time goes on. Not being able to project a trend we are reduced to determining an average rate of improvement, and that comes down to deciding from what past period we ought to calculate that average. One obvious choice is the whole record—the 13 life tables that are available for Canada from 1920-1922 to 1980-2. Other choices are the last half of the period, that shows somewhat more improvement than the first half, or the more recent interval, from 1975-77 to 1980-2, that shows phenomenal improvement. To the series can now be added 1985-7. More sophisticated would be weighting the recent life tables more than the earlier ones, but it is hard to say what improvement that would make.

The curve fitting approach has on the whole been little used, even though strongly recommended (Keyfitz, 1984). Two curves that provide good fits have shown up: that due to Pollard and Heligman and that due to Perks, the former better for the whole period 1921-81, the latter better for the latter half of the period. The difficulty comes in the projecting the time series of individual parameters, in several of which the future would be crucially dependent on what part of the past one works from. This seems to apply less to the Brass method than to the others; at least its two constants $\alpha$ and $\beta$ seem to exhibit a steady trend.

Given all this, what is the recommendation? Especially if simplicity in the explanation to the public is a consideration, one could project the $q_x$ by a geometric progression whose initial point is the last existing life table, and whose ratio is the average ratio of change over five year periods in the historic record.

A GAUSS program that produces most of the numbers in this paper is given as an appendix. It can be readily applied to each of the two sexes, to provinces, and to other population groupings, as well as to individual causes of death if they are needed.

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Appendix Program for
EXPERIMENTS IN THE PROJECTION OF MORTALITY

```
@BRING IN SET OF 13 EARLIER LIFE TABLES, 1920-22 to 1980-82@
load lx;output file=out1 reset;FORMAT /M1 /rd;

@CALCULATE px AND qx FOR EARLIER LIFE TABLES@
px=zeros(19,13);px[1,.]=lx[1,.];px[2:19,.]=trim(lx,1,0)./trim(lx,0,1);
qx=1-px;

@TRANSFORMATIONS@
fn f(x)=ln((1-x)./x)/2;fn ff(x)=1/(1+exp(2*x));
@fn f(x)=x;fn ff(x)=x;@

@BRASS METHOD FOR PROJECTING MORTALITY@
b=zeros(2,13);i=13;YY=f(lx);              @THE iTH TABLE IS TAKEN AS STANDARD@
j=0;do while j<13;j=j+1;
        xxx=ones(19,1)YY[.,i];yyy=yy[.,j];
        b[.,j]=inv(xxx'xxx)*xxx'yyy;
    endo;

t=ones(13,1)seqa(1,1,13);beta=inv(t't)*(t'b');
tfut=t|(ones(9,1)seqa(14,1,9));bfut=tfut*beta;yfut=YYzeros(19,9);
j=13;do while j<22;j=j+1;
         yfut[.,j]=YY[.,13]*bfut[j,2]+bfut[j,1];
     endo;
lxfut=ff(yfut);lxfut=ones(1,22)|lxfut;

"This uses the Brass method";goto NEXT;

@PROJECTING qx AS GEOMETRIC PROGRESSION FROM KNOWN LIFE TABLES@
"Averaging all 13 previous life tables";rrx=(qx[.,13]./qx[.,1])^(1/12);
"Using last two life tables only";rrx=qx[.,13]./qx[.,12];
@"Averaging ratios of last seven life tables";rrx=(qx[.,13]./qx[.,7])^(1/6);@
qxfut=zeros(19,22);I=13;do while i<22;i=i+1;qxfut[.,i]=qx[.,13].*rrx^(i-13);
                     endo;pxfut=1-qxfut;
@IN ORDER TO VARY THE LIFE TABLE FOR THE PROJECTION, ALL THAT IS NEEDED IS
TO CHOOSE A DIFFERENT RANGE OF YEARS FOR THE RATIO.  ONE ALTERNATIVE IS TO
USE THE LAST INTERVAL FOR THE RATIO, THAT GIVES HIGHER SURVIVORSHIPS AND A
HIGHER PROJECTED POPULATION THAN THE AVERAGE OF ALL YEARS THAT APPEARS ABOVE.@

@CALCULATE FUTURE lx FROM px@
lxfut=zeros(20,22);lxfut[1,.]=ones(1,22);lxfut[2:20,1:13]=lx;
i=1;do while i<20;i=i+1;
        ppxfut=pxfut[.,14:22]|zeros(1,9);
        lxfut[i,14:22]=lxfut[i-1,14:22].*ppxfut[i-1,.];
    endo;
"This uses the geometric progression of the qx";LXFUT[18,22]*1000;@goto NEXT@;

@ALTERNATIVE: EXTRAPOLATE 1-lx BY GEOMETRIC PROGRESSION@
    comp@LEMENT@=1-lx;
"Averaging all 13 previous life tables";
    rat@IO@=((comp[.,13]./comp[.,1])^(1/12))|.99;
"Using last two life tables only";
    rat@IO@=(comp[.,13]./comp[.,12])|.99;
"Averaging ratios of last seven life tables";
```

```
    rat@IO@=((comp[.,13]./comp[.,7])^(1/6))|.99;
    i=0;do while i<9;i=i+1;lxfut[.,i+13]=(1-lxfut[.,i+12]).*rat;
           lxfut[.,i+13]=1-lxfut[.,i+13];
         endo;format 5,3;
"This uses the geometric progression of the complement of the lx";LXFUT[18,22];

NEXT:

@CALCULATE FUTURE Lx FROM lx@

FORMAT 5,0;"Survivorships lx";seqa(1921,5,22)';seqa(-5,5,20)lxfut*10000;@PRINT@
z=zeros(1,22);zz=zeros(2,22);
l=lxfut;u1=l|zz|zz;u2=z|l|z|zz;u2=zz|l|zz;u3=zz|z|l|z;u4=zz|zz|l;
LLxfut=(u2+u3)*65/24-(u1+u4)*5/24;LLxfut=Trim(LLxfut,4,3);
       LLxfut[1,.]=.9+lxfut[2,.]*2.5+1.6*lxfut[3,.];
       LLxfut=(LLxfut|LLxfut[17,.]*.6)|(LLxfut[17,.]*.3);
(seqa(1921,5,22)sumc(LLxfut))';
surv=trim(LLxfut,1,0)./trim(LLxfut,0,1);surv=surv|z;

survfut=surv[.,14:22];"Life table numbers living LLx";
FORMAT 5,0;"          " seqa(1921,5,22)';
seqa(0,5,19)LLxfut*10000;"total    " sumc(LLxfut)'*1000;           @PRINT@

@RATIO OF 65 AND OVER TO 20-64 FOR FUTURE LIFE TABLES@
old=sumc(LLxfut[14:19,.])'*10000;young=sumc(LLxfut[5:13,.])'*1000;
"65+     " old;"20-64   " young;"Ratio    " old./young*1000;            @PRINT@

@RATIO OF 70 AND OVER TO 20-69 FOR FUTURE LIFE TABLES@
old=sumc(LLxfut[15:19,.])';young=sumc(LLxfut[5:14,.])';
"70+     " old;"20-69   " young;"Ratio    " old./young*1000;            @PRINT@

@DATA FOR PROJECTION@
load dddall;ddd=dddall[.,1];
      bx=ddd[1:7,.]*5;bbx=bx/sumc(bx);
      imx=ddd[26:43,.];immx=imx/sumc(imx);
      popu=ddd[44:61,.];
      ii=ddd[62,.]+200;
      start=ddd[63,.];
p=zeros(19,9);p[.,1]=popu|0;

@CARRY OUT PROJECTION@
i=1;do while i<9;i=i+1;
p[.,i]=p[.,i-1].*(survfut[.,i-1]);
birbir=((p[.,i-1]+p[.,i])/2); birbir=birbir[4:10,.]'bbx*.9;
p[.,i]=((birbir|trim(p[.,i],0,1)))+((immx*350)|0);
endo;

"Estimated future population 1986-2026";
FORMAT 6,0;"         " seqa(1986,5,9)';
seqa(0,5,19)p;"Total    " sumc(p)';                                     @PRINT@

"Ratio of 65 and over to 20-64 for future population";
old=sumc(p[14:19,.])';young=sumc(p[5:13,.])';
"65+    " old;"20-64   " young;"Ratio   " old./young*1000;              @PRINT@
```