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Abstract—Natural gas will be the dominant primary energy source during the next 50 years and consequently the main source of CO_2 emissions. A procedure is proposed where natural gas is steam reformed to CO_2 and H_2, using nuclear heat from HTR nuclear reactors. CO_2 is permanently stored underground, e.g. by reinjection in the original gas field. Hydrogen is piped to the final consumer neat, or mixed with natural gas. A joint venture is proposed between Western Europe and the Soviet Union, where a set of HTR reactors would reform increasing shares of the natural gas piped to Western Europe. The location of the plants in Western Ukrainia would permit using CO_2 for tertiary recovery in old oil fields there. Some economics are given.

INTRODUCTION

The preoccupation for the radiation balance of the earth's atmosphere, and the effects of human activities on it has started about three decades ago, giving rise to a small set of very interesting work and a huge amount of "recycled paper" in the wake of a successful keyword.

A sample of this literature collected by the Institute for Scientific Information (ISI) of Philadelphia, analysed using the "fashion wave equations", shows that the peak of this literature wave was reached in 1984 and the ebb is beginning (Fig. 1). I did not have the possibility to monitor the real work, of which I give an interesting example in Fig. 2, where air trapped in glacier ice has been measured for its CO_2 content (Neftel *et al.*, 1985).

The methodology followed is very interesting because it permits monitoring air "put in the freezer" way back in the past to get the grand view and try to establish a correlation between CO_2 content and climate. This is also because climate models are still too rough and too complicated to generate the fine print of what happens *if*. History may serve to calibrate them, as many situations did occur in the most various context (Bolin, 1985).

The fact is, the CO_2 content of the atmosphere as shown in Fig. 2 did start increasing in 1800 from a fairly stable ≈ 280 ppm (volume) in historical times to the present 330 ppm. As in 1800, world population was expanding in number and in activity levels, and the connection cause-effect is considered obvious, although the share of emissions between various forms of activity, like forest clearing and fossil fuel combustion and others, is still under discussion.

The last point is important for our purposes, because we will propose *active solutions for the control of* CO_2 *emissions linked to energy use*. Forest clearing would require other means of control. Looking at the past, the share can be calculated checking ^{14}C concentration in tree rings. Fossil fuels have no ^{14}C and appear as diluents. The results are reported in Fig. 3 (Bolin, 1985) and show some interesting features, e.g. that land clearance was the largest contributor of CO_2 to date (265×10^9 tons) and fossil fuels generated a little more than half that value (170×10^9 tons). The situation, however, has drastically changed since World War II, with fossil fuels becoming the dominant emitters.

The cure then has to be applied to fossil fuels. The next question is the share of the different fossils, coal, oil, gas, in the consumption of fossils during the next 50 or 100 years, so that the cure can be applied to the dominant ones, to simplify the procedures and maximize the effect of the measures taken.

This forecast in terms of shares can be done in a very robust form by using the Volterra substitution model employed at IIASA to map the dynamics of primary energy markets during the last 100 years (Fig. 4). The model is parsimonious and predictive, even long term, when the dates for new competitors (e.g. fusion) entering the market are available. This is possible using a more abstract model dealing with innovation (Marchetti, 1980), but in our case this is not necessary as we deal with fossil fuels only. Their future is sealed by nuclear penetration alone.

Nuclear systems (or solar for those who believe in it as a future large source of primary energy) will finally close the fossil fuel era, as the Volterra substitution model, applied to big envelopes: wood and renewables, fossils, nuclear and solar, shows (Fig. 5). So the question boils down to the mix of coal, oil and gas. This comes from Fig. 4, but it will be shown in a more detailed way later on. It will be in any case clear that the *dominant primary fossil energy* in the next 100 years will be *natural gas*, and that most of the CO_2 will come out of its combustion, even if it is the fuel with lowest carbon content and highest efficiency in use. *The cure has to be applied there in order to get maximum effect.*

The model shows market shares, but we need absolute quantities for the CO_2 released. For that purpose we

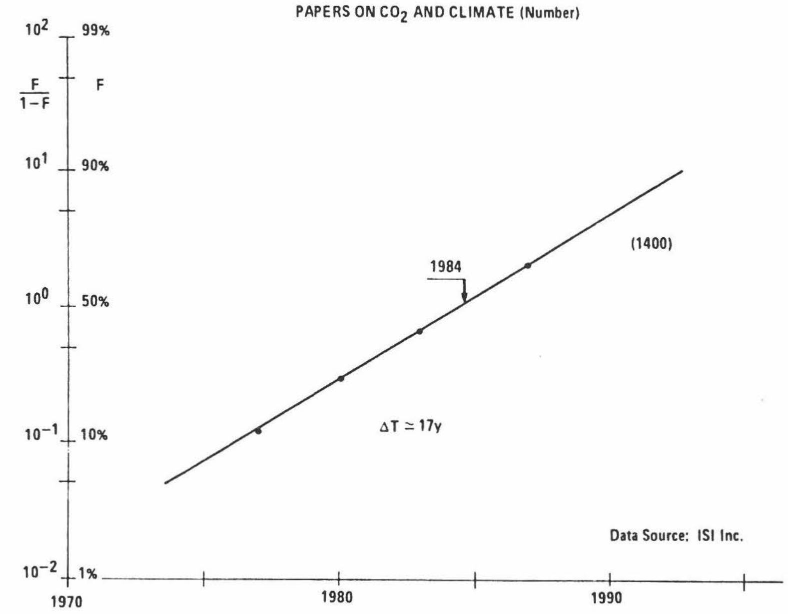

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Fig. 1. Cumulative number of papers on CO_2 and climate as reported in ISI Inc. (1988). Their number is fitted to a logistic equation (see Mathematical Appendix) and the best fit leads to a calculated saturation level of 1400 papers. The point of maximum production of papers, i.e. the flex of the logistic, was passed already in 1984 and we are now on the ebb side of the wave. The time constant being 17 years means in ≈ 1992 90% of the papers on the subject will have been written. This logistic shape of publication waves applies to all sort of subjects.

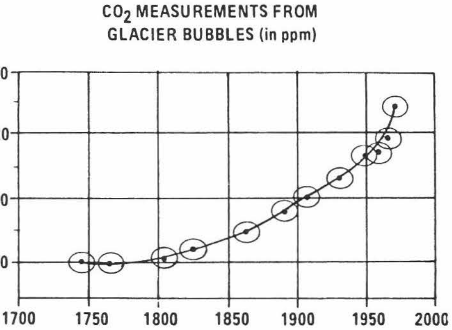


Fig. 2. Historical series of the concentration of CO_2 in air can be produced *today* by looking at air bubbles trapped in glaciers. This methodology may permit to go back perhaps 100,000 years and compare CO_2 levels with prevalent climatic situations that can be evaluated by various types of analyses of sediments (and tree rings for the last 1500 years). This reconstruction may help calibrating the climatic models over which much of the CO_2 controversy is based (Bolin, 1985).

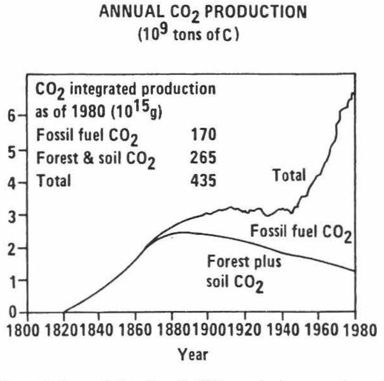
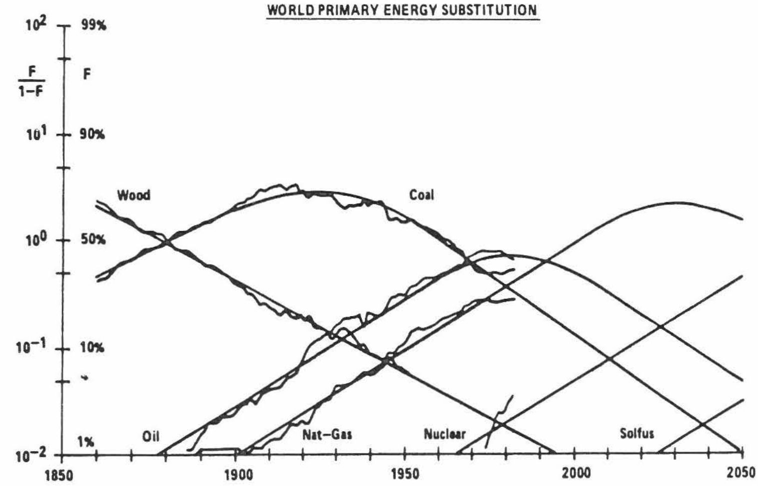


Fig. 3. Knowledge of the fossil CO_2 emissions and analysis of tree rings for ^{14}C and ^{13}C permits a reasonable reconstruction of the amounts of CO_2 put into the atmosphere by changes in the level of carbon storage in standing forests and soil. From these calculations it appears that the integrated amount of CO_2 that burdens CO_2 levels in air is due mostly to activities related to agriculture and forests. Only after World War II emissions from fossil fuels have become dominant (Bolin, 1985).



N. Nakicenovic 1984

Fig. 4. The idea that primary energies compete for the energy market like the varieties of a species for the resources of a niche, give a conceptual framework and a mathematics to deal with the evolution of the energy markets. The excellent fitting of the equations (smooth lines) with the statistical data for more than one hundred years give much weight to their use for forecasting. The fast rise of nuclear by respect to a business as usual market penetration equation is probably due to the fact that nuclear sells wholesale and has not the necessity of laying its own distribution grid. A similar phenomenon did occur when natural gas started diffusing in countries where city gas distribution nets did exist already (Marchetti and Nakicenovic, 1979).

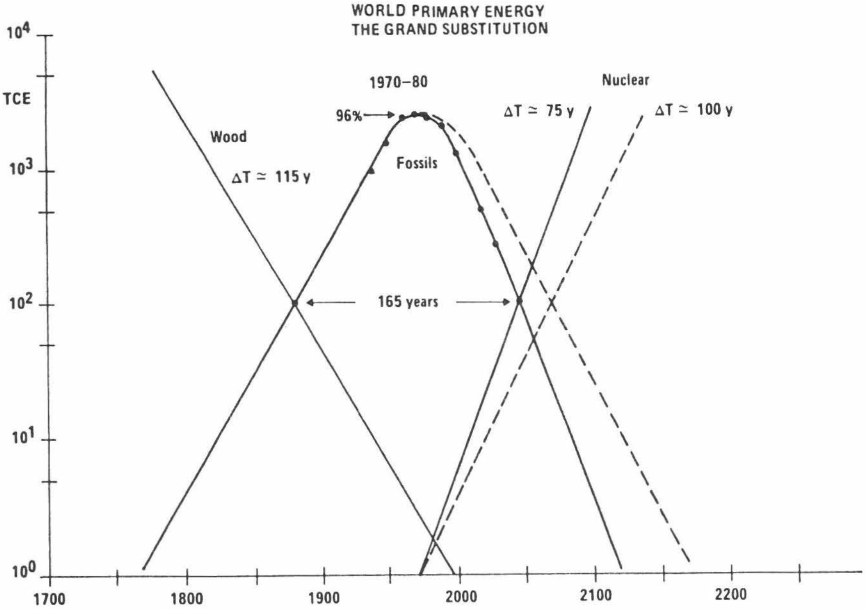


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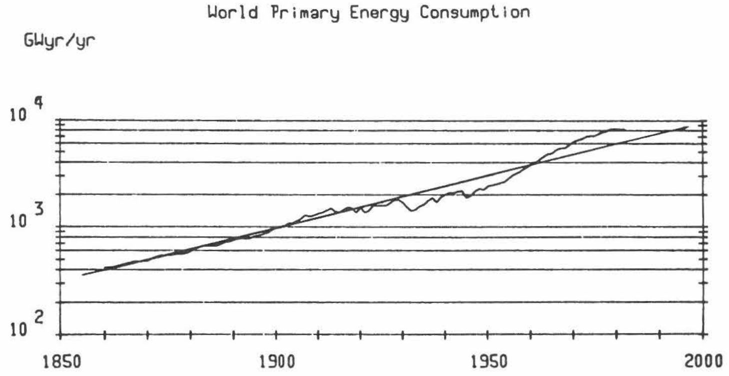
Fig. 5. Fossil fuels can be lumped together by summing their energy contribution to the energy market. We obtain then a line for phasing out wood and other renewable energies. Fossils have a "product life cycle" of about 400 years, after which they will be substituted by nuclear energy in various forms. We gave two time constants for the penetration of nuclear energy to show their effect on the phase out of the fossil fuels.



N. Nakicenovic 1984

Fig. 6. Although some essential conclusion of our study, like when zero growth of atmospheric CO_2 level can be reached by appropriate processing of natural gas need only shares, an insight into quantities is necessary to get a physical feeling of the size of the equipment to be put in place. Here the *primary world energy consumption* is reported including renewable energy and it is fitted with a 2.3% per year growth line. It can be shown that the deviations can be reduced to Kondratiev oscillations with a period of about 55 years. For the following consideration the 2.3% per year growth line will be used.

must make an hypothesis about the growth of energy consumption at world level. By smoothing from Kondratiev oscillation world energy consumption during the last 150 years, we find a steady increase by 2.3% annually (Fig. 6). We may then take this *business as usual* as an *hypothesis* for growth in the next 150 years. This is in a sense arbitrary, but at this rate the final consumption per head will bring developing countries *then* at the same level of the U.S. *today*.

Having set the context Fig. 7(a,b,c) shows the careers of coal, oil and gas seen as *product cycles*. Figure 8(a,b,c) shows actual demand and integrated demand for each of them in absolute amounts, with the hypothesis of 2.3% growth of global energy markets. It is clear form the figures that *gas consumption will integrate at a level one order of magnitude larger than oil consumption.*

From Fig. 8, the actual quantities (10^9 tons) of CO_2 emitted can be calculated for each of the primary energies (Fig. 9). The share of CO_2 emitted from methane is reported in Fig. 10. The analysis is done using smoothed data, i.e. the Kondratiev oscillation is taken away which may change somehow the fine print. However, the date when 50% of CO_2 emissions will come from natural gas will not be very different from the year 2010 that appears in the chart.

Another idea which is implicit to the competition game, as shown in the chart of Fig. 5, is that in due time all primary energy will come from nuclear sources, be it fission or fusion, and consequently also chemical fuels will be produced from them. We all know that hydrogen from water is candidate number one for that role, but we also know that the technologies to make it directly from water on the *scale* and the *cost* it will be required to be initially competitive with the fossil fuels, even in specialized market niches, will take time to develop.

However there are possible *short cuts for a transition period*, which may last for tens of years, and *I will talk about them, also in view of alleviating the problem of* CO_2 *emissions*. It comes out that doing the two things together the marginal cost of eliminating a substantial share of CO_2 emisions will be marginal if not nil, and that is why I did title this presentation as a "solution without tears".

The point is that in order to "inject" nuclear heat into the chemical fuel system, the easiest way to date is to steam-reform natural gas. The reaction is endothermic and absorbs heat at temperatures a High Temperature Reactor (HTR) can provide. The reaction and its heat balances are shown in Fig. 11. Essentially two molecules of water are decomposed per molecule of methane, and the oxygen goes into the CO_2. All the energy of the methane, plus about 30% deriving from nuclear heat, appear in the form of hydrogen. The use of HTR nuclear heat to reform methane has been studied for twenty years at the Kernforschungsanlage Jülich in the F.R.G., and the pilot plants are called EVA (R. Harth *et al.*, 1985; Kernforschungsanlage Jülich GmbH, 1985).

If we now look at world maps for production and transport of natural gas, we find that most of it travels long distance through relatively few *large pipelines*. It is then natural to think that *nuclear power complexes can be put some place along these pipelines and a share or the totality of the gas is processed to hydrogen.* The energy throughput as said would then increase by about 30%.

Naturally the hydrogen does not need to be pure. It could be an hydrogenous mixture, containing also CO, CO_2, N_2 and CH_4. The main scope of the operation would be to *introduce nuclear energy beyond electricity*, on one side, and on the other to reduce fossil fuel demand, beyond the fraction which is going into electricity where the use of nuclear reactors for primary energy is well established.

Penetration of nuclear energy in that area has been fast and the market may be basically exhausted around 2020 or so (Fig. 12). In order to continue general penetration as depicted in Fig. 5, the use of nuclear energy for these reforming processes should be started soon because these technologies have long induction times and slow (if exponential) initial growth. So one should start early to catch up in 2020 at a level that will

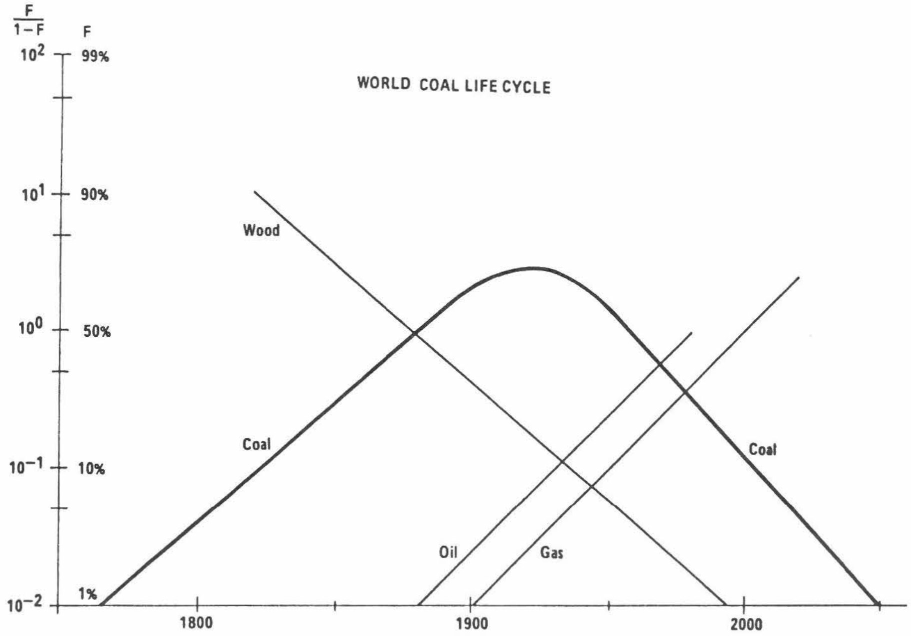


Fig. 7(a).

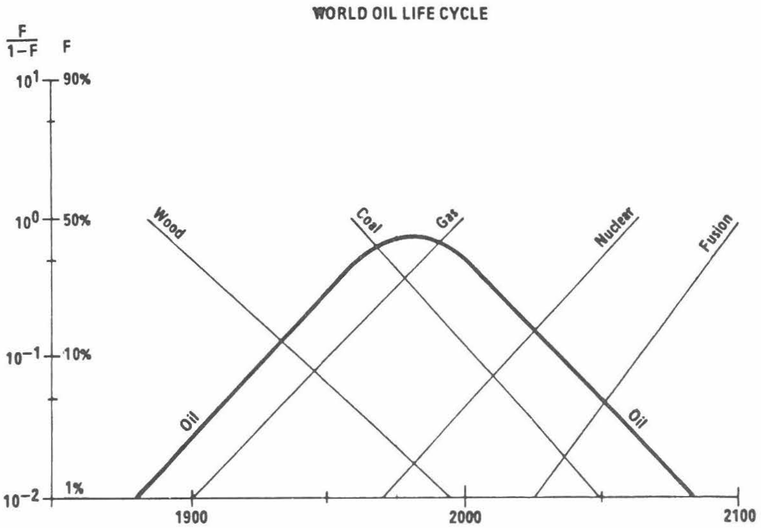


Fig. 7(b).

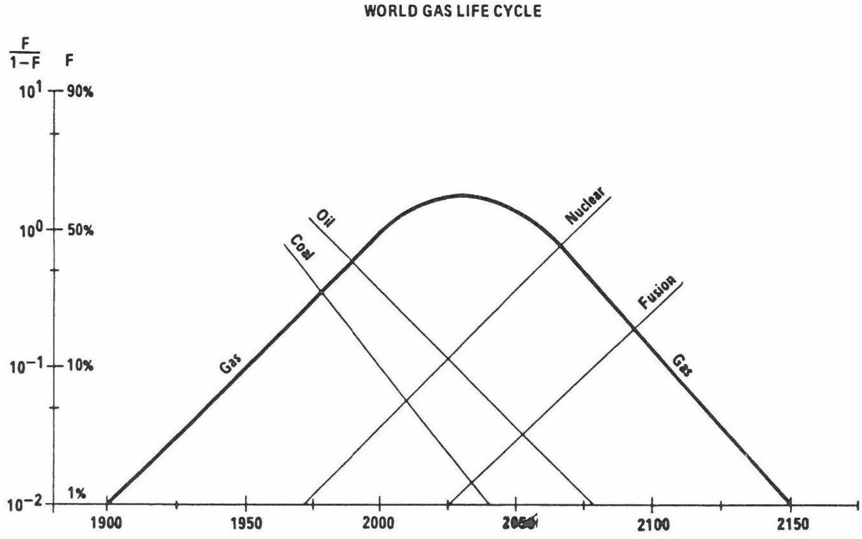


Fig. 7(c).

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Fig. 7(a–c). This is a different version of Fig. 4, where the career of each primary fossil fuel energy is reported in form of a product life cycle. The charts cover the period from 1% of the market at the beginning of market penetration to the final 1% of the market in the phase out process. These products life times cover periods of 200 to 300 years.

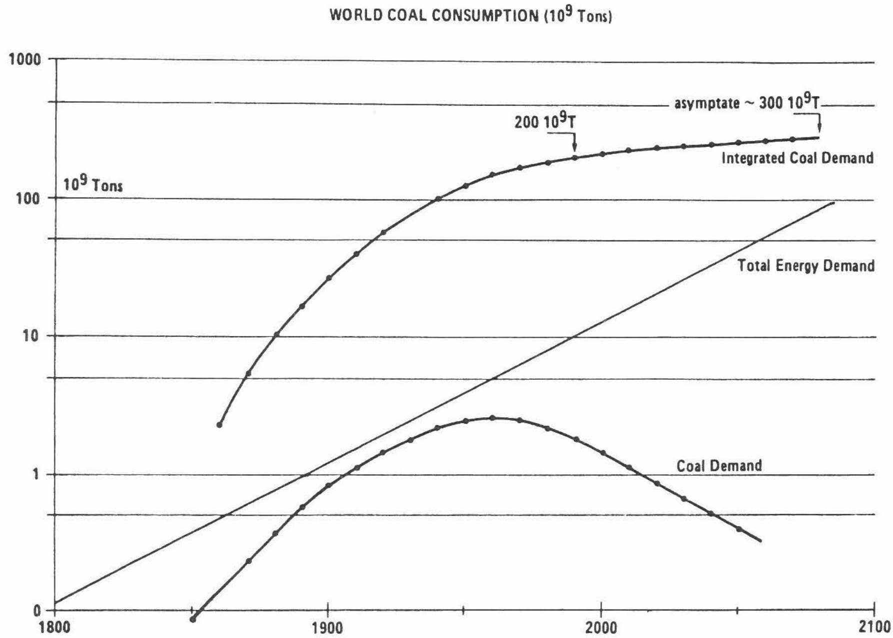

Fig. 8(a).

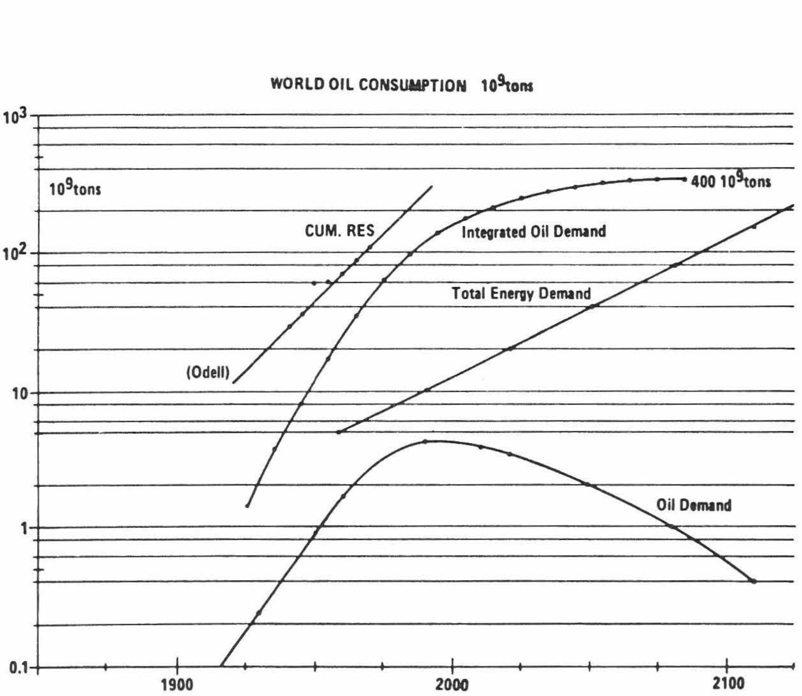

Fig. 8(b).

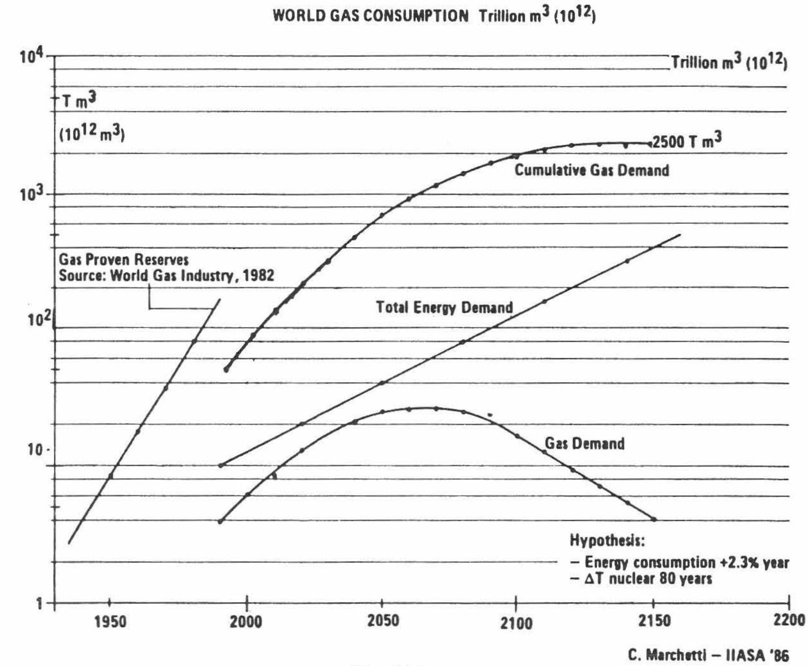

Fig. 8(c).

Fig. 8(a–c). Using absolute amounts of Fig. 6 (smoothed) and actual fractions of Fig. 7(a–c), one can construct the career of each primary energy in terms of actual consumption (demand). Also the integrated amounts are reported to give an idea of the final resources to be activated. It is clear from these charts that natural gas not only is going to be the dominant primary fossil sources from the year 2000 on, but also that the integrated amounts extracted will be by far the largest in comparison with the other primary fuels.

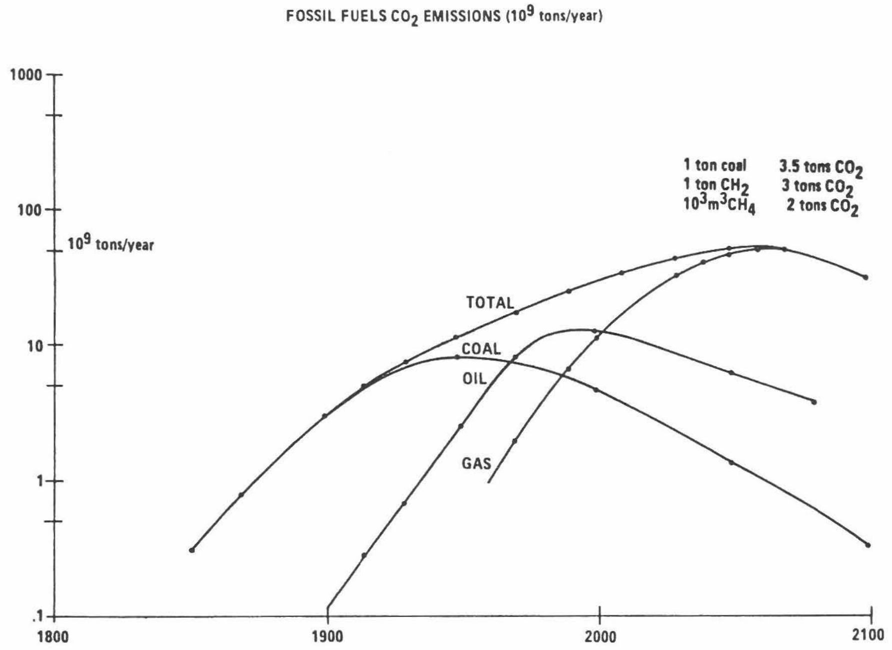

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Fig. 9. The evolution of CO_2 emissions can then be calculated from Fig. 8(a–c), applying an appropriate emission factor to the fuels. As the fuels are fairly inhomogeneous, these factors are only indicative. A finer analysis is not worthwhile in view of the use to which we are going to put the results.

keep nuclear industry growing smoothly as from the chart in Fig. 5.

Let us come back to the process. EVA reforms CH_4 using nuclear heat and with an efficiency of about 50%. Larger and more developed processes may have higher efficiencies, but for the sake of the argument, let us stick to that round figure. In the current EVA experimental set up, the reforming leads to almost equal amounts of CO and CO_2. For the purpose I have in mind, CO_2/CO should be maximized, because I want to extract CO_2. This will permit its disposal outside the atmosphere as we will see.

To have some simple figures to visualize, the Soviet Union is exporting about 50×10^9 m^3 of natural gas to Europe, coming through two sets of pipelines. This is roughly equivalent to a mean power of 50 GW. Reforming may add about 30% of that, i.e. 15 GW, and the nuclear reactors providing the primary energy should have double that power as the efficiency in the reforming is assumed to be 50%. This means these lines only may employ 30 *GW(th) of nuclear power*. The system *could* produce 100×10^9 m^3 *per year of hydrogen from water*, an equal amount coming from the breaking down of CH_4. Also 50×10^9 m^3 of CO_2 would be produced.

The same operation at world scale would represent around the year 2000 a *potential market* for nuclear energy and hydrogen production *two orders of magnitude* larger. This means 3000 GW of primary nuclear heat. At present, nuclear plants produce about 1000 GW of primary heat. The potential for electricity production in the year 2000 is again about 3000 GW of nuclear heat, and the addition of the reforming market would certainly be welcome by the nuclear industry. Incidentally, the present almost stoppage in nuclear plants orders is related to the Kondratiev cycle (Marchetti, 1985).

The secondary, but possibly very important, sidelines of this operation is that CO_2 *should be separated and disposed, away from the atmosphere*. Separation has to be done anyway, as transporting a CO_2 ballast for thousands of kilometers certainly does not pay. Disposal can be done in various ways, as I proposed in old papers of mine (Marchetti, 1977; Marchetti 1979). It can be done by injecting into the underground, or, in an appropriate form, into the oceans.

For the Soviet Union case I would choose *underground injection*. But where? The most obvious solution would be to reinject CO_2 in the fields where CH_4 comes from. Volume for volume. As CO_2 is a much less ideal gas than CH_4, it could even be injected as a liquid or a near critical gas. There is enough space underground, and an appropriate reinjection could even increase the final output of the fields.

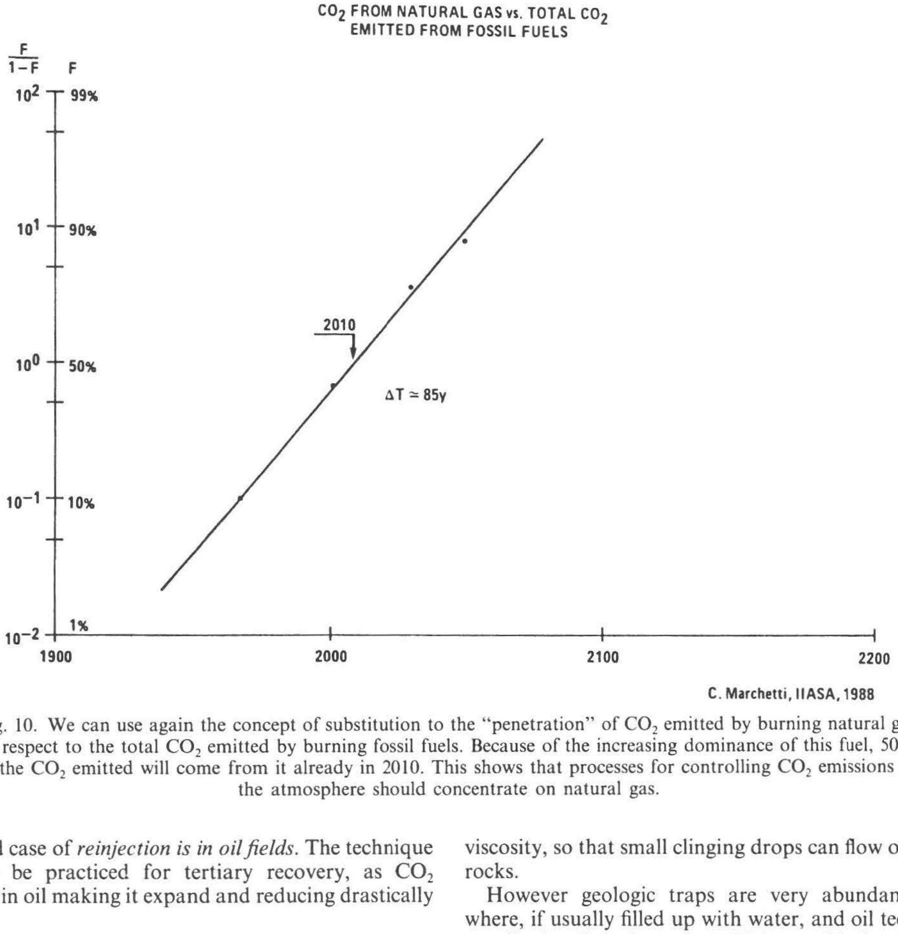

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Fig. 10. We can use again the concept of substitution to the "penetration" of CO_2 emitted by burning natural gas by respect to the total CO_2 emitted by burning fossil fuels. Because of the increasing dominance of this fuel, 50% of the CO_2 emitted will come from it already in 2010. This shows that processes for controlling CO_2 emissions to the atmosphere should concentrate on natural gas.

Second case of *reinjection is in oil fields.* The technique starts to be practiced for tertiary recovery, as CO_2 dissolves in oil making it expand and reducing drastically viscosity, so that small clinging drops can flow out of the rocks.

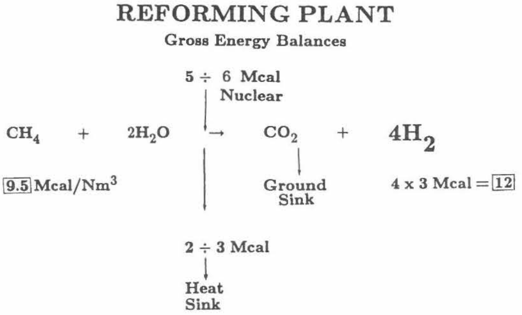

Fig. 11. A skeleton description of the steam reforming process with the help of nuclear heat is here given to show the energy balances. Basically, reforming adds about 30% to methane's energy input. This extra energy obviously comes from the nuclear heat, with an efficiency of 50% or more. This process appears relatively simple and very suited to introduce large amounts of nuclear energy into the fuel system.

However geologic traps are very abundant everywhere, if usually filled up with water, and oil techniques usually localize them first. This would decouple the reforming complex from oil provinces, but would require extra drilling and infrastructures. So it is to recommend only in very special cases. *The best compromise would be in my opinion to spot an oil province midway to the consumer and localize all the processing there* (see the Case History).

Another reason for putting it near an oil field is that the complex would release large amounts of heat, at temperatures interesting for tertiary recovery. The availability of cooling water otherwise requires a river or a medium size lake.

Coming to gross estimates about the cost of this operation, I would say that *if it pays to make hydrogen by reforming natural gas with nuclear heat*, then the marginal cost of reinjecting CO_2 may be very small or even negative. American oilmen pay up to $3 per million cubic feet of CO_2 for their tertiary recovery, which is more than the price of an equal volume of CH_4.

The last question is how fast such technology would penetrate if one finds it fits the basic requirements for a

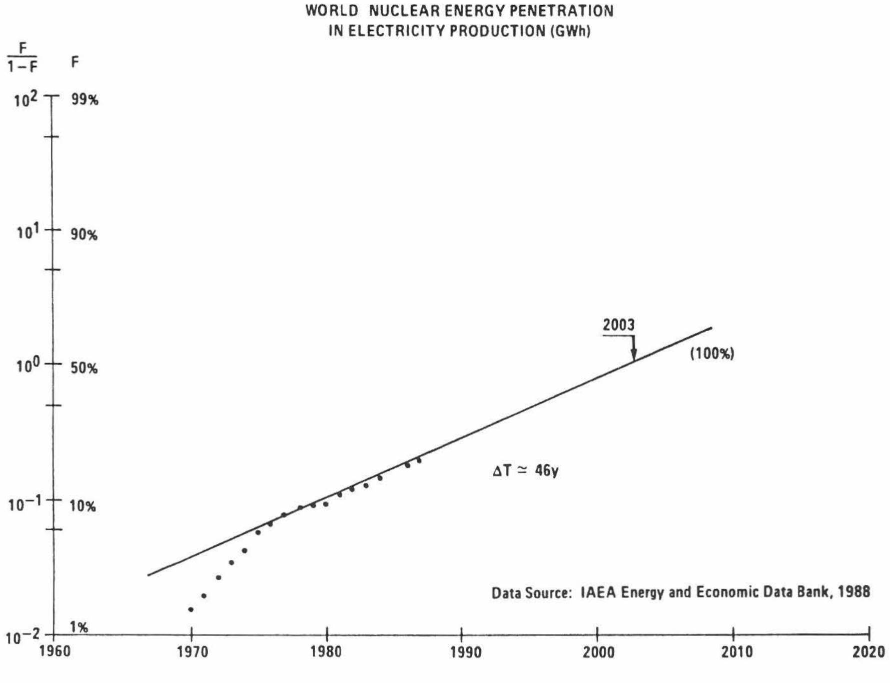

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Fig. 12. Nuclear energy penetration in the market of electricity production (GWh) is here reported. In spite of the very moody descriptions, the situation does not seem to be bleak. Penetration proceeds at a slow but consistent pace and has reached (1987) about 18% of all electricity produced (including hydro). The fitting is done assuming a 100% penetration as a maximum level. The low penetration level reached to date impedes the calculation of a more realistic saturation point (75%?). The analysis is at world level. The chart shows that by 2020 the "conquest" of the electrical system will be substantially concluded, and that, if penetration has to follow the lines of Fig. 4, new very important uses have to be found in the meantime.

commercial start which are usually much more intricate than bare economic break even and profits. The normal penetration times for such things can be 40 years. This means 50% of the market could be covered in 2030. At that time CO_2 emission from CH_4 combustion will be about 75% of total CO_2 emissions, and the effect of our initiative would be to reduce them by 40%. In 2040 the reduction would be above 50%.

This is a magic figure, because *at present about* 50% *of the* CO_2 *emitted into the atmosphere is reabsorbed by the ocean sink*. This sink has an extremely large capacity, and the bottleneck is in the kinetic of CO_2 absorption and transport. The amounts depend on the concentration of CO_2 in the atmosphere and at current rates of concentration increases, it is likely that it will remain the same in the future (Bolin, 1985).

This means our remedy can stop CO_2 *increase in the atmosphere around* 2040, *without any sacrifice in energy consumption*. And lead to a production of hydrogen in the order of 10^{14} m^3 y^{-1}.

Let's start at once.

CASE HISTORY: A REFORMING COMPLEX IN WEST UKRAINE

The Soviet Union is a large gas exporter to Europe. At present, the amount is in the range of 50×10^9 m y^{-1}. Although speculative, the maximum *possible* level of export has been estimated to be an order of magnitude larger.

The location of the large oil and gas provinces in the USSR is shown in Fig. 13. The most important, from the point of view of gas, is the West Siberian province (Tyumen). A set of large gas pipelines moves from there to the Moscow area and to Europe (Fig. 14). Gas is exported also from the south with lines going north through Kiev, and meeting the other ones in northwestern Ukraine (Fig. 15).

There is an oil province, indicated as "West Ukraine" on the map of Fig. 13, which could be a very promising site for the installation of a large nuclear gas reforming complex. The fields in fact have passed their prime production life, and are the natural target for tertiary

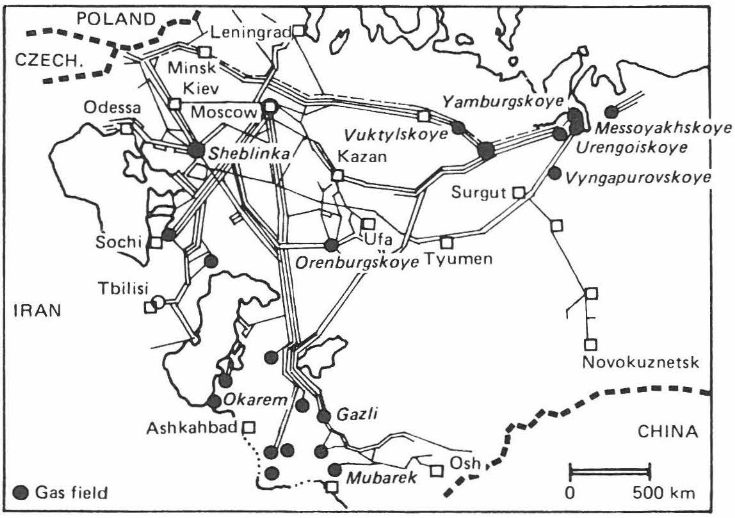

Fig. 13. This map shows the main gas pipeline system in the Soviet Union.

recovery. This means they can be very good customers for CO_2 to be used for flooding. Furthermore, the complex would produce large amounts of heat at temperatures interesting for thermal stimulation. The region has also a system of large rivers, in the Dnjestr basin, and cooling water could be available when necessary.

Complete reforming of the 50×10^9 m^3 of natural gas exported, would require the installation of about 30 GW(th) of nuclear power on the site. The Soviet Union has much experience in graphite moderated reactors, but a limited one in HTR which is the variety very useful for that job. They have been developed mostly in the FRG. Also very large HTRs have not been designed yet, the tendency being toward a modular fail safe system.

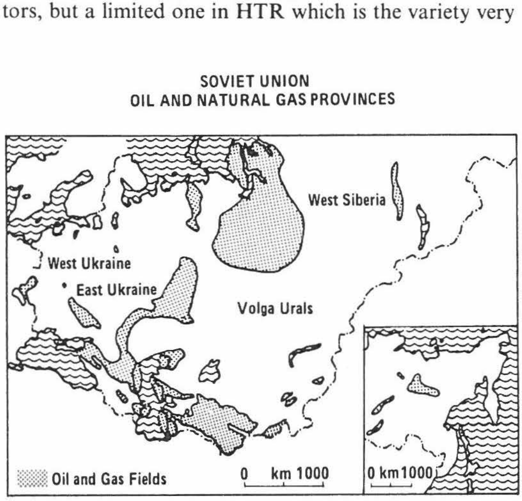

Fig. 14. This map shows the main oil and gas provinces of the Soviet Union. Attention to the area here marked "West Ukraine" at the triple point of the Soviet Union, Czechoslovakia and Poland.

The complete reforming would lead to a production of 200×10^9 m^3 y^{-1} of H_2 and 50×10^9 m^3 of CO_2 to be injected for oil recovery *or* for permanent storage. The cost of the reactors should be reabsorbed in the extra caloric value of the gas sold, if not in the higher commercial value of the hydrogen.

CO_2 for oil recovery however has a premium value. Shell has recently completed the construction of a 500 mile CO_2 pipeline from wells in Colorado to a West Texas field in order to recover an expected 40 million tons of oil through CO_2 flooding. The pipeline operates at a pressure of 140 atm and can transport 2.5×10^9 m^3 of CO_2 per year. Its cost has been 3.3 billion dollars.

Assuming the cost of operation, amortization and interest rates on capital to be around 20%, this brings the indicative value of CO_2 to about 20 ¢US m^{-3}. Current international prices of natural gas in the order of 3$ $MBtu^{-1}$ correspond to a round price for natural gas of 10 ¢US m^{-3}.

CO_2 may be assumed to be worth the double of natural gas, obviously in the appropriate location and amounts. This is certainly true for the first GWs of reforming installed, and could be *the driving force for a start*. Furthermore, adding 5 or 10% hydrogen to natural gas changes only marginally its characteristics, in the sense that the consumer would use the same equipment and get the same effect.

This may be a golden opportunity for the West Ukrainian area to get a large reforming complex that can develop into a large petrochemical complex ($CO + H_2$ available!). The fields would get the CO_2 to enhance their

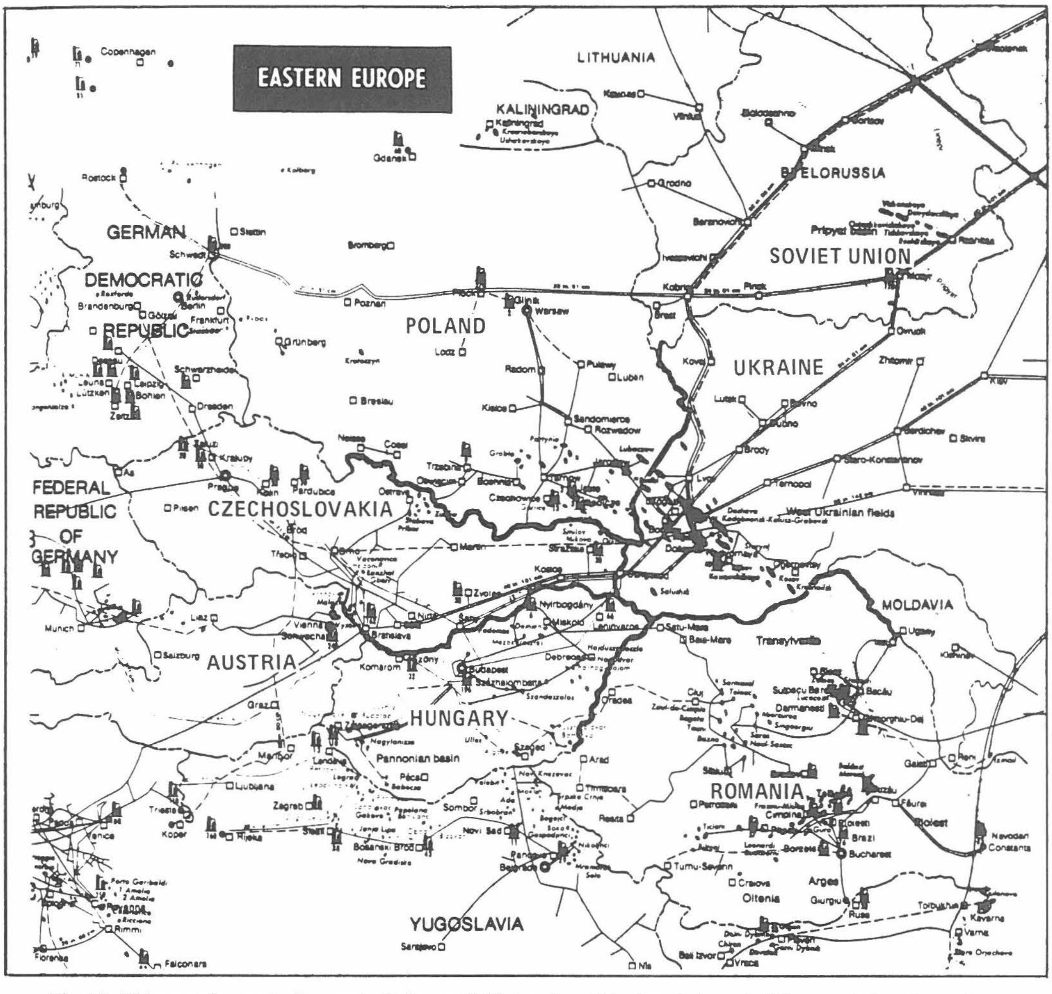

Fig. 15. This map shows pipelines and oil plus gas fields together. It is clear that most of the exported gas is passing through or near the west Ukrainian fields, where a large reforming plant can be situated as explained in the text.

production. The Germans would get the extra energy coming from reactors they may probably produce, having for the moment the best know how in that area.

A mega joint venture in the spirit of perestroika?

MATHEMATICAL APPENDIX

The equations for dealing with different cases are reducible to the general Volterra–Lotka equations

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = K_i N_i + \beta_i^{-1} \sum_{n}^{j=1} a_{ij} N_i N_j, \tag{1}$$

where N_i is the number of individuals in species i, and a, β and K are constants. The equation says a species grows (or decays) exponentially, but for the interactions with other species. A general treatment of these equations can be found in Montroll and Goel (1971) and Peschel and Mende (1986). Since closed solutions exist only for the case of one or two competitors, these treatments mainly deal with the general properties of the solutions.

In order to keep the analysis at a physically intuitive level, I use the original treatment of Verhulst (1845) for the population in a *niche* (Malthusian) and that of Haldane (1924) for the competition between two genes of different fit. For the multiple competition, we have developed a computer package which works perfectly for actual cases (Marchetti and Nakicenovic, 1979), but whose identity with the Volterra equations is not fully proven (Nakicenovic, 1979).

Most of the results are presented using the coordinates for the linear transform of a logistic equation originally introduced by Fisher and Pry (1970).

The Malthusian case

This modeling of the dynamics of population systems started with Verhulst in 1845, who quantified the Malthusian case. A physically very intuitive example is given by a population of bacteria growing in a bottle of broth. Bacteria can be seen as machinery to transform a set of chemicals in the broth into bacteria. The rate of this transformation, *coeteris paribus* (e.g. temperature), can be seen as proportional to the number of bacteria (the transforming machinery) and the concentration of the transformable chemicals.

Since all transformable chemicals will be transformed finally into bacterial bodies, to use homogeneous units one can measure broth chemicals in terms of bacterial bodies. So $N(t)$ is the number of bacteria at time t, and $\bar{N}$ is the amount of transformable chemicals at time 0, before multiplication starts. The Verhulst equation can then be written

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN(\bar{N} - N), \tag{2}$$

whose solution is

$$N(t) = \frac{\bar{N}}{1 - \mathrm{e}^{-(at+b)}}, \tag{3}$$

with b an integration constant, sometimes written as t_0, i.e. time at time 0; a is a rate constant which we assume to be independent of the size of the population. This means that there is no "proximity feedback". If we normalize to the final size of the system, $\bar{N}$, and explicate the linear expression, we can write equation (2) in the form suggested by Fisher and Pry (1970).

$$\log \frac{F}{1-F} = at + b, \quad \text{where } F = \frac{N}{\bar{N}}. \tag{4}$$

Most of the charts are presented in this form. $\bar{N}$ is often called the *niche*, and the growth of a population is given as the fraction of the niche it fills. It is obvious that this analysis has been made with the assumption that *there are no competitors*. A single species grows to match the resources ($\bar{N}$) in a Malthusian fashion.

The fitting of empirical data requires calculation of the three parameters $\bar{N}$, a and b, for which there are various recipes (Oliver, 1964; Blackman, 1972; Bossert, 1977). The problem is to choose the physically more significant representation and procedure.

I personally prefer to work with the Fisher and Pry transform, because it operates on *ratios* (e.g. of the size of two populations), and ratios seem to me more important than absolute values, both in biology and in social systems.

The calculation of $\bar{N}$ is usually of great interest, especially in economics. However, the value of $\bar{N}$ is very sensitive to the value of the data, i.e. to their errors, especially at the beginning of the growth. The problem of assessing the error on $\bar{N}$ has been studied by Debecker and Modis (1986), using numerical simulation.

The Malthusian logistic must be used with great precautions because it contains implicitly some important hypotheses:

—That there are no competitors in sight.
—That the size of a niche remains constant.
—That the species and its boundary conditions (e.g. temperature for the bacteria) stay the same.

The fact that in multiple competition the starts are always logistic may lead to the presumption that the system is Malthusian. When the transition period starts there is no way of patching up the logistic fit.

The fact that the niches keep changing, due to the introduction of new technologies, makes this treatment, generally speaking, unfit for dealing with the growth of human populations, a subject where Pearl (1924) first applied logistics. Since the treatment sometimes works and sometimes not, one can find much faith and disillusionment among demographers.

One-to-one competition

The case was studied by Haldane for the penetration of a mutant or of a variety having some advantage in respect to the preexisting ones. These cases can be described quantitatively by saying that variety (1) has a reproductive advantage of k, over variety (2). Thus, for every generation the ratio of the number of individuals in the two varieties will be changed by $1/(1-k)$. If n is the number of generations, starting from $n = 0$, then we can write

$$\frac{N_1}{N_2} = \frac{R_0}{(1-k)^n}, \quad \text{where } R_0 = \frac{N_1}{N_2} \text{ at } t = 0. \tag{5}$$

If k is small, as it usually is in biology (typically 10^{-3}), we can write

$$\frac{N_1}{N_2} = \frac{R_0}{\mathrm{e}^{kn}}. \tag{6}$$

We are then formally back to square one, i.e. to the Malthusian case, except for the very favorable fact that we have an initial condition (R_0) instead of a final condition ($\bar{N}$). This means that in *relative terms* the evolution of the system is not sensitive to the size of the niche, a property that is extremely useful for forecasting in multiple competition cases. Since the generations can be assumed equally spaced, n is actually equivalent to time.

As for the biological case, it is difficult to prove that the "reproductive advantage" remains constant in time, especially when competition lasts for tens of years and the technology of the competitors keeps changing, not to speak of the social and organizational context. But the analysis of hundreds of cases shows that systems behave exactly *as if*.

Multiple competition

Multiple competition is dealt using a computer package originally developed by Nakicenovic (1979). A simplified description says that all the competitors start in a logistic mode and phase out in a logistic mode. They undergo a transition from a logistic-in to a logistic-out during which they are calculated as "residuals", i.e. as the difference between the size of the niche and the sum of all the *ins* and *outs*. The details of the rules are found in Nakicenovic (1979). This package has been used to treat about one hundred empirical cases, all of which always showed an excellent match with reality.

An attempt to link this kind of treatment to current views in economics has been made by Peterka (1977).

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