THE FUTURE

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The Future.

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> Tu ne quaesieris, scire nefas quem mihi
> quem tibi finem di dederint, Leuconoe,
> nec Babylonios temptaris numeros ...
>
> (HORACE, *Odes,* Book I, Ode XI)

Looking into the future has been a popular sport since time began. Fortunetellers, prophets and forecasters have even found a professional niche in the field. On the other hand, *the touchstone of a scientific theory is its capacity to predict*.

This drive into the future is not peculiar to man. A living creature develops according to the functions it will explicate in the future: a wing is a very sophisticated forecast on the dynamic properties of air. ARISTOTLE was so puzzled by this that he invented a special word for it, *ἐντελέχεια*, the « final cause ».

EINSTEIN would not have been very happy with causes working backward in time, but MENDEL and CRICK found the appropriate compromise. DNA contains implicitly a model of the external world and keeps readjusting it *a posteriori* through selection. It obviously profits from the assumption that rules exist and that they do not change too fast in time. Physicists are in the same ball park.

I think the most important function of the nervous system is to forecast. A rushing lion has to forecast complex trajectories and to appropriately time thousands of muscles in order to reach *in the future* the *entelecheia* of a good meal. More long term, orangutans have mental maps for where and when the fruits of each of the variegated bunch of trees in their territory *will* ripen. It is not really necessary to incommodate the hunter who solves implicitly many complex equations in order to plant his arrow into a running deer.

All these forms of forecasting activity *assume* the existence of invariants of some sort: we can call them laws. To stay more general, I would say the system must be self-consistent.

The subject of this paper is *forecasting within social systems*, from man to humanity. I will show that, by choosing the appropriate indicator and processors, social systems reveal an extraordinary level of self-consistency which,

for certain things, allows accurate forecasting for even a thousand years ahead.

The technique I use is of baffling simplicity, if of high pedigree. I assume that every system can be decomposed into competing subsystems whose dynamics are described by the Volterra-Lotka equations of ecological competition. HERACLIT said that competition is the creative ($\pi\alpha\tau\acute{\eta}\varrho$) and regulatory ($\beta\alpha\sigma\iota\lambda\varepsilon\acute{\upsilon}\varsigma$) force ($\Pi\acute{o}\lambda\varepsilon\mu o\varsigma \ \pi\bar{\alpha}\nu\tau\omega\nu \ \mu\grave{\varepsilon}\nu \ \pi\alpha\tau\acute{\eta}\varrho \ \grave{\varepsilon}\sigma\tau\acute{\iota}, \ \pi\bar{\alpha}\nu\tau\omega\nu \ \delta\acute{\varepsilon} \ \beta\alpha\sigma\iota\lambda\varepsilon\acute{\upsilon}\varsigma$). DARWIN simply readjusted the words into *change* (mutation) and *selection*. VOLTERRA just wrote down the music. The curious thing is that the Volterra equations are of brutal simplicity in spite of the enormous complexity of the system they encompass. There is only one rate term for each competitor and a « cross-section » for each pair of competitors.

I started with this paradigm at the economic level, using operations quite familiar to the physicist, but very arrogant for an economist. I assumed that what people think and say is irrelevant, *only what people do is relevant. Action is the observable.* Physics has passed through various phases of this methodological purification, which is because thinking into the object is a very natural tendency, hard to resist. Even my fans still resent this *objectivation of the subject.* Because the results are good, the procedure *is* correct, in a Darwinian sense.

Coming to brass tacks, I found that the close general solutions of the Volterra equations are not really necessary (they have not been found yet!), as all the cases I have analysed—about one thousand to date—can be described using Malthusian logistic and multiple logistic competition, described in the appendix. This is probably because our socioeconomic systems are very innovative, so that there is no time for oscillatory states to be established. In fact, a new « species » is usually introduced before the preexisting one has fully occupied its niche.

At this point I should also give a definition of the *niche*, but this is not too easy. Since DARWIN, the concept has been very stimulating at the heuristic level in biology. But a bulletproof definition has not been found so far, perhaps because the concept is used statically and yet is intrinsically very dynamic. The ecological *niche* is the intrinsic capacity of an ecological system to host a given species. As we are territorial animals, the concept of niche can be taken as primitive.

Because the applicability of the model is very general, the best way to present the results is by category of objects, with a touch of history. The system analysed first, in 1975, was the primary energy market at world level (fig. 1). I must have shown this chart a thousand times, but, after all, we have only one big bang. Figure 1 depicts the market shares, *in calories*, of the primary energies *vs.* time. The smooth lines are the fitting equations, and it is remarkable that only two parameters are required to initially define each of them. The deviations of the data are never more then a few percent over a century, and they are always reabsorbed elastically. This calls for a barely credible *self-*
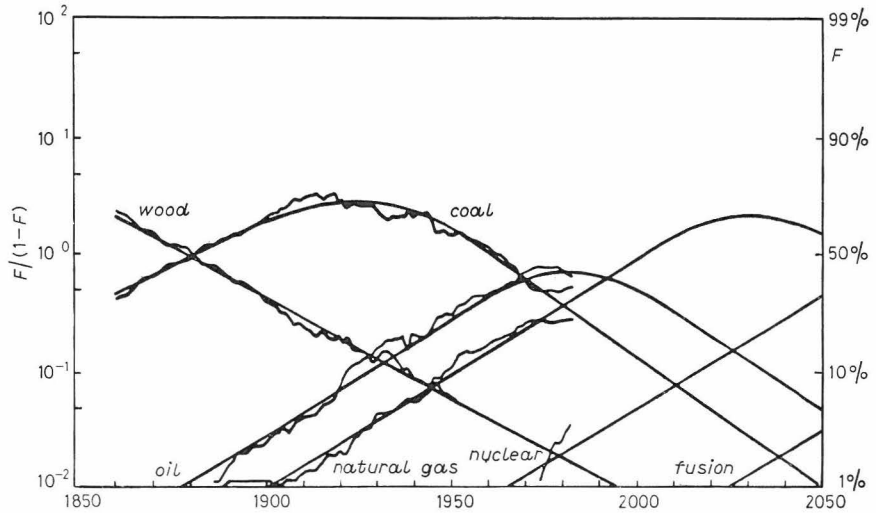
Fig. 1. – Market share of primary energies in caloric units, fitted with a set of inter-active logistics [1].

consistency of the system during a period of violent turnovers in the geopolitical and technological panorama: but that is.

The equations can be extrapolated beyond the data base, so the first question is: *How good are they for predicting the future?* The time constants in this particular example are fairly long, of the order of a century. The time constant is the time required to go from 1 % to 50 % of the market. Waiting 50 years may be boring, so we have experimented with past data.

As shown in fig. 2a), we took the period 1900-1920 as the data base. This included World War I, but daring sometimes helps. From this base we fitted the equations (fig. 2b)) and then superposed the data beyond the base (fig. 2c)). The market share of oil in 1970, after World War II and 50 years ahead into the future, could have been predicted in 1920 with a precision of 2 or 3 %!!! The same applies for coal. The only forecast that did not fit well was that for gas, but the reason is that gas was born within the data base and had reached only 2 % of the market in 1920. Penetration rates usually stabilize above 5 %; below 1 % the system is in a nucleation phase and the behaviour is fairly erratic.

Figure 2 does not contain the « new species », nuclear energy, and that is an obvious weakness in forecasting, although the effects are not noticed for a long period due to the long-time constants of the system. But this weakness has been removed by examining a larger system in which the « comets » are also included, as I will explain later.

The message in fig. 2 can be read in philosophical terms. The present can be unrolled into the past and the future. It is a form of deconvolution having
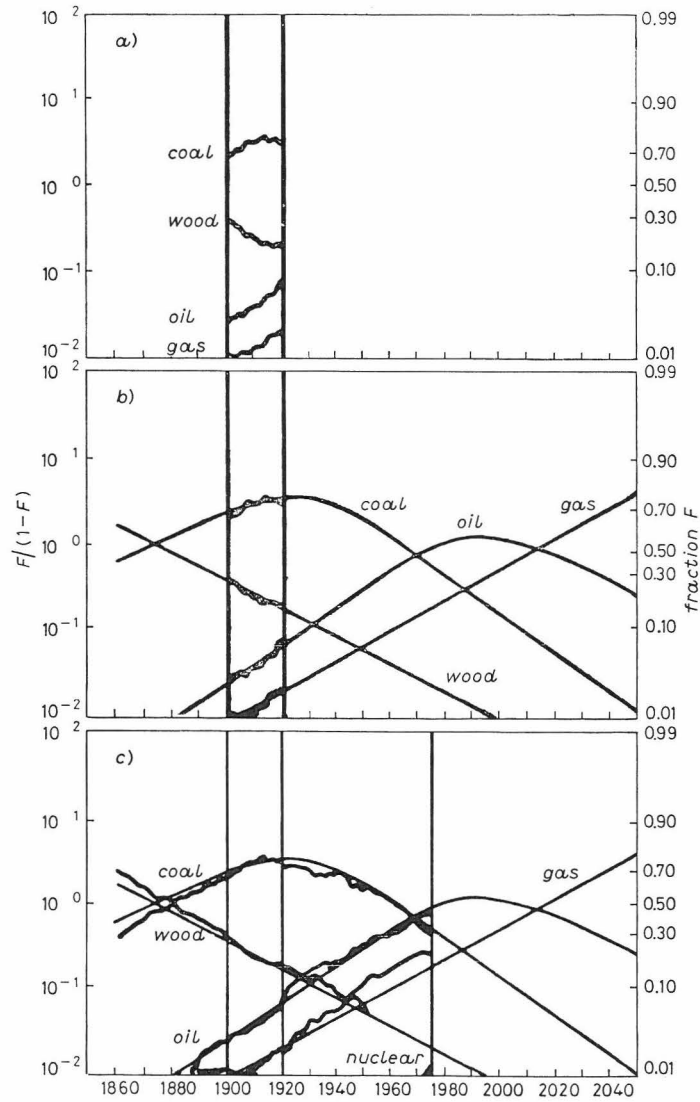
Fig. 2. – Forecasting and backcasting. A set of interactive logistics is fitted on a data base (a)) of 20 years and extrapolated 50 years ahead and backward (b)). Superposing the actual statistical data (c)), the good quality of the forecasting can be ckecked. Oil market share in the 1970s could have been predicted in 1920 with a precision of a few percent.

a certain analogy with that of an hologram. From a thin present we can construct hazy pasts and futures, but from a thick one the image generated is quite crisp. Our 2 % after 50 years is still part of the noise of energy statistics.

Primary energies can be seen as large technological baskets, containing

research, mining, transportation, processing and use. We can try to examine the other side of the coin, and see, *e.g.*, how energy is used for the propulsion of ships. The technologies in competition during the period studied were sail, steam and internal combustion (diesel or gas turbines). The result is reported in fig. 3, and carries the message of a final victory for the internal-combustion engine. Apart from predictions, the fit between our supersimple model and the data over almost two centuries is remarkable, especially for a very complex system like shipping.
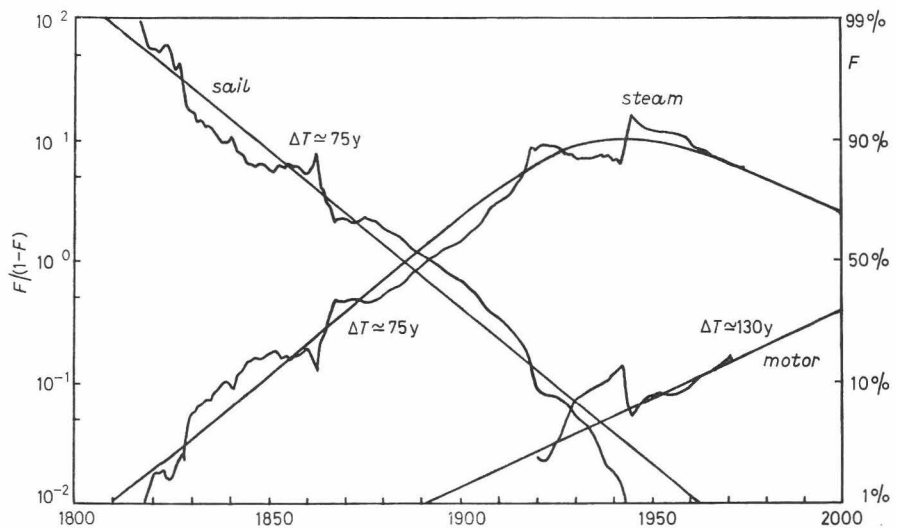

Fig. 3. – The kind of analysis applied to energy substitution can be applied to technological substitution as well. Here we have sail, steam and internal-combustion engines competing for the market of ship propulsion [1].

One first observation is that our frenetic society is finally poking ahead slowly. It took almost 70 years for steam to take 50% of the market from sail. It will take almost double that for internal combustion to go from 1% to 50% of the market. The complete substitution from sail only to motor only will take about three centuries—in the USA! This observation is of quite a general character. The system is by far much slower than we perceive it. Internal combustion will have half the market in 2025 and for the final victory it will have to crank along till the year 2100; if no new basic innovation occurs.

Ship propulsion can be considered an energy use, but also a (relatively small) technological basket. So it is quite natural to see if other technologies compete in the same way: they do. The case of how steel is made from pig iron is given in fig. 4. The problem here is to reduce by oxidation the carbon content of the iron from the blast furnace. The share of the steel produced by the various techniques and their substitution in time is reported in the chart,
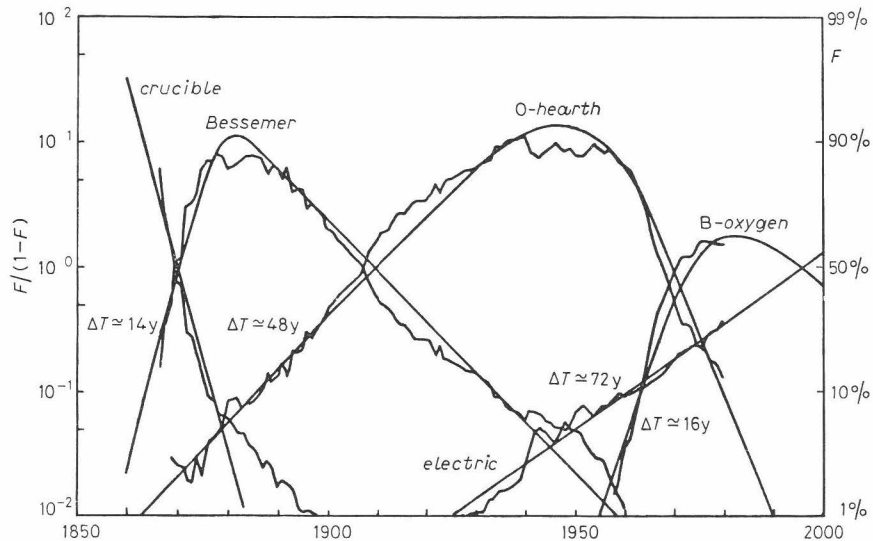
Fig. 4. – One could consider that ship propulsion is still an energy use. In this case we examine the substitution of metallurgical processes to make steel [1].

with the time constants indicated. The final victor seems to be the electric furnace, which basically recycles scrap iron, perhaps pointing to an end of the iron age, at least in the USA. It will have 50 % of the market around 1995.

The formally simplest solution of the Volterra equations is that of a Malthusian population, growing freely in a limited niche. One can see this as a competition between the members of the population for the limited resources of the niche. The equation is a three-parameter logistic. The same result is applicable for finding the final winner of a sequence of competitors, as in the case of electric steel, or when interference from other competitors is not effective, as in the initial penetration phase. Mainframe computers can be considered to be in one of these situations and their penetration in the Japanese market is reported in fig. 5. The size of the niche is calculated by best fit and should be around 140 000. Saturation will arrive ten years from now, around 1995. The complete process, from 1 % to 99 %, takes a bare 30 years: but will their total number be static forever? Here is the moment to move one notch up in abstraction. This will, incidentally, help solve the problem of when a new primary energy source will be introduced, and why the number of mainframe computers in Japan saturates so fast—it is because the system approaches the end of a Kondratieff box (1995).

Forecasting the conceptual phase is an essential step forward in the trade of forecasting. If we take all the basic innovations that appeared during the last couple of centuries and we label them by the date they appeared as products on the markets, we see that they developed in bunches. Three of them are
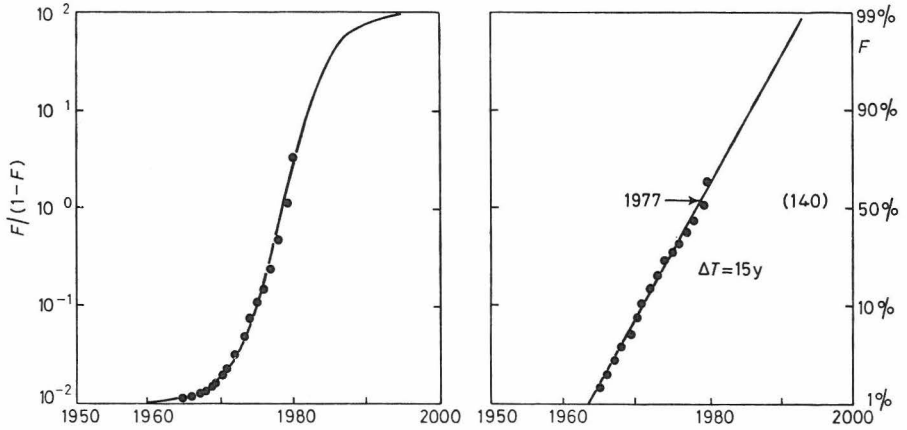
Fig. 5. – Mainframe computers grow in a niche where there are no direct competitors. So we apply the logistic function representing the growth of a Malthusian population. The members of the population compete between themselves for « limited resources ». A population of bacteria growing in a broth follows the same pattern [2]. *The size of the niche or saturation point is given in parenthesis.*

well delineated in the raw data [3]. The jump in abstraction consists of assuming that innovations are products moving to fill a niche, like cars or computers—*the innovation niche*. Each innovation is counted as one unit, independently of what it looks like, simply as one car or one computer. We can then fit the bunches using logistics, which is very easy since the saturation point, or the size of the niche, is already known, the operation being done *a posteriori*. The result is reported in fig. 6. The change in abstraction level did
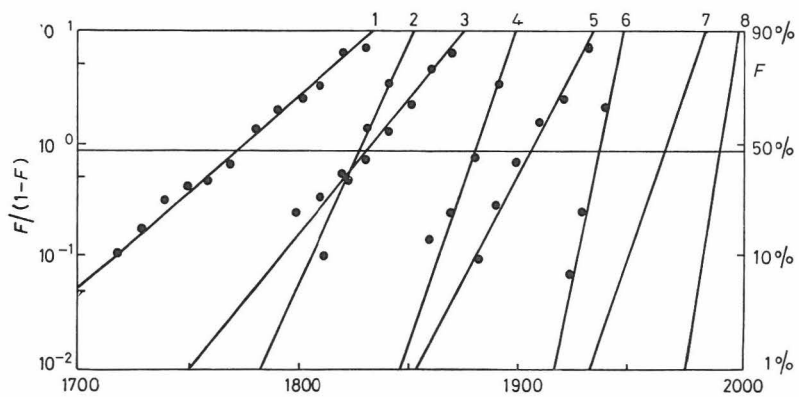

Fig. 6. – Basic inventions and innovations appear to come in pulses. Assuming they are « products » that satisfy a (limited) demand, they can be considered as Malthusian populations. Their cumulative numbers behave, in fact, like that. Even-numbered lines depict innovations and the odd ones the inventions that preceded them. Numbers 7 and 8 are calculated and map the present invention-innovation wave.
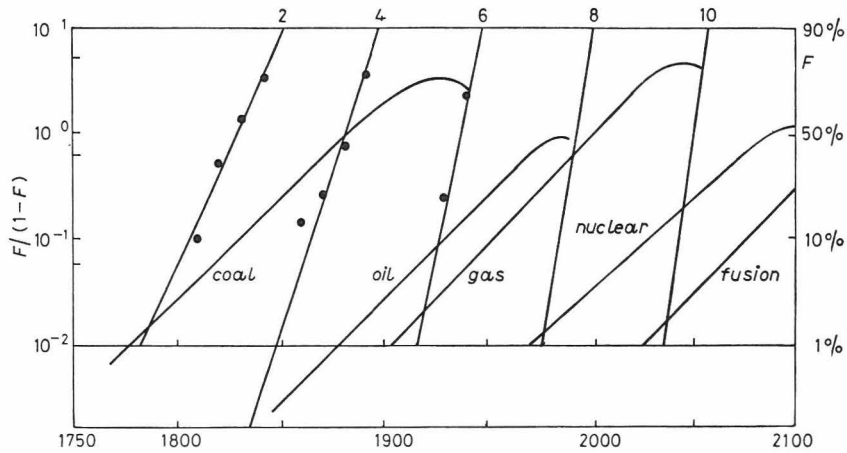
Fig. 7. – Innovation comes in waves with a periodicity of 54 years between their centre points (50%) and the total system pulsates with that periodicity. Every wave is associated with the introduction of a new primary energy—the present wave with nuclear energy and the next one presumably with fusion. The forecast is that fusion will have 1% of the energy market in 2025.

not change the mechanisms, and innovations behave just like products (or rabbits!) diffusing into a niche. Furthermore, the centres of the pulses (50 %) are equally spaced, every 54 years, the period of a Kondratieff cycle. The time constants are so related that one can construct the shape of the next pulse
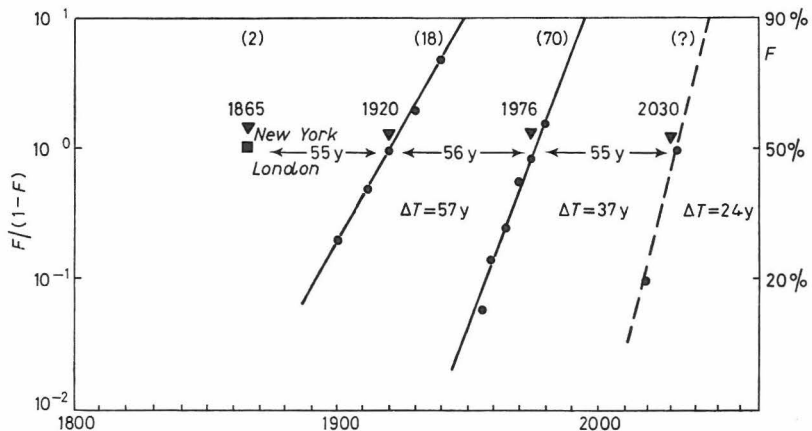

Fig. 8. – Examples of a 54-year pulsation period in societal activity can be found in every field. Here the case of the start of new underground urban transportation networks is analysed. The equations organize the cumulative numbers of « first lines » inaugurated at world level. The numbers in parenthesis represent the size of the niches, i.e. the total numbers of new networks started. It is fascinating to observe the world co-ordination of point decisions, like that to provide a certain city with underground transportation [4].

from the previous ones. As one innovation pulse is running now, I am actually forecasting the present and, a little, the past; but to verify this I have to wait till the year 2000, to be sure that the innovations I count will be successful.

Because the charts are all normalized, I can simply superpose part of fig. 1 onto fig. 6 and make an interesting discovery (fig. 7). Every innovation pulse is preceded by the introduction of a new primary energy. «Our» pulse is preceded by *nuclear* in the right topographic position, which is a good start for the verification. Because I can forecast also the next innovation pulse, I can say the next primary energy source, presumably *fusion*, will enter the market (1 %!) around 2025. Put oil in your lamps, since the vigil will be long. Operating at this level of abstraction permits a swift change of scenery. We can look at cities with a subway network, and at the date when the first line was opened (the first service was sold!). The set of dates bunches, and the bunches are analysed in fig. 8. Curiously enough, the same pattern reappears, and with the same time period of 54 years. Obviously the world is a single system and its parts are beautifully co-ordinated. I conclude this after examining dozens of similar examples.

Another interesting example in that direction is given by the «starters» of railway networks worldwide (see fig. 9). All networks were started during the Kondratieff that formally ended in 1887. They were extended during the following one, ending in 1941, but no new network initiated. The starter dates organize in a perfect logistic. It requires a little variation from the current
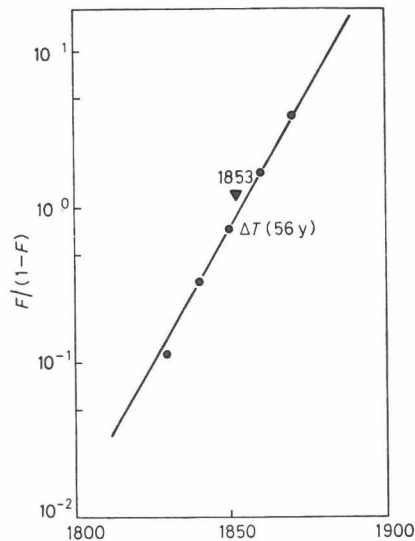

Fig. 9. – A co-ordination at world level can be observed by looking at the cumulative number of railway networks started. (The starting signal is the inauguration of the first line.) Here we have a single pulse, however, with a time constant of 56 years. No networks were started after 1900 [4].

paradigms to view a railway network as a product in the same sense as a car or an apple, and to view the world as a *single* marketplace where apples are sold only during apple-day! But it is a good exercise. Incidentally, the development of Gothic cathedrals also follows this pattern (fig. 10).
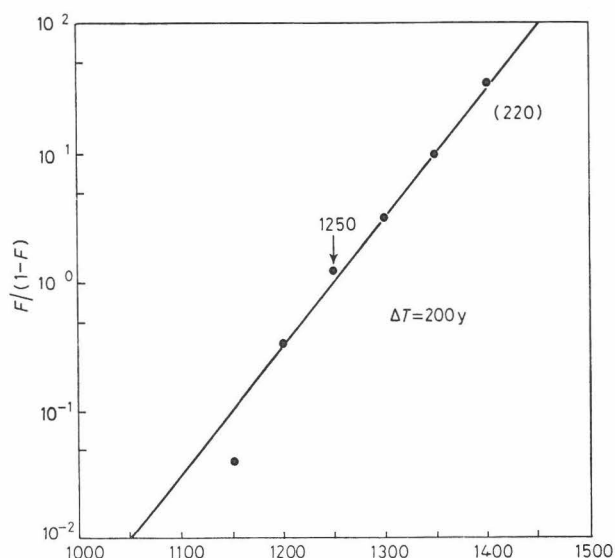

Fig. 10. – That such a behaviour is not specific to our Western technological societies is shown by the analysis of the time distribution of cathedral construction. Here is reported the cumulative number of « first stones » that developed later into Gothic cathedrals. The self-consistency of the process is remarkable, especially if we remember that it spanned about four centuries. The informational and cultural links between the constructors made them operate like a single body [5]!

Having reached the end of the forest, let us zoom in on the tree. The limits to growth for complex systems can be internal—the niche is in the program. Curiously, the growth of an individual or of a tree has the same pattern as the growth of a population in a niche: the usual Malthusian logistic. We can apply the idea to the growth of a company to check if it fits the essentials of an individual—it does. The case of Lufthansa is reported in fig. 11. The indicator chosen is the number of aircraft, its exoskeleton, but one can use also the passenger-km transported per year, a performance indicator (see fig. 12). Both fit the equations extraordinarily well, which permits easy forecasting of the limits to growth of that company.

These limits to growth are « perceived » in the sense that growth evolves as if that were the limit, but usually the limits are not reached as the case of Fiat shows (fig. 13), presumably because of multiple competition effects. At the moment this limits precision forecasting in this particular field, because
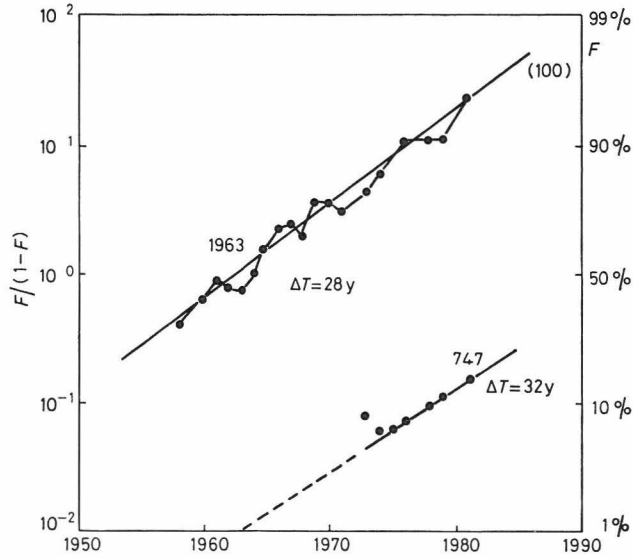
Fig. 11. – A company is also a « single body » kept together by informational interactions and company culture. It grows, in fact, very consistently as if it had a DNA program from the start (the founder message?). Here is the development of Lufthansa, monitored through the number of airplanes it has in service [6].
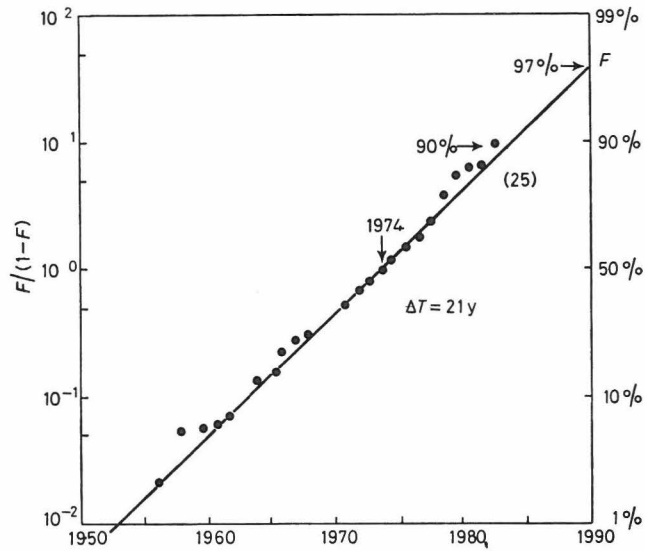

Fig. 12. – Again Lufthansa, but using its output as a size indicator. The founder message seems to establish a niche into which the company grows in a Malthusian pattern [6].

it is difficult to quantify these effects. From the case histories I have examined, it seems that the critical age occurs when about 80 or 85 % of the potential is reached. Another very important limitation is the Kondratieff time box. Many companies die when the end of the Kondratieff cycle (1995!) is approaching and many new ones are created. Research into this population dynamics is due.

A company can be formally seen as a special case of an organization, and the most striking feature for me is the self-consistency of the structure during its lifetime. Where the pertinent « DNA » is located and how it is explicated could be a good subject for « informational sociology » research.

Informationally linked associations, incidentally, operate the same way. The cumulative number of people killed by the Red Brigades fits splendidly a logistic (fig. 14). The flex of the equation, indicating the maximum drive, coincides with the Moro affair. The dominant commander of the Red Brigades, MORETTI, said exactly the same thing in his interviews with BOCCA. The analytical diagnosis coincides with the situation as perceived from within the structure. Thinking in terms of « future », the fading-out of the Red Brigades could have been calculated at the time of Moro with a precision of a couple of years, and a few percent in the quantity of actions executed.

As seems to happen in physics, when we zoom in on the « elementary particle », we find it to be a world in itself; also, in my case, zooming in on the unit component of all social systems, the human being, is no help. He is a system himself, bearing much resemblance to the aggregate he forms as a social animal. After all, his body and his brain are societies made of billions of members, and there must be rulers and rules.

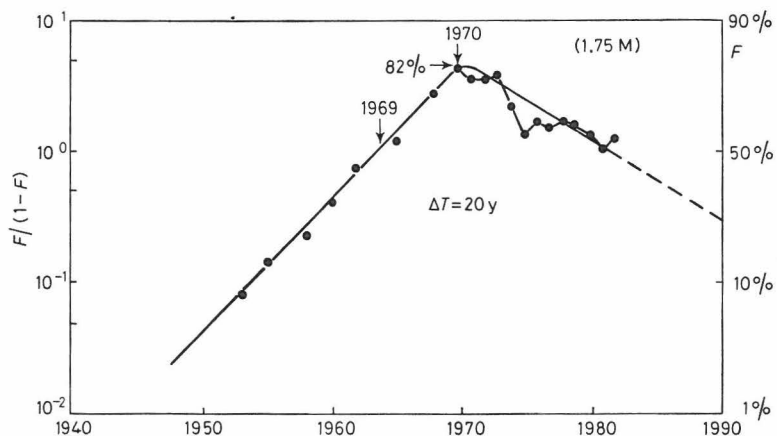The physical growth of a man is logistic, which is a good starting point.

Fig. 13. – Growth is not forever, and sometimes the full potential is not completely expressed. We have here the case of Fiat, where a process of senescence and phase-out has been installed already [7].

Fig. 14. – The concept of organization and organism can be quite loose, as we have seen with cathedral builders and air transporters. The Red Brigades seem to fit the scheme as more loose terrorism operators. The fascinating side here is that their actions can be forecast in time with remarkable precision [8].

Fig. 15. – Zooming into smaller and smaller organizations I found that the single individual does not escape the rules. Here the cumulative number of publications by LOTKA (Volterra-Lotka!) fits the Malthusian logistic. This kind of behaviour is general and I have analysed scientists, artists and criminals [9].

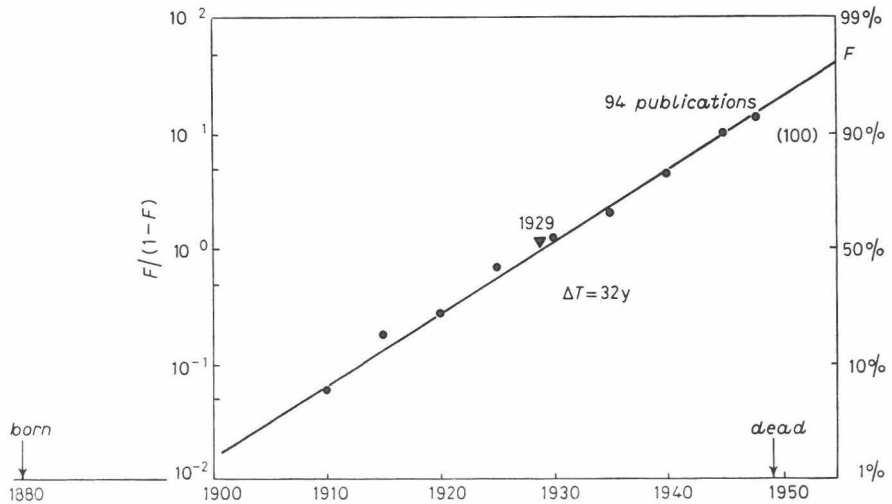But what about his social output? The problem, as usual, is what to measure and how to measure it. In fig. 15 the cumulative number of publications by LOTKA are reported. They fit perfectly a logistic! It looks like LOTKA had a niche, a potential of creativity since the beginning, and his publications are like rabbits multiplying till the niche is occupied. Actually, people die when the niche is 95 % or so occupied—just like our companies. The more one zooms, the more of the same: but, after all, « unification » is the job of the scientist.

At this point the obvious conclusion would be to « explain » why everything works in this way. The answer will be easy *a posteriori*, but I do not have it yet. What I think I have shown is that DNA and syntactic language perform in the same way. The whys for DNA are only partly clear, and thousands of researchers are digging out the intricacies of the clever mechanisms. I am sitting and watching, waiting for their solutions, which I will try to « transcode » into my informational networks, operating on syntactic language.

In the meantime I keep exploring and *forecasting*. During the last ten years not a single forecast has been wrong, which, in a Darwinian optic, says I am on a good track, and, in an intellectual optic, says economy and sociology may finally become sciences. *The touchstone of science being the capacity to see into the future.*

APPENDIX

The equations for dealing with different cases are reducible to the general Volterra-Lotka equations

$$(\text{A.1}) \qquad \frac{\mathrm{d}N_i}{\mathrm{d}t} = K_i N_i + \beta_i^{-1} \sum_n^{j=1} a_{ij} N_i N_j,$$

where N_i is the number of individuals in species i, and α, β and K are constants. The equation says a species grows (or decays) exponentially, but for the interactions with other species. A general treatment of these equations can be found in [10] and [11]. Since closed solutions exist only for the case of one or two competitors, these treatments mainly deal with the general properties of the solutions.

In order to keep the analysis at a physically intuitive level, I use the original treatment of Verhulst [12] for the population in a *niche* (Malthusian) and that of Haldane [13] for the competition between two genes of different fitness. For the multiple competition, we have developed a computer package which works perfectly for actual cases [14], but whose identity with the Volterra equations is not fully proven [15].

Most of the results are presented using the co-ordinates for the linear transform of a logistic equation originally introduced by FISHER and PRY [16].

The Malthusian case. This modelling of the dynamics of population systems started with VERHULST in 1845, who quantified the Malthusian case. A

physically very intuitive example is given by a population of bacteria growing in a bottle of broth. Bacteria can be seen as machinery to transform a set of chemicals in the broth into bacteria. The rate of this transformation, *coeteris paribus* (*e.g.*, temperature), can be seen as proportional to the number of bacteria (the transforming machinery) and the concentration of the transformable chemicals.

Since all transformable chemicals will be transformed finally into bacterial bodies, to use homogeneous units one can measure broth chemicals in terms of bacterial bodies. So $N(t)$ is the number of bacteria at time t, and $\bar{N}$ is the amount of transformable chemicals at time 0, before multiplication starts. The Verhulst equation can then be written

(A.2)
$$\frac{dN}{dt} = \alpha N(\bar{N} - N) \,,$$

whose solution is

(A.3)
$$N(t) = \frac{\bar{N}}{1 - \exp\left[-(\alpha t + b)\right]} \,,$$

with b an integration constant, sometimes written as t_0, *i.e.* time at time 0; α is a rate constant which we assume to be independent of the size of the population. This means that there is no « proximity feedback ». If we normalize to the final size of the system, $\bar{N}$, and explicate the linear expression, we can write eq. (A.2) in the form suggested by FISHER and PRY [16]:

(A.4)
$$\log\frac{F}{1-F} = \alpha t + b \,, \qquad\qquad \text{where } F = \frac{N}{\bar{N}} \,.$$

Most of the charts are presented in this form. $\bar{N}$ is often called the *niche*, and the growth of a population is given as the fraction of the niche it fills. It is obvious that this analysis has been made with the assumption that *there are no competitors*. A single species grows to match the resources ($\bar{N}$) in a Malthusian fashion.

The fitting of empirical data requires calculation of the three parameters $\bar{N}$, a and b, for which there are various recipes of [17-19]. The problem is to choose the physically more significant representation and procedure.

I personally prefer to work with the Fisher and Pry transform, because it operates on *ratios* (*e.g.*, of the size of two populations), and ratios seem to me more important than absolute values, both in biology and in social systems.

The calculation of $\bar{N}$ is usually of great interest, especially in economics. However, the value of $\bar{N}$ is very sensitive to the value of the data, *i.e.* to their errors, especially at the beginning of the growth. The problem of assessing the error on $\bar{N}$ has been studied by DEBECKER and MODIS [20], using numerical simulation.

The Malthusian logistic must be used with great precaution because it contains implicitly some important hypotheses:

That there are no competitors in sight.

That the size of a niche remains constant.

That the species and its boundary conditions (*e.g.*, temperature for the bacteria) stay the same.

The fact that in multiple competition the starts are always logistic may lead to the presumption that the system is Malthusian. When the transition period starts, there is no way of patching up the logistic fit.

The fact that the niches keep changing, due to the introduction of new technologies, makes this treatment, generally speaking, unfit for dealing with the growth of human populations, a subject where PEARL [21] first applied logistics. Since the treatment sometimes works and sometimes not, one can find much faith and disillusionment among demographers.

One-to-one competition. The case was studied by HALDANE for the penetration of a mutant or of a variety having some advantage in respect to the preexisting ones. These cases can be described quantitatively by saying that variety 1 has a reproductive advantage of k over variety 2. Thus, for every generation, the ratio of the number of individuals in the two varieties will be changed by $1/(1-k)$. If n is the number of generations, starting from $n = 0$, then we can write

$$\text{(A.5)} \qquad \frac{N_1}{N_2} = \frac{R_0}{(1-k)^n}, \qquad \text{where } R_0 = \frac{N_1}{N_2} \text{ at } t = 0.$$

If k is small, as it usually is in biology (typically 10^{-3}), we can write

$$\text{(A.6)} \qquad \frac{N_1}{N_2} = \frac{R_0}{\exp[kn]}.$$

We are then formally back to square one, *i.e.* to the Malthusian case, except for the very favourable fact that we have an initial condition (R_0) instead of a final condition $(\bar{N})$. This means that in *relative terms* the evolution of the system is not sensitive to the size of the niche, a property that is extremely useful for forecasting in multiple-competition cases. Since the generations can be assumed equally spaced, n is actually equivalent to time.

As for the biological case, it is difficult to prove that the «reproductive advantage» remains constant in time, especially when competition lasts for tens of years and the technology of the competitors keeps changing, not to speak of the social and organizational context. But the analysis of hundreds of cases shows that systems behave exactly *as if.*

Multiple competition. Multiple competition is dealt using a computer package originally developed by NAKICENOVIC [15]. A simplified description says that all the competitors start in a logistic mode and phase out in a logistic mode. They undergo a transition from a logistic in to a logistic out during which they are calculated as «residuals», *i.e.* as the difference between the size of the niche and the sum of all the *ins* and *outs*. The details of the rules are to be found in [15]. This package has been used to treat about one hundred empirical cases, all of which showed an excellent match with reality.

An attempt to link this kind of treatment to current views in economics has been made by PETERKA [22].

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