Working Paper

On the Reconstruction of a Parameter for a Hyperbolic System

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WP-91-54
December 1991

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Foreword

This paper which was presented during the author's visit to the SDS Program of IIASA is related to the problem of on-line identification of a parameter of a distributed hyperbolic system through available continuous measurements. The solution is achieved here by introducing an adjoint dynamic model with feedback control developed on the basis of the observation data. The suggested on-line reconstruction algorithm ensures numerical stability of the procedure and leads to effective simulation results.

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November 23, 1991

The problem of reconstructing a parameter of a hyperbolic system by measuring its states is considered here. Firstly, the suggested reconstruction algorithm is stable with respect to errors of measurement and, secondly, it can be applied for restoring desired parameters in real time (synchronously with a motion of the system).

The problem belongs to the class of the inverse problems of dynamics that are being intensively studied today (see, for example, investigations [1-19], where the corresponding bibliography is given). To solve the problem, the method suggested by A.V. Kryazhimskii and the author (see, for example, [1-6]) is applied. The method is based on (the ideas of) the theory of differential games [20-24] and the theory of ill-posed problems [25]. The essence of the method is the following: an appropriately controlled dynamical system referred to as a model is constructed; the model act simultaneously with the initial dynamical system, the parameter of which is to be reconstructed. The model is controlled *positionally* (by feedback); at each time instant t, a control is formed on the basis of a state of the model and of measurements of the states of the system realized up to t. It becomes clear that for a sufficiently wide class of inverse problems, a control law for the model can be chosen in such a way that a control realization approximates (reconstructs) the unknown parameter of the system in the desired way, namely, stable with respect to the errors of measurement. Thus the initial inverse problem is reduced to a direct problem of control theory.

1. Let us first consider the problem of reconstruction of a "hindrance" for a hyperbolic system.

Denote:

$T = [t_0, \vartheta]$	– a time interval
V and H	– real Hilbert spaces
V^* and H^*	– spaces dual to V and H, respectively,

$(\cdot,\cdot)$ and $|\cdot|_V$ ($(\cdot,\cdot)$ and $|\cdot|_H$) – the scalar product and the corresponding norm in $V(H)$, U – a uniformly convex real Banach space. Suppose that V is densely imbedded in H. We identify spaces H and H^*. The notions introduced below without comment are given, for example in [26-28].

Let the following objects be given:

$\phi : V \to (-\infty, +\infty]$ – a convex, proper, lower semi-continuous function;

$a(\cdot,\cdot)$ – a continuous bilinear form on V satisfying, with a certain $c_1 > 0$, the condition:

$$a(w,w) \geq c_1|w|_V^2, \quad w \in V; \tag{1.1}$$

$B : U \to H$ – a linear continuous operator; $f \in L^2(T,H)$, $\dot{f} \in L^2(T,H)$ ($\dot{f}$ denotes the derivative of f).

The continuous linear symmetric operator corresponding to the form $a(\cdot,\cdot)$ will be denoted by A. The measurability and integrability of a function will always be understood as defined by Lebesgue, differentiability will be understood as in the theory of distributions.

Consider the hyperbolic system the evolution of which is described by the following variational inequality: for almost all $t \in T$ and all $w \in V$

$$(\dot{y}(t), (t) - w)_H \; + \; a(y(t), y(t) - w) \; +$$

$$+ \; \varphi(\dot{y}(t)) \; - \; \varphi(w) \; \leq \; (Bu(t) + f(t), y(t) - w)_H, \tag{1.2}$$

and

$$y(t_0) = y_0, \; Ay_0 \in H \tag{1.3}$$

$$y(t_0) = y_0 \in \mathcal{D}(\varphi) = \{w \in V : \varphi(w) < +\infty\} \in$$

$$\in L^2(T, U), \;\; \dot{u}L^2(T, U). \tag{1.4}$$

Here $u \in L^2(T, U)$, $u \in L^2(T, U)$. We call function u a disturbance (implying certain concrete systems).

Under the imposed conditions [27-29]

$$y \in C(T, V), \; \dot{y} \in C(T, H) \cap C(T, V_\omega),$$

$$\dot{y} \in L^\infty(T, H). \tag{1.5}$$

Here V_ω is the space V equipped with a weak topology. We also assume that

$$Ay(t) \in H, \quad t \in T. \tag{1.6}$$

(For conditions on a and φ ensuring (1.6), see [29], p. 139-143). When necessary, we write $y = y_u$ emphasizing that y depends on u.

The problem considered is the following. u is and unknown disturbance. It is known only that almost all values $u(t)$ belong to a bounded convex set $P \subset U$. The aim is to form an approximation v_h to the disturbance u knowning the results $\xi(t)$ of the measurements of the velocity $\dot{y}(t)$ of the evolution of the system with error h:

$$|\dot{y}(t) - \xi(t)|_H \leq h, \quad t \in T. \tag{1.7}$$

The mean square approximation is implied:

$$\int_T |u(t) - v_h(t)|_U^2 dt \to 0 \quad \text{as} \quad h \to 0. \tag{1.8}$$

Let us state the problem more precisely.

Let N be the set of all functions from $L^2(T, P)$ such that for almost all $t \in T$ all $w \in V$

$$(\dot{y}_u(t), \dot{y}_u(t) - w)_H + a(y_u(t), y(t) - w) +$$

$$+\varphi(\dot{y}_u(t)) - \varphi(w) \leq (Bv(t) + f(t), \ \dot{y}_u(t) - w)_H.$$

Thus N is the set of all disturbances v that can generate the evolution produced by the disturbance u, but in general, does not satisfy the inclusions $\dot{v} \in L^2(T, H)$. Obviously, N is convex and closed in $L^2(T, P)$.

For $v \in L^2(T, P)$, we put

$$J(v) = \min_{p \in N} |v - p|_{L^2(T,P)}. \tag{1.9}$$

Let $\delta > 0$, $\delta \leq \vartheta - t_0$ and $D : (0, \delta] \times L^2(T, H) \to L^2(T, P)$. Now we formulate the initial problem as follows:

Problem 1.1 Find an operator D such that

$$\sup J_\xi(D(h, \xi)) \to 0 \text{ as } h \to 0. \tag{1.10}$$

Here sup is calculated over all $\xi L^2(T, H)$ such that $|\xi(t) - \dot{y}(t)| \leq h$, $t \in T$. An operator D satisfying (1.10) will be called a reconstruction algorithm.

Remark 1.1 If N is one element, i.e. it contains a single element u, then the algorithm D reconstructs a mean square approximation to the disturbance u. It is also easily seen that the algorithm D reconstructs a mean square approximation to an element of N with a minimal $L^2(T, U)$-norm.

Remark 1.2. Let (1.2) be equivalent to a variational inequality. Then the conditions $\dot{u} \in L^2(T, U)$, $\dot{f} \in L^2(T, U)$, the second condition (1.3), and the condition (1.6) can be omitted.

Let us construct an algorithm D.

Fix an $h, 0 < h \leq \delta$, and a ξ (see (1.10)). Decompose the interval T by points t_i:

$$t_0 < t_1 < \ldots < t_m = \vartheta, \quad m = m(h),$$

$$\Delta = \Delta(h) = \max_i (t_{i+1} - t_i) \leq c_1 h,$$

where $c_1 > 0$ is fixed.

Consider the control system on T (call it a model) described by the following conditions: for almost all $t \in T$ and all $w \in V$.

$$(\dot{z}(t),\ \dot{z}(t) - w)_H \ + \ a(z(t), y(t) - w) +$$

$$+\varphi(\dot{z}(t)) \ - \ \varphi(w) \leq (Bv_h(t) \ + \ f(t), \dot{z}(t) - w)_H \tag{1.11}$$

and $z(t_0) = y_0$, $\dot{z}(t_0) = \dot{y}_0$.

Here $v_h : T \to U$ is a piecewise constant control formed by the rule:

$$v_h(t) = v_i, \; t_i \le t < t_{i+1}, \; i = 0, \dots, m-1. \tag{1.12}$$

Here v_i is an element of P such that

$$\Phi(v_i) \;=\; \min\{\Phi(v), v \in P\}, \tag{1.3}$$

$$\Phi(v) \;=\; 2(Bv, z(t_i) \;-\; \xi(t_i))_H \;+\; \alpha(h)|v|_U^2, \tag{1.14}$$

$\alpha(h)$ is a nonnegative function on $(0, \delta)$ such that $h/\alpha(h) \to 0$ as $h \to 0$. (According to the terminology of the theory of ill-posed problems [25], $\alpha(h)$ is a regularization parameter).

Now define D as the mapping, putting in correspondence to each pair (h, ξ) a function v_h from (1.11)-(1.14): $D(h, \xi) = v_h$.

Theorem 1.1. The operator D solves the Problem 1.1.

The proof of the theorem follows those of analogous statements from [2]. It is based on the following:

Lemma 1.1. Let

$$\Lambda(t) \;=\; |\dot{y}(t) - \dot{z}(t)|_H^2 \;+\; a(y(t) - z(t),\; y(t) - z(t)) +$$

$$+\alpha(h)|v_h|^2_{L_2([t_0,t],U)} \;-\; \alpha(h)|u|^2_{L_2([t_0,t],U)}. \tag{1.15}$$

There exists a $c_3 > 0$ such that

$$\Lambda(t) \;\le\; c_3(\Delta(h) \;+\; h), \quad t \in T.$$

Remark 1.3. According to the terminology of [2], Λ is a stabilized Lyapunov functional.

Remark 1.4 Note that to form the control v_h only the measurement $\xi(t_i)$ at time t_i is used. Thus the constructed algorithm can be applied to reconstructing the parameter in real time. In [3], algorithms of this kind are called dynamical (positional).

2. Now consider the following problem of reconstruction of a parameter for a hyperbolic system.

Let an operator $A : V \to V^*$ depending on $p \in P \subset U$,

$$A = A[p],$$

be given. Suppose that for each p the operator $A[p]$ is linear, continuous and self-adjoint. Denote by $a(p; \cdot, \cdot)$ the bilinear form on V corresponding to $A[p]$. Suppose that for certain $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\lambda_3 \geq 0$, and all $p \in P$

$$\lambda_3 |w|_V^2 \geq a(p; w, w) + \lambda_1 |w|_H^2 \geq \lambda_2 |w|_V^2, \quad w \in V.$$

Let a function $f \in L^2(T, H) \cap C(T, V^*)$ be given. Consider the typerbolic system ([27], p. 281-282)

$$\dot{y}(t) = A[u(t)]y(t) + f(t), \quad t \in T,$$

$$y(t_0) = y_0 \in V, \quad \dot{y}(t_0) = \dot{y}_0 \in H. \tag{2.1}$$

Here $u : T \to P$ is a measurable function such that for any $w_1 \in V$, $w_2 \in V$ the function $a(u(t); w_1, w_2)$ is continuously differentiable on T.

A solution to (2.1) satisfied the conditions [27]

$$y \in C(T, V), \;\; \dot{y} \in C(T, H), \;\; \dot{y} \in L^2(T, V^*).$$

The parameter $u(t)$, $t \in T$, in (2.1) is unknown. It is to be reconstructed (mean square approximated, see Section 1) on the basis of measurements $\xi_1(t)$, $\xi_2(t)$ of values $y(t)$, $\dot{y}(t)$, with error h:

$$|\xi_1(t) - y(t)|_V \leq h, \quad |\xi_2(t) - y(t)|_H \leq h, \quad t \in T \tag{2.2}$$

Let us state the problem more precisely.

Suppose that following conditions are fulfilled:

(1) there exists a nonnegative function $\sigma_1(\varepsilon)$, $\varepsilon > 0$, $\sigma(\varepsilon) \to 0$, such that for $t_1 \in T$, $t_2 \in T$

$$|a(u(t_1); w, w) \ - \ a(u(t_2); w, w)| \leq$$

$$\sigma_1(|t_1 \ - \ t_2|)\ |w|_V^2, w \in V; \tag{2.3}$$

(2) if a sequence $\{v^{(k)}\}$ converges weakly to v^0 in $L^2(T, P)$, then for any $\psi \in C(T, V)$ and $t \in T$ the sequence

$$\{\int_{t_0}^{t} A[v^{(k)}(\tau)]\psi(t)d\tau\}$$

converges to

$$\int_{t_0}^{t} A[v^0(\tau)]\psi(\tau)d\tau.$$

in V^*.

Introduce a set analogous to the set N from Section 1. Let N_1 be the set of all functions v from $L^2(T, P)$ such that for almost all $t \in T$

$$\dot{y}_u(t) \;=\; A[v(t)]y_u(t) \;+\; f(t).$$

The condition (2) implies that N_1 is a weak compactum in $L^2(T,P)$.

For $v \in L^2(T,P)$ we put

$$J_1(v) \;=\; \min_{p\in N_1} |v-p|_{L^2}(T,P).$$

Let $D_1 : (0,\delta] \times L^2(T,V) \times L^2(T,H) \to L^2(T,P)$, where $\sigma > 0$, $\sigma \le \vartheta - t_0$ is fixed.

Problem 2.1. Find an operator D_1 such that

$$\sup_{\xi_1,\xi_2} J_1(D_1(h,\xi_1,\xi_2)) \;\to 0 \;\text{ as }\; h \to 0. \tag{2.4}$$

Here the sup is calculated over all $\xi_1 \in L^2(T,V)$, $\xi_2 \in L^2(T,H)$ satisfying (2.2).

An operator D_1 is satisfying (2.4) will be called a reconstruction algorithm.

Remark 2.1. The algorithm D_1 reconstructs the unknown parameter u of the system (forms a mean square approximation to u), provided N_1 is one element. In general, D_1 reconstructs an element from N_1 the $L^2(T,U)$-norm of which is minimal.

Let us construct the algorithm D_1.

Introduce the following condition:

(3) for each $w_1 \in V, w_2 \in V$ the mapping $a(v;w_1,w_2) : P \to (-\infty,\infty)$ is weakly upper-semicontinuous. Following Section 1, introduce the control system

$$\dot{z}(t) = A[v_h(t)]\xi_1^*(t) + f(t), \quad t \in T,$$

$$z(t_0) = y_0, \quad \dot{z}(t_0) = \dot{y},$$

$$\xi_1^*(t) = \xi_1(\tau_i), \; t_i \leq t < t_{i+1}, \; i = 0, \dots, m-1. \tag{2.5}$$

Here $v_h(t)$ is a piecewise constant control on T formed by the rule

$$v_h(t) = v_i, \quad t_i \leq t < t_{i+1}, \quad i = 0, m-1$$

where v_i is an element from P such that

$$\Phi_1(v_i) = \min\{\Phi_1(v), v \in P\}, \tag{2.6}$$

$$\Phi_1(v) = -2a(v; \xi_1(t_i), \dot{z}(t_i) - \xi_2(t_i)) + \alpha_1(h)|v|_U^2.$$

Define the operator D_1 to be the mapping which puts correspondence to each triplet (h, ξ_1, ξ_2) the function v_h from (2.5), (2.6).

Theorem 2.1. Let $\sigma_2(\varepsilon)$ be a module of continuity of f in V^* and $\alpha_1(h)$ be such that

$$[h + \sigma_1(\Delta(h)) + \sigma_2(\Delta(h))] \, / \, \alpha_1(h) \to 0 \text{ as } h \to 0.$$

Then the operator D_1 solves the problem 2.1.

The proof of the Theorem follows those of the analogous statements from [1-3]. It is based on the following

Lemma 2.1. Let

$$\Lambda_1(t) = |\dot{y}(t) - \dot{z}(t)|^2_{V^*} + \alpha(h) \cdot |y_h|^2_{L_2([t_0,t],U)} -$$

$$\alpha(h) \cdot |u|^2_{L_2([t_0,t],U)}.$$

There exists a $\lambda_4 \leq 0$ such that

$$\Lambda_1(t) \leq \lambda_4 \cdot (\sigma_1(\Delta(h)) + \sigma_2(\Delta(h)) + h), \quad t \in T.$$

Remark 2.2. Note that the control v_h on $[t_i, t_{i+1})$ is formed on the basis of measurements $\xi_1(t_i)$, $\xi_2(t_i)$ at time t_i. Therefore, the constructed algorithm allows us to reconstruct the parameter u in real time

3. There follow below several examples.

Example 1. For the system (1.1), (1.2) we have

$$V = H_0^1(0,\pi), \; H = L^2(0,\pi), \; U = R^1, \; T = [0.1],$$

$$P = [-1,1], \; \varphi \equiv 0, \; y_0 = 0, y_0 = \kappa \cdot \sin x, \; \kappa = \sqrt{2/\pi},$$

$$f = 0, \; a(w_1,w_2) = \int_0^\pi \frac{\partial w_1}{\partial x} \cdot \frac{\partial w_2}{\partial x}\, dx, \; Bu(t) = u(t) \cdot \kappa \cdot \sin x.$$

Consequently the system is described by the question

$$\begin{cases} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + u(t) \cdot \kappa \cdot \sin x, \; x \in (0,\pi) \\ y(t,0) = 0 = y(t,\pi), \; t \in T = [0,1] \\ y(0,x) = 0, \; \frac{\partial y}{\partial t}(0,x) = \kappa \cdot \sin x \end{cases}$$

The set N contains the single element $u = u(t)$, $t \in T$.

Below, the results of the reconstruction of the concrete parameter $u(t) = t$ are shown. The measurement was made in the form

$$\xi(t,x) = y(t,x) = h \cdot \kappa \cdot \sin x$$

where p is a given number. The parameter of regularization was determined by $\alpha = h^{1/2}$. The function v_h approximating u was calculated according to the following rule:

$$v_h(t) = v_i, \ t_i \le t < t_{i+1}, \ i = 0, \dots, m-1,$$

$$v_i = \begin{cases} -1, \gamma_i < -1, \\ \gamma, -1 \le \gamma_i \le 1, \\ 1, 1 \le \gamma_i, \end{cases}$$

$$\gamma_i = \alpha^{-1} \cdot \kappa \cdot \int_0^{\pi} \sin x \cdot \left(\frac{\partial z}{\partial t}(t_i, x) - \frac{\partial \xi}{\partial t}(t_i, x) \right) dx.$$

The results of computer simulations are given in the table and in the figures. S stands for the mean square error of approximation.

Example 2. For system (2.1) we have

$$V = \dot{H}^1(0,\pi),\ H = L^2(0,\pi),\ U = R^1,\ y_0 = \kappa \cdot \sin x,$$

$$\dot{y}_0 = 0,\ \kappa = \sqrt{2/\pi},\ f = 0,\ A[u(t)]y = u(t) \cdot \frac{\partial y}{\partial x^2}.$$

Consequently, the system is described by the equation

$$\begin{cases} \frac{\partial^2 y}{\partial t^2} = u(t) \cdot \frac{\partial^2 y}{\partial x^2},\ x \in (0,\pi) \\ y(t,0) = 0 = y(t,\pi),\ t \in T \\ y(0,x) = \kappa \cdot \sin x,\ \frac{\partial y}{\partial t}(0,x) = 0. \end{cases}$$

The set N_1 contains the single element $u = u(t)\ t \in T$. Below, the results of the reconstruction of the parameters

1) $u(t) = 1 + t^2 \in P = [0,5],\ t \in T = [0,2]$,

and

2) $u(t) = 2 + \sin 2\pi t \in P = [1,3],\ t \in T = [0,1.5]$.

are given. The measurements were made in the form

$$\xi_1(t,x) = y(t,x) + h \cdot m_1 \cdot \kappa \cdot \sin p_1 t \cdot \sin x,$$

$$\xi_2(t,x) = \dot{y}(t,x) + h \cdot m_2 \cdot \kappa \cdot \sin p_2 t \cdot \sin x,$$

where m_1 and m_2 are given numbers. The parameter of regularization was determined by $\alpha = h$. The function v_h approximating u was calculated according to the rule (2.6). The results of the computer simulations are given in the figures.

Example 3. For the system (2.1) we have

$$V = \dot{H}^1(0,1),\ H = L^2(0,1),\ f = 0,\ y_0 = \sin \pi x,\ \dot{y}_0 = 0,$$

$$P = \{p \in H^2(0,1) : 1 \le p(x) \le 2,\ 0 \le x \le 1\},$$

$$U = H^2(0,1),\ A[u(t)]y = \frac{\partial}{\partial x}\,(a(x)\cdot\frac{\partial y}{\partial x}),$$

$$u(t,x) = a(x),\ t \in T = [0,1],\ x \in [0,1].$$

The parameter $a = a(x)$, $0 \le x \le 1$, not depending on time to be reconstructed. Below, the results of the reconstruction of the concrete parameter

$$a(x) = 1 + x,\ 0 \le x \le 1$$

are given. The set N_1 contains a single element. The measurements

$$\xi_1(t,x) = y(t,x) + h\cdot m_1\cdot \sin p_1 t,$$

$$\xi_2(t,x) = \dot{y}(t,x) + h\cdot m_2\cdot \sim p_2 t,$$

where m_1, m_2, p_1, and p_2 are given, were considered. The initial infinite dimensional problem was approximated by a finite dimensional one using the method of lines, the decompostions step in x was equal to 0.02. The parameter of regularization was defined by $\alpha = h^{1/6}$. The function v_h was calculated according to the rule (2.6). The functions $u_*(t) = a(x_*)$,$t \in T$, and the construction results $v_h^*(t) = v_h(t,x_*)$, $t \in T$ for $x_* = 0.5$ and $x_* = 0.75$ are shown in the figures.

The author is grateful to A.V. Kryazhimskii and A.I. Korotkii for helpful discussions and for their assistance.

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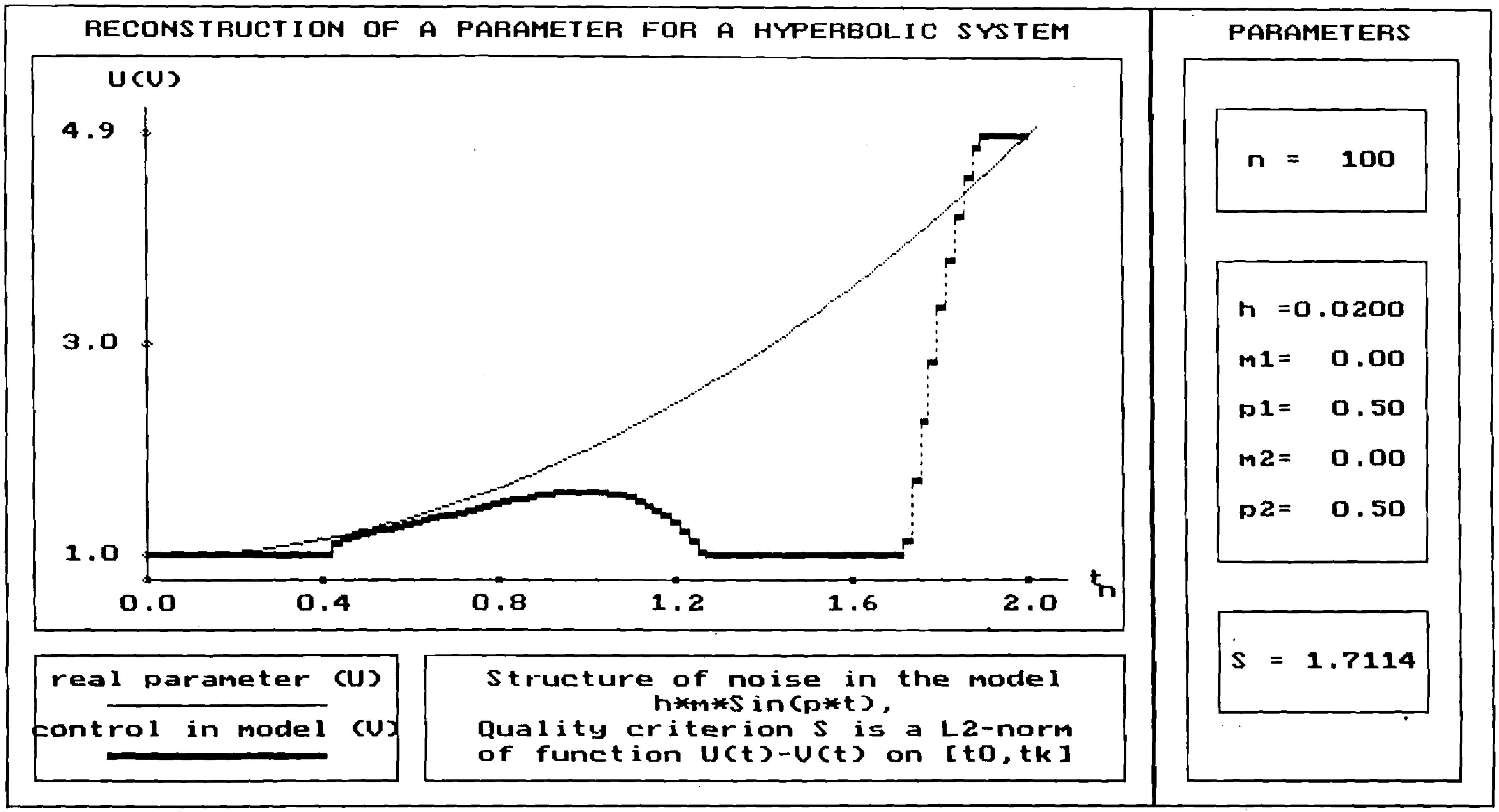
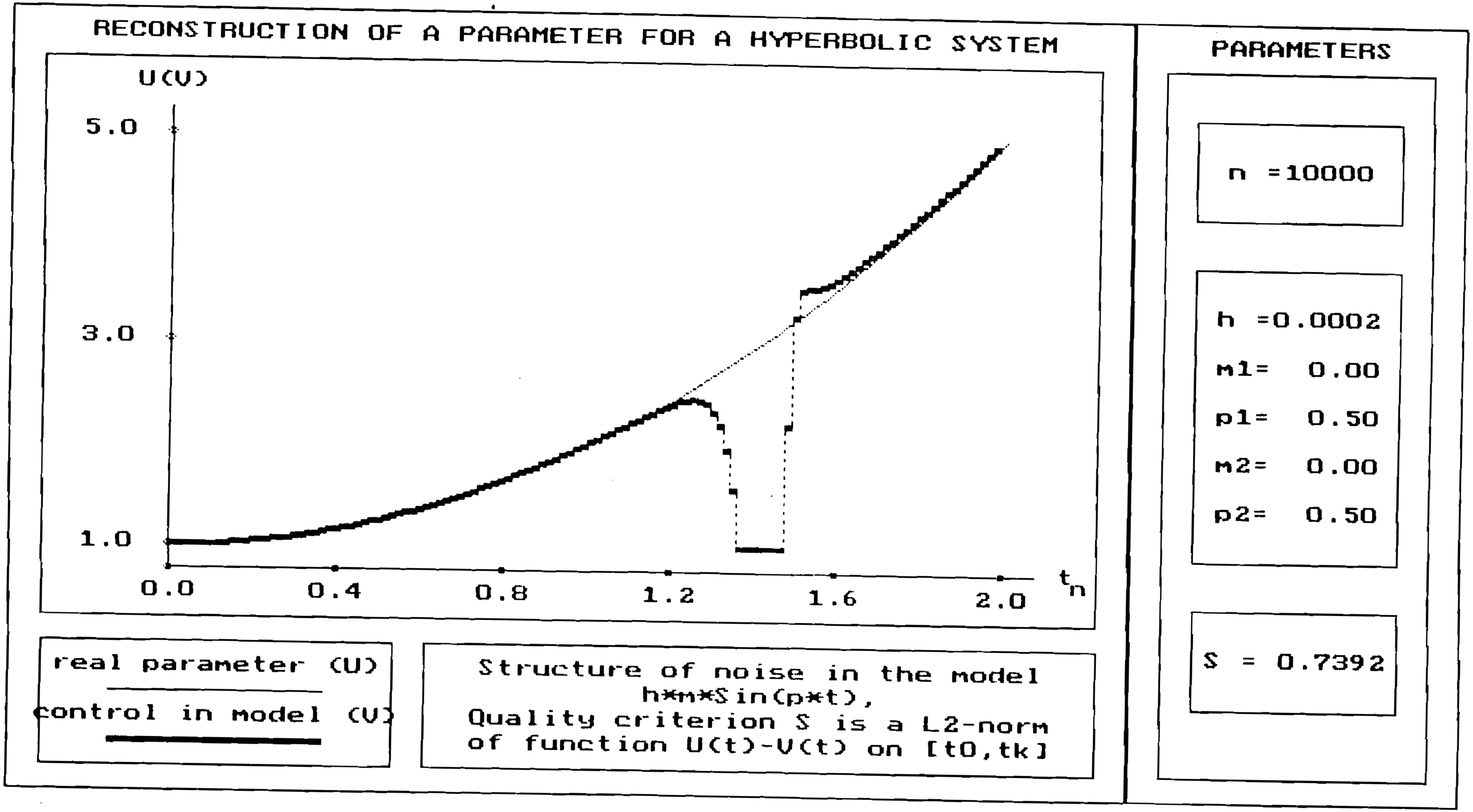
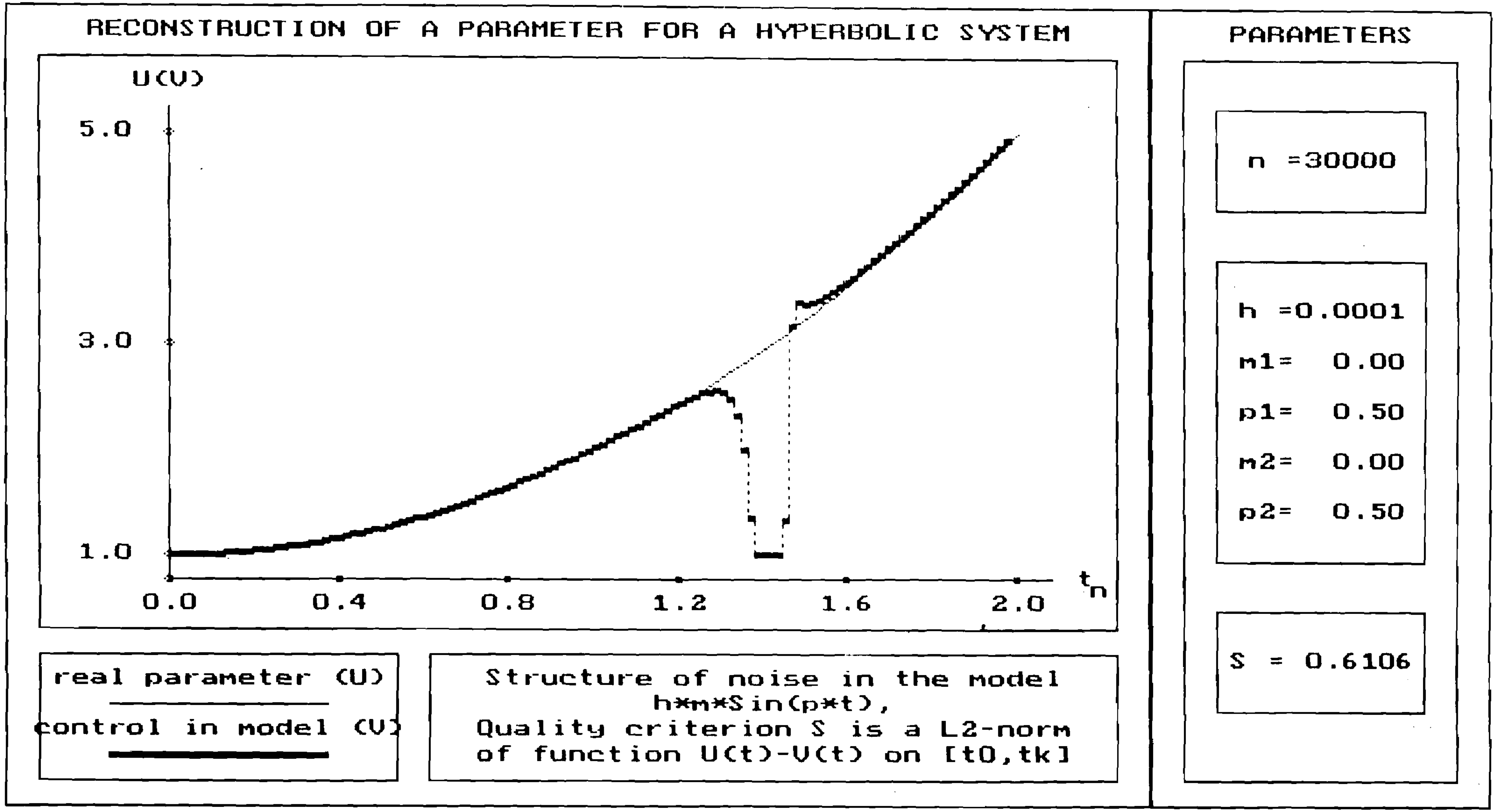
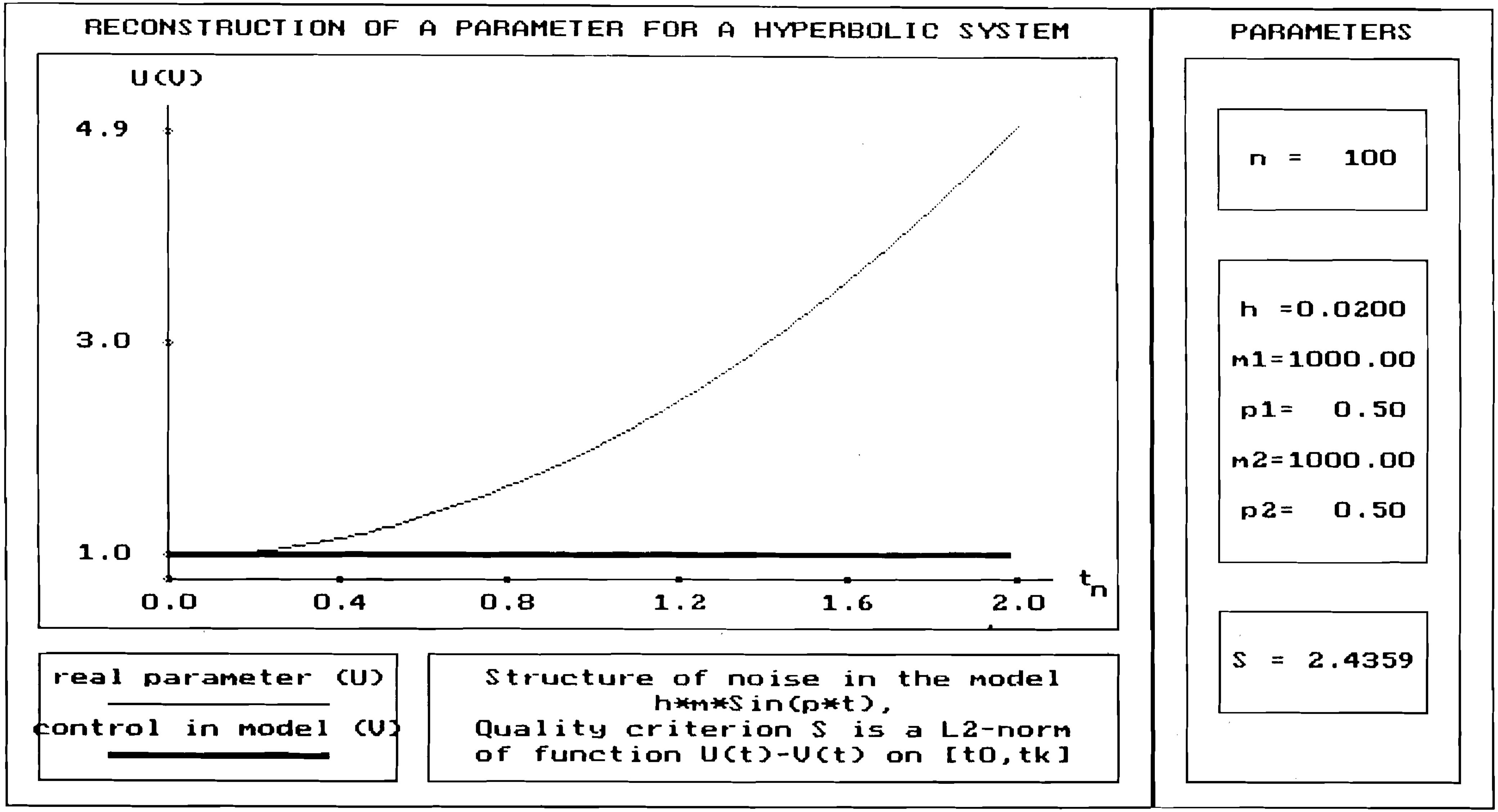
Sverdlovsk 620219
U.S.S.R.

M	10	20	30	40	50	60	70	80	90
S	0.38512	0.27209	0.22301	0.19364	0.17347	0.15849	0.14677	0.13728	0.12938
M	100	110	120	130	140	150	160	170	180
S	0.12268	0.11689	0.11182	0.10734	0.10334	0.09975	0.09649	0.09352	0.09079
M	190	200	210	220	230	240	250	260	270
S	0.08828	0.08597	0.08381	0.08181	0.07994	0.07818	0.07653	0.07498	0.07351
M	280	290	300	310	320	330	340	350	360
S	0.07213	0.07081	0.06957	0.06838	0.06725	0.06617	0.06514	0.06416	0.06322
M	400	600	700	800	900	1000	1400	1800	2200
S	0.05981	0.04833	0.04458	0.04158	0.03910	0.03702	0.03111	0.02736	0.02469
M	2600	3000	3400	3600	3800	4000	4200	4400	4600
S	0.02269	0.02111	0.01981	0.01926	0.01873	0.01827	0.01782	0.01742	0.01702

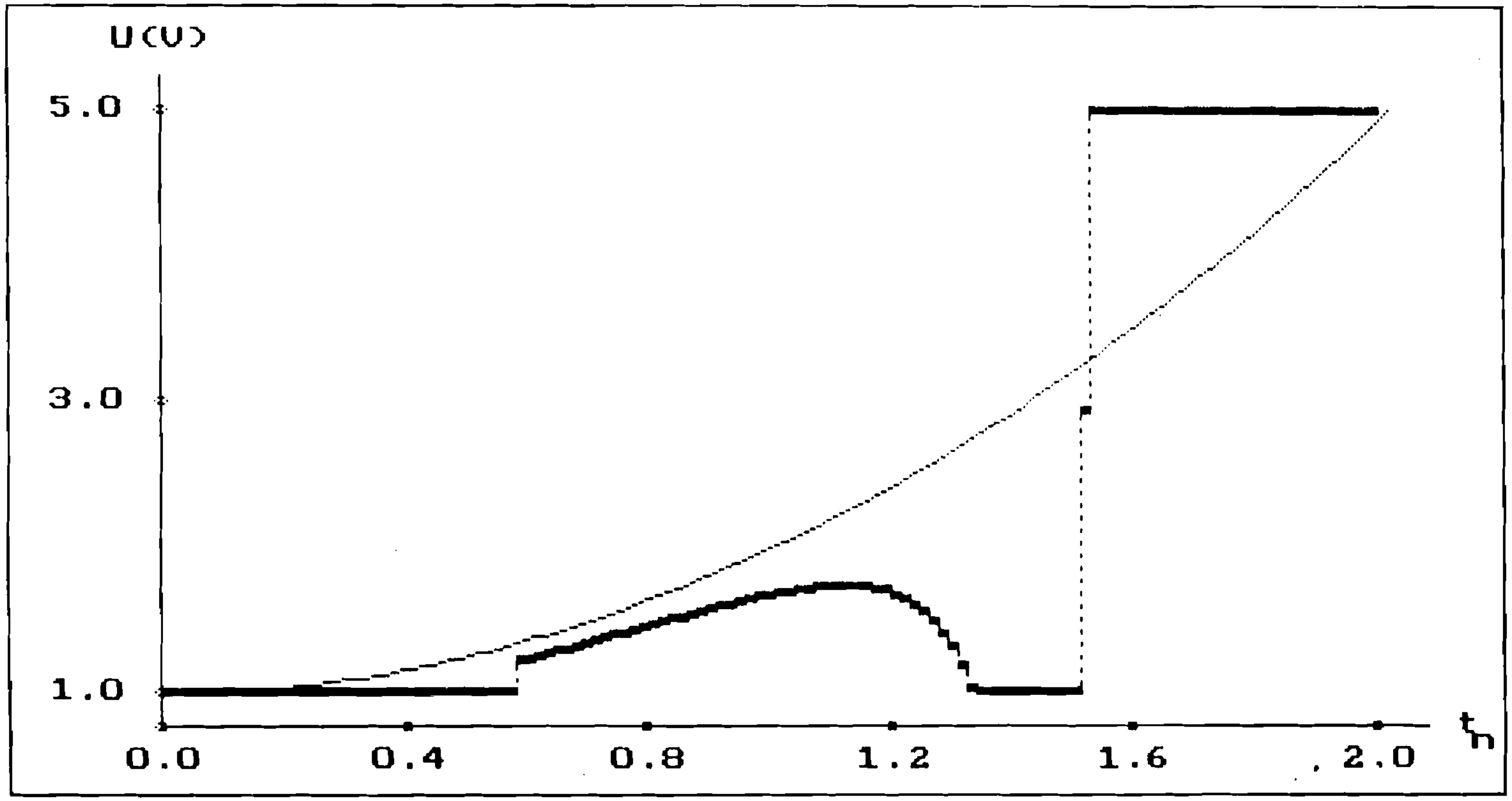

RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
4.9
3.0
1.0
0.0
0.4
0.8
1.2
1.6
2.0
t_n
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n = 100
h =0.0200
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50
S = 1.7114


RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
5.0
3.0
1.0
0.0
0.4
0.8
1.2
1.6
2.0
tn
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n =10000
h =0.0002
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50
S = 0.7392


RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
5.0
3.0
1.0
0.0
0.4
0.8
1.2
1.6
2.0
t_n
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n =30000
h =0.0001
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50
S = 0.6106


RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
4.9
3.0
1.0
0.0
0.4
0.8
1.2
1.6
2.0
tn
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n = 100
h =0.0200
m1=1000.00
p1= 0.50
m2=1000.00
p2= 0.50
S = 2.4359

RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM



real parameter (U)

control in model (V)

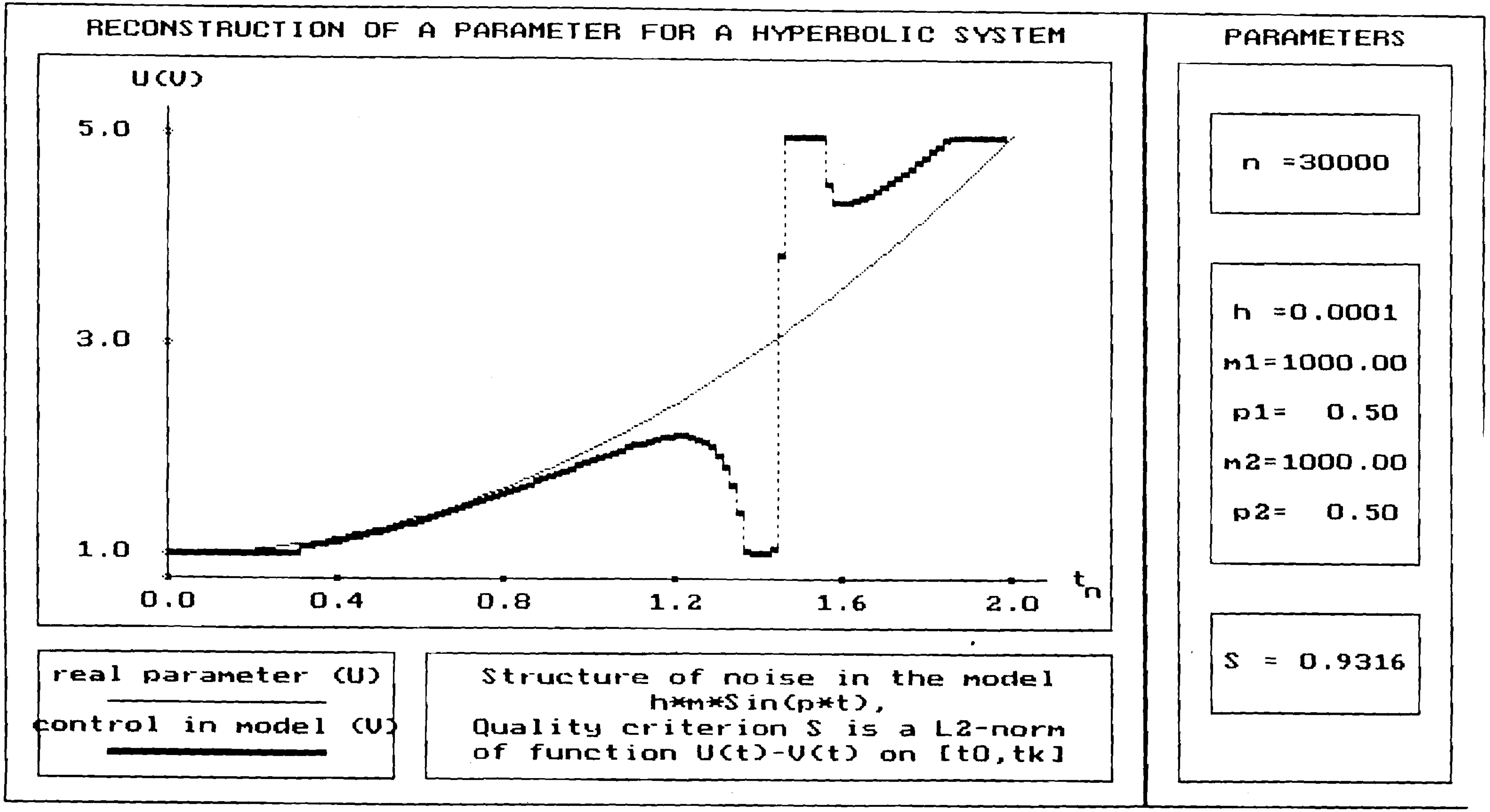
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]

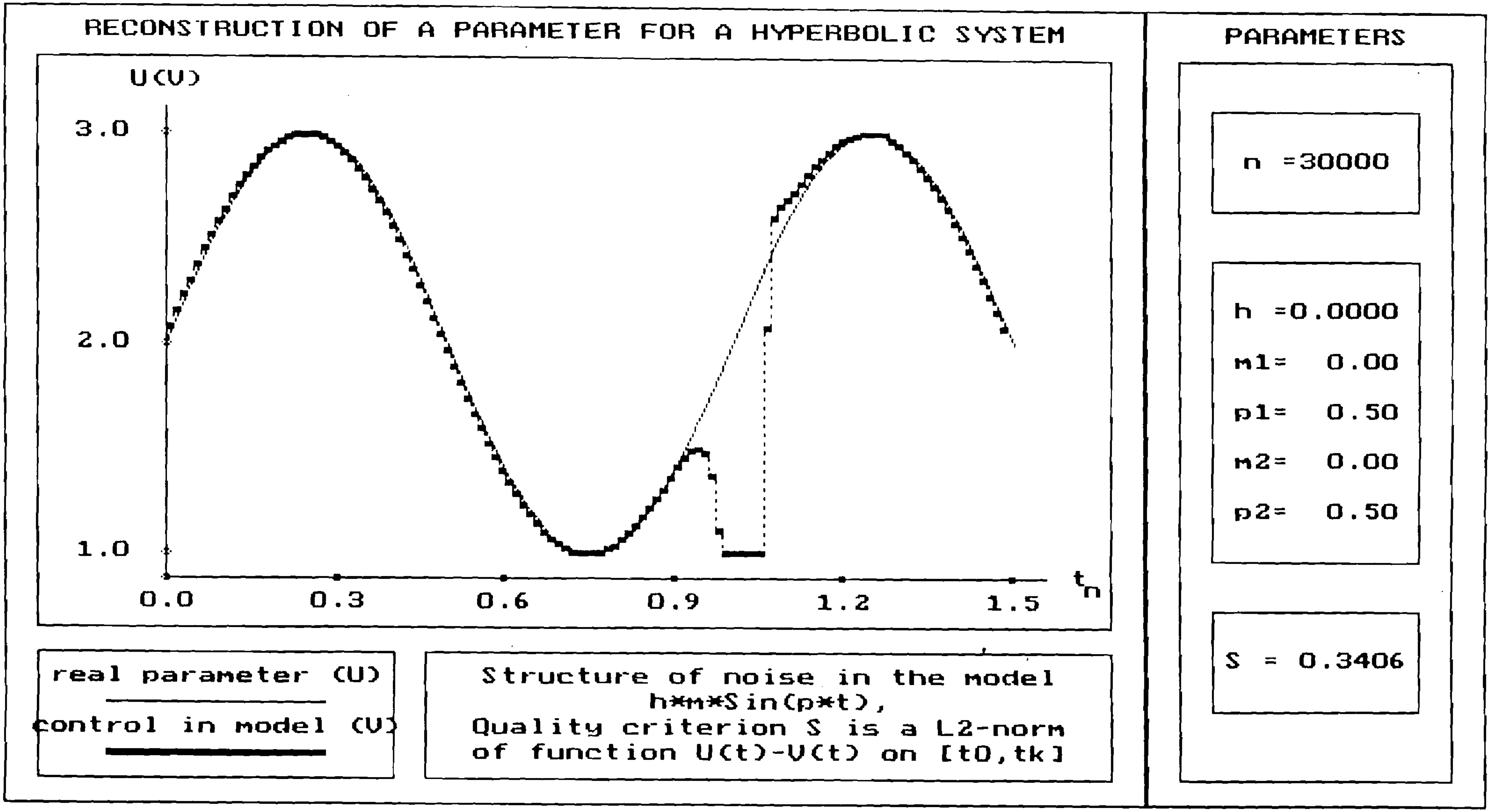
PARAMETERS

n =10000

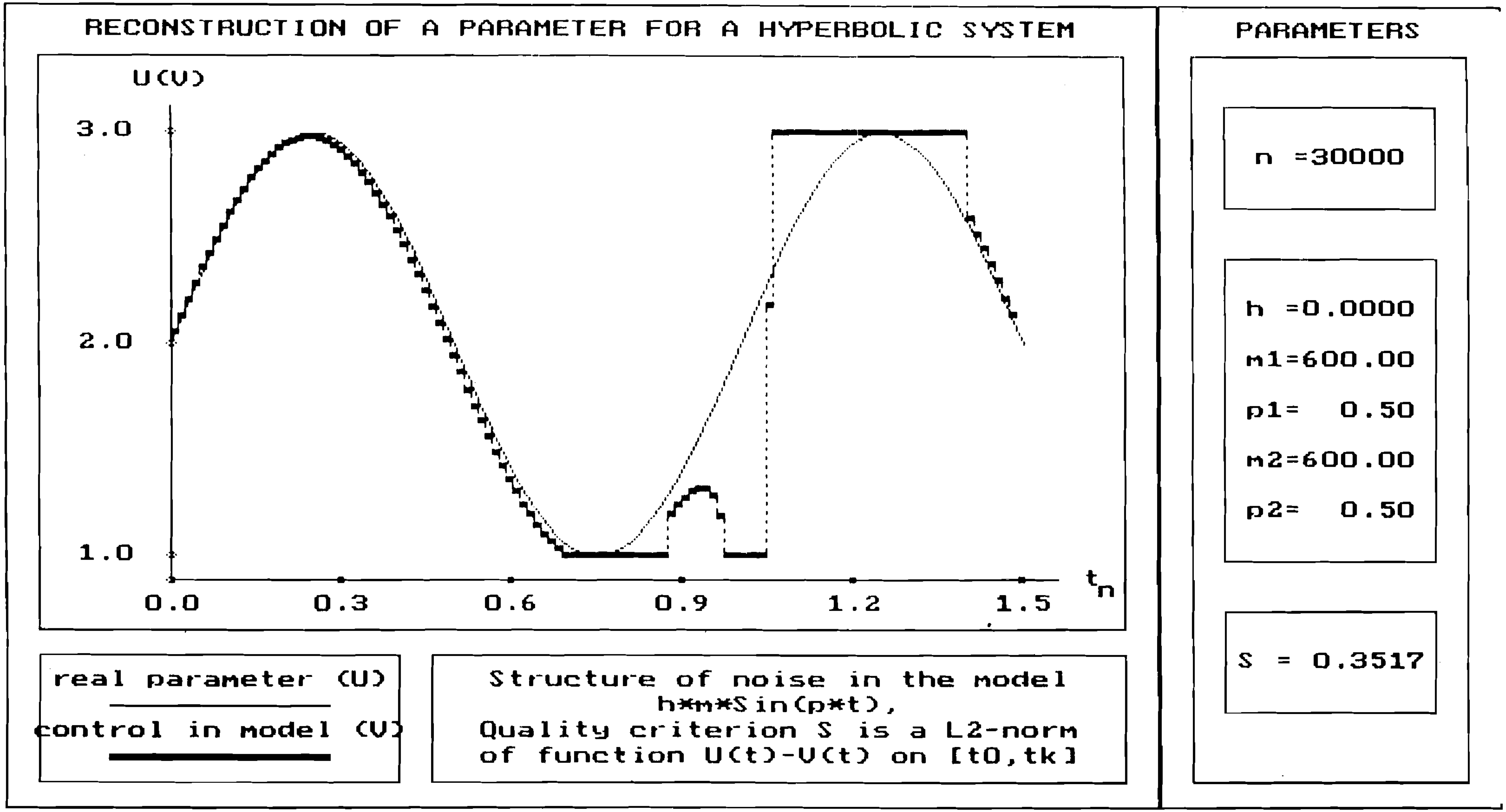
h =0.0002
m1=1000.00
p1= 0.50
m2=1000.00
p2= 0.50

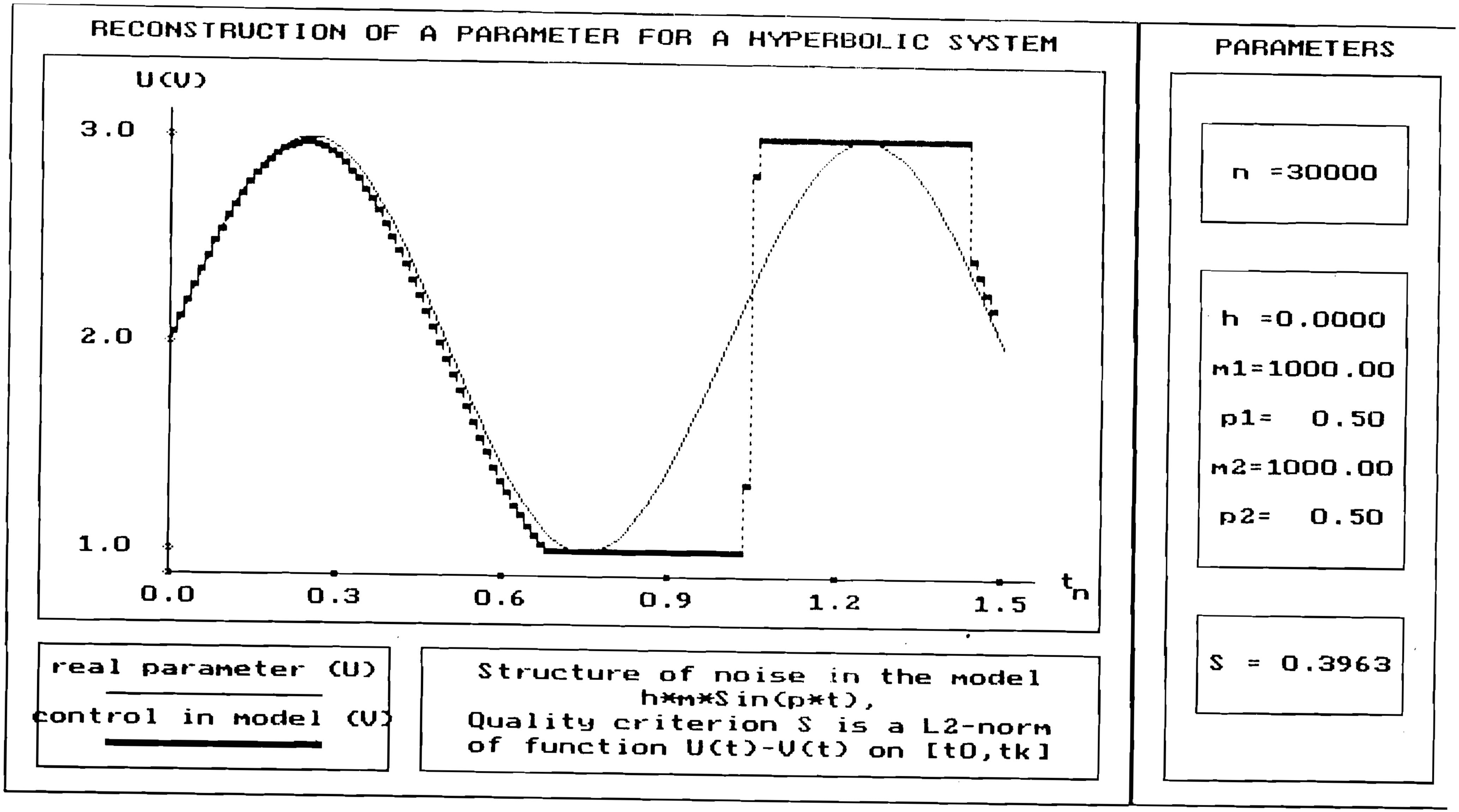
S = 1.2250

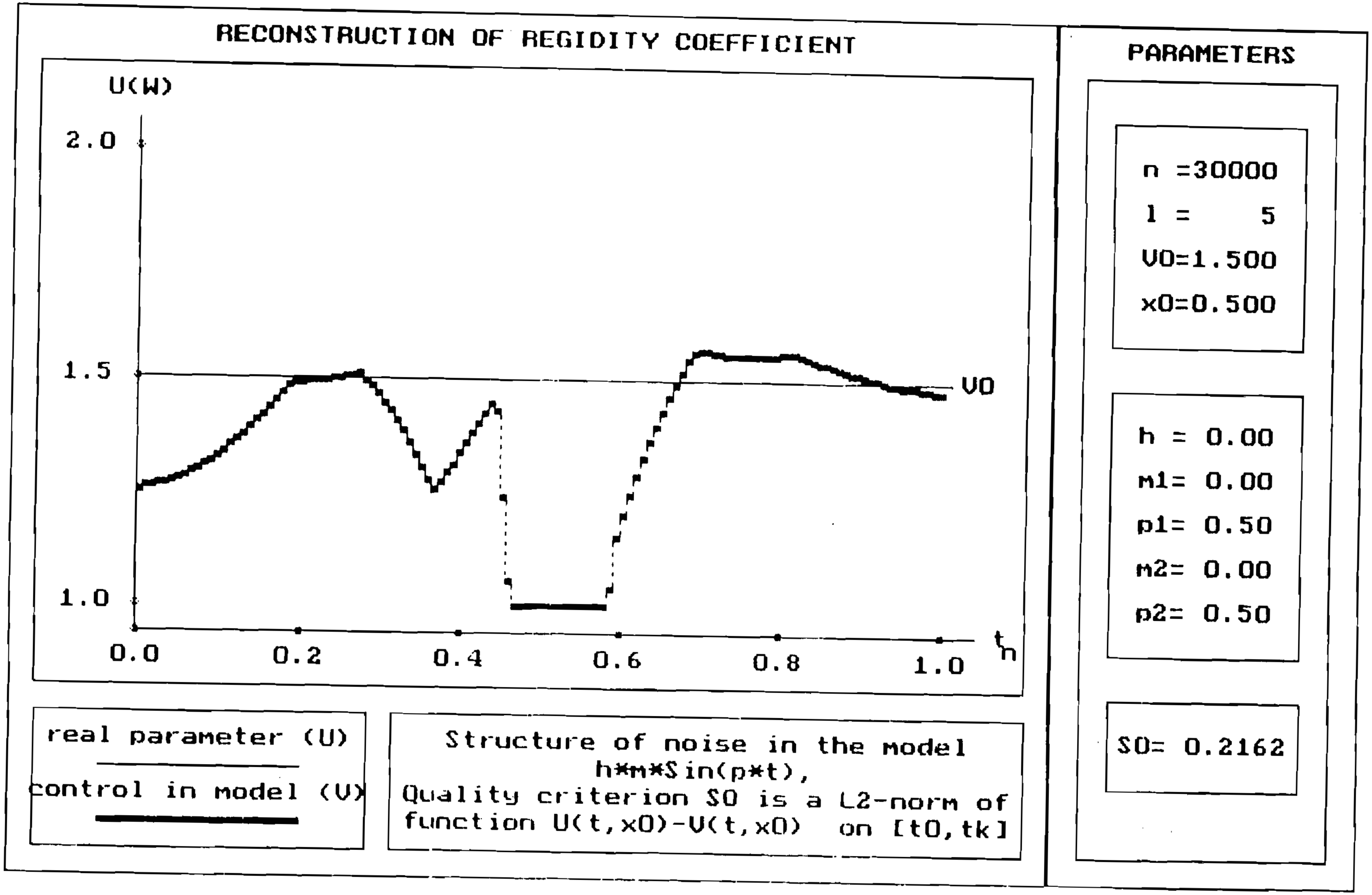

RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
5.0
3.0
1.0
0.0
0.4
0.8
1.2
1.6
2.0
tn
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n =30000
h =0.0001
m1=1000.00
p1= 0.50
m2=1000.00
p2= 0.50
S = 0.9316



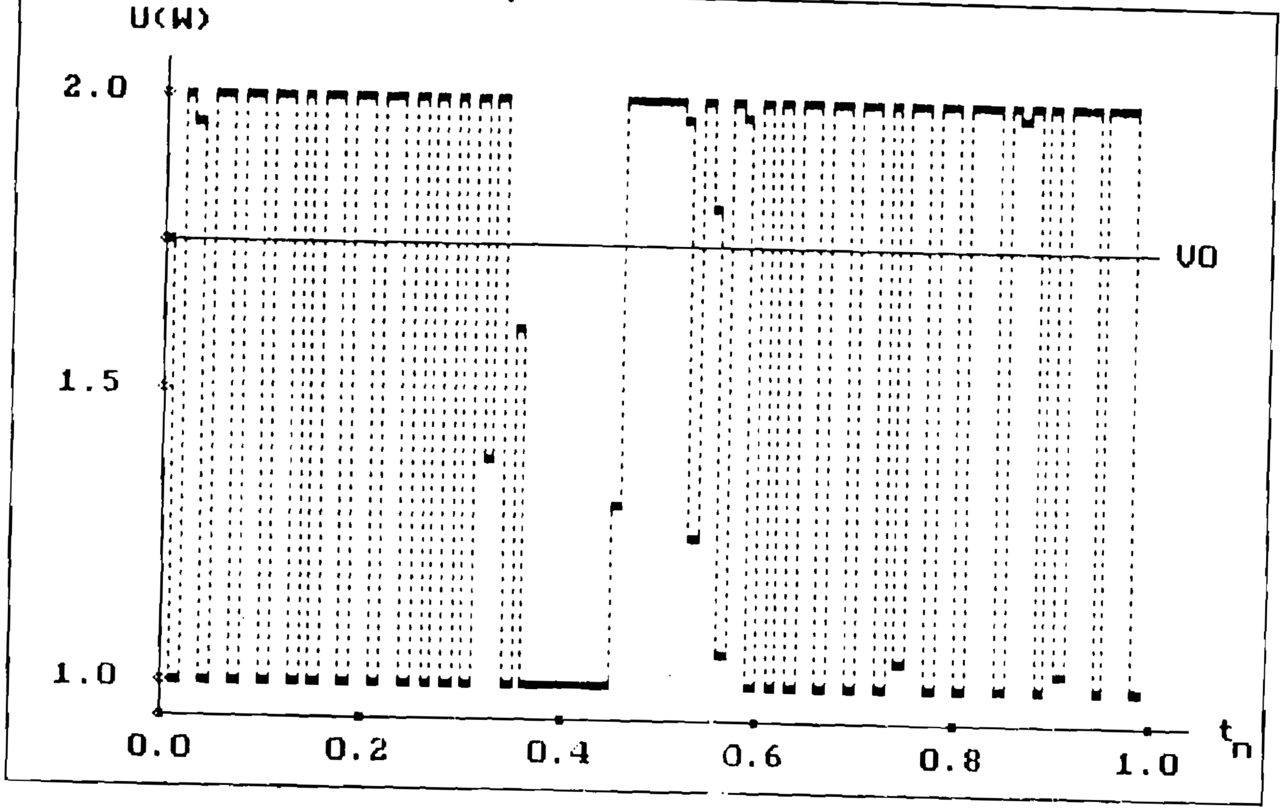
RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
3.0
2.0
1.0
0.0
0.3
0.6
0.9
1.2
1.5
t_n
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n =30000
h =0.0000
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50
S = 0.3406


RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
3.0
2.0
1.0
0.0
0.3
0.6
0.9
1.2
1.5
tn
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n =30000
h =0.0000
m1=600.00
p1= 0.50
m2=600.00
p2= 0.50
S = 0.3517


RECONSTRUCTION OF A PARAMETER FOR A HYPERBOLIC SYSTEM
U(V)
3.0
2.0
1.0
0.0
0.3
0.6
0.9
1.2
1.5
t_n
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S is a L2-norm
of function U(t)-V(t) on [t0,tk]
PARAMETERS
n =30000
h =0.0000
m1=1000.00
p1= 0.50
m2=1000.00
p2= 0.50
S = 0.3963


RECONSTRUCTION OF REGIDITY COEFFICIENT
U(W)
2.0
1.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0
V0
t
n
real parameter (U)
control in model (V)
Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S0 is a L2-norm of
function U(t,x0)-V(t,x0) on [t0,tk]
PARAMETERS
n =30000
l = 5
V0=1.500
x0=0.500
h = 0.00
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50
S0= 0.2162

RECONSTRUCTION OF REGIDITY COEFFICIENT



real parameter (U)

control in model (V)

Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S0 is a L2-norm of
function U(t,x0)-V(t,x0) on [t0,tk]

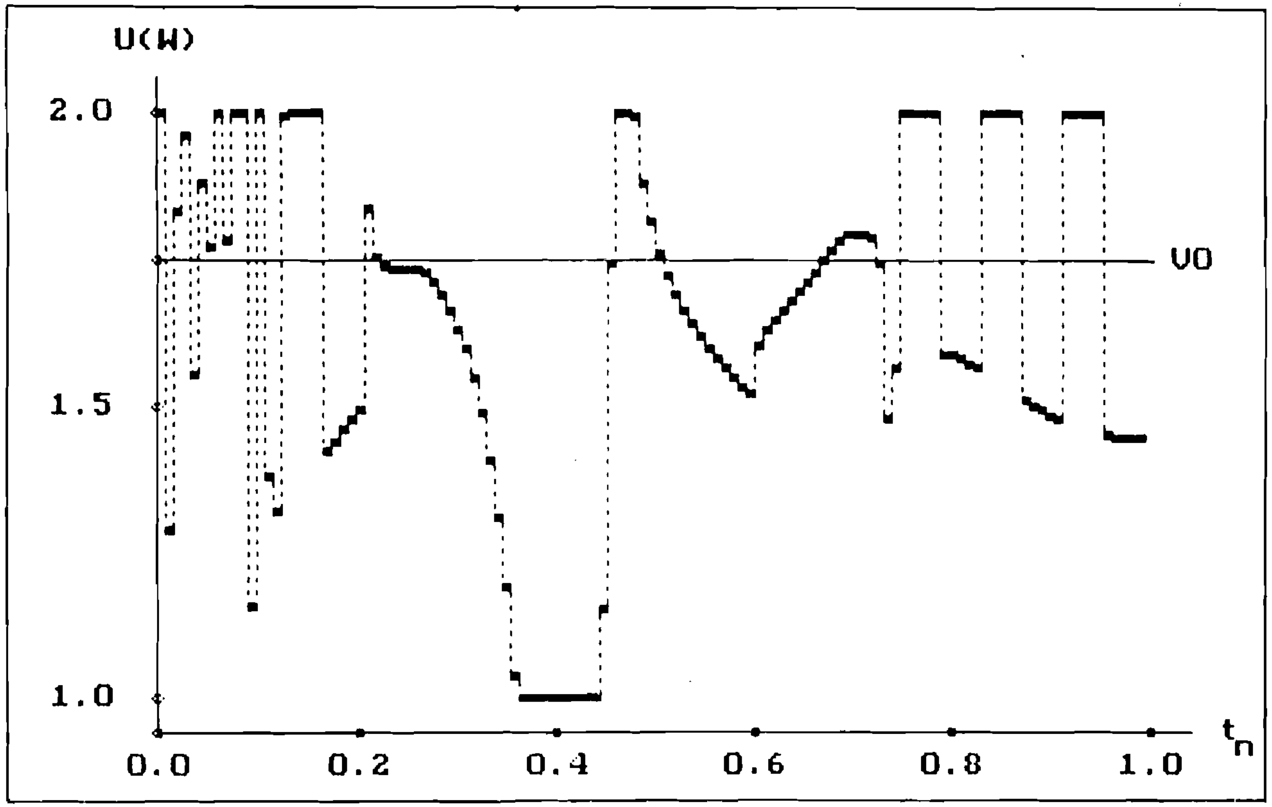
PARAMETERS

n = 100
l = 5
V0=1.750
x0=0.750

h = 0.01
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50

S0= 0.4965

RECONSTRUCTION OF REGIDITY COEFFICIENT



real parameter (U)

control in model (V)

Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S0 is a L2-norm of
function U(t,x0)-V(t,x0) on [t0,tk]

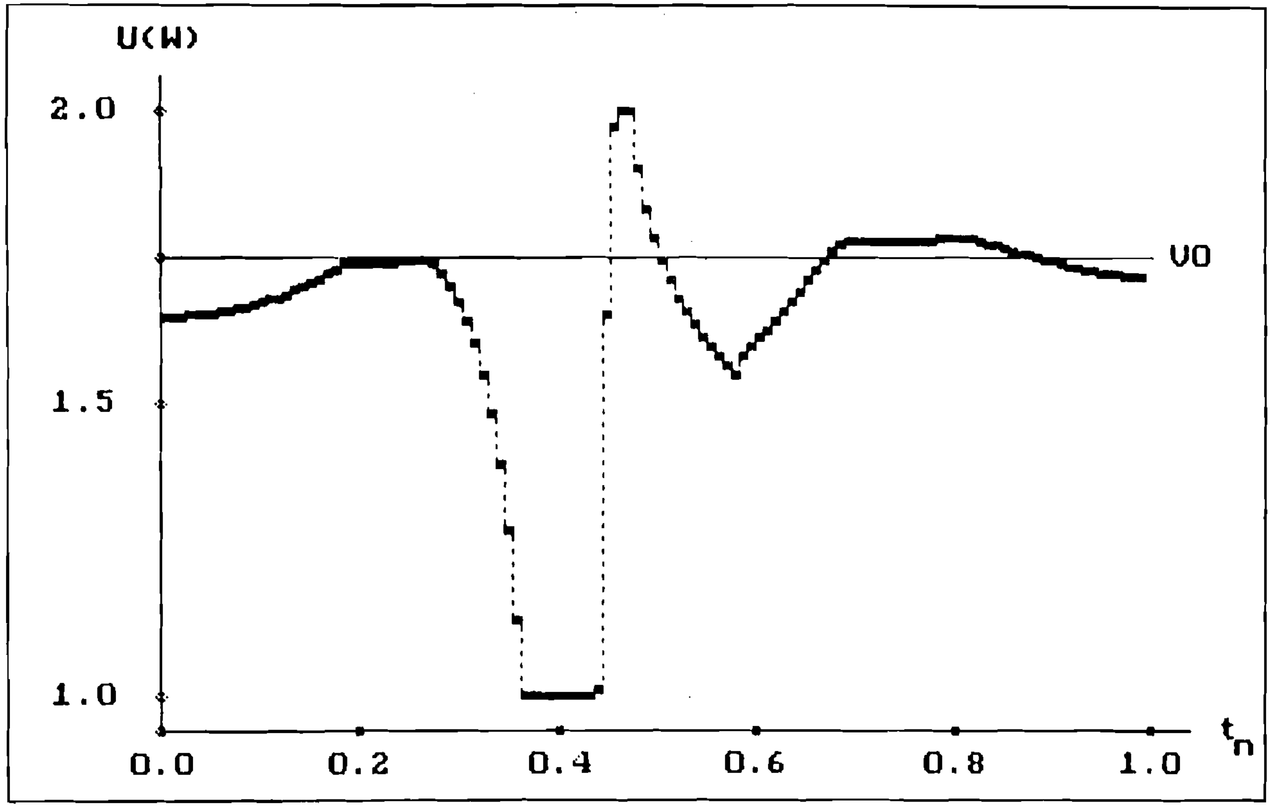
PARAMETERS

n = 1000
l = 5
V0=1.750
x0=0.750

h = 0.00
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50

S0= 0.3260

RECONSTRUCTION OF REGIDITY COEFFICIENT



real parameter (U)

control in model (V)

Structure of noise in the model
h*m*Sin(p*t),
Quality criterion S0 is a L2-norm of
function U(t,x0)-V(t,x0) on [t0,tk]

PARAMETERS

n =30000
l = 5
V0=1.750
x0=0.750

h = 0.00
m1= 0.00
p1= 0.50
m2= 0.00
p2= 0.50

S0= 0.2415